PHY321: Classical Mechanics 1

First mditerm project, due Wednesday March 11

Feb 25, 2020

Practicalities about homeworks and projects.

1. You can work in groups (optimal groups are often 2-3 people) or by yourself. If you work as a group you can hand in one answer only if you wish. **Remember to write your name(s)**!
2. How do I(we) hand in? You can hand in the paper and pencil exercises as a hand-written document. Alternatively, you can hand in everyhting (if you are ok with typing mathematical formulae using say Latex) as a jupyter notebook at D2L. The numerical part should always be handed in as a jupyter notebook by the deadline at D2L.

Introduction to the first midterm project. The relevant reading background is

1. chapters 2-5 of Taylor (there are many good examples there)
2. chapters 6-14 of Malthe-Sørenssen.

In this midterm project we will start with a potential similar to the one we discussed in exercise 4 in homework 6. There are some elements of that exercise which are repeated here, similarly, a good fraction of the codes you have developed for homeworks 4-6 can be used here. We start with a one-dimensional potential and motion. Thereafter we extend this potential model to a two-dimensional model and study the numerical solution of the corresponding problem.

Particle in a potential. We consider a particle of mass m moving in a one-dimensional potential,

$$V(x) = \frac{V_0}{d^4}\left(x^4 - 2x^2d^2 + d^4\right).$$

We will assume all other forces on the particle are small in comparison, and neglect them in our model. The parameters V_0 and d are known constants.

- Midterm-1a (5pt) Plot the potential and find the equilibrium points (stable and unstable) by requiring that the first derivative of the potential is zero. Make an energy diagram (see for example Malthe-Sørenssen chapter 11.3) and mark the equilibrium points on the diagram and characterize their stability. The position of the particle is x.
- Midterm-1b (5pt) Choose two different energies that give two distinct types of motions, draw them into the energy diagram, and describe the motion in each case.
- Midterm-1c (5pt) If the particle starts at rest at $x = 2d$, what is the velocity of the atom at the point $x = d$?
- Midterm-1d (5pt) If the particle starts at rest at $x = d$ with velocity v_0, how large must v_0 be for the particle to reach the point $x = -d$?
- Midterm-1e (5pt) Use the above potential to set up the total forces acting on the particle. Find the acceleration acting on the particle. Is this a conservative force? Calculate also the **curl** of the force $\boldsymbol{\nabla} \times \boldsymbol{F}$ in order to validate your conclusion.
- Midterm-1f (5pt) Are linear momentum and angular momentum conserved?
- Midterm-1g (5pt) Write a numerical algorithm to find the position and velocity of the particle at a time $t + \Delta t$ given the position and velocity at a time t. Here we aim at the **Euler-Cromer and the Velocity Verlet algorithms**.
- Midterm-1h (10pt) Implement the numerical algorithms in a program to find the position of the particle as function of time from $t = 0$ to $t = 30$s using a mass $m = 1.0$ kg, the parameter $V_0 = 1$ J and $d = 0.1$m. Make a plot of two distinct positions with initial conditions $x_0 = d$ and $v_0 = 0.5$m/s, $x_0 = d$ and $v_0 = 1.5$m/s, and $x_0 = d$ and $v_0 = 2.5$m/s. Plot also the velocity. Perform calculations with and without the term x^4 in the potential. Do you see a difference? Compare and discuss the results obtained with the Velocity Verlet algorithm and the Euler-Cromer algorithm. This problem does not have an analytical solution.
- Midterm-1i (5pt) Describe the behavior of the particle for the three initial conditions and sketch the motion in an energy diagram. Is energy conserved in your simulations?
- Midterm-1j (5pt) Repeat the calculations from 1h with the full potential and all three initial conditions but now with a damping force $-\beta v$. Find the modified acceleration using $\beta/m = 0.1$. Discuss your results and sketch again the motion in an energy diagram,

We move then to two dimensions. Our particle/object interacts with a surface potential given by

$$V(r) = \frac{V_0}{d^4}\left(r^4 - 2r^2d^2 + d^4\right),$$

where $r = \sqrt{x^2 + y^2}$ is the distance to the origin.

- Midterm-1k (5pt) Show that the acceleration is now

$$\boldsymbol{a} = -\frac{4V_0}{md^4}\left(r^3 - rd^2\right)\frac{\boldsymbol{r}}{r}.$$

- Midterm-1l (10pt) Rewrite your program to find the velocity and position of the atom using the new expression for the force $\boldsymbol{F}$. Use vectorized expressions in your code as you did in homework 4 for the Earth-Sun system. See eventually the code from the lectures. We recommend to revisit the Earth-Sun problem from homework 4 since it has several similarities with the problem here.

- Midterm-1m (10pt) Plot the motion of a particle starting in $\boldsymbol{r}_0 = (d, 0)$ from $t = 0$s to $t = 20$s for the initial velocities $\boldsymbol{v}_0 = (0, 0.5)$m/s, $\boldsymbol{v}_0 = (0, 1)$m/s, and $\boldsymbol{v}_0 = (0, 1.5)$m/s. The parameters d and V_0 are as before.

- Midterm-1n (10pt) Is energy conserved?

- Midterm-1o (10pt) Can you choose initial conditions r_0 and v_0 in such a manner that the particle moves in a circular orbit with a constant radius? If so, what initial conditions are those? Plot the motion for these conditions.