Wireless Communications System Design, Modeling, and Implementation
Homework #1

Due on May 3rd, 2020 at 11:59pm

Prof. Danijela Cabric

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PART I: The MATLAB code used

The code should be clearly written, and it will be copied from the PDF file for validation.

The recourse code is listed in the end of the document.

PART II: Packet detection

In this part, we will assume only an AWGN channel, i.e., $h = 1$. Your tasks for this problem are the following:

Question 1

Extract the repeating sequence $z_{\mathrm{STF}}(n)$, $n = 0, 1, ..., 15$, from the STF (first 16 samples in x_{STF}). Plot the magnitude of this sequence, i.e., plot $|z_{\mathrm{STF}}(n)|$ versus n.

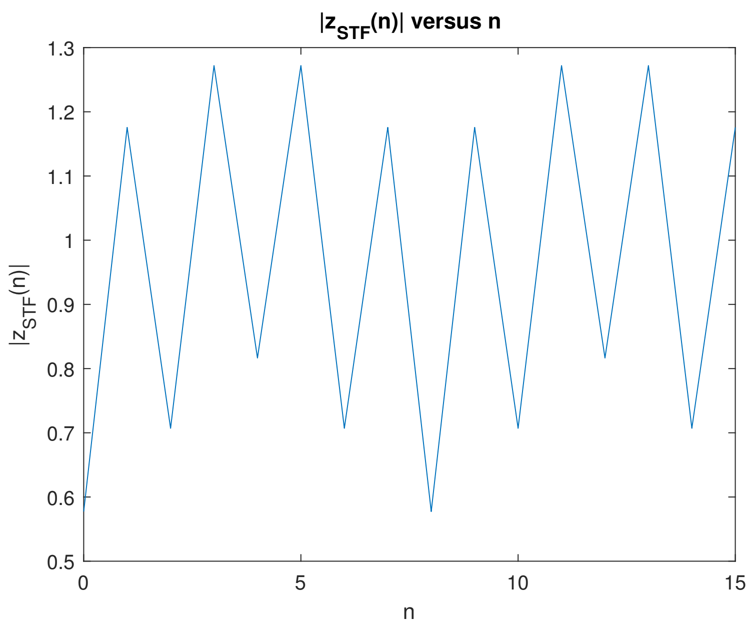

Figure 1: $|z_{\mathrm{STF}}(n)|$ versus n

Question 2

Plot the phase of this sequence, i.e., plot $\angle z_{\mathrm{STF}}(n)$ versus n.

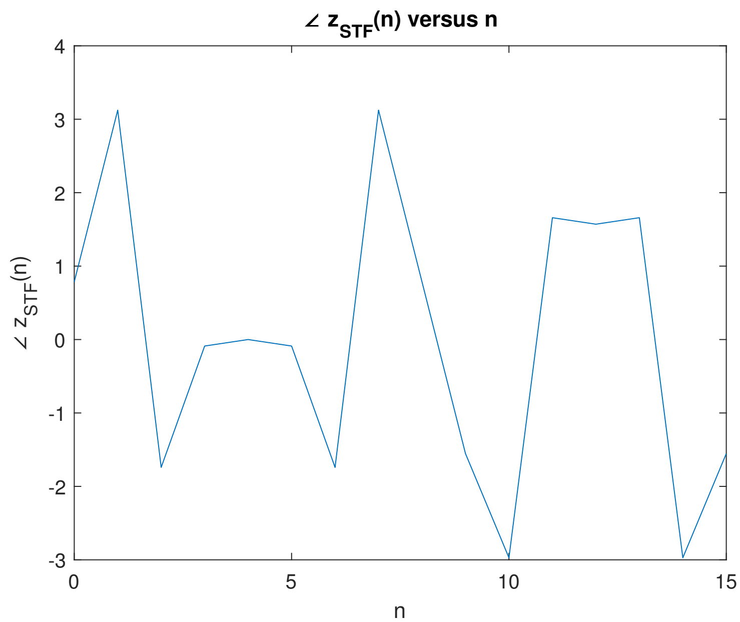

Figure 2: $\angle z_{\mathrm{STF}}(n)$ versus n

Question 3

Use the correlation of x_{STF} with the received signal $y(n)$ for packet detection. The correlation function is given as

