Homework 12

Problem 1

In[89]:=
```
q[sigma_] := sigma + Y
```

In[99]:=
```
L = {{v}, {e}};
L // MatrixForm
Q = {{u}, {d}};
Q // MatrixForm
H = {{psiplus}, {psi0}};
H // MatrixForm
```

Out[100]//MatrixForm=

$$\begin{pmatrix} v \\ e \end{pmatrix}$$

Out[102]//MatrixForm=

$$\begin{pmatrix} u \\ d \end{pmatrix}$$

Out[104]//MatrixForm=

$$\begin{pmatrix} \text{psiplus} \\ \text{psi0} \end{pmatrix}$$

In[114]:=
```
Solve[0 == q[1/2], Y][[1]][[1]];
YupL = Y /. %
Solve[-1 == q[-1/2], Y][[1]][[1]];
YdownL = Y /. %
```

Out[115]= $-\frac{1}{2}$

Out[117]= $-\frac{1}{2}$

In[118]:=
```
Solve[2/3 == q[1/2], Y][[1]][[1]];
YupQ = Y /. %
Solve[-1/3 == q[-1/2], Y][[1]][[1]];
YdownQ = Y /. %
```

Out[119]= $\frac{1}{6}$

Out[121]= $\frac{1}{6}$

In[122]:=
```
Solve[1 == q[1/2], Y][[1]][[1]];
YupH = Y /. %
Solve[0 == q[-1/2], Y][[1]][[1]];
YdownH = Y /. %
```

Out[123]= $\frac{1}{2}$

Out[125]= $\frac{1}{2}$

In[132]:=
```
newL = {{YupL}, {YdownL}};
newL // MatrixForm
newQ = {{YupQ}, {YdownQ}};
newQ // MatrixForm
newH = {{YupH}, {YdownH}};
newH // MatrixForm
```

Out[133]//MatrixForm=

$$\begin{pmatrix} -\frac{1}{2} \\ -\frac{1}{2} \end{pmatrix}$$

Out[135]//MatrixForm=

$$\begin{pmatrix} \frac{1}{6} \\ \frac{1}{6} \end{pmatrix}$$

Out[137]//MatrixForm=

$$\begin{pmatrix} \frac{1}{2} \\ \frac{1}{2} \end{pmatrix}$$

They are all identical for each individual matrix! It works.

In[144]:=
```
HyperchargeofL = newL[[1]][[1]]
HyperchargeofQ = newQ[[1]][[1]]
HyperchargeofH = newH[[1]][[1]]
```

Out[144]= $-\frac{1}{2}$

Out[145]= $\frac{1}{6}$

Out[146]= $\frac{1}{2}$

Problem 2

In[276]:=
```
Clear[Vud, Vus, Vcd, Vcs, Vuddag, Vcddag, Vusdag, Vcsdag];
```

In[222]:=
```
V = {{Vud, Vus}, {Vcd, Vcs}};
Vdag = {{Vuddag, Vcddag}, {Vusdag, Vcsdag}};
V // MatrixForm
Vdag // MatrixForm
```

Out[224]//MatrixForm=

$$\begin{pmatrix} Vud & Vus \\ Vcd & Vcs \end{pmatrix}$$

Out[225]//MatrixForm=

$$\begin{pmatrix} Vuddag & Vcddag \\ Vusdag & Vcsdag \end{pmatrix}$$

Remember that the identity matrix looks like this:

In[196]:=
```
Id = {{1, 0}, {0, 1}};
Id // MatrixForm
```

Out[197]//MatrixForm=

$$\begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$$

This means that...

