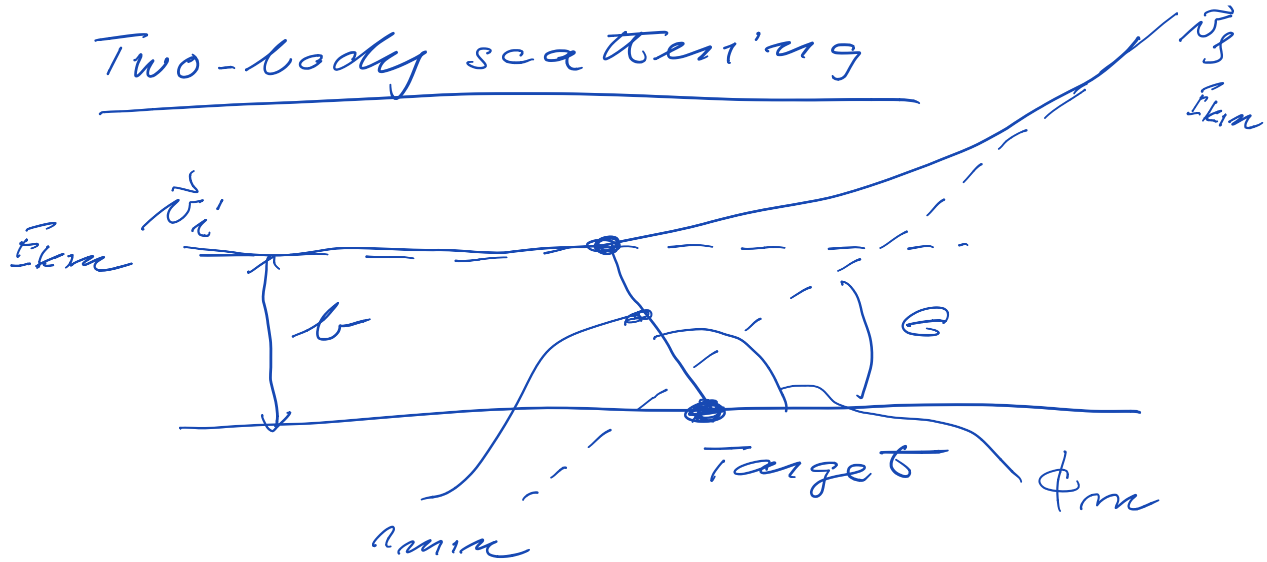
Two-body scattering



$$r(\phi) = \frac{c}{1+\varepsilon\cos\phi} \qquad r_{min} \quad \phi = 0$$

$$\frac{d\sigma}{d\Omega} = \frac{b}{\sin\theta}\left|\frac{db}{d\theta}\right|$$

$$c = \left(\frac{\mu\alpha}{L^2 A}\right)^{-1} \qquad \Bigg| \quad r(\phi) = \frac{1}{\frac{\mu\alpha}{L^2} + A\cos\phi}$$

$$A = \sqrt{\left(\frac{\mu\alpha}{L^2}\right)^2 + \frac{2\mu E}{L^2}}$$

Conservation of Angular mom

$$L^2 = \mu^2 b^2 v^2 = 2\mu b^2 E$$

$$A = \sqrt{\left(\frac{\mu\alpha}{L^2}\right)^2 + \frac{1}{b^2}}$$

From geometry

$$\boxed{\sin\frac{\theta}{2} = \frac{\mu\alpha}{\ldots}} \qquad a = \frac{\alpha}{2E}$$

$$\left(\qquad\qquad A L^2 \right)$$

$$= \frac{a}{\sqrt{a^2+b^2}}$$

$$\left(d\sigma = 2\pi \, b \, db \right)$$

$$\frac{1}{2} \cos\frac{\theta}{2}\, d\theta = \frac{a\, b\, db}{(a^2+b^2)^{3/2}}$$

$$= \frac{b\, db}{a^2} \sin^3 \theta/2$$

$$\frac{\pi a^2}{\sin^3(\theta/2)} \cos \theta/2 \, d\theta$$

$$= \frac{\pi a^2}{2 \sin^4 \theta/2} \sin\theta \, d\theta$$

$$\boxed{\frac{d\sigma}{d\Omega} = \frac{a^2}{4 \sin^4 \theta/2}} \qquad a = \frac{\alpha}{2E}$$

same for attractive and repulsive potential.

