Homework 5

Voting and Elections

Notes from Chapter 7

Majority voting: A winner is decided simply by who receives the most votes.

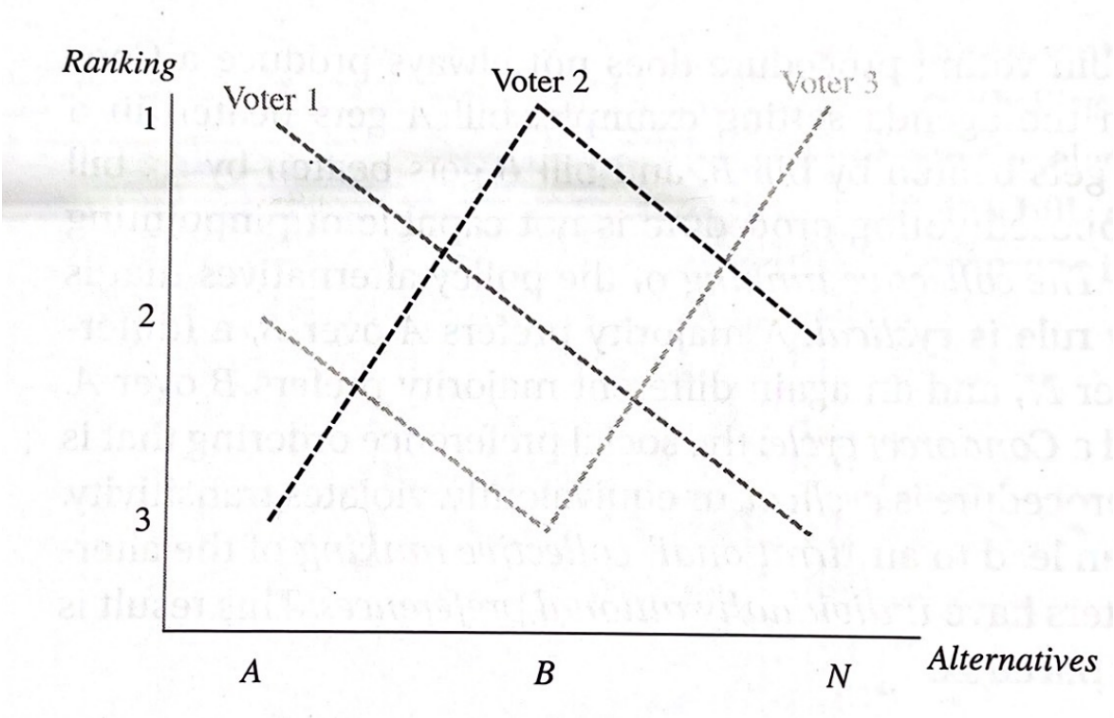
Agenda Setting : When a candidate can set the agenda so that their favorable outcome will win.

Condorcet Winner: An alternative that is never beaten by another when pitted against it for a majority vote.

Condorcet Paradox: Each individual voter having transitive preferences does not imply that this is true of the electorate as a whole.

Single - Peaked Preferences

A voter's preferences are single-peaked if moving away from his or her top choice makes all other alternatives less preferred.



Voter B is not single-peaked because as he moves from N to B his preferences decrease, but as he moves from B to A they increase.

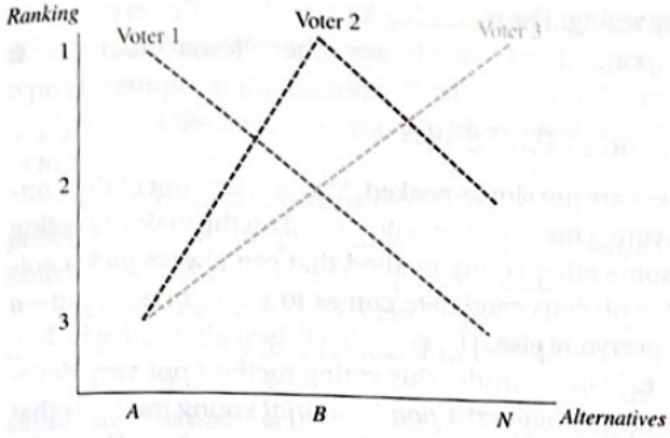


FIGURE 7.2.

We argue that when the preferences are single-peaked then the favorite alternative of the *median voter*—loosely speaking the favorite alternative[11] of the voter that is in the middle of the distribution—is the Condorcet winner. Adding some precision, label the (single) peak of voter i as alternative m_i. We now define the *median voter.*

Definition. Agent j is the median voter for the preference profile $(\succeq_1, \ldots, \succeq_N)$ if no more than half of the agents have a peak higher than m_j and if no more than half of the agents have a peak lower than m_j. Written succinctly:[12]

$$\#\{i : m_i > m_j\} \leq \frac{N}{2} \quad \text{and} \quad \#\{i : m_i < m_j\} \leq \frac{N}{2}$$

It turns out that if the individual preferences of all voters are single-peaked preferences with respect to some lineup, then there always exists a Condorcet winner: the bliss point of the median voter.

Theorem. *Median voter theorem* If the preference profile of $(\succeq_1, \ldots, \succeq_N)$ of N agents is single-peaked on a lineup of M alternatives, then the ideal point of the median voter j, m_j, is never beaten by any other alternative by majority voting: the alternative m_j is the Condorcet winner.

The Hotelling-Downs Model

The Hotelling-Downs Model implies that if one places his stance slightly right on the political spectrum, then he will get all the voters to the right of him if he is the furthest right candidate. The same logic applies for one who places their stance slightly left.

