Revised Mar 27, 2012.

Homework Sets for Math 273 Spring 2012 Section 2

Each problem set has two parts. Write up and turn in the exercises from both parts. From the textbook exercises in part 1turn in only those marked with an *. Problems in Part 2 will be graded. However, the grader has a few points to give or withhold for completeness of the entire assignment (Parts 1 and 2), neatness, and clarity of your solutions.

Solutions to exercises in Part 2 will be posted to the course website shortly after the due date. Note that correct mathematical notation is expected and you must show appropriate work. **No late homework will be accepted.**

Homework 1 due Jan 19 (4 points for completeness, neatness and clarity of solutions)

Part 1

Section 10.5 # 5*, 6, 21, 27*, 28, 31
Section 10.6 # (3, 4)*, 5, 7, 11*, 13, 17*
Section 10.7 # 1, 2*, 5, (6, 7)*, 9, 17-22, 33, 35, (45, 49, 51)*
Section 10.8 # 1, 3*, 7*
Section 10.9 # 1*, 2, 3*, 7, 11, 12*

Part 2

1. [3 pts] For the curve C given by $x = 2 - t^3$, $y = 2t - 1$, $z = \ln t$:
   (a) Find the point where C intersects the *xz*-plane.
   (b) Find parametric equations of the tangent line at $(1,1,0)$.
   (c) Find an equation for the normal plane to C at $(1,1,0)$.
2. [3 pts] Find the traces of the surface $z^2 = \frac{x^2}{4} - y^2$ in the planes $x = k$, $y = k$, $z = k$. Then sketch the surface.

Homework 2 due Jan 26 (2 points for completeness, neatness and clarity of solutions)

Part 1

Section 11.1 # 1, 2, 5*, 7-8, 9*, 13-19, 25*, 27, (29, 33)*, 35, (41-43)*

Part 2

1. [4 pts] Which of the following is true? If false, find a counterexample.
   (a) The values of contour lines are always 1 unit apart.
   (b) Any contour diagram that consists of parallel lines comes from a plane.
   (c) The contour diagram of any plane consists of parallel lines.
   (d) Contour lines can never cross.
   (e) The closer the contours, the steeper the graph of the function.

2. [4 pts] For the function $f(x, y) = \sqrt{16 - x^2 - 16y^2}$, do the following.
   (a) Find the domain and range of the function f. Sketch the domain.
   (b) Sketch several level curves of the function f (indicate the height k of each curve).
   (c) Use the information obtained in part (b) to sketch the graph of f.

Homework 3 due Feb 2 (4 points for completeness, neatness and clarity of solutions)

Part 1

Section 11.1 # (37, 50)*, 51, 52, 54* Use Maple or other program for #37 and #54.

Section 11.3 # 1, 3, 7*, 8-10, (15, 19, 20, 21, 26, 29, 31)*, 37, 39*, 41, (43, 47, 51)*

Part 2

[2 pts each] From Section 11.3 do # 2, 6 and 42.

Homework 4 due Feb 9 (4 points for completeness, neatness and clarity of solutions)

Part 1

Section 11.4 # 1-4, (5-6, 11)*, 13, (15, 19)*, 21, 23*

*Sec*tion 11.5 # 3*, 4, 5, (7, 9, 13)*, 15, 17, (19, 23, 25)*

Also do the following as practice with solving systems of equations. For each function $f(x, y)$ below, find all (x, y) values that satisfy both equations $f_x = 0$ and $f_y = 0$.

(a) $f(x, y) = 3x^2 - 2xy + y^2 - 8y$
(b) $f(x, y) = x^2y - 2x^2 - 2y^3 + 2y^2 - 3$

Part 2

1. [2 pts] Consider the function $g(x, y) = x^3 - 7xy + e^y$.
   (a) Explain why the function g is differentiable at $(-1,0,0)$.
   (b) Find an equation for the tangent plane at $(-1,0,0)$.
   (c) Find the linearization of g at $(-1,0,0)$.
2. [2 pts] If $v = x^2\sin(y) + ye^{xy}$, where $x = s + 2t$, and $y = st$, use the Chain Rule to find $\partial v/\partial s$ and $\partial v/\partial t$ when $s = 0$ and $t = 1$.
3. [2 pts] For $f(x, y) = x^2 + y^3 - 6xy + 3x + 6y$, find all (x, y) values that satisfy both $f_x = 0$ and $f_y = 0$.

Homework 5 due Feb 16 (4 points for completeness, neatness and clarity of solutions)

Part 1

Section 11.6 # 3, 5*, 6, (7, 13, 17, 23, 33)*

A differentiable function $f(x, y)$ has the property that $f(1,3) = 7$ and $\nabla f(1,3) = \langle 2, -5\rangle$.

