Homework 2

Strategic Form Games and Dominant Strategies, as well as Dominance Solvability.

Notes from Chapter 3

Strategic Form Games involve three things

1. The **list** of players in the game
2. The set of **strategies** available to each player
3. The **payoffs** associated with any strategic combination (one strategy per player)

Matrix Form

General :

Example :

Dominant Strategy

Stratgey s_i' strongly dominates all other strategies of player i if the payoff to s_i' is strictly greater than the payoff to any other strategy, regardless of which strategy is chosen by the other player(s) (s_{-i}):

$\pi_i (s_i', s_{-i}) > \pi_i (s_i, s_{-i})$

where s_{-i} is a strategy vector choice of players other than i.

Weakly dominates if:

$\pi_i (s_i', s_{-i}) \geq \pi_i (s_i^w, s_{-i})$

$\pi_i (s_i', \hat{s}_{-i}) > \pi_i (s_i^w, \hat{s}_{-i})$

Dominant Strategy Solution: A combination of strategies is said to be a dominant strategy solution if each player's strategy is a dominant strategy.

Problems from Chapter 3

Problems 3.19 - 3.23

3.19

Strategic Form

- Players: The two players who volunteer their spare time working in the garden.
- Strategies: A player's strategy is to maximize their utility.
- Outcomes: Both players will receive utility a certain amount of utility once they finish their work volunteering. The one who works more will be tired and will experience a dis-utility.
- Payoffs: Each player receives a payoff at the end of the game. Player 1's payoff will be the result of how hard both volunteers worked minus the time he spent working: $\sqrt{x+y}-x$. Player 2's payoff will be the result of how hard both volunteers worked minus the time he spent working: $\sqrt{x+y}-y$.

3.20

Volunteering for one hour weekly dominates volunteering for two hours.

