HW 6 - The Commons Problem

Chapter 8 Notes

Two characteristics of commons

1. Access to (nearly) everyone: it is infeasible or undesirable to restrict access.
2. Resource depletability: The more people use the resource (or the more intensively each person uses the resource) the less there is of the resource in the future.

Two sources of externality due to the commonality of access.

1. Current externality : one person' s usage may decrease the benefits to usage for another today.

2. Future externality : one person' s usage today may decrease the benefits to usage for another in the future.

Tragedy of the commons: The consequence of the fact that the resource is commonly owned is that it might get overused.

Tragedy of the Commons Game Theory Model

Period 1 of Commons Consumption

If $c_A + c_B \geq y$ then each consume $\frac{y}{2}$; otherwise, each consume c_i.

Period 2

In the second period, the share the reminder $y - c_A - c_B$ from the first period: $\frac{y - c_A - c_B}{2}$.

Consumer A gets utility based on the log of how much he consumes in period 1 and period 2:
$u_A = \log(c_A) + \log\left(\frac{y - c_A - c_B}{2}\right)$

The Best Response

Best response

a. FOC: $\frac{\partial u_A}{\partial c_A} = \frac{1}{c_A} - \frac{1}{y - c_A - c_B} = 0 \Leftrightarrow c_A = y - c_A - c_B \Leftrightarrow c_A^{br} = \frac{y - c_B}{2}$

b. SOC: $-\frac{1}{c_A^2} - \frac{2}{(y - c_A - c_B)^2} < 0$

$c_A{}^{\text{br}} = \frac{y - c_B}{2}$ and $c_B{}^{\text{br}} = \frac{y - c_A}{2}$. Then plug in the $c_B{}^{\text{br}}$ equation for c_B in the first equation and solve for $c_A{}^*$.

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Equilibrium

a. $c_A^{br} = \frac{1}{2}\left[y - \frac{1}{2}(y - c_A)\right] = \frac{1}{4}y + \frac{1}{4}c_A$

b. $\frac{3}{4}c_A^* = \frac{1}{4}y \Leftrightarrow c_A^* = \frac{1}{3}y$; symmetrically, $c_B^* = \frac{1}{3}y$

c. Each consumes 1/3 now, 1/6 later.

Social Optimality

Social planner

a. Maximize $u_A + u_B = \log c_A + \log c_B + 2\log\left(\frac{y - c_A - c_B}{2}\right)$

b. FOC: $\frac{1}{c_A} - \frac{2}{y - c_A - c_B} = 0 \Leftrightarrow 2c_A = y - c_A - c_B \Leftrightarrow c_A = \frac{1}{3}(y - c_B)$

   i. Symmetrically, $c_B = \frac{1}{3}(y - c_A)$

c. SOC: $-\frac{1}{c_A^2} - \frac{2}{(y - c_A - c_B)^2} < 0$

d. Solution: $c_A^* = c_B^* = c^* \Leftrightarrow c^* = \frac{1}{3}(y - c^*) \Leftrightarrow \frac{2}{3}c^* = \frac{1}{3}y \Leftrightarrow c^* = \frac{1}{2}y$

Each consumes $\frac{1}{4}$ now and $\frac{1}{4}$ later. Without the planner, individuals over-consume.

Other Applications

Drinking water, fisheries, cave stalactites, bandwidth, public library books, ward budget, family budget.

A Large Population Problem

Large n

a. Utility $u_1 = \log c_1 + \log\left(\frac{y - c_1 - c_2 \ldots - c_n}{n}\right)$

b. FOC: $\frac{\partial u_1}{\partial c_1} = \frac{1}{c_1} - \frac{1}{y - c_1 - c_2 \ldots - c_n} = 0 \Leftrightarrow c_1 = y - c_1 - c_2 \ldots - c_n$

c. Symmetric equilibrium: $c^* = y - nc^* \Leftrightarrow c^* = \frac{1}{n+1}y$

d. Planner: $W = \log c_1 + \log c_2 + \cdots + \log c_n + n\log\left(\frac{y - c_1 - c_2 \ldots - c_n}{n}\right)$