$$r(n) = \sum_{m=0}^{15} y^{*}(n+m)\, z_{\mathrm{STF}}(m) \tag{3.1}$$

Declare that the packet is DETECTED if there are at least 9 peaks in $|r(n)|$ separated by 16 samples. The sample $r(n)$ is denoted as a peak if $|r(n)| \geq \mu_{|r|} + 2\sigma_{|r|}$, where $\mu_{|r|} = \frac{1}{160}\sum_n |r(n)|$ and $\sigma_{|r|} = \sqrt{\frac{1}{160}\sum_n \left(|r(n)| - \mu_{|r|}\right)^2}$. In MATLAB, inbuilt functions mean and std can be used to find $\mu_{|r|}$ and $\sigma_{|r|}$, respectively. Plot the probability of packet detection as a function of SNR over the range 0:2:20 dB. To find the probability, run 10000 Monte Carlo trials for each SNR value, count the number of detected packets for each SNR value, and then calculate the probability by dividing the number of detected packets by the number of trials.

Question 3 continued on next page...

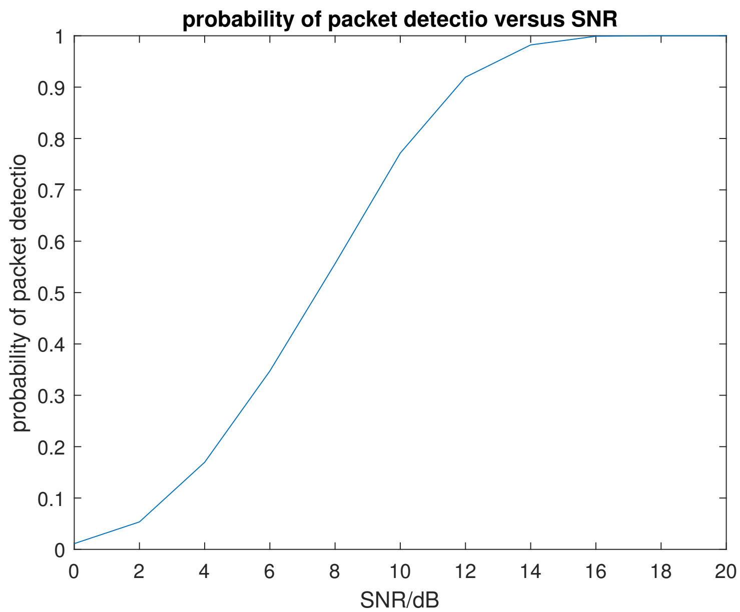

Figure 3: probability of packet detectio versus SNR

PART III: Channel estimation

In this part, we will assume an AWGN channel with three multipath components, i.e., $h = [1,\ 0.9,\ 0.5]$. Let H_k be the channel coefficient for subcarrier k. The true value of H_k is at the k-th element of the 64-point FFT of h. This coefficient can be estimated as follows.

1. From the received signal $y(n)$, extract 160 samples of the LTF field. Then, extract samples 33 to 96 from the LTF field. Denote these 64 samples by $v_n,\ n = 0, 1, ..., 63$. 2. For k-th subcarrier, obtain the received LTF symbol using

$$\hat{L}_k = \frac{1}{8} \sum_{n=0}^{63} v_n e^{-j2\pi k \Delta f n T_s} \tag{4.1}$$

3. Then use the least-squares estimate of H_k, i.e., compute

$$\hat{H}_k = \sqrt{\frac{52}{64}} \frac{\hat{L}_k}{L_k} \tag{4.2}$$

Your tasks for this problem are the following:

Question 1

Plot the mean of normalized squared estimation error $\mathbb{E}\left[\frac{|\hat{H}_k - H_k|^2}{|H_k|^2}\right]$ versus the SNR for subcarriers $k = 10$ and $k = 22$, where the SNR range is 0:2:20 dB. To find the mean error, run 10000 Monte Carl trials for each SNR value, calculate $\hat{H}_k$ and the normalized squared estimation error $\frac{|\hat{H}_k - H_k|^2}{|H_k|^2}$ for each trial, and then average the error over all trials.