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In[134]:= func1[x_, y_] := √(x + y) - x
          func2[x_, y_] := √(x + y) - y
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In[87]:= PopupMenu[a, {a = "Example Calculations", bothonehour = N[func1[1, 1]],
   p1onehourxutility = N[func1[1, 2]], p1onehouryutility = N[func2[1, 2]],
   p2onehourxutiliry = N[func1[2, 1]], p2onehouryutility = N[func2[2, 1]],
   bothtwohours = N[func1[2, 2]], onethree1 = func1[1, 3] // N,
   onethree2 = func2[1, 3] // N, onefour1 = func1[1, 4] // N, onefour2 = func2[1, 4] // N,
   twothree1 = func1[2, 3] // N, twothree2 = func2[2, 3] // N, twofour1 = func1[2, 4] // N,
   twofour2 = func2[2, 4] // N, fourone1 = func1[1, 4] // N, fourone2 = func2[1, 4] // N}]
```

Out[87]= Example Calculations

P1 / P2	1 Hour	2 Hours
1 Hour	(0.414, 0.414)	(0.7321, −0.2680)
2 Hours	(−0.2680, 0.7321)	(0, 0)

3.21

Volunteering for one hour weekly dominates the strategy of volunteering for two hours. In every circumstance, volunteering for one hour yields equal to or greater utility for Player 1 than every other strategy.

P1 / P2	1 Hour	2 Hours	3 Hours	Hours
1 Hour	(0.414, 0.414)	(0.7321, −0.2680)	(1, −1)	(1.2361, −1.
2 Hours	(−0.2680, 0.7321)	(0, 0)	(0.2361, −0.7639)	(0.4495, −1.
3 Hours	(−1, 1)	(−0.7639, 0.2361)	(−0.5505, −0.5505)	(−0.3542, −1
4 Hours	(−1.764, 1.2361)	(−1.551, 0.4495)	(−1.354, −0.3542)	(−1.1716, −1

