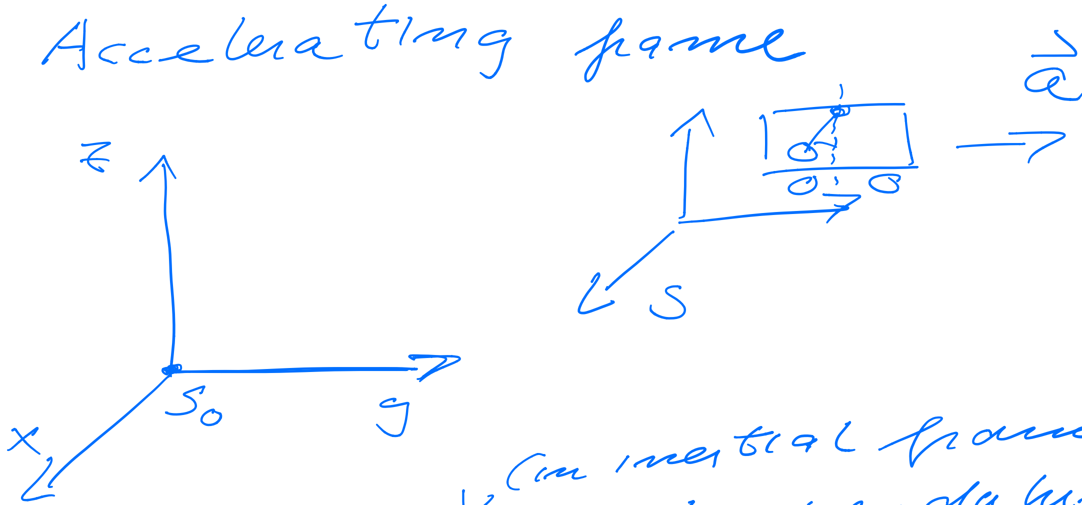
Accelerating frame



Two forces (in inertial frame) on the pendulum

$$\vec{F} = \vec{T} + m\vec{g}$$

Tension

choose to work in non-inertial

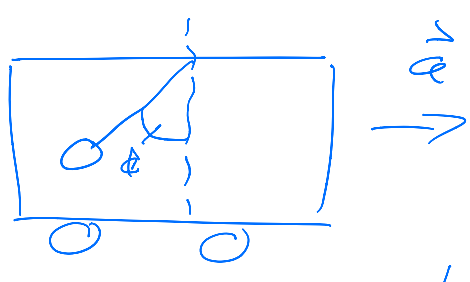
we need to add $-m\vec{a}$

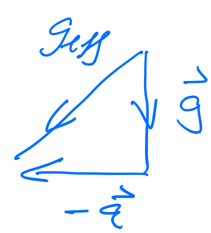
(Taylor's text 9.1-9.2, 9.3-9.4)

$$m\ddot{\vec{r}}' = \vec{T} + m\vec{g} - m\vec{a}$$

$$= \vec{T} + m\vec{g}_{eff}$$

$$\vec{g}_{eff} = \vec{g} - \vec{a}$$

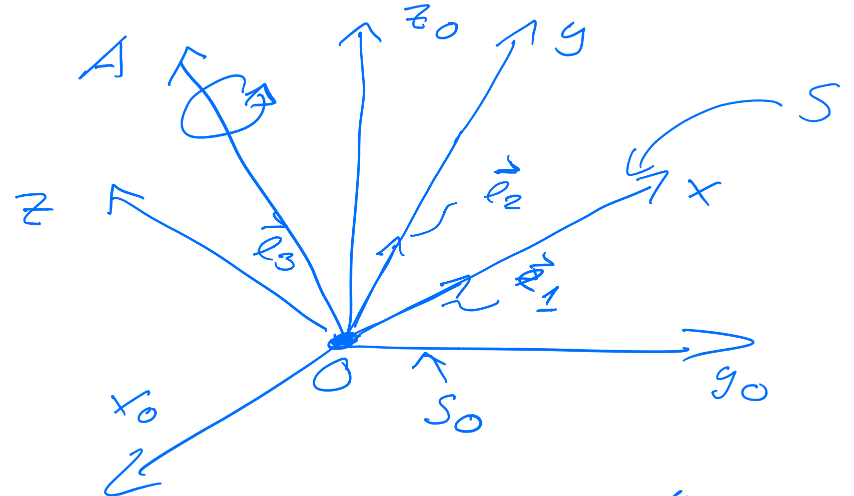




$$m\ddot{\vec{r}}' = 0 \implies \phi = \tan^{-1}(a/g)$$

Rotating

Euler's theorem: The general displacement of a rigid body with one point fixed is a rotation about some axis.



