Name:

Computational Physics

Final Exam

Due Midnight 8 May 2019

Instructions: You may use your books, notes, previous codes (in fact I encourage it!) and the internet, but not your classmates. Any questions should be directed to me.

1. **Hubble's Law**

   Hubble's law is the relationship between the recessional velocity of galaxy calculated with Doppler Shift and its distance from us, determined using a variety of methods. It is given the form $v = H_o d$ where v is the velocity in $km\ s^{-1}$, d is the distance in Mpc and H_o is Hubble's constant. You can assume that at a distance of zero, the recessional velocity is also zero. With your exam you have been given a data file which contains distances in Mpc (millions of parsecs, where 1 parsec = 3.26 light years) and velocities in $km\ s^{-1}$ for a set of galaxies.

   (a) Using **Monte Carlo techniques** determine a the best fit value of Hubble's constant and the standard deviation. Plot the data, the best fit line for Hubble's Law, and a shaded region for Hubble's Law for 1 and 2 standard deviations above and below the best fit value.

   (b) Hubble's constant can be used to measure the age of the Universe, τ where $\tau = 1/H_o$. Using you best fit H_o and its standard deviations to calculation the age of the Universe with error bars. We know that the oldest stellar populations are around 13.5 Gyr old. Is that knowledge consistent with your value of H_o? Does it constrain your value further?

2. **'Real' projectile motion:**

   Many elementary mechanics problems deal with the physics of objects moving through the air, but they almost always ignore friction and air resistance. Consider a spherical cannon ball shot through air. The air resistance on a moving sphere is a force in the opposite direction with the magnitude:

   $$F = \frac{1}{2}\pi R^2 \rho C v^2 \tag{1}$$

   where R is the sphere's radius , ρ the density of air, v is the velocity, and C is the coefficient of drag (dependent of the shape of the object).

   (a) Starting from Newton's second law, $F = ma$, show that the equations of motion for the position (x,y,) of the cannonball are:

   $$\frac{d^2x}{dt^2} = -\frac{\pi R^2 \rho C}{2m}\frac{dx}{dt}\sqrt{\left(\frac{dx}{dt}\right)^2 + \left(\frac{dy}{dt}\right)^2},\quad \frac{d^2y}{dt^2} = -g - \frac{\pi R^2 \rho C}{2m}\frac{dy}{dt}\sqrt{\left(\frac{dx}{dt}\right)^2 + \left(\frac{dy}{dt}\right)^2} \tag{2}$$

   where m is the mass of the cannonball, g is the acceleration due to gravity.

(b) Change these two second-order equations into four first-order equations using the methods you have learned, then write a program that solves the equations for a cannon ball of:

density (Pb)	=	11.3 g/cm^3,
radius	=	6 cm ,
firing angle	=	38° to the horizontal,
firing elevation	=	12 m
initial velocity	=	110 m s^{-1}.
density of air ρ	=	1.19 kg m^{-3}
coefficient of drag C	=	0.47

(Assume all other ground is flat beside the firing platform)

Measure the total distance travelled by the projectile and plot the projectiles paths.

(c) Now experiment with different projectile materials to determine which would fire the farthest.

(W) ρ = 19.3 g cm^3
(Ni) ρ = 8.9 g cm^3
produce plots and comparisons. Explain your findings.

3. **The temperature of a light bulb:**

An incandescent light bulb is a simple device—it contains a resistive filament, usually made of tungsten, heated by the flow of electricity until it becomes hot enough to radiate thermally. Essentially all of the power consumed by such a bulb is radiated as electromagnetic energy, but some of the radiation is not in the desired wavelengths, which means it is useless for desired lighting purposes.

Let us define the efficiency of a light bulb to be the fraction of the radiated energy that falls in a specific band. It's a good approximation to assume that the radiation obeys the Planck radiation law, meaning that the power radiated per unit wavelength λ obeys

$$I(\lambda) = 2\pi A h c^2 \frac{\lambda^{-5}}{e^{hc/\lambda k^B T} - 1} \tag{3}$$

where A is the surface area of the filament, T is the temperature, h is Planck's constant, c is the speed of light, and k_B is Boltzmann's constant. For a specific use, the desired wavelengths run from $\lambda_1 = 450$ **nm to** $\lambda_2 = 750$ **nm**, so the total energy radiated in the visible window is $\int_{\lambda_1}^{\lambda_2} I(\lambda) d\lambda$ and the total energy at all wavelengths is $\int_o^\infty I(\lambda) d\lambda$. Dividing one expression by the other and substituting for $I(\lambda)$ from above, we get an expression for the efficiency η of the light bulb thus:

$$\eta = \frac{\int_{\lambda_1}^{\lambda_2} \lambda^{-5}/(e^{hc/\lambda k^B T} - 1) d\lambda}{\int_o^\infty \lambda^{-5}/(e^{hc/\lambda k^B T} - 1) d\lambda} \tag{4}$$

where the leading constants and the area A have canceled out. Making the substitution $x = hc/\lambda k_B T$, this can also be written as

$$\eta = \frac{\int_{hc/\lambda_2 k_B T}^{hc/\lambda_1 k_B T} x^3/(e^x - 1)dx}{\int_o^\infty x^3/(e^x - 1)dx} = \frac{15}{\pi^4} \int_{hc/\lambda_2 k_B T}^{hc/\lambda_1 k_B T} \frac{x^3}{e^x - 1} dx \tag{5}$$

where we have made use of the known exact value of the integral in the denominator.

(a) Write a Python function that takes a temperature T as its argument and calculates the value of η for that temperature from the formula above. The integral in the formula cannot be done analytically, but you can do it numerically using **Simpson's Rule**. Ensure that you are choosing a high enough number of steps that your answer is convergent. Use your function to make a graph of η as a function of temperature between 300 K and 10,000 K. You should see that there is an intermediate temperature where the efficiency is a maximum. What is that maximum temperature?

(b) Is it practical to run a tungsten-filament light bulb at the temperature you found? If not, why not?