Two-body scattering

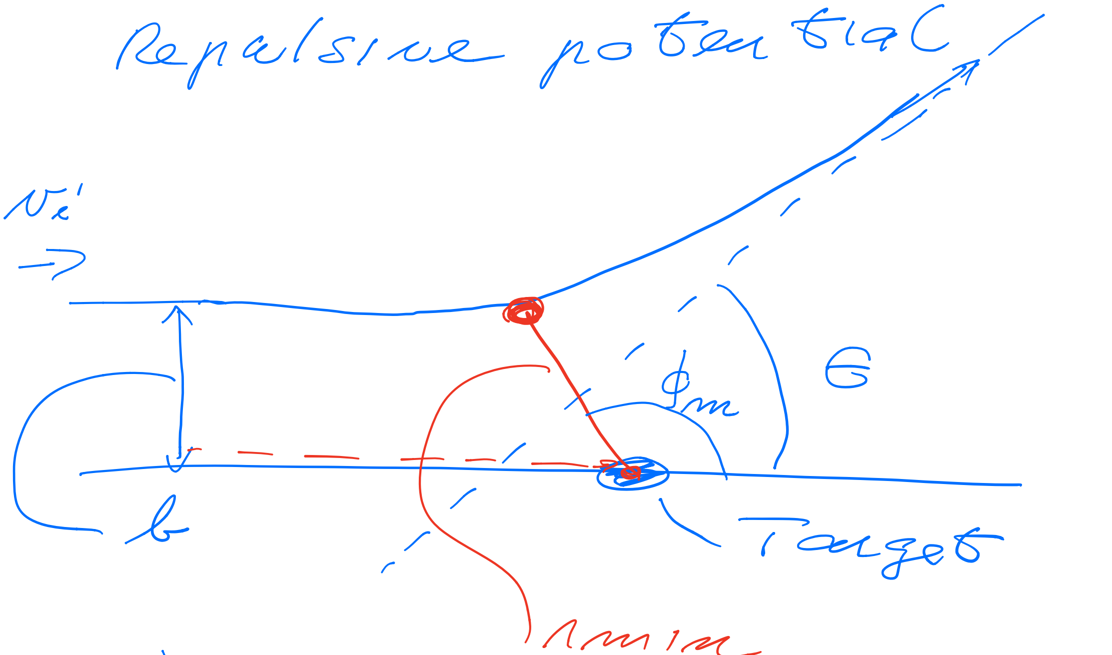
scattering angle θ

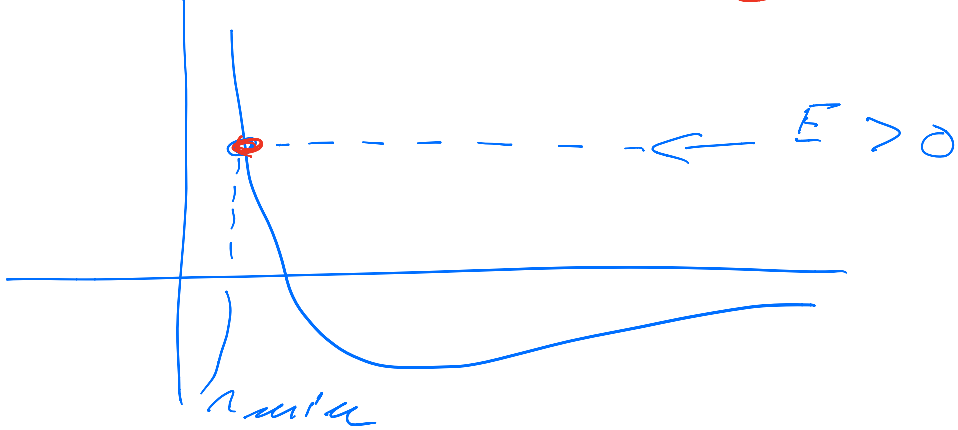
differential cross-section

$$\frac{d\sigma}{d\Omega} = \frac{b}{\sin\theta}\left|\frac{db}{d\theta}\right|$$

