2.3 L1: Collected definition

Syntax

Booleans $b \in \mathbb{B} = \{\textbf{true}, \textbf{false}\}$
Integers $n \in \mathbb{Z} = \{..., -1, 0, 1, ...\}$
Locations $\ell \in \mathbb{L} = \{l, l_0, l_1, l_2, ...\}$

Operations $op ::= + \mid \geq$

Expressions

$$\begin{array}{rcl} e & ::= & n \mid b \mid e_1 \;\; op \;\; e_2 \mid \textbf{if} \;\; e_1 \;\; \textbf{then} \;\; e_2 \;\; \textbf{else} \;\; e_3 \mid \\ & & \ell := e \mid !\ell \mid \\ & & \textbf{skip} \mid e_1; e_2 \mid \\ & & \textbf{while} \;\; e_1 \;\; \textbf{do} \;\; e_2 \end{array}$$

Operational semantics

Note that for each construct there are some *computation* rules, doing 'real work', and some *context* (or *congruence*) rules, allowing subcomputations and specifying their order.

Say *stores* s are finite partial functions from $\mathbb{L}$ to $\mathbb{Z}$. Say *values* v are expressions from the grammar $v ::= b \mid n \mid \textbf{skip}$.

$$\text{(op +)} \quad \langle n_1 + n_2, s\rangle \longrightarrow \langle n, s\rangle \quad \text{if } n = n_1 + n_2$$

$$\text{(op } \geq\text{)} \quad \langle n_1 \geq n_2, s\rangle \longrightarrow \langle b, s\rangle \quad \text{if } b = (n_1 \geq n_2)$$

$$\text{(op1)} \quad \frac{\langle e_1, s\rangle \longrightarrow \langle e_1', s'\rangle}{\langle e_1 \;\; op \;\; e_2, s\rangle \longrightarrow \langle e_1' \;\; op \;\; e_2, s'\rangle}$$

$$\text{(op2)} \quad \frac{\langle e_2, s\rangle \longrightarrow \langle e_2', s'\rangle}{\langle v \;\; op \;\; e_2, s\rangle \longrightarrow \langle v \;\; op \;\; e_2', s'\rangle}$$

$$\text{(deref)} \quad \langle !\ell, s\rangle \longrightarrow \langle n, s\rangle \quad \text{if } \ell \in \mathrm{dom}(s) \text{ and } s(\ell) = n$$

$$\text{(assign1)} \quad \langle \ell := n, s\rangle \longrightarrow \langle \textbf{skip}, s + \{\ell \mapsto n\}\rangle \quad \text{if } \ell \in \mathrm{dom}(s)$$

$$\text{(assign2)} \quad \frac{\langle e, s\rangle \longrightarrow \langle e', s'\rangle}{\langle \ell := e, s\rangle \longrightarrow \langle \ell := e', s'\rangle}$$

$$\text{(seq1)} \quad \langle \textbf{skip}; e_2, s\rangle \longrightarrow \langle e_2, s\rangle$$

$$\text{(seq2)} \quad \frac{\langle e_1, s\rangle \longrightarrow \langle e_1', s'\rangle}{\langle e_1; e_2, s\rangle \longrightarrow \langle e_1'; e_2, s'\rangle}$$

$$\text{(if1)} \quad \langle \textbf{if} \;\; \textbf{true} \;\; \textbf{then} \;\; e_2 \;\; \textbf{else} \;\; e_3, s\rangle \longrightarrow \langle e_2, s\rangle$$

$$\text{(if2)} \quad \langle \textbf{if} \;\; \textbf{false} \;\; \textbf{then} \;\; e_2 \;\; \textbf{else} \;\; e_3, s\rangle \longrightarrow \langle e_3, s\rangle$$

$$\text{(if3)} \quad \frac{\langle e_1, s\rangle \longrightarrow \langle e_1', s'\rangle}{\langle \textbf{if} \;\; e_1 \;\; \textbf{then} \;\; e_2 \;\; \textbf{else} \;\; e_3, s\rangle \longrightarrow \langle \textbf{if} \;\; e_1' \;\; \textbf{then} \;\; e_2 \;\; \textbf{else} \;\; e_3, s'\rangle}$$

$$\text{(while)} \quad \langle \textbf{while} \;\; e_1 \;\; \textbf{do} \;\; e_2, s\rangle \longrightarrow \langle \textbf{if} \;\; e_1 \;\; \textbf{then} \;\; (e_2; \textbf{while} \;\; e_1 \;\; \textbf{do} \;\; e_2) \;\; \textbf{else} \;\; \textbf{skip}, s\rangle$$