# Problem 1

In[579]:=
```
SetDirectory[NotebookDirectory[]]
```

Out[579]=
```
C:\Users\carte\OneDrive\Documents\Wolfram Mathematica\513\HW
```

In[532]:=
```
T1 = 1/2 * {{0, 1, 0}, {1, 0, 0}, {0, 0, 0}} // MatrixForm

T2 = 1/2 * {{0, -I, 0}, {I, 0, 0}, {0, 0, 0}} // MatrixForm

T3 = 1/2 * {{1, 0, 0}, {0, -1, 0}, {0, 0, 0}} // MatrixForm

T4 = 1/2 * {{0, 0, 1}, {0, 0, 0}, {1, 0, 0}} // MatrixForm

T5 = 1/2 * {{0, 0, -I}, {0, 0, 0}, {I, 0, 0}} // MatrixForm

T6 = 1/2 * {{0, 0, 0}, {0, 0, 1}, {0, 1, 0}} // MatrixForm

T7 = 1/2 * {{0, 0, 0}, {0, 0, -I}, {0, I, 0}} // MatrixForm

T8 = 1/(2 * Sqrt[3]) * {{1, 0, 0}, {0, 1, 0}, {0, 0, -2}} // MatrixForm
```

Out[532]//MatrixForm=

$$\begin{pmatrix} 0 & \frac{1}{2} & 0 \\ \frac{1}{2} & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}$$

Out[533]//MatrixForm=

$$\begin{pmatrix} 0 & -\frac{i}{2} & 0 \\ \frac{i}{2} & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}$$

Out[534]//MatrixForm=

$$\begin{pmatrix} \frac{1}{2} & 0 & 0 \\ 0 & -\frac{1}{2} & 0 \\ 0 & 0 & 0 \end{pmatrix}$$

Out[535]//MatrixForm=

$$\begin{pmatrix} 0 & 0 & \frac{1}{2} \\ 0 & 0 & 0 \\ \frac{1}{2} & 0 & 0 \end{pmatrix}$$

Out[536]//MatrixForm=

$$\begin{pmatrix} 0 & 0 & -\frac{i}{2} \\ 0 & 0 & 0 \\ \frac{i}{2} & 0 & 0 \end{pmatrix}$$

Out[537]//MatrixForm=

$$\begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & \frac{1}{2} \\ 0 & \frac{1}{2} & 0 \end{pmatrix}$$

Out[538]//MatrixForm=

$$\begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & -\frac{i}{2} \\ 0 & \frac{i}{2} & 0 \end{pmatrix}$$

Out[539]//MatrixForm=

$$\begin{pmatrix} \frac{1}{2\sqrt{3}} & 0 & 0 \\ 0 & \frac{1}{2\sqrt{3}} & 0 \\ 0 & 0 & -\frac{1}{\sqrt{3}} \end{pmatrix}$$

In[540]:=
```
T1 = 1/2 * {{0, 1, 0}, {1, 0, 0}, {0, 0, 0}};
T2 = 1/2 * {{0, -I, 0}, {I, 0, 0}, {0, 0, 0}};
T3 = 1/2 * {{1, 0, 0}, {0, -1, 0}, {0, 0, 0}};
T4 = 1/2 * {{0, 0, 1}, {0, 0, 0}, {1, 0, 0}};
T5 = 1/2 * {{0, 0, -I}, {0, 0, 0}, {I, 0, 0}};
T6 = 1/2 * {{0, 0, 0}, {0, 0, 1}, {0, 1, 0}};
T7 = 1/2 * {{0, 0, 0}, {0, 0, -I}, {0, I, 0}};
T8 = 1/(2 * Sqrt[3]) * {{1, 0, 0}, {0, 1, 0}, {0, 0, -2}};
```

# Check that they are Antisymmetric

In[453]:=
```
T1.T2 - T2.T1 // MatrixForm
T2.T1 - T1.T2 // MatrixForm
T1.T3 - T3.T1 // MatrixForm
```

Out[453]//MatrixForm=

$$\begin{pmatrix} \frac{i}{2} & 0 & 0 \\ 0 & -\frac{i}{2} & 0 \\ 0 & 0 & 0 \end{pmatrix}$$

Out[454]//MatrixForm=

$$\begin{pmatrix} -\frac{i}{2} & 0 & 0 \\ 0 & \frac{i}{2} & 0 \\ 0 & 0 & 0 \end{pmatrix}$$

Out[455]//MatrixForm=

$$\begin{pmatrix} 0 & -\frac{1}{2} & 0 \\ \frac{1}{2} & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}$$

