Precomputed Hash Chains

- Let *P* be a Finite set of passwords and H a pre-image resistant hash function.
- **Goal**: Given hash *h*, locate a password *p in P* such *that H(p)=h*, or no *p in P*
- **Trivial solutions**:
  - compute H(p) for all p in P until H(p)=h
    - PRO: no memory involved
    - CONS: compute hashes for all P in P.
  - Store all the possible pair (p,H(p)) of n-bitpasswords and their associated hash passwords
    - PRO: very fast to check
    - CONS:|P|n bits stored
  - Infeasible if P is large, for example P={0,1}^{80} , i.e., all the strings of length n=80 (|P|=2^{80})

Precomputed Hash Chains

Can we optimize the computation/space ratio?

Example

- *P* = 6-digit lowecase letters
- Hash *H*: *6-digit lowercase letters -> {0,1}*^{32} (HEX repr.)

Idea: Define a *Reduction function R: {0,1}*^{32} *-> 6-digit lowercase letters* (<u>Not the inverse of H)</u>

$$\texttt{281DAF40} \xrightarrow[R]{} \texttt{sgfnyd}$$

Precomputed Hash Chains

- Choose a random subset of words in P
- Compute a chain of length k and save only the first and the last element for each chain, for example, if we have

$$\texttt{aaaaaa} \xrightarrow[H]{} \texttt{281DAF40} \xrightarrow[R]{} \texttt{sgfnyd} \xrightarrow[H]{} \texttt{920ECF10} \xrightarrow[R]{} \texttt{kiebgt}$$

we need to only store `(aaaaaa,kiebgt).`

Let h=`920ECF10`, applying R we see that $\texttt{920ECF10} \xrightarrow[R]{} \texttt{kiebgt}$

By reapplying the H,R,... from `aaaaaa` we notice that `sgfnyd` is a correct password: $\texttt{sgfnyd} \xrightarrow[H]{} \texttt{920ECF10}$

Precomputed Hash Chains

- Chains could merge, for example `FB107E70` also leads to `kiebgt`

$$\mathtt{FB107E70} \xrightarrow[R]{} \mathtt{bvtdll} \xrightarrow[H]{} \mathtt{0EE80890} \xrightarrow[R]{} \mathtt{kiebgt}$$

- The chain starting from `aaaaaa` will never reach `FB107E70`
- **False alarm**: the chain of `FB107E70` will be extended for another match
- **No match found**: password never produced by any of the chains

Precomputed Hash Chains

- Longer chains: more computation
  - Trade-off: chain length, lookup table size

- Problems
  - **collisions**
    - Same value in different chain at different position
  - Pick the correct *R*
    - Depends on the plaintext distribution

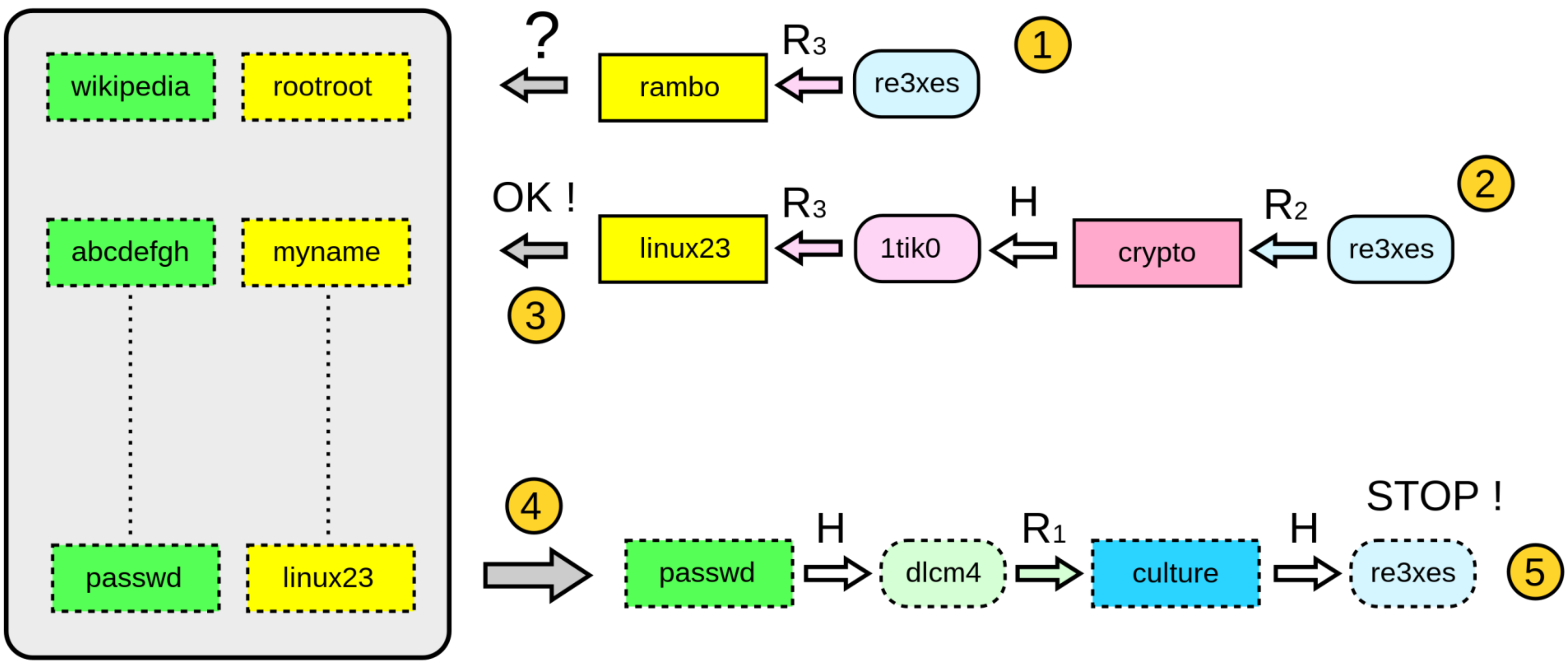
Rainbow Tables

- **Idea** to avoid collision:
  - Replace R with a sequence $R_1,\ldots,R_k$ to reduce the prob. that an hash is a result of the same reduction function
  - **To collide**: same value in the same iteration

- To find an hash, compute one of the following until an entry in the rainbow table is found
  - R_k
  - R_{k-1} ->H->R_k
  - R_{k-2} ->H->R_{k-1} ->H->R_k
  - …

Rainbow Tables

- Reduction functions R_1 R_2 R_3
- h=re3xes



Source: Wikipedia

Countermeasures

- Use **salt**, for example H(pwd+salt) or H(H(pwd))+salt
  - Large salt: need to precompute a table for each salt
  - Public salt
- **Key stretching**: hash multiple times with salts and intermediate values
  - More time to verify hash: Brute-force attacks harder
- **Key strenghtening**: private salt
- **Longer passwords**: 14 characters.