Defined effective potential

$$V_{eff}(r) = V(r) + \frac{L^2}{2\mu r^2}$$

$$E = \underbrace{\frac{1}{2}\mu \dot{r}^2 + \frac{L^2}{2\mu r^2}}_{\text{kinetic energy.}} + V(r)$$

HW 1 $\quad V(r) = \frac{1}{2}k(x^2+y^2) = \frac{1}{2}k r^2$

$$V(r) = -\alpha/r \qquad r = |\vec{r}_1 - \vec{r}_2|$$

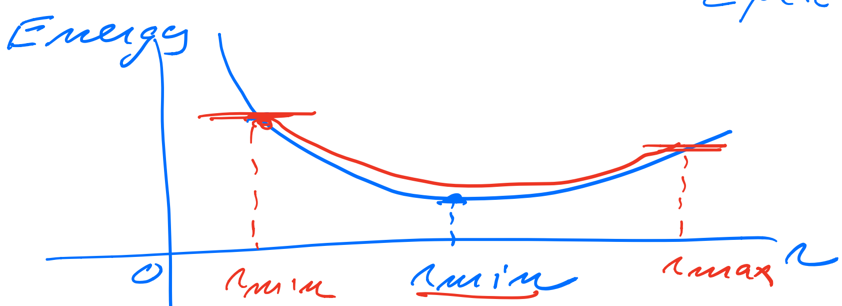
$$\alpha = G m_1 m_2$$

$$\alpha = \gamma q_1 q_2$$

$$V(r) = \pm \beta r$$

$$V(r) = \delta / r^2$$

$$V_{eff}(r) = \frac{1}{2} r^2 k + \frac{L^2}{2\mu r^2}$$



$r \in [0, \infty)$

$x \in (-\infty, \infty)$

$y \in (-\infty, +\infty)$

$$\frac{d V_{eff}(r)}{dr} = 0$$

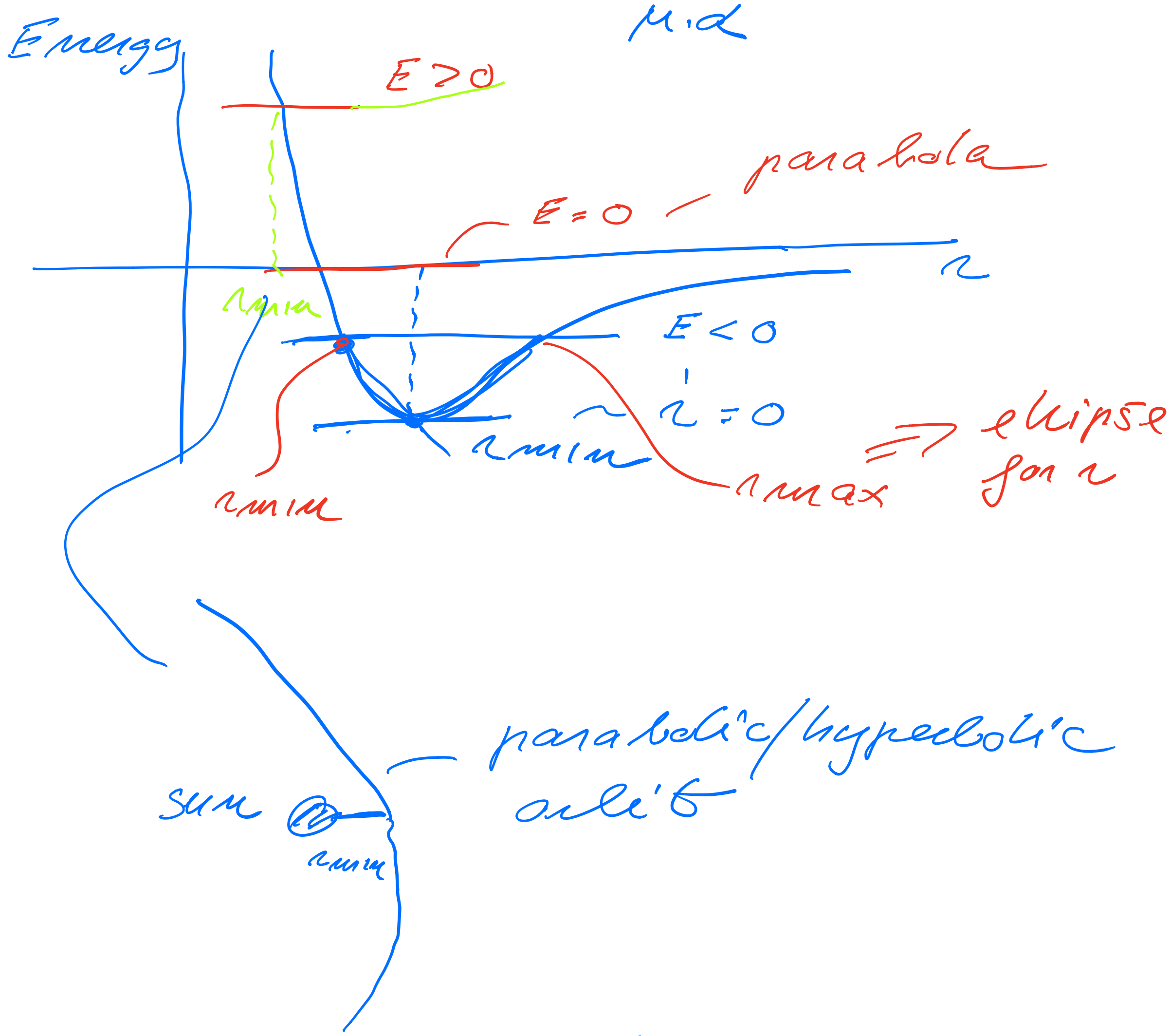
$$rk - \frac{L^2}{\mu r^3} = 0 \Rightarrow$$

