Wireless Communications System Design, Modeling, and Implementation
Homework #2

Due on May 24th, 2020 at 11:59pm

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PART I: The MATLAB code used

The code should be clearly written, and it will be copied from the PDF file for validation.

The recourse code is listed in the end of the document.

PART II: Antenna spacing and beamforming

In this question, we study the impact of antenna spacing on beamforming. We will consider a transceiver with N antennas and spacing d between them. The normalized beamforming vector $\mathbf{f}$ depends on the angle ϕ, wavelength λ, and antenna spacing d as follows

$$\mathbf{f}(\phi) = \frac{1}{\sqrt{N}} \left[1, e^{-j2\pi\frac{d}{\lambda}\sin(\phi)}, \ldots, e^{-j2\pi(N-1)\frac{d}{\lambda}\sin(\phi)}\right] \tag{1.1}$$

Question 1

In one figure, plot the normalized beam pattern in logscale in polar coordinates in the range $\left[-\frac{\pi}{2}, \frac{\pi}{2}\right]$ for the following cases

(a) Number of antennas $N = 16$, angle $\phi = -30°$, and spacing $d = \frac{\lambda}{2}$ (critically spaced antennas)
(b) Number of antennas $N = 16$, angle $\phi = -30°$, and spacing $d = 2\lambda$ (critically spaced antennas)
(c) Number of antennas $N = 16$, angle $\phi = -30°$, and spacing $d = \frac{\lambda}{8}$ (critically spaced antennas)

What can you conclude about the impact of antenna spacing on beamforming? Explain your reasoning.

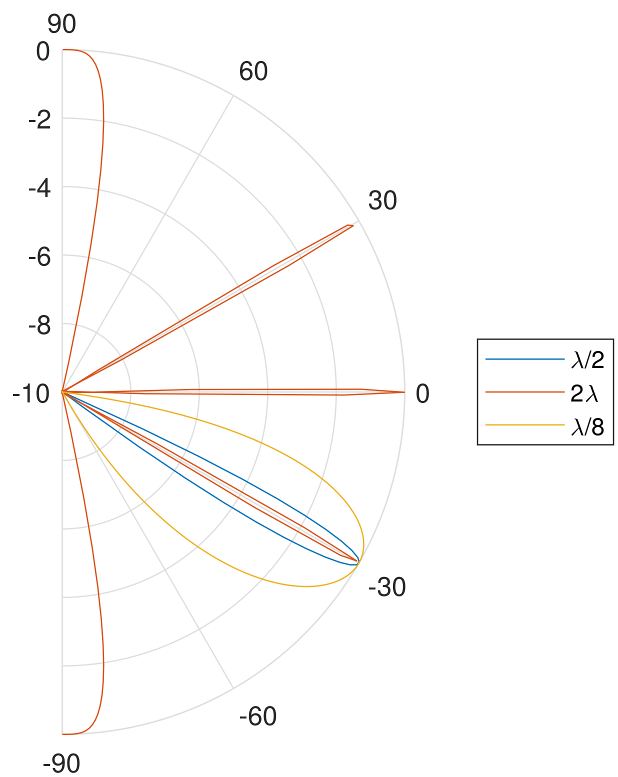

Figure 1: Normalized beamforming of different antennas space

As can be seen from the normalized antenna lobe diagram, generally speaking, as the antennas spacing increases, the main lobe becomes narrower. The energy is concentrated and the transmission will be farther. So as the directivity, it is also stronger. But as the spacing further increases, grating lobes may appear, which will interfere with other directions.

PART III: Channel capacity and achievable rate

We will study channel capacity and achievable rate in the system. In particular, we will study them in the channels with 1) rich scattering, and 2) with a few multipath components. We assume narrowband channels, i.e., we assume that the channels are frequency flat with AWGN.

