

PHY321: Introduction to Classical Mechanics

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Introduction

Classical mechanics is a topic which has been taught intensively over several centuries. It is, with its many variants and ways of presenting the educational material, normally the first **real** physics course many of us meet and it lays the foundation for further physics studies. Many of the equations and ways of reasoning about the underlying laws of motion and pertinent forces, shape our approaches and understanding of the scientific method and discourse, as well as the way we develop our insights and deeper understanding about physical systems.

There is a wealth of well-tested (from both a physics point of view and a pedagogical standpoint) exercises and problems which can be solved analytically. However, many of these problems represent idealized and less realistic situations. The large majority of these problems are solved by paper and pencil and are traditionally aimed at what we normally refer to as continuous models from which we may find an analytical solution. As a consequence, when teaching mechanics, it implies that we can seldomly venture beyond an idealized case in order to develop our understandings and insights about the underlying forces and laws of motion.

On the other hand, numerical algorithms call for approximate discrete models and much of the development of methods for continuous models are nowadays being replaced by methods for discrete models in science and industry, simply because **much larger classes of problems can be addressed** with discrete models, often by simpler and more generic methodologies.

As we will see below, when properly scaling the equations at hand, discrete models open up for more advanced abstractions and the possibility to study real life systems, with the added bonus that we can explore and deepen our basic understanding of various physical systems

Analytical solutions are as important as before. In addition, such solutions provide us with invaluable benchmarks and tests for our discrete models. Such benchmarks, as we will see below, allow us to discuss possible sources of errors and their behaviors. And finally, since most of our models are based on various algorithms from numerical mathematics, we have a unique opportunity to gain a deeper understanding of the mathematical approaches we are using.

With computing and data science as important elements in essentially all aspects of a modern society, we could then try to define Computing as **solving scientific problems using all possible tools, including symbolic computing, computers and numerical algorithms, and analytical paper and pencil solutions**. Computing provides us with the tools to develop our own understanding of the scientific method by enhancing algorithmic thinking.

The way we will teach this course reflects this definition of computing. The course contains both classical paper and pencil exercises as well as computational projects and exercises. The hope is that this will allow you to explore the physics of systems governed by the degrees of freedom of classical mechanics at a deeper level, and that these insights about the scientific method will help you to develop a better understanding of how the underlying forces and equations of motion and how they impact a given system. Furthermore, by introducing various numerical methods via computational projects and exercises, we aim at developing your competences and skills about these topics.

These competences will enable you to

- understand how algorithms are used to solve mathematical problems,
- derive, verify, and implement algorithms,
- understand what can go wrong with algorithms,
- use these algorithms to construct reproducible scientific outcomes and to engage in science in ethical ways, and
- think algorithmically for the purposes of gaining deeper insights about scientific problems.

All these elements are central for maturing and gaining a better understanding of the modern scientific process *per se*.

The power of the scientific method lies in identifying a given problem as a special case of an abstract class of problems, identifying general solution methods for this class of problems, and applying a general method to the specific problem (applying means, in the case of computing, calculations by pen and paper, symbolic computing, or numerical computing by ready-made and/or self-written software). This generic view on problems and methods is particularly

important for understanding how to apply available, generic software to solve a particular problem.

However, verification of algorithms and understanding their limitations requires much of the classical knowledge about continuous models.

A well-known examples to illustrate many of the above concepts

Before we venture into a reminder on Python and mechanics relevant applications, let us briefly outline some of the abovementioned topics using an example many of you may have seen before in for example CMSE201. A simple algorithm for integration is the Trapezoidal rule. Integration of a function $f(x)$ by the Trapezoidal Rule is given by following algorithm for an interval $x \in [a, b]$

$$\int_a^b (f(x)dx = \frac{1}{2} [f(a) + 2f(a+h) + \dots + 2f(b-h) + f(b)] + O(h^2),$$

where h is the so-called stepsize defined by the number of integration points N as $h = (b-a)/(n)$. Python offers an extremely versatile programming environment, allowing for the inclusion of analytical studies in a numerical program. Here we show an example code with the **trapezoidal rule**. We use also **SymPy** to evaluate the exact value of the integral and compute the absolute error with respect to the numerically evaluated one of the integral $\int_0^1 dx x^2 = 1/3$. The following code for the trapezoidal rule allows you to plot the relative error by comparing with the exact result. By increasing to 10^8 points one arrives at a region where numerical errors start to accumulate.

This example shows the potential of combining numerical algorithms with symbolic calculations, allowing us to

- Validate and verify their algorithms.
- Including concepts like unit testing, one has the possibility to test and test several or all parts of the code.
- Validation and verification are then included *naturally* and one can develop a better attitude to what is meant with an ethically sound scientific approach.
- The above example allows the student to also test the mathematical error of the algorithm for the trapezoidal rule by changing the number of integration points. The students get **trained from day one to think error analysis**.
- With a Jupyter notebook you can keep exploring similar examples and turn them in as your own notebooks.

In this process we can easily bake in

1. How to structure a code in terms of functions
2. How to make a module
3. How to read input data flexibly from the command line
4. How to create graphical/web user interfaces
5. How to write unit tests (test functions or doctests)
6. How to refactor code in terms of classes (instead of functions only)
7. How to conduct and automate large-scale numerical experiments
8. How to write scientific reports in various formats (L^AT_EX, HTML)

The conventions and techniques outlined here will save you a lot of time when you incrementally extend software over time from simpler to more complicated problems. In particular, you will benefit from many good habits:

1. New code is added in a modular fashion to a library (modules)
2. Programs are run through convenient user interfaces
3. It takes one quick command to let all your code undergo heavy testing
4. Tedious manual work with running programs is automated,
5. Your scientific investigations are reproducible, scientific reports with top quality typesetting are produced both for paper and electronic devices.

