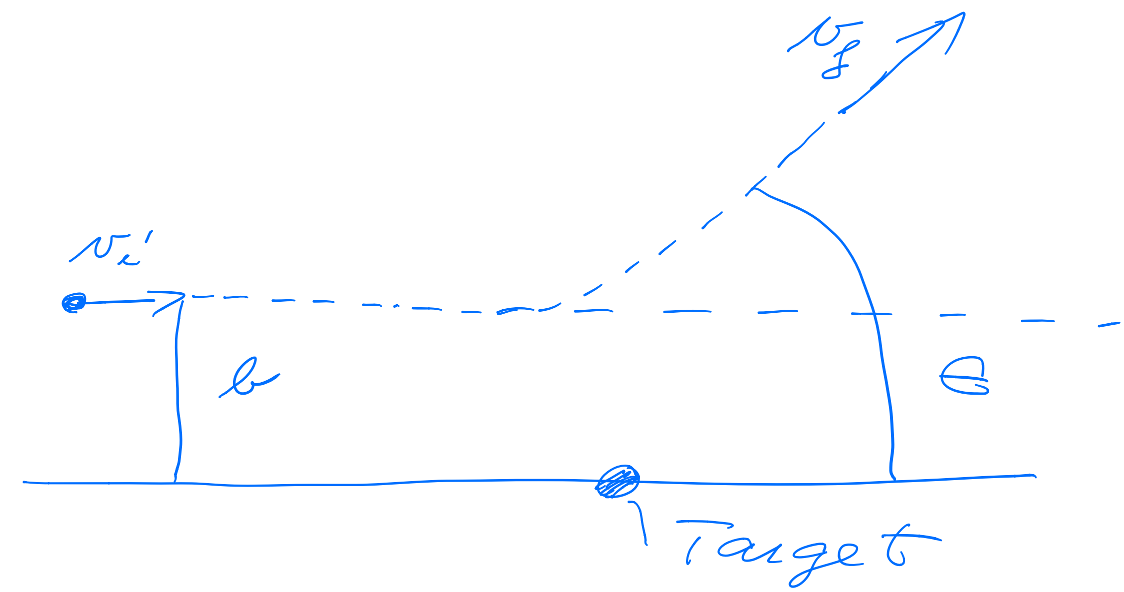
Two-body scattering with central forces (Taylor chap 14)

Two important parameters

- scattering angle θ
- impact parameter $-b-$

How can we relate what we have done till now with our "experimental" quantity like θ and the cross section and $-b-$?



1) scattering angle θ: angle between incoming and outgoing velocities

2) b - impact parameter perpendicular distance

from the incoming straight-line axis to a parallel axis through the center of the target.

$b = 0$, means we have a head-on collision
$\theta = \pi$

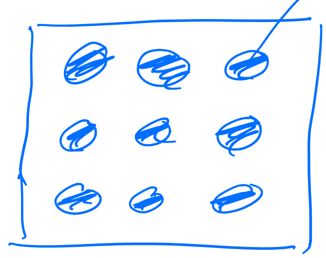
b very large, then θ will be very small.

later $L^2 = (\mu \cdot v)^2 b^2$

$\theta(b)$ or $b(\theta)$

Collision cross sectional

Target



crossectional area or just cross section

~ total area A

Target density
$\rho_t \longrightarrow$ # target per area

targets $= \rho_t \cdot A$

Likelihood of making a hit

cross sectional area

$$\sigma = \pi \cdot R^2$$

Total area of all targets

$$= \sigma \cdot \rho_t \cdot A$$

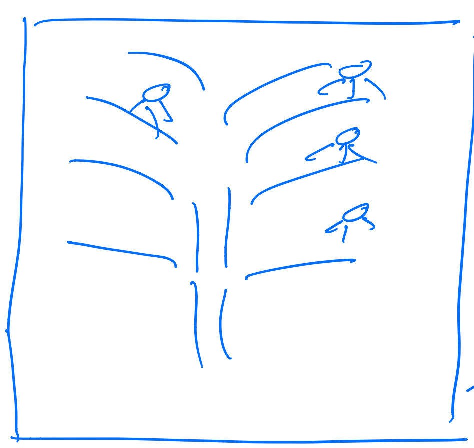
probability of hit =

$$\frac{\text{area of occupied targets}}{\text{total Area}}$$

$$= \frac{\rho_t \cdot A \cdot \sigma}{A} = \rho_t \cdot \sigma$$

$$N_{scatter} = N_{incoming} \cdot \rho_t \cdot \sigma$$

<u>Example</u>



oak Tree

$A = 150\,ft^2$

50 pigeons, each with

$$\sigma = \tfrac{1}{2}\, ft^2$$

Fire 60 bullets at random

How pigeons will we hit?