<u>Example</u>

particle of mass m and charge z with kinetic energy E scatters on a heavy nucleus of mass M and charge Z and Radius <u>R</u>.

what is the angle where the classical scattering formula breaks down?

$$\alpha = \frac{Zze^2}{4\pi\varepsilon_0}$$

$$\sin\theta/2 = \frac{a}{\sqrt{a^2+b^2}}$$

Point of closest approach

$b = R$. can find it from conservation of angular momentum

$$L = \underline{\mu \cdot v_i \cdot b} = b\sqrt{2\mu E} = \underline{\mu v_f R}$$

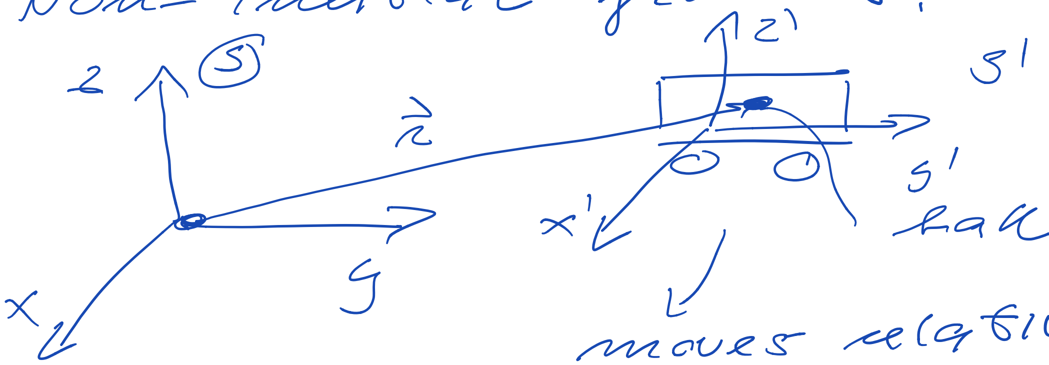
$$= R\sqrt{2\mu(E-\alpha/R)}$$

$$b = R\,\frac{\sqrt{2\mu(E-\alpha/R)}}{\sqrt{2\mu E}}$$

$$\underline{b} = R\sqrt{1-\frac{\alpha}{ER}}$$

$$\theta = 2\sin^{-1}\left[\frac{a}{\sqrt{a^2+b^2}}\right]$$

Non-inertial frames:



moves relative to S with velocity $\vec{V}'$

$\vec{a}' = \dfrac{d\vec{V}'}{dt}$

S is an inertial frame

Newton's 2nd law

$$m\ddot{\vec{r}} = F$$

$\vec{r}$ is the ball's position relative to S

The ball's motion relative to the accelerating frame S' is determined by $\vec{r}'$

$$\dot{\vec{r}} = \dot{\vec{r}}' + \vec{V}'$$

$\vec{V}'$

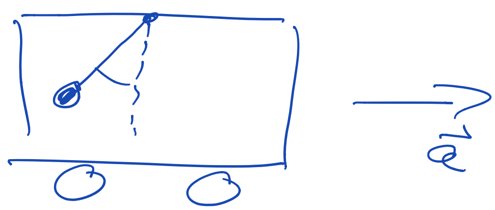
Ball's velocity relative to S

= Ball's velocity relative to train car + car's velocity relative to S.

$$\ddot{\vec{r}}\,' = \ddot{\vec{r}} - \vec{a}\,'$$

$$\boxed{m\ddot{\vec{r}} = \underbrace{\vec{F}}_{\text{sum of all forces in inertial system}} \underbrace{- m\vec{a}\,'}_{\text{Extra force / inertial force}}}$$

Example



Taylor <u>9.1 - 9.3</u>