Question 1

In this question, we compare channel capacities in channels with 1) rich scattering, and 2) sparse scattering. If bandwidth B is used for communication, capacity in rich scattering channel is calculated as follows

$$c_r = B\sum_{k=1}^{K}\log_2\left(1 + N_t N_r \frac{P_t}{K}\frac{\left|\mathbf{w}_k^{\mathrm{H}}\mathbf{H}_r\mathbf{f}_k\right|^2}{BN_0}\right) \tag{1.1}$$

where K is the channel rank, P_t is the transmit power, and N_0 is the noise spectral density. The rank K, normalized combining vector $\mathbf{w}_k$, and normalized precoding vector $\mathbf{f}_k$ are obtained through the SVD decomposition of $\mathbf{H}_r$. Similarly, in a sparse MIMO channel with the channel matrix $\mathbf{H}_s$, the corresponding capacity c_s is calculated through the SVD decomposition of $\mathbf{H}_s$. Plot capacities of the channels with rich and sparse scattering versus N_r. For each value of c_s, run 1000 Monte Carlo trials and generate new channel matrices $\mathbf{H}_r$ and $\mathbf{H}_s$ in each trial, use SVD decomposition to find the ranks and normalized combining and precoding vectors, calculate capacities in each trial, and then find the averages. Compare the capacities and explain why they are the same/different.

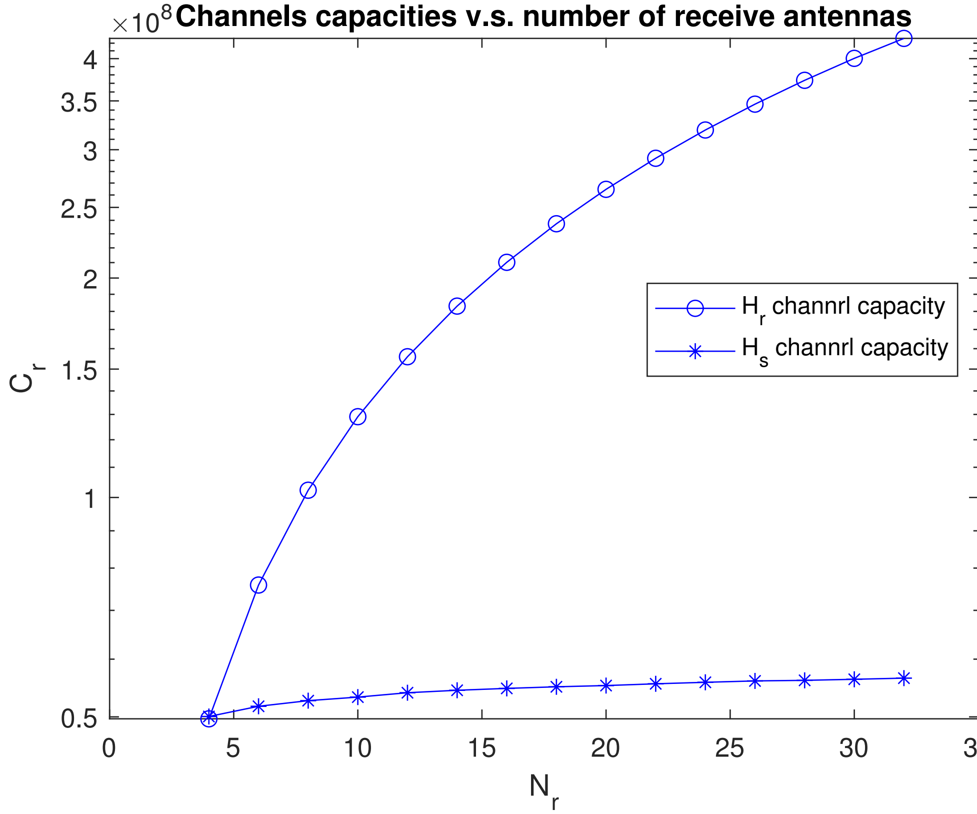

Figure 2: Channels capacities versus the number of receive antennas

Shannon pointed out in his article *Communication in the Presence of Noise* that the best form of signal for effective and reliable communication should be a signal with statistical characteristics of white noise, because white noise signals have ideal autocorrelation characteristics. However, in the sparse scattering matrix, the matrix is sparse, so the performance is significantly lower than that of the rich scattering matrix. Because of the "multiplexing" characteristics of MIMO, as the number of antennas increases, the channel capacity will also increase in logarithmic order.

