Announcements

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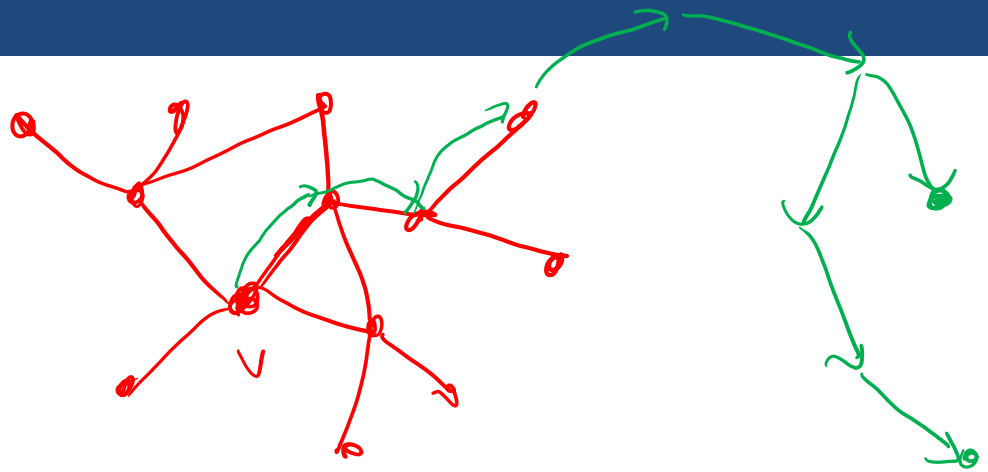
- HW1 grades are done
  - Note: the TAs are new and forgot to leave feedback!
  - They are going to try to go back and add some
  - In the meantime, check the posted solutions on Blackboard
  - If you can't figure out why you lost points, e-mail the TAs and ask
- HW2 is due ~~today~~ March 4
  - Any questions on HW2?
- HW3 has been posted – March 11



Graphs

Depth First Search

N = # nodes
M = # edges
adjacency matrix

```
procedure explore(G, v)
Input:   G = (V, E) is a graph; v ∈ V
Output:  visited(u) is set to true for
         all nodes u reachable from v

previsit(v)
visited(v) = true
for each edge (v, u) ∈ E:
  if not visited(u): explore(u)
postvisit(v)
```

called N times, O(N)

O(N)

Ignore for now

O(M)

O(M)

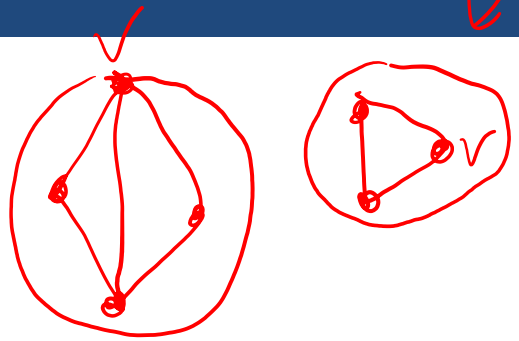
O(N)

O(N)

What is the running time of Depth First Search? O(N+M)

Depth First Search

nodes are indexed 1 ... N



DFS(G):

