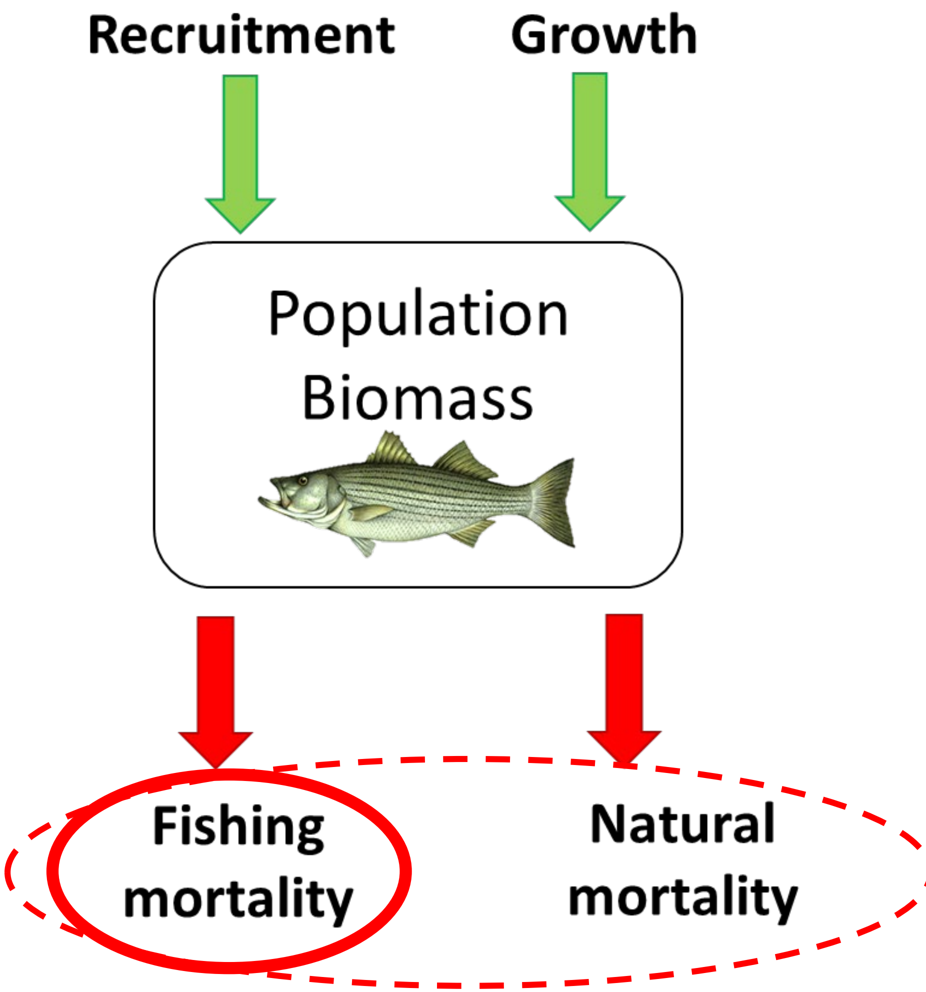
Effects of harvest on populations

Reading:

Jennings et al. 2001 – Chapter 7

Acknowledgment: M. Wilberg

Conceptual population model



Review of exponential mortality

- If following a single cohort, then we can track the mortality losses to the cohort

$$\frac{dN}{dt} = -ZN$$

$$\boxed{N_t = N_0 e^{-Zt}}$$

- Useful for modeling changes in a cohort over time; very common in fisheries science

Derive equations for the following:

S – annual survival (proportion living after one year)

A – the annual mortality rate (proportion dying per year)

Review of exponential mortality

Formulas for converting from instantaneous to annual rates:

