# Controlling atomic wave packets at the quantum speed limit

Manolo Rivera Lam,[1] Natalie Peter,[1] Thorsten Groh,[1] Wolfgang Alt,[1] Carsten Robens,[2]
Dieter Meschede,[1] Antonio Negretti,[3] Tommaso Calarco,[4] and Andrea Alberti[1]

[1]*Institut für Angewandte Physik, Universität Bonn, Wegelerstraße 8, D-53115 Bonn, Germany*
[2]*Departement of Physics, Massachusetts Institute of Technology, Cambridge, MA 02139*
[3]*Zentrum für Optische Quantentechnologien and The Hamburg Centre for
Ultrafast Imaging, Universität Hamburg, Luruper Chaussee 149, D-22761 Hamburg*
[4]*Forschungszentrum Jülich, Wilhelm-Johnen-Straße, 52428 Jülich, Germany*

In quantum computing and related quantum technologies, executing quantum processes fast is key to overcome the adverse effects caused by decoherence. There is, however, a minimum time necessary to carry out a given quantum process. We use a single-atom Mach-Zehnder interferometer to reveal such a quantum speed limit in the process of splitting and recombining atomic wave packets by means of movable optical potentials. Relying on quantum optimal control theory, we steer the movable potentials so as to minimize motional excitations of the atomic wave packet after transport. The fidelity of the process measured as a function of the transport duration exhibits a behavior that is believed to be universal. These results give a paradigmatic demonstration of the existence of a quantum speed limit to which quantum technologies are subject, and constitute an important step on the way to quantum computing with neutral atoms in optical lattices.

Quantum computers process information more efficiently than classical computers by harnessing the power of quantum superposition states. These states, though, are extremely vulnerable to environment-induced noise, which entangles the system's degrees of freedom with those of the environment, leading to rapid decoherence. Fortunately, the detrimental effects caused by decoherence can be cured using quantum error correction protocols [1–9]. For fault-tolerant quantum computation, however, quantum error correction protocols require that the fidelity of the individual quantum processes involved in the computation reaches a minimal threshold level [10].

Thus, several techniques have been put forward in order to enhance the fidelity of quantum processes. These include spin-echo pulses [11–13] to suppress nearly static noise, dynamical decoupling [14–19] of the quantum system from its noisy environment, decoherence-free subspaces [20–22] for magnetic-field insensitive qubits, holonomic quantum gates [23, 24] relying on fault-tolerant geometrical effects, surface codes [25] relying on topological robustness, and fast driving protocols [26, 27] allowing one to mitigate the effects of decoherence by outrunning it. Among these techniques, fast driving protocols hold center stage, for they can work in conjunction with virtually any of the other techniques, and are applicable to all quantum systems. It is thus relevant to determine how fast quantum processes can be performed and what defines their *quantum speed limit.* Moreover, controlling a system at its quantum speed limit is not only relevant for quantum technologies, but also because it helps us better understand fundamental aspects about the evolution of quantum systems.

It has long been known in classical physics that physical processes require a minimal time to carry out a certain transformation, as exemplified by Bernoulli's famous brachistochrone problem [28]. Such a speed limit reflects the fact that the energy available to control any process is ultimately finite. By the same token, it is thus natural to expect that an analogous speed limit also applies to quantum processes. Indeed, such a quantum speed limit has been recognized by Mandelstam and Tamm [29], observing that the maximum rate of change of a quantum state is bound by its energy uncertainty. In fact, according to the generalization of this result to time-dependent quantum systems, the following inequality holds [30]:

$$\frac{\cos^{-1}(|\langle \psi_A | \psi_B \rangle|)}{\tau} \leq \frac{\langle \Delta E \rangle}{\hbar}, \tag{1}$$

where $\tau$ is the duration of a quantum process transforming a pure state $|\psi(0)\rangle := |\psi_A\rangle$ into another pure state $|\psi(\tau)\rangle := |\psi_B\rangle$, $\langle \Delta E \rangle := \tau^{-1} \int_0^\tau dt\, \langle \psi(t)| \hat{H}(t) |\psi(t)\rangle$ is the time-averaged energy uncertainty associated with the Hamiltonian $\hat{H}(t)$ of the system, and $\hbar$ is the reduced Planck constant. It should be noted that the inverse cosine in Eq. (1) represents the Fubini-Study distance between $|\psi_A\rangle$ and $|\psi_B\rangle$ [31], quantifying of how much the evolved state differs from the initial one. Importantly, this bound can be saturated only in a two-level system [32], where the initial and final states are exclusively coupled to each other through a Rabi coupling. When this occurs, the system is said to have evolved along a quantum brachistochrone [33], which has been experimentally verified in effectively two-level setups [34, 35]. In all other cases, the Mandelstamm-Tamm bound (with its generalization comprising the Margolus-Levitin bound [36]) can only be attained asymptotically [37].