e. Planner FOC: $\frac{1}{c_1} - \frac{n}{y - c_1 - c_2 \ldots - c_n} = 0 \Leftrightarrow \frac{1}{c^{**}} - \frac{n}{y - nc^{**}} = 0 \Leftrightarrow nc^{**} = y - nc^{**} \Leftrightarrow c^{**} = \frac{1}{n}\frac{y}{2}$

   i. First equivalence because FOC for $c_1, c_2, \ldots, c_n$ together imply $\frac{1}{c_1} = \frac{1}{c_2} = \cdots = \frac{1}{c_n}$

As n becomes large, a vanishingly small amount of the resources become available in the second period. The tragedy of the commons is exacerbated in large population.

Public Good Contributions

Volunteerism example (from earlier HW)

Utilities $u_1 = \sqrt{x+y} - x$ and $u_2 = \sqrt{x+y} - y$, where $x, y \in [0,4]$ (Homework was simpler: $\{0,1,2,3,4\}$)

$$x^{br}(y) = \begin{cases} \frac{1}{4} - y \ \ if\ y \le \frac{1}{4} \\ 0 \ \ if\ y > \frac{1}{4} \end{cases}$$

a. Given y, maximize u_1. FOC: $\frac{\partial u_1}{\partial x} = \frac{1}{2}(x+y)^{-\frac{1}{2}} - 1 = 0$

   $\Leftrightarrow (x+y)^{-\frac{1}{2}} = 2 \Leftrightarrow (x+y) = \frac{1}{4} \Leftrightarrow x^{br}(y) = \frac{1}{4} - y$

b. SOC: $\frac{\partial^2 u_1}{\partial x^2} = -\frac{1}{4}(x+y)^{-\frac{3}{2}} < 0$

c. Similarly, $y^{br}(x) = \begin{cases} \frac{1}{4} - x \ \ if\ x \le \frac{1}{4} \\ 0 \ \ if\ x > \frac{1}{4} \end{cases}$

Equilibria

a. $x^{br}(y) \in \left[0, \frac{1}{4}\right]$ for any y and $y^{br}(x) \in \left[0, \frac{1}{4}\right]$ for any x, so (x^*, y^*) is equilibrium only if $x^*, y^* \in \left[0, \frac{1}{4}\right]$

b. Best response conditions then imply $x^* = \frac{1}{4} - y^*$ and $y^* = \frac{1}{4} - x^*$. But these are same equation.

c. Thus, any (x^*, y^*) pair with $x^* + y^* = .25$ is an equilibrium.

   i. Many equilibria! (But way more (x, y) pairs that are not equilibria.)

   ii. Examples: $(0, .25)$, $(.25, 0)$, $(.125, .125)$, $(.2, .05)$, but not $(0.3, 0.3)$ or $(0,0)$.

d. Symmetric equilibrium: $(.125, .125)$

The Environment as a Tragedy of the Commons

In this case, the cost of consumption in the future for the firms is g.

Greenhouse emissions $g = g_1 + g_2$ created by consumption $g_1 = c_1$, $g_2 = c_2$

Utility $u_1 = \log c_1 - g = \log c_1 - c_1 - c_2$

FOC: $\frac{\partial u_1}{\partial c_1} = \frac{1}{c_1} - 1 = 0$ if $c_1 = 1$; symmetrically, $c_2 = 1$ so utility $u_1 = u_2 = -2$

a. Nash equilibrium is maybe more intuitive in continuous model: from arbitrary starting point, both players gradually optimize. E.g., status quo $(0.1, 1.8)$, player 2 cuts back on pollution, player 1 notices pollution but is so hungry he/she makes a campfire to cook on, player 2 cuts back on pollution even more, etc...

Planner: $W = \log c_1 + \log c_2 - 2(c_1 + c_2)$; FOC $\frac{1}{c_1} - 2 = 0 \Leftrightarrow c_1^{**} = \frac{1}{2}$, $u_1^{**} = \log\left(\frac{1}{2}\right) - 1 \approx -1.7$; everyone consumes less but is happier.