~~pick a~~

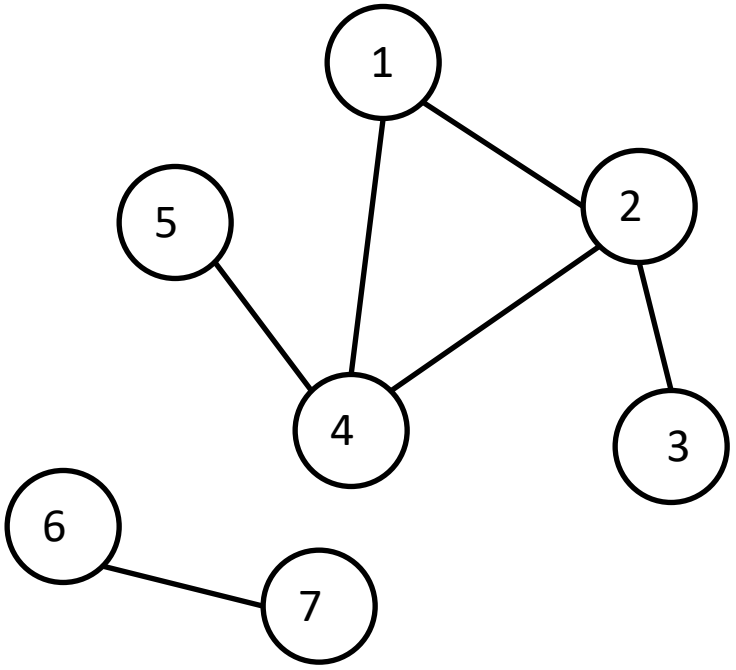
initialize visited = False

while there are ~~still~~ unvisited nodes:

pick an unvisited node v

explore(v)

Depth First Search: Example with Stacks

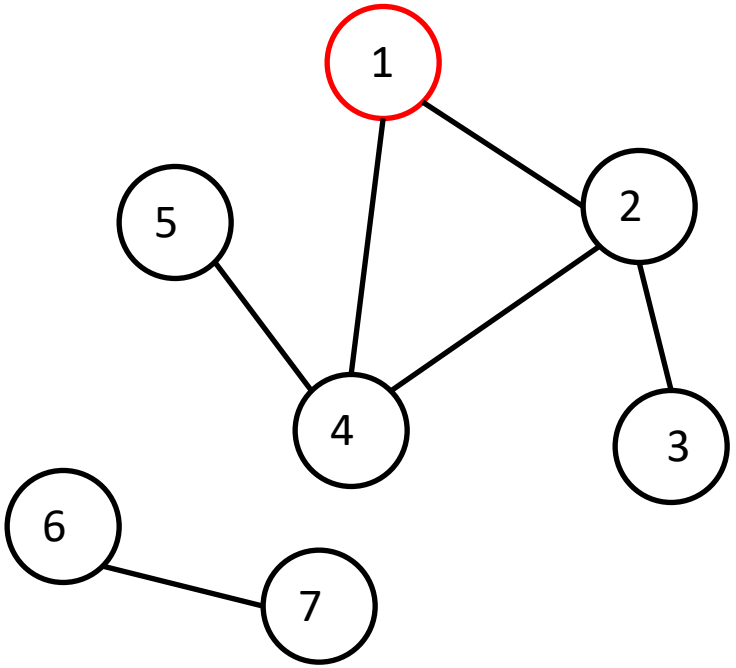


v = 1

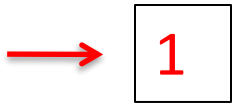
Start by putting 1 on the stack

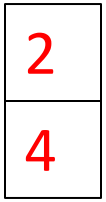
1

Depth First Search: Example with Stacks

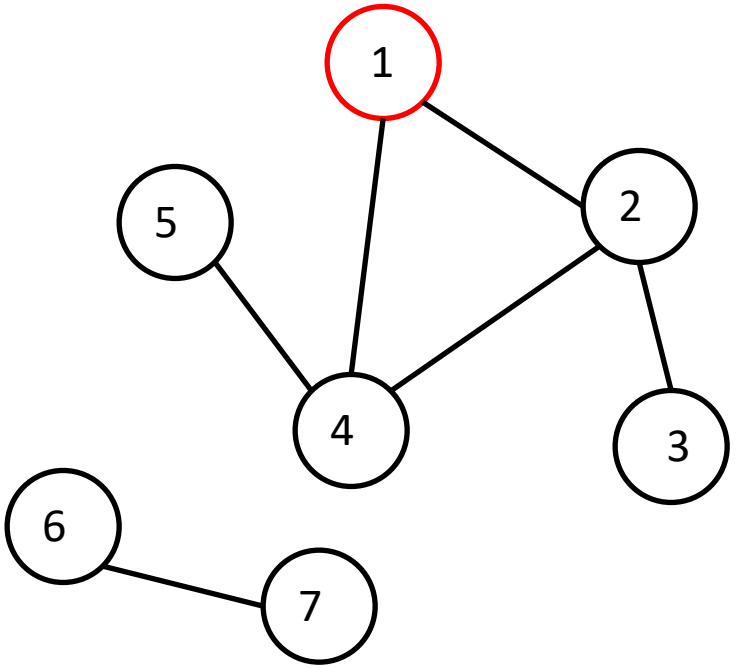


Which node is first on the stack? Pop it from the stack, mark it as explored, put its unexplored neighbors on the stack.