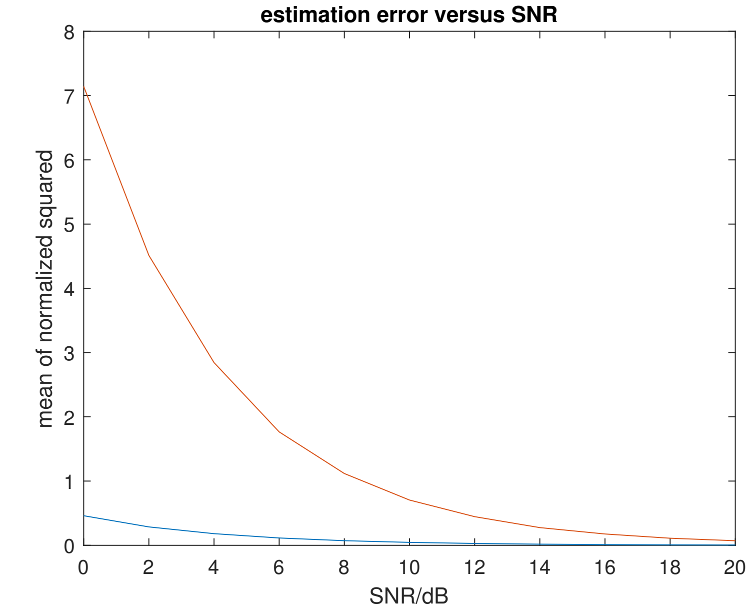

Figure 4: Estimation error versus SNR

Question 2

Which subcarrier, $k = 10$ or $k = 22$, has a higher channel estimation error? Why? (Hint: plot magnitude of 64-point FFT of h to investigate)

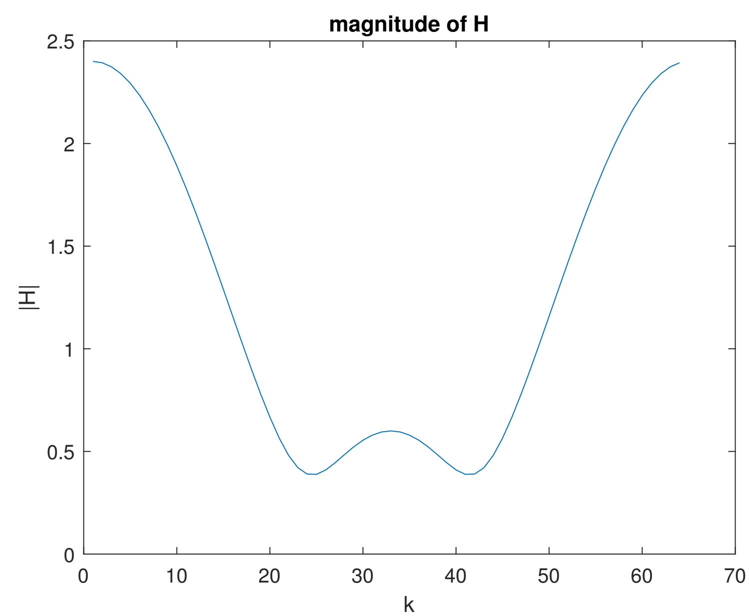

Figure 5: magnitude of H

The subcarrier $k = 22$ will have a higher channel estimation error. Figure 5 shows the magnitude of 64-point

FFT of h. It is clear that $|H_{10}| > |H_{22}|$. What's more, we have the relationship between estimation error and magnitude:

$$Y_k = H_k X_k + W_k \tag{2.1}$$

$$\hat{H}_k = \frac{Y_k}{X_k} = H_k + \frac{W_k}{X_k} \tag{2.2}$$

$$\frac{\left|\hat{H}_k - H_k\right|^2}{|H_k|^2} = \frac{|W_k|^2}{|X_k|^2 |H_k|^2} \tag{2.3}$$

which W_k is the additive white Gaussian noise in the frequency domain, and X_k is the LTF signal. Neither of these two formulas will affect the result. That means H_{22} will have a have a higher channel estimation error because it have a lower magnitude of frequency response function of channel.