Question 2

Achievable rate is calculated similarly to capacity in (5), but with two key differences. First, when achievable rate is calculated, there are P, $P \leq K$, terms in the summation, and in each term, the transmit power P_t is divided by P instead of K. The number P depends on the transmitter and receiver architectures, and we will assume that $P = 2$. Second, the way normalized beamforming vectors $\mathbf{f}_k$ and $\mathbf{w}_k$ are obtained depends on the knowledge of the channel.
When the channel matrix is perfectly known at the transmitter/receiver, normalized beamforming vectors are obtained through the SVD decomposition of the channel matrix, just as in the calculation of capacity.
When the channel matrix is unknown, normalized beamforming vectors are obtained through extensive beam training at both the transmitter and receiver. The transmitter uses N_t normalized precoding vectors $\mathbf{f}_{\mathrm{tr},1}, \ldots \mathbf{f}_{\mathrm{tr},N_t}$, each corresponding to one angle from the set $\left\{-\frac{\pi}{2}, -\frac{\pi}{2} + \frac{\pi}{N_t}, \ldots, -\frac{\pi}{2} + (N_t - 1)\frac{\pi}{N_t}\right\}$, to scan the angular space. Similarly, the receiver uses Nr normalized combining vectors $\mathbf{w}_{\mathrm{tr},1}, \ldots \mathbf{w}_{\mathrm{tr},N_r}$, each corresponding to one angle from the set $\left\{-\frac{\pi}{2}, -\frac{\pi}{2} + \frac{\pi}{N_r}, \ldots, -\frac{\pi}{2} + (N_r - 1)\frac{\pi}{N_r}\right\}$, to scan the angular space. For

example, when $\mathbf{f}_{tr,1}$ and$\mathbf{w}_{tr,1}$ are used in a noiseless channel with rich scattering, the received signal is

$$y = \mathbf{w}_{\mathrm{tr},1}^{\mathrm{H}} \mathbf{H}_r \mathbf{f}_{\mathrm{tr},1} \tag{2.1}$$

Among $N_t N_r$ pairs of normalized beamforming vectors, the pairs that result in the highest received signal power $|y|^2$ are selected and used in the next phase.
Use results from Question 1 to plot channel capacities for both types of channels in a new figure. Use the same figure to plot the achievable data rate versus N_r in both channels, when channel matrices $\mathbf{H}_r$ and $\mathbf{H}_s$ are known (SVD for both channels) and when they are not known (beam training for both channels). The total number of curves in the figure is six: two for capacities, two for achievable rates when channel matrices are known, two for achievable rates when channel matrices are unknown. To find the achievable rates, run 1000 Monte Carlo trials, generate new channel matrices in each trial, perform the SVD and beam training for both channels, and calculate rates in each iteration, then find the averages.
Compare the curves. Which channel provides higher achievable data rate, and is the difference in data rates significant? How does beam training perform compared to the SVD approach?