- Z = total *instantaneous* mortality rate
- S = *annual* survival rate (proportion surviving in a year)

$$S = e^{-Z} \qquad\qquad Z = -\ln(S)$$

- A = *annual* mortality rate (proportion dying in a year)

$$A = 1 - e^{-Z}$$

Deterministic theory of fishing

- For fished populations, total mortality (Z) is composed of mortality due to harvest and mortality due to natural causes:

$$\boxed{Z = F + M}$$

- F = instantaneous fishing mortality rate
- M = instantaneous natural mortality rate
  - Includes all non-fishing mortality

Deterministic theory of fishing

- Restate our exponential mortality equations:

$$\frac{dN}{dt} = -(F + M)N$$

$$\boxed{N_t = N_0 e^{-(F+M)t}}$$

Baranov Catch Equation

- How can we calculate how many fish die from fishing (or from natural causes)?
- <u>**Baranov catch equation:**</u>

$$C_t = \frac{F}{Z}(1 - e^{-Z})N_t$$

  - Where C_t = catch in year t
  - What does this mean in words?

Baranov Catch Equation

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- **Baranov catch equation:**

$$\boxed{C_t = \frac{F}{Z}(1 - e^{-Z})N_t}$$

  - Where C_t = catch in year t
  - What does this mean in words?
    - Catch = (fraction of mortality due to fishing)*(Proportion dying)*abundance
    - Catch = (fraction of mortality due to fishing)*(total number of deaths)
  - What would the equation be for fraction of fish dying from natural causes?

Extra

Baranov Catch Equation

- Other forms:

$$C_t = \frac{F}{Z}(N_t - N_{t+1})$$

$$C_t = \frac{F}{Z}(N_t - N_t e^{-Z})$$

$$C_t = \frac{FA}{Z} N_t$$

$$C = F\bar{N}$$

N(bar) is the average abundance over the course of a year

Converting to annual rates

- Annual rates
  - **A** = proportion of population dying in 1 year
  - ***u*** = ***exploitation rate*** (e.g., proportion of population dying from fishing in 1 year)
  - ***v*** = proportion of population dying from natural causes in one year

$$A = u + v$$

Converting to annual rates

- **<u>Type II fishery</u>** <u>(continuous fishery)</u>
  - fishing and natural mortality continuous throughout the year; more common
  - Calculating *u* requires info on 2 of the following: F,M,Z

$$\boxed{u = \frac{F}{Z}(1 - e^{-Z}) = \frac{FA}{Z}}$$

$$\boxed{v = \frac{M}{Z}(1 - e^{-Z}) = \frac{MA}{Z}}$$

- **<u>Type I fishery</u>** <u>(pulse fishery)</u>
  - Fishing occurs in short pulse (days/weeks), natural mortality elsewhere; less common

$$u = 1 - e^{-F} \qquad v = 1 - e^{-M}$$

Catch in Difference Model

- Catch (C) is the product for the difference equation:

$$\boxed{C_t = u_t N_t}$$

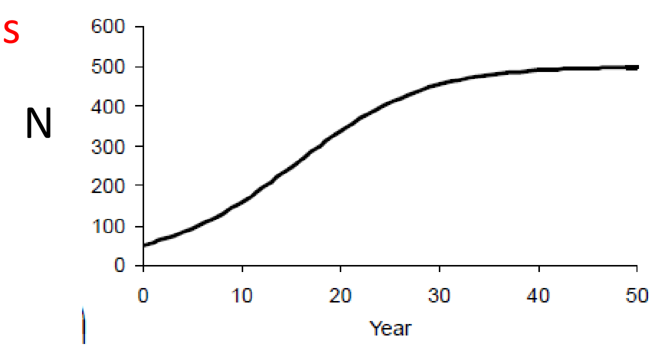
We can incorporate Catch into the logistic model…

  - u_t = exploitation rate at time t
  - N_t = abundance at time t
  - (Assumes catch occurs in one pulse)

Note:
Difference equation – time is viewed as discrete points
Differential equation – time is used as a continuous variable

