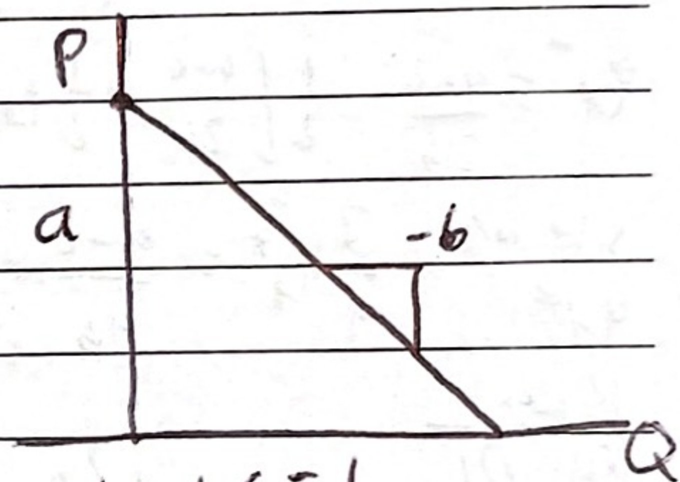
Cournot (1838) duopoly model

Aggregate demand $P = a - bQ$

e.g. $P = 10 - Q$

where $Q = q_A + q_B$



Profit $\pi_A = Pq_A - cq_A$

$= (P-c)q_A$ — Constant marginal cost $c = 1$

FOC: $\frac{\partial \pi_A}{\partial q_A}$ → recall $P = a - b(q_A + q_B)$ $\frac{\partial P}{\partial q_A} = -b$

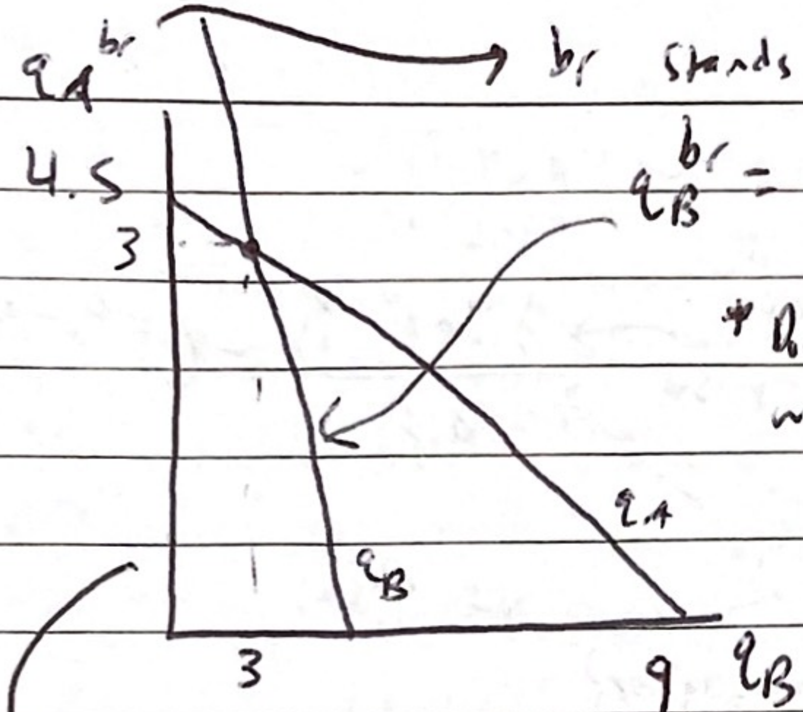
$\frac{\partial \pi_A}{\partial q_A} = -bq_A + P - c = 0$ SOC: $\frac{\partial^2 \pi_A}{\partial q_A^2} = -b - b$ → always negative so this is a max

$= -bq_A + a - bq_A - bq_B - c = 0$

$\frac{a - bq_B - c}{2b} = q_A$ $q_A^{br} = \underset{\substack{\parallel \\ 4.5}}{\frac{a-c}{2b}} - \frac{1}{2}q_B$

4.5 cause $a = 10$

$c = 1$



br stands for best response

$$q_B^{br} = \frac{a-c}{2b} - \frac{1}{2} q_A$$

* Dominant strategy, no matter what you are doing, I should always play this:

→ a horizontal line is the best response function that will be a dominant strategy

$$q_A^{br} = \frac{a-c}{2b} - \frac{1}{2} q_B^{br} \rightarrow q_A^{br} = \frac{a-c}{2b} - \frac{1}{2}\underbrace{\left[\frac{a-c}{2b} - \frac{1}{2} q_A\right]}$$

$$q_A^{*} = \frac{a-c}{2b} - \frac{1}{2}\left[\frac{a-c}{2b} - \frac{1}{2} q_A^{*}\right]$$

best response to best response change to q_A^*

Solve for q_A^* $\quad \frac{3}{4} q_A^{*} = \frac{a-c}{4b} \qquad q_A^{*} = \frac{a-c}{3b} = \frac{10-1}{3(1)} = 3$

Social ~~[illegible]~~ Planner

$W = \pi_A + \pi_B = \Pi$ wants to maximize total profits

↳ welfare $= [P(q_A, q_B) - c](q_A + q_B)$

for max

$$\frac{\partial W}{\partial q_A} = (-b)(q_A+q_B) + [a - bq_A - bq_A - c][1] = 0$$

$= a - 2bq_A - 2bq_B - c = 0$

$$\frac{\partial W}{\partial q_B} = a - 2bq_A - 2bq_B - c = 0$$

solve for (q_A^{**}, q_B^{**})

infinite solutions $\xrightarrow{\text{add}}$ $q_A = q_B = q$

now $a - 4bq - c = 0 \rightarrow$ $\boxed{q_A^{**} = q_B^{**} = q^{**} = \frac{a-c}{4b}} = \frac{q}{4}$

$\boxed{Q^{**} = \frac{a-c}{2b}} = \frac{q}{2}$

↳ aggregate quantity

If we both follow planner

$\pi_A^{**} = \pi_B^{**} = (a/2 - c/2)(a-c/4b) = 1/8b\ (a-c)^2 = 81/8 = 10.125$

$P^{**} = a - bQ^{**} = a/2 + c/2 = 5.5$

↳ we get a higher price when following planner

$\pi_A^{**} = \pi_B^{**} = 10.125$ → higher profits. → best profit if both of us max profits at economic equilibrium

If I want to know my best response to 2.25

$R_A(2.25) = R_A(q_B) = \frac{a-c}{2b} - \frac{1}{2} q_B = 4.5 - 1.125 = 3.375$

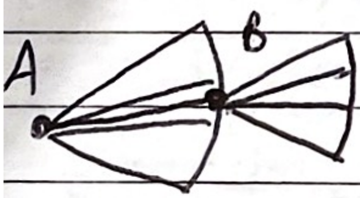
↳ I get even higher profits if I cheat out my partner and do a best response to firm B if they are producing at the Social Planner's / Equilibrium's quantity.

A better word for Social Planner is Cartel. The Social Planner / Cartel is also what a Monopoly would produce if using aggregate quantity.

↳ Thus, Monopoly would have Q^{**}, P^{*}, and $2\pi_A^{**} = 2\pi_B^{**}$

Stackleberg Model (1934)

Same as before, but A goes first, then B has to go



→ to solve these kinds of models, work backwards

$q_B = R_B(q_A) = \underbrace{\frac{a-c}{2b} - \frac{1}{2} q_A}$ ← a knows that this is how player b will respond

↳ q_B is no longer exogenous to A

$\pi_A = [P(q_A, q_B) - c]\, q_A$

↳ q_B is now a function of q_A

$$\pi_A = \left[a - bq_A - b\left(\frac{a-c}{2b} - \frac{1}{2}q_A\right) - c\right]q_A$$

$$= \left[\frac{1}{2}(a-c) - \frac{1}{2}bq_A\right]q_A$$

$$\frac{\partial \pi_A}{\partial q_A} = \frac{1}{2}(a-c) - bq_A = 0 \qquad q_A^* = \frac{1}{2b}(a-c) = 4.5$$

$$q_B^* = \frac{a-c}{2b} - \frac{1}{2}q_A^* = 2.25 \qquad Q^* = 6.75 \qquad P^* = 10 - 6.75 = 3.25$$

$$\pi_A^* = (P^* - c)q_A^* = (3.25 - 1)(4.5) = 10.125$$

$$\pi_B^* = (P^* - c)\,q_B^* = 5.0625$$

Oligopoly

$$P(Q) = a - b(q_1 + q_2 + \dots + q_n)$$

$$\pi_1 = [P(Q) - c]\,q_1$$

$$\frac{\partial \pi}{\partial q} = a - bq_1 - bq_2 - \dots - bq_N = 0 \qquad q_1 =$$