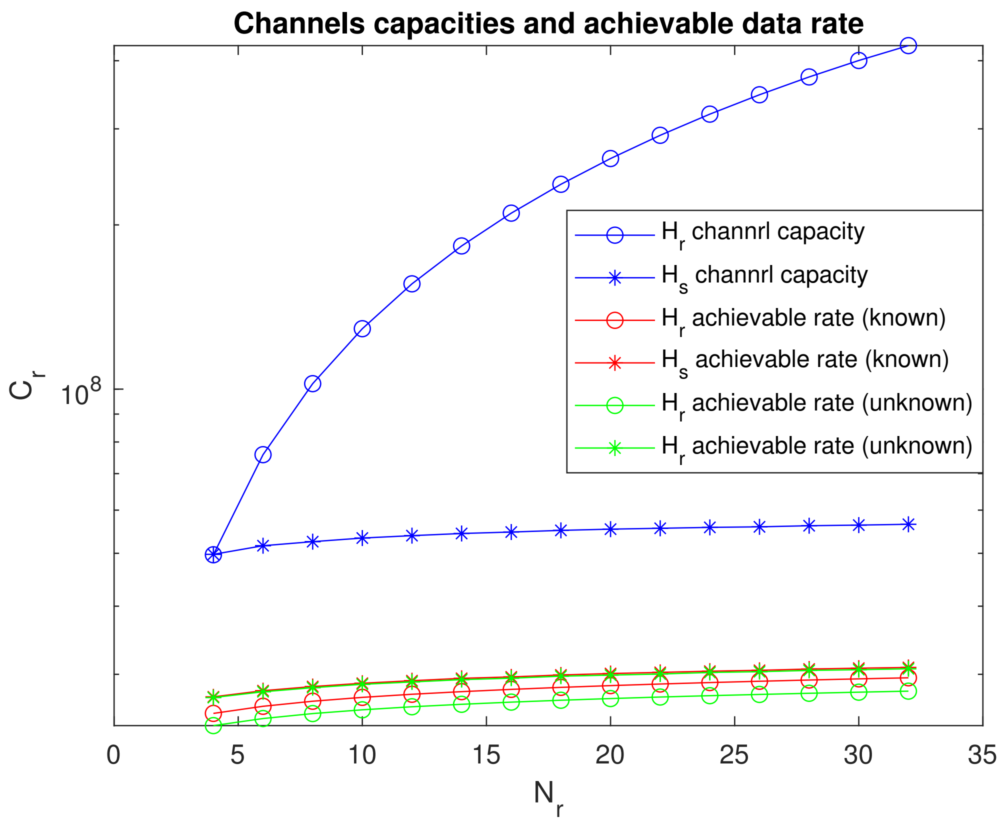

Figure 3: Channel capacities and achievable rates

Shannon's theoretical limit rate of channel capacity expression, so the true achievable rate will undoubtedly be lower than the theoretical rate. What is obtained through SVD is the best situation when the channel is known, so the rate obtained through training will also be lower than the SVD rate, but the difference between the two is not quite significant. In addition, the conclusion of the first question is still true. The rich scattering matrix is always better than the sparse scattering matrix under the same conditions, and the gap between the two is also not large.

PART IV: Adaptive loading and power allocation

We consider line-of-sight channels between the BS and UEs with unit path-loss. The $1 \times M$ channel vector between the BS antenna array and UE-1 is denoted by

$$\mathbf{h}_1 = \left[1, e^{-j\pi \sin(\phi_1)}, e^{-j\pi 2 \sin(\phi_1)}, \ldots, e^{-j\pi (M-1) \sin(\phi_1)}\right]. \tag{3.1}$$

Similarly, the $1 \times M$ channel vector between the BS antenna array and UE-2 is denoted by

$$\mathbf{h}_2 = \left[1, e^{-j\pi \sin(\phi_2)}, e^{-j\pi 2 \sin(\phi_2)}, \ldots, e^{-j\pi (M-1) \sin(\phi_2)}\right]. \tag{3.2}$$

Upsample and Pulse-shaping: Upsample the sequence of 4-QAM symbols $s_1(n)$ by a factor of 4 and apply root-raised cosine transmit filter with roll-off factor 0.5 and filter span of 8 symbols to obtain the sequence $\tilde{s}_1(n)$. Similarly, obtain $\tilde{s}_2(n)$ from the sequence of 4-QAM symbols $s_2(n)$.
Use inbuilt MATLAB functions upsample and rcosdesign for upsampling and pulse-shaping, respectively.
Pre-coding: The pre-coder takes pulse shaped sequence $\tilde{s}_1(n)$ and $\tilde{s}_2(n)$ and applies a zero-forcing pre-coder $\mathbf{P}$ to transmit the signals to the UEs. The precoding matrix $\mathbf{P}$ is obtained as follows:

$$\mathbf{P} = \mathbf{H}^{\mathrm{H}}\left(\mathbf{H}\mathbf{H}^{\mathrm{H}}\right)^{-1} \tag{3.3}$$

where $\mathbf{H} = \left[h_1^T, h_2^T\right]^T$ is the equivalent MIMO channel between BS and UEs and $\mathbf{H}^{\mathrm{H}}$ is the Hermitian (conjugate transpose) of the matrix $\mathbf{H}$. The output of pre-coder is $\mathbf{x}(n) = \mathbf{P}\tilde{\mathbf{s}}(n)$, where $\mathbf{x}$ is an $M \times 1$ vector and $\tilde{\mathbf{s}}(n) = [\tilde{s}_1(n), \tilde{s}_2(n)]^T$ is a 2×1 vector.
DAC model: Consider that the DAC quantizes 2^b uniformly spaced levels between the range $\left[-\frac{1}{M}, \frac{1}{M}\right]$, where b is the number of bits and M is the number of antennas. Each antenna chain m includes a DAC that quantizes $x_m(n)$ to give output $y_m(n)$, where $x_m(n)$ is the m-th element of the vector $\mathbf{x}(n)$. The quantizer is applied on real and imaginary parts of $x_m(n)$ separately to get quantized real and imaginary parts of $y_m(n)$.
PA model: We consider the PA in each antenna chain adds a third order non-linearity in the transmitted signal using the following model:

$$z_m(n) = \beta_1 y_m(n) + \beta_3 y_m(n) \left|y_m(n)\right|^2, \; m = 1, 2, \ldots, M, \tag{3.4}$$

The $M \times 1$ transmitted signal vector from the antenna array is $\mathbf{z}(n) = [z_1(n), z_2(n), \ldots, z_M(n)]^T$.
In the following, in order to observe the impact of DAC quantization and PA non-linearity on the transmitted signal, we are going to transmit 4-QAM signal to UE-1 and set $s_2(n) = 0$ for $n = 1, 2, \ldots, 1000$.

Question 1

For this part, assume that PA is ideal, i.e., $\beta_1 = 1$ and $\beta_3 = 0$. Use inbuilt MATLAB function periodogram to plot the power spectral density of the signal transmitted at the first antenna, i.e., $z_1(n)$ for the following cases

(a) Number of antennas $M = 8$ and number of DAC bits $b = 12$
(b) Number of antennas $M = 8$ and number of DAC bits $b = 4$
(c) Number of antennas $M = 32$ and number of DAC bits $b = 12$
(d) Number of antennas $M = 32$ and number of DAC bits $b = 4$

Under which of the 4 cases, do you see maximum and minimum spectral regrowth (out-of-band transmission)?

Question 1 continued on next page…

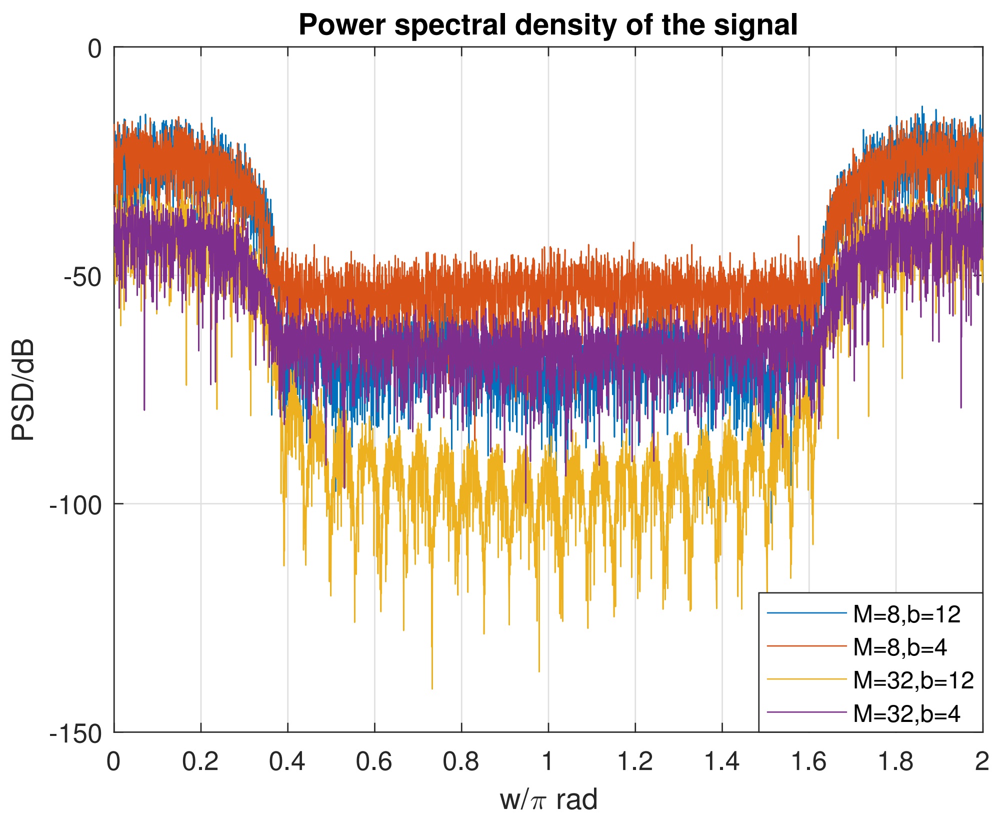

Figure 4: The power spectral density of the signal transmitted at the first antenna

In the Figure 4, we can see case (b) has the maximum and case (c) minimum spectral regrowth (out-of-band transmission). However, they are quite similar, the difference is not as significant as Question 3.