In[462]:=
```
T2.T3 - T3.T2 // MatrixForm
T3.T2 - T2.T3 // MatrixForm
T2.T1 - T1.T2 // MatrixForm
```

Out[462]//MatrixForm=

$$\begin{pmatrix} 0 & \frac{i}{2} & 0 \\ \frac{i}{2} & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}$$

Out[463]//MatrixForm=

$$\begin{pmatrix} 0 & -\frac{i}{2} & 0 \\ -\frac{i}{2} & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}$$

Out[464]//MatrixForm=

$$\begin{pmatrix} -\frac{i}{2} & 0 & 0 \\ 0 & \frac{i}{2} & 0 \\ 0 & 0 & 0 \end{pmatrix}$$

In[474]:=
```
T3.T4 - T4.T3 // MatrixForm
T4.T3 - T3.T4 // MatrixForm
T3.T5 - T5.T3 // MatrixForm
```

Out[474]//MatrixForm=

$$\begin{pmatrix} 0 & 0 & \frac{1}{4} \\ 0 & 0 & 0 \\ -\frac{1}{4} & 0 & 0 \end{pmatrix}$$

Out[475]//MatrixForm=

$$\begin{pmatrix} 0 & 0 & -\frac{1}{4} \\ 0 & 0 & 0 \\ \frac{1}{4} & 0 & 0 \end{pmatrix}$$

Out[476]//MatrixForm=

$$\begin{pmatrix} 0 & 0 & -\frac{i}{4} \\ 0 & 0 & 0 \\ -\frac{i}{4} & 0 & 0 \end{pmatrix}$$

In[480]:=
```
T4.T6 - T6.T4 // MatrixForm
T6.T4 - T4.T6 // MatrixForm
T4.T2 - T2.T4 // MatrixForm
```

Out[480]//MatrixForm=

$$\begin{pmatrix} 0 & \frac{1}{4} & 0 \\ -\frac{1}{4} & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}$$

Out[481]//MatrixForm=

$$\begin{pmatrix} 0 & -\frac{1}{4} & 0 \\ \frac{1}{4} & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}$$

Out[482]//MatrixForm=

$$\begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & -\frac{i}{4} \\ 0 & -\frac{i}{4} & 0 \end{pmatrix}$$

In[486]:=
```
T4.T7 - T7.T4 // MatrixForm
T7.T4 - T4.T7 // MatrixForm
T4.T1 - T1.T4 // MatrixForm
```

Out[486]//MatrixForm=

$$\begin{pmatrix} 0 & \frac{i}{4} & 0 \\ \frac{i}{4} & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}$$

Out[487]//MatrixForm=

$$\begin{pmatrix} 0 & -\frac{i}{4} & 0 \\ -\frac{i}{4} & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}$$

Out[488]//MatrixForm=

$$\begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & -\frac{1}{4} \\ 0 & \frac{1}{4} & 0 \end{pmatrix}$$

In[510]:=
```
T8.T4 - T4.T8 // MatrixForm
T4.T8 - T8.T4 // MatrixForm
T8.T5 - T5.T8 // MatrixForm
```

Out[510]//MatrixForm=

$$\begin{pmatrix} 0 & 0 & \frac{\sqrt{3}}{4} \\ 0 & 0 & 0 \\ -\frac{\sqrt{3}}{4} & 0 & 0 \end{pmatrix}$$

Out[511]//MatrixForm=

$$\begin{pmatrix} 0 & 0 & -\frac{\sqrt{3}}{4} \\ 0 & 0 & 0 \\ \frac{\sqrt{3}}{4} & 0 & 0 \end{pmatrix}$$

Out[512]//MatrixForm=

$$\begin{pmatrix} 0 & 0 & -\frac{i\sqrt{3}}{4} \\ 0 & 0 & 0 \\ -\frac{i\sqrt{3}}{4} & 0 & 0 \end{pmatrix}$$

In[575]:=
```
T3.T7 - T7.T3 // MatrixForm
T7.T3 - T3.T7 // MatrixForm
T3.T6 - T6.T3 // MatrixForm
```

Out[575]//MatrixForm=

$$\begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & \frac{i}{4} \\ 0 & \frac{i}{4} & 0 \end{pmatrix}$$

Out[576]//MatrixForm=

$$\begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & -\frac{i}{4} \\ 0 & -\frac{i}{4} & 0 \end{pmatrix}$$

Out[577]//MatrixForm=

$$\begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & -\frac{1}{4} \\ 0 & \frac{1}{4} & 0 \end{pmatrix}$$