Volunteering for two hours over three hours and three hours over four hours are other examples of weakly dominated strategies. Two weakly dominates three and three weakly dominates four. The lower volunteering weakly dominates the greater volunteering because one's own action of volunteering has a greater dis-utitlity than utility according to the functions above. The dominate strategy for player one is almost to always work for one hour. However, there is one catch. Working for two hours when Player 2 works for 4 hours is actually preferable because that gives Player 1 a utility of .4495. Recall that the utility player 1 receives if they both work for one hour is 0.414.

3.22

The strategic form will be the same, except for the outcome, which will include Player 1's new payoff of $2\sqrt{x+y}-x$.

The new one and two hour matrix would look like this. Working for one hour does not weakly dominate working for two hours, as if both players work for one hour, Player 1 receives a utility of 1.828. However, if both players were to work for 2 hours, Player 1 would receive a utility of 2.

P1 / P2	1 Hour	2 Hours
1 Hour	(1.828, 0.414)	(2.46, -0.2680)
2 Hours	(1.464, 0.7321)	(2, 0)

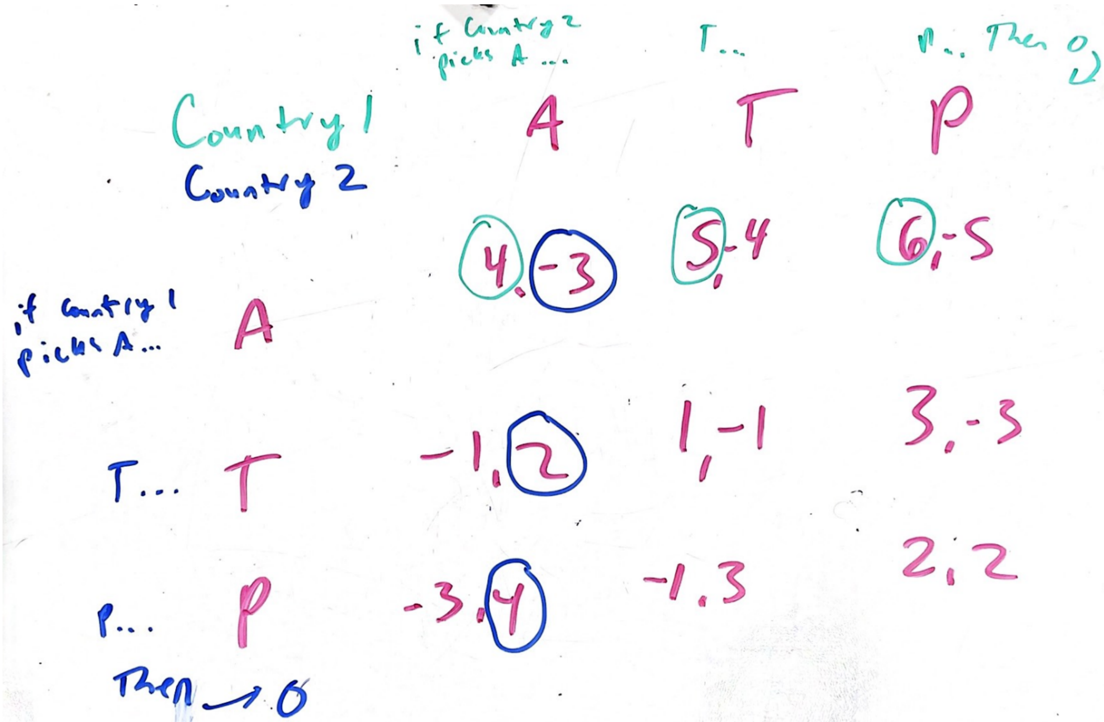
I am not going to calculate the entire matrix again. However, this time it seems that it is much harder to find a dominate strategy. There are no weakly dominated strategies because Player 1's and Player 2's utility are never equal. Additionally, working for one hour is not preferable to working for even three hours because Player 1's utility if each person works for three hours is 1.89898, which is greater than if both players work for one hour. However, working for one hour does dominate working for four hours. Working for one hour both weakly and strongly dominates working for four hours. There is no dominate strategy for Player 1.