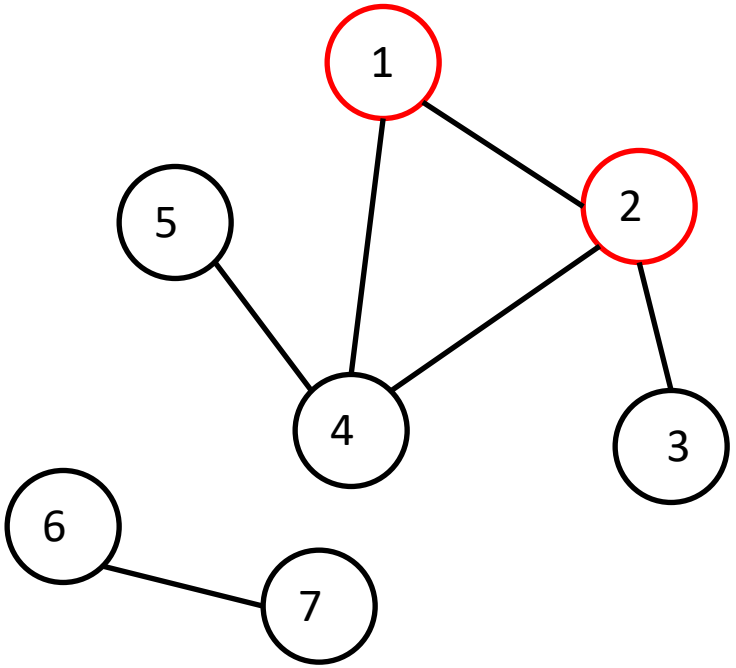
Depth First Search: Example with Stacks



Which node is first on the stack? Pop it from the stack, mark it as explored, put its unexplored neighbors on the stack.

2
4

Depth First Search: Example with Stacks



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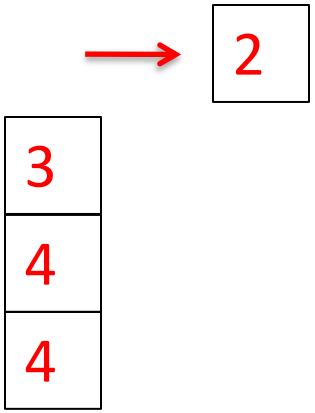
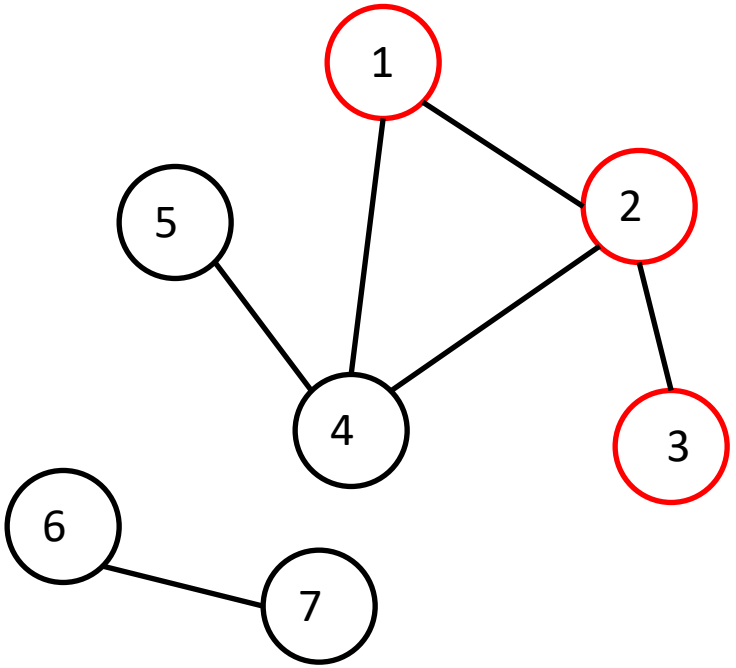
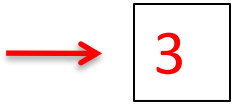