In[560]:=
```
T1.T6 - T6.T1 // MatrixForm
T6.T1 - T1.T6 // MatrixForm
T1.T5 - T5.T1 // MatrixForm
```

Out[560]//MatrixForm=

$$\begin{pmatrix} 0 & 0 & \frac{1}{4} \\ 0 & 0 & 0 \\ -\frac{1}{4} & 0 & 0 \end{pmatrix}$$

Out[561]//MatrixForm=

$$\begin{pmatrix} 0 & 0 & -\frac{1}{4} \\ 0 & 0 & 0 \\ \frac{1}{4} & 0 & 0 \end{pmatrix}$$

Out[562]//MatrixForm=

$$\begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & -\frac{i}{4} \\ 0 & -\frac{i}{4} & 0 \end{pmatrix}$$

In[566]:=
```
T2.T7 - T7.T2 // MatrixForm
T7.T2 - T2.T7 // MatrixForm
T2.T5 - T5.T2 // MatrixForm
```

Out[566]//MatrixForm=

$$\begin{pmatrix} 0 & 0 & -\frac{1}{4} \\ 0 & 0 & 0 \\ \frac{1}{4} & 0 & 0 \end{pmatrix}$$

Out[567]//MatrixForm=

$$\begin{pmatrix} 0 & 0 & \frac{1}{4} \\ 0 & 0 & 0 \\ -\frac{1}{4} & 0 & 0 \end{pmatrix}$$

Out[568]//MatrixForm=

$$\begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & \frac{1}{4} \\ 0 & -\frac{1}{4} & 0 \end{pmatrix}$$

In[572]:=
```
T2.T5 - T5.T2 // MatrixForm
T5.T2 - T2.T5 // MatrixForm
T2.T7 - T7.T2 // MatrixForm
```

Out[572]//MatrixForm=

$$\begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & \frac{1}{4} \\ 0 & -\frac{1}{4} & 0 \end{pmatrix}$$

Out[573]//MatrixForm=

$$\begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & -\frac{1}{4} \\ 0 & \frac{1}{4} & 0 \end{pmatrix}$$

Out[574]//MatrixForm=

$$\begin{pmatrix} 0 & 0 & -\frac{1}{4} \\ 0 & 0 & 0 \\ \frac{1}{4} & 0 & 0 \end{pmatrix}$$

```
(* And Continued *)
```

# Proof that it normalizes to the $\frac{1}{2}$ Kronecker Delta

In[414]:=
```
Tr[T1.T1]
Tr[T2.T2]
Tr[T3.T3]
Tr[T4.T4]
Tr[T5.T5]
Tr[T6.T6]
Tr[T7.T7]
Tr[T8.T8]
```

Out[414]= $\frac{1}{2}$

Out[415]= $\frac{1}{2}$

Out[416]= $\frac{1}{2}$

Out[417]= $\frac{1}{2}$

Out[418]= $\frac{1}{2}$

Out[419]= $\frac{1}{2}$

Out[420]= $\frac{1}{2}$

Out[421]= $\frac{1}{2}$

In[358]:=
```
Tr[T1.T2]
Tr[T1.T3]
Tr[T1.T4]
Tr[T1.T5]
Tr[T1.T6]
Tr[T1.T7]
Tr[T1.T8]
```

Out[358]= 0

Out[359]= 0

Out[360]= 0

Out[361]= 0

Out[362]= 0

Out[363]= 0

Out[364]= 0

In[365]:=
```
Tr[T2.T1]
Tr[T2.T3]
Tr[T2.T4]
Tr[T2.T5]
Tr[T2.T6]
Tr[T2.T7]
Tr[T2.T8]
```

Out[365]= 0

Out[366]= 0

Out[367]= 0

Out[368]= 0

Out[369]= 0

Out[370]= 0

Out[371]= 0

In[372]:=
```
Tr[T3.T1]
Tr[T3.T2]
Tr[T3.T4]
Tr[T3.T5]
Tr[T3.T6]
Tr[T3.T7]
Tr[T3.T8]
```

Out[372]= 0

Out[373]= 0

Out[374]= 0

Out[375]= 0

Out[376]= 0

Out[377]= 0

Out[378]= 0

In[379]:=
```
Tr[T4.T1]
Tr[T4.T2]
Tr[T4.T3]
Tr[T4.T5]
Tr[T4.T6]
Tr[T4.T7]
Tr[T4.T8]
```