Question 2

For this part, assume that PA is ideal. Due to quantization, some part of the signal $s_1(n)$ is transmitted toward the angle $\phi_2 = 40°$, causing interference to UE-2. To measure the interference, we plot angular response $g(\phi) = \frac{1}{N}\sum_n \left|\mathbf{a}^{\mathrm{H}}(\phi)\, z(n)\right|^2$, where $\phi \in (-90°, 90°)$, $\mathbf{a}(\phi) = \left[1, e^{-j\pi\sin(\phi)}, e^{-j\pi 2\sin(\phi)}, \ldots, e^{-j\pi(M-1)\sin(\phi)}\right]^T$ is an $M \times 1$ steering vector in the direction ϕ, and N is the number of samples $z(n)$. The interference transmitted toward UE-2 is $g(40°)$.
Plot $g(\phi)$ versus ϕ for the following cases and find the interference toward UE-2.

(a) Number of antennas $M = 8$ and number of DAC bits $b = 12$
(b) Number of antennas $M = 8$ and number of DAC bits $b = 4$
(c) Number of antennas $M = 32$ and number of DAC bits $b = 12$
(d) Number of antennas $M = 32$ and number of DAC bits $b = 4$

Under which of the 4 cases, do you see maximum and minimum interference to UE-2?

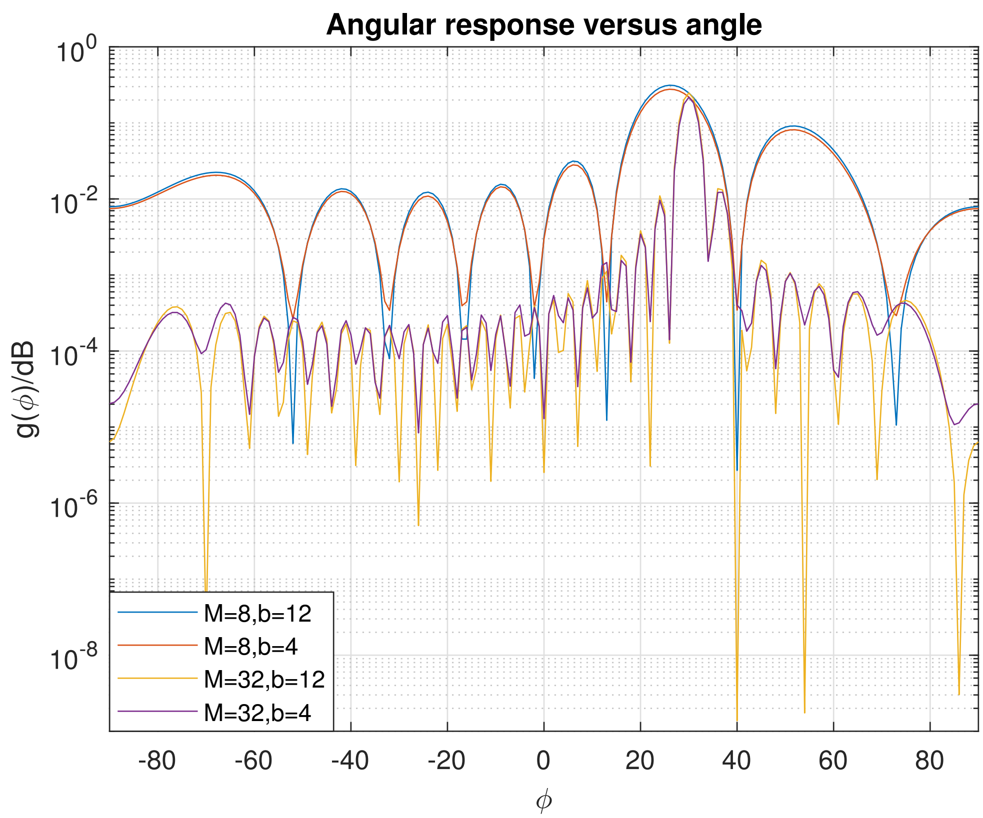

Figure 5: Angular response $g(\phi)$ versus angle ϕ

cases	$g(40°)$
case (a)	$2.69025855017031 \times 10^{-6}$
