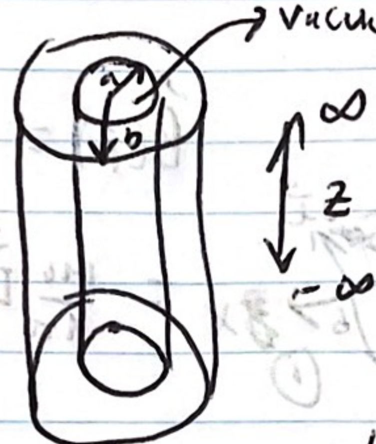
18-2



$J = \frac{k}{s^3}\hat{z}$

$\vec{J} = \frac{d\vec{I}}{da_\perp}$ $\quad \vec{J} = \rho\vec{v}$

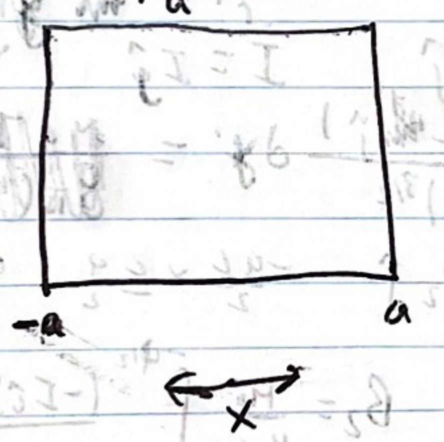
$\vec{I} = \int_S J\,da_\perp = \int_S \vec{J}\cdot d\vec{a}$

$da_\perp = s\,ds\,d\phi$

$$I = \int\left(\frac{k}{s^3}\right)(s\,ds\,d\phi) = 2\pi k\left(\int_a^b s^{-2}\,ds\right)$$

$$= 2\pi k\left(-s^{-1}\right)_a^b = 2\pi k\left(-\frac{1}{b} + \frac{1}{a}\right) = \boxed{\left(\frac{2\pi k}{a} - \frac{2\pi k}{b}\right)\hat{z}}$$

b.



∞
y
−∞

$\vec{K} = x^2 k\hat{y}$

$\vec{K} = \frac{d\vec{I}}{de_\perp}$ $\quad \vec{K} = \sigma\vec{v}$

$\vec{I} = \int K\,de_\perp$

$$\vec{I} = \int_{-a}^{a} x^2 k\,\hat{y} = k\left[\frac{x^3}{3}\right]_{-a}^{a} = k\left[\frac{a^3}{3} - -\frac{a^3}{3}\right]$$

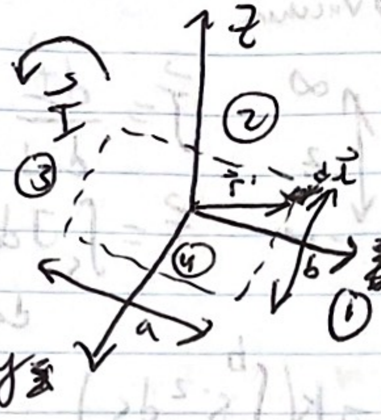
$$\boxed{\vec{I} = \frac{2}{3}a^3 k\,\hat{y}}$$

18-3

$\vec{R} = \vec{r} - \vec{r}\,'$

$\vec{r} = z\hat{n}$

$\vec{r}\,' = x'\hat{\imath} + y'\hat{\jmath}$



$$\vec{B}(\vec{r}) = \frac{\mu_0}{4\pi}\int \frac{\vec{I}\times\hat{\mathscr{r}}}{\mathscr{r}^2}\,dl'$$

$$= \frac{\mu_0}{4\pi}\vec{I}\int \frac{d\vec{l}\,'\times\hat{\mathscr{r}}}{\mathscr{r}^2}$$

$$I\times(\vec{r}-\vec{r}\,') = Iz\hat{\imath} + \frac{b}{2}I\hat{n}$$
$$= m$$

(1) $\vec{r}\,' = a\hat{\imath} + y\hat{\jmath}$ $\quad -\frac{1}{2}b \le y \le b/2$ $\quad dl = dx$

$$\vec{B}(\vec{r}) = \frac{\mu_0}{4\pi}\int \frac{\vec{I}\times(\vec{r}-\vec{r}\,')}{|\vec{r}-\vec{r}\,'|^3}\,dl' \qquad \vec{B}(\vec{r}) \quad \vec{r} = 0$$

