

P1

a.  $f(n) = 12n - 5$ ,  $g(n) = 1235813n + 2017$ .

For  $O(g(n))$ , find  $c$ ,  $n_0$ ,

make  $f(n) \leq cg(n)$ ,

$12n - 5 \leq 1235813n + 2017$ ,

$(12 - 1235813c)n \leq 2017c + 5$ ,

$n \leq (2017c + 5) / (12 - 1235813c)$

hence, contradiction as  $n$  cannot hold all  $n > n_0$ ,  $f$  is not order of  $g(n)$ .

for  $\omega(g(n))$ , find  $c$ ,  $n_0$ ,

make  $f(n) \geq cg(n)$ ,

same with above,

$n \geq (2017c + 5) / (12 - 1235813c)$ ,

hence, exist  $n_0$  makes  $f(n)$  is  $\omega$  of  $g(n)$ .

b.  $f(n) = n \log n$ ,  $g(n) = 0.00000001n$ .

assume the base is  $a$ ,

For  $O(g(n))$ , find  $c$ ,  $n_0$ ,

$n \log n \leq 0.00000001nc$ ,

$\log n \leq 0.00000001c$ ,

$n \leq a^{(0.00000001c)}$ ,

hence, contradiction as  $n$  cannot hold all  $n > n_0$ ,  $f$  is not order of  $g(n)$ .

for  $\omega(g(n))$ , find  $c$ ,  $n_0$ ,

same with above,

$n \geq a^{(0.00000001c)}$ ,

hence, exist  $n_0$  makes  $f(n)$  is  $\omega$  of  $g(n)$ .

c.  $f(n) = n^{2/3}$ ,  $g(n) = 7n^{3/4} + n^{1/10}$ .

For  $O(g(n))$ , find  $c$ ,  $n_0$ ,

$n^{1/2} - cn^{2/15} \leq 7c$ ,

for  $n^{1/2} - cn^{2/15}$  part, assume  $c = 1$ ,

$n^{1/2} - n^{2/15} \leq 7$ ,

The derivation of the left side is  $1/2 * n^{-1/2} - 2/5 * n^{-13/15}$ , Is  $15 / (30 * n^{15/30}) - 4 / (30 * n^{26/30})$ ,

It's bigger than 0 when  $n$  is sufficiently large, which means value of the left side will increase,

Let's set the  $n = 10^{10}$ , the result is larger than 7,

hence, when  $n$  is big, there has a condition that does not suit for the function, contradiction as  $n$  cannot hold all  $n > n_0$ ,  $f$  is not order of  $g(n)$ .

for  $\omega(g(n))$ , find  $c, n_0$ ,  
 same with above,  
 $n^{(1/2)} - cn^{(2/15)} \geq 7c$ ,  
 the left side is increasing and there exist  $n$ 's that let it larger than 7,  
 hence, exist  $n_0$  makes  $f(n)$  is  $\omega$  of  $g(n)$ .

d.  $f(n) = n^{1.0001}, g(n) = n \log n$ .

For  $O(g(n))$ , find  $c, n_0$ ,

$1/c \leq n^{(-0.0001)} \log n$ , noted as (1), assume the base is  $a$ ,

Assume  $c = 10,000, a = 10$ , when  $n = 100,000,000$ , the function works,

$1/10000 \leq 1/10,000 * 8$

For the right side of (1), the derivation for it is  $n^{(-1/10000)} * 1/(n * \ln 10) - 1/10000 * n^{(-10001/10000)} * \log n$ , both of them are larger than 0 when  $n$  is large enough, I don't know where the result of the derivation might below the 0, which means the value of (1) may decrease till smaller than  $1/c$ , so complicated by hands.

Hence, I don't know.

e.  $f(n) = n6^n, g(n) = (3n)^2$ .

For  $O(g(n))$ , find  $c, n_0$ ,

$n * (2/3)^n \leq c$ ,

The derivation of the left side is  $n * (2/3)^n * \ln(2/3) + (2/3)^n$ , larger than 0 when  $n$  is big enough. Hence,  $n * (2/3)^n$  will larger than  $c$  somehow.

Hence, contradiction as  $n$  cannot hold all  $n > n_0$ ,  $f$  is not order of  $g(n)$ .

Reversely, for  $\omega(g(n))$ ,

exist  $n_0$  makes  $f(n)$  is  $\omega$  of  $g(n)$ .

P2

$\log(n!) = \theta(n \log n)$ , base: 2

$f(n) = \log(n!), g(n) = n \log n$ ,

For  $O(g(n))$ , find  $c, n_0$ ,

$\log(n!) = \log n + \log(n-1) + \dots + \log(1) \leq \log n + \dots + \log n = n \log n$ ,

Hence,  $f(n) \leq cg(n)$ ,  $f$  is order of  $g(n)$ .

for  $\omega(g(n))$ , find  $c, n_0$ ,  
 $\log(n!) = \log n + \log(n-1) + \dots + \log(1) \geq n/8 * \log(n/8) \geq c \log n$ ,  
 Then we need to find at least a “c”,

$n \geq 16$ ,  
 $\log n \geq 4$ ,  
 $1/16 * n \log n \geq 1/4 * n$ ,  
 $1/8 * n \log n - 1/4 * n \geq 1/16 * n \log n$ , all positive,  
 Hence, when  $c = 1/8$ , and  $n > n_0 = 16$ ,  $\log(n!)$  is larger than  $c \log n$ ,  
 Which means  $f(n)$  is  $\omega$  of  $g(n)$ .

Hence,  $f(n)$  is  $\theta$  of  $g(n)$ .

P3

```

1
2 def trans(x):
3     if x == 1 or x == 0:
4         return x
5     else:
6         print(x%2)
7         return trans(x//2)
8
9 print(trans(5))

```

PROBLEMS OUTPUT DEBUG CONSOLE TERMINAL 1: Python

```

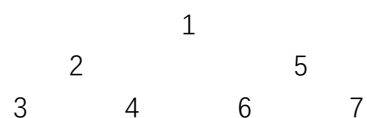
+ conda activate base
+ ~~~~~
+ CategoryInfo          : ObjectNotFound: (conda:String) [], CommandNotFoundException
+ FullyQualifiedErrorId : CommandNotFoundException

PS C:\Users\GR> & C:/Users/GR/Anaconda3/python.exe d:/GRADUATE_OSU/Fall_2020/CS325_retaken/pa1_3.py
PS C:\Users\GR> & C:/Users/GR/Anaconda3/python.exe d:/GRADUATE_OSU/Fall_2020/CS325_retaken/pa1_3.py
1
0
1

```

P4

Starting from the root node, and visit the left node of itself. Creating a new small tree and treat the node we mentioned as a new root. Keep doing that until the bottom left leaf. Then visit right leaf and back.



Postorder would find the most left and bottom one first and the right node match with it, then back to the subroot node.

