

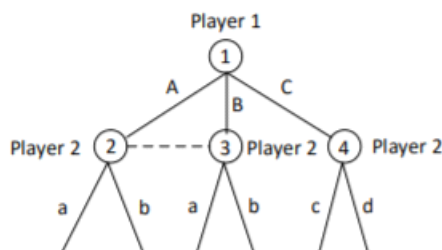
Carson Stevens CSCI 455 Homework 2

Thursday, October 1, 2020

1. [Information Set Video Resource](#)

(10 + 10) Consider the following game:

- (a) List all the information sets for each player.
- (b) Identify all the subgames.

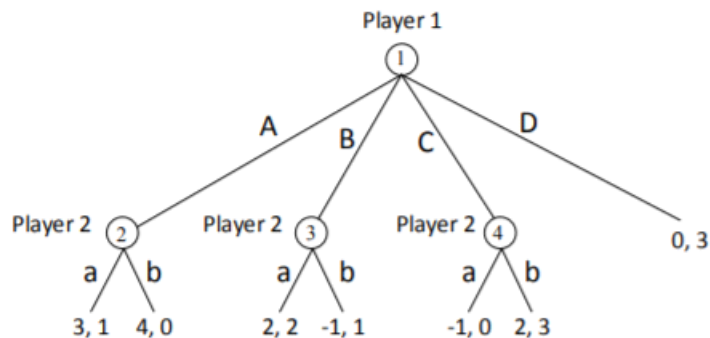


- a. Player 1:
 - {A, B, C}
- Player 2:
 - {Aa, Ab, Ba, Bb, Cc, Cd}
- b. Subgames:
 - (A, B; (a, b)), (C; (c, d))

2.

(10 + 10 + 10) Consider the following game:

- (a) Identify all the subgames.
- (b) Write one strategy for each of Player 1 and Player 2. Any will do.
- (c) Find the subgame perfect Nash equilibrium. Write down the whole strategy for each player.



- a. Subgames:

- (A; (a, b)), (B; (a, b)), (C; (a, b)), (D)
- b. Player 1 chooses A, Player 2 chooses a
- c. SPNE: (A; a)
 - Player 1 chooses A, Player 2 chooses a

3. (10) Suppose two firms produce slightly different products; suppose, as a consequence, Firm 1's price is more sensitive to its own product than Firm 2. In particular, letting the outputs 1 be denoted by q_1 and q_2 and the prices set by each firm be denoted by p_1 and p_2 , suppose the two demand curves are

$$p_1 = a - bq_1 - dq_2$$

$$p_2 = a - bq_2 - dq_1$$

where $b > 0$ and $d > 0$ (and any quantity, including fractions, can be produced). Suppose also that the costs of producing a unit of output is the same for both firms and is equal to c dollars where $a > c$. Assume that Firm 1 decides q_1 first, and Firm 2 observes q_1 and then decides q_2 . What is the SPNE?

$$\text{Profit } F1 = U1 = (a - bq_1 - dq_2) * q_1 - c * q_1$$

$$\text{Profit } F2 = U2 = (a - bq_2 - dq_1) * q_2 - c * q_2$$

Since Firm 2 chooses q_2 according to q_1 ,
Best Response Firm 2 (BR2) to q_1 :

$$q_2 = \text{BR2}(q_1) = \frac{(a - c - (b + d) * q_1)}{2b}$$

Since Firm 1 knows that Firm 2 will choose q_2 , Firm 1 now maximizes

$$q_1^* = \max(U1(q_1, q_2(q_1))) = \frac{(a - c - (b + d) * (q_1 + (a - c - (b + d) * q_1))}{2b} * q_1$$

