Homework 1

Problem 1.1

What makes a game?

- Players
- Actions/strategies, timing
- Payoffs, objectives
- Information, nature of interaction

Examples of games in my everyday life:

- I love playing board games such as chess. In chess, there are two players who each have the objective of winning, usually to get the payoff of bragging rights. To win, one must employ effective strategies such as thinking ahead to what the other player will do and adjusting one's gameplay accordingly. The nature of interaction/information is the set of rules that govern chess gameplay, such as how many spaces each piece can move.
- Going out on outings with my roommates (all of my roommates with cars are the players) is another kind of game. You see, most of us would rather be driven than have to drive, especially with the snow. Therefore, our objective is to be one of the ones who does not have to drive. To do this, we have to think about what other people will do and how we can get out of driving. For example, one strategy we may use is to fail to bring our keys with us outside. Basically any strategy that helps save everyone time is the best way to go (the nature of the interaction is fast-paced and based on the assumption that we are all college drivers).
- Buying stock is another game that I play in my everyday life. The players are everyone in the stock market. The objective is to make the most money possible. Strategies involve short selling, keeping up with company news, diversifying one's portfolio, buying a stock when it is low and selling when it is high. The nature of the interaction is wholly governed by the stock market broking agency. They determine when one can buy and sell.

Problem 1.2

Economic and political examples that are not games:

- Monopolies are considered to be political or economic problems by many. However, a monopolist’s actions are not governed by game theory because they involve only one player.
- The choices of price-taking firms are also not determined by game theory, as these have little to no impact on the grand scheme of things. Since none of them can change prices, there are no game theory choices to be made.
- Things that are based on nothing but luck are also not games. For example, the gambling industry (minus games like poker where one is player against other players and not just the odds) may have a great impact on the economy, but that does not mean that it is a game because it is based on luck.

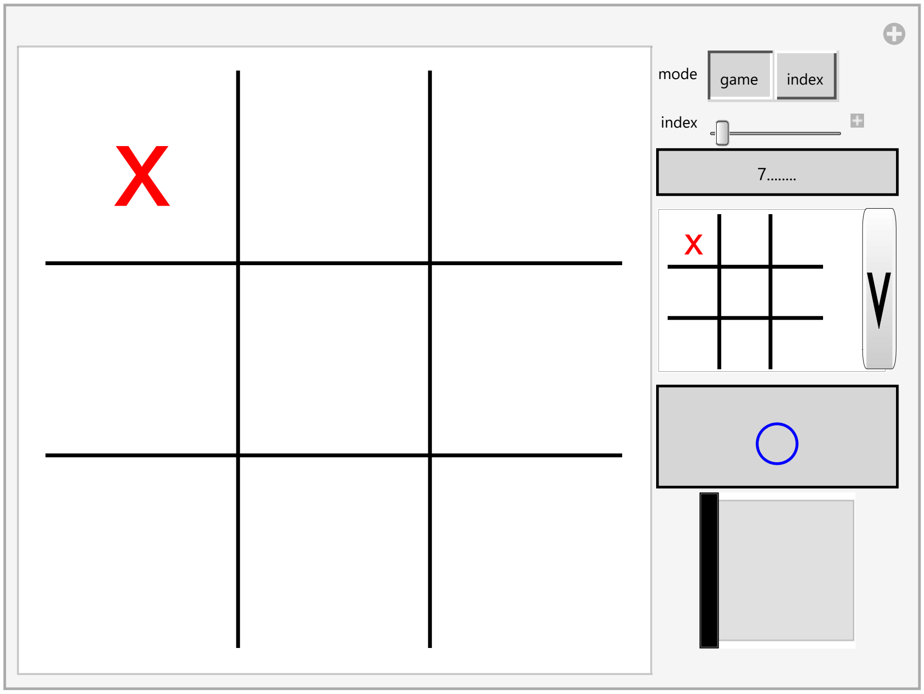
Problem 1.3

Economic and political examples that are games:

