Evolutionary Game Theory

Real Estate Overvaluation, Stable Development, & Government Intervention

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Abstract

The continuance of overvaluation in the real estate market introduces a number of considerations for appraisers and regulatory bodies, which include various trade-offs. This paper aims to use evolutionary game theory models involving real estate appraisers and government supervisors to analyze overvaluation in the real estate market. By presenting a two-player asymmetric bi-matrix game with time-evolving environments, we examine the influence of policy on appraisers' decisions to overvalue or fairly price a real estate property. Further, we propose a secondary model – a multiplayer game involving cooperators and defectors – to analyze the impact of a novel regulatory policy to promote fair valuation of real estate.

Introduction

Before a real estate transaction takes place (especially involving a mortgage loan), the value of the property is assessed by an professional appraiser, who walks the property, inspecting interior and exterior spaces. The appraisal determines what is known as the 'fair market value' (FMV) of the the property. While government supervisors strive to ensure there is fair valuation of real estate, overvaluation is prevalent in practice, leading to appraisal fraud.

Appraisal fraud is a type of mortgage fraud, in which the appraised value of a property is intentionally inflated. The overstated value of the property is typically used to induce a higher loan for the mortgagee and/or mortgagor, yet may also be used to artificially increase the market price for a seller.

To protect from appraisal fraud, banks often elect to use a preferred appraiser when underwriting a mortgage or loan refinance. Nonetheless, overvaluation in the real estate market is still frequent, as an appraiser's commission is closely related to the appraised value; an appraiser may boost his/her income by overstating the value of a given real estate property.

In the short term, all involved parties benefit from the overvaluation of real estate. However, if a critical mass of appraised properties are overvalued, a bubble in the real estate market is likely to develop, alongside long term systematic financial risks.

The stable development of real estate is essential to a successful economy, both nationally and internationally. In recent years (along with the past), there has been increased speculation in the real estate market, marked by overvaluation of property, leading to heightened uncertainty in the sector. As such, it appears as though diligent government intervention is necessary at this time to provide stability to the real estate market.

The Savings & Loan (S&L) Crisis

Overview

Prior to investigating recent issues with the stable development of the real estate market, we will look at the Savings & Loan Crisis of the 1980s to 1990s to identify the potential consequences of overvaluation.

What was the S&L Crisis?

According to Kenton, the Savings & Loan Crisis was the “build-up and extended deflation of a real estate lending bubble in the United States from the early 1980s to the early 1990s... [culminating] in the collapse of hundreds of savings and loans institutions and the insolvency of the Federal Savings and Loan Insurance Corporation.”

A volatile interest rate climate, leading to stagflation and slow growth in the 1970s precipitated the crisis, which likely contributed to the recession of 1990-91, as nearly a third of the 3,234 savings and loans associations collapsed between 1986 and 1995. Estimates of the total cost are as high as $160 billion, and it is estimated that $132 billion was taken on by taxpayers.

Why did the S&L crisis occur?

The S&L crisis developed in response to excessive lending, speculation, and risk-taking driven by the moral hazard created by deregulation and taxpayer bailout guarantees. Not only was there a mismatch of regulations to market conditions and speculation, but there was also outright corruption and fraud.

How did the S&L crisis unfold?

In 1932, the Federal Home Loan Bank Act placed restrictions on S&Ls, including caps on interest rates on deposits and loans. As the economy slowed with inflation, S&Ls could not compete with other lenders. A recession, sparked by high-interest rates from the Fed (to reduce inflation) left S&Ls with portfolios of low-interest mortgage loans. In 1982, S&Ls had a tight revenue stream and were losing $4.1 billion per year.

Later that year, President Ronald Reagan signed the Garn-St. Germain Depository Institutions Act, eliminating loan-to-value ratios and interest caps, further allowing S&Ls to hold 30% of their assets in consumer loans and 40% in commercial loans. This meant that S&Ls were no longer governed by Regulation Q, so they began to invest in risky projects, assuming they would pay off in higher returns. If the returns did not materialize, the taxpayers would be bailing them out via the The Federal Savings and Loan Insurance Corporation (FSLIC).

Thus, S&Ls were encouraged to take greater risks, leading to rapid growth in the industry, with increased speculative risk. According to Kenton, “[deregulated] lending and capital requirements along with a taxpayer-funded guarantee backstop created an enormous moral hazard.” By 1987, the FSLIC had become insolvent, and the government opted to recapitalize the corporation, rather than letting it and the S&Ls fail as they were supposed to.

How does the S&L crisis relate to appraisal fraud?

During the S&L crisis, insiders developed a strategy of fraudulent transactions to elicit profits. The strategy was typically performed as follows. Two partners would "conspire with an appraiser to buy land using S&L loans and flip it to extract profits." The first partner would buy the parcel at its appraised fair market value (FMV), then the appraiser would reappraise the property at a higher price. The second partner would then buy the parcel using a loan from an S&L, later defaulting on the loan. The partners and appraiser would share the profits following the series of transactions.