$$\rho_{pigeons} = \frac{50}{150} = \tfrac{1}{3}\, ft^{-2}$$

$$N_{inc} = 60$$

$$N_{hit} = N_{inc} \cdot \rho_{pigeons} \cdot \sigma$$

$$= 60 \times \left(\tfrac{1}{3}\, ft^{-2}\right) \times \left(\tfrac{1}{2}\, ft^{2}\right)$$

$$= 10 \text{ pigeons}$$

Differential cross-section

$$\frac{d\sigma}{d\Omega} = \frac{b}{\sin\theta} \left| \frac{db}{d\theta} \right|$$

$$\sigma = \int \frac{d\sigma}{d\Omega}\, d\Omega =$$

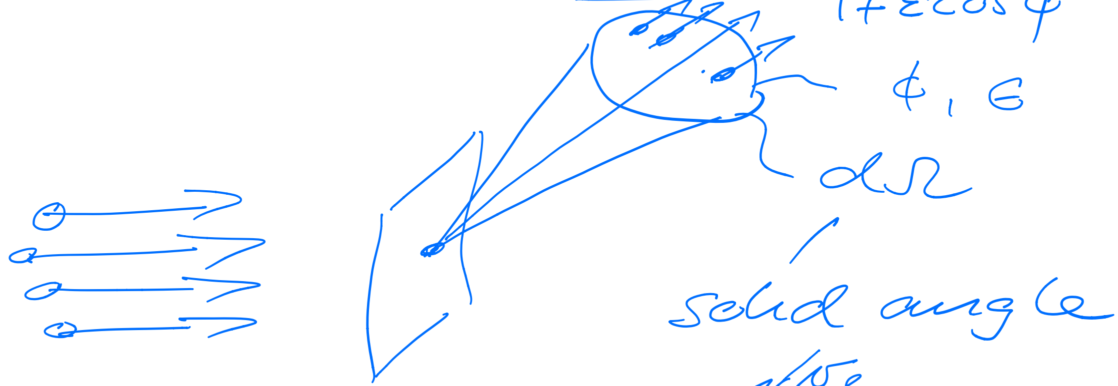
$$\int_0^{\pi} \int_0^{2\pi} \frac{d\sigma}{d\Omega}(\theta,\phi)\, d\phi \cdot d(\cos\theta)$$

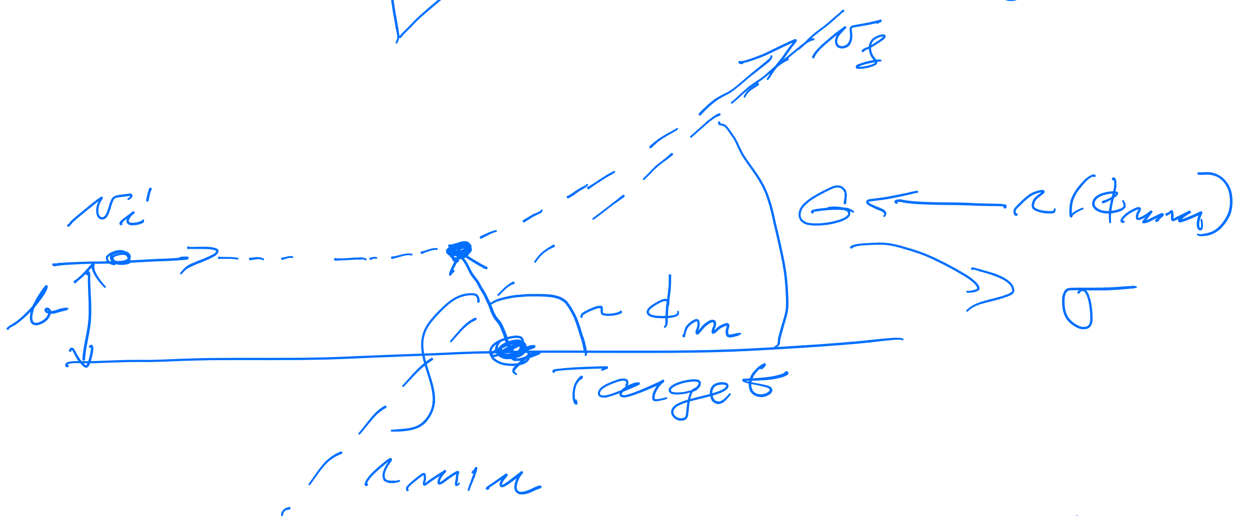
$$= \int_0^{\pi} \sin\theta\, d\theta \int_0^{2\pi} d\phi\, \frac{d\sigma}{d\Omega}(\theta,\phi)$$

How do we link this to

something like $r(\phi')$?

$F = \pm \alpha / r^2$ $\quad r(\phi) = \dfrac{c}{1+\varepsilon\cos\phi'}$





How to relate θ to ϕ_m