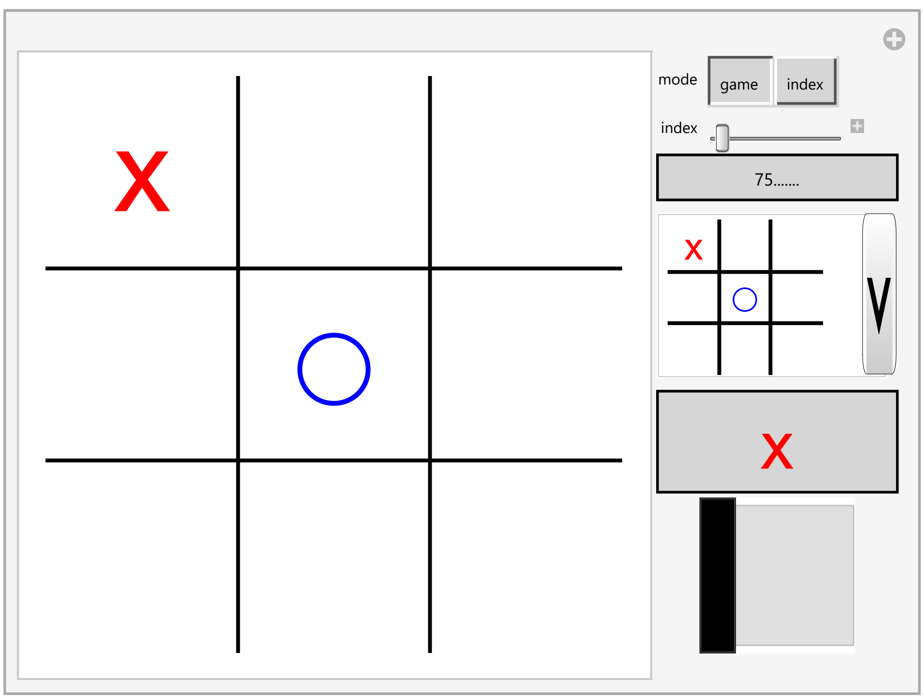
- Political campaigns are excellent examples of games because they involve players (those running for office), objectives (to get elected/gain power), strategies (playing against the other player, making them look bad, making themselves look good, etc), and information (election rules, voting, press conferences, etc).
- Oligopolies are also good economic examples of game theory interactions. An oligopoly means that a market is governed by two or more major players (companies). These players must predict what the others’ will do (strategy) and adjust their prices accordingly (prices and price-adjusting is the nature of interaction). The objective of each company is to make the biggest profit.
- Passing laws is also an example of a game. The players that pass laws are the members of the Legislative, Executive, and Judicial branches. The Legislative players have to pass laws that they believe will get enough votes while not getting vetoed by the Executive branch, which involves strategy and the passing/vetoing and other government procedures are the nature of interaction. The objective of each player is to pass laws that will benefit their viewpoints/beliefs/political party.

