

# Stock-recruitment

## Part II

Reading:

Jennings et al. 2001. Marine Fisheries Ecology, Chapter 4  
(section 4.2)

Advanced: Quinn and Deriso 1999, Chapter 3

“Stock recruitment  
funny”



*“They’re the ideal temp workers! Easy to train, industrious, punctual... And with a 13-day life-span, they conveniently die before collecting a paycheck!”*

# Recap: Stock-recruitment models

## Models

- 1. Density Independent
  - 2. Beverton-Holt
  - 3. Ricker
  - 4. Shepherd
  - 5. Hockey stick
  - Others...
- What did BH model look like?
  - What is compensation?
  - How does density dependence play into BH model?



William E. Ricker

### 3. Ricker Model

- Based on idea of density-dependence acting on the **adults**
  - Change in juv. abundance (N) affected by total mortality, Z
  - Assume Z described by linear function of *spawner abundance (S)*, affected by density independent (a) and density dependent (b) parameters
  - Leads to nonlinear differential equation (see Quinn and Deriso for solution)

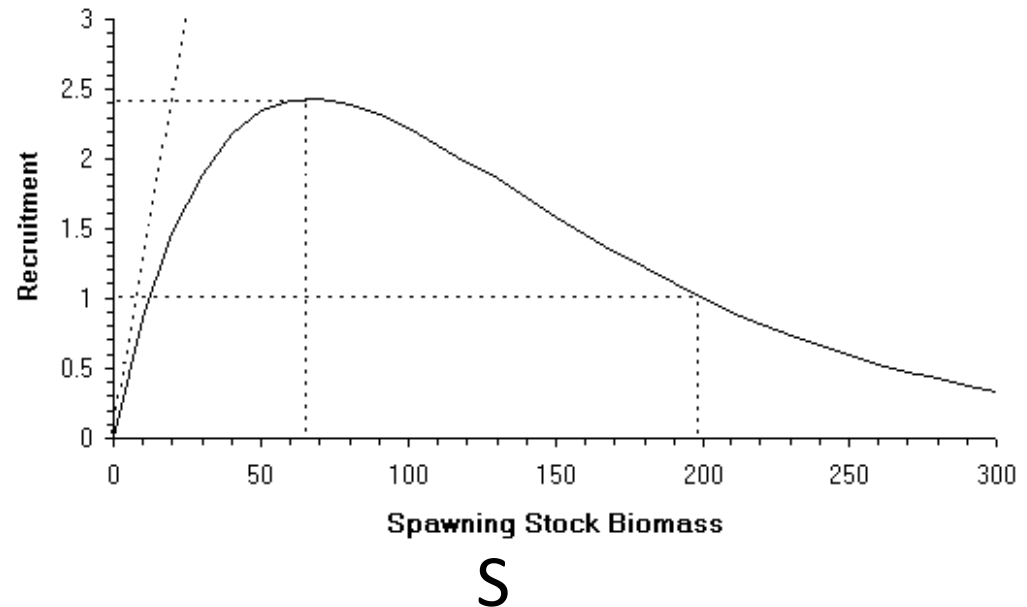
$$\frac{dN}{dt} = -ZN$$

$$Z = a + bS$$

$$\frac{dN}{dt} = -(a + bS)N$$

# 3. Ricker Model

$$R = aSe^{-bS}$$



- R = number (or biomass) of recruiting individuals
- S = number (or biomass) of spawners
- a = productivity parameter (number of R per S at low S)
- b = parameter for degree of density dependence
- Basic property: “hump” shaped with declining R at higher S