$$\text{Firm 1 chooses } q_1^* = \frac{a - c}{2b}$$

$$q_2^* = \frac{a - c}{4b}$$

$$U1 = \frac{(a - c)^2}{8b}$$

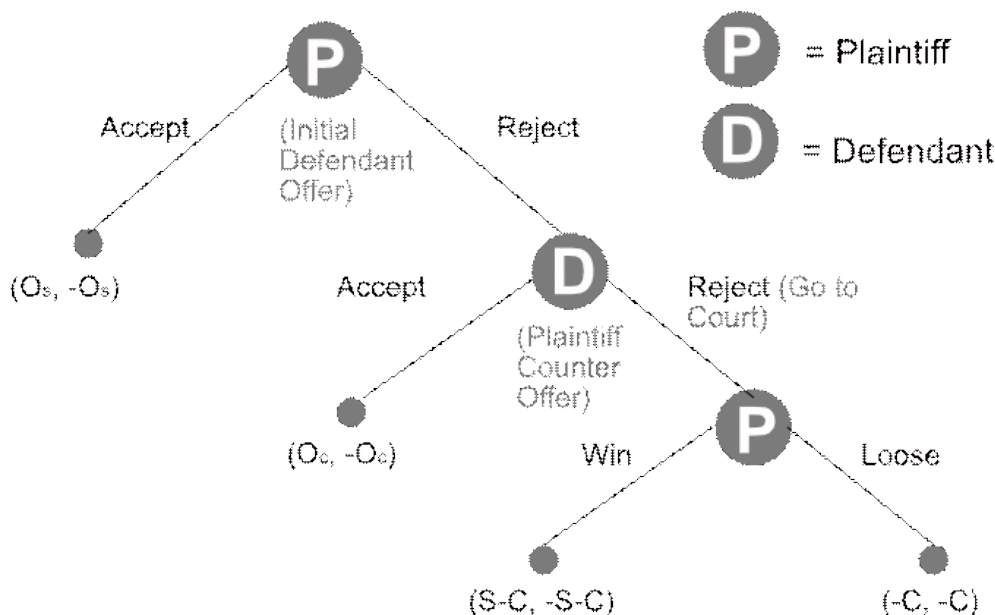
$$U2 = \frac{(a - c)^2}{16b}$$

SPNE is: $(q_1^*, \text{BR2}(q_1^*))$

4. (10 + 20) This problem deals with a lawsuit by a family (plaintiff) against a company (defendant). Both the company and the family know that, should the case go to court, the court will award the family damages S with certainty. In addition, the court will assess court fees of C which both the plaintiff and the defendant will have to pay. Both parties have obtained lawyers, and have agreed to participate in the following bargaining mechanism. First, the defendant will make a settlement offer O_S to the plaintiff. If the plaintiff accepts the offer, the lawsuit is dropped and the defendant pays the agreed-upon settlement to the plaintiff. If the plaintiff rejects the offer, the plaintiff then gets to make a counter offer O_C . The defendant can either accept the offer or reject it. If the defendant

accepts it, the defendant will pay the new offer to the plaintiff. If the defendant rejects the offer, the parties go to court. As was known to both parties previously, the court assesses a settlement S which the defendant pays to the plaintiff. The court also assesses court fees C to each party.

- a. Write this situation as a game using a game tree.



- b. Solve this game using backward induction. Write the SPNE strategies as well as the equilibrium outcome. You may assume that if either party is just indifferent between accepting and rejecting an offer, they will accept it.
- SPNE strategies: (Accept_P) , $(\text{Reject}_P; (\text{Accept}_D, (\text{Reject}_D; (\text{Win}_P, \text{Lose}_P))))$
 - Equilibrium Outcome: Defendant rejects Plaintiff Counter Offer and goes to court.

5.

(10) Consider the game represented by the table below:

		Player 2	
		L	R
Player 1	U	1, 2	4, 1
	D	0, 5	3, 4

Table 1: Game 1

What is the SPNE of the game repeated 3 times? Be sure to explicitly specify the strategies.

Backward induction:

In the last period (T-3), Up is the dominant strategy for Player1 and Left is the dominant strategy for Player 2. So the NE is (U,L)

In (T-1) period, we know at T, no matter what, Player 1 plays U and Player 2 plays L.

Therefore, the **unique SPNE is (U,L)** for *each* period