# PHYS 50733 - Computational Physics
# Homework 2

Homework 2 is due at **midnight** on **Sunday 3 February 2019**. Along with your jupyter notebook you should include copies of any plots generated and any analytical derivations or calculations required in a folder labeled Homework_2 in your github repository. Remember to comment your code. You will be graded on whether your notebook runs on my computer and produces the correct results as well as your use of functions when possible.

**Problem 1**

In physics calculations we often encounter functions which do not have a nice closed form expression. One of these is the gamma function.

$$\Gamma(a) = \int_0^\infty x^{a-1} e^{-x} dx \tag{1}$$

For this problem feel free to use scipy integrate where appropriate, but you must explicitly choose your integrator and not let scipy choose for you.

a) Write a program to make a graph of the value of the integral given for the bounds x=0-5 and for a = 2. On the same axis plot the result for ours five integration methods (Trapezoid, Gaussian Quadrature, Simpson's, Romberg and Monte Carlo. On two other sets of axis show the value as a function of N=8,16,32,63,128,256 for each method and plot the estimated error as a function of N.

b) Choose what you think is the best integrator for this method and the N required for accuracy. Justify your choices and write a program to make a graph of the value of the integral given from x=0-5 for a = 2,3, and 4 on the same axis.

c) In order to numerically integrate $x = 0 - \infty$ we need to change variables. Since most of value of the integral is near the peak, what is the change of variables which shifts the range of integration from 0 to 1 and places the peak near 1/2.

d) We next need to transform the integral to make it a bit easier to evaluate. This can be done by writing $x^{a-1}$ as $e^{(a-1)lnx}$. Using this new transformation calculate to write a user defined function $\Gamma(a)$ which will calculate $\Gamma$ for an arbitrary a value. Test your function with the known value of $\Gamma(1.5) = 0.5\sqrt(\pi)$.

e) Plot the Gamma function for a = 1-10.

f) Show that for integer values of a the Gamma function is equal to the factorial of a. (HINT: Write a user defined function to calculate the factorial of an integer).

**Problem 2**

Let us assume we have a distribution of electric charges and want to calculate the resulting electric field, remembering that the electric field is the gradient of the electric potential. Calculate

and plot the electric potential and the electric field for the following charge distributions on a 1 m x 1 m grid. You can assume the distributions are centered on the origin. Make sure to justify your choice of integrator and N.

a) Three equal charges arranged in an equilateral triangle 10 cm on a side.

b) Two rings of equal charge with constant charge density in the same plane. One with a radius of 2 cm and one with a radius of 15 cm.

c) A plane of charge in the xy plane with a constant change density which is 10 cm on a side surrounded by a ring in the xz plane with constant charge density and a radius of 20 cm. You can assume the plane and the ring have the same total charge. For this one calculate and plot the electric potential and the electric field in three dimensions.