Discussion of hw7

Exercises 3, 4 + 5.

E in polar coordinates

$$E = \frac{1}{2}\mu\left[\dot{r}^2 + r^2\dot{\phi}^2\right] + V(r)$$

$r \in [0, \infty)$ — kinetic — $\propto r^a$, $a = 2, -1, -2$

$$L = \mu r^2 \dot{\phi}$$

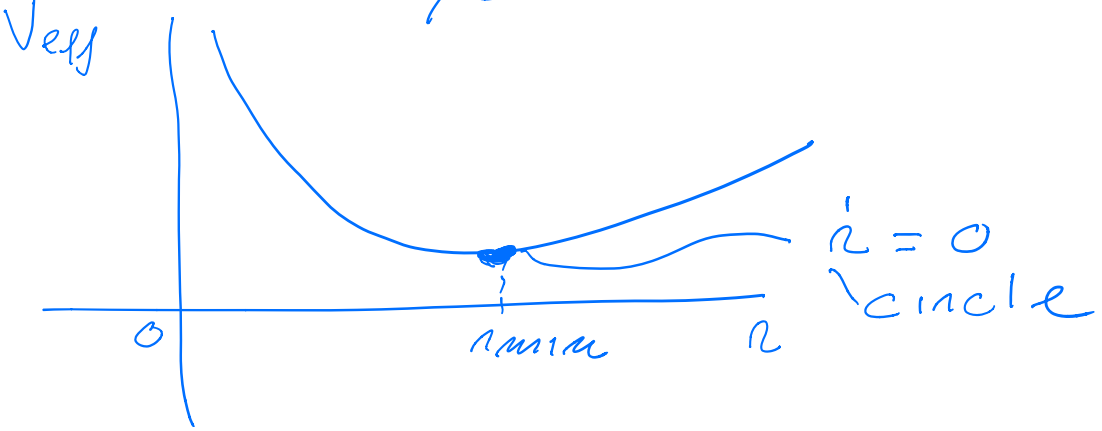
$$V_{eff}(r) = \frac{L^2}{2\mu r^2} + V(r)$$

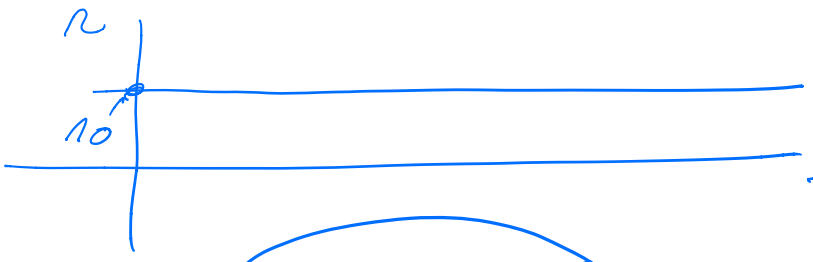
Equations of motion

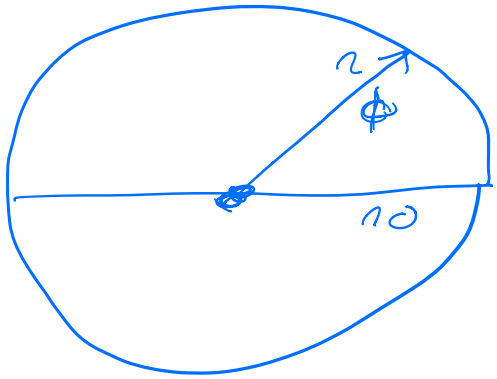
$$\ddot{r} = \frac{1}{\mu}\left[-\frac{dV(r)}{dr} + \mu r\dot{\phi}^2\right]$$