Question 3

Use estimates $\hat{H}_k$ from (a) to determine the data symbols transmitted on subcarriers $k = 10$ and $k = 22$ in x_{DATA}. Assume that all subcarriers use 4-QAM modulation. Plot the symbol error rate versus SNR over the range 0:2:20 dB. To find the symbol error rate, run 10000 Monte Carlo trials for each SNR value, generate new 4QAM symbols at all subcarriers in each trial, use the corresponding $\hat{H}_k$ from (a) in each trial to estimate the data symbols, and then find the symbol error rate over all trials.

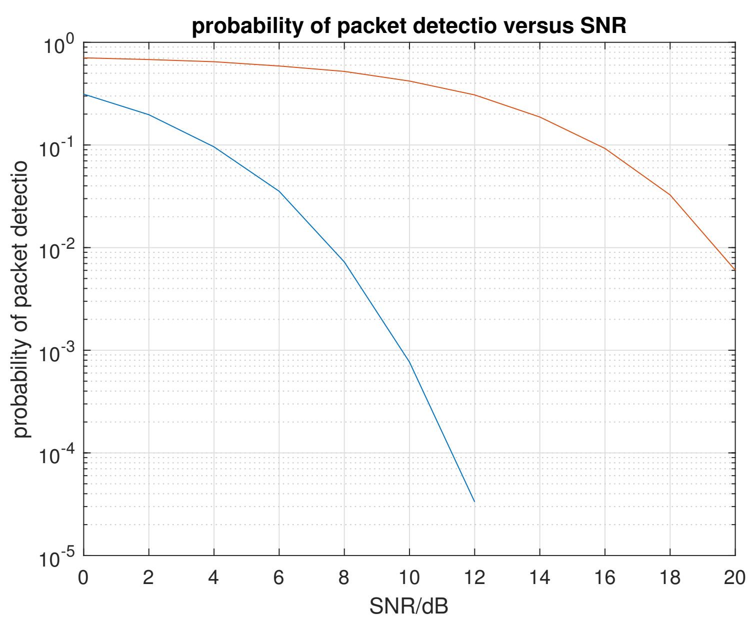

Figure 6: magnitude of h

PART IV: Adaptive loading and power allocation

In this part, we will assume the same AWGN channel with three multipath components as in Part 3, so the channel impulse response is again $h = [1,\ 0.9,\ 0.5]$. Further, we assume static environment, i.e., that the channel does not change.

The modulation types that can be used on the subcarriers in 802.11a are BPSK, 4QAM, 16QAM, and 64QAM. Note that the modulation size can be $M = 2$, 4, 16, or 64. The same modulation must be used for all subcarriers at any given time. For simplicity, we will consider modulation selection for only two subcarriers: $k = 10$ and $k = 22$. We assume that the fracton $\alpha \in [0, 1]$ of power budget is allocated to subcarrier $k = 10$, while the remaining fraction $1 - \alpha$ is allocated to subcarrier $k = 22$.
The modulation order is selected as follows

1. Calculate the spectral efficiency for both subcarriers.

$$\eta_{10} = \log_2\left(1 + \left|\hat{H}_{10}\right|^2 \alpha \mathrm{SNR}\right) \tag{4.1}$$

$$\eta_{22} = \log_2\left(1 + \left|\hat{H}_{22}\right|^2 (1-\alpha)\,\mathrm{SNR}\right) \tag{4.2}$$

2. Select the modulation with the maximum size for which

$$\eta_{10} \geq \log_2(M) \text{ and } \eta_{22} \geq \log_2(M). \tag{4.3}$$

3. If $\eta_{10} < \log_2(M)$ or $\eta_{22} < \log_2(M)$ for every M, the modulation size is set to $M = 1$. Note that $M = 1$ does not represent a proper modulation size, and it is only used to indicate low spectral efficiency and zero data rate for all subcarriers.