The best place to be on the political spectrum is in the middle because there one gets the most people on their side to the right and to the left. Therefore, the Nash Equilibrium lies right on the middle of the political spectrum. Policial candidates will therefore “court” the median voter and will keep nudging

toward the middle of the spectrum until they hit the Nash Equilibrium.

Problems From Chapter 7

Problem 7.5

First Choice

- A = 10
- B = 10
- C = 9
- D = 11

The Condorcet winner is D!

Problems 7.2 - 7.4

7.2

If all legislators approve all bills, then every bill will get a *yes* vote and one of the three bills will be chosen at random. Therefore, the likelihood of one's bill being chosen would be 33%. That is not a Nash Equilibrium, as no bill has a dominant strategy over the others, so there can be no best response.

7.3

B is the Nash Equilibrium because voter 1 prefers B to N and voter 3 prefers B to A. Therefore, the best response to ensure that a bill that one hates does not get picked is to vote for B.

7.4

- A beats B, N once
- N beats A twice, N once
- B beats A, N twice

B is never beaten when compared to the others will a majority vote. A and N are beaten at least once. B is the Condorcet winner.

Problems 7.13 and 7.15

The Borda counting method: The worst candidate gets one point, the second worst gets two points, and so on. The player with the highest aggregate score after this counting method, wins!

7.13

Joe Burrow

```
843 * 3 + 41 * 2 + 3 = 2614
```

Jalen Hurts

```
6 * 3 + 271 * 2 + 187 = 747
```

Justin Fields

```
12 * 3 + 231 * 2 + 264 = 762
```

Chase Young

```
20 * 3 + 205 * 2 + 173 = 643
```

Joe Burrow is the Heisman Trophy winner. Justin Fields is the runner-up. Jalen Hurts receives third place.

7.15

A

```
10 * 4 + 15 * 3 + 6 * 2 + 9 = 106
```

B

```
10 * 4 + 13 * 3 + 6 * 2 + 11 = 102
```

C

```
9 * 4 + 6 * 3 + 23 * 2 + 2 = 102
```

D

```
11 * 4 + 6 * 3 + 5 * 2 + 18 = 90
```

A is the winner of the election according to the Borda counting method. The complete ranking of the candidates is A > B = C > D.

Problems 7.19 - 7.21

7.19 Borda

B

```
60 * 3 + 40 * 2 + 80 = 340
```

M

```
40 * 3 + 140 * 2 = 400
```

S

```
80 * 3 + 100 = 340
```

M is the Borda winner.

7.20 Plurality

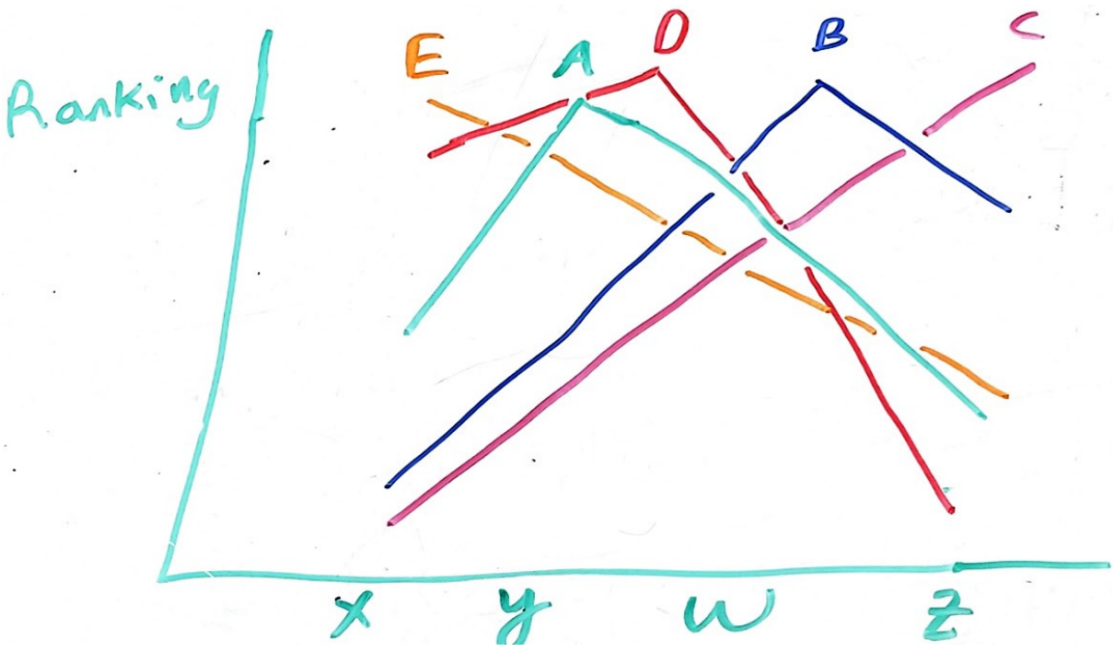
Plurality rule: The top-ranked candidate of each voter receives a vote of one, all other candidates receive a score of zero.

S wins by plurality rule. S has the most voters who chose it as their number-one choice.

7.21

B would win the runoff election. First, S and B would win the first round because S has 80 votes and B has 60 votes. However, B will win in the second round because 40 of the voters voted for M in the first round, and they would vote for B in the second round, because more voters picked B as their second and third picks than S.

Problem 6 From the Homework



w is preferred by the median voter, as you can see all the lines intersect over w.