Variational calculus &
Lagrangian formalism.

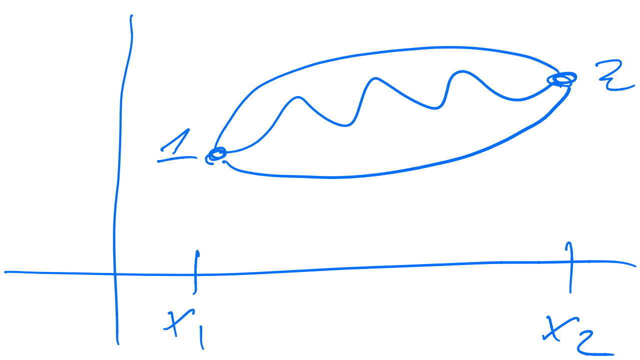
- Work-energy theorem

$$W_{ab}^{Net} = \int_{t_a}^{t_b} \vec{F}^{net}(\vec{r},\vec{v},t)\,\vec{v}(t)\,dt$$

$$= \int_C F^{net}(\vec{r})\,d\vec{r}$$

$$= \frac{1}{2}mv_b^2 - \frac{1}{2}mv_a^2$$

what is the path which minimizes W_{ab}^{Net} (maximize)



The calculus of variations deals with finding the min or max of a quantity that is expressable as an integral.

1-dim

$$\mathcal{L} = \mathcal{L}(x(t), v(t), t)$$

(Lagrangian)

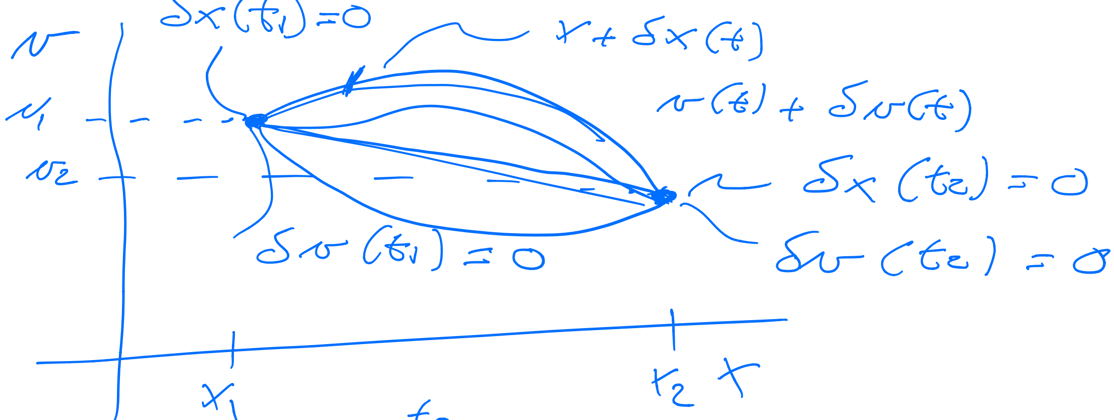
$$\mathcal{L} = K - V \quad \text{(kinetic} - \text{potential)}$$

quantity to min/max

$$S = \int_{t_1}^{t_2} \mathcal{L}(x, v, t)\, dt$$

$\Rightarrow$ Euler-Lagrange equation

$\Rightarrow$ equations of motion



$$\delta S = \int_{t_1}^{t_2} \left[\mathcal{L}(x + \delta x, v + \delta v, t) \right] dt - \int_{t_1}^{t_2} \mathcal{L}(x, v, t)\, dt$$

δx and δv are small.

$$\mathcal{L}(x + \delta x, v + \delta v, t) = \mathcal{L}(x, v, t)$$

$$+ \frac{\partial \mathcal{L}}{\partial x}\delta x + \frac{\partial \mathcal{L}}{\partial v}\delta v + o(\delta x^2, \delta v^2)$$

$$\delta v = \frac{d}{dt}\delta x \qquad v = \frac{dx}{dt}$$

$\rightarrow$ integration by parts

$$\Rightarrow \boxed{\frac{\partial \mathcal{L}}{\partial x} - \frac{d}{dt}\frac{\partial \mathcal{L}}{\partial v} = 0}$$

Euler–Lagrange equation

$$\mathcal{L} = \frac{1}{2}mv^2 - \frac{1}{2}kx^2$$

$$\boxed{m\frac{dv}{dt} = -kx}$$

Taylor 6,7