Owing to its universal character and its practical implications in quantum technologies, the quantum brachistochrone problem has attracted great interest by the scientific physics community in recent years [38]. However, the foregoing definition of quantum brachistochrone relying on the Mandelstam-Tamm bound has drawn criticism [39], since a direct Rabi coupling between initial and final state is only relevant for a limited number of systems. In the majority of quantum systems, instead, only local couplings are physically realizable, which thus excludes the

possibility to directly couple states that are nonlocally connected. For these systems, the bound in Eq. (1) fails to capture the true quantum speed limit. In this work, we give an experimental demonstration of quantum control of a physical system at its quantum speed limit, where the initial and final states are not locally connected. More specifically, we consider the problem of transporting an atomic matter wave from a site A to a site B of an optical lattice potential, where $\langle \psi_A | \psi_B \rangle = 0$. Naturally, our experimental work only involves direct coupling between locally connected states. In addition, we assume a physical constraint on the maximum trap depth of our optical lattice potential, as this reflects the fact that there is eventually a maximum energy available to control the quantum process. The problem of transporting atomic matter waves has direct practical interest, and has been experimentally investigated to ... [40] [41] [42] [43]

---

These are quantum processes represented by a unitary operator that acts in general onto a large Hilbert space.

- This is shown by the problem of transport ... - Cite paper by D.G.Odelin, Wineland, Schmidkaler, and also J.Sherson.

Here we report on the experimental realization of fast, high-fidelity transport of atomic wave packets in deep optical lattices. The goal is to transport atoms by one or more lattice sites in the shortest time allowed by quantum mechanics, under the constraint that no motional excitation is created after transport, and the optical lattice depth does not exceed a maximum value given by the available resources (e.g., finite laser power). To achieve fast atom transport, we use quantum optimal control, which allows several motional excitations to be created during the transport process, and yet refocus them back into the motional ground state with a fidelity > 99%. Optimizing the process for various transport times, we clearly observe a minimum time below which transport operations unavoidably create motional excitations. This minimal time defines the *quantum speed limit* for the transport operation.

To be cited: [44] for preserving quantum information during transport. Quantum computing with neutral atoms [45].

Also to be cited [39] for QSL along geometric path, and [30] for geometric interpretation of QSL.

These transformations must be performed with high fidelity, because this allows quantum error correction to be put in place. There is in fact a threshold to the fidelity, below which corrections can be applied repeatedly.

transform quantum states with a high quantum fidelity. In most quantum hardware, one defines a conventional set of states forming the computational basis. This is however a simplification.

require controlling and processing quantum states. in large Hilbert spaces which involve a large number of internal and external degrees of freedom.

Keywords:

General introduction:

- Importance of quantum technologies, which requires complex transformation of states including both internal and external degrees of freedom. Here it is important to reach high fidelity.

- State what our definition of quantum speed limit is (which is inspired by relevant quantum technologies). Explain that this definition is compatible with the one assumed by most of scientific literature. - Explain the idea of a S-like transition between quantum controllable and non controllable system, stressing that this is a rather universal behavior.

System:

- We control a nontrivial Hilbert space with quantum optical control, involving 10 motional states. - We show quantum application of this concept controlling both internal and external degrees of freedom (spin-dependent optical lattices play an important role for that) - Beyond adiabatic concept, we aim at reaching high fidelity in the minimal time allowed by Schrödinger equation.

- Define basic problem of transport, introducing atomic potentials for both spin states. Refer to Fig.1(a) which exemplifies the problem for one spin species. - Explain that fast transport is possible only through a highly wiggling ramp, far away from the adiabatic limit. Fig.1(b) - Show that this is indeed realized experimentally in our system, where we need to overcome the problem of the limited bandwidth. We do it by deconvolving the theoretical ramp with the response function of the system. - Insight about the shape of the ramp, explaining for example why at the beginning the ramp is steeper. - Mentioning that this fast transport creates many excitations and explores a large Hilbert space, Fig. 1(c).

Experimental apparatus:

List of questions:

- How does it look like the fidelity for highly excited states. Can one optimize transport for both ground state and first excited state? - What is the best fidelity in transport? Either measure or estimate from current measurements.

## I. INTRODUCTION

- In the past decades has been a tremendous experimental and theoretical progress towards coherent control of quantum technologies such as atomic clocks, quantum communications and quantum signal processing.

- Powerful methods has been discovered to overcome the problem of decoherence such as quantum error correcting codes [46] or identifying shortcuts [34, 47] and decoherence-free subspaces.

- All processes gain from fast processes and robust control. Besides combating decoherence, quantum

computer architectures also gain by speeding up quantum information protocols [48–51].

- Two major challenges of high-speed operations are to overcome technical limitations in order to reach an extremely high degree of quantum control and to determine and realize optimal solutions for systems with many-degrees of freedom.