Review of Logistic Model

Sidenote: equations work with B or N



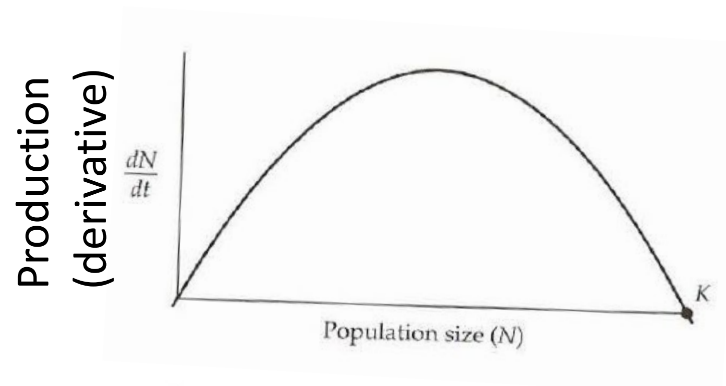
Logistic growth model

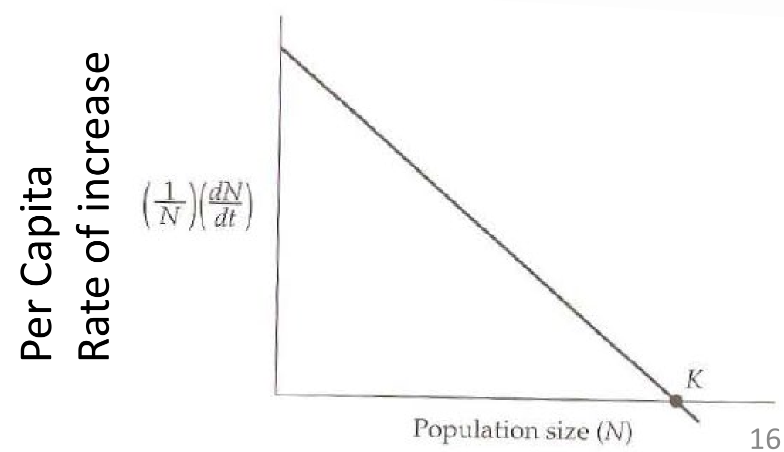
$$\frac{dN}{dt} = rN\left(1 - \frac{N}{K}\right)$$

Discrete version

$$N_{t+1} = N_t + rN_t\left(1 - \frac{N_t}{K}\right)$$

- Density-dependent
  - Per-capita growth rate varies with pop. size
- Key parameters:
  - r=intrinsic rate of increase
    K=carrying capacity
- Equilibrium = carrying capacity
- Shape: logistic ("S-shaped")
- For fisheries models:
  - Foundation for production models





Surplus production

$$B_{t+1} = B_t + rB_t\left(1 - \frac{B_t}{K}\right)$$

- Biomass$_{t+1}$ = Biomass$_t$ + **surplus production (SP)**

$$SP = rB_t\left(1 - \frac{B_t}{K}\right)$$

- **<u>Surplus production</u>** = the excess biomass generated above what is needed to maintain the population at its current biomass.

Logistic model **with harvest**

- Logistic growth model adjusted for fishing mortality (F) or exploitation rate (u)

$$\begin{pmatrix}\text{Biomass}\\\text{next year}\end{pmatrix}=\begin{pmatrix}\text{Biomass}\\\text{this year}\end{pmatrix}+\begin{pmatrix}\text{Surplus}\\\text{production}\end{pmatrix}-(\text{Catch})$$

Catch

Discrete version (difference equation):

