# HW 9

P2] $m^2 \mathrm{Tr}[A_M A^M]$ $\quad V = e^{-i\theta^a T^a}$

$A_M = A_M^a T^a$

$m^2 A_M^a A_a^M \mathrm{Tr}[T^a T^a]$ $\quad \mathrm{Tr}[T^a T^a] = \frac{1}{2}\delta^{aa}$

$m^2\left(\left(V A_M^a V^{-1} + i/g\, \partial_M(V) V^{-1}\right)\left(V A_a^{M'} V^{-1} + i/g\, \partial_M(V) V^{-1}\right)\right)$

$m^2\left(\left(e^{-i\theta^a T^a} A_M^{a'} e^{i\theta^a T^a} + i/g\, \partial_M\left(e^{-i\theta^a T^a}\right) e^{-i\theta^a T^a} e^{i\theta^a T^a}\right)(\ldots)\right)$

$m^2\left(\left(A_M^{a'} + i/g\, \partial_M\left(e^{-i\theta^a T^a}\right)\right)\left(A_a^{M'} + i/g\, \partial_M\left(e^{-i\theta^a T^a}\right)\right)\right)$

$m^2\Big(\Big(A_M^{a'} A_a^{M'} + A_M^{a'}\, i/g\, \partial_M\left(e^{-i\theta^a T^a}\right) + i/g\, \partial_M\left(e^{-i\theta^a T^a}\right) A_a^{M'}$

$-1/g^2\, \partial_M^2\left(e^{-i\theta^a T^a}\right)\Big)\frac{1}{2}$ → From $\mathrm{Tr}[T^a T^a]$

Not gauge invariant

P31 $\alpha_s(Q) = \frac{\alpha_s(Q_0)}{1 + \alpha_s(Q_0)/2\pi (11 - 2/3 n_f) \ln Q/Q_0}$

$\alpha_s(m_z) = 0.1179$ $\quad m_z = 91.1876\ GeV$

$\alpha_s(m_b) \rightarrow$ find it $\quad m_b = 4.18\ GeV \quad n_f = 5$

$$\alpha_s(m_b) = \frac{\alpha_s(m_z)}{1 + \alpha_s(m_z)/2\pi (11 - 2/3 n_f) \ln m_b/m_z}$$

$$\alpha_s(m_b) = \frac{0.1179}{1 + 0.1179/2\pi (11 - 2/3(5)) \ln 4.18\ GeV/91.1876}$$

$$\boxed{\alpha_s(m_b) = 0.211846}$$

$m_c = 1.28\ GeV$

$n_f = 4$

$$\alpha_s(m_c) = \frac{\alpha_s(m_z)}{1 + \alpha_s(m_z)/2\pi (11 - 2/3\, n_f) \ln m_c/m_z}$$

$$\alpha_s(m_c) = \frac{0.1179}{1 + 0.1179/2\pi (11 - 2/3(4)) \ln 1.28\ GeV/91.1876\ GeV}$$

$$\boxed{\alpha_s(m_c) = 0.354142}$$

$$\frac{1}{\alpha_s(\Lambda_{QCD})} = \frac{1 + 0.1179/2\pi (11 - 2/3(3)) \ln(\Lambda_{QCD}/91.1876)}{0.1179}$$

(3 quarks)

$$-1 = 0.1179/2\pi (11 - 2/3(3)) \ln(\Lambda_{QCD}/91.1876)$$

$$91.1876 \exp\left(\frac{-2\pi}{0.1179} \frac{1}{(11 - 2/3(3))}\right) = \frac{\Lambda_{QCD}}{91.1876}$$

$$\boxed{\Lambda_{QCD} = 0.244517}$$