impact parameter - b -

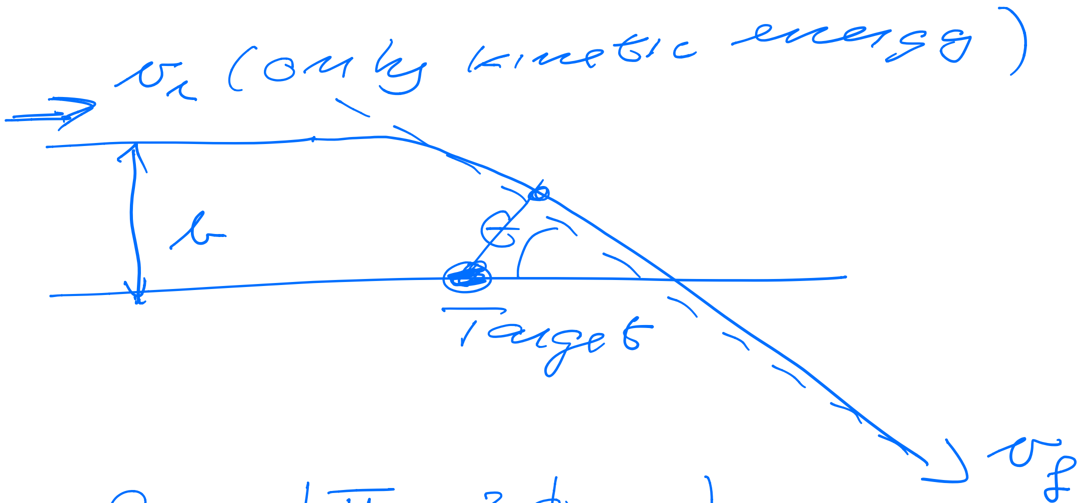
Repulsive potential





potential

an attractive potential



$$\Theta = |\pi - 2\phi_m|$$

$$V(r) = -\frac{\alpha}{r}$$

$$r(\phi) = \frac{1}{\frac{\mu\alpha}{L^2} + A\cos\phi} = \frac{C}{1 + \varepsilon\cos\phi}$$

$$\varepsilon = \frac{AL^2}{\mu\alpha}$$

Θ is defined when $r(\phi) \to \infty$

This happens

$$\underline{\text{if} \quad 1 + \varepsilon\cos\phi' = 0}$$

$$\frac{d\sigma}{d\Omega} = \frac{b}{\sin\Theta}\left|\frac{db}{d\Theta}\right|$$