$$B_{t+1} = B_t + rB_t\left(1 - \frac{B_t}{K}\right) - C_t$$

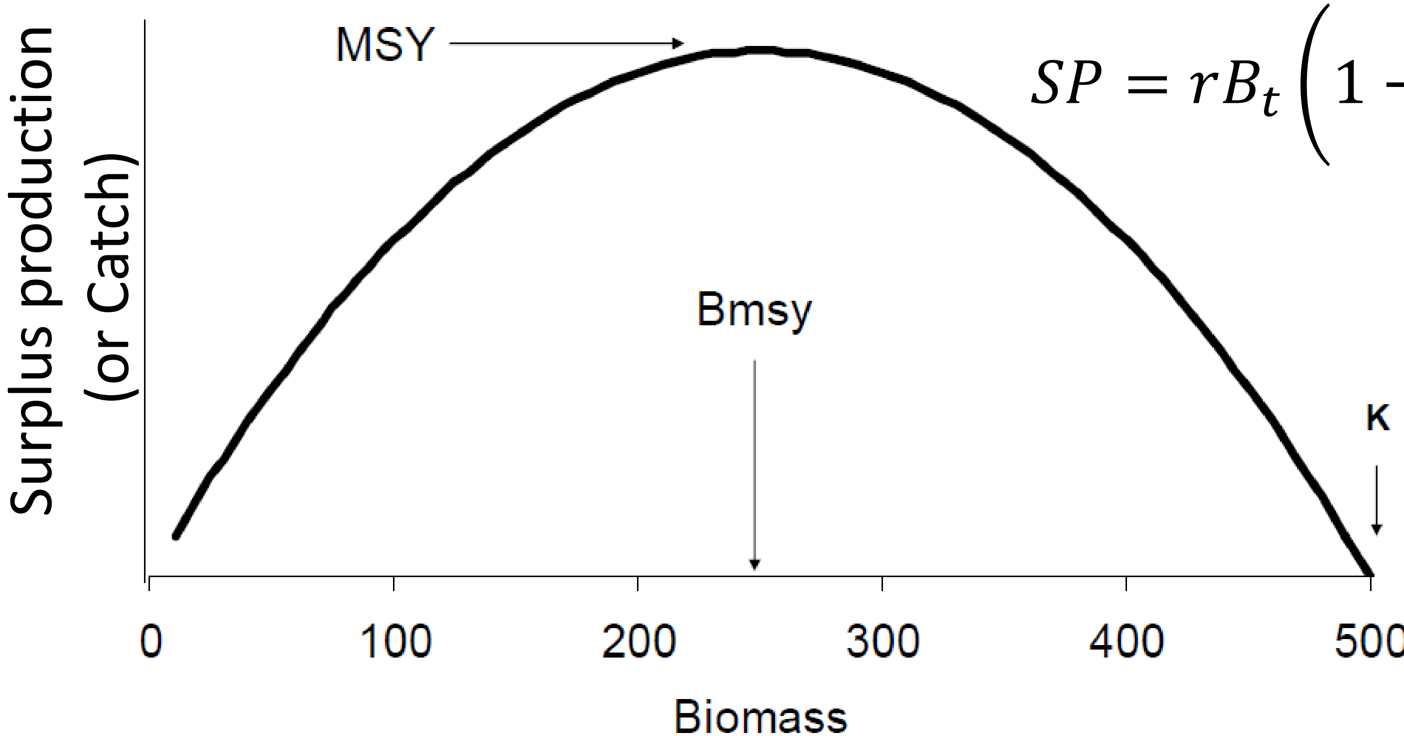
$$B_{t+1} = B_t + rB_t\left(1 - \frac{B_t}{K}\right) - uB_t$$

How can we ensure the Catch doesn't deplete a stock?