Problems 3.27 - 3.30

Problem 3.27

Choosing A is always preferable for Country 1. Choosing A is a dominate strategy. Country 2 does have a dominate strategy. Choosing A is a dominant strategy, as the A column always has better options than the T and P columns no matter what Country 1 chooses.

Problem 3.28

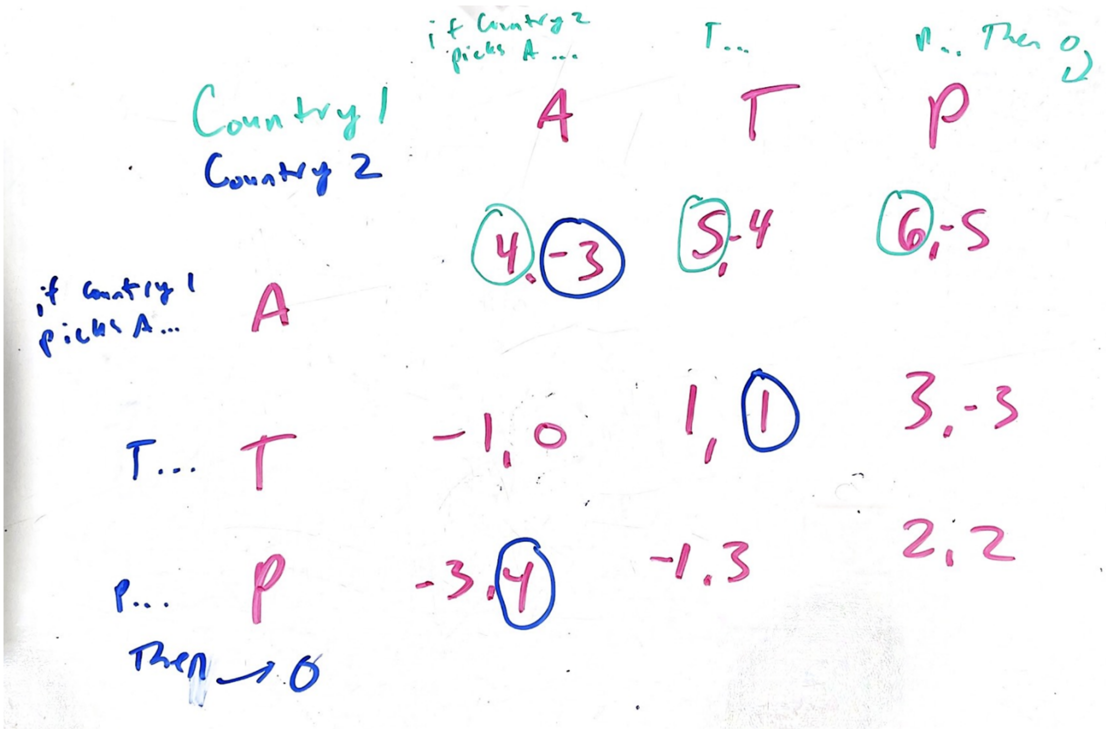


The dominant strategy equilibrium is at A, A or 4,-3.

Problem 3.29

Choosing A is always preferable for Country 1. Choosing A is a dominate strategy. Country 2 does not have a dominate strategy of any kind. It gets close. However, the second column holds a better option in its second row than does the first column in its second row. If this were not the case, than choosing A would always be the best option for Country 2.

Problem 3.30



The dominant strategy equilibrium is still at A, A or 4, -3.

Problems from Chapter 4

Problems 4.8 - 4.12

Q = 6 – p

- two firms
- p is the lower of the two prices
- the lower prices firm meets all the demand
- if both firms set the same price, they each get half: $\frac{6-p}{2}$
- prices are in dollar units and go from (1,2,3,4,5,6)

Problem 4.8

Posting a price of $0 and $6 are dominated strategies because posting $0 means that you are making no revenue and posting $6 means that the other firm can just post at $5 and take all the money. The two firms will both attempt to beat each other out by posting lower and lower prices until they both settle at 1$. 2$ and 5$ will be dominated by 1$ because both firms will settle there and they will make half the profit of the market.

