HW10

1

a)

1/2	Left	Right
Up	1,0,1	2,0,2
Down	0,2,2	0,0,0

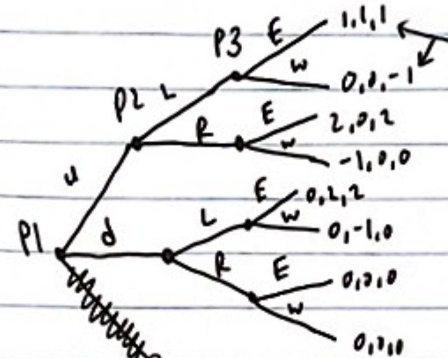
Player 3 East

1/2	Left	Right
Up	0,0,-1	-1,0,0
Down	0,-1,0	0,0,0

Player 3 West

The pure strategy Nash Equilibria is one of Down, Right, West and Up, Left, East. Player 3 will chose $\boxed{\text{Up, Left, East}}$

b) No, the preceding strategy is not a pure strategy Nash Equilibria Subgame perfect Nash Equilibrium because ULW is not a best-response for Player 3. Player 3 should never play ULW opposed to ULE. It is never in their best interest. So, it cannot be a Nash Equilibrium within the subgame.



ULW. no ULW
ULE DRW

b) Sorry if the above answer is hard to read. I said no. It is not a subgame perfect Nash Equilibria because ULW is not a best-response for player 3. He should never play ULW because he'll get -1. He could chose East and get 1 util. Since it is not a best response, it cannot be a Nash Equilibria.

c) The pure strategy subgame perfect Nash Equilibria include any combination of the two pure-strategy Nash Equilibria (ULE & DRW). It can be them in any order, or it could simply be ULE twice and DRW twice. Another subgame perfect Nash Equilibria can be obtained with the "Grim Trigger" strategy. For example, DLE threatening DRW is one. URE threatening DRW is another. For each of these 2 of the players are better off playing the first version and the other player is indifferent because they will get 1 util either way with ULE or if the others play DRW the second round.

in each case, the players will play VLE in the second round unless the first round is not DLE or VRE, then they will play DRW. That is it.

d) I explained that in the paragraph above. The two that require threats/promises are DLE threatening DRW and VRE threatening DRW.

2)

a) Reduced multiprice auction

1/2	75, h	75, ℓ
75, h	$\underline{50\pi_h}, \underline{50\pi_h}$	$75\pi_h, 25\pi_\ell$
75, ℓ	$\underline{25\pi_\ell}, \underline{75\pi_h}$	$\underline{50\pi_\ell}, \underline{50\pi_\ell}$

we assume $50\pi_\ell > 75\pi_h$

Two Nash Equilibria

Since 50π is better than $75\pi_h$ no matter if it is $50\pi_h$ or $50\pi_\ell$, the 50's will always be a best response no matter what. Since they are both Nash Equilibria, any combination of high and low prices will be a subgame perfect Nash Equilibria as long as both the bidders pick the same price.

15.16-15.22 → Grim Trigger

b) Although there are two equilibria in our subgame, the ℓ,ℓ is the "better" equilibrium because it allows the bidders to earn more profits. A bidder would be better off by playing h out of turn if their opponent plays l as well. However, the Grim Trigger strategy eliminates that incentive. If a bidder plays h out of turn, then the strategies will be h,h from then on. In the last T^* auctions, there will be no more rounds to the game so there is no potential punishment. Therefore, the bidders have no incentive to bid other than ℓ.

c) Before the last T^* auctions, odd #'s are h,ℓ even # are ℓ,h. One auction phase = 2 rounds

$h,\ell \to E(1) = 75\pi_h \cdot 1/2 \quad E(2) = 25\pi_\ell \cdot 1/2$

$\ell,h \to E(1) = 25\pi_\ell \cdot 1/2 \quad E(2) = 75\pi_h \cdot 1/2$

$$\boxed{E_{tot}\text{ (for one round)} = \frac{75\pi_h}{2} + \frac{25\pi_\ell}{2} = \frac{75\pi_h + 25\pi_\ell}{2}}$$

d) After departing from the strategy, one would bid h when they were supposed to bid ℓ.

$h, h \rightarrow E(1) = 50\pi_h \cdot 1/2 \qquad E(2) = 50\pi_h \cdot 1/2$

$$\boxed{E_{tot} = 50\pi_h/2 + 50\pi_h/2 = 50\pi_h}$$

2 rounds, $h,h \rightarrow E(1) = 50\pi_h \cdot 1/2 \qquad E(2) = 50\pi_h \cdot 1/2$ — add round one and two

↳ always h.h from now on (Grim Trigger)

e) Before the last T^* auctions, we would get the profit

$$\frac{25\pi_\ell + 75\pi_h}{2}(T - T^*) + 50\pi_\ell T^* \text{ if we kept strategy}$$

if not, we would get

$$\cancel{50\pi_h} + 50\pi_h(T - T^*) + 50\pi_h T^*. \text{ Calculate the loss.}$$

Sorry, these are current gains only include future

$$\frac{25\pi_\ell + 75\pi_h}{2}(T - T^*) + 50\pi_\ell T^* - \cancel{50\pi_h} - 50\pi_h(T - T^*) - 50\pi_h T^*$$

$$\boxed{= \left(\frac{25\pi_\ell + 75\pi_h}{2} - 50\pi_h\right)(T - T^*) + T^*(50\pi_\ell - 50\pi_h)}$$

f) If P1's turn to bid high

$P1 = 75\pi_h$

If P1 bids low instead (and P2 keeps strategy 3 also bids low)

$P1 = 50\pi_\ell$

$$\therefore \boxed{\text{profit} = \underbrace{50\pi_\ell}_{\text{current bid}} - \underbrace{75\pi_h}_{\text{opportunity cost}}}$$

low to high

$P1_{initial} = 25\pi_\ell$

$P1_{switch} = 50\pi_h \rightarrow \boxed{50\pi_h - 25\pi_\ell = \text{Profit}}$

g) Future loss $= (T - T^*)\left(\frac{25\pi_\ell + 75\pi_h}{2} - 50\pi_h\right) + T^*(50\pi_\ell - 50\pi_h)$

present gain $= 50\pi_\ell - 75\pi_h$ or $50\pi_h - 25\pi_\ell$

if $T^* = T$ the $FL = T(50\pi_\ell - \underbrace{50\pi_h}_{\text{bigger}}) > 50\pi_\ell - \underbrace{75\pi_h}_{\text{smaller}}$

and $T(\underbrace{50\pi_\ell}_{\text{bigger}} - \underbrace{50\pi_h}_{\text{bigger}}) > \underbrace{50\pi_h}_{\text{smaller}} - \underbrace{25\pi_\ell}_{\text{smaller}}$

But, with a large enough T, the left side > right side!

h) I feel like we already established that with all the preceding questions. In e), we showed that alternating is more profitable $\left(\frac{25\pi_L + 75\pi_H}{2} > 50\pi_H\right)$. That means that the two buyers will alternate bids (high and low) at the first of the game and before the last T^* options. We also showed that never party has the incentive to deviate from l in the last T^* options in b). Therefore, the Treasury first sees pairs of high and low bids all year except toward the last T^* when all bids are low.