Part (a)

data independent

inner for-loop: $T(n) = \sum_{j=0}^{n-1} \theta(1)$

outer: $T(n) = \sum_{i=0}^{k} \theta(1)$

iteration	i	n
1	0	n
2	1	$n-\sqrt{n}$
3	2	$n-2\sqrt{n}$
4	3	$n-3\sqrt{n}$
k	k-1	$n-k\sqrt{n}$

stop @ $i = n-1$

$k-1 = (n-k\sqrt{n})-1$

$k = n-k\sqrt{n}$

$k(1+\sqrt{n}) = n$

$k = \frac{n}{1+\sqrt{n}}$

$$T(n) = \sum_{k=0}^{\frac{n}{1+\sqrt{n}}} \sum_{j=0}^{n-1} \theta(1)$$

$$= \sum_{k=0}^{\frac{n}{1+\sqrt{n}}} \theta(n) \quad = \theta\left(\frac{n^2}{1+\sqrt{n}}\right)$$

Part (b)

Data dependent

$$T(n,i) = \sum_{i=1}^{n} \sum_{k=1}^{n} \left(\theta(1) + O\left(\sum_{k=} \theta(1)\right)\right)$$

no need for O bc it's possible for A[k] to equal i every time

iteration	$m+m$
1	$2m$
2	$4m$
3	$8m$
k	$2^k m$

stop @ $m=n$

$2^k = n$

$k = \log n$

$$T(n) = \sum_{i=1}^{n} \sum_{k=1}^{n} \left(\theta(1) + \sum_{v=1}^{\log n} \theta(1)\right)$$

$$T(n) = \sum_{i=1}^{n} \sum_{k=1}^{n} \left(\theta(1) + \theta(\log n)\right)$$

$$T(n) = \sum_{i=1}^{n} \left(\theta(n) + \theta(n \log n)\right)$$

$$= \theta(n^2) + \theta(n^2 \log n)$$

$$= \theta(n^2 \log n)$$

Part (c)

$$\text{1st: } T(n) = T(n-2) + \theta(1) + T(n-2); \quad T(0) = \theta(1), \; T(1) = \theta(1)$$

return if $n=0$ or $n=1$

recursion called

do something that takes $O(1)$

another reccursion

2nd: $T(n) = \theta(1) + (\theta(1) + T(n-4) + T(n-4)) +$
$(\theta(1) + T(n-4) + T(n-4))$

$= \theta(3) + 4T(n-4)$

3rd: $T(n) = \theta(3) + 4(\theta(1) + 2T(n-6))$
$= \theta(7) + 8\,T(n-6)$

4th: $T(n) = \theta(7) + 8(\theta(1) + 2T(n-8))$
$= \theta(15) + 16T(n-8)$

kth: $T(n) = \theta(2^k - 1) + 2^k\,T(n-2k)$

stop @ $n - 2k = 1$
$k = \frac{n-1}{2}$

$T(n) = \theta\left(2^{\left(\frac{n-1}{2}\right)} - 1\right) + 2^{\frac{n-1}{2}}\,T(0)$

ignore constant bc lower

$\theta(1)$

$T(n) = \theta\left(2^{\left(\frac{n-1}{2}\right)}\right) + \theta\left(2^{\frac{n-1}{2}}\right)$

$= \theta\left(2^{\left(\frac{n-1}{2}\right)}\right)$ or $\theta\left(\frac{\sqrt{2^n}}{\sqrt{2}}\right)$

Part (d)

outer loop: $\sum_{i=0}^{n-1} \theta(1)$ when $i \neq size$

$\theta(1) + \sum_{j=0}^{size-1} \theta(1) + \theta(1)$ when $i = size$

iteration	size
0 - 9	10
10-39	40
40-159	160

size always stays constant, no variable n

we want worst case so assume if happens

size < n bc i < n so otherwise i cannot = size

so assume

$$T(n) = \sum_{i=0}^{n-1} \left(\theta(1) + \sum_{j=0}^{n-1} \theta(1) + \theta(1)\right)$$

$$= \sum_{i=0}^{n-1} \left(\theta(1) + \theta(n) + \theta(1)\right)$$

$$T(n) = \Theta(n) + \Theta(n^2) + \Theta(n)$$

$$= \Theta(n^2)$$