Definition. A *repeated game* is defined by a stage game G and the number of its repetitions, say T. The stage game G is a game in strategic form: $G = \{S_i, \pi_i; i = 1, \ldots, N\}$ where S_i is player i's set of strategies and π_i is his payoff function [and it depends on $(s_1, s_2, \ldots, s_N)$].

If T is finite, we call the game a *finitely repeated game*, whereas if the repeated game has no fixed end we call it an *infinitely repeated game*. It is customary to compute the payoffs of a finitely repeated game as the sum of the payoffs in each stage. For the infinitely repeated game, a common way to compute payoffs is to add the (discounted) expected payoffs in the manner shown in Equation (15.1). In the remainder of this chapter we will discuss finitely repeated games, and in Chapter 16 we will turn to infinitely repeated games.

Buyer 1 \ Buyer 2	*50, h*	*75, h*	*50, l*	*75, l*
50, h	$50\pi_h, 50\pi_h$	$40\pi_h, 60\pi_h$	$50\pi_l, 50\pi_l$	$50\pi_l, 50\pi_l$
75, h	$60\pi_h, 40\pi_h$	$50\pi_h, 50\pi_h$	$75\pi_l, 25\pi_l$	$75\pi_l, 25\pi_l$
50, l	$50\pi_l, 50\pi_l$	$25\pi_l, 75\pi_l$	$50\pi_l, 50\pi_l$	$40\pi_l, 60\pi_l$
75, l	$50\pi_l, 50\pi_l$	$25\pi_l, 75\pi_l$	$60\pi_l, 40\pi_l$	$50\pi_l, 50\pi_l$

For example, if buyer 1 bids a high price against 50, *l*, then he gets all of his quantity at a low price; the price is low because there is no demand for all 100 units at price h. If he bids 75, l (against 50, l), then he gets 60 units allocated at the low price. Note furthermore that, at any price, it is always better to ask for a larger quantity—75, l dominates 50, l, and 75, h dominates 50, h. The reason is simple: 75, l dominates 50, l because at the consequent low price a higher quantity demand means a higher (and more profitable) allocation. Also, 75, h dominates 50, h; it has the same effect on price, and its quantity allocation is at least as high.

Similarly, the strategic form of a multiprice auction looks like this:

Buyer 1 \ Buyer 2	*50, h*	*75, h*	*50, l*	*75, l*
50, h	$50\pi_h, 50\pi_h$	$40\pi_h, 60\pi_h$	$50\pi_h, 50\pi_l$	$50\pi_h, 50\pi_l$
75, h	$60\pi_h, 40\pi_h$	$50\pi_h, 50\pi_h$	$75\pi_h, 25\pi_l$	$75\pi_h, 25\pi_l$
50, l	$50\pi_l, 50\pi_h$	$25\pi_l, 75\pi_h$	$50\pi_l, 50\pi_l$	$40\pi_l, 60\pi_l$
75, l	$50\pi_l, 50\pi_h$	$25\pi_l, 75\pi_h$	$60\pi_l, 40\pi_l$	$50\pi_l, 50\pi_l$

For precisely the same reason as before, again 75, l dominates 50, l, and 75, h dominates 50, h. From the point of view of stage-game Nash equilibrium analysis we can therefore look at the *reduced single-price auction*:

Buyer 1 \ Buyer 2	*75, h*	*75, l*
75, h	$50\pi_h, 50\pi_h$	$75\pi_l, 25\pi_l$
75, l	$25\pi_l, 75\pi_l$	$50\pi_l, 50\pi_l$

And this is the *reduced multiprice auction:*

Buyer 1 \ Buyer 2	*75, h*	*75, l*
75, h	$50\pi_h, 50\pi_h$	$75\pi_h, 25\pi_l$
75, l	$25\pi_l, 75\pi_h$	$50\pi_l, 50\pi_l$

(Note that the profits are different in the two cases when one bidder bids h and the other bids l.) On one hand, the Treasury would like the price to be high because a high price produces a larger revenue. On the other hand, the financial institutions would like the price to be low because a low price enables them to make a larger profit. Which is it going to be? And does it make a difference whether the auction is single price or multiprice? There are two cases to consider.