In[198]:=
```
identity1 = V.Vdag;
identity1 // MatrixForm
```

Out[199]//MatrixForm=

$$\begin{pmatrix} Vud\ Vuddag + Vus\ Vusdag & Vcddag\ Vud + Vcsdag\ Vus \\ Vcd\ Vuddag + Vcs\ Vusdag & Vcd\ Vcddag + Vcs\ Vcsdag \end{pmatrix}$$

In[200]:=
```
identity2 = Vdag.V;
identity2 // MatrixForm
```

Out[201]//MatrixForm=

$$\begin{pmatrix} Vcd\ Vcddag + Vud\ Vuddag & Vcddag\ Vcs + Vuddag\ Vus \\ Vcd\ Vcsdag + Vud\ Vusdag & Vcs\ Vcsdag + Vus\ Vusdag \end{pmatrix}$$

We want both of these to look like the identity matrix. Thus,

In[202]:=
```
identity1[[1, 1]] == 1
identity1[[2, 2]] == 1
identity2[[1, 1]] == 1
identity2[[2, 2]] == 1
```

Out[202]= Vud Vuddag + Vus Vusdag == 1

Out[203]= Vcd Vcddag + Vcs Vcsdag == 1

Out[204]= Vcd Vcddag + Vud Vuddag == 1

Out[205]= Vcs Vcsdag + Vus Vusdag == 1

Recall that $VV^{+} = |V|^2$. Thus, all the above can be translated to

$|V_{ud}|^2 + |V_{us}|^2 = 1$

$|V_{cd}|^2 + |V_{cs}|^2 = 1$

$|V_{ud}|^2 + |V_{cd}|^2 = 1$

$|V_{us}|^2 + |V_{cs}|^2 = 1$

In[206]:=
```
identity1[[1, 2]] == 0
identity2[[1, 2]] == 0
identity1[[2, 1]] == 0
identity2[[2, 1]] == 0
```

Out[206]= Vcddag Vud + Vcsdag Vus == 0

Out[207]= Vcddag Vcs + Vuddag Vus == 0

Out[208]= Vcd Vuddag + Vcs Vusdag == 0

Out[209]= Vcd Vcsdag + Vud Vusdag == 0

One and three and two and four are identical so...

In[210]:=
```
identity1[[1, 2]] == 0
identity2[[2, 1]] == 0
```

Out[210]= Vcddag Vud + Vcsdag Vus == 0

Out[211]= Vcd Vcsdag + Vud Vusdag == 0

These can be written as
$V_{ud} V_{cd}^* + V_{us} V_{cs}^* = 0$
$V_{ud} V_{us}^* + V_{cd} V_{cs}^* = 0$

Now we will check to see if these identities are satisfied by the criteria below.

In[227]:=
```
Clear[Vud, Vus, Vcd, Vcs, Vuddag, Vcddag, Vusdag, Vcsdag];
```

In[277]:=
```
Vud = Exp[I * alpha] * Cos[theta];
Vus = -Exp[I * (alpha + gamma)] * Sin[theta];
Vcd = Exp[I * beta] * Sin[theta];
Vcs = Exp[I * (beta + gamma)] * Cos[theta];
Vuddag = Exp[-I * alpha] * Cos[theta];
Vusdag = -Exp[-I * (alpha + gamma)] * Sin[theta];
Vcddag = Exp[-I * beta] * Sin[theta];
Vcsdag = Exp[-I * (beta + gamma)] * Cos[theta];
```

In[270]:=
```
Simplify[Vud * Vuddag + Vus * Vusdag]
Simplify[Vcd * Vcddag + Vcs * Vcsdag]
Simplify[Vud * Vuddag + Vcd * Vcddag]
Simplify[Vus * Vusdag + Vcs * Vcsdag]
Simplify[Vud * Vcddag + Vus * Vcsdag]
Simplify[Vud * Vusdag + Vcd * Vcsdag]
```

Out[270]= 1

Out[271]= 1

Out[272]= 1

Out[273]= 1

Out[274]= 0

Out[275]= 0

They indeed are.

Problem 3

```
V = {{Vud, Vus}, {Vcd, Vcs}};
uLnew = ul;
cLnew = Exp[-I * (beta - alpha)]; (* put a negative because we want complex conjugate *)
UL = {uLnew, cLnew};
dLnew = Exp[-I * alpha] * dl;
sLnew = Exp[-I * (alpha + gamma)] * sl;
DL = {dLnew, sLnew};
```

```
In[368]:= Simplify[UL.V.DL]
Out[368]= (sl + dl ul) Cos[theta] + (dl - sl ul) Sin[theta]
```

All of the phases are eliminated and we just have a real function of theta! You cannot remove all 6 phases when there are three generations involved. You cannot only remove 5. There is one left over “physical” phase.