Sheet 6

Case 1

$\delta = 1$ Lyapunov condition yields

$$\begin{aligned}\frac{1}{\sigma_n^{2+1}} \sum_{i=1}^{n} \mathbb{E}(|X_{ni}|^{2+1}) &= \frac{1}{\sigma_n^3} \sum_{i=1}^{n} \lambda \\ &= \frac{1}{(n\lambda)^{3/2}} (n\lambda) \\ &= \frac{1}{\sqrt{n\lambda}} \xrightarrow[n\to\infty]{} 0\end{aligned}$$

Case 2

Newly computing the variance:

$$\begin{aligned}\sigma_n^2 = Var(S_n) &= Var\left(\sum_{i=1}^{n} X_i\right) \\ &= \sum_{i=1}^{n} \frac{\lambda}{n} \\ &= \lambda\end{aligned}$$

Again Lyapunov-condition:

$$\begin{aligned}\frac{1}{\sigma_n^3} \sum_{i=1}^{n} \mathbb{E}(|X_{ni}|^3) &= \frac{1}{\lambda^{3/2}} \sum_{i=1}^{n} \frac{\lambda}{n} \\ &= \frac{1}{\lambda^{3/2}} \lambda \\ &= \frac{1}{\sqrt{\lambda}}\end{aligned}$$

This does not converge to zero as $n \to \infty$.

Case 3

Variance can be computed by:

$$\begin{aligned}
\sigma_n^2 = Var(S_n) &= Var\left(\sum_{i=1}^{n} X_{ni}\right) \\
&= (n-1)\lambda + \frac{n-1}{n-1}\lambda \\
&= (n-1+1)\lambda \\
&= n\lambda
\end{aligned}$$

From here it is analog to case 1