Consider an arbitrary vector $\vec{A}$ (position, force, ...)

$$\vec{A} = A_1 \vec{e}_1 + A_2 \vec{e}_2 + A_3 \vec{e}_3 = \sum_{i=1}^{3} A_i \vec{e}_i$$

Need to find relation between the derivative wrt time of $\vec{A}$ as measured in S_0 to the corresponding derivative in S

$$\left[\frac{d\vec{A}}{dt}\right]_{S_0} = \text{rate of change in } S_0$$

$$\left[\frac{dA}{dt}\right]_S = \text{rate of change of } \vec{A} \text{ relative to the rotating frame } S$$

in S:

$$\vec{A} = \sum_{i=1}^{3} A_i \vec{e}_i$$

in S, $\vec{e}_i$ are fixed.
For an observer in S_0, these vectors are rotating.

in S, $\vec{e}_i$ are constants:

$$\left[\frac{d\vec{A}}{dt}\right]_S = \sum_i \frac{dA_i}{dt} \vec{e}_i$$

Seen from S_0

$$\left[\frac{d\vec{A}}{dt}\right]_{S_0} = \sum_i \frac{dA_i}{dt} \vec{e}_i + \sum_i A_i \left[\frac{d\vec{e}_i}{dt}\right]_{S_0}$$

<u>Later</u> (9.3 Taylor)

$$\left[\frac{d\vec{e}_i}{dt}\right]_{S_0} = \vec{\Omega} \times \vec{e}_i$$

angular velocity

$$\sum_i A_i \left[\frac{d\vec{e}_i}{dt}\right]_{S_0} = \sum_i A_i \left[\vec{\Omega} \times \vec{e}_i\right]$$

$$= \vec{\Omega} \times \sum A_i \vec{e}_i = \hat{\vec{\Omega}} \times \vec{A}$$

$$\boxed{\left[\frac{d\vec{A}}{dt}\right]_{S_0}^{\vec{v}_{S_0}} = \left[\frac{d\vec{A}}{dt}\right]_S + \vec{\Omega} \times \vec{A}}$$

any vector $\vec{A}$ measured in S_0 to the corresponding derivative in a rotating frame S

$$\vec{A} \Rightarrow \vec{r} \qquad \frac{d\vec{r}}{dt} = \vec{v}$$

$$- \left[\frac{d\vec{r}}{dt}\right]_{S_0} = \left[\frac{d\vec{r}}{dt}\right]_S + \vec{\Omega} \times \vec{r}$$

$$\vec{A} = \vec{\Omega}$$

$$\boxed{\left[\frac{d\vec{\Omega}}{dt}\right]_{S_0} = \left[\frac{d\vec{\Omega}}{dt}\right]_S} + \underbrace{\vec{\Omega} \times \vec{\Omega}}_{=0}$$

$$\frac{d\vec{\Omega}}{dt} = \dot{\vec{\Omega}}$$

$$\left[\frac{d\vec{r}}{dt}\right]_{S_0} = \vec{v}_{S_0} \qquad \left[\frac{d\vec{r}}{dt}\right]_S = \vec{v}_S$$

$$\underline{\vec{v}_{S_0} = \vec{v}_S + \vec{\Omega}\times\vec{r}}$$

$$\left[\frac{d^2\vec{r}}{dt^2}\right]_{S_0} = \left[\frac{d}{dt}\right]_{S_0} \underbrace{\left[\frac{d\vec{r}}{dt}\right]_{S_0}}_{\vec{v}_{S_0}}$$

$$= \left[\frac{d}{dt}\right]_{S_0} \underbrace{\left[\left[\frac{d\vec{r}}{dt}\right]_S + \vec{\Omega}\times\vec{r}\right]}_{\vec{v}_{S_0}}$$

$$\left[\frac{d\vec{v}_{S_0}}{dt}\right]_{S_0} = \left[\left[\frac{d\vec{v}_{S_0}}{dt}\right]_S + \vec{\Omega}\times\vec{v}_{S_0}\right]$$

$$= \left[\frac{d}{dt}\right]_S \left[\left[\frac{d\vec{r}}{dt}\right]_S + \vec{\Omega}\times\vec{r}\right]$$

$$+ \vec{\Omega}\times\left[\underbrace{\left[\frac{d\vec{r}}{dt}\right]_S}_{v_S} + \vec{\Omega}\times\vec{r}\right]$$

$$= \left[\frac{d}{dt}\vec{v}_S\right]_S + \left[\frac{d\vec{\Omega}}{dt}\right]_S$$

$$+ \vec{\Omega}\times\left[\frac{dr}{dt}\right]_S + \vec{\Omega}\times\vec{v}_S$$

$$\underbrace{\qquad}_{\vec{v}_s}$$

$$+\ \vec{\Omega}\times(\vec{\Omega}\times\vec{r})$$

$$\left[\frac{d\vec{v}_{so}}{dt}\right]_{so} = \vec{a}_{so} = \underbrace{\left[\frac{d\vec{v}_s}{dt}\right]_s}_{\vec{a}_s} + \dot{\vec{\Omega}}\times\vec{r} + 2\vec{\Omega}\times\vec{v}_s + \vec{\Omega}\times(\vec{\Omega}\times\vec{r})$$

$$\vec{a}_{so} = \vec{a}_s + \dot{\vec{\Omega}}\times\vec{r} + 2\vec{\Omega}\times\vec{v}_s + \vec{\Omega}\times(\vec{\Omega}\times\vec{r})$$

$$m\vec{a}_s = \underbrace{\vec{F}}_{m\vec{a}_{so}} + \underbrace{m\,\dot{\vec{\Omega}}\times\vec{r}}_{\neq 0} + \underbrace{2m\,\vec{v}_s\times\vec{\Omega}}_{\text{coriolis}} + \underbrace{m(\vec{\Omega}\times\vec{r})\times\vec{\Omega}}_{\text{centrifuga force}}$$