Surplus production and MSY

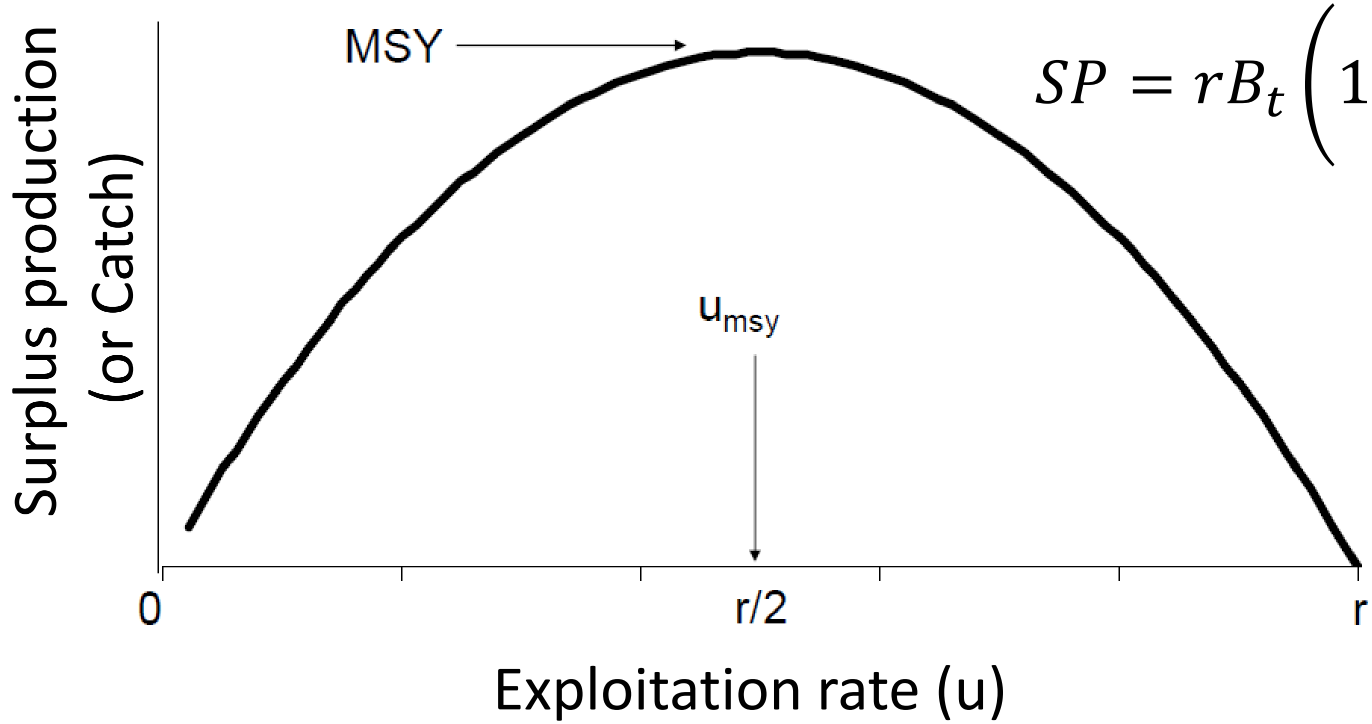
Set Catch to SP

$$SP = rB_t\left(1 - \frac{B_t}{K}\right)$$



- **MSY** = largest catch (yield) that can be supported by the population over an indefinite period
- **B_{msy}** = Biomass at which MSY is generated