Your have the following tasks in this problem:

Question 1

In one figure, plot the mean sum rate over the two subcarriers versus the fraction α for three SNR levels, including SNR = 0 dB, SNR = 10 dB, and SNR = 20 dB. Assuming the code rate of $\frac{3}{4}$, the mean sum rate in Mbps can be calculated as follows

$$R = \mathbb{E}\left[\frac{3}{4}\frac{2\log_2(M)\,\Delta f}{10^6}\right] \tag{1.4}$$

To find the mean sum rate for all three SNR levels, you need to run 10000 Monte Carlo trials for each SNR and α pair, use the corresponding channel estimates $\hat{H}_k$ from Part 3(a) in each trial to calculate the spectral efficiencies, determine the modulation size M, calculate the sum rate, and then average the sum rate over all trials.

Question 1 continued on next page...

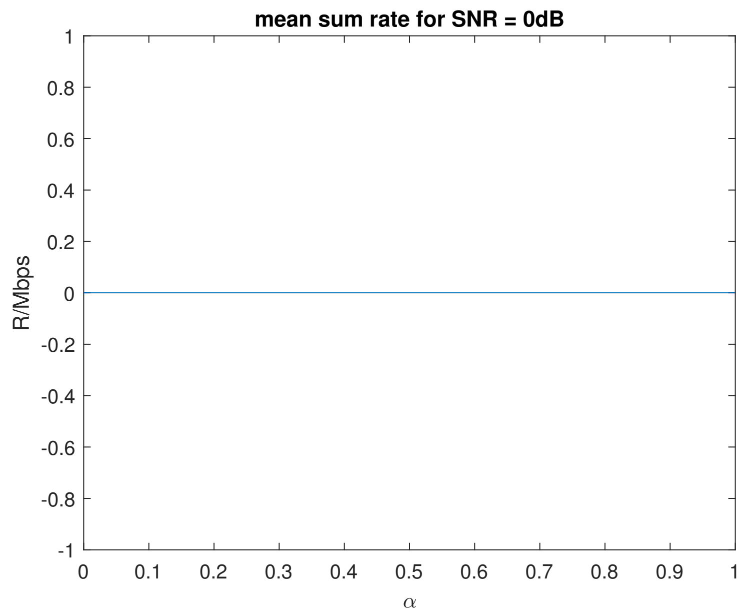

Figure 7: mean sum rate for SNR = 0 dB

