1) Order does matter

i.e. putting 's' before 't' is different than 't' before 's'

for 's' and 't', it's 4 choose 2

s, t, | |

stars bars

_ _ _ slots

start w/ s, 3 locations possible

next t, 2 locations available

now 1 location but 26 possible entries

so $3! \cdot 26$

3! ways to place 3 letters in 3 spots

26 options · 1 location

need to account for repeats

if random letter is 's' ...

then 3 repeats

sts

sst

tss

same for if random letter is 't'

so 6 total repeats

$(3! \cdot 26) - 6 = 150$

2)

3 people
2 people w/ 5 options
1 person (Alice) w/ 4 options

addition of 2 stars & bars problems

P1: 2 stars: Bob, Carlos
4 bars ; 5 possible floors (2-6)

$\frac{6!}{2!\,4!}$

P2: 1 star: Alice
3 bars: 4 floor options

$\frac{4!}{3!\,1!} = \frac{4!}{3!} = 4$

so final

$\frac{6!}{2!\,4!} + 4$

Alice: 4 floor options
Bob: 5 floor options
Carlos: 5 floor options

Order does not matter

so $5 \cdot 5 \cdot 4$ options

$= 100$

3.1) 7 water type
5 fire type
9 other type

21 choose 3 gives total # of 3 card combos

$$C(21,3) = \frac{21!}{18!\,3!}$$

subtract all w/ 2 or 3 fire types

$-$ (5 choose 2)$\cdot$(16 choose 1) $\quad 16 \cdot C(5,2)$
$-$ 5 choose 3 $\quad C(5,3) = \frac{5!}{3!\,2!}$

subtract all w/ 2 or 3 water types

$-$ (7 choose 2)$\cdot$(14 choose 1) $\quad 14 \cdot C(7,2)$
$-$ 7 choose 3 $\quad C(7,3) = \frac{7!}{3!\,4!}$

so

$$C(21,3) - 16 \cdot C(5,2) - C(5,3) - 14 \cdot C(7,2) - C(7,3)$$

$$\frac{21!}{18!\,3!} - 16\cdot\frac{5!}{2!\,3!} - \frac{5!}{2!\,3!} - 16\cdot\frac{7!}{2!\,5!} - \frac{7!}{3!\,4!}$$

$$1330 - 160 - 10 - 294 - 35 = 831$$

3.2) stars + bars problem

10 stars: 10 pokemon cards
2 bars: 3 types (water, fire, other)
w | f | o

so $C(12,2) = C(12,10) =$

$$\frac{12!}{10!\,2!} = 66$$

4) total combos:

6 stars: # of copies
3 bars: 4 copy machines

$C(9,6) = C(9,3)$

$$\frac{9!}{6!\,3!}$$

account for all cases where a single copier makes >5 copies (i.e. a single machine makes all 6)

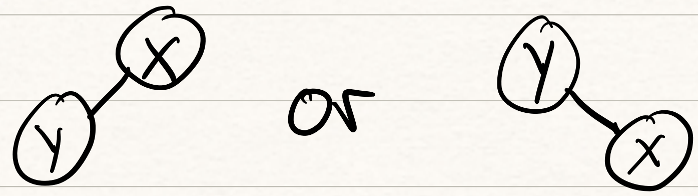
4 ways this is possible
all stars in one section

so

$C(9,6) - 4$

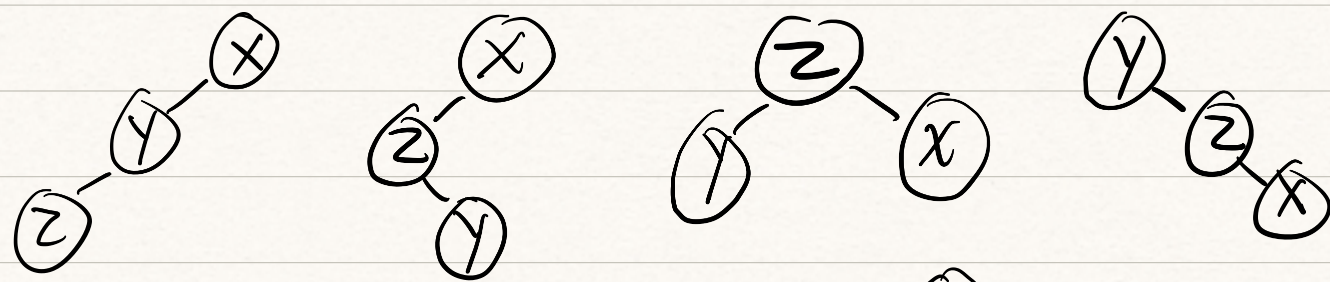
$$\frac{9!}{6!\,3!} - 4 = 84 - 4 = 80$$

5) 2 nodes: $x > y$



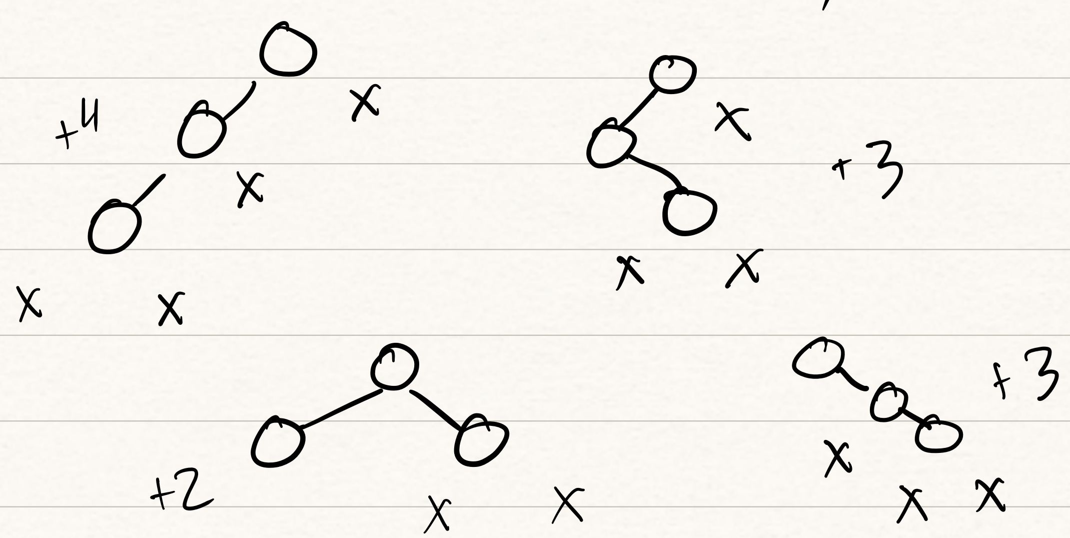
2 combos

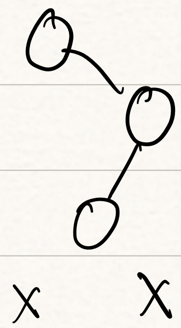
3 nodes: $x > y > z$



5 combos

4 nodes: based of 3, don't add repeats





+2

14 combos

8