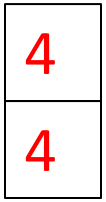
Depth First Search: Example with Stacks



Which node is first on the stack? Pop it from the stack, mark it as explored, put its unexplored neighbors on the stack.

No unexplored neighbors!





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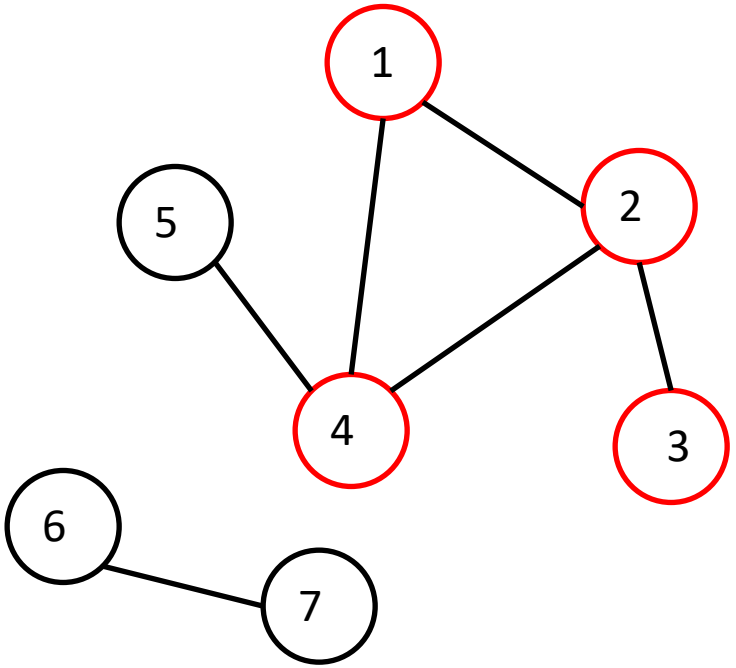

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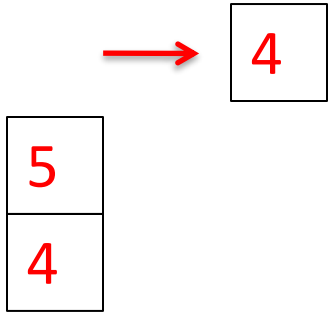

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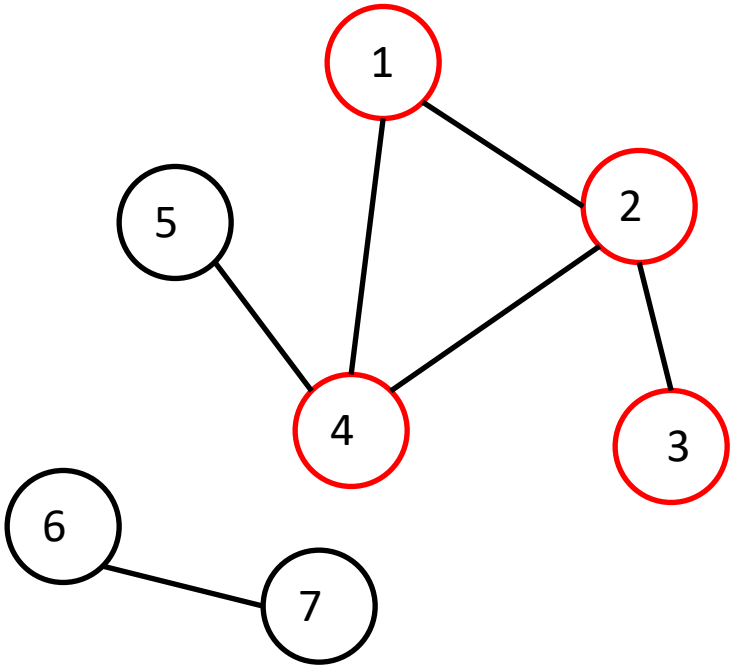