(a) Find the equation of the tangent line to the level curve of f through the point $(1,3)$.
(b) Find the equation of the tangent plane to the surface $z = f(x, y)$ at the point $(1,3,7)$.

Part 2

1. [3 points] The temperature at the point (x, y, z) is given, in degrees Celsius, by $T(x, y, z) = e^{-(x^2+y^2+z^2)}$.
   (a) Describe in words the shapes of the surfaces in which the temperature is constant.
   (b) Find ∇T.
   (c) You travel from the point $(1,0,0)$ to the point $(2,1,0)$. Find the rate of change of the temperature as you leave the point $(1,0,0)$. Give units.

2. [3 points] Find an equation of the tangent plane at (2,3,1) to the surface given by $x^2 + y^2 - xyz = 7$. Do this in two ways:
   (i) Viewing the surface as the level set of a function $F(x, y, z)$ of three variables.
   (ii) Viewing the surface as the graph of a function of two variables $z = f(x, y)$.

Can both ways be used to find the tangent plane to the surface $\cos(x+z) - e^{xz+2} = 0$? Explain.

Homework 6 due Feb 23 (4 points for completeness, neatness and clarity of solutions)

Part 1

Section 11.7 # 2, (3, 5)*, 7, 8, 15* (include graph or level curves), 25, 33

Part 2

1. [2 pts] Find the local maximum and minimum values and saddle point(s) of the function $f(x,y) = e^{-y}(x^2 - y^2)$.

2. [2 pts] A closed rectangular box with faces parallel to the coordinate planes has one bottom corner at the origin and the opposite corner in the first octant on the plane $3x + 2y + z = 1$.
   (a) Find a function $V(x,y)$ that gives the volume of the box.
   (b) Find all critical points of $V(x,y)$.
   (c) Use the second derivative test to find the maximum volume of such a box.

3. [2 pts] A critical point (a,b) is degenerate if $D(a,b) = 0$. Under what conditions on the constant k will the function $f(x,y) = kx^2 - 2xy + ky^2$ have a local nondegenerate minimum at $(0,0)$? What about a local maximum?

Homework 7 due Mar 2 (3 points for completeness, neatness and clarity of solutions)

Part 1

Section 11.7 # 25, 26, 33, 34*
Section 12.1 # 7, 11*, 15, (17, 18, 20)*, 21, 22*, 23, (24, 25)*, 33, (37, 40)*

Part 2

1. [3 pts] Find the absolute maximum and minimum of the function $f(x,y) = 4xy^2 - x^2y^2 - xy^3$ on the closed triangular region D in the xy-plane with vertices $(0,0)$, $(0,6)$, and $(6,0)$.

2. [2 pts] The integral $\iint_R (6-2y)\,dA$, where $R = [0,3] \times [0,2]$, represents the volume of a solid. Sketch the solid, and evaluate the integral by computing the volume.

3. [2 pts] Find the volume of the solid bounded on top by the surface $z = y\sqrt{x+y}$, on the bottom by the xy-plane, and on the sides by the planes $x = 0$, $x = 1$, $y = 0$, and $y = 1$.

Homework 8 due Mar 8 (4 points for completeness, neatness and clarity of solutions)

Part 1

Section 12.2 # 1, 2, 3*, 6, 7, (9, 15)*, 16-18, 19*, 21, 23, 31, (35, 37, 39)*, 40, 43, (44, 45)*, 46
Section 12.3 # 1*, 2, 3*, 4, 5*, 6, 7-25 odds: (9, 13, 17, 19, 23, 25)*

Part 2

[2 pts each] From section 12.2 do #24 and #34

[2 pts] From section 12.3 do #16.

Homework 9 due Mar 22 (4 points for completeness, neatness and clarity of solutions)

Part 1

Section 12.5 # 3-20, 25-35 odds: (9, 11, 13, 19, 25, 29)*
Section 12.6 # 3*, 5, 6*, 7, (9, 11)*, 17, 19, 21, (22, 27)*, 28
Section 12.7 # 1, 3*, 5-7, 9, (11, 17, 19)*, 20, 21, (23, 25, 35)*

Part 2

1. [2 pts] Find the volume of the solid region bounded by the parabolid $z = x^2 + y^2$ and the plane $z = 5$ in two ways.
   (a) Express the volume as a double integral.
   (b) Express the volume as a triple integral.

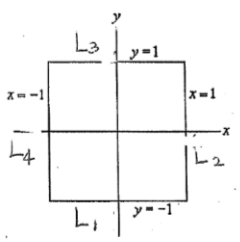
2. [2 pts] Compute $\iiint_E yz\, dV$, where E lies above the plane $z = 0$, below the plane $z = y$, and inside the cylinder $x^2 + y^2 = 4$.