Brief introduction of our system:

- In previous work, we have demonstrated fast, high-precision transport of cesium atoms in two state-dependent optical lattices [52] and ... (gives some nice applications of our previous results).

- Here, we report the realization of optimal transport by means of optimal control theory under the condition to preserve the motional ground state and identify the quantum speed limit as a sudden decrease of the transport fidelity in good agreement with the numerical and analytical expectation.

Detailed explanation of our setup:

- describing our system in more details [53] and give some recent results. We do ultra-fast, optimal control transport across the quantum speed limit of a cesium atom in an optical lattice over one lattice site as shown in Fig. 1 a) while maintaining the ground state after the process.

- then I want to describe the control parameters and its realization in the system: depth $U_0(t)$, phase $\varphi(t)$

$$\hat{H} = \frac{\hat{p}^2}{2m} + U_0(t)\cos^2(k\hat{x} - \varphi(t)/2), \qquad (2)$$

$$\hat{U}_{\text{evo}}(T) = e^{-\frac{i}{\hbar}\int_0^T \hat{H}(t')dt'} \qquad (3)$$

- and give a description of its experimental realization with phase and amplitude ramps Fig. 1 b) and our challenges. There I want to mention that it is difficult to realize ramps on such short time scales (impulse response, deconvolution, ...). Another challenge is how to measure the transport fidelity. Here we present our measurement protocol from Belmechri [54]), Fig. 2 a), to determine the fidelity directly (MW sideband cooling with 99 % efficiency [52]

- the we discuss the measurement result, Fig. 2 b). The challenge is the number of repetitions vs. decoherence.

- Then I describe the major challenge how to find transport ramps, since there exists no analytic solution for a transport ramp. We approach the QSL numerically by using optimal control theory [55–57]

- then I describe how we get optimal control solutions from simulation (cost function, discrete parametrization, ...)

$$\mathcal{F} = |\langle\psi_{\text{target}}|\hat{U}_{\text{evo}}(T)|\psi_{\text{init}}\rangle|^2 \qquad (4)$$

$$
\begin{aligned}
U(t) &= U_0 &&+ \sum_{m=0}^{M} a_m \sin(\nu_m t), \quad t \in [0, T] \quad, \\
\varphi(t) &= \frac{2\pi}{T}t &&+ \sum_{m=0}^{M} b_m \sin(\nu_m t), \quad t \in [0, T] \quad.
\end{aligned}
\qquad (5)
$$

## RESULTS

The first paragraph discuss the result of the linear and quantum speed limit measurement. In detail:

- then discuss the measurement linear transport vs. optimal control, Fig. 3 a). Linear transport was used by many other groups to speed up their transport, but we can do it better. The measurement with optimal control solution Fig. 3 b) demonstrates a significant improvement and we can identify the quantum speed limit as a sharp border.

- The data is well explained by our theoretical model, which is sensitive to the finite radial temperature as the only free parameter = thermometer

- in related work they have shown a single measurement close to the expected quantum speed limit with Bose-Einstein condensates [34, 58].

- Linear transport in the non-adiabatic regime has been demonstrated in previous work to speed up transport operations with ions [41, 42, 59] and neutral atoms [40, 47].

- In addition discuss the analytic solution of the QSL, which scales with inverse of the trap frequency (formula)

- then discuss the fact, that the classical speed limit has the same scaling

$$\tau^{\text{classical}} = \sqrt{\frac{2\,d\,m\,\lambda}{\pi\,U_0}}. \qquad (6)$$

The second paragraph describes the interferometer as a nice application. In detail:

- first describe an atom interferometer [60] Fig. 4 a), measurement of Ramsey fringe contrast, our challenge is the cross talk of the lattices, which we have to compensate

- discuss measurement result Fig. 4 b), when using the interferometer at the QSL

## CONCLUSION AND OUTLOOK

- Then I want to describe the source of the QSL, the Heisenberg's uncertainty relation for energy and time [30, 38]

- There exist few papers of state-of-the-art experiments measuring close to the quantum speed limit, which I want to cite [34, 58]

- one sentence conclusion, importance for all experiments, which suffer from decoherences (examples)

- outlook: macroscopically separated quantum superpositions on the millimeter scale for interferometry, quantum walks [61], sensing of external forces, molecule formation [62], but also for fundamental tests of quantum mechanics such as proving bosonic versus fermionic statistics [63]

---

[1] A. R. Calderbank and P. W. Shor, "Good quantum error-correcting codes exist," Phys. Rev. A **54**, 1098 (1996).

[2] A. Steane, "Multiple-Particle Interference and Quantum Error Correction," Proc. R. Soc. A **452**, 2551 (1996).

[3] A. M. Steane, "Error Correcting Codes in Quantum Theory," Phys. Rev. Lett. **77**, 793 (1996).