Unfortunately, due to the complexity of such cases (alongside staffing/workload issues), law enforcement did not tend to pursue wrongdoers even when made aware of the fraudulent actions.

What was the result of the S&L crisis?

In 1989, Congress passed the Financial Institutions Reform, Recover, and Enforcement Act, creating the Resolution Trust Corporation, which was responsible for "winding down" the failed S&Ls that regulators had taken control of. By creating minimum capital requirements, raising insurance premiums, and limiting S&L mortgage related holdings to 30%, at least 700 S&Ls were liquidated.

By 1989, more than 1,000 S&Ls had failed, ending what "had been one of the most secure sources of home mortgages." The recession of 1990-91 followed, as new home starts fell to a new low, since World War II.

Modern Day

Overview

To further understand the significance of addressing real estate overvaluation, and to investigate a recent issue with real estate appraisals, we will perform a literature review of "Accuracy of Appraisals Is Spotty, Study Says" (2012) by Julie Satow.

Literature Review

According to the article, excessive purchase prices based on "faulty property appraisals" were a major factor contributing to the economic crisis faced by commercial landlords and lenders. A study by KC Conway, an executive managing director at the brokerage firm Colliers International, and Brian F. Olasov, a managing director at the law firm McKenna Long & Aldridge, found "a wide discrepancy between the appraisal values and the eventual sales prices of the properties" when considering data from securitized real estate bonds. In evaluating 2,076 properties, it was determined that 64% were overvalued, with the total discrepancy between appraisal value and sale price at $1.4 billion.

The study, published in CRE Finance World "based on data from the research company Trepp L.L.C. for March 2007 through September 2011", highlights the inaccuracy of real estate appaisals and the tendency of overvaluation. In 121 instances, "the appraised value was more than double the sale price," indicating that a number of appraisals are more or less a shot in the dark.

Jay A. Neveloff, a partner at the law firm Kramer Levin Naftalis & Frankel, highlights that appraisals are "important in nearly every aspect of a real estate deal, whether it is originating a loan, working out a loan, the decision to buy or sell a property, and even bankruptcy." John Cicero, a managing principal of the appraisal firm Miller Cicero indicates "[the] appraisal industry has become commoditized, where lenders see appraisals as simply a commodity to be purchased by a vendor and where more emphasis is placed on the price of an appraisal than the expertise of the appraiser." This calls attention to a fundamental flaw in the modern appraisal industry.

Functions Of Appraisers

As indicated previously, appraisers serve multiple functions: they are used by lenders to determine the size of a loan or by lawyers in litigation. As such, they should be experienced professionals accustomed to properly conducting appraisals. This involves visiting property sites, speaking to brokers in the area, reviewing financial statements (income and expense histories), and examining individual leases. Unfortunately, many appraisers rush through the process, leading to inaccuracies with broad implications for the real estate market.

Impact On Banking Institutions

The impact of real estate overvaluation on banking institutions in particularly pronounced, according to Mr. Olasov, who states that "when you take a look at the 400-plus bank failures that have taken place going back to 2009, the precipitating cost was declining appraisal values on real estate portfolios." This reflects the situation in which a real estate bubble is created, followed by a correction that forces banks to take write-downs, charge-offs, and increase loan loss reserves which drain capital, leading to bank failures.

The consensus is that there needs to be a systematic way to fairly appraise real estate properties, considering a wide variety of market factors, along with government intervention when necessary.

Model #1 - Asymmetric Bi-Matrix Game

Overview

In this section, we present a two-player asymmetric bi-matrix game with time-evolving environments to examine the influence of policy on appraisers' decisions to overvalue or fairly price a real estate property. We derive an evolutionary game theory model with theoretical replicator dynamics, applying the modeling analysis to model changes in appraiser and government supervisor preferences as appropriate.

Replicator Equations For Asymmetric Bi-Matrix Games

Following along "Evolutionary Dynamics of Gig Economy Labor Strategies under Technology, Policy and Market Influence" by Kevin Hu and Dr. Feng Fu, we utilize the replicator equation, "a differential equation that determines the evolving composition of strategies in a population" in considering real estate overvaluation.

In particular, we are interested in how appraiser and government supervisor preferences evolve over interactions. The general replicator equation is

$$\dot{x}_i = x_i \left(\pi_i - \overline{\pi}\right)$$

where x_i denotes the proportion of strategy type i in the population, π_i is the fitness of strategy type i, and $\overline{\pi}$ represents the average payoff across the entire population. The fitness of strategy type i is equivalent to the expected payoff for that strategy.

Accordingly, for asymmetric bi-matrix games, the replicator equations take the following form:

$$\dot{x}_i = x_i \left(\left(A\vec{y}\right)_i - \vec{x} \cdot \left(A\vec{y}\right)\right)$$

$$\dot{y}_i = y_i \left(\left(B\vec{x}\right)_i - \vec{y} \cdot \left(B\vec{x}\right)\right)$$

where $\dot{x}_i$ denotes the evolution for player 1 strategies and $\dot{y}_i$ denotes the evolution for player 2 strategies. In our model, player 1 is the appraiser and player 2 is the government supervisor, while A and B represent the respective payoffs in matrix form for both players. Further, $\vec{x}$ and $\vec{y}$ denote the strategies for each player, which takes a value in the domain $[0, 1]$ and represents the probability that the strategy is selected.