Problem 1.12

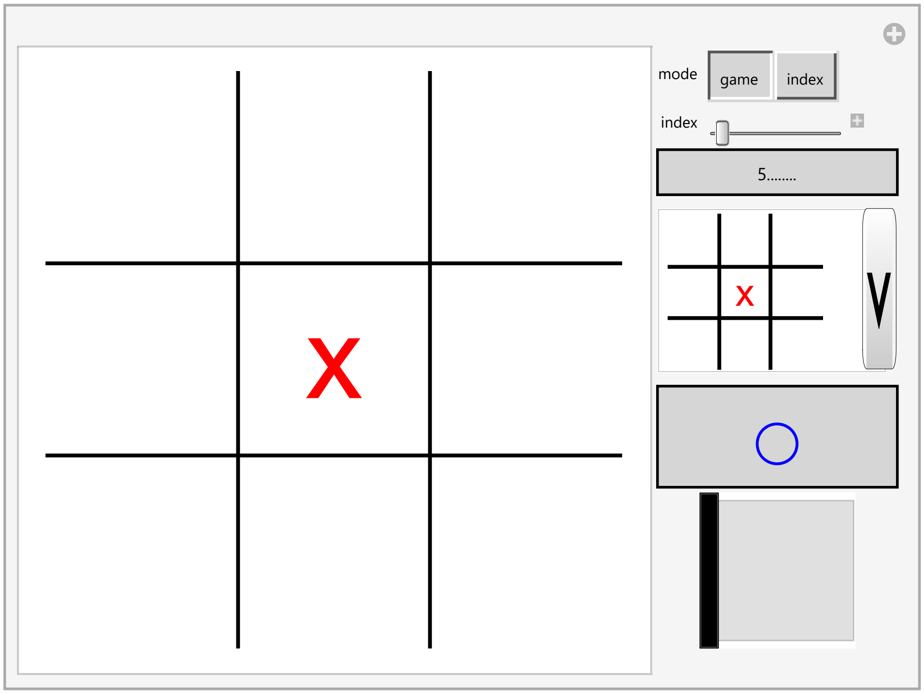
The game of tic-tac-toe has no winning strategy because it involves two players making marks on a 3-by-3 matrix, with the objective of successfully getting three of one’s own mark in a row. There is no winning strategy because as long as both players pick safe moves, there is no chance that either of them will win. For example, the best places to move involve the corners and the center. Should player one place their mark in a corner



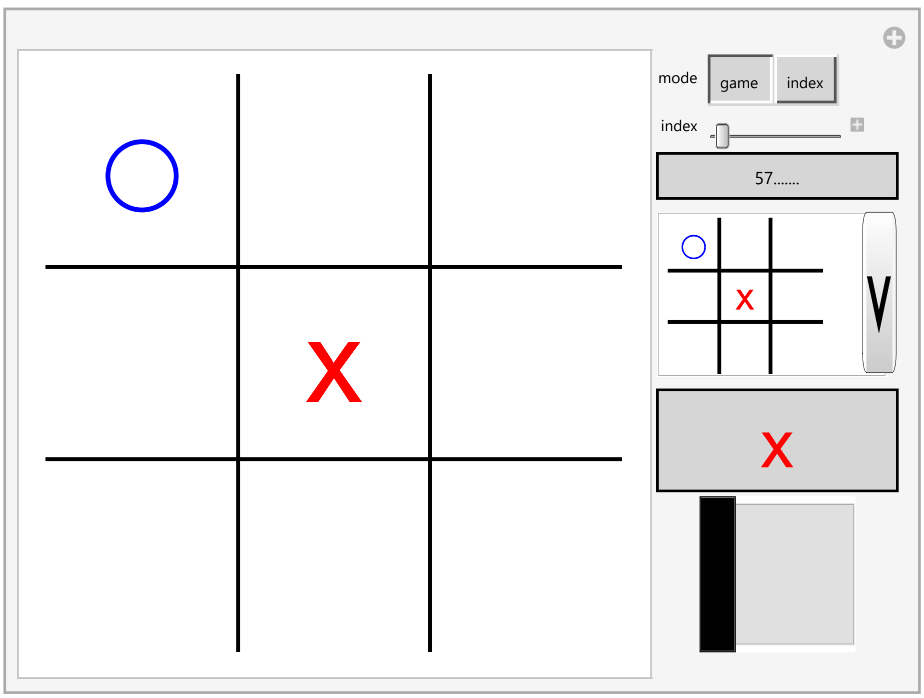
player two must put their mark in the center so that player one cannot exploit the extra space. If player two does this, neither of them will win.



Similarly, if player one places their mark in the center



player two should put theirs in a corner to best block player one and win the game.



Therefore, if both players are playing rationally, tic-tac-toe is a zero sum game.