Surplus production and MSY



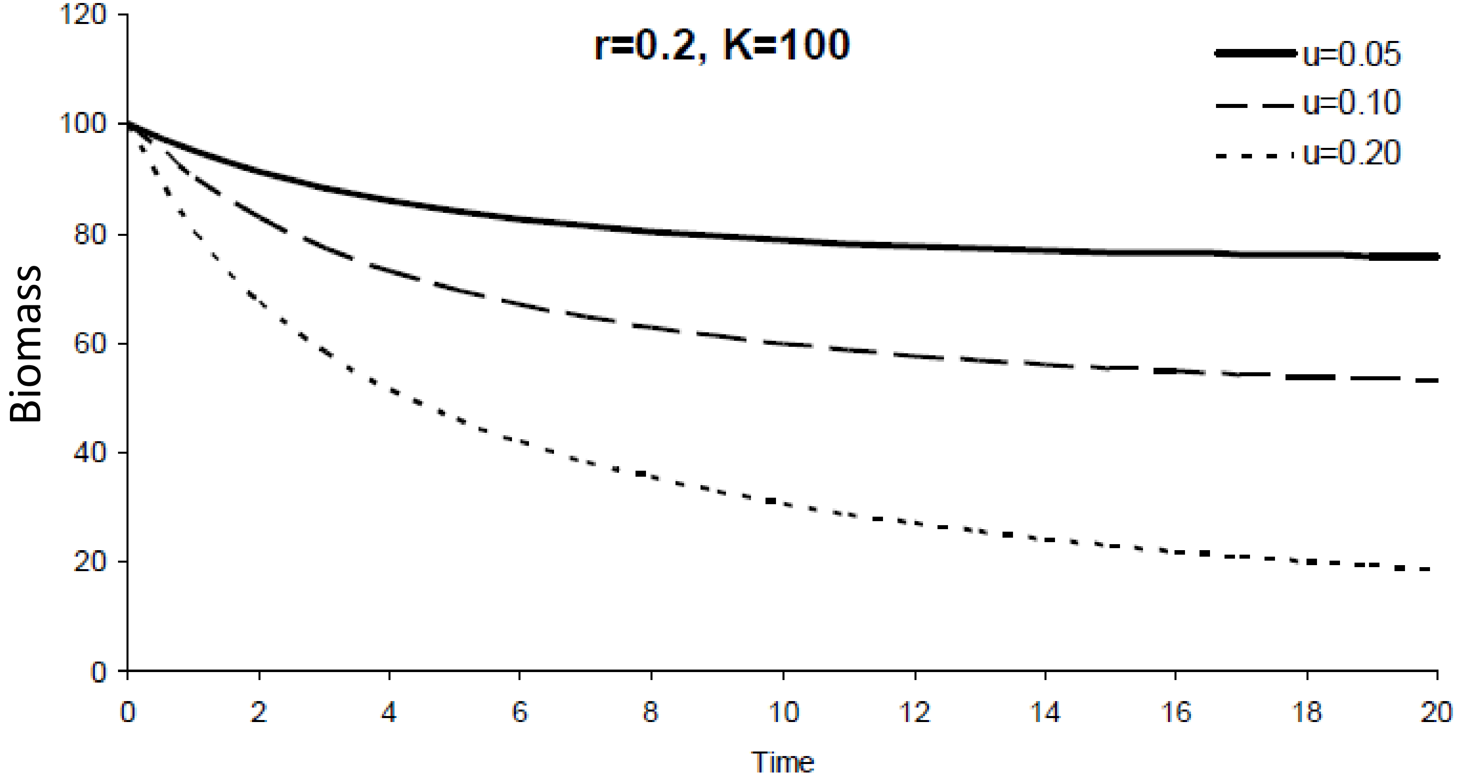
$$SP = rB_t\left(1 - \frac{B_t}{K}\right)$$

How hard should we fish to get MSY and keep the pop. at B_{MSY}?

- $\mathbf{u_{msy}}$ = exploitation rate that generates MSY
- $\mathbf{F_{msy}}$ = instantaneous fishing mortality rate that generates MSY