Now, we continue along the lines of "Evolutionary Dynamics of Gig Economy Labor Strategies under Technology, Policy and Market Influence". The following is taken directly from this paper, with modifications made to fit our model:

> Selection intensity, denoted with $\omega \in [0, 1]$, represents the frequency in which appraisers and government supervisors interact. When government supervisors choose to involve themselves in the appraisal market, appraiser decisions are determined based on respective payoff incentives, and the composition of appraisers evolves accordingly.
>
> Selection intensity may further be understood as the rate at which appraisers and government supervisors realize external factors that cause them to shift strategy preferences. In evolutionary game theory, this social learning process can be modeled as the Moran process, which is stochastic in nature.

Set Up

Transitioning to a tactile theoretical approach, we must remind ourselves of the overarching issue. Appraisers are likely to overvalue a given real estate property, yielding a higher commission and larger loan for mortgage. While this results in short term growth, bubbles develop in the real estate market, alongside long term systematic risks. As a solution, we consider the implementation of government supervision.

The model we use for the appraisal market consists of two player groups: appraisers (denoted by A) and government supervisors (denoted by B). The strategies for the appraisers are to overvalue (O) or fairly price (F) a given property. The strategies for the government supervisors are to supervise (S) or not supervise (NS) the appraisal process.

Variables

The variables for the payoff matrix are as follows. The appraisers may gain rewards from an overstated value (denoted by v), but must pay a fine/penalty (denoted by f) if they are caught doing so. There is a cost to government supervision (c), which covers both inspection and finding evidence of overvaluation. When overvaluation is caught, the penalty paid by the appraisers acts as government revenue.

As we have set up the asymmetric bi-matrix game, $\{v, c, f\} \geq 0$, as there is not a negative value/reward to overvaluation, nor a negative cost of government supervision, nor a negative fine/penalty to over appraising if caught.

Payoff Matrix

The payoff matrix of the asymmetric bi-matrix game following the conditions set forth in the set up is as follows:

Model #1 - Asymmetric Bi-Matrix Game		B (Government Supervisors)	
		Supervise	Do Not Supervise
A (Appraisers)	Overvalue	*(v - f, -c + f)*	*(v, 0)*
	Fair Price	*(0, -c)*	*(0, 0)*

Evidently, if $v > f$ appraisers have a dominant strategy to overvalue a given property. If $f < c$, government supervisors have a dominant strategy to not supervise.

Evolutionary Stable Strategies (Dynamo 2x2 Results)

Now, after outlining the derivation and characteristics of our evolutionary model, we analyze the phase diagrams of given situations using Dynamo 2x2. The Dynamo 2x2 results presented demonstrate the trajectories and evolutionary stable strategies for each case, based on the model formulated.

The Dyanmo 2x2 results also present a heat map to demonstrate the speed of attraction/repulsion to/from a given point on the 2x2 phase plane. Red is representative of faster movement, whereas blue is representative of slower movement.

On the Dynamo 2x2 phase plot shown on the following pages, you will notice each corner is appropriately labeled by the strategies for appraisers and government supervisors, with O representing "Overvalue", F representing "Fair Price", S representing "Supervise", and NS representing "Do Not Supervise".

The Nash equilibria and ESS values presume $(1, 0)$ indicates a pure strategy of "Overvalue" for appraisers and a pure strategy of "Supervise" for government supervisors, while $(0, 1)$ indicates a pure strategy of "Fair Price" for appraisers and a pure strategy of "Do Not Supervise" for government supervisors.

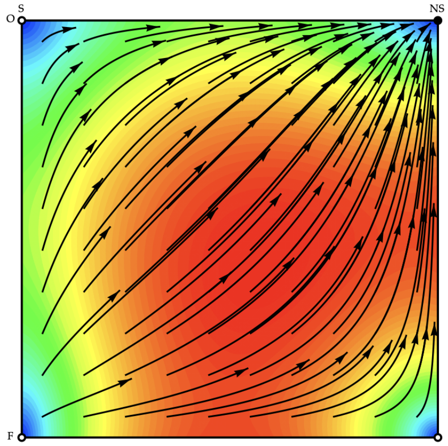


Figure 1

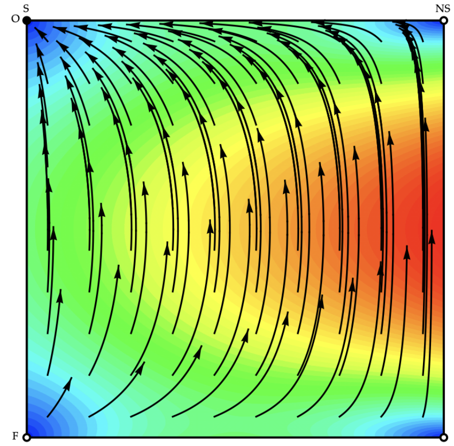


Figure 2

Figure 1 displays the case where $v = 2, c = 2, f = 1$, that is $v > f$ and $f < c$. Thus, the payoff matrix is of the form

$$\begin{pmatrix} & \text{Supervise} & \text{Do Not Supervise} \\ \text{Overvalue} & (+,-) & (+,0) \\ \text{Fair Price} & (0,-) & (0,0) \end{pmatrix}$$

The dominant strategy is chosen by both appraisers and government supervisors, producing a Nash equilibrium at $(1, 0)$ (Overvalue) for appraisers and $(0, 1)$ (Do Not Supervise) for government supervisors. The evolutionary stable strategy (ESS) matches the Nash equilibrium.