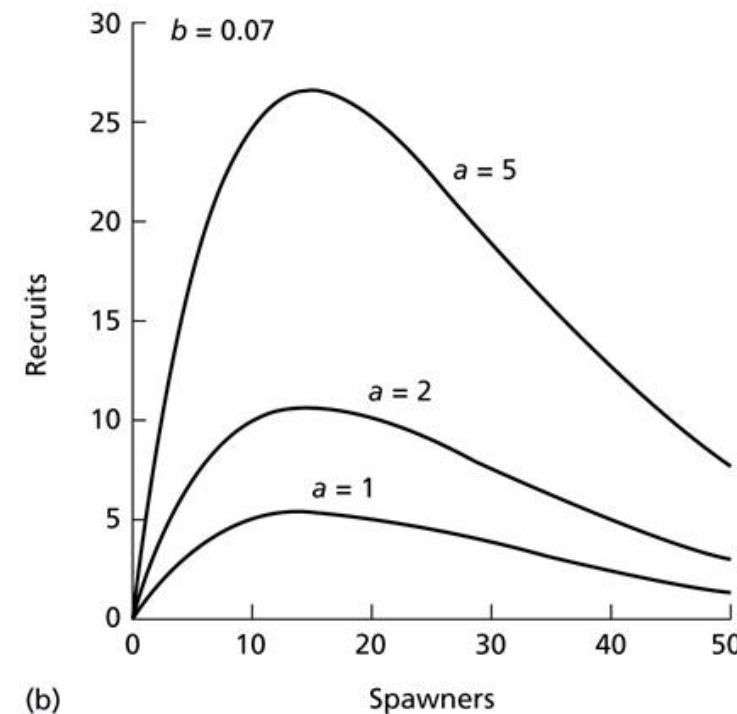
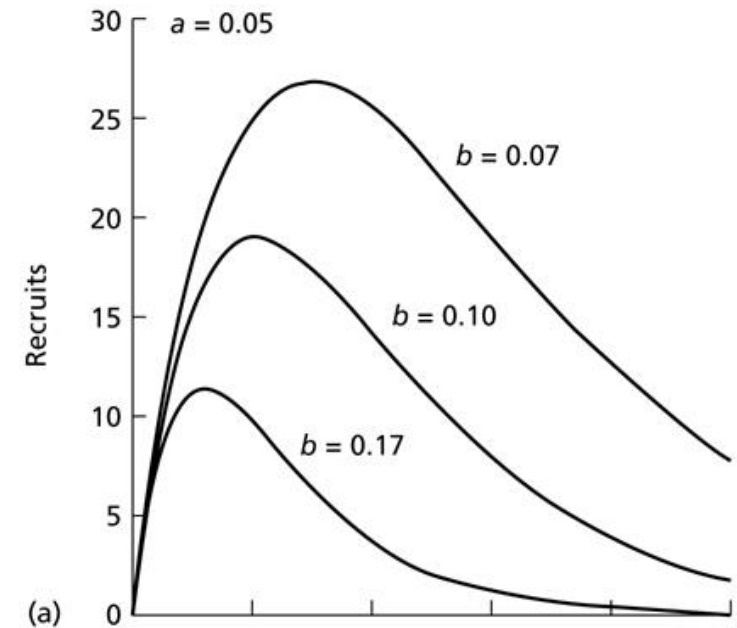
### 3. Ricker Model

- Effects of changing parameters

$$R = aSe^{-bS}$$

- Maximum mean R occurs at  $S=1/b$
- Alternative parameterization you might see:

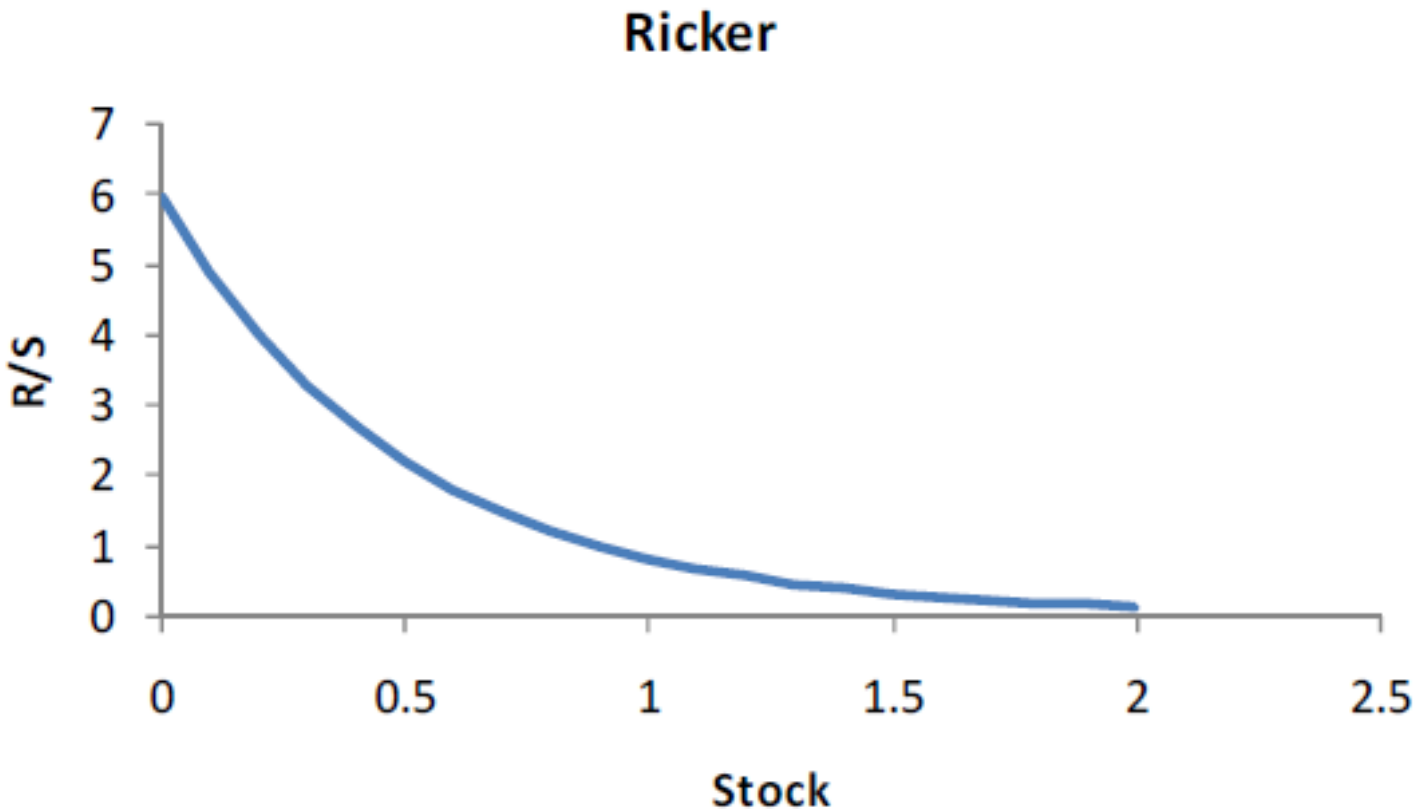
$$R = Se^{a'-bS}$$



(b) Spawners

### 3. Ricker Model

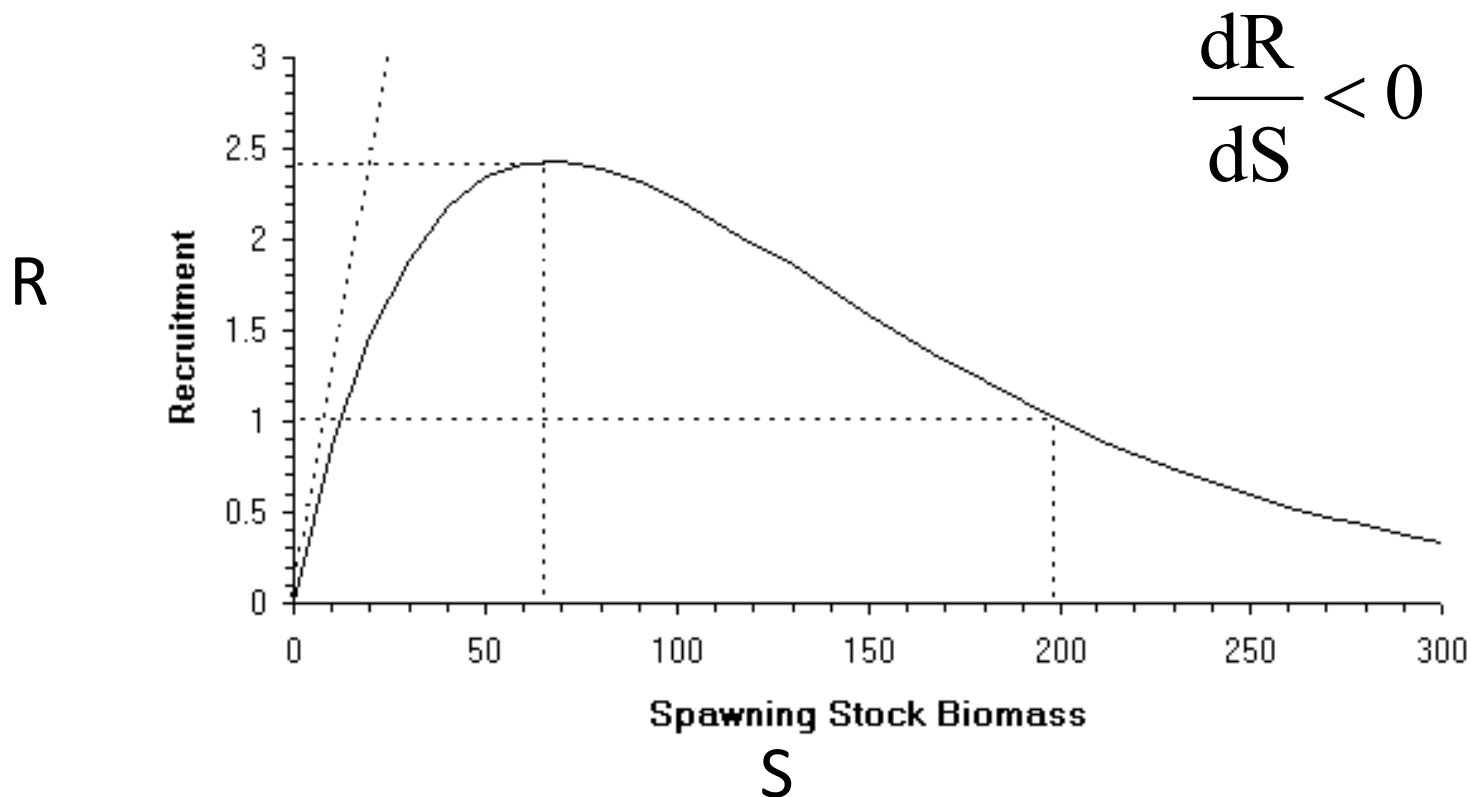
- What does survival proxy ( $R/S$ ) look like?
  - Like, BH, has stabilizing effect on population at low  $S$