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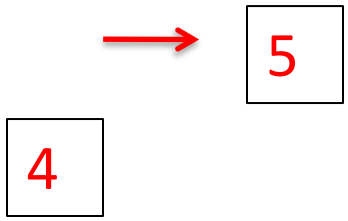

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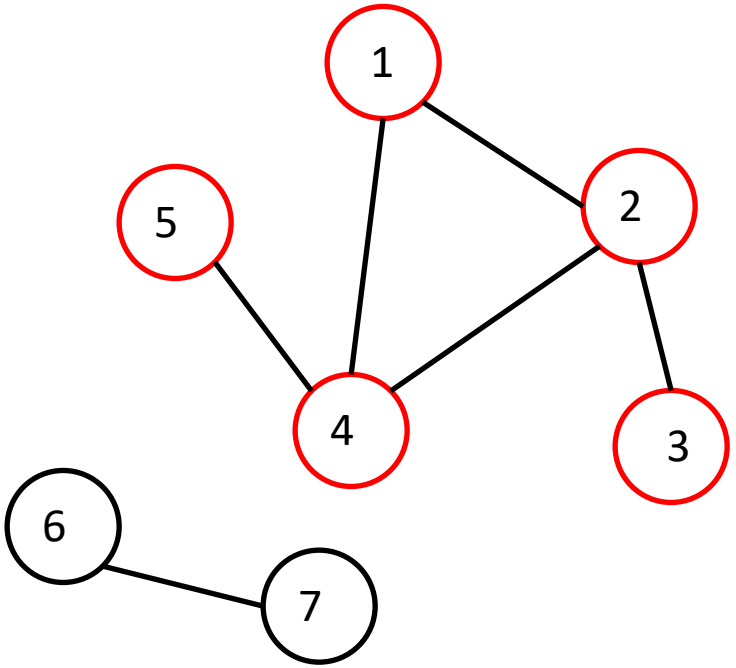
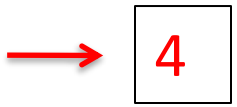


Which node is first on the stack? 4 is, but it has already been explored!

Done!



Timekeeping

```
procedure previsit(v)
pre[v] = clock
clock = clock + 1

procedure postvisit(v)
post[v] = clock
clock = clock + 1
```

pre[*u*] to post[*u*] is the time *u* is on the stack

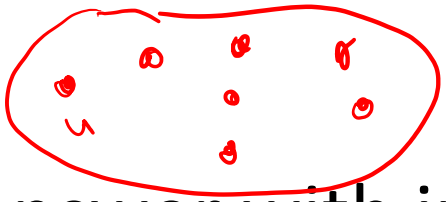
For all nodes *u*, *v*, either [pre[*u*], post[*u*]] is completely within [pre[*v*], post[*v*]], or the other way around, or there is no overlap. Why?

In-class Exercise

explore(u)

Suppose we start at node *u*. Are we guaranteed to find all nodes reachable from *u*?

IF v is reach. from u, there is a path from u→v



Let p_v be the length of shortest path from $u \to v$

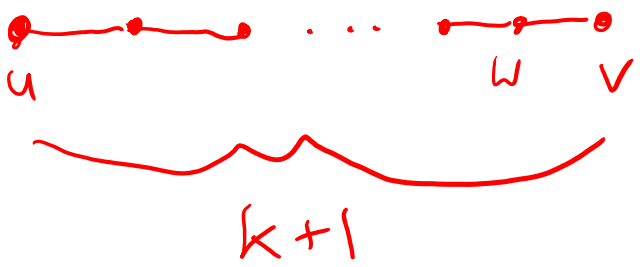
Prove your answer with induction!

Base case: $p_v = 0$. The only node within 0 edges of u is u itself. explore(u) marks u as visited. ✓

I.H.: Assume all v with $p_v \leq k$ are marked. as visited.