In this scenario, we see a situation in which appraisers quickly take advantage of the benefit awarded from overvaluation with minimal penalty, while government supervisors move toward no supervision in light of high costs.

Figure 2 displays the case where $v = 3, c = 1, f = 2$, that is $v > f$ and $!(f < c)$. Thus, the payoff matrix is of the form

$$\begin{pmatrix} & \text{Supervise} & \text{Do Not Supervise} \\ \text{Overvalue} & (+,+) & (+,0) \\ \text{Fair Price} & (0,-) & (0,0) \end{pmatrix}$$

The dominant strategy is chosen by appraisers, resulting in government supervisors adapting strategies, producing a Nash equilibrium at $(1, 0)$ (Overvalue) for appraisers and $(1, 0)$ (Supervise) for government supervisors. The evolutionary stable strategy (ESS) matches the Nash equilibrium.

In this scenario, we see a situation in which appraisers quickly take advantage of the benefit awarded from overvaluation with minimal penalty, though government supervisors move toward supervision in light of low costs. Despite supervision, appraisers still find it advantageous to overvalue properties, as the fines/penalty incurred are not sufficient to counteract the benefit to overvaluation.

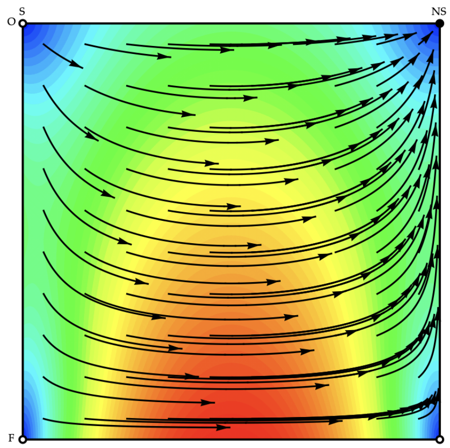


Figure 3

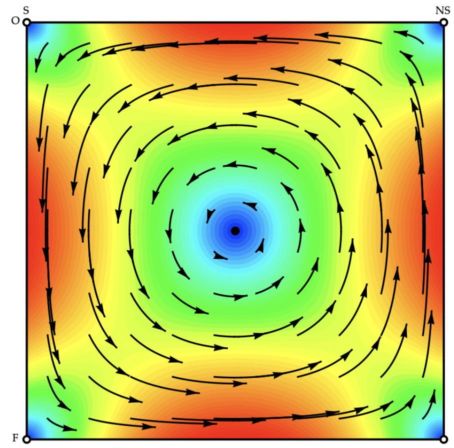


Figure 4

Figure 3 displays the case where $v = 1, c = 3, f = 2$, that is $!(v > f)$ and $f < c$. Thus, the payoff matrix is of the form

$$\begin{pmatrix} & \text{Supervise} & \text{Do Not Supervise} \\ \text{Overvalue} & (-,-) & (+,0) \\ \text{Fair Price} & (0,-) & (0,0) \end{pmatrix}$$

The dominant strategy is chosen by government supervisors, resulting in appraisers adapting strategies, producing a Nash equilibrium at $(1,0)$ (Overvalue) for appraisers and $(0,1)$ (Do Not Supervise) for government supervisors. The evolutionary stable strategy (ESS) matches the Nash equilibrium.

In this scenario, we see a situation in which government supervisors move away from supervision in light of high costs, allowing appraisers to take advantage of the benefit awarded from overvaluation with minimal penalty.

Figure 4 displays the case where $v = 1, c = 1, f = 2$, that is $!(v > f)$ and $!(f < c)$. Thus, the payoff matrix is of the form

$$\begin{pmatrix} & \text{Supervise} & \text{Do Not Supervise} \\ \text{Overvalue} & (-,+) & (+,0) \\ \text{Fair Price} & (0,-) & (0,0) \end{pmatrix}$$

No dominant strategy is chosen, so appraisers and government supervisors adapt strategies as appropriate. There is a mixed Nash equilibrium at $(\frac{1}{2}, \frac{1}{2})$ (Overvalue/Fair Price) for appraisers and $(\frac{1}{2}, \frac{1}{2})$ (Supervise/Do Not Supervise) for government supervisors. There is no evolutionary stable strategy (ESS), as strategies constantly change.

