UK Maths Trust

SENIOR MATHEMATICAL CHALLENGE
Tuesday 1 October 2024
Organised by the United Kingdom Mathematics Trust

proudly sponsored by **[XTX]**
MARKETS

INSTRUCTIONS

1. Do not open the paper until the invigilator tells you to do so.

2. Time allowed: **90 minutes**. No answers, or personal details, may be entered after the allowed time is over.

3. The use of blank paper for rough working is allowed; **squared paper, calculators and measuring instruments are forbidden**.

4. **Use a B or an HB non-propelling pencil**. Mark at most one of the options, A, B, C, D, or E, on the Answer Sheet for each question. Do not mark more than one option.

5. **Do not expect to finish the whole paper in the time allowed**. The questions in this paper have been arranged in approximate order of difficulty with the harder questions towards the end. You are not expected to complete all the questions during the time. You should bear this in mind when deciding which questions to tackle.

6. **Scoring rules**: All candidates start with 25 marks; 0 marks are awarded for each question left unanswered; 4 marks are awarded for each correct answer; 1 mark is deducted for each incorrect answer (to discourage guessing).

7. **Your Answer Sheet will be read by a machine**. Do not write or doodle on the sheet except to mark your chosen options. The machine will read all black pencil markings even if they are in the wrong places. If you mark the sheet in the wrong place, doodle, or leave bits of eraser stuck to the page, the machine will interpret the mark in its own way, or reject the answer sheet.

8. **The questions on this paper are designed to challenge you to think, not to guess**. You will gain more marks, and more satisfaction, by doing one question carefully than by guessing lots of answers. This paper is about solving interesting problems, not about lucky guessing.

9. To accommodate candidates sitting at other times, please do not discuss the paper on the internet until 08:00 BST on Thursday 3 October, when the solutions video will be released at ukmt.org.uk/competition-papers. Candidates in time zones more than 5 hours ahead of GMT must sit the paper on Wednesday 2 October (as defined locally).

Enquiries about the Senior Mathematical Challenge should be sent to:

challenges@ukmt.org.uk   *www.ukmt.org.uk*

supported by **6verleaf**

1. What is two-fifths of the recurring decimal $0.\dot{2}\dot{5}$?

A $0.\dot{1}$ B $0.0\dot{1}$ C $0.\dot{0}\dot{1}$ D 0.10 E $0.\dot{1}\dot{0}$

2. A *twip* is a very short unit of length, derived from imperial units, and is equal to approximately 0.000018 metres. A *league* is a long unit of length which is equal to approximately 4800 metres.

Roughly how many twips are there in a league?

A 270 000 000   B 27 000 000   C 2 700 000   D 270 000   E 27 000

3. Two standard dice are placed on a table, with one on top of the other, so that only nine of the faces of the dice may be seen. The touching faces have the same number on them. The sum of the numbers on the visible faces is 33.

What is the number on the touching faces?

A 1   B 2   C 3   D 4   E 6

4. The sizes of the three angles in a triangle, in degrees, are x, $7x$ and x^2.

What is the size of the largest angle?

A $10°$ B $18°$ C $100°$ D $120°$ E $121°$

5. When $4^5 \times 5^4$ is correctly calculated, how many digits are there in the answer?

A 4   B 6   C 10   D 16   E 20

6. One face of a solid polyhedron is an octagon.

What is the smallest possible number of edges the solid could have?

A 9   B 10   C 12   D 16   E 24

7. Which is the largest prime factor of $3^8 - 1$?

A 41   B 37   C 31   D 29   E 23

8. In the following expressions, x is non-zero. When one of these expressions is removed, the mean of the remaining four is $11x$.

Which expression is removed?

A $4x$ B $8x$ C $12x$ D $16x$ E $20x$

9. A palindromic number is one where the digits read the same forwards as backwards, such as 123 321.

What is the hundreds digit of the largest six-digit palindromic number that is divisible by 18?

A 9   B 7   C 5   D 3   E 1

10. The prime factorization of 2024 is $2^3 \times 11 \times 23$.

How many two-digit numbers are factors of 2024?

A 2   B 4   C 6   D 7   E 8

11. Which one of the following expressions is a square number for each positive integer n?

A $n + 1$ B $n(n + 1) + 1$ C $n(n + 1)(n + 2) + 1$
D $n(n + 1)(n + 2)(n + 3) + 1$ E $n(n + 1)(n + 2)(n + 3)(n + 4) + 1$

12. p, q, r and s are two-digit primes which between them use all the non-zero digits except 5.
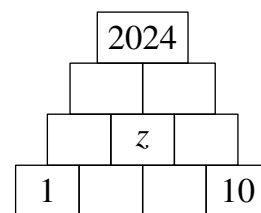
What is the value of $p + q + r + s$?

A 220   B 210   C 200   D 190

E more information needed

13. The diagram shows a partially completed number pyramid. When correctly completed, the number on any brick above the bottom row should be the sum of the two numbers on the two bricks on which it rests.