Out[379]= 0

Out[380]= 0

Out[381]= 0

Out[382]= 0

Out[383]= 0

Out[384]= 0

Out[385]= 0

In[386]:=
```
Tr[T5.T1]
Tr[T5.T2]
Tr[T5.T4]
Tr[T5.T3]
Tr[T5.T6]
Tr[T5.T7]
Tr[T5.T8]
```

Out[386]= 0

Out[387]= 0

Out[388]= 0

Out[389]= 0

Out[390]= 0

Out[391]= 0

Out[392]= 0

In[393]:=
```
Tr[T6.T1]
Tr[T6.T2]
Tr[T6.T4]
Tr[T6.T5]
Tr[T6.T3]
Tr[T6.T7]
Tr[T6.T8]
```

Out[393]= 0

Out[394]= 0

Out[395]= 0

Out[396]= 0

Out[397]= 0

Out[398]= 0

Out[399]= 0

In[400]:=
```
Tr[T7.T1]
Tr[T7.T2]
Tr[T7.T4]
Tr[T7.T5]
Tr[T7.T6]
Tr[T7.T3]
Tr[T7.T8]
```

Out[400]= 0

Out[401]= 0

Out[402]= 0

Out[403]= 0

Out[404]= 0

Out[405]= 0

Out[406]= 0

In[407]:=
```
Tr[T8.T1]
Tr[T8.T2]
Tr[T8.T4]
Tr[T8.T5]
Tr[T8.T6]
Tr[T8.T7]
Tr[T8.T3]
```

Out[407]= 0

Out[408]= 0

Out[409]= 0

Out[410]= 0

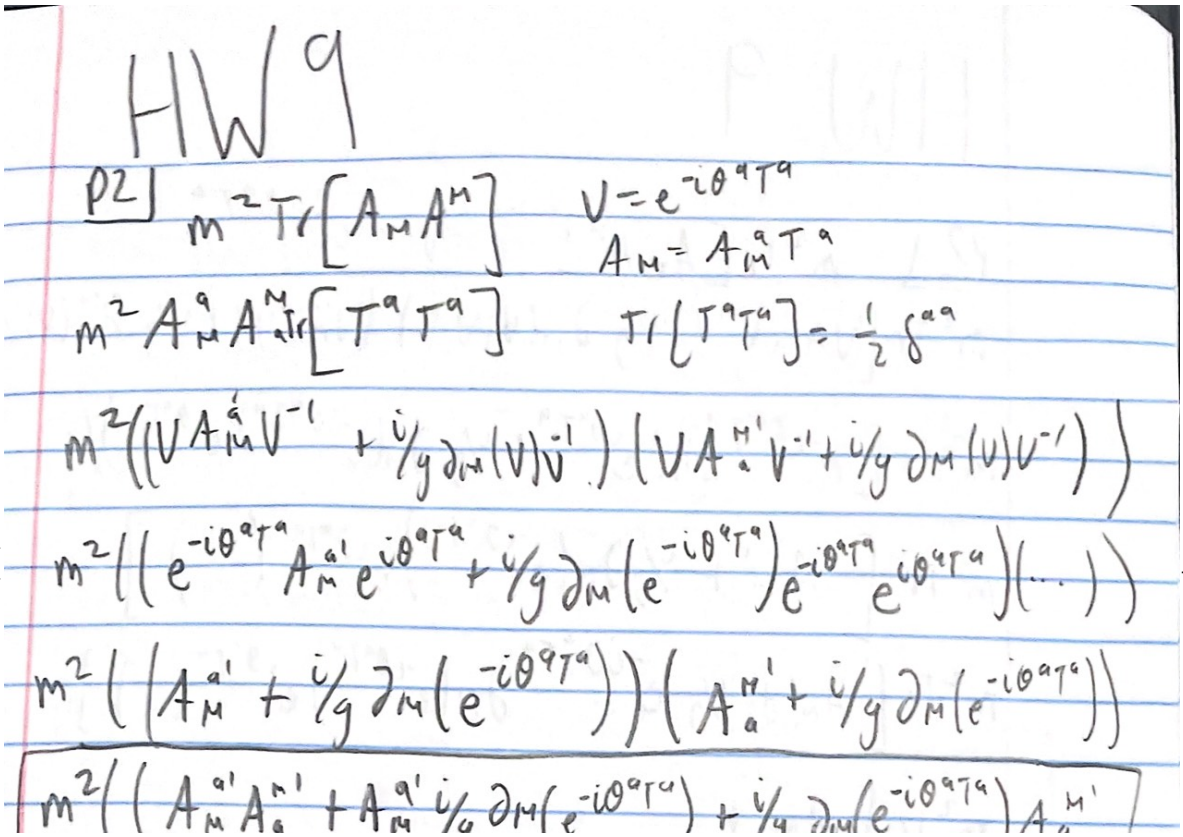
Out[411]= 0

Out[412]= 0

Out[413]= 0

In[582]:=
```
Import["problem9_1.pdf"]
```

Out[582]=



$-\frac{1}{g^2}\partial_\mu^2\left(e^{-i\theta^a T^a}\right)\frac{1}{2}$ → From $T_r[T^a T^a]$