In this scenario, we see a situation in which neither appraisers nor government supervisors have a dominant strategy. Thus, there is a constant cycle between overvaluation vs. fair pricing for appraisers and supervision vs. no supervision for government supervisors.

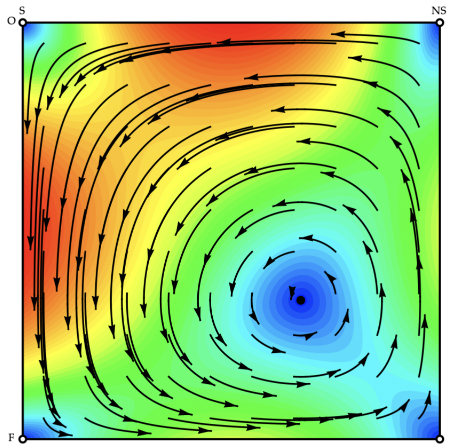


Figure 5

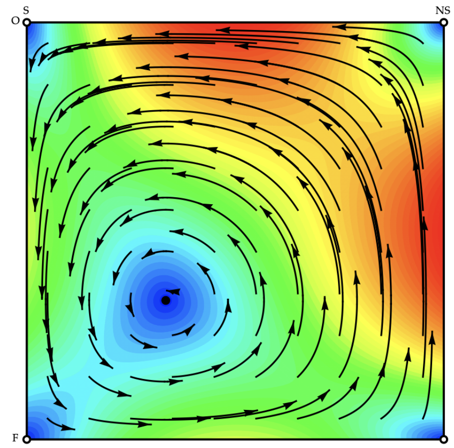


Figure 6

Figure 5 displays the case where $v = 1, c = 1, f = 3$, that is $!(v > f)$ and $!(f < c)$. Thus, the payoff matrix is of the form

$$\begin{pmatrix} & \text{Supervise} & \text{Do Not Supervise} \\ \text{Overvalue} & (-,+) & (+,0) \\ \text{Fair Price} & (0,-) & (0,0) \end{pmatrix}$$

No dominant strategy is chosen, so appraisers and government supervisors adapt strategies as appropriate. There is a mixed Nash equilibrium at $(\frac{1}{3}, \frac{2}{3})$ (Overvalue/Fair Price) for appraisers and $(\frac{1}{3}, \frac{2}{3})$ (Supervise/Do Not Supervise) for government supervisors. There is no evolutionary stable strategy (ESS), as strategies constantly change.

In particular, the phase plot highlights that increasing the fine/penalty faced by appraisers who overvalue has a positive effect overall; the stable point shifts closer to a fair price without government supervision.

Figure 6 displays the case where $v = 2, c = 1, f = 3$, that is $!(v > f)$ and $!(f < c)$. Thus, the payoff matrix is of the form

$$\begin{pmatrix} & \text{Supervise} & \text{Do Not Supervise} \\ \text{Overvalue} & (-,+) & (+,0) \\ \text{Fair Price} & (0,-) & (0,0) \end{pmatrix}$$

No dominant strategy is chosen, so appraisers and government supervisors adapt strategies as appropriate. There is a mixed Nash equilibrium at $(\frac{1}{3}, \frac{2}{3})$ (Overvalue/Fair Price) for appraisers and $(\frac{2}{3}, \frac{1}{3})$ (Supervise/Do Not Supervise) for government supervisors. There is no evolutionary stable strategy (ESS), as strategies constantly change.

In particular, the phase plot highlights that increasing the benefit/reward to overvaluation while simultaneously increasing the fine/penalty faced by appraisers who overvalue has a somewhat positive effect overall; the stable point shifts closer to a fair price, yet requires greater government intervention.

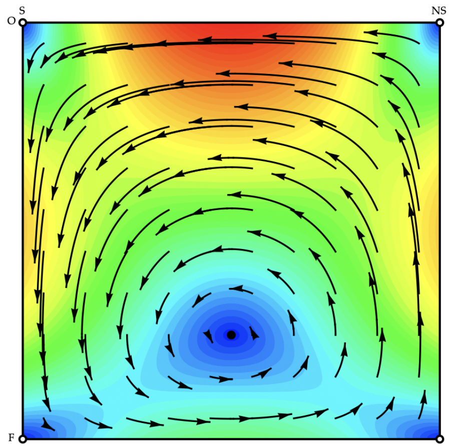


Figure 7

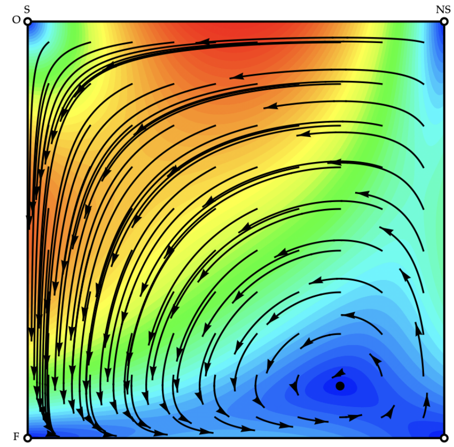


Figure 8

Figure 7 displays the case where $v = 1, c = 0.5, f = 2$, that is $!(v > f)$ and $!(f < c)$. Thus, the payoff matrix is of the form

$$\begin{pmatrix} & \text{Supervise} & \text{Do Not Supervise} \\ \text{Overvalue} & (-,+) & (+,0) \\ \text{Fair Price} & (0,-) & (0,0) \end{pmatrix}$$