What number should appear on the brick marked 'z'?

A 176   B 617   C 671   D 716   E 761

14. P, Q, R, S and T are the digits 1, 2, 3, 4 and 5 in some order. 'PRT' and 'QRS' are both three-digit primes.

Which digit is R?
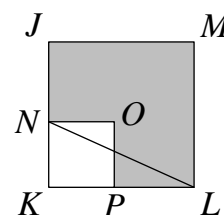
A 1   B 2   C 3   D 4   E 5

15. The diagram shows two squares, $JKLM$ and $NKPO$.
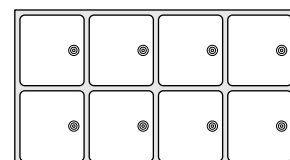The length of NL is 10 cm. The shaded region has area 62 cm^2.

What is the length of KN in cm?

A 3   B $\sqrt{18}$ C $\sqrt{19}$ D $\sqrt{22}$ E 5

16. A set of cupboards containing eight identical blue doors is arranged in a 2 by 4 grid as shown. A fussy decorator wishes to paint three of the doors red such that at least one door in each row is painted red and at least two of the four corners are painted red.

How many ways are there to do this?

A 12   B 24   C 36   D 40   E 56

17. A bag contains four balls each of which is coloured either red or white. If one ball is drawn at random from the bag but not replaced and then a second ball is drawn at random, the probability that both balls are red is $\frac{1}{2}$.
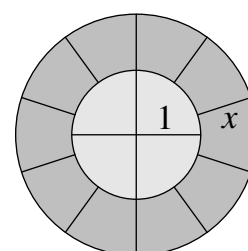
What is the probability that both balls are white?

A $\frac{1}{2}$ B $\frac{1}{3}$ C $\frac{1}{4}$ D $\frac{1}{6}$ E 0

18. The diagram shows two concentric circles divided by radial lines into 14 pieces of equal area. The radius of the smaller circle is 1.

What is the length, x, of an outer radial line?

A $\sqrt{14} - 1$ B $\sqrt{14} - 2$ C $\dfrac{\sqrt{14}}{2} - 1$ D $\dfrac{\sqrt{14}}{2} - 2$ E $\dfrac{\sqrt{14} - 1}{2}$

19. Five friends are dealt two cards each from a set of twelve cards. The cards are numbered 1 to 12 inclusive. In turn, the friends declare the sum of the values of their two cards. Paolo scores 4, Quinn scores 11, Romy scores 16, Stephen scores 19 and Thomas scores 20.

Which of the following statements is true?

    A Paolo has card 2   B Quinn has card 3   C Romy has card 5   D Stephen has card 7
    E Thomas has card 11

20. Let x and y be positive integers such that $\dfrac{1}{x} + \dfrac{1}{y} = \dfrac{1}{20}$. What is the maximum possible value of y?

    A 40   B 60   C 240   D 420   E 480

21. The crossnumber is to be filled with eight of the digits 1 to 9, which are each used once.

Across
1. A multiple of 9
3. A square

Down
1. A multiple of 11
2. A multiple of 13 and of 19
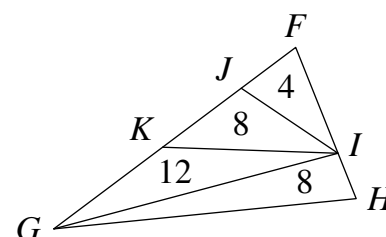

Which digit is not used?

    A 9   B 8   C 5   D 3   E 2

22. As shown in the diagram, triangle FGH is divided into four smaller triangles which have areas 4, 8, 12 and 8 respectively.
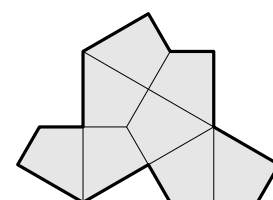
What is the area of triangle IKH?

    A 4   B 5   C 6   D 7   E 8

23. The plane can be tiled using the 'hat tile' shown here. This tile can be subdivided into eight congruent kites. The area of the hat tile is $8\sqrt{3}$.

What is the perimeter of the hat tile?

    A $8 + 12\sqrt{3}$ B $16 + 6\sqrt{3}$ C $8 + 8\sqrt{3}$ D $6 + 8\sqrt{3}$
    E $8 + 6\sqrt{3}$

24. A function f satisfies the equation $f(x) + f\left(\dfrac{1}{1-x}\right) = 24x$ for all real values of x except $x = 0$ and $x = 1$.

What is the value of $f(3)$?

    A 40   B 42   C 45   D 48   E 50

25. Three semicircles, each of area 24, overlap as shown in the diagram. The centres of the arcs are X, Y and Z and $\angle ZXY = 30°$.

What is the total area of the shaded regions?

    A 12   B $6\sqrt{3}$ C 15   D 18   E $8\sqrt{3}$