[4] D. G. Cory, M. D. Price, W. Maas, E. Knill, R. Laflamme, W. H. Zurek, T. F. Havel, and S. S. Somaroo, "Experimental Quantum Error Correction," Phys. Rev. Lett. **81**, 2152 (1998).

[5] J. Chiaverini, D. Leibfried, T. Schaetz, M. D. Barrett, R. B. Blakestad, J. Britton, W. M. Itano, J. D. Jost, E. Knill, C. Langer, R. Ozeri, and D. J. Wineland, "Realization of quantum error correction," Nature **432**, 602 (2004).

[6] P. Schindler, J. T. Barreiro, T. Monz, V. Nebendahl, D. Nigg, M. Chwalla, M. Hennrich, and R. Blatt, "Experimental Repetitive Quantum Error Correction," Science **332**, 1059 (2011).

[7] M. D. Reed, L. Dicarlo, S. E. Nigg, L. Sun, L. Frunzio, S. M. Girvin, and R. J. Schoelkopf, "Realization of three-qubit quantum error correction with superconducting circuits," Nature **482**, 382 (2012), 1109..

[8] J. Kelly, R. Barends, A. G. Fowler, A. Megrant, E. Jeffrey, T. C. White, D. Sank, J. Y. Mutus, B. Campbell, Y. Chen, Z. Chen, B. Chiaro, A. Dunsworth, I. C. Hoi, C. Neill, P. J. J. O'Malley, C. Quintana, P. Roushan, A. Vainsencher, J. Wenner, A. N. Cleland, and J. M. Martinis, "State preservation by repetitive error detection in a superconducting quantum circuit," Nature **519**, 66 (2015).

[9] N. M. Linke, M. Gutierrez, K. A. Landsman, C. Figgatt, S. Debnath, K. R. Brown, and C. Monroe, "Fault-tolerant quantum error detection." Sci. Adv. **3**, e1701074 (2017).

[10] E. Knill, R. Laflamme, and W. H. Zurek, "Resilient quantum computation: error models and thresholds," Proc. R. Soc. A **454**, 365 (1998).

[11] M. F. Andersen, A. Kaplan, and N. Davidson, "Echo Spectroscopy and Quantum Stability of Trapped Atoms," Phys. Rev. Lett. **90**, 23001 (2003).

[12] S. Kuhr, W. Alt, D. Schrader, I. Dotsenko, Y. Miroshnychenko, A. Rauschenbeutel, and D. Meschede, "Analysis of dephasing mechanisms in a standing-wave dipole trap," Phys. Rev. A **72**, 023406 (2005).

[13] G. Kleine Büning, J. Will, W. Ertmer, E. Rasel, J. Arlt, C. Klempt, F. Ramirez-Martinez, F. Piéchon, and P. Rosenbusch, "Extended Coherence Time on the Clock Transition of Optically Trapped Rubidium," Phys. Rev. Lett. **106**, 240801 (2011).

[14] L. Viola and S. Lloyd, "Dynamical suppression of decoherence in two-state quantum systems," Phys. Rev. A **58**, 2733 (1998).

[15] E. M. Fortunato, L. Viola, J. Hodges, G. Teklemariam, and D. G. Cory, "Implementation of universal control on a decoherence-free qubit," New. J. Phys. **4**, 5 (2002).

[16] E. Fraval, M. J. Sellars, and J. J. Longdell, "Dynamic Decoherence Control of a Solid-State Nuclear-Quadrupole Qubit," Phys. Rev. Lett. **95**, 30506 (2005).

[17] M. J. Biercuk, H. Uys, A. P. VanDevender, N. Shiga, W. M. Itano, and J. J. Bollinger, "Optimized dynamical decoupling in a model quantum memory," Nature **458**, 996 (2009).

[18] J. Du, X. Rong, N. Zhao, Y. Wang, J. Yang, and R. B. Liu, "Preserving electron spin coherence in solids by optimal dynamical decoupling," Nature **461**, 1265 (2009).

[19] Y. Sagi, I. Almog, and N. Davidson, "Process Tomography of Dynamical Decoupling in a Dense Cold Atomic Ensemble," Phys. Rev. Lett. **105**, 53201 (2010).

[20] J. P. Home, D. Hanneke, J. D. Jost, J. M. Amini, D. Leibfried, and D. J. Wineland, "Complete methods set for scalable ion trap quantum information processing." Science **325**, 1227 (2009).

[21] N. Timoney, I. Baumgart, M. Johanning, A. F. Varón, M. B. Plenio, A. Retzker, and C. Wunderlich, "Quantum gates and memory using microwave-dressed states," Nature **476**, 185 (2011).

[22] Y. Wang, A. Kumar, T. Y. Wu, and D. S. Weiss, "Single-qubit gates based on targeted phase shifts in a 3D neutral atom array," Science **352**, 1562 (2016).

[23] J. Pachos and P. Zanardi, "Quantum Holonomies for Quantum Computing," Int. J. Mod. Phys. B **15**, 1257 (2001).