$$\dot{\phi} = \frac{L}{\mu r^2}$$

$$\ddot{r} = \frac{1}{\mu} \times (-1)\frac{dV_{eff}(r)}{dr}$$







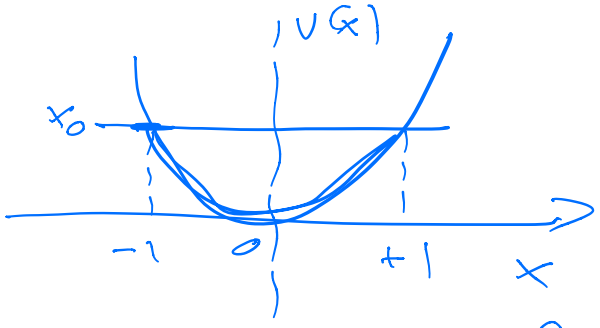
circle

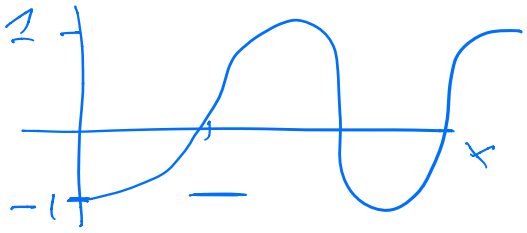
cartesian coordinates

$x \in (-\infty, +\infty)$

$y \in (-\infty, +\infty)$

$$V(x,y) = \frac{1}{2} k (x^2 + y^2) = \frac{1}{2} k r^2$$





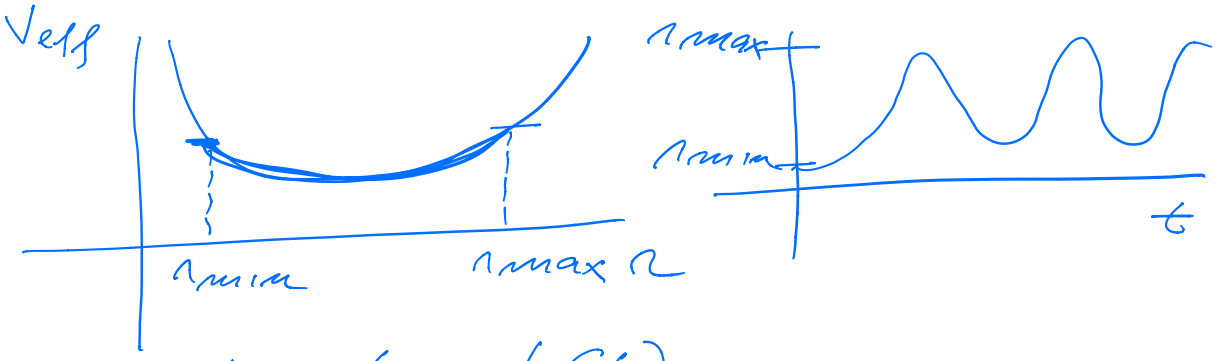
$$x(t) = A \cos(\omega t) + B \sin(\omega t)$$

$$y(t) = C \cos(\omega t) + D \sin(\omega t)$$

$$\ddot{x} = -\omega^2 x$$

$$\ddot{y} = -\omega^2 y$$

polar coordinates r, ϕ



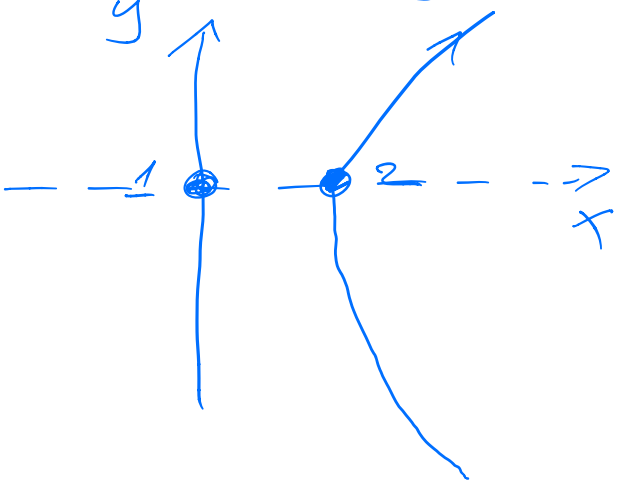
Extract $\phi(t)$

plot r and ϕ we get an ellipse.

