Nasdaq

"bid" price b → sell it to ~~market~~ broker

"ask" price a → buy it from ~~market~~ broker

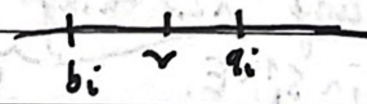
Price increments in $\$1/8$

Christy & Schultz 1994: 85% of prices ended in .00 .25 .50 .75

15% of the time they ended in .125 .375 .625 and .875

n market makers

each chose a_i, b_i



investor buys at price $a = \min_i a_i$

investor sells at price $b = \max_i b_i$

Supply & demand volumes $S(b), D(a)$

True asset value $v = p$

broker payoff:

$(v-a)D(a)$ or

$(b-v)S(b)$

Suppose

$D(a) = 120 - 5a$

$S(b) = -80 + 5b$

quantities in 10k shares

If single price $a = b = p$

market would clear at

$S(p) = D(p)$

$-80 + 5p = 120 - 5p$

$10p = 200$

$p = 20$

Collusion?/Monopoly

Choose b to maximize:

$$S(b)(v-b) = (5b-8)(20-b)$$

$$\text{FOC: } 5(20-b) - (5b-80) = 0$$

$$\text{if } 100-5b = 5b-80$$

$$180 = 10b$$

$$18 = b$$

Choose a to maximize:

$$(a-v)D(a) = (120-5a)(a-20)$$

$$\text{FOC: } -5(a-20)+(120-5a) = 0$$

$$120 - 5a = 5a - 100$$

$$220 = 10a$$

$$22 = a$$

$\therefore$ $b_i = 18$ $\quad v = 20$ $\quad a_i = 22$

$$S(b) = -80 + 5b = 5(18) - 80 = 10$$

$$D(a) = 120 - 5a = 120 - 5(22) = 10$$

$$\Pi \text{ (Profit)} = \text{broker payoff}_{\text{for selling}} = S(b)(v-b) = (\underbrace{5\overset{=18}{b}-80}_{10})(\underbrace{20-\overset{=18}{b}}_{2})$$

$$20 =$$

Profit for monopolist: 20 profit for n producers $20/n$

$$\Pi \text{ (Profit)} = \text{for } \text{buying} = D(a)(a-v) = (\underbrace{120-5a}_{10})(\underbrace{a-20}_{2})$$

$$20 =$$

Profit for monopolist: 20 Profit for n producers $20/n$

Total = profit buying + profit selling $= 20/n + 20/n = 40/n$

Deviation Profit

$b_i = 19$

$a_i = 21$

Profit from buying $= [5(19) - 80](20 - 19) = 15$

Profit from selling $= [120 - 5(21)](21 - 20) = 15$

Total profit $= 30$

Trigger Pricing: v

Collusion payoff: $\dfrac{40}{n(1-\delta)}$ Deviation payoff: $30 + \dfrac{0\delta}{1-\delta}$

Sustain collusion $\dfrac{40}{n(1-\delta)} > 30 \rightarrow \dfrac{4}{3} > n(1-\delta)$

Avg # of market makers $n = 11$

$\delta > \dfrac{29}{33} = 0.88$

if $n = 50$

$\delta > \dfrac{73}{75} = 0.97$

Trigger Pricing: match $\begin{matrix} b = 19 \\ a = 21 \end{matrix}$

Collusion payoff: $\dfrac{40}{n(1-\delta)}$ Deviation payoff: $30 + \dfrac{30/n\ \delta}{1-\delta}$

$n(1-\delta) < \dfrac{4}{3} - \delta$ sustain: $n(1-\delta) < \dfrac{1}{3}$

if $n = 11$

$\delta > \dfrac{32}{33} = 0.97$

n big market workers

m small market makers (half as large)

shares: $\frac{2}{2n+m}$ vs. $\frac{1}{2n+m}$ of surplus

OPEC

$q_{SA} \in \{8, 10\}$ Costs \$20/barrel

$q_V \in \{5, 7\}$ Demand < good / bad

$Q \in \{13, 15, 17\}$

\$100, \$88, \$76 → good

\$64, \$60, \$56 → bad

Good

SA \ V	5	7
8	640,400	544,476
10	680,340	560,392

Bad

SA \ V	5	7
8	352,220	320,280
10	400,200	360,252

SA profit stream $\frac{640}{1-\delta} \geq 680 + 560\delta/(1-\delta)$

collude as long as $\delta > 1/3$

V profit $\frac{400}{1-\delta} \geq 476 + \delta 392/(1-\delta)$

collude as long as $\delta > 19/21$

Probability p demand good

$1-p$ demand bad

Strategies:
- high for sure
- high if good
- low if good
- low q for sure

V

SA

Cartel: high q in bad years

low q in good years

$$\frac{[640p + 360(1-p)]}{1-\delta} \quad \text{deviation} \to 680 + \frac{560p + 360(1-p)}{1-\delta}$$

Sustain collusion if

$$\left.\begin{aligned} \delta &> \frac{1}{1+2p} \\ &\approx \frac{1}{3} \text{ if } p \approx 1 \end{aligned}\right\} SA$$

$$\left.\delta > \frac{19}{19+2p}\right\} V$$