Problems 1.22-1.26

We have three voters with preferences and three outcomes (A, B, and N):

Voter 1: A > B > N

Voter 2: B > N > A

Voter 3: N > A > B

Problem 1.22

The first round of voting consists of a choice between A and B. The second round is a choice between the winner of the first round and N. Either way, every voter should vote truthfully during the second round. Why? Because whatever outcome gets the most votes in the second round gets passed. To vote for something one does not want in the second round would be to risk their least wanted outcome becoming a reality.

Problem 1.23

Suppose Voter 3 is now N > B > A. Truthful voting would mean that B would win over A in the first round and the second round. With strategic voting it would be a different story. Voter 3 should pick A in the first round so that A wins. Then, they should pick N in the second round because Voter 2 will vote for N over A and N would win.

Problem 1.24

Now suppose A and N are voted on first and then their winner faces B in the second round. In this case, truthful voting would result in N winning in the first round and B winning in the second round. Now let’s do some analysis. If A wins in the first round then A will be passed. If N wins in the first round then B will be passed. Therefore, the voting is between A and B. Since Voter 3 wants A more than B, A will be passed in strategic voting.

Problem 1.25

Now suppose B and N are voted on first and then their winner faces A in the second round. In this case, truthful voting would result in B winning in the first round and A winning in the second round. Now let's do some analysis. If B wins in the first round then A will be passed. If N wins in the first round then N will be passed. Therefore, the voting is between A and N. Since Voter 2 wants N more than A, N will be passed in strategic voting.

Problem 1.26

Suppose each outcome is an option to be voted for in the first round. If all outcomes receive the same number of votes, N will be passed. The outcome of truthful voting would be N, as each voter would vote for the law they like most. There would be one vote for A, B, and N each. Now, let’s do some more analysis. A cannot beat N, N cannot beat B, B cannot beat A. Therefore, it is a little muddy to determine the outcome of strategic voting in this case. A could choose B in the first round, due to the fact that Voter 1 hates N. However, Voter 3 could vote for A knowing this, then A would get passed in the second round. Similar responses occur for each voter. If they all did this, then Voter 1 would choose B, Voter 2 would choose N, and Voter 3 would choose A. Since all outcomes were chosen, N would get passed after strategic voting as well.

Chapter 2 Notes

Here are some of my notes from chapter 2.

Games

Games are played by a set of rules that involve four things

- Who is playing: the group of players that strategically interacts.
- What they are playing with: the alternative actions or choices and the strategies that each player has available.
- When each player gets to play (in what order).
- How much they stand to gain (or lose) from the choices made in the game.

In Game Theory we assume **common knowledge about the rules:** that every player knows the rules of the game and that fact is commonly known.

Turn-taking Games

Extensive form: a pictorial representation of the rules. The main pictorial form is called the game tree, which is made up of a root (a choice player 1 has to make) and branches (the varies choices he can make) arranged in order. Extensive form answers the questions of

- Who, each node on the game tree represents a player in the game.
- What, each branch represents the different choices one can make.
- When, nodes that are a certain number of places removed from the root must be played in that order.

Simultaneous Games

Information set: a collection of decision nodes that a player cannot distinguish between (shown with an oval).