[24] L. M. Duan, J. I. Cirac, and P. Zoller, "Geometric Manipulation of Trapped Ions for Quantum Computation," Science **292**, 1695 (2001).

[25] E. Dennis, A. Kitaev, A. Landahl, and J. Preskill, "Topological quantum memory," J. Math. Phys. **43**, 4452 (2002).

[26] W. C. Campbell, J. Mizrahi, Q. Quraishi, C. Senko, D. Hayes, D. Hucul, D. N. Matsukevich, P. Maunz, and C. Monroe, "Ultrafast Gates for Single Atomic Qubits," Phys. Rev. Lett. **105**, 90502 (2010).

[27] V. M. Schäfer, C. J. Ballance, K. Thirumalai, L. J. Stephenson, T. G. Ballance, A. M. Steane, and D. M. Lucas, "Fast quantum logic gates with trapped-ion qubits,"
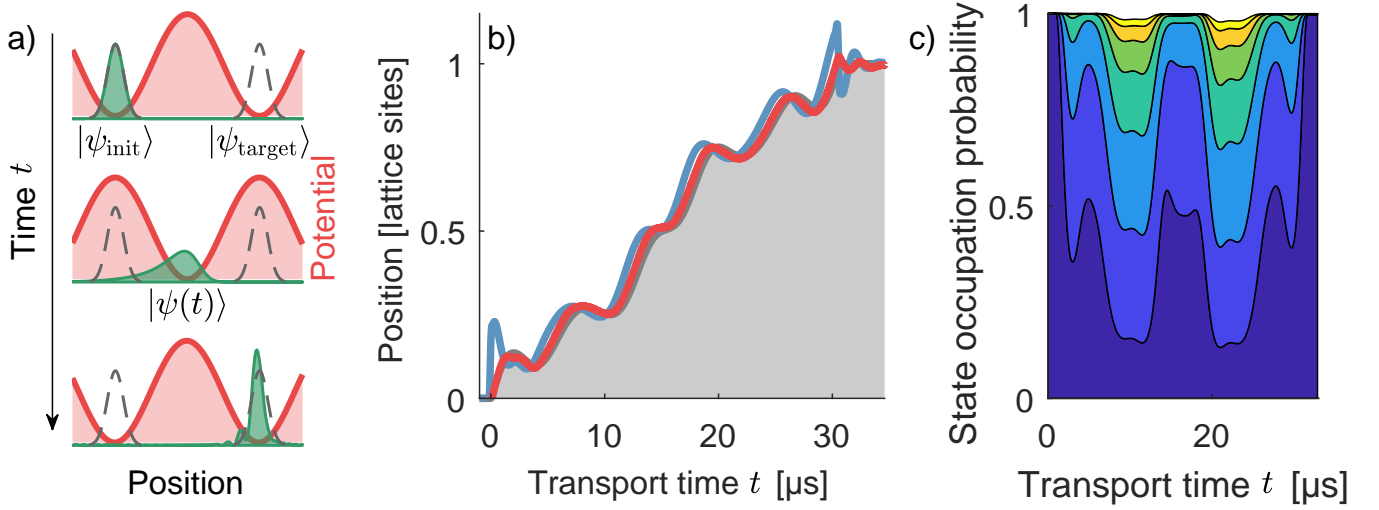
Figure 1. Scheme of atom transport in an optical lattice (red): a) shows the probability distribution $|\psi(t)\rangle$ of an atom (green) transported over one lattice site. The fidelity to transfer the atom from the initial to the final motional ground state in a sinusoidal potential is 50 %. b) compares the optimal control solution with a linear and adiabatic phase ramp in order to reach a transport fidelity of 99% in the shortest possible duration $T$ for a constant trap depth $U_0 = 25\,k_B\,\mu\text{K}$.
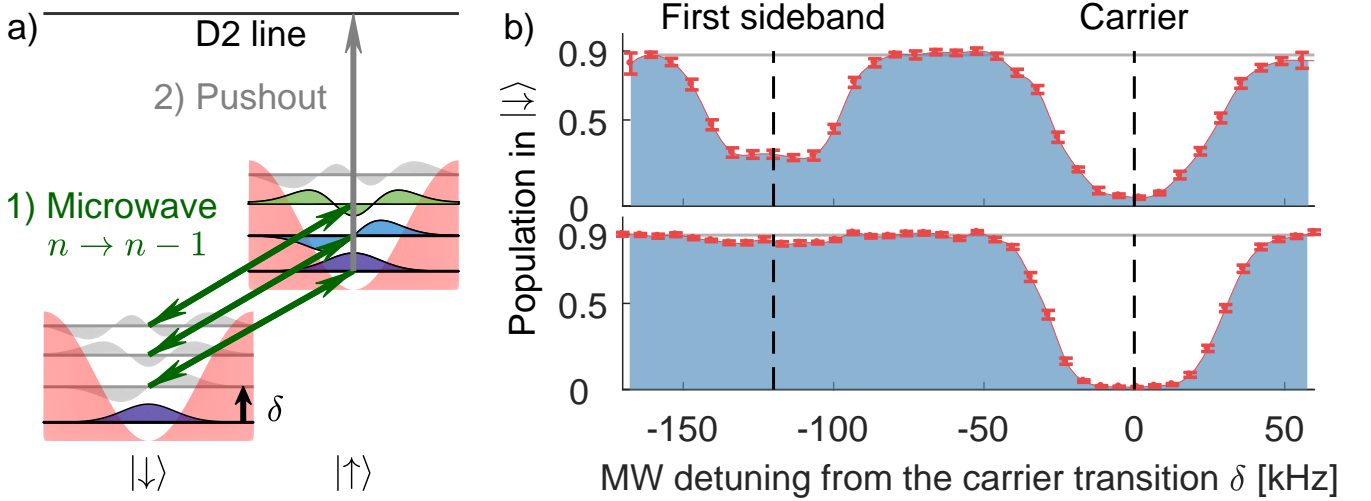


