Project Template

Mariana Martínez Aguilar
Professor: Laura Grigori

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Abstract

This is optional, an abstract is just a small summary of the important things in a project. Makes people interested in what they're about to read.

1 Introduction

To learn how to write mathematical texts you can refer to Nick Higham's book "Handbook of Writing for the Mathematical Sciences".

An example of a centered but unnumbered equation

$$(1-\varepsilon)\|Ax\|_2 \leq \|\Omega Ax\|_2 \leq (1+\varepsilon)\|Ax\|_2.$$

An example of a lemma

Lemma 1.1 (Johnson-Lindenstrauss)**.** *Given $\varepsilon \in (0,1)$, a set X of n points in $\mathbb{R}^m$, i.e. $X \in \mathbb{R}^{m\times n}$, and an integer $l > 8\log(n)/\varepsilon^2$, there is a linear map $f: \mathbb{R}^m \to \mathbb{R}^l$ such that*

$$(1-\varepsilon)\|x-y\|_2 \leq \|f(x)-f(y)\|_2 \leq (1+\varepsilon)\|x-y\|_2$$

for all $x, y \in X$.

1.1 Subsection 1

To cite from the bibliography you can use this command [**?**]. If you need to cite more than one paper you can put them together [**?**, **?**, **?**, **?**].

An example of a definition.

Definition 1.1 (Leverage scores homogenizers (LSH))**.** *Let $V \in \mathbb{R}^{m\times n}$ be an orthogonal matrix such that $m \geq n$. A matrix $\hat{H} \in \mathbb{R}^{r\times m}$ with $m \geq r \geq n$ is called a leverage scores homogenizer (LSH) if the preconditioned matrix $U = \hat{H}V$ satisfies the following properties*

1. $\mathbb{E}\left[U^\top U\right] = I_{n\times n}$
2. $\mathbb{E}\left[U\right] = 0$, $\mathbb{E}\left[U_{ij}^2\right] = \frac{1}{r}$
3. *Let $L_U(\delta)$ be a deterministic function that only depends on the size of U and on δ, then $\mathbb{P}\left(\|e_j^\top U\|_2 \leq L_U(\delta)\right) \geq 1-\delta$ for all $j = 1, ..., r$ i.e. $\hat{H}$ homogenizes the leverage scores of V w.h.p.*
4. *We know $\varphi(r,m)$ such that $\|Ux\|_2 \leq \varphi(r,m)\|x\|_2$ (deterministic bound)*

Example of a theorem.

Theorem 1.1 (Some theorem)**.** *Epsilon subspace embeddings are useful.*

To prove something.

Proof. Exercise left to the reader. ∎

To reference a theorem or lemma use theorem 1.1. An example of a corollary.