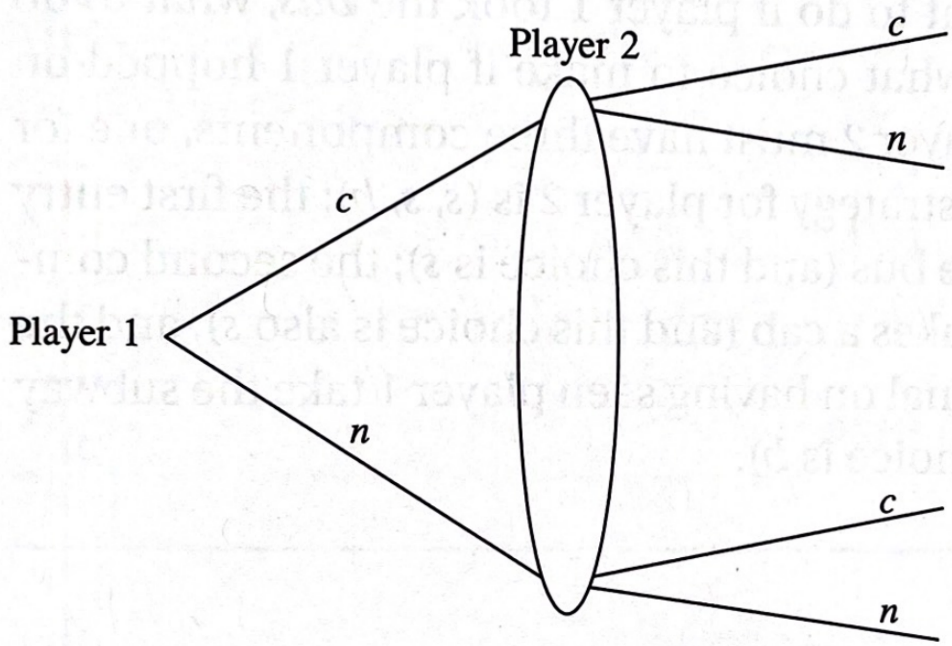


FIGURE 2.4.

Strategy: A strategy for a player specifies what to do at every information set at which the player has to make a choice.

Normal or strategic form: a complete list of who the players are, what strategies are available to each of them, and how much each gets.

TABLE 2.1

Player 1 \ Player 2	*sss*	*ssb*	*ssc*	*bbs*	...	*ccb*	*ccs*	*ccc*
b	N, T	N, T	N, T	T, N		N, T	N, T	N, T
c	T, N	T, N	T, N	T, N		T, N	T, N	T, N
s	T, N	T, N	N, T	T, N		T, N	T, N	N, T

Expected Utility: Preferences satisfy expected utility when the payoff to an uncertain outcome is precisely the average payoff of the underlying certain outcomes.

Example:

Example 1: Nim

Suppose, to begin with, there are two matches in one pile and a single match in the other pile. Let us write this configuration as (2, 1).

Winning is preferred to losing and, hence, the payoff number associated with winning must be higher than the one that corresponds to losing; suppose that these numbers are, respectively, 1 and −1. Figure 2.6 represents the extensive form of this game.[6]

The strategic form representation is as follows:

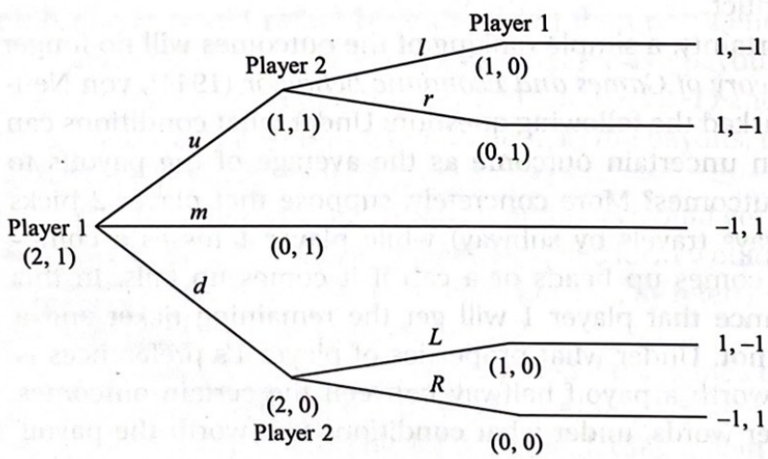
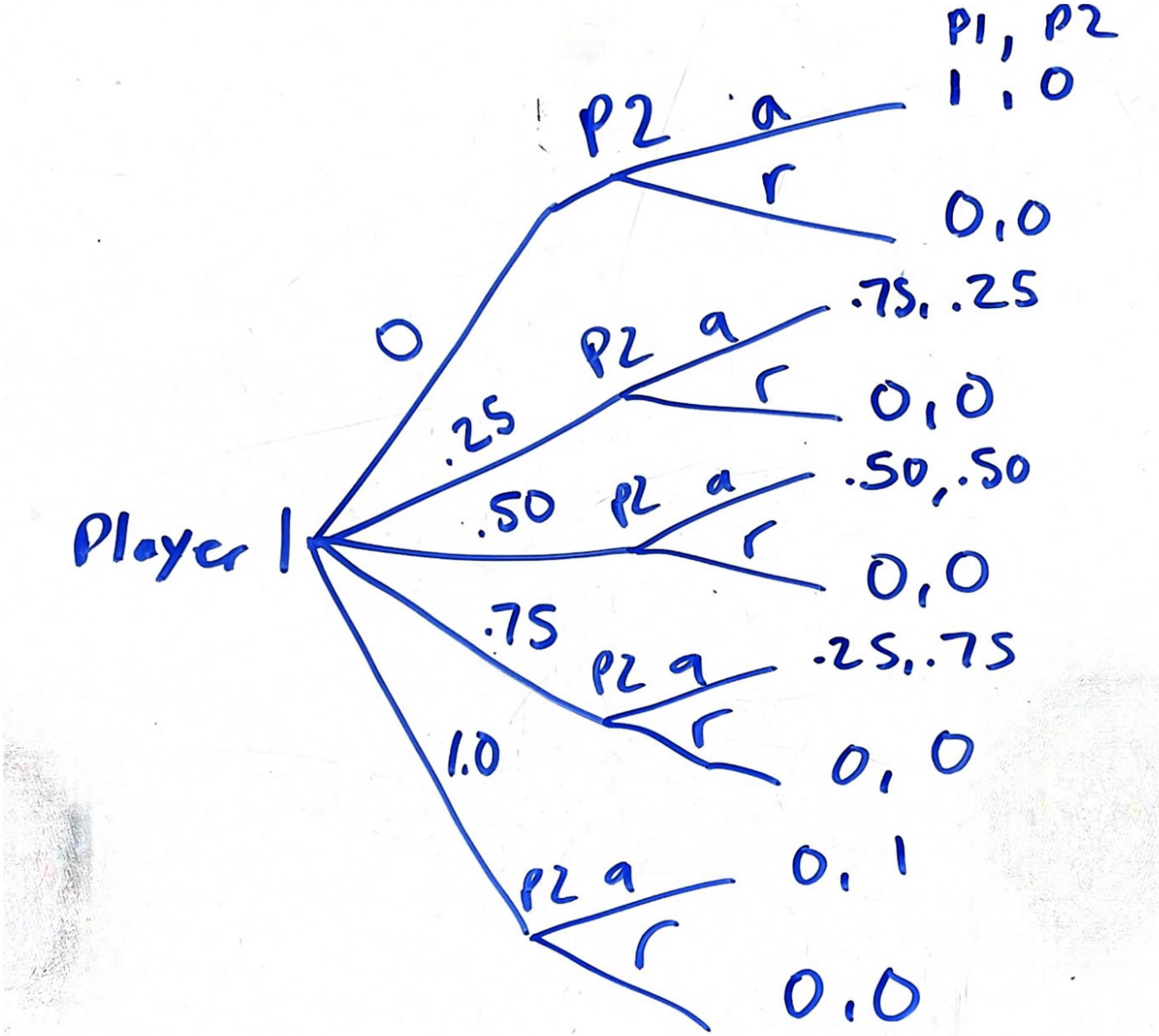