Case 1: The Competitive Case

Suppose that it is less profitable to buy half the quantity even if it is at a low price; that is, suppose that $50\pi_h > 25\pi_l$. Then h is a dominant strategy in the *reduced single-price* auction. Hence the unique Nash equilibrium in the stage game is (h, h). The Treasury is especially happy because (h, h) in every stage is then the unique subgame perfect equilibrium as well. (Why?) Repeating the auction, as the Treasury does, makes no difference to the intensity of competition in the market and does not allow the participants to make credible deals to keep prices low.

Consider now the *reduced multiprice* auction. Now there might be a second Nash equilibrium if the best response to a low price is to also to bid low, that is, if $50\pi_l > 75\pi_h$.[14] In that case, (l, l) is also a Nash equilibrium; that is, the buyers have the incentive to implicitly collude and keep the price low. Hence one subgame perfect equilibrium is for both buyers to bid l all the time.[15]

Case 2: The Collusive Case

Suppose instead that $50\pi_h < 25\pi_l$. It is straightforward to see the following:

CONCEPT CHECK

LOWBALLING

Show that in the *multiprice* auction, l is a dominant strategy (and hence, the buyers stiff the Treasury by offering low bids).

In the *single-price* auction there is still a unique Nash equilibrium, but it is now a mixed-strategy equilibrium.

CONCEPT CHECK

MIXED STRATEGY

Show that in the *single-price* auction, there is a unique mixed-strategy Nash equilibrium in the stage game. Compute this equilibrium (as a function of the parameters π_h and π_l).

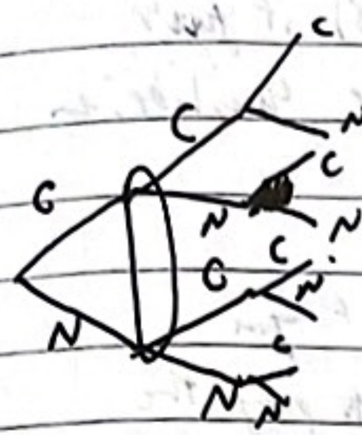
Prisoner's Dilemma

	Confess	not
Confess	0,0	7,-2
not	-2,7	5,5

Modified PD

	Confess	Partial	not
Confess	$\underline{0},\underline{0}$	$\underline{3},-1$	$\underline{7},-2$
Partial	$-1,\underline{3}$	$\underline{3},\underline{3}$	6,0
not	$-2,\underline{7}$	0,6	5,5

What if we repeated PD?



and the tree keeps going.

The dominant strategy (Nash Equilibrium) is to always confess. (CCCCC, CCCCC)

Round 1

Modified Equilibrium

round 1 → C P N → P2's First Choice

P1's first choice

	C	P	N
C	C	C	C
C	C	C	P
C	C	C	N
P	P	N	C
C	N	P	C
N	C	C	P

retaliation for these choices

P1's second choice (a strategy set of possible responses to P2's choices)

round 2

N | CCP → (5+③, 5+③)

N in first round

P in second round

First round

responses based on what other player played in previous (first round)

(N|CCP, N|CCP) → P1 & P2 strategy = (5+3, 5+3) = (8, 8)

Subgame perfect Nash Equilibrium

(C|CPN, P|PNC)
2 +3 -1 +3 = (5,2)

* I'll give you the better equilibrium (3,3) if you're nice to me & I'll give you the worse equilibrium (0,0) if you're not good to me.

If there is one equilibrium then even if you play 100 times, we expect everyone to always play the equilibrium

$3^4 = 81$ strategy options. First round I have 3 options: C, P, N. Then, in the next round I can choose C, P, or N based on each of the three options my opponent chooses, so 3·3·3·3 is my strategy options.

*There is a single strategy that both players want to choose in a symmetric equilibrium.

C P N ← based on 1st round decision if P1 plays

If P1 plays N: CCP

If P2 plays N: CPP

1. First round, both pick N: +5
2. Second round, both pick P: +3

P1: 5+3=8

P2: 5+3=8

still plays P if P1 plays P first round

Not best response in this case

P1 can play P first round and P2 won't punish him: 6+3=9

this is best response

P2 would get 0+3=3

Not Nash Equilibrium

By repeating the game we can introduce new behaviors that are Nash Equilibrium that were not before.

What if?

N ¦ CCN ¦ CCN ¦ .... ¦ CCN ¦ CCP

S + S + S + S + S + 3

.... + 6 + 0 → If I play P here,

I get 1 extra util but then

zeros after that, so the Nash Equilibrium is this

Playing Nash Equilibrium in each round of a repeated game is a subgame perfect Nash Equilibrium for the entire repeated game.