# Ricker – overcompensation

- **Overcompensation** – decrease in recruitment with increasing spawning stock



# Ricker Overcompensation

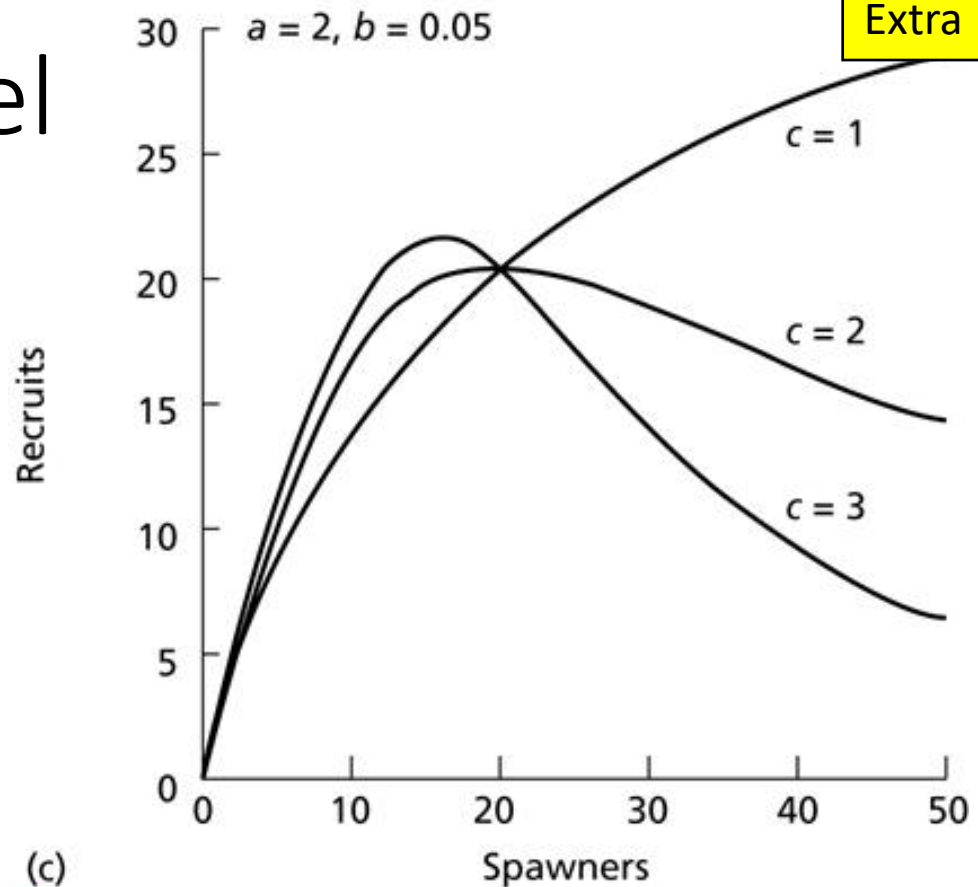
- Recruitment decreases at large stock sizes
- **Some possible causes:**
  - 1. Cannibalism of juveniles by adults
  - 2. Disease transmission from adults to juveniles
  - 3. Oxygen limitations due to heavy egg deposit that affects all eggs
  - 4. Spawning site damage by adults
  - 5. Density dependent growth with size-dependent predation