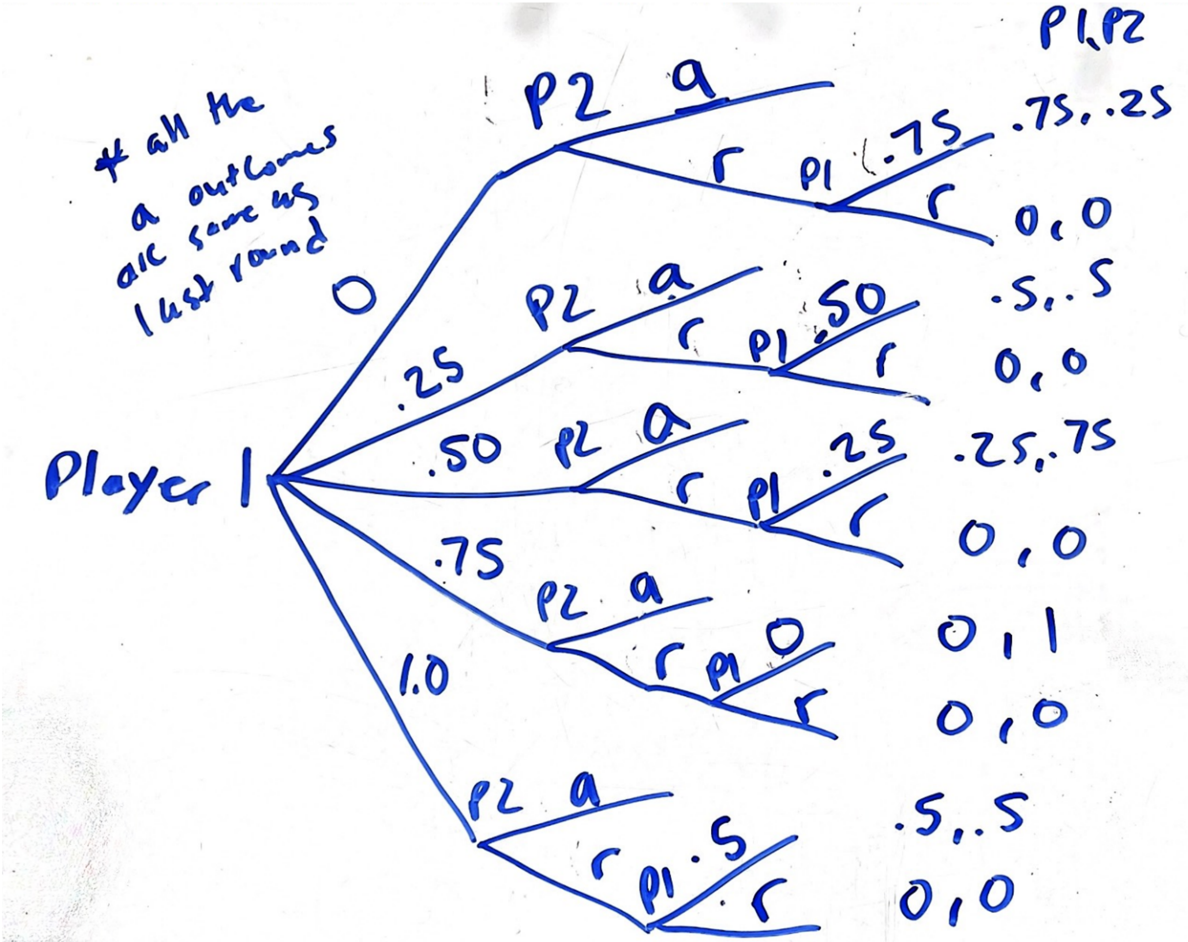