3. [2 pts] Find the volume of the solid that lies within the sphere $x^2 + y^2 + z^2 = 4$, above the xy-plane, and below the cone $z = \sqrt{x^2 + y^2}$.

Homework 10 due Mar 29 (4 points for completeness, neatness and clarity of solutions; 3 problems to be graded)

1. Find the local maximum and minimum values, and saddle points of $f(x, y) = 2xy - x^2y - y^3$.

2. Find the absolute maximum and minimum values of $f(x, y) = x^2 + y^2 + x^2y + 4$ on the region $D = \{(x, y)| \ \ |x| \le 1, \ \ |y| \le 1\}$. Below are the absolute maximum and minimum values on three of the sides.



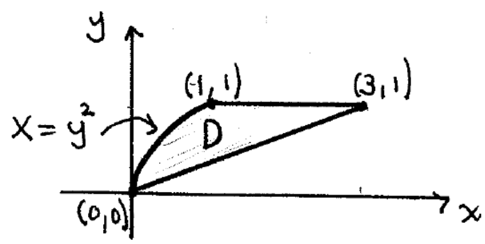
On the side L_1, $f = 5$ for any point along that side.

	Maximum on that side	Minimum on that side
On side L_2	$f = 7$ attained at $(1,1)$	$f = \frac{19}{4}$ attained at $\left(1, -\frac{1}{2}\right)$
On side L_4	$f = 7$ attained at $(-1,1)$	$f = \frac{19}{4}$ attained at $\left(-1, -\frac{1}{2}\right)$

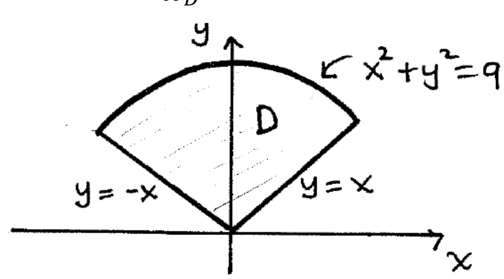
Note that you are given the maximum and minimum values of f along three sides. You only need to find the local maximum or minimum inside the region D, and the maximum and minimum along the side L_3. Using these information on the inside and boundary you can determine the absolute maximum and minimum over D.

3. For each of the regions shown below set up, but do not evaluate, the integral $\iint_D (x^2y + y^3)\, dA$.

(a)



(b)



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4. For the integral $\iint_D x\cos(y^2)\,dA = \int_0^2 \int_{x^2}^4 x\cos(y^2)\ dy\,dx$.
   (a) Sketch the region of integration D. Clearly label the sides and vertices.
   (b) Evaluate the integral by first reversing the order of integration.

5. For the integral $\iiint_G \frac{1}{y^2}\,dV = \int_2^4 \int_0^1 \int_0^{1-x} \frac{1}{y^2}\ dz\,dx\,dy$.
   (a) Evaluate the integral.
   (b) Describe and sketch the region of integration G.
   (c) Fill in the missing limits of integration using the order of integration shown:

$$\iiint_G \frac{1}{y^2}\,dV = \int \quad \int \quad \int \quad \frac{1}{y^2}\ dy\,dz\,dx$$

6. Let E be the solid region enclosed by the paraboloid $z = 1 + x^2 + y^2$, the cylinder $x^2 + y^2 = 5$, and the plane $z = 0$. Evaluate $\iiint_E e^z\,dV$.

7. Evaluate $\iiint_F z\,dV$, where F is the region inside the sphere $x^2 + y^2 + z^2 = 1$ in the first octant.

Homework 11 due April 5 (3 points for completeness, neatness and clarity of solutions)

Part 1

Section 12.8 # (2, 5, 7, 9, 10, 11, 13, 15, 19, 21)*, 22, 23

Part 2

1. Consider the iterated integral $\int_0^1 \int_{y/2}^{(y/2)+2} (2x - y)\,dxdy$.
   (a) [2 pts] Evaluate this integral and sketch the region D of integration in the xy-plane.
   (b) [1 pt] Let $u = 2x - y$ and $v = y$. Find the region D^* of integration in the uv-plane that corresponds to D.
   (c) [2 pts] Evaluate this integral by using the substitution $u = 2x - y$ and $v = y$.

2. [2 pts] Evaluate $\iint_D (2x + y)^2 e^{x-y}\ dA$ where D is the region enclosed by $2x + y = 1$, $2x + y = 4$, $x - y = -1$, and $x - y = 1$.