# 4. Shepherd Model

- Generalizing equation

$$R = \frac{aS}{1 + (bS)^c}$$

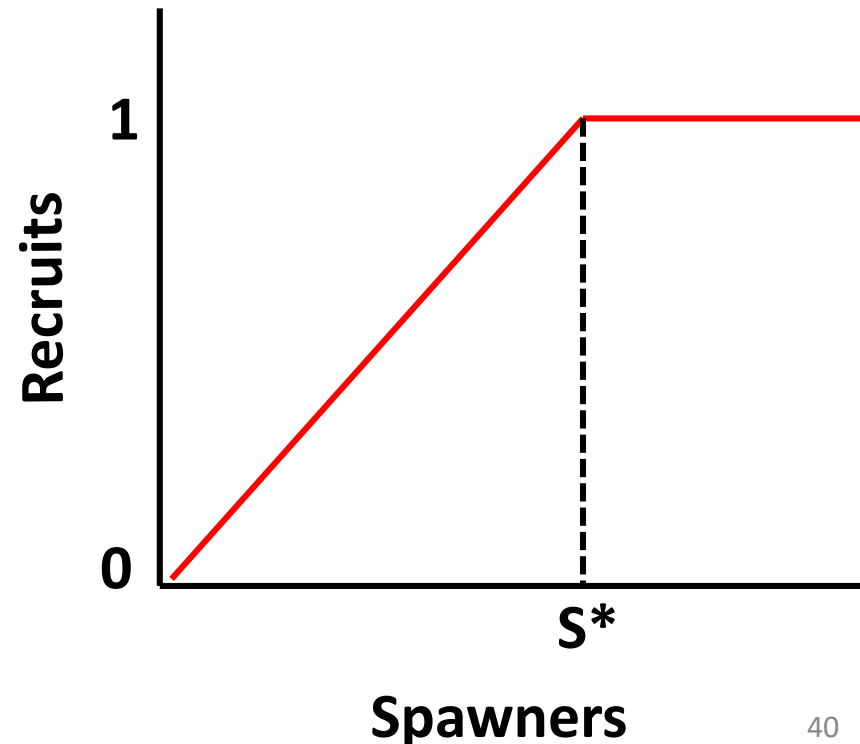


- $a$  = productivity parameter (number of R per S at low S)
- $b$  = parameter for degree of density dependence
- $c$  = shape parameter
  - $c < 1$ : density-independent;  $c = 1$ : Beverton-holt;  $c > 1$ : Ricker shape
- Basic property: generalizing equation for other model shapes

## 5. Hockey stick model

- Segmented (change-point) regression
  - Slope  $a > 0$  at the origin;
  - Slope  $a = 0$  beyond pivotal spawner level,  $S^*$

$$R_t = \begin{cases} aS_t & \text{if } S_t < S^* \\ aS^* & \text{if } S_t \geq S^* \end{cases}$$



# Stock-recruitment models

- Most common
  - Ricker
  - Beverton-Holt
- Others
  - Shepherd
  - Deriso-Schnute
  - Cushing
  - “Hockey-stick”
  - Unnormalized gamma density
  - Many others

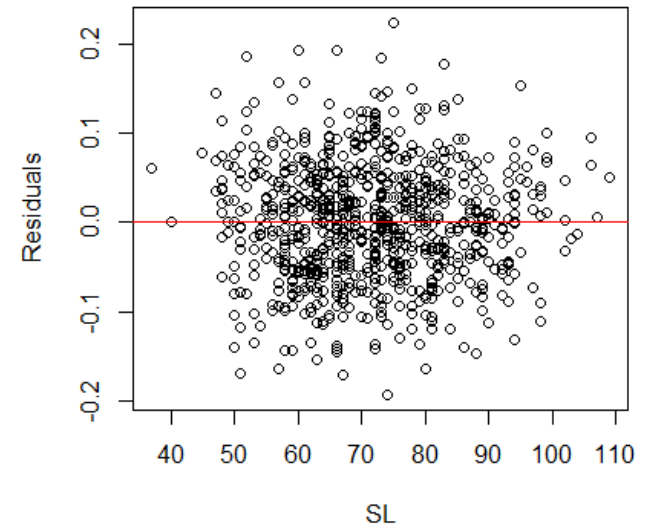
# Fitting Stock-recruitment models

- Recommend using nonlinear regression
- Making the following assumptions:
  - No error in our estimate of  $S$
  - Independent errors\*
- But, model equation will depend on whether error is assumed to be additive or multiplicative

\*see Quinn and Deriso 1999 (section 3.2) for “measurement error approach” and “autocorrelated errors” if these assumptions are grossly inappropriate

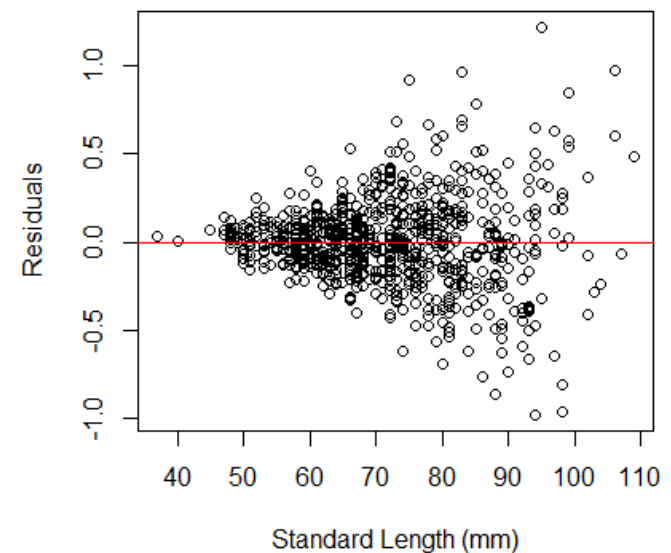
# Additive vs. multiplicative error

- Additive error
    - for a given model, the residuals will tend to have constant variance
    - Doesn't violate regression assumption of Homogeneity of Variance (HOV)
- $$Y = f(X) + \varepsilon$$



