PHYS 50733 - Computational Physics
Homework #4

In physics the problem of two bodies orbiting one another can be solved neatly, and analytically. Likewise, approximates can be made for a 3 or 4 body system assuming that the additional bodies have masses much smaller than the original 2. However, as soon as we deal with the problem of more than 2 bodies of equivalent mass things become chaotic and require numerical methods to solve them. This is the N-body problem is physics.

The physics of the N-body problem is governed by the equations of gravity so that for the i^{th} body of the system the force from the other bodies in a system of N-bodies is

$$\mathbf{F}_i = M_i\mathbf{a}_i = M_i\frac{d^2\mathbf{r}}{dt^2} = \sum_{1,i!=j}^{N} \frac{GM_iM_j}{|\mathbf{r_i} - \mathbf{r_j}|^2} \tag{1}$$

You have the following system, two stars of equal mass orbit around their common center of mass a distance of 50 AU apart. A third star with mass of 1/2 that of the inner two stars orbits the center of mass of the system with a pericenter of 30 AU and a apocenter of 2000 AU. The orbit of the 3^{rd} star is also inclined relative to the plane of the inner two stars by 13 degrees. Do the calculation for a clockwise and counterclockwise rotation for M_1 and M_2.

Start your calculation with 1000 timesteps and then explore what occurs when you increase or decrease the number of timesteps. Do you notice any computational glitches? Do you believe your answers?

For each case make the following plots.

- Magnitude of the velocity and acceleration of the stars as a function of time.
- Orbits of the three stars as a function of time.

Required for the graduate students, option for undergraduates.

- We have initially assumed $M_3 = 1\ M_{\odot}$. Run the code assuming $M_3 = [0.1, 1.0, 2.0, 5.0]\ M_{\odot}$. What differences do you see?