Problem 4.9

$$Q = 6 - p$$

$$\pi = pQ$$

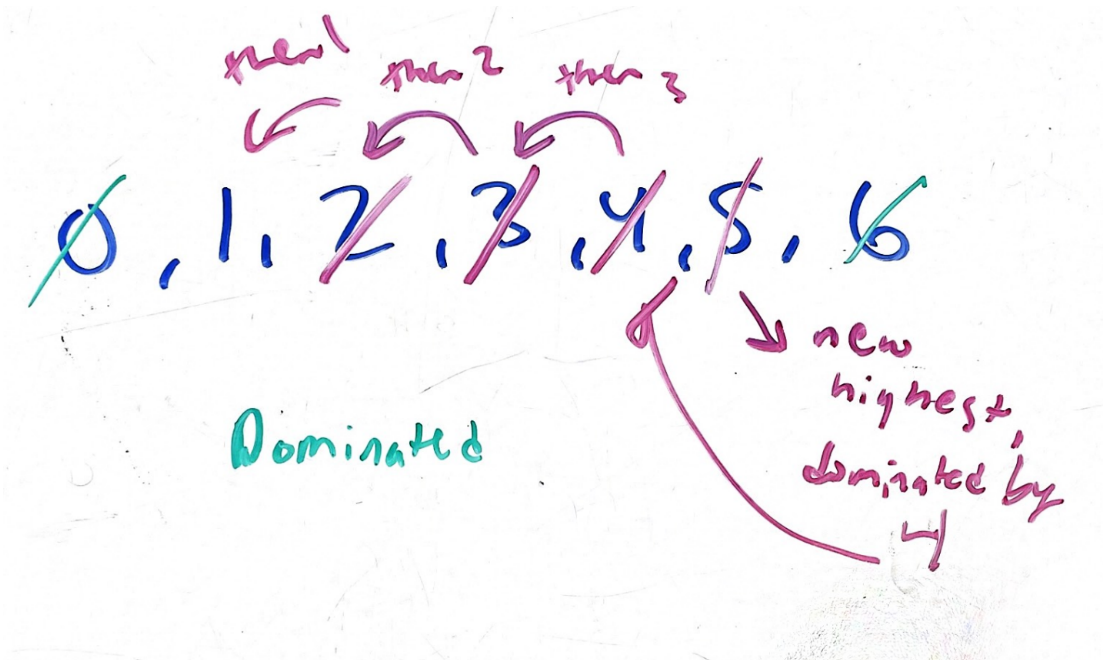
$$pQ = 6p - p^2$$

$$\frac{\partial \pi}{\partial p} = 6 - 2p = 0$$

$$6 = 2p \quad p = \$3$$

Problem 4.10, 4.11, 4.12

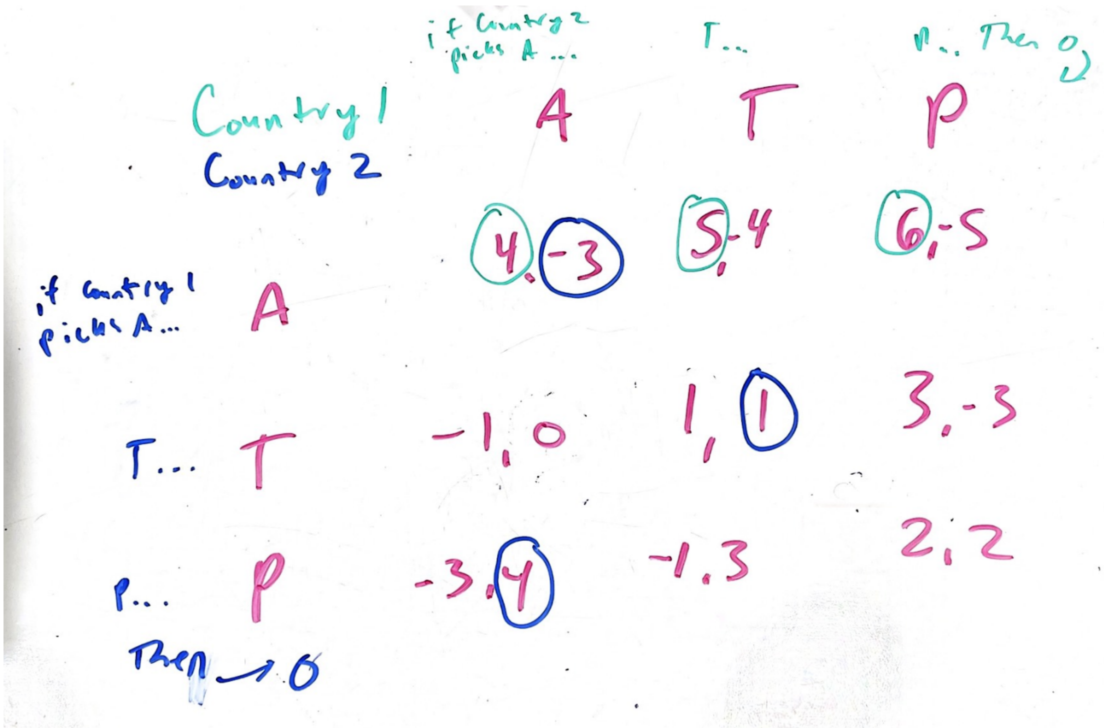
A duopoly would never want to charge above the monopoly price because there would be no demand for its products at that price. A duopoly firm could only make profits when charging above the monopoly price if they distributed a necessary good and were working in tandem with the other firm. However, that will never work because each firm is looking out for its own interests and will try to make the most money by offering a lower price. If we restricted prices to \$1, 2\$, and 3\$, both firms would want to charge at the price that would make them the most money. However, if one firm chose \$3, the other would have a dominant strategy of pricing at \$2, because they would have control of the entire market, yielding them the most profit. Therefore, \$3 will always be a dominated strategy. The unique outcome to IEDS in this model would be \$1. This is due to the fact that the highest price is always a dominated strategy. Once one firm prices at \$5, it is a dominant strategy for the other firm to price at \$4. Then from \$4, the dominant strategy is to price at \$3. This continues all the way down to \$1.



Problems 4.18 - 4.23

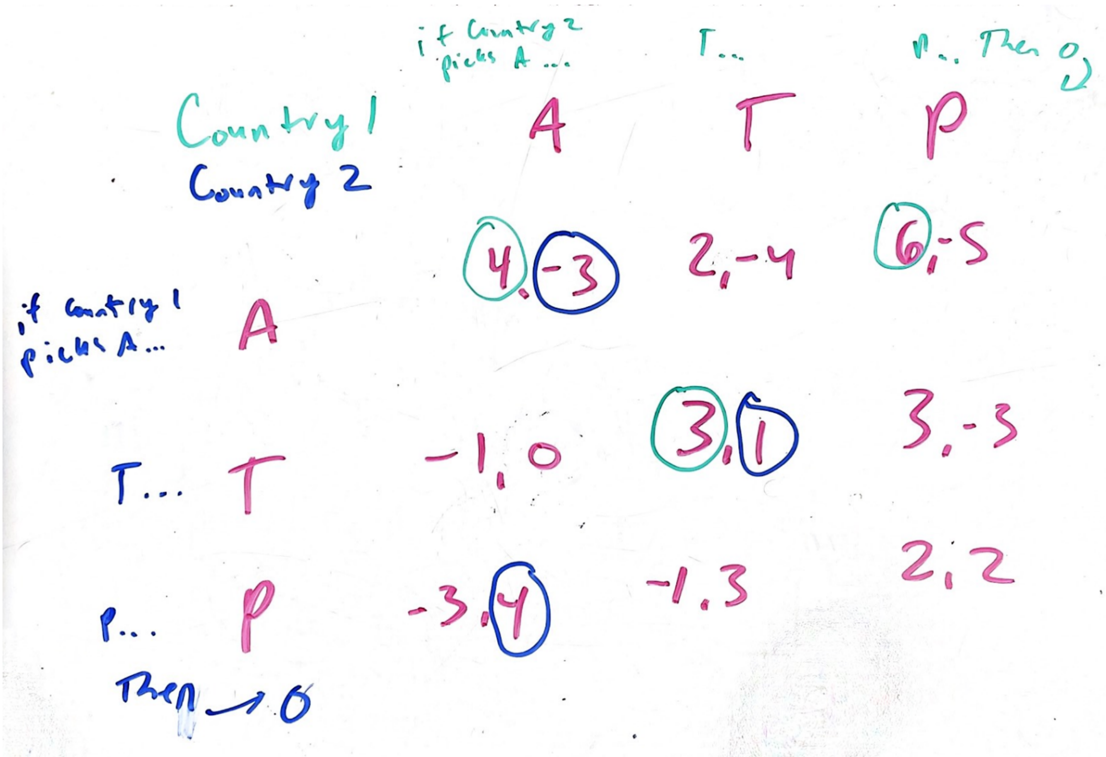