Corollary 1.1 (A corollary)**.** *Math things.*

1.2 Another subsection

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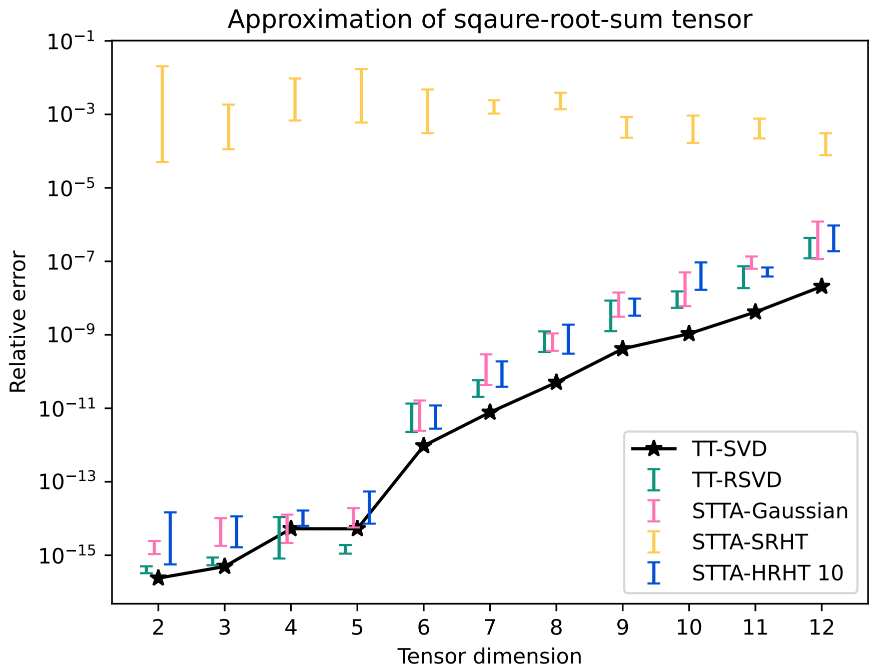
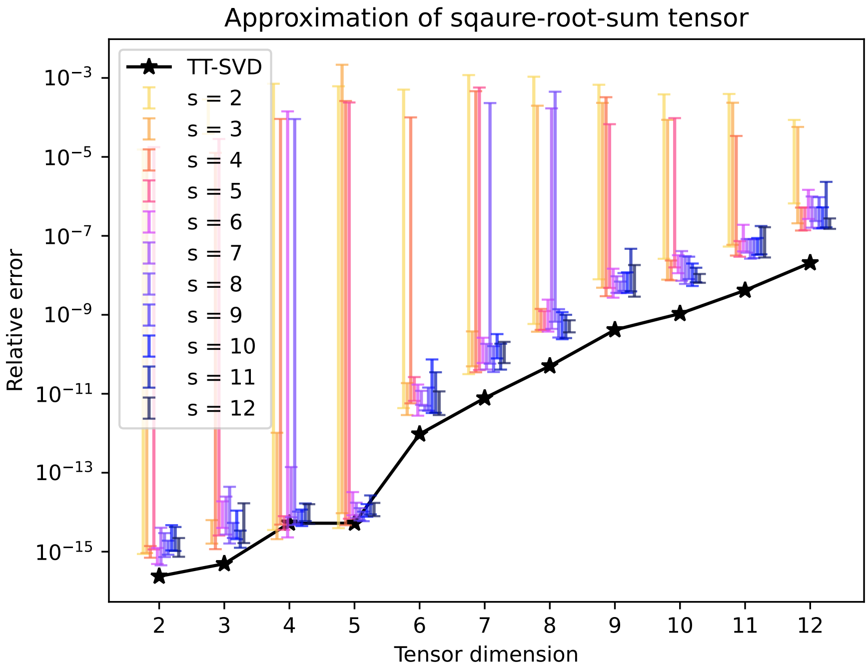
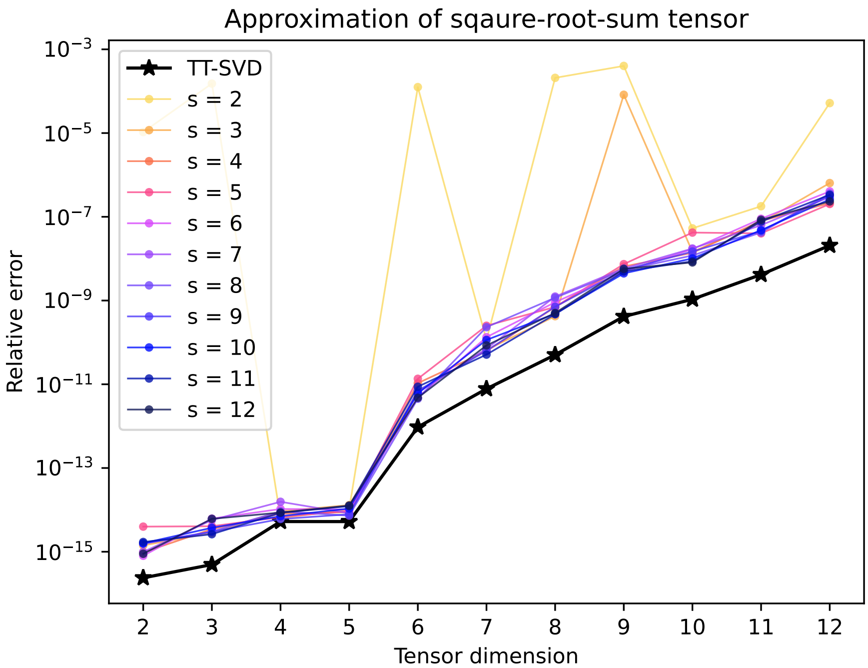


Figure 1: Different methods applied to tensor train rounding

This option also works with subfigures.



(a) Sample standard deviation for different s



(b) Median errors for different. s

Figure 2: Comparison different sizes of s

2 Pseudocode

An example of pseudocode in a very simple format. The packages used are in the .sty file.

Algorithm 1 RQRCP

Input: $A \in \mathbb{R}^{m\times n}$, $\Omega \in \mathbb{R}^{l\times m}$, k $l > k$
Output: I_{02}, indices of the columns of A from which to build the low rank approximation
  Compute $B = \Omega A$, $B \in \mathbb{R}^{l\times n}$.
  Compute k steps of QRCP on B and select k columns.
  Return k selected columns, with indices saved in I_{02}.

3 Code

You should upload your code as a separate file but this is an option just in case there is a moment where you need to input code as text.

```
# Exercise I: Matrices in Python
import numpy as np
from time import perf_counter as tic
# We try different ways of assigning the matrix (these are some of the options)

size = (2,4)

# Manually entering the elements
t0 = tic()
M0 = np.empty(size)
c = 1
for i in range(size[0]):
    for j in range(size[1]):
        M0[i, j] = c
        c += 1
t1 = tic() - t0
print("Time taken entering manually each element: " + f"{t1:.2e}" + " s.")

# Filling the matrix row wise
t0 = tic()
M1 = np.empty(size)
for i in range(size[0]):
    M1[i, :] = np.arange(i*size[1] + 1, (i+1)*size[1] + 1)
t1 = tic() - t0
print("Time taken filling the matrix row wise: " + f"{t1:.2e}" + " s.")

# Filling the matrix column wise
t0 = tic()
M2 = np.empty(size)
for j in range(size[1]):
    M2[:, j] = np.arange( j+1, (size[0]-1)*size[1] + j + 2, size[1] )
t1 = tic() - t0
print("Time taken filling the matrix row wise: " + f"{t1:.2e}" + " s.")


# Using np arrange and np resize
t0 = tic()
M1 = np.resize(np.arange(1, 9), (4,2))
```

```
t1 = tic() - t0
print("Time taken using np.arange and np.resize: " + f"{t1:.2e}" + " s.")

# Using linspace
t0 = tic()
M2 = np.linspace( (1, 5), (4, 8), 4 ).T
t1 = tic() - t0
print("Time taken using np.linspace: " + f"{t1:.2e}" + " s.")
```

A An example of an appendix

Things written in the appendix are considered to be optional for the reader. Your text should be self contained and the appendix should only include extra information.

B An example of another appendix