- Multiplicative error
  - for a given model, the residuals will increase with higher values of X
  - This violates the HOV assumption for regression → so log-transform equation to use regression

$$Y = f(X) \cdot e^{\varepsilon}$$



# Log transforming to deal with multiplicative error

- Recall our example using W-L allometric model:

$$W = aL^b e^\varepsilon$$



*Log-transform both sides of equation*

$$\log(W) = \log(aL^b e^\varepsilon)$$



*Algebra*

$$\log(W) = \log(a) + b \cdot \log(L) + \varepsilon$$

Now, the error is  
“additive” in our model,  
so we can use  
regression and OLS



# Fitting Stock-recruitment models

- Our approach
  - use nonlinear regression (and OLS)
  - The equation we fit with `nls()` will depend on whether we assume additive or multiplicative error
  - *Multiplicative error is typically more appropriate for SR data*
- If using multiplicative error (& log transformation) → must use bias correction
  - Back transforming (i.e. exponentiating) estimates from log space introduces bias
  - *Bias correction*: multiply the back-transformed predicted values by a correction factor (CF), which depends on the standard error of the estimate (SEE; aka Residual SE)

# Fitting Stock-recruitment models

- Our approach

## Equations to fit using nonlinear regression

More  
common!



### Additive error

### Multiplicative error

(log of  $R=f(S)e^\varepsilon \rightarrow \log(R)=\log(f(S))+\varepsilon$ )

Density  
Independ.

$$R = aS + \varepsilon$$

$$\ln(R) = \ln(aS) + \varepsilon$$

Beverton  
Holt

$$R = \frac{aS}{1 + bS} + \varepsilon$$

$$\ln(R) = \ln(aS / (1 + bS)) + \varepsilon$$

Ricker

$$R = aSe^{-bS} + \varepsilon$$

$$\ln(R) = \ln(aSe^{-bS}) + \varepsilon$$

Note: “ln” is “natural log”, which in R, is written just as “log()”

# Fitting Stock-recruitment models

## Example: Ricker Model with Multiplicative Error

$$R = aSe^{-bS} e^{\varepsilon}$$



*Log-transform*

$$\ln(R) = \ln(aSe^{-bS}) + \varepsilon$$



*Fit using nonlinear regression,  
and estimate parameters:*

$$\hat{a}, \hat{b}, \hat{\sigma}_{\varepsilon}^2$$



*Back-transform & bias-correct*

$$\hat{R} = \hat{a}Se^{-\hat{b}S} \cdot e^{(\hat{\sigma}_{\varepsilon}^2 / 2)}$$

$$SEE = \hat{\sigma}_{\varepsilon}^2 = \sqrt{\frac{\Sigma(\text{obs.} - \text{pred.})^2}{n - (\text{nos. parameters})}}$$

Note: In R, SEE is the  
“residual standard error”.  
Value stored in:  
`summary(MyModel)$sigma`

$$CF = e^{(\hat{\sigma}_{\varepsilon}^2 / 2)} = e^{(SEE / 2)}$$

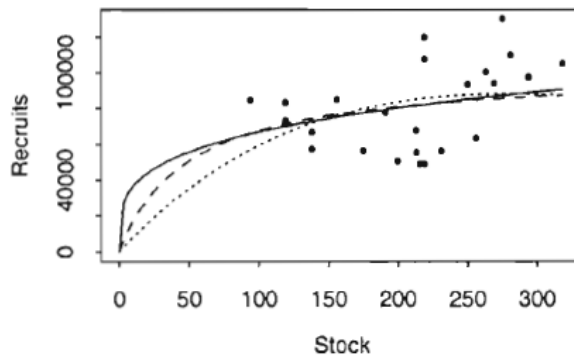
# Examples of SR model fits

Thoughts?

Why are fits so poor?

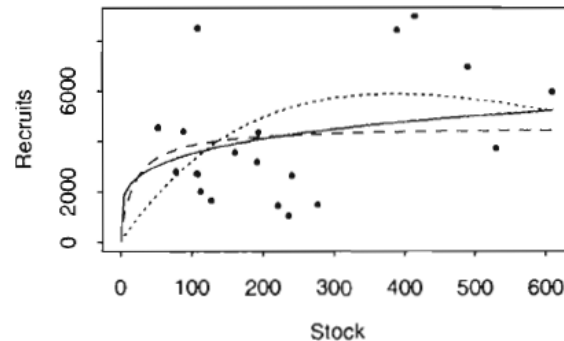
Yellowfin tuna  
(*Thunnus albacares*)