I.S.: We need to show that if a node v has $p_v = k+1$, v gets marked.

In-class Exercise



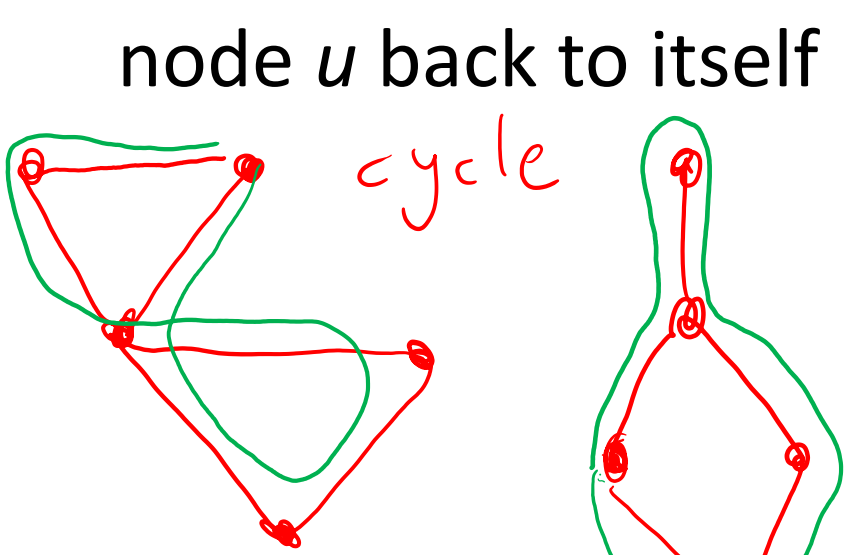
$p_v = k+1$

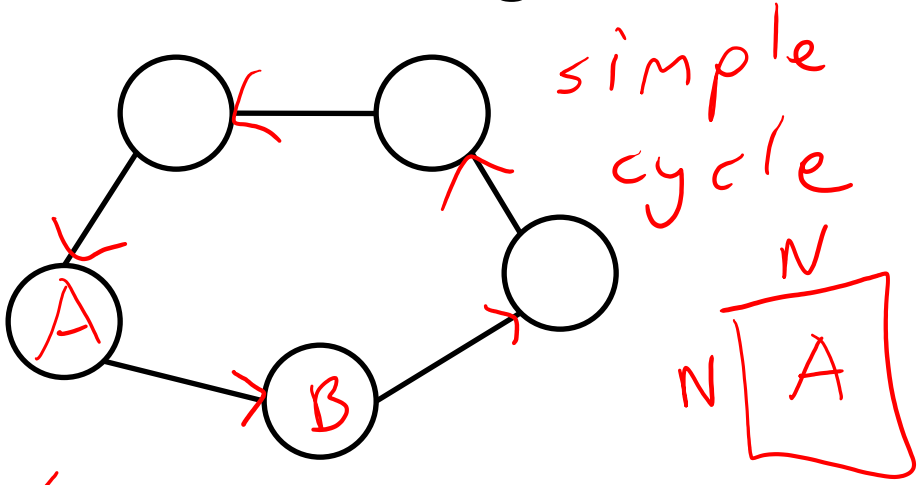
$p_w = k$

Let w be the node right before v on path.
$p_w = k$. If $p_w < k$, then $p_v < k+1$. If $p_w > k$,
then we could use the above path to find
a shorter path. Because $p_w = k$, it get marked
(by I.H.). Only time a node is marked is
when we call explore on it. As part of explore(w),
we call explore(v), unless v is already marked.
Either way, it gets marked. □

