

CSCI-400: Lambda Calculus Homework

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- A **printed** copy is due in class on Thursday, February 21.
 - Handwritten or typed answers are acceptable. If handwritten, please write clear and legibly.
1. (50 points) Fully beta reduce each of the following lambda terms. Check your answer using the `lc` tool shown in class. **Do not use any shorthands in your evaluation.**

Note: it is acceptable to skip currying steps. You should not skip any other steps: show all of your work!

(a) $(\lambda p. \lambda q. pqp)(\lambda x. \lambda y. y)(\lambda x. \lambda y. x)$

$$\begin{aligned} & \xrightarrow{\beta} (\lambda q. (\lambda x. \lambda y. q)(\lambda x. \lambda y. q))(\lambda x. \lambda y. x) \\ & \xrightarrow{\beta} ((\lambda x. \lambda y. q)(\lambda x. \lambda y. x))(\lambda x. \lambda y. q) \\ & \xrightarrow{\beta} ((\lambda x. \lambda y. q)(\lambda y. (\lambda x. \lambda y. q))) \end{aligned}$$

(b) $(\lambda p. \lambda q. ppq)(\lambda x. \lambda y. y)(\lambda x. \lambda y. x)$

$$\begin{aligned} & \xrightarrow{\beta} (\lambda q. (\lambda x. \lambda y. q)(\lambda x. \lambda y. q))(\lambda x. \lambda y. x) \\ & \xrightarrow{\beta} (\lambda x. \lambda y. q)(\lambda x. \lambda y. q)(\lambda x. \lambda y. x) \end{aligned}$$

(c) $(\lambda p. p(\lambda x. \lambda y. y)(\lambda x. \lambda y. x))(\lambda x. \lambda y. y)$

$$\xrightarrow{\beta} ((\lambda x. \lambda y. y)(\lambda x. \lambda y. y))(\lambda x. \lambda y. x)$$

$$(d) (\lambda p. \lambda a. \lambda b. \underline{pab}) (\lambda x. \lambda y. y) (\lambda f. (\lambda x. f x)) (\lambda f. (\lambda x. f(fx)))$$

$$\xrightarrow{\beta} (\lambda x. \lambda y. y) (\lambda f. (\lambda x. f x)) (\lambda f. (\lambda x. f(fx)))$$

$$\xrightarrow{\beta} (\lambda x. \lambda y. y) (\lambda x. (\lambda f. (\lambda x. f(fx)) x))$$

$$\xrightarrow{\beta} (\lambda x. \lambda y. y) (\lambda f. (\lambda x. f(fx)))$$

$$(e) (\lambda c. c(\lambda x. \lambda y. y)) (\lambda x. \lambda y. \lambda f. fxy) (\lambda f. \lambda x. f(fx)) (\lambda f. \lambda x. f(f(fx)))$$

$$\xrightarrow{\beta} (\lambda x. \lambda y. \lambda f. fxy) (\lambda x. \lambda y. y) (\lambda f. \lambda x. f(fx)) (\lambda f. \lambda x. f(f(fx)))$$

$$\xrightarrow{\beta} (\lambda x. \lambda y. y) (\lambda f. \lambda x. f(fx)) (\lambda f. \lambda x. f(f(fx)))$$

$$\xrightarrow{\beta} (\lambda x. \lambda y. y) (\lambda x. (\lambda f. \lambda x. f(f(fx))) ((\lambda f. \lambda x. f(f(fx))))))$$

2. (20 points) Identify which variables are free and which are bound in each of the lambda terms:

(a) $\lambda x. xy$

Bound: x, y

Free:

(b) $(\lambda x. x)m$

Bound: x

Free: m

3. (10 points (bonus)) Write a lambda calculus abstraction which, when a Church numeral n is applied, evaluates to the Church numeral $n - 1$. Note that this is the inverse of the **SUCC** function. Then, explain how your abstraction works and provide an example for the Church numeral 5. (Note that this abstraction need not handle 0.)