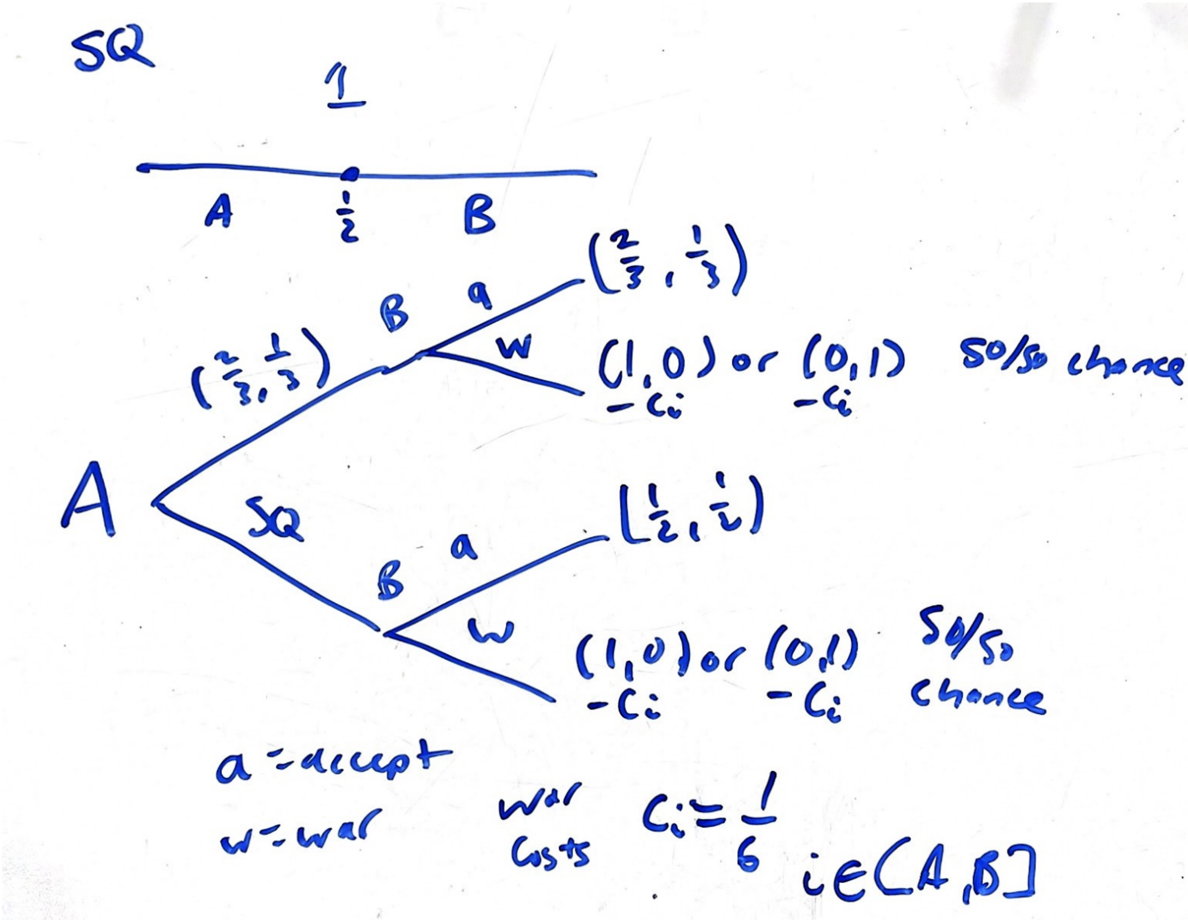
FIGURE 2.6.

Problems 2.10-2.11


P1, P2
Player 1
0
.25
.50
.75
1.0
P2
a
r
1,0
0,0
.75, .25
0,0
.50, .50
0,0
.25, .75
0, 0
0, 1
0,0


P1,P2
all the a outcomes are same as last round
Player 1
0
.25
.50
.75
1.0
P2 a r
P1 .75 r
.75, .25
0, 0
P2 a r
P1 .50 r
.5, .5
0, 0
P2 a r
P1 .25 r
.25, .75
0, 0
P2 a r
P1 0 r
0, 1
0, 0
P2 a r
P1 .5 r
.5, .5
0, 0

Problem 2.13


SQ
1
A
1/2
B
A
B
a
w
(2/3, 1/3)
(2/3, 1/3)
(1,0) or (0,1) 50/50 chance
-Ci -Ci
SQ
B
a
w
(1/2, 1/2)
(1,0) or (0,1) 50/50 chance
-Ci -Ci
a = accept
w = war
war costs Ci = 1/6
i∈[A,B]