Infinitely Repeated Games

		N Don't Snitch	S Snitch
N	Don't Snitch	5, 5	-2, 7
S	Snitch	7, -2	0, 0

$$U[(N,N)] = 5T \xrightarrow[T\to\infty]{\lim} \infty$$

↳ # of periods we play this for

$$U[(N,C)] + U[(C,N)] = \frac{(7-2)}{2} T \xrightarrow[T\to\infty]{\lim} \infty$$

$(N,C) \times 5$ then $(N,N) \times 2$

$5(-2)$ + $2(5)$ $= 0$ in seven periods

$$1 \underbrace{(2 \; 3 \; 4 \; 5 \; 6 \; 7 \; 8)}_{0} \underbrace{(9 \; 10 \; 11 \; 12 \; 13 \; 14 \; 15)}_{0}$$

-2 + 0 + 0 = -2

Discounting

\$100 today $\approx$ \$101 next year

$= 1.01$(\$100 next year)

\$100 Next year = δ \$100 this year

$\delta < 1$

$$\delta^0 \pi_{i0} + \delta \pi_{i1} + \delta^2 \pi_{i2} + \delta^3 \pi_{i3} + \cdots$$

Let's say every period is the same payoff

$$= \pi \sum_{k=0}^{\infty} \delta^k = \pi\left(\frac{1}{1-\delta}\right)$$

$$S = 1 + \delta + \delta^2 + \delta^3 + \cdots$$

$$1 + \delta S = 1 + \delta + \delta^2 + \delta^3 + \delta^4 + \cdots$$

$$1 + \delta S = S$$

$$1 = (1-\delta)S \longrightarrow \frac{1}{1-\delta} = S$$

(Grim)
Trigger strategy: play (n,n) until someone deviates, then play (c,c) thereafter.

NC, NC forever is SPNE

$$\underset{\text{Grim}}{EU(eq)} = 5 + \delta 5 + \delta^2 5 + \dots = \frac{5}{1-\delta}$$

$$EU(\text{deviate}) = 7 + \delta 0 + \delta^2 0 + \delta^3 0 + \dots = 7$$

when is $EU(n,n) > EU(\text{deviate})$

$$\frac{5}{1-\delta} > 7 \rightarrow 5 > 7 - 7\delta \qquad 7\delta > 2$$

$\delta > \frac{2}{7} \rightarrow$ if $\delta > 2/7$ we have a subgame perfect Nash Equilibrium

With infinitely repeated games we can sustain cooperation

Forgiving trigger strategy: play (n,n) until someone deviates, then (c,c) for T periods then back to (n,n)

NC, NC forever is SPNE

$EU(n,n)$ still $\frac{5}{1-\delta}$

$$EU(\text{deviate}) = 7 + \delta 0 + \delta^2 0 + \dots + \delta^T 0 + \delta^{T+1} 5 + \delta^{T+2} 5 + \delta^{T+3} 5 + \dots$$

$$= 7 + \frac{\delta^{T+1} 5}{1 - \delta}$$

$$\frac{5}{1-\delta} > 7 + \frac{\delta^{T+1} 5}{1-\delta} \qquad 5 > 7 - 7\delta + \delta^{T+1} 5$$

or $\frac{5(1-\delta^{T+1})}{1-\delta} > 7 \qquad 5 \lim_{\delta \to 1} \frac{1-\delta^{T+1}}{1-\delta}$

if δ is close to one

$$= 5 \lim_{\delta \to 1} \frac{-(T+1)\delta^T}{-1} = 5(T+1) \qquad \boxed{\text{so } 5(T+1) > 7}$$

(N,C) → not sustainable

(C,C) → is sustainable and a Nash Equilibrium

* Something that is sustainable is also a Nash Equilibrium

* Subgame perfect Nash Equilibria are Nash Equilibria that are credible

Tit for tat:

Play NC if opponent played NC last period

Play C if opponent played C last period

Non-deviation ((NC,NC)) ... ((NC),(NC))

$$\delta + \delta 5 + \delta^2 5 + \dots = 5/1-\delta$$

NC,NC forever is SPNE

Deviate: (C,NC) + (C,C)

7 + 0

(C,NC) + (NC,C) + (NC,NC) + (NC,NC)

$$7 + \delta(-2) \quad \delta^2 5 \quad + \delta^3 5$$

Choosing this over this depends on how high delta is.

New Strategy: (NC,C), (NC,C), (NC,C)

↳ there is nothing worse that P2 could threaten P1 with here, so it is not worth it.

↳ not sustainable as an equilibrium

What about: (NC,NC), (NC,C), (C,N), (C,C), (NC,NC) ...

SPNE → Follow: $\dfrac{5 + \delta(-2) + \delta^2(7) + \delta^3 0 + \delta^4 5}{1 - \delta^4}$ divided by

Better than

Deviates $\underbrace{7 + \delta 0 + \delta^2 0 + \delta^3 0 + \dots}_{7}$

Another one: (C, NC), (C, C), (C, NC), (NC, NC), repeat

P2 Follow: $-2 + \delta 0 + \delta^2(-2) + \delta^3 5 + \dots$

Deviate: $0 + \delta 0 + \delta^2 0 + \delta^3 0 + \dots$

if $\delta \approx 1$ then first 4 rounds $= 1$ → SPNE

if $\delta \approx 1$ then first 4 rounds $= 0$

"Folk" theorem:

I δ is high enough (close enough to one) then $\underbrace{\text{any individually rational behavior}}$ can be sustained in SPNE.

→ not the lowest possible payoff that you can impose on me.

*So anything that gives you better than 0 can be sustained in a SPNE. (Because 0 is the lowest possible payoff one can impose on me without my permission)

We are assuming perfect enforcement, which implies perfect observation.

Longer punishment paths are more protective of the good behavior.