$$r_{min} = \left(\frac{L^2}{\quad}\right)^{1/4}$$

$$V(r) = -\alpha/r \quad \left(\mu k\right) \quad + \frac{L^2}{2\mu r^2}$$

$$\frac{dV_{eff}}{dr} = \alpha/r^2 - \frac{L^2}{\mu r^3} = 0$$

$$\Rightarrow r_{min} = \frac{L^2}{\mu \cdot \alpha}$$



Analytical solution of radial motion:

$$\boxed{\mu \ddot{r} = -\frac{dV_{eff}}{dr}} \qquad F = -\alpha/r^2$$

$$= -\alpha/r^2 + \frac{L^2}{\mu^2 r^3}$$

I want $r(\phi)$

$$\frac{dr}{dt} = \frac{dr}{d\phi}\frac{d\phi}{dt} = \frac{dr}{d\phi}\underbrace{\dot\phi}_{\frac{L}{\mu r^2}}$$

$$= \frac{dr}{d\phi}\frac{L}{\mu r^2}$$

$$\ddot r = \frac{d^2 r}{dt^2} = \frac{d^2 r}{d\phi^2}\dot\phi^2 + \frac{dr}{d\phi}\left[\frac{d}{dr}\frac{L}{\mu r^2}\right]\dot r$$

$$= \frac{d^2 r}{d\phi^2}\left[\frac{L}{\mu r^2}\right]^2 - \frac{2}{r}\left[\frac{dr}{d\phi}\right]^2\left[\frac{L}{\mu r^2}\right]^2$$

$$= -\frac{\alpha}{r^2\mu} + \frac{L^2}{\mu^2 r^3}$$

This works for all types of radial equations

$F(r)/\mu$

$$r = \frac{1}{u} \qquad \frac{dr}{d\phi} = -\frac{1}{u^2}\frac{du}{d\phi}$$

$$\frac{d^2 r}{d\phi^2} = \frac{2}{u^3}\left[\frac{du}{d\phi}\right]^2 - \frac{1}{u^2}\frac{d^2 u}{d\phi^2}$$