Reinterpret: litter, water pollution, noise pollution

Policy Solutions

- Privatize the Commons.
- Tax/Subsidy: park use fees, carbon tax, subsidies for green appliances.
- Regulate Number of Users: zero vandalism limit, cap and trade, national parks lottery.
- Persuade: Negative shaming, positive preaching, preach environmentalism or preach love thy neighbor.
- Global Cooperation: In case of greenhouse gases, clean oceans, etc., collective actions within US insufficient: production merely moves foreign and continues. Need global cooperation.

Problems From Chapter 8

8.1

One example of a resource that not everybody has access to is designer clothing. Only the people with enough money can buy designer clothes.

8.2

Solar energy is a resource that is not depletable. It does not matter how many people have solar panels on their homes, they do not cause the availability of solar energy to decrease by using it for themselves.

8.3

There is not a tragedy of the commons for either of the resources, because it is impossible for either of them to get overused. A resource has to be commonly accessible and depletable for it to be overused. Since designer clothing is too expensive, there will never be enough people to overuse it: hence it is not commonly accessible. Furthermore, since solar energy cannot be depleted, there will never not be enough for anybody who wanted to use it, so it cannot be overused.

8.4

I believe that the resources devoted to public education are examples common property resources because they are commonly accessible (for all intents and purposes) in the United States and they are depletable. I do believe there must be qualifications on the resources, however. For example, public education may not be a common property resource in third-world countries because it may not be commonly accessible.

8.5

An example of the tragedy of the commons in my daily life is store bought candy left on the counter in my apartment. Anybody in the apartment can go buy candy, so it is commonly accessible (and we do go buy candy often). Furthermore, once one of use puts the candy on the counter, it is a sign that anybody can eat it. Once this happens, there is never enough to go around. It is definitely depletable, as one cannot get the candy back once it is eaten without some serious drawbacks. The candy eventually gets overused, eaten too quickly, and the cycle repeats.

Renewable Resources Problem: Problems 8.7, 8.8, and 8.15

(Calculus problem) The resource that we analyzed in the text is an exhaustible resource. To see that the phenomenon of the tragedy of the commons can also arise if the resource is *renewable,* consider the following variant of the model in the text. Each player extracts an amount c_i in the first period, $i = 1, 2$. Whatever is not extracted, that is, the amount $y - c_1 - c_2$, regenerates and becomes an amount equal to $\sqrt{y - c_1 - c_2}$ in period 2.[11] The rest of the model will be identical to that in the text; in particular, the utility function will be $\log c$, and the allocation rule (if the total desired is more than what is available) will be to give half to each player.

8.6. Write down the best-response problem for player 1.

8.7. Show that the best-response function is given by[12]

$$R_1(c_2) = \frac{2(y - c_2)}{3}$$

8.7

$$u_1 = \log(c_1) + \log\left(\frac{\sqrt{y-c_1-c_2}}{2}\right)$$

$$u_1 = \log(c_1) \quad + (\log(\sqrt{y-c_1-c_2}) - \log 2)$$

$$\text{FOC: } \frac{1}{c_1} - \frac{1}{2}\,\frac{1}{y-c_1-c_2} = 0$$

$$c_1 = \frac{2(y-c_1-c_2)}{1}$$

$$c_1 + 2c_1 = \frac{2(y-c_2)}{1}$$

$$3c_1 = \frac{2(y-c_2)}{1}$$

$$c_1 = \frac{2}{3}\left(\frac{y-c_2}{1}\right)$$

$$R_1(c_2) = \frac{2(y-c_2)}{3}$$

8.8

$$R_1(c_2) = \frac{2(y-c_2)}{3} \qquad R_2(c_1) = \frac{2(y-c_1)}{3}$$

$$c_1^{br} = \frac{2y}{3} - \frac{2}{3}\left[\frac{2(y-c_1)}{3}\right]$$

$$c_1^{br} = \frac{2y}{3} - \frac{4}{9}y + \frac{4}{9}c_1$$

$$\frac{5}{9}c_1 = \frac{2y}{3} - \frac{4}{9}y$$

$$\frac{5}{9}c_1 = \frac{2}{9}y$$

$$c_1^* = \frac{2}{5}y$$

Symmetric

$$c_2^* = \frac{2}{5}y$$

8.15

$$u_1 = \log(c_1) + \log\left(\sqrt{\frac{y - c_1 - c_2 \cdots - c_n}{n}}\right)$$