Figure 2. Precision measurement of ground state occupation. Avoid filling of non occupied motional state in the lower potential.

Nature **555**, 75 (2018).

[28] L. Haws and T. Kiser, "Exploring the Brachistochrone Problem," Amer. Math. Monthly **102**, 328 (1995).

[29] L. Mandelstam and I. Tamm, "The Uncertainty Relation Between Energy and Time in Non-relativistic Quantum Mechanics," J. Phys. USSR **9**, 249 (1945).

[30] J. Anandan and Y. Aharonov, "Geometry of quantum evolution," Phys. Rev. Lett. **65**, 1697 (1990).

[31] I. Bengtsson and K. Życzkowski, *Geometry of Quantum States: An Introduction to Quantum Entanglement*, 2nd ed. (Cambridge University Press, Cambridge, UK, 2017).

[32] A. Uhlmann and B. Crell, "Geometry of State Spaces," in *Entanglement and Decoherence*, Lecture Notes in Physics, Vol. 768, edited by A. Buchleitner, C. Viviescas, and M. Tiersch (Springer, Berlin, Heidelberg, 2009) p. 1.

[33] A. Carlini, A. Hosoya, T. Koike, and Y. Okudaira, "Time-optimal quantum evolution." Phys. Rev. Lett. **96**,

060503 (2006).

[34] M. G. Bason, M. Viteau, N. Malossi, P. Huillery, E. Arimondo, D. Ciampini, R. Fazio, V. Giovannetti, R. Mannella, and O. Morsch, "High-fidelity quantum driving," Nat. Phys. **8**, 147 (2012).

[35] A. Vepsäläinen, S. Danilin, and G. S. Paraoanu, "Superadiabatic population transfer in a three-level superconducting circuit," Sci. Adv. **5**, eaau5999 (2019).

[36] N. Margolus and L. B. Levitin, "The maximum speed of dynamical evolution," Physica. D **120**, 188 (1998).

[37] L. B. Levitin and T. Toffoli, "Fundamental limit on the rate of quantum dynamics: the unified bound is tight." Phys. Rev. Lett. **103**, 160502 (2009).