Not gauge invariant

P31 $\alpha_s(Q) = \frac{\alpha_s(Q_0)}{1+\alpha_s(Q_0)/2\pi\,(11-\frac{2}{3}n_f)\ln Q/Q_0}$

$\alpha_s(m_z) = 0.1179$ $\quad m_z = 91.1876\ GeV$

$\alpha_s(m_b)$ → find it $\quad m_b = 4.18\ GeV \quad n_f = 5$

$\alpha_s(m_b) = \frac{\alpha_s(m_z)}{1+\alpha_s(m_z)/2\pi\,(11-\frac{2}{3}n_f)\ln m_b/m_z}$

$\alpha_s(m_b) = \frac{0.1179}{1+0.1179/2\pi\,(11-\frac{2}{3}(5))\ln 4.18\ GeV/91.1876}$

$\boxed{\alpha_s(m_b) = 0.211846}$

$m_c = 1.28\ GeV$, $n_f = 4$

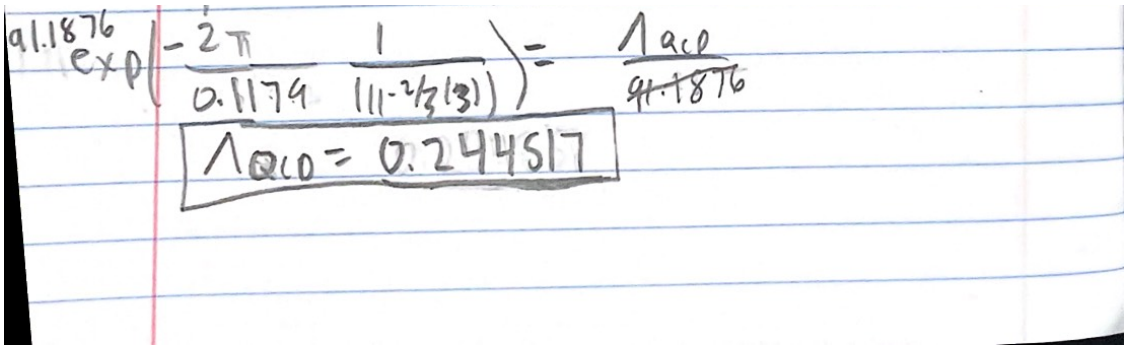
$\alpha_s(m_c) = \frac{\alpha_s(m_z)}{1+\alpha_s(m_z)/2\pi\,(11-\frac{2}{3}n_f)\ln m_c/m_z}$

$\alpha_s(m_c) = \frac{0.1179}{1+0.1179/2\pi\,(11-\frac{2}{3}(4))\ln 1.28\ GeV/91.1876\ GeV}$

$\boxed{\alpha_s(m_c) = 0.354142}$

$\frac{1}{\alpha_s(\Lambda_{QCD})} = \frac{1+0.1179/2\pi\,(11-\frac{2}{3}(3))\ln(\Lambda_{QCD}/91.1876)}{0.1179}$ ← 3 quarks

$-1 = 0.1179/2\pi\,(11-\frac{2}{3}(3))\ln(\Lambda_{QCD}/91.1876)$




# Problem 3

In[284]:=
$$m_b = \frac{0.1179}{1 + \frac{0.1179}{2*Pi} * \left(11 - \frac{2}{3} * 5\right) * Log\left[\frac{4.18}{91.1876}\right]}$$

$$m_c = \frac{0.1179}{1 + \frac{0.1179}{2*Pi} * \left(11 - \frac{2}{3} * 4\right) * Log\left[\frac{1.28}{91.1876}\right]}$$

Out[284]= 0.211846

Out[285]= 0.354142

In[449]:=
$$Delta_{QCD} = 91.1876 * Exp\left[-2 * \frac{Pi}{0.1179} * \frac{1}{\left(11 - \frac{2}{3} * 3\right)}\right]$$

Out[449]= 0.244517