$$\text{FOC: } \frac{\partial u_1}{\partial c_1} = \frac{1}{c_1} - \frac{1}{2}\,\frac{1}{y - c_1 - c_2 \cdots - c_n} = 0$$

all c's are same so $y - nc^*$

$$\text{symmetric } \quad c_1 = \frac{y - c_1 - c_2 \cdots - c_n}{2}$$

$$c^* = \frac{y - nc^*}{2}$$

$$2c^* = y - nc^*$$

$$c^*(2+n) = y$$

$$c^* = \frac{y}{2+n}$$

$$\lim_{n \to \infty} c^* = \frac{y}{\infty} = 0$$

Yes. All resources will be extracted in the first period.

The Greenhouse Emissions Model: Problems 8.17-8.21

Consider a two-period version of the greenhouse emissions model. The two countries choose consumption in period 1, c_i^1, and in period two, c_i^2. Greenhouse gases accumulate over time. In period 1 they are equal to $g^1 = c_1^1 + c_2^1$ and in period 2: $g^2 = g^1 + c_1^2 + c_2^2$.

8.17. Find the Nash equilibrium of this two-period game.

8.18. Set up and solve the social optimality problem of this two-period greenhouse emissions problem.

Now assume that a fraction σ of the greenhouse gases of period 1 dissipates due to photosynthesis: $g^2 = \sigma g^1 + c_1^2 + c_2^2$, where $\sigma \in [0, 1]$.

8.19. Find the Nash equilibrium of this two-period game.

8.20. Set up and solve the social optimality problem of this two-period greenhouse emissions problem.

8.21. Compare your results with to the situation in which there is no photosynthesis ($\sigma = 1$).

8.17

Period 1: c_i^1 Period 2: c_i^2

$g_1 = c_1^1 + c_2^1$ $g_2 = g_1 + c_1^2 + c_2^2$

$$u_1 = \log(c_1^1) - g^1 + \log(c_1^2) - g_2$$

$$u_1 = \log(c_1^1) - 2c_1^1 - 2c_2^1 + \log(c_1^2) - c_1^2 - c_2^2$$

$$\frac{\partial u_1}{\partial c_1^1} = \frac{1}{c_1^1} - 2 = 0 \qquad c_1^1 = \frac{1}{2}$$

$$\frac{\partial u}{\partial c_1^2} = \frac{1}{c_1^2} - 1 = 0 \qquad c_1^2 = 1$$

Symmetric

$$c_2^1 = \frac{1}{2}$$

$$c_2^2 = 1$$

$$\boxed{\begin{array}{l} (c_1^{1*}, c_1^{2*}) = (\frac{1}{2}, 1) \\ (c_2^{1*}, c_2^{2*}) = (\frac{1}{2}, 1) \end{array}}$$

8.18

Social Optimality

$$U = \log(c_1^1) - 2c_1^1 - 2c_2^1 + \log(c_1^2) - c_1^2 - c_2^2$$
$$+ \log(c_2^1) - 2c_1^1 - 2c_2^1 + \log(c_2^2) - c_1^2 - c_2^2$$

$$U = \log(c_1^1) + \log(c_1^2) + \log(c_2^1) + \log(c_2^2)$$
$$-4c_1^1 - 4c_2^1 - 2c_1^2 - 2c_2^2$$

$$\frac{\partial U}{\partial c_1^1} = \frac{1}{c_1^1} - 4 = 0 \quad c_1^1 = \frac{1}{4}$$

