1.1) total ways to pick 3 = $\frac{7!}{4!}$
order matters

ways to get admitted:
e = empathetic, i = innovative, a = analytical
x = anything else

Case 2: choose e, i, x $3! = 6$ arrangements
Case 1: choose ia, x $2! = 2$ arrangements

multiply each case by $C(5,1) = 5$ to get the last letter
$6 \cdot 5 = 30$ $2 \cdot 5 = 10$

subtract 2 duplicates (eia & iae)
$30 + 10 - 2 = 38$ ways to get admitted

probability of getting admitted $= \frac{\text{ways to be admitted}}{\text{ways to select 3 words}}$

$$\frac{38}{\frac{7!}{4!}} = \frac{38 \cdot 4!}{7!} = \frac{19}{105} \approx 18.1\%$$

1.2) Probability of each applicant being accepted

$P = \frac{19}{105}$

$C(10,4) = \frac{10!}{4!6!} = 210$

Bernoulli Trials: 10 trials
4 success

$210 \cdot P^4 \cdot (1-P)^6$

$210 \cdot \left(\frac{19}{105}\right)^4 \cdot \left(\frac{86}{105}\right)^6 \approx 6.8\%$

2.1) $P(A \cap B)$ = probability of 2 consec heads & tail first or last

$2^4 = 16$ possible combos

HHHT HHTT THHH TTHH
THHT

5 instances where both occur

$P(A \cap B) = \frac{5}{16}$

$P(A)$: 2 consec H start @ pos 1-3

$HHxx = 2^2 = 4$ options

$THHx = 2^1 = 2$ options

$xTHH = 2^1 = 2$ options

$\overline{8}$

$P(A) = \frac{8}{16} = \frac{1}{2}$

$P(B) = P(\text{not start/end w/ T}) = 1 - P(\text{start \& end w/ H})$

2^2 permutations for 1st & last

$= 1 - \frac{1}{2^2} = 1 - \frac{1}{4}$ $\quad P(B) = \frac{3}{4}$

$\frac{P(A \cap B)}{P(A)} = \frac{P(A)P(B)}{P(A)} = P(B)$ for independent

$\frac{5/16}{1/2} \simeq \frac{1/2 \cdot 3/4}{1/2} = \frac{3}{4}$

$\frac{10}{16} = \frac{6}{8} = \frac{3}{4}$

$\frac{5}{8} \neq \frac{3}{4} = \frac{3}{4}$

$\therefore$ not independent

2.2) 0 to 3 possible pairs of consecutive heads from 4 flip

permutations of X and Y for 4 letters

$2^4 = 16$

$P(X=0) = 1/2$
$P(X=1) = 5/16$ HHTx, xTHH, THHT
$P(X=2) = 1/8$ THHH, HHHT
$P(X=3) = 1/16$ HHHH

$E(X) = 0 \cdot 1/2 + 1(5/16) + 2(1/8) + 3(1/16)$

$= \frac{5}{16} + \frac{4}{16} + \frac{3}{16} = \frac{12}{16} = \frac{3}{4}$

3) min 1 move (startig guess)

NASTY:

$P(X=1) = 1/5$

*get next NASTY → HASTY
*get next NASTY → MATHS
NASTY → BOARD or HOARD

guess 1/2
50/50 chance you get it right
50/50 chance you get it next try

$2 \cdot \frac{1}{2}$ on guess 2, $2 \cdot \frac{1}{2}$ on guess 3

$P(X=2) = 3/5$
$P(X=3) = 1/5$

$E(X) = 1 \cdot (1/5) + 2(3/5) + 3(1/5) = \frac{1}{5} + \frac{6}{5} + \frac{3}{5} = 10/5$

2 moves

HASTY:

$P(X=1) = 1/5$

HASTY → NASTY
HASTY → MATHS

HASTY → HOARD
HASTY → BOARD

$P(X=2) = 4/5$

$E(X) = 1 \cdot \left(\frac{1}{5}\right) + 2\left(\frac{4}{5}\right) = \frac{1}{5} + \frac{8}{5} = 9/5$

$9/5 < 2$ so

HASTY is better

4.1) 100 people match description

1 guilty, 99 innocent

$\frac{99}{100}$

4.2) 99999 innocent people
99 match description

$$\frac{99}{99999} = \boxed{\frac{11}{11111}}$$

4.3) C(100000, 1000) combos of people investigated

1 − combos where none match

$\approx$

combos where 1+ match

99900 won't match description

so combos where none match

$$\frac{C(999000, 1000)}{C(100000, 1000)}$$

$$\boxed{1 - \frac{C(999000, 1000)}{C(100000, 1000)}}$$

$$\frac{999{,}000 \text{ choose } 1{,}000}{100{,}000 \text{ choose } 1{,}000}$$

5) 30% chance only has the 2

70% chance it has 1 more

↳ can either be rocky or giant

but not giant because 100% chance it would be detected if there

80% chance rocky was not detected

if there is another planet, it will be rocky

P(x) = 3 planets, 2 giant 1 rocky

3/8 GGR, GRG, RGG 2^3 total

$P(X) = 70\% \cdot \frac{3}{8} = \frac{21}{80}$

P(Y) = w/ 2 planets, both giant

1/4 GG 2^2 total

$P(Y) = 30\% \cdot 1/4 + 70\% \cdot 3/8 \cdot 80\%$

$= \frac{57}{200}$

$$P(X|Y) = \frac{P(X \cap Y)}{P(Y)} = \frac{70\% \cdot 3/8 \cdot 80\%}{\frac{57}{200}}$$

$$= \frac{14}{19} \approx 73.7\%$$

$$70\% \cdot 80\% = \frac{14}{25} = 56\%$$