1) $x_1 = (13 \cdot -5 + 7) \bmod 12 = 2$
$x_2 = (13 \cdot 2 + 7) \bmod 12 = 9$
$x_3 = (13 \cdot 9 + 7) \bmod 12 = 4$
$x_4 = (13 \cdot 4 + 7) \bmod 12 = 11$
$x_5 = (13 \cdot 11 + 7) \bmod 12 = 6$

2, 9, 4, 11, 6

2) # of trailing 0s depends on # of $(2 \cdot 5)$s bc thats what creates a trailing 0

$(2 \cdot 5) = 10$ 1 $(2 \cdot 5)$ so 1 0

more factors of 2 than factors of 5 so just count 5s

100! has $\frac{100}{5} = 20$ terms divisible by 5^1

has $\frac{100}{25}$ terms divisible by 5^2

$5^3 > 100$ so stop there

$20+4=24$ factors of 5 in $100!$

so 24 zeros

3) proof by induction

n divisible by 5 if $n \bmod 5 = 0$

BC: $n = 0$

$$0^5 - 5(0)^3 + 4(0) = 0$$

$$0 \bmod 5 = 0 \checkmark$$

Assume: $(n^5 - 5n^3 + 4n) \bmod 5 = 0$

for all $k \geq 0$

$$\left((k+1)^5 - 5(k+1)^3 + 4(k+1)\right) \bmod 5 = 0$$

$$= (k+1)\left((k+1)^4 - 5(k+1)^2 + 4\right)$$

$$= (k+1)\left((k+1)^2 - 4\right)\left((k+1)^2 - 1\right)$$

$$= (k+1)(k-1)(k+3)(k)(k+2)$$

By definition: product of n consecutive integers is divisible by $n!$

have 5 consecutive ints

$k-1, k, k+1, k+2, k+3$

then divisible by $5! = 120$

$\therefore$ divisible by 5 as well

Proven by Induction

4) $1333 \bmod 11 = 2$ so

$1333^{42} \bmod 11 = (1333 \bmod 11)^{42} \bmod 11$

$= 2^{42} \bmod 11 = (2^{21})^2 \bmod 11$

$= (2\cdot(2^{10})^2)^2 \bmod 11 = (2\cdot(((2)^5)^2)^2)^2 \bmod 11$

$= (2\cdot((32)^2)^2)^2$ $\qquad 32 \bmod 11 = 10$

$(2\cdot((10)^2)^2)^2 \bmod 11 = (2\cdot(100)^2)^2 \bmod 11$

$100 \bmod 11 = 1$

$(2\cdot(1)^2)^2 = 2^2 = 4 \bmod 11$ $\qquad 4$

5) $309/112 = 2$ remainder 85

$112/85 = 1$ r 27

$85/27 = 3$ r 4

$27/4 = 6$ r 3

$4/3 = 1$ r 1

$3/1 = 3$ r 0

since remainder $= 0$, 1 is greatest common divisor of 309 and 112

$\therefore$ by definition, 309 & 112 are relatively prime

6) Diophantine Equations 2

$r0\,x + r1\,y = \gcd(r0, r1)$

$r0 = 54 \quad r1 = 16$

$54 = 16 \cdot 3 + 6 \Rightarrow 6 = 54 - 16 \cdot 3 = r_0 - 3r_1$

$16 = 2 \cdot 6 + 4 \Rightarrow 4 = 16 - 2 \cdot 6 = r_1 - 2(r_0 - 3r_1)$

$= r_1 + 6r_1 - 2r_0$

$= 7r_1 - 2r_0$

$6 = 1\cdot 4 + 2 \Rightarrow 2 = 6 - 1\cdot 4 = r_0 - 3r_1 - 1\cdot(7r_1 - 2r_0)$
$= 3r_0 - 10r_1$

$\gcd(54, 16) = 2 = 6 - 1\cdot 4 = 3\cdot 54 - 10(16)$

$x = 3 \quad y = 10$

7) $x = 33 \bmod 112$

$r0 = 112 \quad r1 = 33$

find $\gcd(112, 33)$

$112 = 33\cdot 3 + 13 \Rightarrow 13 = 112 - 33\cdot 3 = r_0 - 3r_1$
$33 = 2\cdot 13 + 7 \Rightarrow 7 = 33 - 2\cdot 13 = r_1 - 2(r_0 - 3r_1)$
$= 7r_1 - 2r_0$
$13 = 1\cdot 7 + 6 \Rightarrow 6 = 13 - 1\cdot 7 = r_0 - 3r_1 - 1\cdot(7r_1 - 2r_0)$
$= 3r_0 - 10r_1$

$7 = 6\cdot 1 + 1 \Rightarrow 1 = 7 - 6\cdot 1 = 7r_1 - 2r_0 - 1(3r_0 - 10r_1)$
$= 17r_1 - 5r_0$

$1 = 17(33) - 5(112)$

$x = 17$