More Terminology

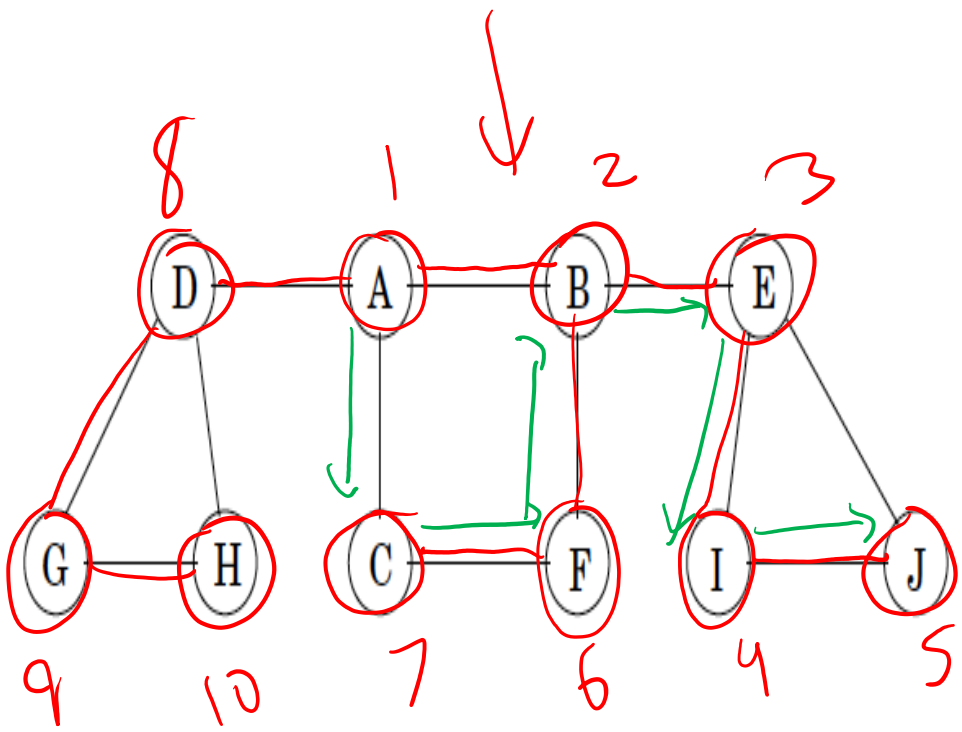
- A cycle is a path with at least two edges from a node *u* back to itself