No dominant strategy is chosen, so appraisers and government supervisors adapt strategies as appropriate. There is a mixed Nash equilibrium at $(\frac{1}{4}, \frac{3}{4})$ (Overvalue/Fair Price) for appraisers and $(\frac{1}{2}, \frac{1}{2})$ (Supervise/Do Not Supervise) for government supervisors. There is no evolutionary stable strategy (ESS), as strategies constantly change.

In particular, the phase plot highlights that decreasing the cost of supervision by the government has a mostly positive effect overall; the stable point shifts closer to a fair price with the same government intervention.

Figure 8 displays the case where $v = 1, c = 0.5, f = 4$, that is $!(v > f)$ and $!(f < c)$. Thus, the payoff matrix is of the form

$$\begin{pmatrix} & \text{Supervise} & \text{Do Not Supervise} \\ \text{Overvalue} & (-,+) & (+,0) \\ \text{Fair Price} & (0,-) & (0,0) \end{pmatrix}$$

No dominant strategy is chosen, so appraisers and government supervisors adapt strategies as appropriate. There is a mixed Nash equilibrium at $(\frac{1}{8}, \frac{7}{8})$ (Overvalue/Fair Price) for appraisers and $(\frac{1}{4}, \frac{3}{4})$ (Supervise/Do Not Supervise) for government supervisors. There is no evolutionary stable strategy (ESS), as strategies constantly change.

In particular, the phase plot highlights that decreasing the cost of supervision by the government while simultaneously increasing the fine/penalty faced by appraisers who overvalue has an extreme positive effect overall; the stable point shifts closer to a fair price without government intervention.

Model #2 - Cooperators & Defectors (Multiplayer Game)

Overview

In this section, we propose an alternative model, involving cooperators and defectors in a multiplayer game, implementing a novel regulatory policy to promote fair valuation of real estate.

Set Up

As before, the model we use for the appraisal market consists of two player groups: cooperators (denoted by C) and defectors (denoted by D). Cooperators follow a single strategy of fairly pricing a real estate property, while defectors follow a single strategy of overvaluing a real estate property.

The regulatory policy we propose is as follows: the government supervises with probability p and penalizes defectors if caught. The revenue brought in from the fines covers the cost of government supervision before being equally re-distributed to cooperators.

In this way, cooperators are incentivized to fairly appraise property values at the appropriate market value, as they potentially receive a benefit from others choosing not to do so. Thus, this policy shows promise for discouraging overvaluing assets of real estate.

Variables

The variables associate with such a model are as follows. The defectors may gain rewards from an overstated value (denoted by v, but must pay a fine/penalty (denoted by f) if they are caught doing so. There is a cost to government supervision (c), which covers both inspection and finding evidence of overvaluation. When overvaluation is caught, the penalty paid by the appraisers acts as government revenue, up to the cost of supervision, before being redistributed to cooperators equally.

As we have set up the model, $\{v, c, f\} \geq 0$, as there is not a negative value/reward to overvaluation, nor a negative cost of government supervision, nor a negative fine/penalty to over appraising if caught.

In the case of this multiplayer game, the cost of government supervision is not on a case-by-case basis, rather representing the total cost required to supervise all appraisers. This indicates that government supervision is not selective; the government chooses to unanimously supervise all appraisers for a given period of time. Perhaps by doing so, the government may utilize economies of scale to maintain a low cost of supervision.

Payoffs

To determine the replicator/selection dynamics for this model, we first must calculate the payoffs for cooperators and defectors. Let us assume we have a population of x cooperators and y defectors, where $\{x, y\} \in [0, 1]$, so $x + y = 1$. This implies that x and y are actually percentages of the overall population.

As defectors receive a benefit from overvaluation (v), yet must pay a fine/penalty (f) with probability

p, their payoff is represented as follows:

$$\pi_D = v - pf$$

As the penalty paid by y appraisers acts as government revenue, up to the cost of supervision, before being redistributed to cooperators equally, the payoff of cooperators is represented as follows:

$$\pi_C = p\left(fy - c\right)/x$$

Given that $x + y = 1$, this may be expressed as

$$\pi_C = p\left(f\left(1 - x\right) - c\right)/x$$

Replicator Dynamics

Now, we utilize the general replicator equation, which is given as follows:

$$\dot{x}_i = x_i\left[f_i(x) - \phi(x)\right] \qquad \phi(x) = \sum_{j=1}^{n} x_j f_j(x)$$