[38] S. Deffner and S. Campbell, "Quantum speed limits: from Heisenberg's uncertainty principle to optimal quantum control," J. Phys. A:. Math. Theor. **50**, 453001 (2017).
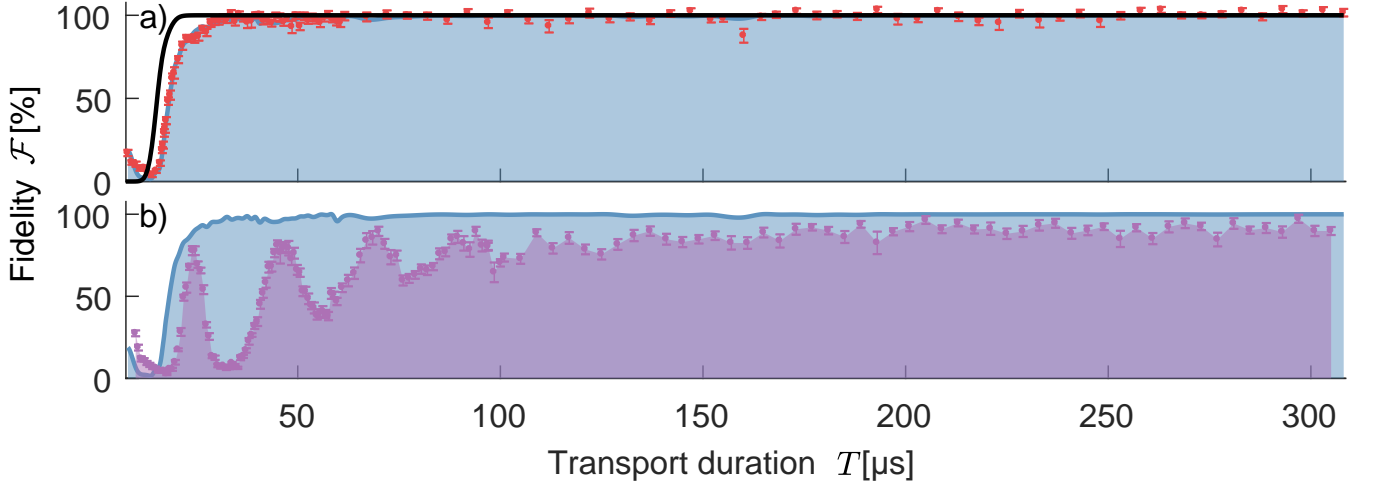
Figure 3. Measurement of the quantum speed limit (red): a) The fidelity for optimal control solutions suddenly reduces below a certain transport duration, which is identified as the quantum speed limit and shows as a significant improvement in comparison to linear driving ramps (purple). The result is verified by the theoretical expectation for a finite radial temperature (blue) and is even further decreased for a three-dimensional ground state (black).
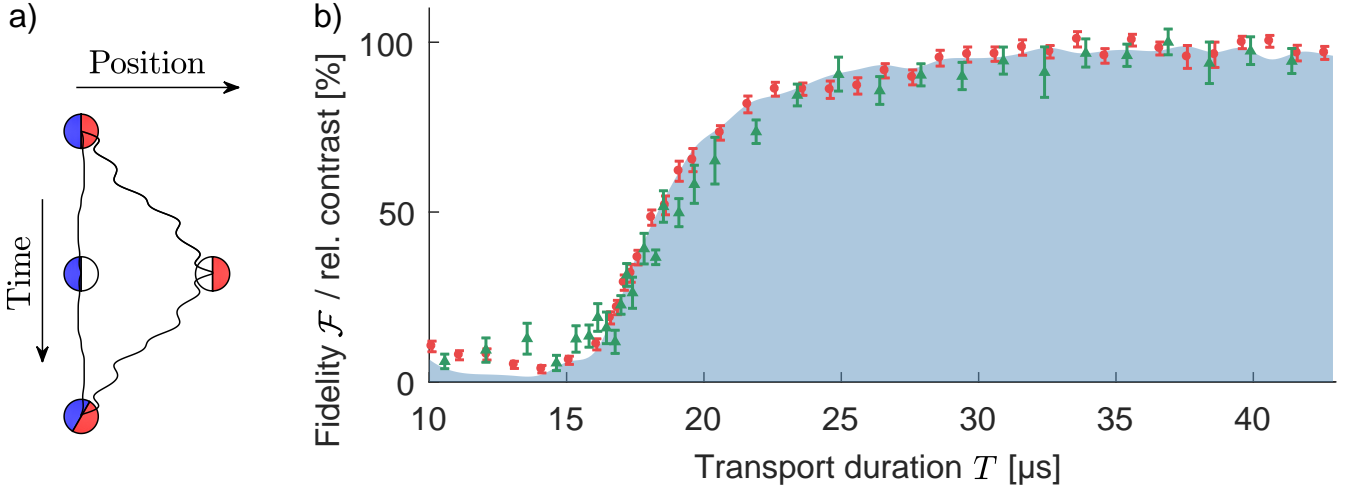


Figure 4. Interferometer: a) shows the Ramsey fringe contrast (green) of an atom interferometer by applying the ramps, which improves with the transport fidelity. b) The interferometer operates by spatially delocalizing an atom in a superposition of two states (blue and red colored circle) and a coherent recombination. The black lines describe the movement of the two state-dependent lattices.

[39] M. Bukov, D. Sels, and A. Polkovnikov, "Geometric Speed Limit of Accessible Many-Body State Preparation," Phys. Rev. X **9**, 011034 (2019).

[40] A. Couvert, T. Kawalec, G. Reinaudi, and D. Guéry-Odelin, "Optimal transport of ultracold atoms in the non-adiabatic regime," Europhys. Lett. **83**, 13001 (2008).

[41] R. Bowler, J. Gaebler, Y. Lin, T. R. Tan, D. Hanneke, J. D. Jost, J. P. Home, D. Leibfried, and D. J. Wineland, "Coherent Diabatic Ion Transport and Separation in a Multizone Trap Array," Phys. Rev. Lett. **109**, 257 (2012).

[42] A. Walther, F. Ziesel, T. Ruster, S. T. Dawkins, K. Ott, M. Hettrich, K. Singer, F. Schmidt-Kaler, and U. Poschinger, "Controlling Fast Transport of Cold

Trapped Ions," Phys. Rev. Lett. **109**, 80501 (2012).

