Math 369: Lab 10

Prof. Schaeffer, TA: Linan Zhang

Background: For Lab 10, you will write a program which solves a system of ordinary differential equations using the forward Euler method. Systems of ODEs are one of the most popular ways to model a variety of phenomena in sciences and for predictive models in technology. Examples of such models include: chemical reactions, population dynamics, particle dynamics, satellite orbital dynamics, and recently deep networks and AI.

In the upcoming labs, we will program various numerical solvers, building-off of the previous week's lab. This week we will start with the forward Euler method applied to population dynamics and the Lorenz system (3D). The Lorenz system is a simplified model for weather which emits a chaotic attractor; see figure below.

Figure 1: Viewing a Lorenz attractor from different angles.

Problem Statement: You will be asked to code two examples: one scalar ODE and one 3D system of ODEs. For the first example, consider the logistic growth model:

$$y'(t) = y(t) - y^2(t), \tag{1}$$

where $y(t)$ is the population at time t. Solve Equation (1) using the forward Euler method, with $t \in [0, 10]$ and $h = 0.01$. Submit your code (`MyLogisticModel.m`) and two plots, one with $y(0) = 0.01$ and one with $y(0) = 0.1$. (Note that $y(t) \in [0, 1]$.)

For the second problem, consider the Lorenz system:

$$\begin{cases} \dot{x}_1 &= \sigma(x_2 - x_1), \\ \dot{x}_2 &= x_1(\rho - x_3) - x_2, \\ \dot{x}_3 &= x_1 x_2 - \beta x_3. \end{cases} \tag{2}$$

where $(x_1(t), x_2(t), x_3(t))$ is the location of a particle in space at time t. Solve Equation (2) using the forward Euler method, with $t \in [0, 100]$ and $h = 0.01$. Submit your code (`MyLorenzModel.m`) and the plot (set the view angle to be $[5, -5]$). Use the vectorized forward Euler to have no more than one loop in your code.