Homework 3

Nash Equilibrium

Notes from Chapter 5

A **Nash Equilibrium** is a best response to a best response.

Definition. A strategy s_i^* is a *best response* to a strategy vector s_{-i}^* of the other players if[1]

$$\pi_i(s_i^*, s_{-i}^*) \geq \pi_i(s_i, s_{-i}^*), \quad \text{for all } s_i$$

In other words, s_i^* is a *dominant strategy* in the very weak sense that it is a best strategy to play *provided the other players do in fact play* the strategy combination s_{-i}^*. We need a condition to ensure that player i is correct in his conjecture that the other players are going to play s_{-i}^*. And, likewise, the other players are correct in their conjectures. This analysis gives us the following definition:

Definition. The strategy vector $s^* = (s_1^*, s_2^*, \ldots, s_N^*)$ is a *Nash equilibrium* if

$$\pi_i(s_i^*, s_{-i}^*) \geq \pi_i(s_i, s_{-i}^*), \text{ for all } s_i \text{ and all } i \tag{5.1}$$

Equation (5.1) says that each player i, in playing s_i^*, is playing a best response to the others' strategy choice. This one condition includes the two requirements of Nash equilibrium that were intuitively discussed earlier:

1. Each player must be playing a best response against a conjecture.
2. The conjectures must be correct.

It includes the first requirement because s_i^* is a best response against the conjecture s_{-i}^* for every player i. It includes the second because no player has an incentive to change his strategy (from s_i^*). Hence, s_i^* is stable—and each player's conjecture is correct.

Consider the case of two players, 1 and 2, each with two strategies, a^1 and a^2 for player 1, b^1 and b^2 for player 2. Here (a^2, b^1), for example, is a Nash equilibrium if and only if

$$\pi_1(a^2, b^1) \geq \pi_1(a^1, b^1)$$

$$\pi_2(a^2, b^1) \geq \pi_2(a^2, b^2)$$

Example :

Example 1: War and Peace in Congress

Recall that a Democrat and a Republican need to choose which bill to vote for: a progressive bill, P, or a conservative bill, C. The payoffs are given in the following table:

Democrat \ Republican	P	C
P	3, 1	0, 0
C	0, 0	1, 3

Here a best response of player 1 (the Democrat) to a play of P by 2 (the Republican) is to play P; denote this choice as $b^1(P) = P$. Likewise, $b^1(C) = C$. For player 2, the best response can be written as $b^2(P) = P$ and $b^2(C) = C$. Note that an alternative definition of a Nash equilibrium in a two-player game is that it is a pair of strategies in which each strategy is a best response to the other one in that pair. In the War and Peace in Congress game, (P, P) is a Nash equilibrium because

$$P = b^1(P)$$

$$P = b^2(P)$$

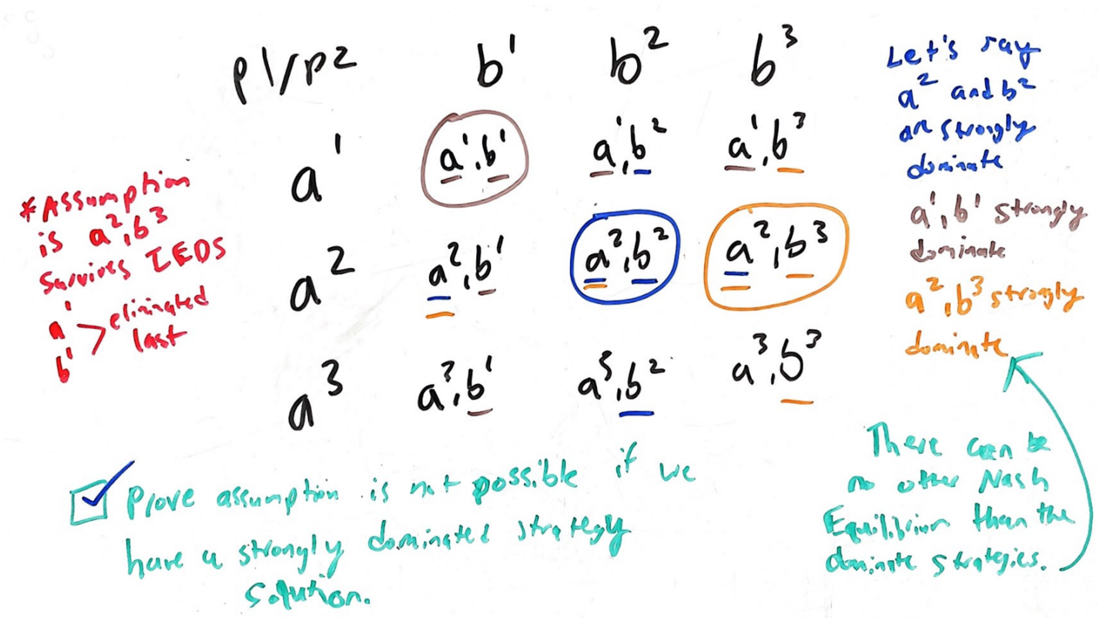
How can we make it to the Nash Equilibrium?

- Play Prescription: Players are told the Nash Equilibrium and choose it because it will give them the highest payoffs.
- Preplay Communication: Players talk to each other before they play to determine what they will both pick.
- Rational Introspection: Player think to themselves what the outcome of the game will be.
- Focal Point: There is one option that has an emphasis or a focus above the rest.
- Trial and Error: After getting it wrong once, twice, or more, players will eventually settle on the Nash Equilibrium.

We need to know if the Nash Equilibrium exists and if it is unique.

