Controlling atomic wave packets at the quantum speed limit

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Controlling quantum processes faster than the decoherence rate is a fundamental challenge for any technology based on quantum mechanics. Ultimately, however, quantum mechanics sets limits on the shortest time required to carry out a quantum process with ideal fidelity, which depends on the distance between the initial and final state, and on the energy available to control the process. We here demonstrate the existence of such a quantum speed limit by transporting single atoms in optical potentials between sites situated far apart. By splitting and recombining the wavefunction of each atom in a Mach-Zehnder type interferometer, we also verify that quantum coherence is preserved during the fast transport operations. Relying on quantum optimal control theory, we steer the movable potentials so as to maximize the overlap of the transported wave packet with the ground state wavefunction in the target site. The fidelity of the process measured as a function of the transport duration exhibits a behavior that is believed to be universal. These results are a paradigmatic demonstration of the existence of a quantum speed limit to which quantum technologies are subject, and constitute an important step on the way to quantum computing with neutral atoms in optical lattices.

Quantum computers process information more efficiently than classical computers by harnessing the power of quantum superposition states. These states, though, are extremely vulnerable to environment-induced noise, which entangles the system's degrees of freedom with those of the environment, leading to rapid decoherence. Fortunately, the detrimental effects caused by decoherence can be cured using quantum error correction protocols [1–9]. For fault-tolerant quantum computation, however, quantum error correction protocols require that the fidelity of the individual quantum processes involved in the computation reaches a minimal threshold level [10].

Thus, several techniques have been put forward in order to enhance the fidelity of quantum processes. These include spin-echo pulses [11–13] to suppress nearly static noise, dynamical decoupling [14–19] of the quantum system from its noisy environment, decoherence-free subspaces [20–22] for magnetic-field insensitive qubits, holonomic quantum gates [23, 24] relying on fault-tolerant geometrical effects, surface codes [25] relying on topological robustness, and fast driving protocols [26, 27] allowing one to mitigate the effects of decoherence by outrunning it. Among these techniques, fast driving protocols hold center stage, for they can work in conjunction with virtually any of the other techniques, and are applicable to all quantum systems. It is thus relevant to determine how fast quantum processes can be performed and what defines their *quantum speed limit*. Moreover, controlling a system at its quantum speed limit is not only relevant for quantum technologies, but also because it helps us better understand fundamental aspects about the evolution of quantum systems.

It has long been known in classical physics that physical processes require a minimal time to carry out a certain transformation, as exemplified by Bernoulli's famous brachistochrone problem [28]. Such a speed limit reflects the fact that the energy available to control any process is ultimately finite. By the same token, it is thus natural to expect that an analogous speed limit also applies to quantum processes. Indeed, such a quantum speed limit has been recognized by Mandelstam and Tamm [29], observing that the maximum rate of change of a quantum state is bound by its energy uncertainty. In fact, according to the generalization of this result to time-dependent quantum systems, the following inequality holds [30]:

$$\frac{\cos^{-1}(|\langle\psi_A|\psi_B\rangle|)}{\tau} \leq \frac{\langle \Delta E\rangle}{\hbar}, \tag{1}$$

where τ is the duration of a quantum process transforming a quantum state $|\psi_{\mathrm{A}}\rangle := |\psi(0)\rangle$ into $|\psi_{\mathrm{B}}\rangle := |\psi(\tau)\rangle$, $\hbar$ is the reduced Planck constant, and $\langle\Delta E\rangle$ is the time-averaged energy uncertainty derived from the system's Hamiltonian $\hat{H}(t)$ [31]. It should be noted that the inverse cosine on the left-hand side of Eq. (1) is a measure of how the evolved state differs from the initial one, expressed in terms of the Fubini-Study metric [32].