For a population structure of two types, with cooperators and defectors, this may alternatively be expressed as

$$\dot{x} = x\left(1 - x\right)\left(\pi_C - \pi_D\right).$$

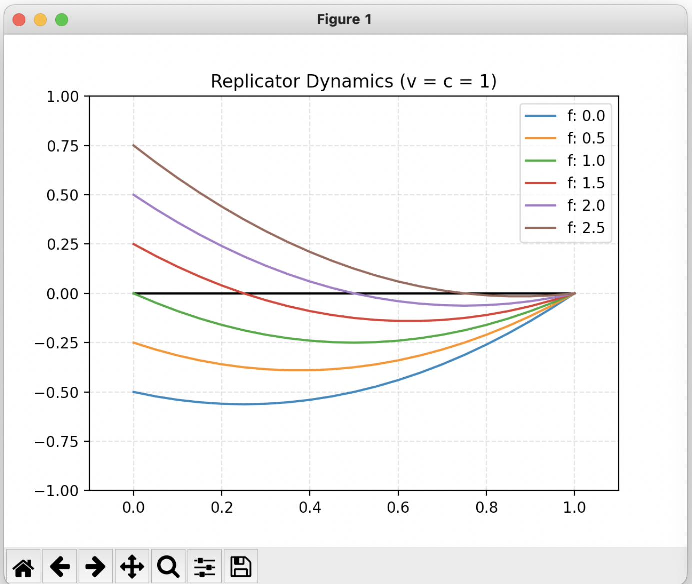
Regardless, we may use the first expression to calculate the selection dynamics for cooperators x for our analysis. This gives

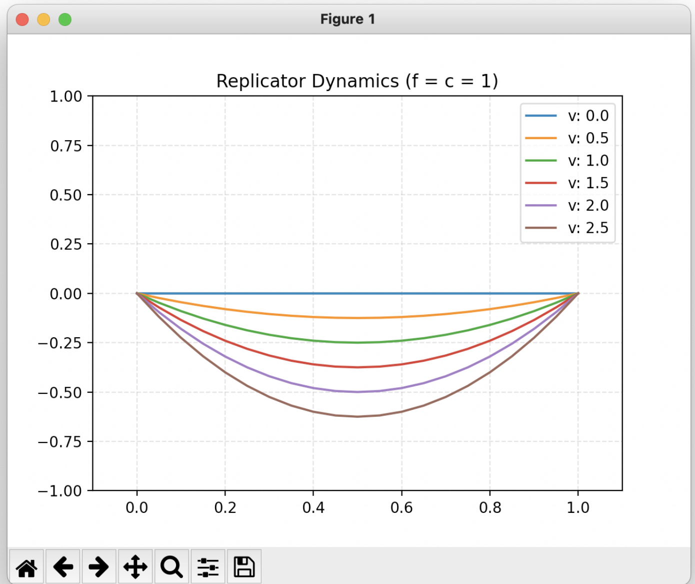
$$\dot{x} = x\left(1 - x\right)\left(\frac{p\left(f - c\right)}{x} - v\right).$$

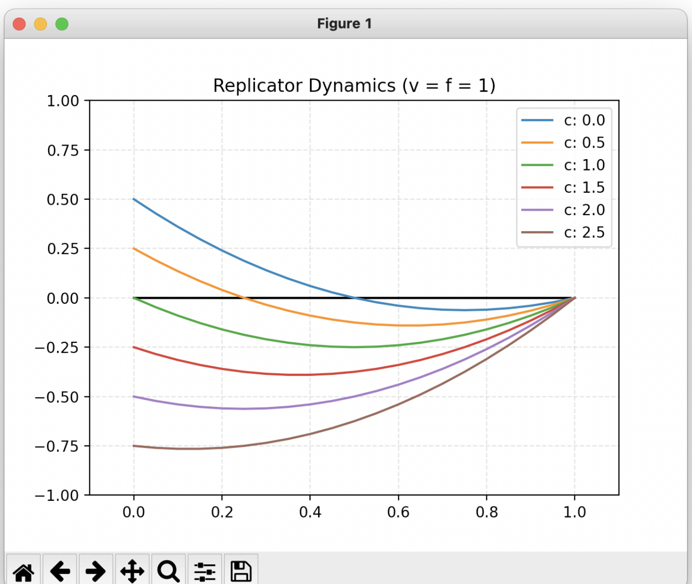
(See Appendix for calculation and simplification.)

Graphical Representation

Now, we may graphically represent such selection dynamics, considering changing values of v, c, and f. The following plots were created using a Python script, included in the Appendix.







From the selection dynamics curves, it is evident that decreasing the cost of supervision by the government shifts the equilibrium point farther to the right, indicating there will be a higher percentage of cooperators. This may serve as an effective means to correct overvaluation in the real estate market.

Further, increasing the benefit/reward to overvaluation does not shift the equilibrium point at all, except in the case where the benefit to defectors is zero. The speed of convergence to a stable resting point is determined by the position of a point away from the x-axis. For this case, we find that increasing the benefit to defectors further encourages appraisers to overvalue a property. This is not the aim of the policy implemented, so if this situation is presented, appropriate countermeasures must be put in place to decrease the number of defectors in a population.