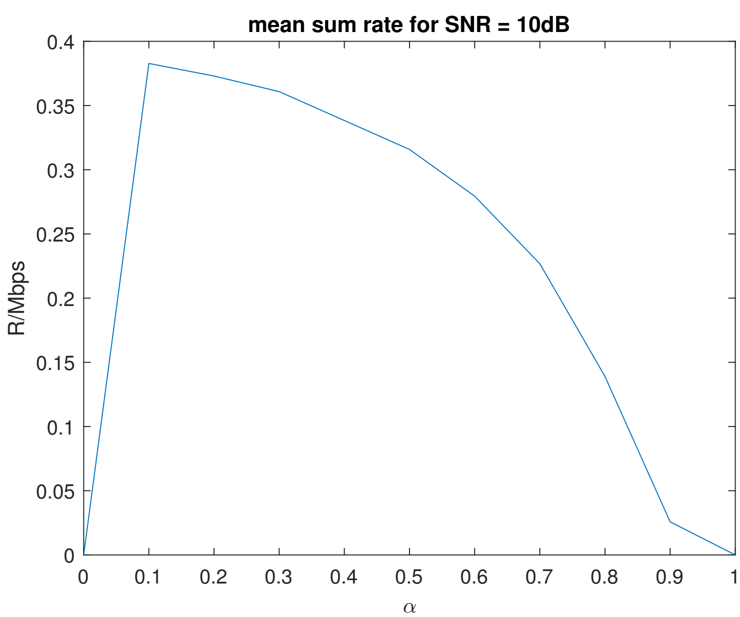

Figure 8: mean sum rate for SNR = 10 dB

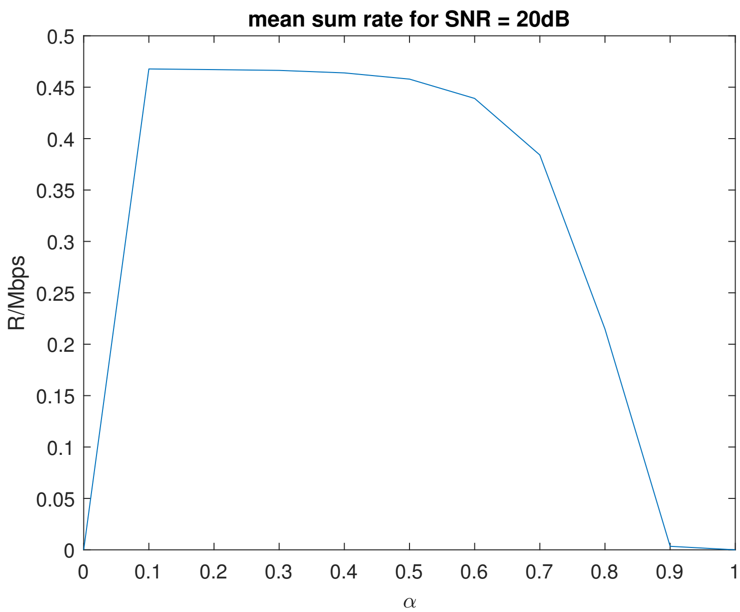

Figure 9: mean sum rate for SNR = 20 dB

Question 2

Assume that you can redesign 802.11a so that the subcarriers can use different modulation types. Let M_k be the modulation size for the subcarrier k. The subcarrier k chooses its maximum M_k based on the corresponding spectral efficiency η_k. If $\eta_k < \log_2(M)$ for every M, then $M_k = 1$. Assuming the code rate of $\frac{3}{4}$, the mean sum rate can be calculated as follows

$$R = \mathbb{E}\left[\frac{3}{4}\frac{\left(\log_2(M_{10}) + \log_2(M_{22})\right)\Delta f}{10^6}\right] \tag{2.1}$$

Create a new figure and plot the mean sum rate versus the fraction α for SNR = 0, 10, 20 dB, as in part (a). To find the mean sum rate, use a similar procedure as in (a).