Owing to its universal character and its practical implications in quantum technologies, the quantum brachistochrone problem has been the subject of intense research in recent years [33, 34]. Importantly, it has been realized that the bound in Eq. (1) can only be saturated in simple two-level systems [33], where the initial and final states can be locally coupled to each other through a Rabi coupling. This was recently verified experimentally using effectively two-level setups based on ultracold atoms [35] and superconducting transmon circuits [36]. In more complex systems, however, the Mandelstamm-Tamm bound (with its generalization comprising the Margolus-Levitin bound [37]) can only be attained asymptotically [38], as the system's dynamics gradually reduces to two quantum states. For this reason, the definition of quantum brachistochrone in terms of the Mandelstam-Tamm bound has drawn criticism [39], since a direct Rabi coupling between initial and final state is only relevant in a limited number of setups. In the majority of quantum systems, instead, only local couplings are physically realizable, which thus excludes the possibility to directly couple states that are nonlocally connected. For these systems, the bound in Eq. (1) fails to capture the true quantum speed limit. In this work, we give an experimental demonstration of quantum control of a physical system at its quantum speed limit, where the initial and final states are not locally connected. More specifically, we consider the problem of transporting an atomic matter wave from a site A to a site B of an optical lattice potential, where $\langle\psi_A|\psi_B\rangle = 0$. Naturally, our experimental work only involves direct coupling between locally connected states. In addition, we assume a physical constraint on the maximum trap depth of our optical lattice potential, as this reflects the fact that there is eventually a maximum energy available to control the quantum process. The problem of transporting atomic matter waves has direct practical interest, and has been experimentally investigated to ... [40] [41] [42] [43]

These are quantum processes represented by a unitary operator that acts in general onto a large Hilbert space.

- This is shown by the problem of transport ... - Cite paper by D.G.Odelin, Wineland, Schmidkaler, and also J.Sherson.

Here we report on the experimental realization of fast, high-fidelity transport of atomic wave packets in deep optical lattices. The goal is to transport atoms by one or more lattice sites in the shortest time allowed by quantum mechanics, under the constraint that no motional excitation is created after transport, and the optical lattice depth does not exceed a maximum value given by the available resources (e.g., finite laser power). To achieve fast atom transport, we use quantum optimal control, which allows several motional excitations to be created during the transport process, and yet refocus them back into the motional ground state with a fidelity $> 99\%$. Optimizing the process for various transport times, we clearly observe a minimum time below which transport operations unavoidably create motional excitations. This minimal time defines the *quantum speed limit* for the transport operation.

To be cited: [44] for preserving quantum information during transport. Quantum computing with neutral atoms [45].

Also to be cited [39] for QSL along geometric path, and [30] for geometric interpretation of QSL.

These transformations must be performed with high fidelity, because this allows quantum error correction to be put in place. There is in fact a threshold to the fidelity, below which corrections can be applied repeatedly.

transform quantum states with a high quantum fidelity. In most quantum hardware, one defines a conventional set of states forming the computational basis. This is however a simplification.

require controlling and processing quantum states. in large Hilbert spaces which involve a large number of internal and external degrees of freedom.

Keywords:

General introduction:

- Importance of quantum technologies, which requires complex transformation of states including both internal and external degrees of freedom. Here it is important to reach high fidelity.

- State what our definition of quantum speed limit is (which is inspired by relevant quantum technologies). Explain that this definition is compatible with the one assumed by most of scientific literature. - Explain the idea of a S-like transition between quantum controllable and non controllable system, stressing that this is a rather universal behavior.

System:

- We control a nontrivial Hilbert space with quantum optical control, involving 10 motional states. - We show quantum application of this concept controlling both internal and external degrees of freedom (spin-dependent optical lattices play an important role for that) - Beyond adiabatic concept, we aim at reaching high fidelity in the minimal time allowed by Schrödinger equation.

- Define basic problem of transport, introducing atomic potentials for both spin states. Refer to Fig.1(a) which exemplifies the problem for one spin species. - Explain that fast transport is possible only through a highly wiggling ramp, far away from the adiabatic limit. Fig.1(b) - Show that this is indeed realized experimentally in our system, where we need to overcome the problem of the limited bandwidth. We do it by deconvolving the theoretical ramp with the response function of the system. - Insight about the shape of the ramp, explaining for example why at the beginning the ramp is steeper. - Mentioning that this fast transport creates many excitations and explores a large Hilbert space, Fig. 1(c).

