PHY321: Classical Mechanics 1

First mditerm project, due Wednesday March 11

Feb 25, 2020

Practicalities about homeworks and projects.

1. You can work in groups (optimal groups are often 2-3 people) or by yourself. If you work as a group you can hand in one answer only if you wish. **Remember to write your name(s)**!
2. How do I(we) hand in? You can hand in the paper and pencil exercises as a hand-written document. Alternatively, you can hand in everyhting (if you are ok with typing mathematical formulae using say Latex) as a jupyter notebook at D2L. The numerical exercise(s) should always be handed in as a jupyter notebook by the deadline at D2L.

Introduction to the first midterm project. The relevant reading background is

1. chapters 2-5 of Taylor (there are many good examples there)
2. chapters 6-14 of Malthe-Sørenssen.

In this midterm project we will start with a potential similar to the one we discussed in exercise 4 in homework 6. There are some elements of that exercise which are repeated here. Thereafter we extend this potential model to a two-dimensional model and study the numerical solution of the problem.

Particle in a potential. We consider a particle of mass m moving in a one-dimensional potential,

$$V(x) = \frac{V_0}{d^4}\left(x^4 - 2x^2 + d^4\right).$$

We will assume all other forces on the particle are small in comparison, and neglect them in our model. The parameters V_0 and d are known constants.

- Midterm-1a (5pt) Plot the potential and find the equilibrium points (stable and unstable) by requiring that the first derivative of the potential is zero. Make an energy diagram (see for example Malthe-Sørenssen chapter 11.3) and mark the equilibrium points on the diagram and characterize their stability. The position of the particle is x.

- Midterm-1b (5pt) Choose two different energies that give two distinct types of motions, draw them into the energy diagram, and describe the motion in each case.

- Midterm-1c (5pt) If the particle starts at rest at $x = 2d$, what is the velocity of the atom at the point $x = d$?

- Midterm-1d (5pt) If the particle starts at rest at $x = d$ with velocity v_0, how large must v_0 be for the particle to reach the point $x = -d$?

- Midterm-1e (5pt) Use the above potential to set up the total forces acting on the particle. Find the acceleration acting on the particle. Is this a conservative force? Calculate also the **curl** of the force $\boldsymbol{\nabla} \times \boldsymbol{F}$.

- Midterm-1f (5pt) Are linear momentum and angular momentum conserved?

- Midterm-1g (5pt) Write a numerical algorithm to find the position and velocity of the particle at a time $t + \Delta t$ given the position and velocity at a time t. Here we aim at the **Euler-Cromer and the Velocity Verlet algorithms**.

- Midterm-1h (5pt) Implement the numerical algorithm in a program to find the position of the particle as function of time from $t = 0$ to $t = 10$s using a mass unit $m = 1.0$ kg, the parameter $V_0 = 1$ J and $d = 0.1$m. Make a plot of two distinct positions with initial conditions $x_0 = d$ and $v_0 = 0.5$m/s and $x_0 = d$ and $v_0 = 1.5$m/s. Plot also the velocity. Perform calculations with and without the term x^4 in the potential. Do you see a difference?

- Midterm-1i (5pt) Describe the behavior of the particle for both initial conditions and sketch the motion in an energy diagram. Is energy conserved in your simulations?

We move then to two dimensions. Our particle/object interacts with a surface potential given by

$$V(r) = \frac{V_0}{d^4}\left(r^4 - 2r^2 + d^4\right),$$

with $r = \sqrt{x^2 + y^2}$ is the distance to the origin.

- Midterm-1j (5pt) Show that the acceleration is now

$$\boldsymbol{a} = -\frac{4V_0}{md^4}\left(r^3 - rd^2\right)\frac{\boldsymbol{r}}{r}.$$

- Midterm-1k (10pt) Rewrite your program to find the velocity and position of the atom using the new expression for the force $\boldsymbol{F}$. Use vectorized expressions in your code as you did in homework 4 for the Earth-Sun system. See eventually the code at.

- Midterm-1l (10pt) Plot the motion of an atom starting in $\boldsymbol{r}_0 = (d, 0)$ from $t = 0$s to $t = 20$s for the initial velocities $\boldsymbol{v}_0 = (0, 0.5)$m/s, $\boldsymbol{v}_0 = (0, 1)$m/s, and $\boldsymbol{v}_0 = (0, 1.5)$m/s.

- Midterm-1m (5pt) Is energy conserved?

- Midterm-1n (10pt) Can you choose initial conditions r_0 and v_0 in such a manner that the particle moves in a circular orbit with a constant radius? If so, what initial conditions are those? Plot the motion for these conditions.