HW 12

17.3 $a = b = p$

$S(p) = D(p)$

$140 - 5p = -60 + 5p \rightarrow 200 = 10p$ $\boxed{p = 20}$

$140 - 5(20) = 40$ $-60 + 5(20) = 40$

40 supplied 40 demanded

$\boxed{\text{quantity transacted at that price: } 40}$

$p = 20$ $q = 40 \rightarrow 10{,}000$ shares $40 \cdot 10000 = 40{,}000$

↳ 1/8 \$ so $20/8 = \$2.5$

$\boxed{p = \$2.5 \quad q = 40000 \text{ shares}}$

17.4 Max $\overset{D(a)}{\cancel{}}\,(\overset{a-v}{\cancel{}}) = (5a - 140)(20 - a)$

FOC: $5(20 - a) - (5a - 140) = 0$

$100 - 5a - 5a + 140 = 0$ $10a = 240$ $\boxed{a = 24}$

Max $S(b)(v - b) = (-60 + 5b)(v - b)$

FOC: $5(20 - b) - (-60 + 5b)$

$100 - 5b + 60 - 5b = 0$ $10b = 160$ $\boxed{b = 16}$

π(profits → selling) $= S(b)(v - b) = (5\underset{16}{b} - 60)(20 - \underset{16}{b}) = 80$

$\boxed{\text{Profit for cartel/collusion}\ \overset{\text{selling}}{\wedge}:\ 80/n}$

π(profit → buying) $= D(a)(a - v) = (140 - 5\underset{24}{a})(\underset{24}{a} - 20) = 80$

$\boxed{\text{Profit for cartel/collusion}\ \overset{\text{buying}}{\wedge}:\ 80/n}$

$\boxed{\text{Total profit } \pi_{buying} + \pi_{selling} = 160/n}$

Yes, the market makers buy as much as they sell!

17.5 $a = \frac{24}{8} \rightarrow \boxed{a = \$3}$ $b = \frac{16}{8} \rightarrow \boxed{b = \$2}$

17.6 6 trading rounds per day / 250 trading days per year

$6 \cdot \frac{160}{n} \cdot 250 \rightarrow \boxed{\frac{240{,}000}{n} \text{ yearly profit}}$ if $n = 20$ $\frac{240000}{20} = \boxed{12{,}000/\text{year}}$

$6 \cdot \frac{160}{n} \rightarrow \boxed{\frac{960}{n} \text{ daily profit}}$

*Recall, this is not in \$ amounts. To get \$ amounts, you would ÷ by 8.

\$ amounts $\boxed{\text{yearly profit per dealer } \$1500 \quad \text{yearly profit } \$\frac{30000}{n} \quad \text{daily } \$\frac{120}{n}}$

17.7 total annual profits $\$\frac{30000}{n} + \frac{30000}{n}\delta^1 + \frac{30000}{n}\delta^2 + \dots$

$$\frac{\$30{,}000}{n}\left(\frac{1-\delta^{T+1}}{1-\delta}\right) = \boxed{\frac{\$30{,}000}{n}\left(\frac{1-.99^{T+1}}{.01}\right)}$$

or non-dollar $\boxed{\frac{240000}{n}\left(\frac{1-.99^{T+1}}{.01}\right)}$ per-dealer would be if $n=20$

17.8 Every broker has a best response to deviate from the collusive price. If it is at $b=16$ and $a=24$, one dealer could charge at $b=17$ and $a=23$ and take all the market. Therefore, deviation is always a best response. So, dealers would bid each other down until they get to the market price $p=20$, like in the Bertrand price model.

17.9

Collusion payoff: $\frac{160}{n(1-\delta)}$ Deviation: $[5(17)-60](20-17) = 75$ profit $=150$

$[140-5(23)](23-20) = 75$

Sustain Collusion $\frac{160}{n(1-\delta)} > 150$ $\frac{160}{150} > n(1-\delta)$ $\boxed{\frac{16}{15} > n(1-\delta)}$

when $n=20$ $\frac{16}{15} > 20-20\delta$ $\boxed{\delta > .94\overline{6}}$

17.11

Collusion: $\frac{160}{n(1-\delta)}$ Deviation: $150 + [5(19)-60](1) = 35$ profit $=75$

$[140-5(21)](1) = 35$

Sustain Collusion $\frac{160}{n(1-\delta)} > 150 + \frac{\frac{75}{n}\delta}{1-\delta}$ $150 + \frac{75/n\,\delta}{1-\delta}$

$\frac{160-\frac{75}{n}\delta}{n(1-\delta)} > 150$ $160-75\delta > 150n - 150\delta n$

$160-150n > -150\delta n + 75\delta$

$-160+150n < \delta(150n-75)$ $\boxed{\delta > \frac{-160+150n}{150n-75}}$

when $n=2$ $\boxed{\delta > 0.971}$ $\rightarrow$ also a SPNE

profits w/ Collusion: $\frac{160}{n(1-\delta)}$	profits w/ deviation: $150 + \frac{75/n\,\delta}{1-\delta}$

17.10 From last problem

Profit at $b=19$, $a=21$ $\rightarrow$ $\boxed{\frac{75}{n} \quad 35/n \text{ selling} \quad 35/n \text{ buying}}$

It is a stage-game Nash Equilibrium because the only way to deviate is to bid down. If one bids up, he will not get any buyers or sellers. However, if one bids down, they are at the market price and receive a profit of 0. So, the best response is to remain at $b=19$, $a=21$ w/ profits $\frac{75}{n}$.

18.1

		S	7 V
SA	8	608, 380	512, 448
	10	640, 320	520, 364

Good

Costs / \$20/barrel

$q_{SA} \in \{8, 10\}$
$q_V \in \{5, 7\}$
$Q \in \{13, 15, 17\}$
96 84 72 → good
64 60 56 → bad

Calculate

18.2 what will maximize joint profits?

Max → $608+380=988$ $\quad$ $640+320=960$
$512+448=960$ $\quad$ $520+364=884$

$\boxed{(8,5) \rightarrow (608, 380) \text{ 13 mbd maximizes profits}}$

$\boxed{\text{Stage-game NE is } (10,7) \rightarrow (520, 364) \text{ 17 mbd}}$

18.3

$\frac{608}{1-\delta}$ Deviation $640 + \frac{520}{1-\delta}$

Sustain $\frac{608}{1-\delta} > 640 + \frac{520}{1-\delta}$ $\quad$ $\frac{608-520}{1-\delta} > 640$

$640 - 640\delta < 88$ $\quad$ $552 < 640\delta$

$\boxed{\delta > .8625}$ → SA

$\frac{380}{1-\delta}$ Deviation: $448 + \frac{364}{1-\delta}$ $\quad$ $\frac{380-364}{1-\delta} > 448$

$\frac{16}{1-\delta} > 448$

$448 - 448\delta < 16$

$432 < 448\delta$

$\boxed{\delta > .9643}$ → V

18.4 Higher prices make it easier to sustain collusion. With higher prices δ was 1/3 for SA & .905 for V. With lower prices δ was .8625 for SA & .9643 for V.

18.5 SA gets entire market

$13 \times \$96 = 1248 \quad 1248 - 13 \cdot 20 = 988$

SA: $\frac{988}{1-\delta} = \frac{988}{.25} = 3952$

Normally V would get

$\frac{380}{1-\delta} = \frac{380}{.25} = 1520$ SA would have to pay V 1,520 for it not to overproduce.

I assumed that SA would be the only produce and that it would produce at 13 mbd for \$96 (as that maximizes profits). If both SA & V produced at 13 mbd, V would get \$1520, so SA would have to pay them at least that much.