Lognormal fits



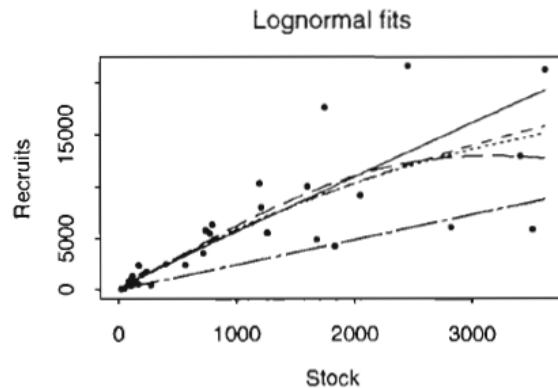
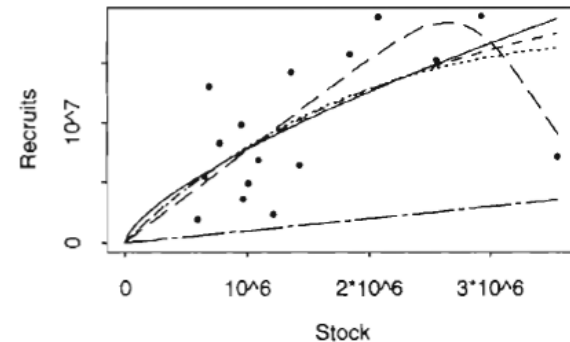
Petrale sole  
(*Eopsetta jordani*)

Lognormal fits

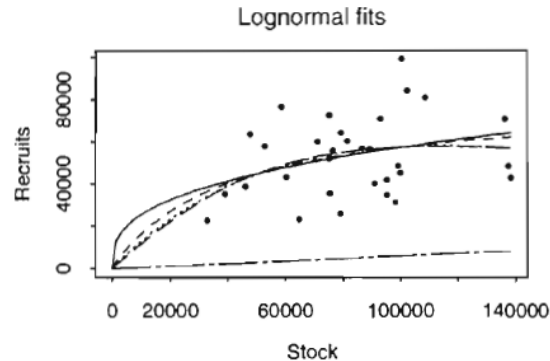


Pink salmon (Fraser river)  
(*Oncorhynchus gorbuscha*)

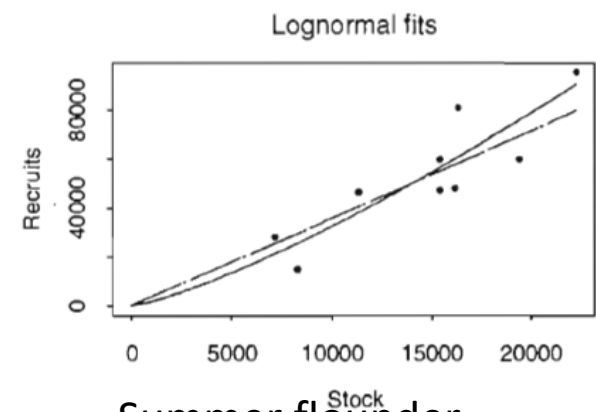
Lognormal fits



CA Sardine  
(*Sardinops sagax*)



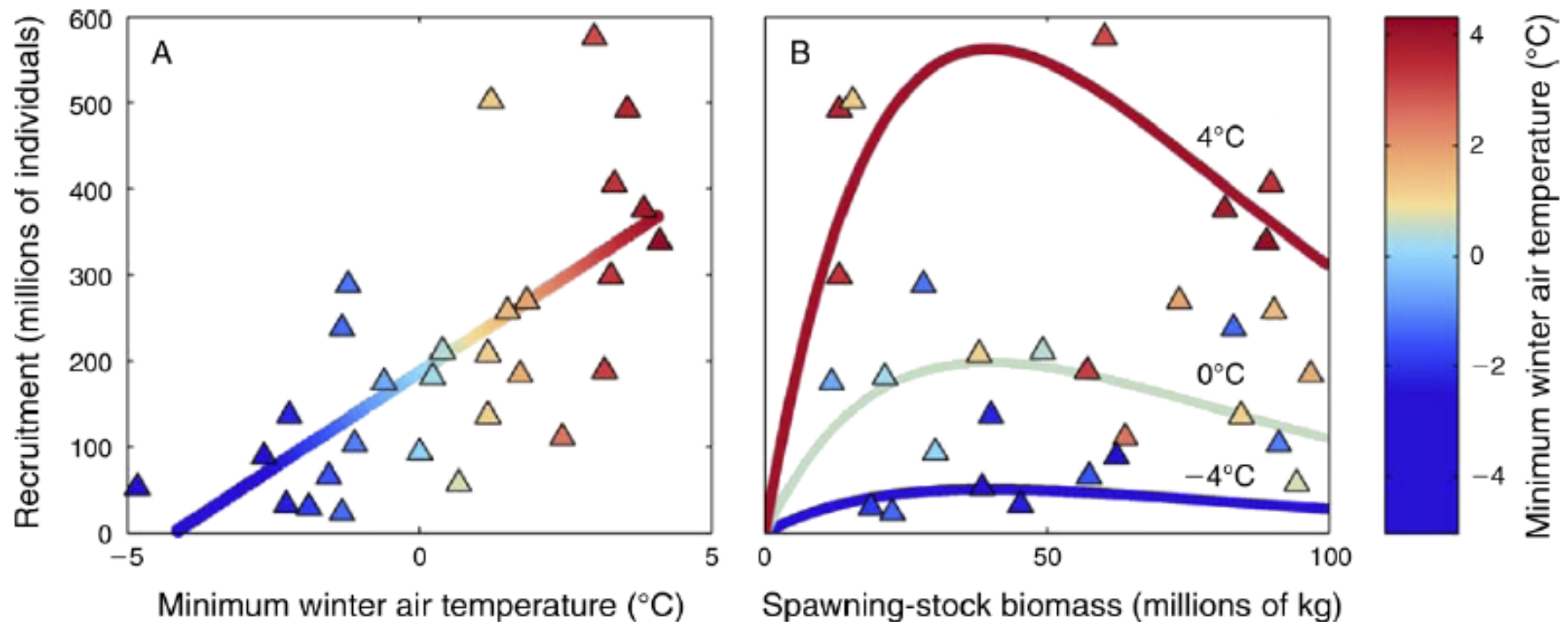
Atlantic cod (NAFO 3Ps)  
(*Gadus morhua*)