$\vec{r}\,' = \quad -\frac{b}{2} \le y' \le b/2$

$\vec{R} = \vec{r} - \vec{r}\,' = -\frac{a}{2}\hat{\imath} - y'\hat{\jmath} \qquad \vec{I} = I\hat{\jmath} \quad \to b/2\,\hat{\imath} + y'\hat{\jmath}$

$$B_1 = \frac{\mu_0}{4\pi}\int_{-b/2}^{b/2} \frac{I\hat{\jmath}\times(-\frac{a}{2}\hat{\imath} - y'\hat{\jmath})}{(y^2 + a^2/4)^{3/2}}\,dy' = \hat{n}\frac{\mu_0 I b}{8\pi}\int_{-b/2}^{b/2}\frac{dy}{(a^2/4+y^2)^{3/2}}$$

(2) $B_2 =$ recall $\vec{r}\,' = x'\hat{\imath} + a/2\,\hat{\jmath}$ $\quad -\frac{a}{2} \le x \le \frac{a}{2}$ $\quad dl = dx \quad I = -I\hat{\imath}$

$$\vec{R} = \vec{r} - \vec{r}\,' = -x'\hat{\imath} - a/2\,\hat{\jmath} \qquad B_2 = \frac{\mu_0}{4\pi}\int_{a/2}^{-a/2}\frac{(-I\hat{\imath})\times(-x'\hat{\imath} - a/2\,\hat{\jmath})}{\sqrt{(\frac{a}{2})^2 + x'^2}}(-dx)$$

$$= \frac{\mu_0}{4\pi}\int_{-a/2}^{a/2}\frac{Ia\,\hat{n}}{2\sqrt{(a/2)^2+x^2}}\,dx = \frac{\mu_0 I a}{8\pi}\hat{n}\int_{-a/2}^{a/2}\frac{dx}{\sqrt{(\frac{a}{2})^2+x^2}}$$

$B_1 = B_3 \qquad B_2 = B_4$

$$B_1 = B_3 \quad \frac{\mu_0 I a}{4\pi}\int_{-b/2}^{b/2}\frac{dy}{(a^2/4+y^2)^{3/2}} \quad +z^2 \qquad B_2 = B_4 \quad \frac{\mu_0 I b}{4\pi}\int_{-a/2}^{a/2}\frac{dx}{(b^2/4+x^2)^{3/2}} \quad +z^2$$

$$\boxed{\vec{B} = \left(\frac{\mu_0 I a}{4\pi}\int_{-b/2}^{b/2}\frac{dy}{\left(\sqrt{a^2/4+y^2+z^2}\right)^3} + \frac{\mu_0 I b}{4\pi}\int_{-a/2}^{a/2}\frac{dx}{\left(\sqrt{b^2/4+x^2+z^2}\right)^3}\right)\hat{k} \quad \cancel{+\left(\frac{\mu_0 I z}{2\pi}\int_{-b/2}^{b/2}\frac{dy}{\sqrt{a^2/4+y^2+z^2}} + \frac{\mu_0 I z}{2\pi}\int_{-a/2}^{a/2}\frac{dx}{\sqrt{b^2/4+x^2+z^2}}\right)\hat{\imath}}}$$

b. $\hat{\imath}$ stuff goes to zero because field is only in z direction

$$\lim_{z\to\infty} = \frac{\mu_0 I a}{4\pi}\left(\frac{2b}{2}\left(\frac{1}{z^3}\right)\right) + \frac{\mu_0 I b}{4\pi}\left(\frac{2a}{2}\left(\frac{1}{z^3}\right)\right) = \frac{2\mu_0 I a b}{4\pi z^3}\hat{k}$$

$$\boxed{\vec{B} = \frac{\mu_0 m}{2\pi z^3}\hat{z} \quad \text{where} \quad m = Iab}$$