case (b)	$3.45142234120797 \times 10^{-4}$
case (c)	$1.37399487083223 \times 10^{-9}$
case (d)	$4.03331017995731 \times 10^{-4}$

Table 1: Comparison $g(40°)$ for different cases.

According to Table 1, the case (d) has the maximum interference to UE-2, and the case (c) has the minimum interference.

Question 3

For this part, ignore DAC quantization, i.e., $y_m(n) = x_m(n)$. Use inbuilt MATLAB function periodogram to plot the power spectral density of the signal transmitted at the first antenna, i.e., $z_1(n)$ for the following cases

(a) Number of antennas $M = 8$, and $\beta_1 = 1$, $\beta_3 = 0$
(b) Number of antennas $M = 8$, and $\beta_1 = 1$, $\beta_3 = -133$
(c) Number of antennas $M = 32$, and $\beta_1 = 1$, $\beta_3 = 0$
(d) Number of antennas $M = 32$, and $\beta_1 = 1$, $\beta_3 = -133$

Under which of the 4 cases, do you see maximum and minimum spectral regrowth (out-of-band transmission)?

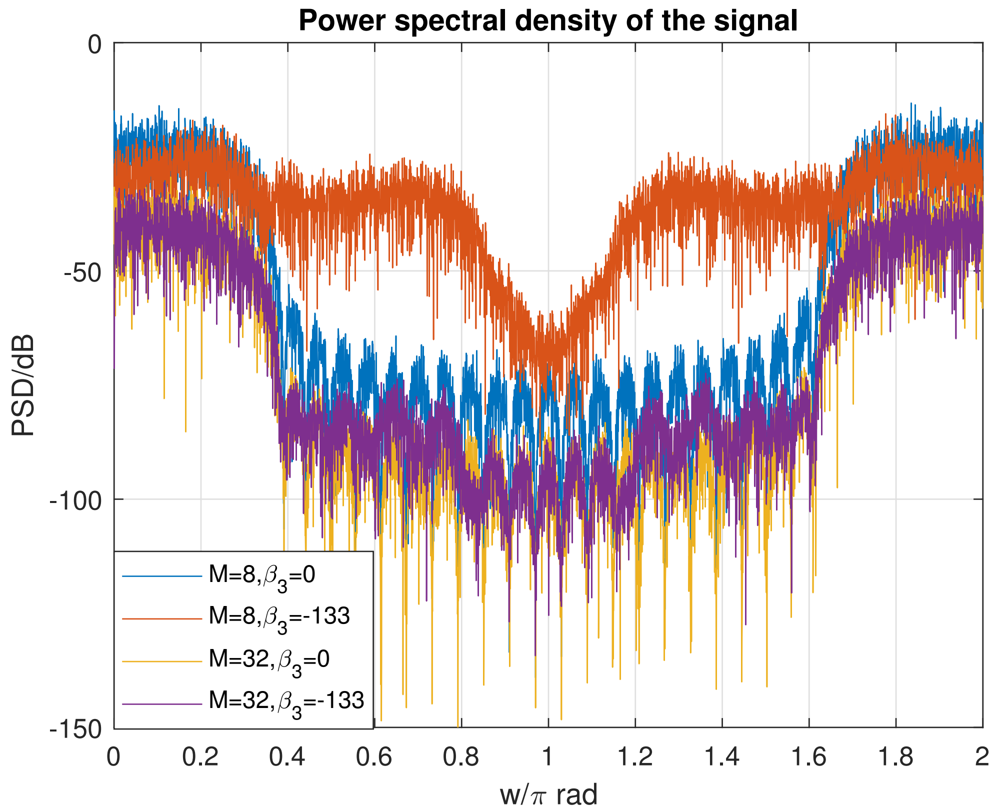

Figure 6: The power spectral density of the signal transmitted at the first antenna

In the Figure 4, we can see case (b) has the maximum and case (c) has the minimum spectral regrowth (out-of-band transmission).