$$\frac{\partial U}{c_1^2} = \frac{1}{c_1^2} - 2 = 0 \quad c_1^2 = \frac{1}{2}$$

$$\frac{\partial U}{c_2^1} = \frac{1}{c_2^1} - 4 = 0 \quad c_2^1 = \frac{1}{4}$$

$$\frac{\partial U}{c_2^2} = \frac{1}{c_2^2} - 2 = 0 \quad c_2^2 = \frac{1}{2}$$

$$\boxed{\begin{aligned} (c_1^{1*}, c_1^{2*}) &= \left(\tfrac{1}{4}, \tfrac{1}{2}\right) \\ (c_2^{1*}, c_2^{2*}) &= \left(\tfrac{1}{4}, \tfrac{1}{2}\right) \end{aligned}}$$

8.19

Now $g^2 = \sigma g^1 + c_1^2 + c_2^2 \quad \sigma \in [0,1]$

$$u_1 = \log(c_1^1) - g^1 + \log(c_1^2) - g_2$$

$$= \log(c_1^1) - c_1^1 - c_2^1 + \log(c_1^2) - \sigma c_1^1 - \sigma c_2^1 - c_1^2 - c_2^2$$

$$\frac{\partial u}{\partial c_1^1} = \frac{1}{c_1^1} - 1 - \sigma = 0 \qquad c_1^1 = \frac{1}{1+\sigma}$$

$$\frac{\partial u}{\partial c_1^2} = \frac{1}{c_1^2} - 1 = 0 \qquad c_1^2 = 1$$

Symmetric

$$c_2^1 = \frac{1}{1+\sigma}$$

$$c_2^2 = 1$$

$$\boxed{\begin{aligned} &(c_1^{1*}, c_1^{2*}) = \left(\frac{1}{1+\sigma}, 1\right) \\ &(c_2^{1*}, c_2^{2*}) = \left(\frac{1}{1+\sigma}, 1\right) \end{aligned}}$$

8.20

Social Optimality

$$u = \log(c_1^1) - c_1^1 - c_2^1 + \log(c_1^2) - \sigma c_1^1 - \sigma c_2^1 - c_1^2 - c_2^2$$
$$+ \log(c_2^1) - c_1^1 - c_2^1 + \log(c_2^2) - \sigma c_1^1 - \sigma c_2^1 - c_1^2 - c_2^2$$

$$\frac{\partial u}{\partial c_1^1} = \frac{1}{c_1^1} - 2 - 2\sigma = 0 \qquad c_1^1 = \frac{1}{2+2\sigma}$$

$$\frac{\partial u}{\partial c_1^2} = \frac{1}{c_1^2} - 2 = 0 \qquad c_1^2 = 1/2$$

$$\frac{\partial u}{\partial c_2^1} = \frac{1}{c_2^1} - 2 - 2\sigma = 0 \qquad c_2^1 = \frac{1}{2+2\sigma}$$

$$\frac{\partial u}{\partial c_2^2} = \frac{1}{c_2^2} - 2 = 0 \qquad c_2^2 = \frac{1}{2}$$

$$(c_1^{1*}, c_1^{2*}) = \left(\frac{1}{2+2\sigma}, \frac{1}{2}\right)$$

$$(c_2^{1*}, c_2^{2*}) = \left(\frac{1}{2+2\sigma}, \frac{1}{2}\right)$$

8.21

if $\sigma = 1$

Nash:

$C_1 \rightarrow (\frac{1}{2}, 1)$ $C_2 \rightarrow (\frac{1}{2}, 1)$

Social Optimality:

$C_1 \rightarrow (\frac{1}{4}, \frac{1}{2})$ $C_2 \rightarrow (\frac{1}{4}, \frac{1}{2})$

Same as in problems 8.17 & 8.18

8.23

To avert crisis, Internet providers could only give internet to a select amount of people. Then, the next day during Internet *rush hour*, the Internet provider could choose new people to be allowed to have internet, giving them a term at fast Internet. That way, the internet would not be commonly accessible to everyone and we would not have a tragedy of the commons. This is not in the interest of the online companies to implement because they would get less money as they would be providing their service less. Also, there is a good chance they would lose a bunch of customers out of annoyance.