$$\boxed{\phi' = \phi_m}$$

$$\varepsilon \cos\phi_m = -1$$

$$\cos\phi_m = -\frac{\mu\alpha}{AL^2}$$

electrostatic

$$\alpha = \frac{q_1 q_2}{4\pi\varepsilon_0}$$

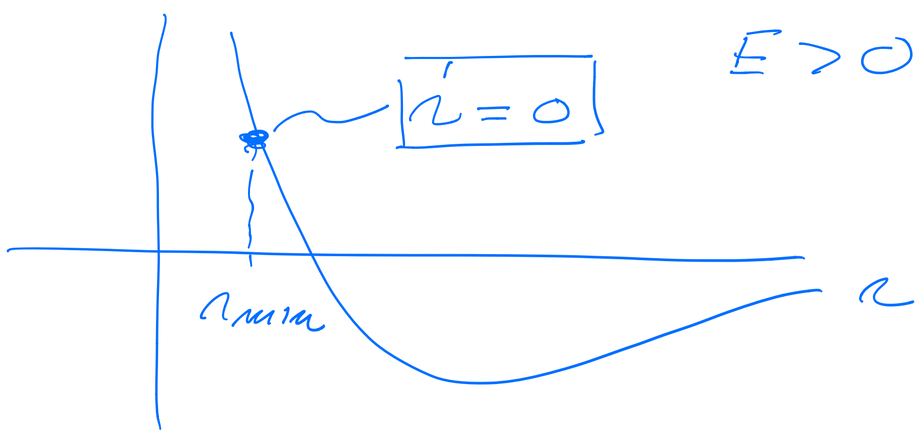
$$\Theta = \pi - 2\phi_m$$

$$\frac{\Theta}{2} = \frac{\pi}{2} - \phi_m$$

$$\sin\frac{\Theta}{2} = -\cos\phi_m = \frac{\mu\alpha}{AL^2}$$

$$\boxed{\sin\frac{\Theta}{2} = \frac{\mu\alpha}{AL^2}} \qquad \frac{d\sigma}{d\Omega} = \frac{b}{\sin\Theta}\left|\frac{db}{d\Theta}\right|$$

Want b in terms of μ, α, A, L^2



$E > 0$ (kinetic energy)

$$E = \frac{1}{2}\dot{r}^2\mu + \frac{1}{2}\mu(r\dot{\phi})^2 + V(r)$$

$$= -\frac{\alpha}{r_{min}} + \frac{L^2}{2\mu r_{min}^2}$$

quadratic equation r_{min}

$$\frac{1}{r_{min}} = \frac{\mu\alpha}{L^2} + \sqrt{\left(\frac{\mu\alpha}{L^2}\right)^2 + \frac{2\mu E}{L^2}}$$

$$r(\phi) = \frac{c}{1+\varepsilon\cos\phi}$$

$$r_{min} = \frac{c}{1+\varepsilon} \qquad \phi = 0$$

$$\frac{1}{r_{min}} = \frac{\mu\alpha}{L^2} + A$$

$$A = \sqrt{\left(\frac{\mu\alpha}{L^2}\right)^2 + \frac{2\mu E}{L^2}}$$

How to introduce b in A?

$$L = \mu r^2\dot{\phi} = \mu r v = \mu \cdot b \cdot v$$

$$L^2 = \mu^2 b^2 v^2 \qquad E = \frac{1}{2}\mu v^2$$

$$= 2\mu b^2 E$$

$$b^2 = \frac{L^2}{2\mu E}$$

$$A = \sqrt{\left(\frac{\mu\alpha}{L^2}\right)^2 + \frac{1}{b^2}}$$

$$\boxed{\sin\frac{\theta}{2} = \frac{\mu\alpha}{A L^2}} \qquad A(b)$$

$$-\frac{d\sigma}{d\Omega} = \frac{b}{\sin\theta}\left|\frac{db}{d\theta}\right|$$

$$\boxed{\frac{d\sigma}{d\Omega} = \frac{a^2}{4\sin^4\theta/2}}$$

$$a = \frac{\alpha}{2E}$$

$$\sqrt{(x-a)^2+y^2} + \sqrt{(x+a)^2+y^2} = 2D$$

$$4D^2 = 2(x^2+y^2) + 2a^2 + \left[(x-a)^2+y^2\right]^{1/2}\left[(x+a)^2+y^2\right]^{1/2}$$

$$= x^2\left[4D^2 - 4a^2\right] + y^2 4D^2 = 4(D^2-a^2)D^2$$

$$/\,4(D^2-a^2)D^2$$

$$\frac{x^2}{D^2} + \frac{y^2}{D^2-a^2} = 1$$

$$r(\phi) = \frac{c}{1+\varepsilon \cos\phi}$$

$$r(1+\varepsilon\cos\phi) = c$$

$\varepsilon = 0$ $\quad \varepsilon = \underline{1}$ $\quad x = r\cdot\cos\phi$

$$r + x = c$$

$$r = c - x \qquad r^2 = x^2 + y^2$$

$$y^2 = c^2 - 2cx$$