Summer flounder  
(*Paralichthys dentatus*)

# Some modifications to S-R Models

- Possible to build in environmental effects (e.g., temp for Atlantic Croaker)
- Account for error in S estimates
  - see Quinn & Deriso, Section 3.2.3



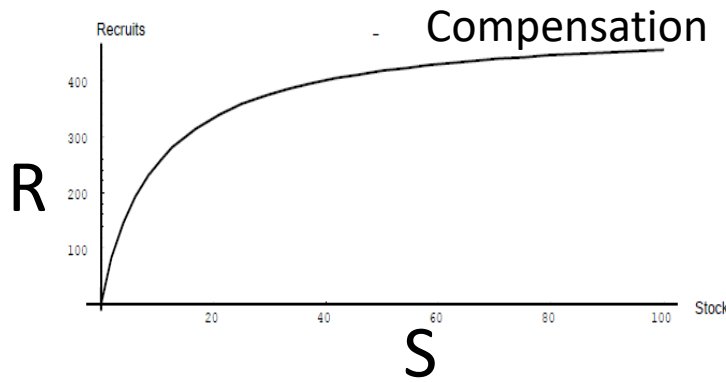
# Summary

- Stock recruitment models
  - Relate the production of recruits to adult spawning stock
  - Critical for forecasting, assessing, and managing populations
  - Typically account for some type of density-dependence (DD)
  - Fits can be rather poor → lots of uncertainty
- Know definitions:
  - **Stock, Recruitment, Density dependence, Compensation**
- Stock recruitment models
  - **Beverton-Holt**
  - **Ricker**
  - Shepherd – Generalization of other models
  - “Hockey-stick”
  - Many others (Deriso-Schnute, Cushing, ...)
- Fitting models
  - For us: assume multiplicative error (if have HOV problem) → log-transform model → use nonlinear regr. → back-transform & bias correct

# Summary of BH and Ricker models

## Beverton Holt

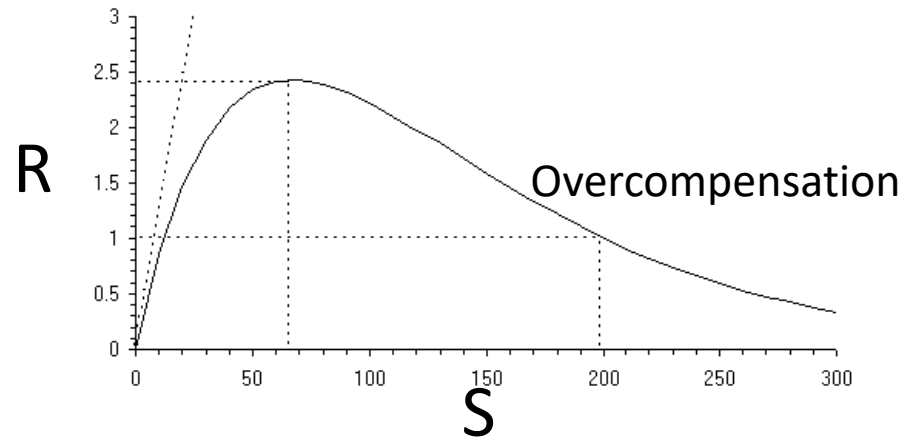
$$R = \frac{aS}{1 + bS}$$



- Density dependence
  - Acts via juvenile stage (*know examples*)
- Parameters
  - $a$  = productivity parameter
  - $b$  = density dependence
- Shape: asymptotic

## Ricker

$$R = aSe^{-bS}$$



- Density dependence
  - Acts via adult stage (*know examples*)
- Parameters
  - $a$  = productivity parameter
  - $b$  = density dependence
- Shape: dome