After some algebra:

$$\frac{d^2u}{d\phi^2} = -u - \frac{F\mu}{L^2u^2}$$

$$F = -\alpha/r^2 = -\alpha u^2$$

$$\boxed{\frac{d^2u}{d\phi^2} = -u + \frac{\mu\alpha}{L^2}}$$

$$L = \mu \cdot r^2 \cdot \dot{\phi} = \mu \cdot \vec{r} \times \dot{\vec{r}}$$

$$\dot{\phi} = \frac{L}{\mu \cdot r^2} \qquad x = r \cdot \cos\phi$$
$$y = r \cdot \sin\phi$$

$$\alpha = 0 \implies F = 0$$

$$\boxed{\frac{d^2u}{d\phi^2} = -u} \quad - A\cos\phi + B\sin\phi$$

$$u(\phi) = A\cos(\phi - \delta)$$

$$r(\phi) = \frac{1}{u(\phi)} = \frac{1}{A\cos(\phi - \delta)}$$

(in polar coordinates this is a straight line)

define $w(\phi) = u(\phi) - \frac{\alpha\mu}{L^2}$

$$\frac{d^2w}{d\phi^2} = -w(\phi)$$

$d\phi$ –

$$w(\phi) = A \cdot \cos(\phi) \Rightarrow$$

$$\underline{u(\phi)} = \frac{\alpha \cdot \mu}{L^2} + A\cos\phi$$

$$= \frac{\alpha\mu}{L^2}(1 + \varepsilon\cos\phi)$$

$$c = \frac{L^2}{\mu\alpha}$$

$$\frac{1}{r(\phi)} = \frac{1}{c}(1 + \varepsilon\cos\phi)$$

$$\boxed{r(\phi) = \frac{c}{1 + \varepsilon\cos\phi}}$$

$$\ddot{r} = -\frac{dV_{eff}}{dr}/\mu$$

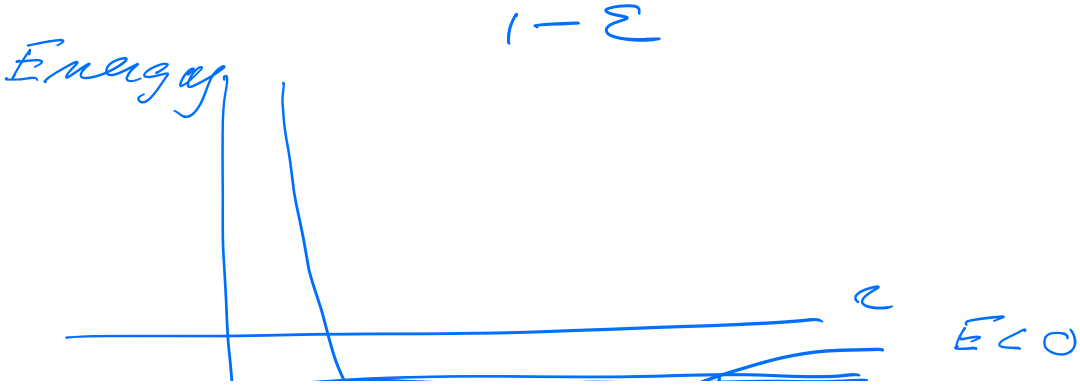
r_{min} $\phi = 0$

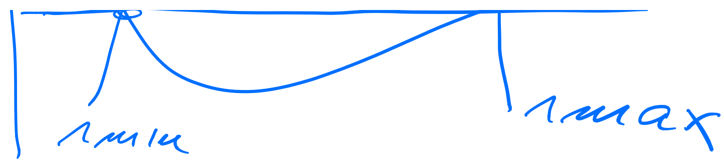
$$r_{min} = \frac{c}{1+\varepsilon}$$

$$\frac{c}{1+\varepsilon}$$

r_{max} $\phi = \pm\pi$

$$r_{max} = \frac{c}{1-\varepsilon}$$





Taylor <u>8.4</u>