Experimental apparatus:

List of questions:

- How does it look like the fidelity for highly excited states. Can one optimize transport for both ground state and first excited state? - What is the best fidelity in transport? Either measure or estimate from current measurements.

I. INTRODUCTION

- In the past decades has been a tremendous experimental and theoretical progress towards coherent control of quantum technologies such as atomic clocks, quantum communications and quantum signal processing.

- Powerful methods has been discovered to overcome the problem of decoherence such as quantum error correcting codes [46] or identifying shortcuts [35, 47] and decoherence-free subspaces.
- All processes gain from fast processes and robust control. Besides combating decoherence, quantum computer architectures also gain by speeding up quantum information protocols [48–51].
- Two major challenges of high-speed operations are to overcome technical limitations in order to reach an extremely high degree of quantum control and to determine and realize optimal solutions for systems with many-degrees of freedom.

Brief introduction of our system:

- In previous work, we have demonstrated fast, high-precision transport of cesium atoms in two state-dependent optical lattices [52] and ... (gives some nice applications of our previous results).
- Here, we report the realization of optimal transport by means of optimal control theory under the condition to preserve the motional ground state and identify the quantum speed limit as a sudden decrease of the transport fidelity in good agreement with the numerical and analytical expectation.

Detailed explanation of our setup:

- describing our system in more details [53] and give some recent results. We do ultra-fast, optimal control transport across the quantum speed limit of a cesium atom in an optical lattice over one lattice site as shown in Fig. 1 a) while maintaining the ground state after the process.
- then I want to describe the control parameters and its realization in the system: depth $U_0(t)$, phase $\varphi(t)$

$$\hat{H} = \frac{\hat{p}^2}{2m} + U_0(t)\cos^2(k\hat{x} - \varphi(t)/2), \tag{2}$$

$$\hat{U}_{\mathrm{evo}}(T) = e^{-\frac{i}{\hbar}\int_0^T \hat{H}(t')dt'} \tag{3}$$

- and give a description of its experimental realization with phase and amplitude ramps Fig. 1 b) and our challenges. There I want to mention that it is difficult to realize ramps on such short time scales (impulse response, deconvolution, ...). Another challenge is how to measure the transport fidelity. Here we present our measurement protocol from Belmechri [54]), Fig. 2 a), to determine the fidelity directly (MW sideband cooling with 99 % efficiency [52]
- the we discuss the measurement result, Fig. 2 b). The challenge is the number of repetitions vs. decoherence.
- Then I describe the major challenge how to find transport ramps, since there exists no analytic solution for a transport ramp. We approach the QSL numerically by using optimal control theory [55–57]
- then I describe how we get optimal control solutions from simulation (cost function, discrete parametrization, ...)

$$\mathcal{F} = |\langle\psi_{\mathrm{target}}|\,\hat{U}_{\mathrm{evo}}(T)\,|\psi_{\mathrm{init}}\rangle|^2 \tag{4}$$

$$\begin{aligned} U(t) &= U_0 &&+ \sum_{m=0}^{M} a_m \sin(\nu_m t), \quad t \in [0,T] \quad , \\ \varphi(t) &= \frac{2\pi}{T}t &&+ \sum_{m=0}^{M} b_m \sin(\nu_m t), \quad t \in [0,T] \quad . \end{aligned} \tag{5}$$

RESULTS

The first paragraph discuss the result of the linear and quantum speed limit measurement. In detail:

- then discuss the measurement linear transport vs. optimal control, Fig. 3 a). Linear transport was used by many other groups to speed up their transport, but we can do it better. The measurement with optimal control solution Fig. 3 b) demonstrates a significant improvement and we can identify the quantum speed limit as a sharp border.
- The data is well explained by our theoretical model, which is sensitive to the finite radial temperature as the only free parameter = thermometer
- in related work they have shown a single measurement close to the expected quantum speed limit with Bose-Einstein condensates [35, 58].
- Linear transport in the non-adiabatic regime has been demonstrated in previous work to speed up transport operations with ions [41, 42, 59] and neutral atoms [40, 47].
- In addition discuss the analytic solution of the QSL, which scales with inverse of the trap frequency (formula)
- then discuss the fact, that the classical speed limit has the same scaling

$$\tau^{\mathrm{classical}} = \sqrt{\frac{2\,d\,m\,\lambda}{\pi\,U_0}}. \tag{6}$$

The second paragraph describes the interferometer as a nice application. In detail:

- first describe an atom interferometer [60] Fig. 4 a), measurement of Ramsey fringe contrast, our challenge is the cross talk of the lattices, which we have to compensate

- discuss measurement result Fig. 4 b), when using the interferometer at the QSL

CONCLUSION AND OUTLOOK

- Then I want to describe the source of the QSL, the Heisenberg's uncertainty relation for energy and time [30, 34]

- There exist few papers of state-of-the-art experiments measuring close to the quantum speed limit, which I want to cite [35, 58]

- one sentence conclusion, importance for all experiments, which suffer from decoherences (examples)

- outlook: macroscopically separated quantum superpositions on the millimeter scale for interferometry, quantum walks [61], sensing of external forces, molecule formation [62], but also for fundamental tests of quantum mechanics such as proving bosonic versus fermionic statistics [63]

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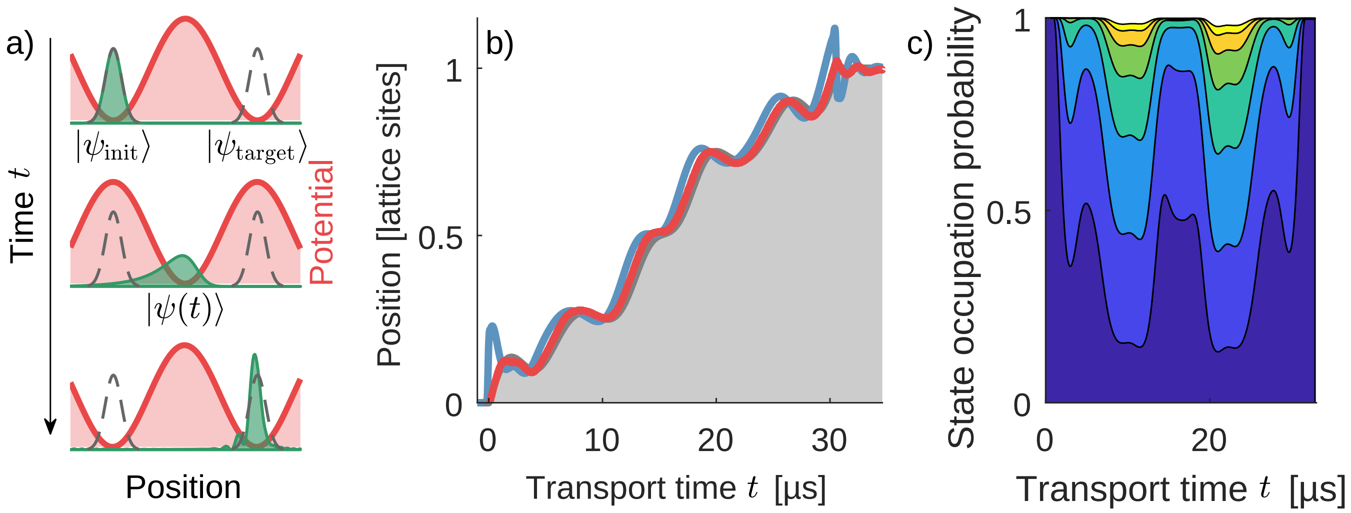

Figure 1. Scheme of atom transport in an optical lattice (red): a) shows the probability distribution $|\psi(t)\rangle$ of an atom (green) transported over one lattice site. The fidelity to transfer the atom from the initial to the final motional ground state in a sinusoidal potential is 50 %. b) compares the optimal control solution with a linear and adiabatic phase ramp in order to reach a transport fidelity of 99% in the shortest possible duration T for a constant trap depth $U_0 = 25\,k_{\rm B}\,\mu$K.