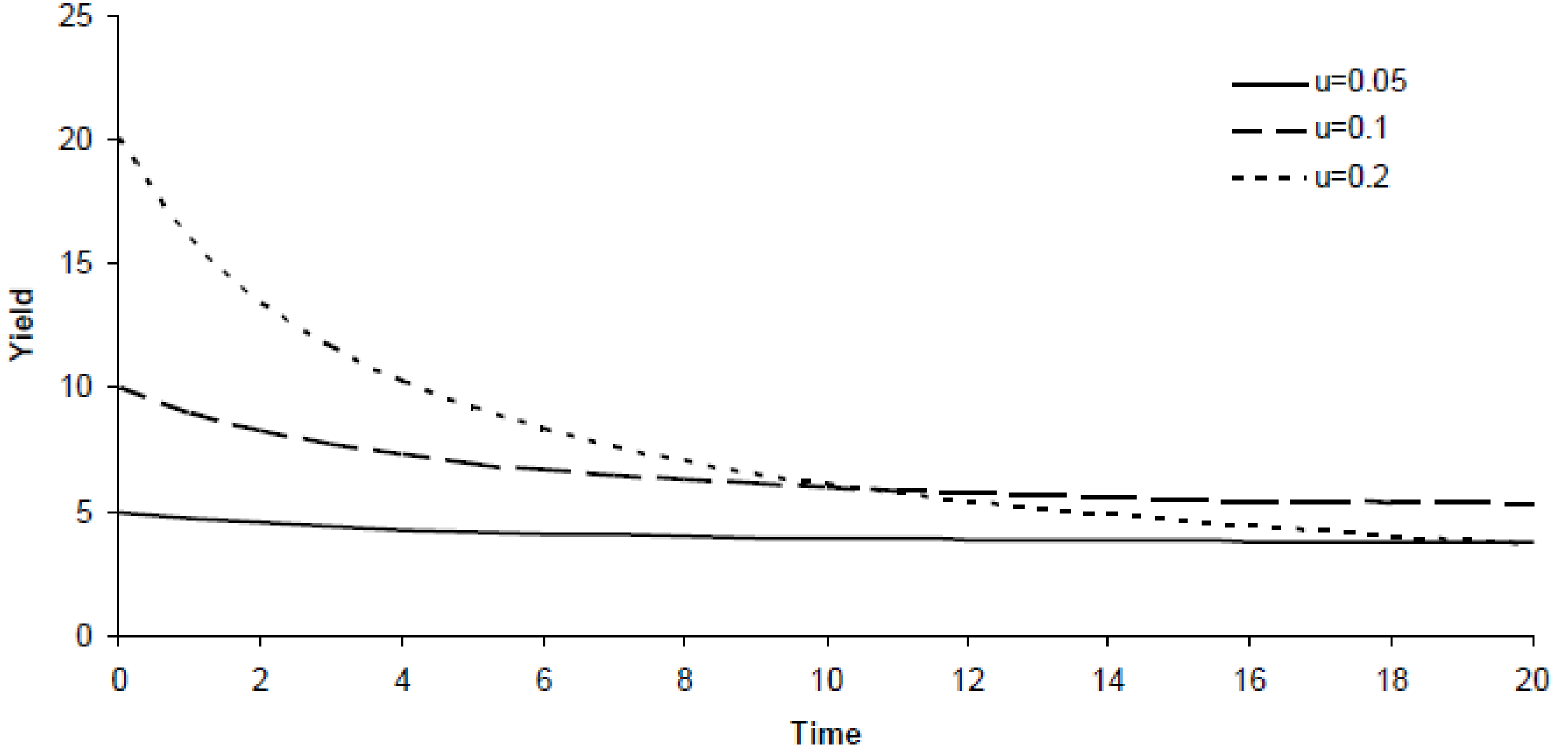
How to calculate MSY?

What happens to pop. through time at different exploitation rates (u)?



- What do you notice about the Biomass?

Yield over time



- What do you notice about the yield over time?
- →MSY is when we maximize this longterm yield

How to calculate MSY

$$B_{t+1} = B_t + rB_t\left(1 - \frac{B_t}{K}\right) - uB_t$$

$$u_{MSY} = \frac{r}{2}$$

$$B_{MSY} = \frac{K}{2}$$

$$\boxed{MSY = u_{MSY}B_{MSY} = \frac{rK}{4}}$$

Management Quantities

To be continued when we get into stock assessments…

Maximum surplus production (MSY)	$rK/4$
Stock size for MSY (B_{MSY})	$K/2$
Rate of exploitation at MSY (u_{MSY})	$r/2$
Effort required to achieve MSY (E_{MSY})	$r/2q$
Maximum rate of exploitation (u_{max})	r
Effort at maximum rate of exploitation (E_{max})	r/q

