Carson Stevens CSCI 445 Homework 1

Thursday, September 10, 2020

1.

(10 × 2) Solve the following games using IESDS.

		Player 2		
		L	C	R
Player 1	U	1, 9	5, 8	8, 4
	M	3, 4	7, 5	4, 1
	D	2, 2	4, 1	9, 0

Table 1: Game 1

		Player 2		
		L	C	R
Player 1	U	5, 5	9, 4	5, 8
	M	4, 9	8, 7	9, 0
	D	1, 4	1, 9	1, 0

Table 2: Game 2

2.

(10) Show that the following game has at least one pure strategy Nash equilibrium.

		Player 2	
		A	B
Player 1	A	a*, a*	b, c
	B	c, b	d*, d*

a > c,d > b

(A,A) and (B,B) and in NE

3.

(10 × 2) In this game, each of two players can volunteer some of their spare time planting and cleaning up the community garden. They both like a nicer garden and the garden is nicer if they volunteer more time to work on it. However, each would rather that the other person do the volunteering. Suppose that each player can volunteer 0, 1, 2, 3, or 4 hours. If player 1 volunteers x hours and 2 volunteers y hours, then the resultant garden gives each of them a payoff equal to $\sqrt{x+y}$. Each player also gets disutility equal to the number of hours spent working in the garden. Hence, the total payoff of player 1 is $\sqrt{x+y} - x$ (and that of player 2 is $\sqrt{x+y} - y$).

(a) Write down the best response of a player to every strategy of the other player.

(b) Determine the Nash equilibria of the game.

a.

	0	1	2	3	4
0	0*, 0*	1*, 0*	1.414, -0.586	1.732*, -1.268	2*, -2
1	0*, 1	0.414, 0.414*	0.732, -0.268	1, -1	1.236,-1.764
2	-0.586, 1.414*	-0.268, 0.732	0, 0	0.236, -0.764	0.449, -1.551
3	-1.268, 1.732*	-1, 1	-0.764, 0.236	-0.551, -0.551	-0.354, -1.354
4	-2, 2*	-1.764, 1.236	1.551*, 0.449	-1.354, -0.354	-1.172, -1.172

b. Nash equilibria: (0,0), (0,1)

4.

(10 × 3) Suppose two firms produce slightly different products. In particular, letting the outputs be denoted by q_1 and q_2 and the prices set by each firm be denoted by p_1 and p_2, suppose the two demand curves are

$$p_1 = a - bq_1 - dq_2$$
$$p_2 = a - bq_2 - dq_1$$

where $b > 0$ and $d > 0$ (and any quantity, including fractions, can be produced). Suppose also that the costs of producing a unit of output is the same for both firms and is equal to c dollars where $a > c$.

(a) Compute the best response strategy of each firm.

(b) Compute the Nash equilibrium.

(c) Compute the payoff of each firm.

Profit firm1: U1(*q1*,*q2*) = D(*q1*,*q2*)**q1* - c**q1*
Profit firm2: U1(*q1*,*q2*) = D(*q1*,*q2*)*q2 - c**q2*

Max*q1*U1(*q1*,*q2*) = max*q1*(a-b*q1*-d*q2*)**q1* -c **q1*
Max*q2*U1(*q1*,*q2*) = max*q2*(a-b*q2*-d*q1*)**q2* -c **q2*

(a - b*q1* - d*q2*)**q1* - c**q1* = 0
b*q1* - a + d*q2* + c = 0
$q1 = \frac{(a - c - dq2)}{b}$

(a - b*q2* - d*q1*)**q2* - c**q2* = 0
b*q2* - a + d*q1* + c = 0
$q2 = \frac{(a - c - dq1)}{b}$

	q	q - 1
p	1, 0	$\frac{a-c}{d}$
1 - p	$\frac{a-c}{d}$	0, 1

a. Firm 1 chooses q1 = BR1(q2) = (a - c - d$q2$) / b
   Firm 2 chooses q2 = BR1(q1) = (a - c - d$q1$) / b

b. q1* = q2* = $\frac{a-c}{d}$
c. Firm 1 Payoff:
   p = 1 - p
   p = 1/2
   Firm 2 Payoff:
   q = 1 - q
   q = 1/2
   ==> ((1/2, 1/2), (1/2, 1/2) is and MSNE

5.

(10 × 2) Find all Nash equilibria (in both pure and mixed strategies) of the following games.

		Player 2	
		C	D
Player 1	C	0, 4	2, 1
	D	6, 3	1, 4

Table 3: Game 1

		Player 2	
		L	R
Player 1	T	3, 2	0, 0
	M	0, 1	3, 3
	D	1, 3	1, 2

Table 4: Game 2

Game 1
UC1 = 4p + 3 -3p => 1p = -3
UD1 = p + 4 - 4p => -3p = -4
P1 = -3, p2= -4
((-3,-4) is an MSNE)

UC2 = 2 - 2q
UD2 = 6q - 1 - q
Q1 = 1, q2 = 1/5
((1,1/5) is and MSNE)

Game 2

UL = $2p1 + p2 + 3 - 3p1 - 3p2 \Rightarrow -p1 - 2p2 + 3 = 0$
UR = $3p2 + 2 - 2p1 - 2p2 \Rightarrow p2 - 2p1 + 2 = 0$
$p1 = 2/5$, $p2 = 7/10$
((-2/5, -7/10, 11/10) is an MSNE)

UT = $3q1$
UM = $3 - 3q1$
UD = 0
$q1 = 0, 1$
((0, 1, 0) is an MSNE)