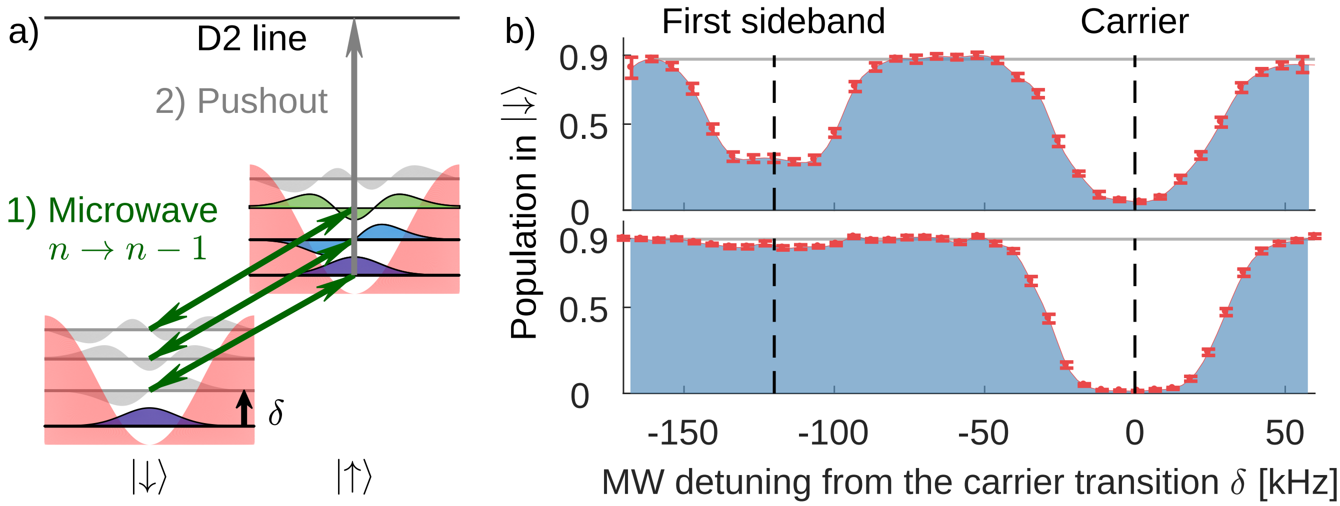

Figure 2. Precision measurement of ground state occupation. Avoid filling of non occupied motional state in the lower potential.

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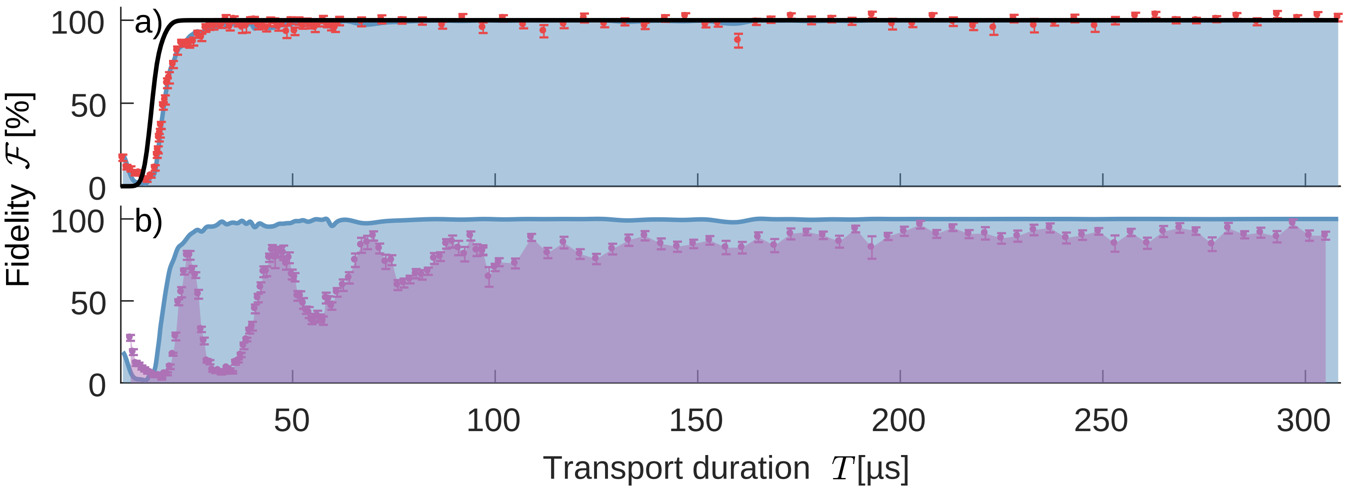