$$\ddot{r} = \frac{d\dot{r}}{dt} = -\frac{dV_{eff}}{dr}$$

$$\frac{dr}{dt} = \dot{r}$$

Two-body scattering (Hyperbola)



$$V(r) = \pm \alpha / r$$

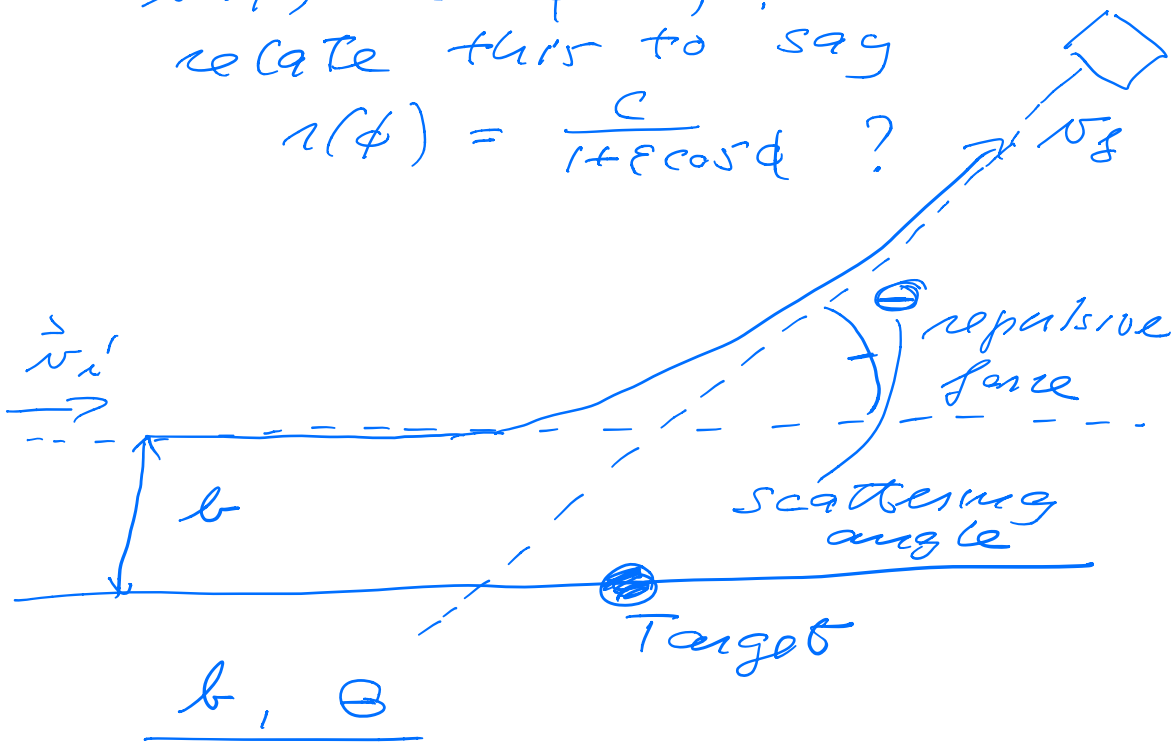
$$r(\phi) = \frac{c}{1 + \varepsilon \cos\phi}$$

reuse these results in scattering problems

can we link these theoretical calculations with experiments?

- scattering angle (expt quantity)
- cross section (expt)
- impact parameter b (deduce from expts)

$b(\phi)$ or $\phi(b)$. How to relate this to say $r(\phi) = \frac{c}{1+\varepsilon\cos\phi}$?



b, θ