[43] J. J. W. H. Sørensen, M. K. Pedersen, M. Munch, P. Haikka, J. H. Jensen, T. Planke, M. G. Andreasen, M. Gajdacz, K. Mølmer, A. Lieberoth, and J. F. Sherson, "Exploring the quantum speed limit with computer games." Nature **532**, 210 (2016).

[44] P. Kaufmann, T. F. Gloger, D. Kaufmann, M. Johanning, and C. Wunderlich, "High-Fidelity Preservation of Quantum Information During Trapped-Ion Transport," Phys. Rev. Lett. **120**, 010501 (2018).

[45] D. S. Weiss and M. Saffman, "Quantum computing with neutral atoms," Phys. Today **70**, 44 (2017).

[46] A. Y. Kitaev, *Quantum Error Correction with Imperfect*

*Gates* (Springer US, 1997).

[47] X. Chen, I. Lizuain, A. Ruschhaupt, D. Guéry-Odelin, and J. G. Muga, "Shortcut to Adiabatic Passage in Two- and Three-Level Atoms," Phys. Rev. Lett. **105**, 123003 (2010).

[48] R. Van Meter and C. Horsman, "A blueprint for building a quantum computer," Commun. ACM **56**, 84 (2013).

[49] D. Kielpinski, C. Monroe, and D. J. Wineland, "Architecture for a large-scale ion-trap quantum computer," Nature **417**, 709 (2002).

[50] C. Monroe and J. Kim, "Scaling the Ion Trap Quantum Processor," Science **339**, 1164 (2013).

[51] C. Weitenberg, S. Kuhr, K. Mølmer, and J. F. Sherson, "Quantum computation architecture using optical tweezers," Phys. Rev. A **84**, 32322 (2011).

[52] C. Robens, S. Brakhane, W. Alt, D. Meschede, J. Zopes, and A. Alberti, "Fast, High-Precision Optical Polarization Synthesizer for Ultracold-Atom Experiments," Phys. Rev. Appl. **9**, 034016 (2018).

[53] M. Karski, L. Förster, J.-M. Choi, A. Steffen, W. Alt, D. Meschede, and A. Widera, "Quantum Walk in Position Space with Single Optically Trapped Atoms," Science **325**, 174 (2009).

[54] N. Belmechri, L. Förster, W. Alt, A. Widera, D. Meschede, and A. Widera, "Microwave control of atomic motional states in a spin-dependent optical lattice," J. Phys. B:. At. Mol. Phys. **46**, 104006 (2013).

[55] P. Treutlein, T. W. Hänsch, J. Reichel, A. Negretti, M. A. Cirone, and T. Calarco, "Microwave potentials and optimal control for robust quantum gates on an atom chip," Phys. Rev. A **74**, 22312 (2006).

[56] R. Bücker, J. Grond, S. Manz, T. Berrada, T. Betz, C. Koller, U. Hohenester, T. Schumm, A. Perrin, and J. Schmiedmayer, "Twin-atom beams," Nat. Phys. **7**, 608 (2011).

[57] S. Rosi, A. Bernard, N. Fabbri, L. Fallani, C. Fort, M. Inguscio, T. Calarco, and S. Montangero, "Fast closed-loop optimal control of ultracold atoms in an optical lattice," Phys. Rev. A **88**, 021601 (2013).

[58] S. van Frank, M. Bonneau, J. Schmiedmayer, S. Hild, C. Gross, M. Cheneau, I. Bloch, T. Pichler, A. Negretti, T. Calarco, and S. Montangero, "Optimal control of complex atomic quantum systems," Sci. Rep. **6**, 34187 (2016).

[59] H. Kaufmann, T. Ruster, C. T. Schmiegelow, M. A. Luda, V. Kaushal, J. Schulz, D. von Lindenfels, F. Schmidt-Kaler, and U. G. Poschinger, "Fast ion swapping for quantum-information processing," Phys. Rev. A **95**, 052319 (2017).

[60] A. Steffen, A. Alberti, W. Alt, N. Belmechri, S. Hild, M. Karski, A. Widera, and D. Meschede, "Digital atom interferometer with single particle control on a discretized space-time geometry," Proc. Natl. Acad. Sci. U.S.A. **109**, 9770 (2012).

[61] T. Groh, S. Brakhane, W. Alt, D. Meschede, J. K. Asbóth, and A. Alberti, "Robustness of topologically protected edge states in quantum walk experiments with neutral atoms," Phys. Rev. A **94**, 013620 (2016).

[62] L. R. Liu, J. D. Hood, Y. Yu, J. T. Zhang, N. R. Hutzler, T. Rosenband, and K. K. Ni, "Building one molecule from a reservoir of two atoms," Science **360**, 900 (2018).

[63] C. F. Roos, A. Alberti, D. Meschede, P. Hauke, and H. Häffner, "Revealing Quantum Statistics with a Pair of Distant Atoms," Phys. Rev. Lett. **119**, 160401 (2017).