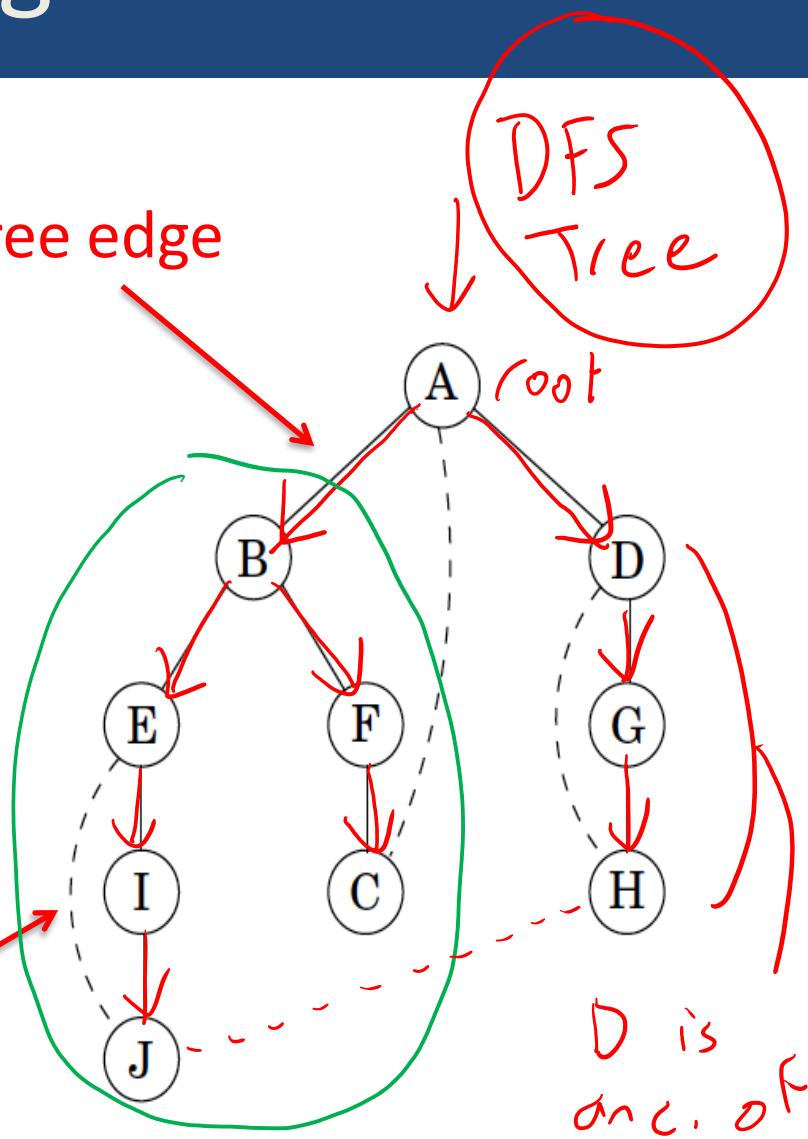




- A tree is a graph without cycles

Tree Edges vs. Back Edges





DFS on Directed Graphs

- We can run DFS on directed graphs, making sure to only follow edges in their correct direction
- The starting node is the root of the DFS tree
- *u* is an ancestor of *v* if there is a path from *u* to *v* in the DFS tree. *v* is a descendant of *u*.
- *u* is the parent of *v* if there is a directed edge from *u* to *v* in the DFS tree. *v* is the child of *u*.

DFS on Directed Graphs

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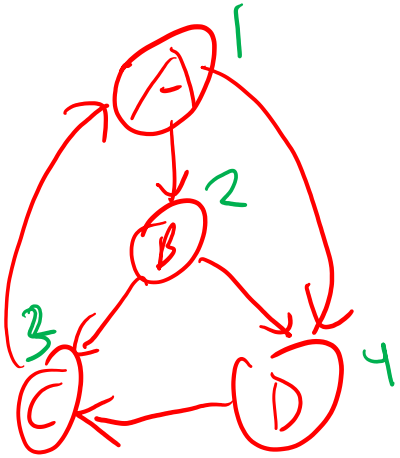

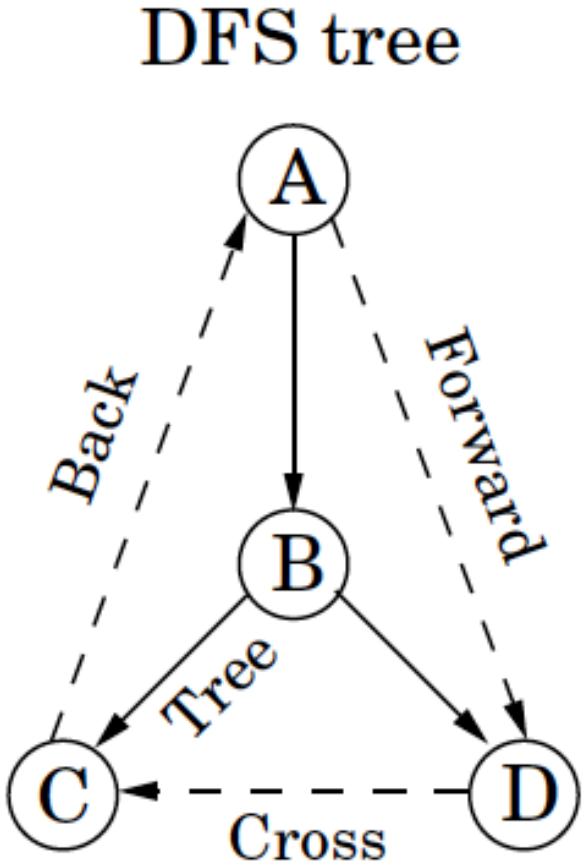

A is the root

B is a child of A

B is a parent of C

A is an ancestor of D

In-Class Exercise

1. How many parents can a node have in the DFS tree?
2. How many children can a node have in the DFS tree?
3. How many ancestors can a node have in the DFS tree?
4. How many descendants can a node have in the DFS tree?

Give examples for your answers!