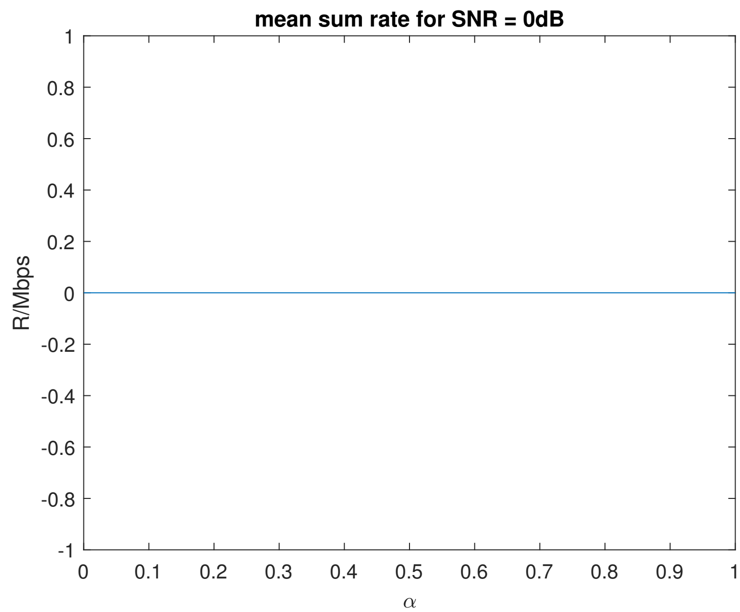

Figure 10: mean sum rate for SNR = 0 dB

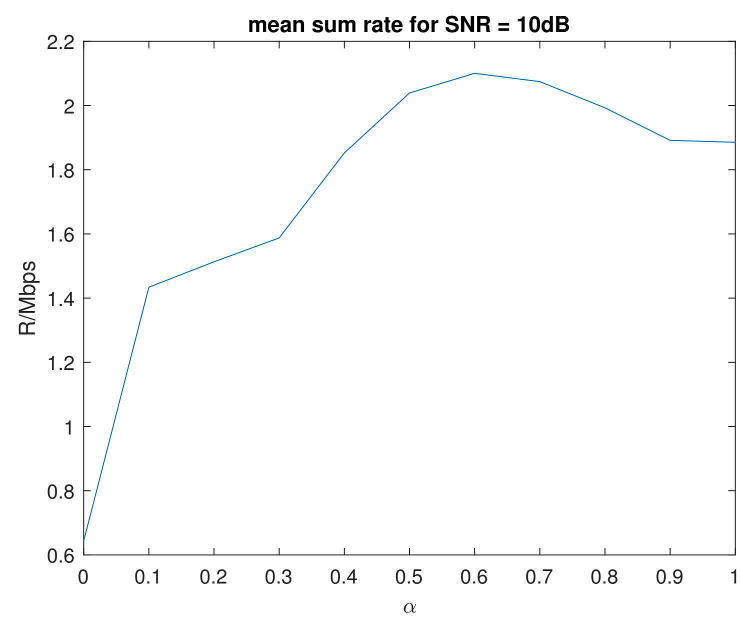

Figure 11: mean sum rate for SNR = 10 dB

Question 2 continued on next page…

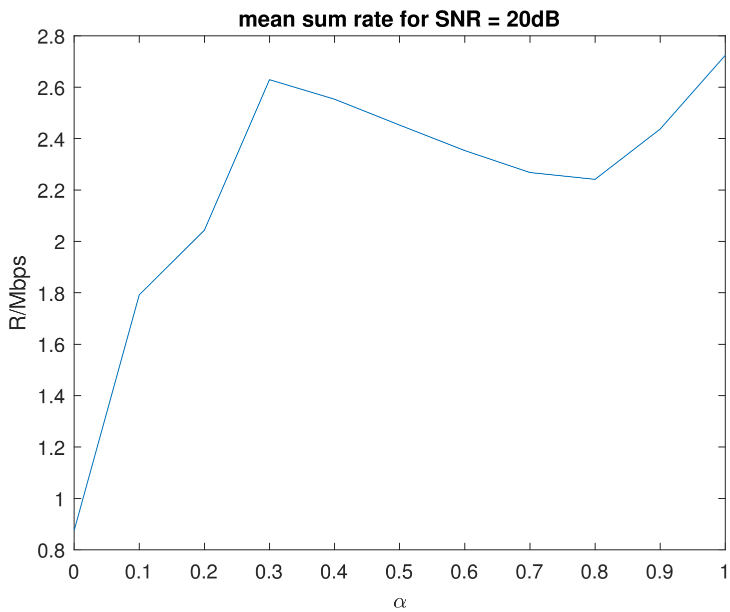

Figure 12: mean sum rate for SNR = 20 dB

Compare and explain the mean sum rates in parts (a) and (b). Find and explain the optimal α for SNR = 20 dB in parts (a) and (b).

It is clear that the mean sum rates in part (b) is higher than part (a). The reason is that the mean sum rates relay on the modulation degree. A higher modulation degree leads to a higher mean sum rate. And the better channel it is, the higher modulation degree could be applied.

As I have already learned in Part 3, the subcarrier $k = 10$ has a better performance that the subcarrier $k = 22$. However, all the subcarriers should use the same modulation, so I have to pay more attention on the weakness according to the "Barrel theory". According to my simulation, $\alpha \in (0, 0.1]$ is the optimal choice. That means the more power assign to the subcarrier $k = 22$, the better performance will be (but not 100%, which will leads to $R = 0$).

And in part (b), the most sensible method is to allocate as much power as possible to $k = 10$ to achieve higher order modulation, so that the optimal α is 1, and the subcarrier $k = 22$ is ignored in such situation. We will give all the recourse to the subcarrier $k = 10$, to let it achieve a high degree of modulation.