Summary 1

- Exponential mortality model
  - describes cohort abund. through time
  - Divide total inst. Mortality into: **Z=M+F**

$$N_t = N_0 e^{-(F+M)t}$$

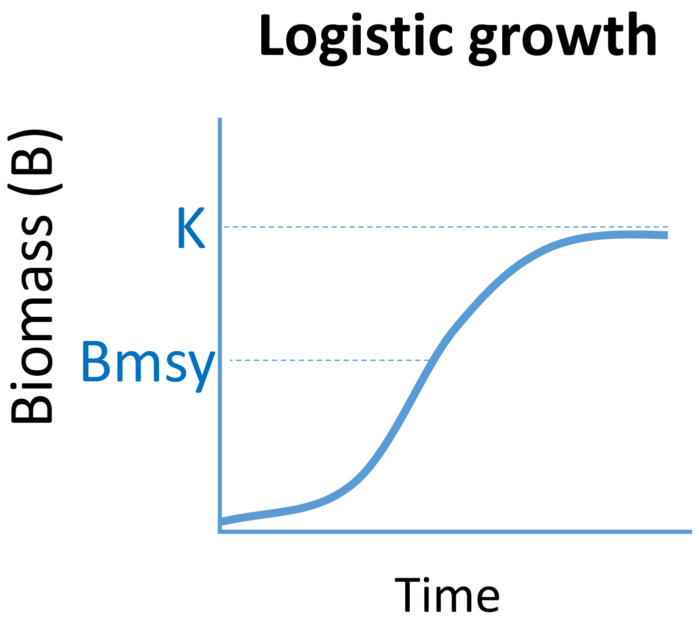
- Definitions
  - A, S, Z, u, v, F, and M
  - Exploitation rate (u)
  - Type I and II fisheries

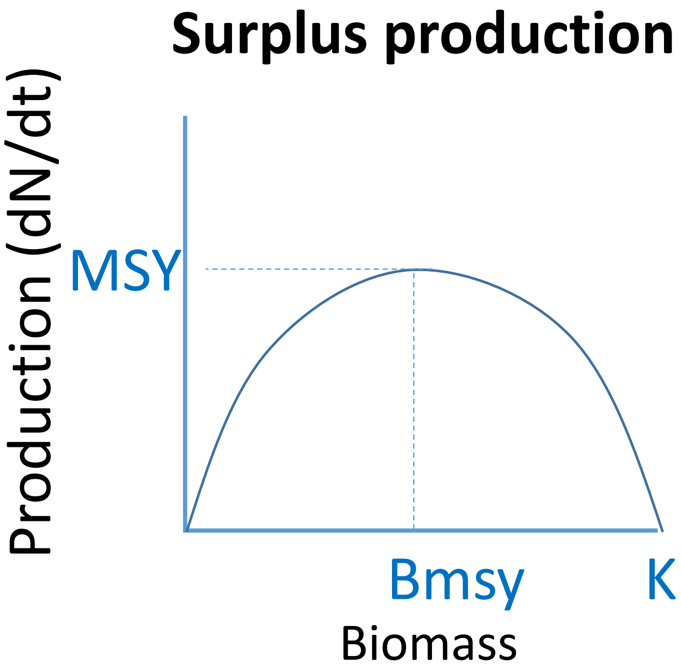
$$C_t = \frac{F}{Z}(1 - e^{-Z})N_t$$

- Baranov Catch Equation (many variants)
  - Catch = (fraction of mortality due to fishing)*(Proportion dying)*abundance
- Know MSY concepts!
  - how MSY related to logistic growth and surplus prod.
  - **MSY, B_{MSY}, u_{MSY}, F_{MSY}**

2 Logistic growth & surplus production

- Key concepts from the simple model:
  - K, r
  - Surplus production
  - MSY, Bmsy
  - u_{MSY} and F_{MSY}





3 Instantaneous & annual mortality

Instantaneous rate	Annual rate (proportion)	Relationship*
Z Total mortality	A Annual mortality	$A = 1\text{-}e^{-Z}$ $S = 1–A = e^{-Z}$ $Z = -\ln(S)$
F Fishing mortality	u Exploitation rate	$u = FA/Z$
M Natural mortality	v Prob. death from natural causes	$v = MA/Z$
$Z = F+M$	$A = u+v$	

For u or v, need 2 of the following: F, M, Z

*Note: u and v relationships assume Type II fishery that operates continuously throughout the year