Figure 3. Measurement of the quantum speed limit (red): a) The fidelity for optimal control solutions suddenly reduces below a certain transport duration, which is identified as the quantum speed limit and shows as a significant improvement in comparison to linear driving ramps (purple). The result is verified by the theoretical expectation for a finite radial temperature (blue) and is even further decreased for a three-dimensional ground state (black).

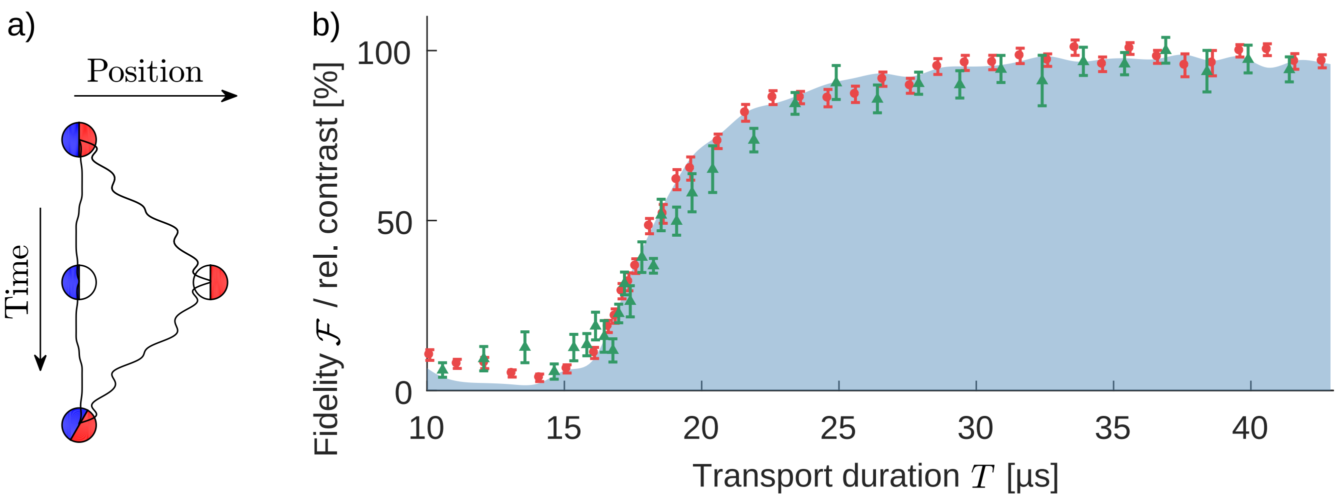

Figure 4. Interferometer: a) shows the Ramsey fringe contrast (green) of an atom interferometer by applying the ramps, which improves with the transport fidelity. b) The interferometer operates by spatially delocalizing an atom in a superposition of two states (blue and red colored circle) and a coherent recombination. The black lines describe the movement of the two state-dependent lattices.

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