Teaching team, grading and other practicalities

Lectures			Location
Monday 3:00-3:50pm	Wednesday 3:00-3:50pm	Friday 3:00-3:50pm	Room 1420 BPS

Instructor	Email	Office	Office phone/cellphone
Morten Hjorth-Jensen	hjensen@msu.edu	Office: NSCL/FRIB 2131	5179087290/5172491375

Office Hours	
Monday/Wednesday 4-5:00pm, Room 2131 NSCL/FRIB	or immediately after class

Homework Grader	Email
Kasun Senanayaka	senanaya@msu.edu

Learning Assistant	Email
Dylan R. Smith	smithdy6@msu.edu

Grading and dates

Activity	Percentage of total score
Homeworks, 10 in total and due Wednesdays the week after	20%
First Midterm Project, due Friday February 28	25%
Second Midterm Project, due Friday April 10	25%
Final Exam, April 29, 5:45-7:45pm	30%
Extra Credit Assignment (Due Friday April 24)	10%

Grading scale
4.0(90height

Possible textbooks and lecture notes

Recommended textbook:

- John R. Taylor, Classical Mechanics (Univ. Sci. Books 2005), see also <https://github.com/mhjensen/Physics321/tree/master/doc/Literature>

Additional textbooks:

- Anders Malthe-Sørenssen, Elementary Mechanics using Python (Springer 2015) and <https://github.com/mhjensen/Physics321/tree/master/doc/Literature>
- Alessandro Bettini, A Course in Classical Physics 1, Mechanics (Springer 2017) and <https://www.springer.com/gp/book/9783319292564>

Lecture notes: Posted lecture notes are in the doc/pub folder here or at <https://mhjensen.github.io/Physics321/doc/web/course.html> for easier viewing. They are not meant to be a replacement for textbook. These notes are updated on a weekly basis and a **git pull** should thus always give you the latest update.

Teaching schedule with links to material (This will be updated asap)

Weekly mails (Wednesdays or Thursdays) with updates, plans for lectures etc will sent to everybody. We use also Piazza as a discussion forum. Please use this sign-up link <https://piazza.com/msu/spring2020/phy321>. The class link is <https://piazza.com/msu/spring2020/phy321/home>

Week 2, January 6-10, 2020.

1. Monday: Introduction to the course and start discussion of vectors, space, time and motion, Taylor chapter 1.2 and lecture notes (<https://mhjensen.github.io/Physics321/doc/pub/Int>)

2. Wednesday: More on time,space, vectors and motion, Taylor chapters 1.2 and 1.3 and lecture notes (<https://mhjensen.github.io/Physics321/doc/pub/Introduction/html/Introduction.html>) first homework available
3. Friday: Forces and Newton's laws of motion. Taylor chapter 1.4 and lecture notes (<https://mhjensen.github.io/Physics321/doc/pub/Introduction/html/Introduction.html>). Introduction to Git and GitHub and getting started with numerical exercises. Installing software (anaconda) and first homework due January 15.

Week 3, January 13-17, 2020.

1. Monday:
2. Wednesday:
3. Friday: 2nd homework, due January 22

Week 4, January 20-24, 2020.

1. Monday: MLK day, no lectures
2. Wednesday:
3. Friday: 3rd homework, due January 29

Week 5, January 27-31, 2020.

1. Monday:
2. Wednesday:
3. Friday: 4th homework, due February 5

Week 6, February 3-7, 2020.

1. Monday:
2. Wednesday:
3. Friday: 5th homework, due February 12

Week 7, February 10-14, 2020.

1. Monday:
2. Wednesday:
3. Friday: 6th homework, due February 19

Week 8, February 17-21, 2020.

1. Monday:
2. Wednesday:
3. Friday: **First midterm project, due February 28, 2020**

Week 9, February 24-28, 2020.

1. Monday:
2. Wednesday:
3. Friday:

Week 10, March 2-6, 2020, Spring break.

1. Monday: No lectures
2. Wednesday: No lectures
3. Friday: No lectures

Week 11, March 9-13, 2020.

1. Monday:
2. Wednesday:
3. Friday: 7th homework, due March 18

Week 12, March 16-20, 2020.

1. Monday:
2. Wednesday:
3. Friday: 8th homework, due March 25

Week 13, March 23-27, 2020.

1. Monday:
2. Wednesday:
3. Friday: 9th homework, due April 1

Week 14, March 30-April 3, 2020.

1. Monday:
2. Wednesday:
3. Friday: **Second midterm project, due April 10, 2020**

Week 15, April 13-17, 2020.

1. Monday:
2. Wednesday:
3. Friday: 10th homework and extra assignments, due April 24

Week 16, April 20-24, 2020.

1. Monday:
2. Wednesday:
3. Friday: Summary and discussions of finals exams

Week 17, April 27- May 1, 2020, Finals week.

1. Final Exam: April 29, 5:45pm - 7:45pm in 1420 Biomedical & Physical Sciences

Space, Time, Motion and Reminder on vectors and other mathematical quantities

Newton's Laws of Motion