Question 4

For this part, ignore DAC quantization. Due to PA non-linearity, some part of the signal $s_1(n)$ is transmitted toward the angle $\phi_2 = 40°$, causing interference to UE-2. Plot $g(\phi)$ versus ϕ for the following cases and find the interference toward UE-2.

(a) Number of antennas $M = 8$, and $\beta_1 = 1$, $\beta_3 = 0$
(b) Number of antennas $M = 8$, and $\beta_1 = 1$, $\beta_3 = -133$
(c) Number of antennas $M = 32$, and $\beta_1 = 1$, $\beta_3 = 0$
(d) Number of antennas $M = 32$, and $\beta_1 = 1$, $\beta_3 = -133$

Under which of the 4 cases, do you see maximum and minimum interference to UE-2?

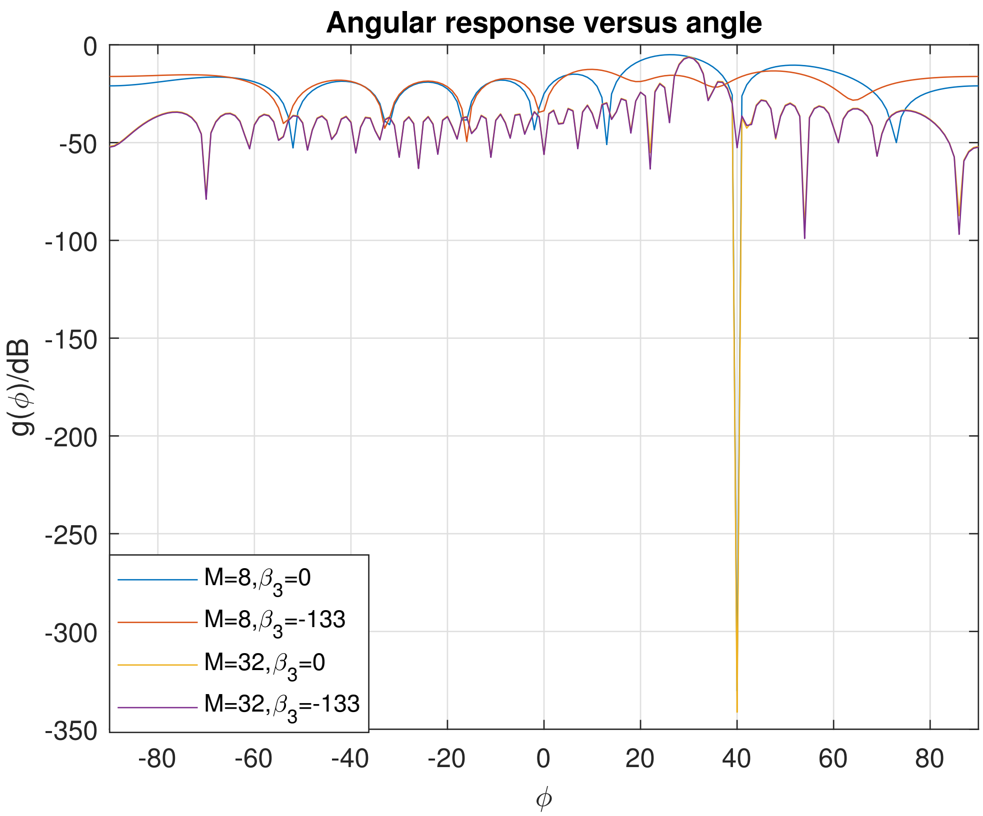

Figure 7: Angular response $g(\phi)$ versus angle ϕ

cases	$g\left(40^{\circ}\right)$
case (a)	$8.99463695219214 \times 10^{-34}$
case (b)	$1.93653580829205 \times 10^{-2}$
case (c)	$7.39374603036034 \times 10^{-35}$
case (d)	$5.47199433258856 \times 10^{-6}$

Table 2: Comparison $g\left(40^{\circ}\right)$ for different cases.

According to Table 2, the case (b) has the maximum interference to UE-2, and the case (c) has the minimum interference.