Problem 4.18, 4.19, and 4.20

Choosing A is always preferable for Country 1. Choosing A is a dominate strategy. Country 2 does not have a dominate strategy of any kind. It gets close. However, the second column holds a better option in its second row than does the first column in its second row. If this were not the case, than choosing A would always be the best option for Country 2. T and P are dominated strategies for Country 1; they are dominated by the strategy of A. For country 2, P is dominated by A and T. A and T are always better options than P. A,A is the only strategy that survives IEDS. For all other strategies, a better play can be made by one of the two countries.



Problems 4.21, 4.22, and 4.23

There are no dominant strategies for Country 1 or 2. Choosing P is a dominated strategy for both countries because both countries can always do better no matter what the other picks. There are two solutions surviving IEDS. These solutions are at A,A (4,-3) and T,T (3,1). It is not dominance solvable because there is not a unique strategy vector that is a solution to IEDS.

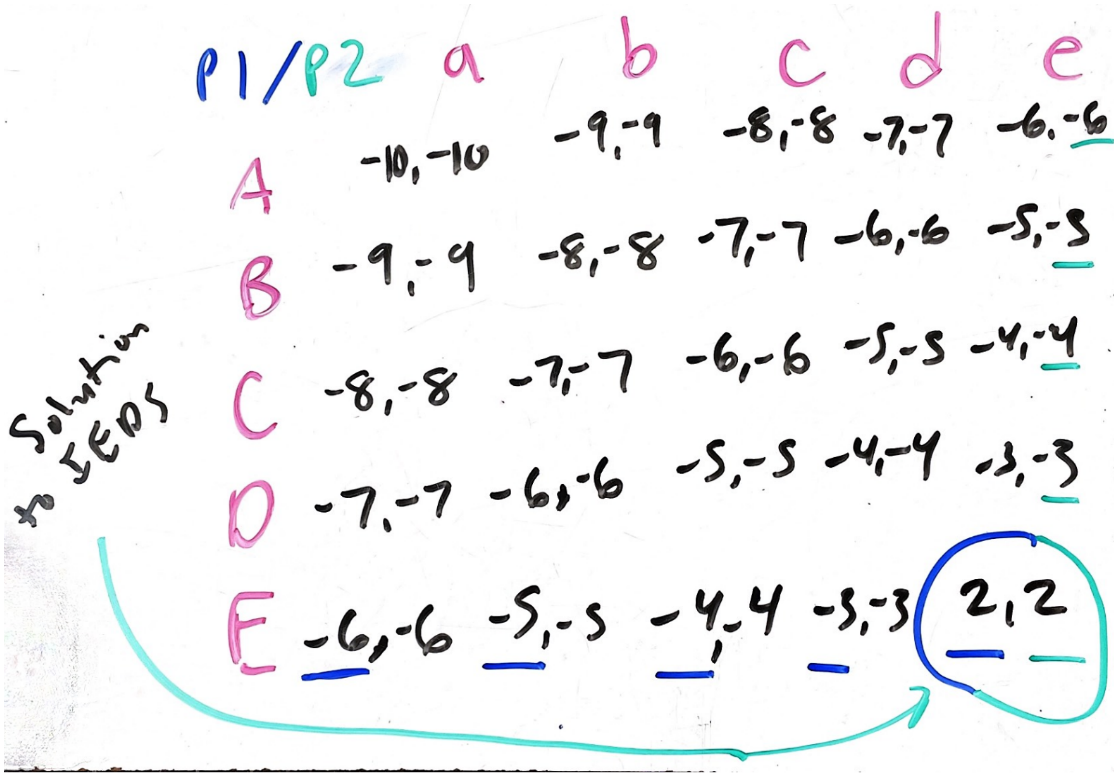


Problems 4.24 - 4.25

There is not a dominant strategy for Andy or Mandy, as there are no choices that are always the best.
The only solution surviving IEDS is H,H (20,20)

A/M	N	L	H
N	50,50	40,40	0,70
L	40,40	30,30	15,35
H	80,0	35,15	20,20

Problem 5 from the Homework



P1/P2	a	b	c	d	e
A	-10,-10	-9,-9	-8,-8	-7,-7	-6,-6
B	-9,-9	-8,-8	-7,-7	-6,-6	-5,-5
C	-8,-8	-7,-7	-6,-6	-5,-5	-4,-4
D	-7,-7	-6,-6	-5,-5	-4,-4	-3,-3
E	-6,-6	-5,-5	-4,-4	-3,-3	2,2