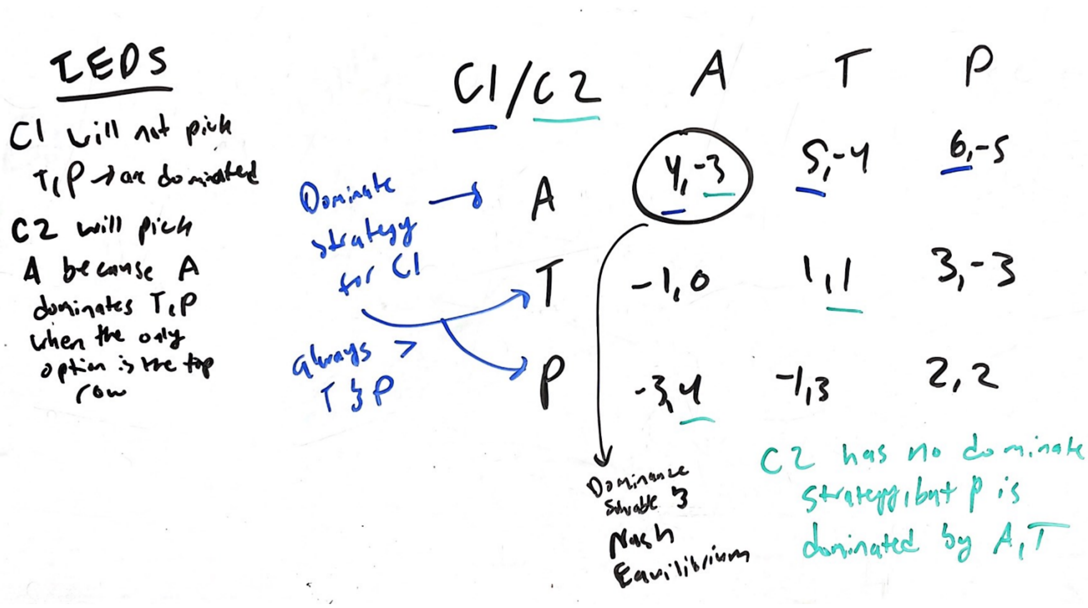
Problem From Chapter 5

Problem 5.25

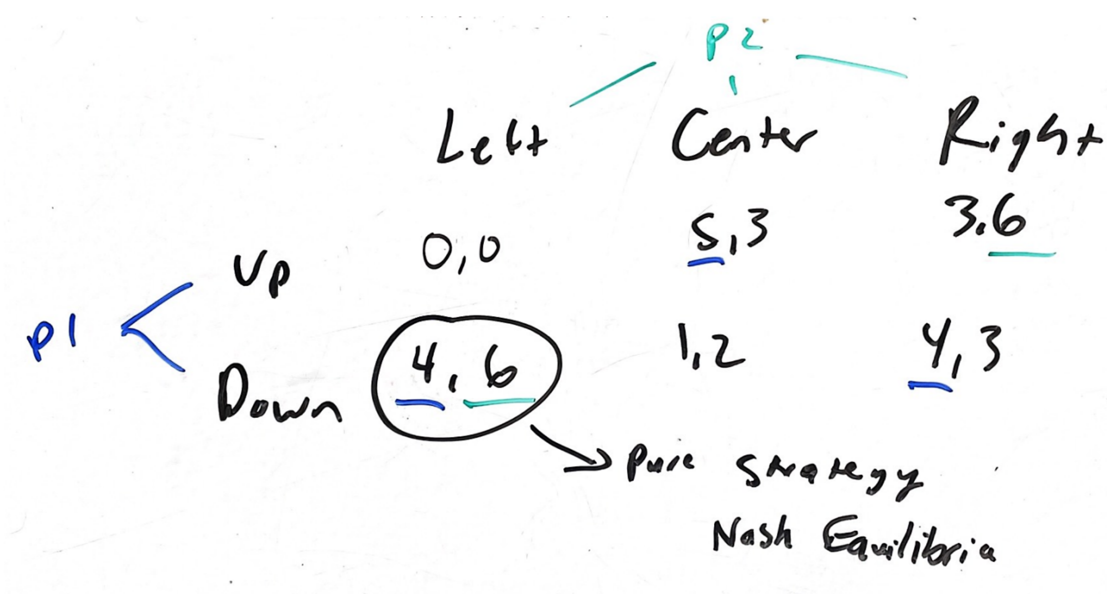


As shown above, there can be no other Nash Equilibrium than the dominate strategies. This is because if there is a strictly dominate strategy, then player 1's only option is to pick that row, and if player 2 has a strictly dominate strategy, then player 2 must also pick that column and the Nash Equilibria will always be the resultant dominate strategies.

Problem 5.26



Problem 2 From the Homework



Problem 3 From the Homework

This game is most similar to the Prisoner's Dilemma. In the Prisoner's Dilemma, each player has the choice to confess or not confess. Players who confess when the other does not get much more utility than those who did not confess. If both players confess, the each get an average amount of disutility. If neither players confess, they get a much lower amount of disuility. Therefore, it would be better for everyone if nobody confessed. This is similar to our work game. The workers who work for 80 hours get much more utility back than those who only work for 40, like the prisoners who confess when their partners-in-crime do not. Similarly, if all workers worked for 80 hours, they would all get a certain amount of disuility for having to work so long, just like when both the criminals confess to the crime. In addition, if all workers came together and only worked for 40 hours, they would get equal pay and would have to work less, so they would get a good deal of utility and would be better off as a whole; this is just like if both the criminals had chosen to not confess.