Finally, increasing the fine/penalty faced by appraisers who overvalue shifts the equilibrium point farther to the right, indicating there will be a higher percentage of cooperators. The shift observed is noticeably greater than than for decreasing the cost of supervision by the government, indicating this strategy should be the aim of future policy in the real estate market.

Conclusion

Significance

In this paper, evolutionary game theory models involving real estate appraisers and government supervisors were used to analyze overvaluation in the real estate market. By presenting a two-player asymmetric bi-matrix game with time-evolving environments, we examined the influence of policy on appraisers' decisions to overvalue or fairly price a real estate property. Further, we proposed a secondary model – a multiplayer game involving cooperators and defectors – to analyze the impact of a novel regulatory policy to promote fair valuation of real estate.

The results demonstrate that high rewards to overvaluation, along with light punishment and costly government supervision are directly responsible for the continuance of overvaluation in the real estate market as a result of appraisal fraud.

Further, the results provide indication as to measures that can be taken to prevent overvaluation in the real estate market and allow for more stable development. The most effective method to prevent overvaluation is to increase the penalty for appraisal fraud. By increasing the penalty, appraisers are highly dissuaded from committing wrongdoing, as they would typically tend to. This presumes that government supervision is present, though it does not necessarily have to be implemented without exception. That is, government supervisors may choose to randomly supervise appraisers, which would yield similar positive results in ensuring fair values for appraisals.

Moreover, a greater penalty for appraisal fraud would yield economic benefits for the government (along with cooperators in the secondary model), which indicates why we see the greatest impact on real estate valuation when modifying the penalty variable.

Lowering the cost of government supervision is shown to be less effective, and may require greater effort and resources. This being said, the government could utilize economies of scale to lower the cost of monitoring/supervision to a reasonable value.

Next Steps

For further investigation, there are several directions to follow. Empirical research should be performed to verify the trends established in this paper. Further, a paper devoted to studying self-discipline in the real estate appraisal industry may reveal an underlying understanding as to why not all appraisals are overvalued.

Beyond that, the next natural question to follow is how much the government should charge as a fine/penalty for overvaluation, which would require significant research. Ideally, a non-linear function would take into account the percentage a given appraisal is overvalued, outputting a dollar amount corresponding to the appropriate penalty/fine. To this end, greater overvaluation would be punished more harshly, encouraging appraisers to carefully review properties and give the fair market value (FMV) when performing an appraisal. Current policy and the implications of imposing such a penalty would require further evaluation.

Final Remarks

The real estate market plays a significant role in the functioning of the overall economy (nationally and internationally). By evaluating overvaluation and government intervention from an evolutionary game theory standpoint, we may identify ways to effectively ensure market stability with direct consequences to the appraisal industry, simultaneously indicating potential future directions for the transformation of the real estate market.

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Appendix

Model #2 - Replicator/Selection Dynamics Calculation

$$\begin{aligned}
\dot{x} &= x\left[p\left(f\left(1-x\right)-c\right)/x - x\left(p\left(f\left(1-x\right)-c\right)/x\right)-\left(1-x\right)\left(v-pf\right)\right] \\
&= p\left(f-fx-c\right)-x\left(p\left(f-fx-c\right)\right)-x\left(1-x\right)\left(v-pf\right) \\
&= \left(1-x\right)\left(p\left(f-fx-c\right)\right)-x\left(1-x\right)\left(v-pf\right) \\
&= \left(1-x\right)\left(pf-pfx-pc-xv+pfx\right) \\
&= \left(1-x\right)\left(pf-pc-xv\right) \\
&= x\left(1-x\right)\left(\frac{p\left(f-c\right)}{x}-v\right)
\end{aligned}$$

Model # 2 - Replicator/Selection Dynamics Python Code

```
import matplotlib.pyplot as plt
import numpy as np

np.seterr(divide = 'ignore', invalid = 'ignore')

plt.axis([-0.1, 1.1, -1, 1])

x_axis = np.arange(0, 1, 0.01)
y_axis = x_axis * 0
plt.plot(x_axis, y_axis, color='black')

v_0 = 1
f_0 = 1
c_0 = 1

v_arr = np.arange(0, 3, 0.5)
f_arr = np.arange(0, 3, 0.5)
c_arr = np.arange(0, 3, 0.5)
p = 0.5

x = np.arange(-0.00001, 1, 0.05)

# for v in v_arr:
#     y = x * (1 - x) * (((p * (f_0 * (1 - x) - c_0)) / x) - (v - p * f_0))
#     plt.plot(x, y, label = 'v: ' + str(v))

# for f in f_arr:
#     y = x * (1 - x) * (((p * (f * (1 - x) - c_0)) / x) - (v_0 - p * f))
#     plt.plot(x, y, label = 'f: ' + str(f))

for c in c_arr:
```

```
    y = x * (1 - x) * (((p * (f_0 * (1 - x) - c)) / x) - (v_0 - p * f_0))
    plt.plot(x, y, label = 'c: ' + str(c))


plt.title('Replicator Dynamics (v = f = 1)')
plt.grid(alpha = 0.4, linestyle = '--')
plt.legend()
plt.show()
```