UCLQ
Quantum Science &
Technology Institute

Quantum Hackathon

Optimal location of renewable sources in the UK

In this hackathon, you will use quantum computers to find where to optimally install renewable energy sources around the UK. This will be done using the Quantum Approximate Optimization Algorithm (QAOA) and you will work on IBM's computers using Qiskit. This document should help you get started to implement your abstract problem onto an algorithm.

It is necessary to rewrite our problem into something that can be given to Qiksit. In the first part of this document, we give you an optimisation framework that enables to run the problem onto the QAOA . The solution will be a variable X that contains the optimal location of each renewable energy source in the UK. A first implementation of the algorithm has been done in this notebook, using example values from this spreadsheet. In the second section of the document, we explain how we found these example values and suggest ways of improvements.

1 Optimisation function

Installing optimally energy sources means two things : (1) we want to **minimise the cost** of installing the renewable energy sources (2) but we also want to **maximise the energy produced** at the solution location, looking at what historical weather forecasts shows to be beneficial. This is equivalent to solving the following optimisation problem:

$$\min_{x_i \in \{0,1\}} f(X = x_1, \ldots, x_n) = \gamma \, f_{\text{cost}}(X) - \lambda f_{\text{yield}}(X)$$

with x_i is a binary variable, being 1 if there is an energy source at location i and 0 otherwise, $\lambda, \gamma \in \mathbb{R}^+$ are weighting constants, and f are functions in pound sterling per year [1].

The decision to use renewables at location i is given by the binary variable x_i, which can be split even further into x_{ij} to describe renewable source j at location i. f_{cost} talks about the cost of installing the renewables at the solution location. Minimising this fulfills requirement (1). f_{yield} talks about the energy (electricity) yielded by the renewables placed at the solution location and is converted in pound sterling. Maximising this fulfills requirement (2).

Under the linear assumption [2], we can rewrite the optimisation problem as :

$$\min_{X \in \{0,1\}^{N \times R}} f(X = x_{11}, \ldots, x_{NR}) = -\lambda \sum_i^N \sum_j^R a_{ij} x_{ij} + \gamma \sum_i^N \sum_j^R b_{ij} x_{ij}$$

where a_{ij} and b_{ij} are the revenues and costs (in pound sterling) of renewable j at location i respectively, N the number of location and R the number of renewable sources.

However, to ensure that this model does not suggest that we place renewable at every location or favour the cheaper renewables, we need to add the energetic constraint, C, which is the total cost of the UK energy consumption in 2019. Therefore we need to minimise

$$\min_{X \in \{0,1\}^{N \times R}} f(X = x_{11}, \ldots, x_{nn}) = \lambda \left(C - \sum_i^N \sum_j^R a_{ij} x_{ij} \right)^2 + \gamma \sum_i^N \sum_j^R b_{ij} x_{ij} \, .$$

This is a complete graph with NR qubits. Therefore, if in solution X, $x_{ij} = 1$, then this tells us to to put renewable energy source j at location i to meet the UK energy needs. For simplicity we reduced the double summation into a single and then expand to yield

$$\min_{X \in \{0,1\}^{N \times R}} f(X = x_{11}, \ldots, x_{nn}) = \sum_i^{NR} \left(\lambda a_i^2 + \gamma b_i - 2\lambda C a_i \right) x_i + 2\lambda \sum_{i<j} a_i a_j x_i x_j + \lambda C^2 \, .$$

This equation can be adjusted to incorporate more complex models e.g. power load.

An example set of values of the parameters, namely that of revenues a_i, costs b_j and energy constraint C, are given to you in the excel spreadsheet. This set of values will have a strong influence on the result of your problem, and hence you should try and find more accurate parameters. In the next part, we explain how we chose these values and give directions to help you improve the model.

[1] Here, you see the parameters can vary depending on the time-frame taken e.g. whether cost of renewables is spread over 5, 10 or 20 years. This will need to be thought about in the assumptions

[2] We consider the cost and yield functions to be linear in the variables.

2 Assumptions

A lot of useful information on renewable energy statistics can be found in the Digest of UK Energy Statistics (DUKE) 2021 available here.

2.1 Tiling of the UK

The problem, as stated in section I, requires a tiling of the UK in N parts. The $N = 12$ regions we used in our example can be seen on the map beside and are : (1) East Midlands, (2) East of England, (3) North East, (4) North West, (5) London, (6) South East, (7) South West, (8) West Midlands, (9) Yorkshire and the Humber, (10) Northern Ireland, (11) Scotland (12) Wales.

These parts could be refined into smaller areas to account for different weathers within each region.

Figure 1: UK regions, from *Savic, M. (2016). Regional Studies, Regional Science, 3(1), 445-454.*

2.2 Renewable energy sources

For the example values, we considered only two possible renewable energy sources: (1) solar (2) wind. You can make the model more realistic by adding other energy sources.

2.3 Revenue estimation

For each energy source, you will need to calculate the revenue a_{ij} per region and unit. You can do that from estimating the amount of renewable energy produced per region, and multiplying by the current price of electricity, in £/GWh for example. In the model given to you, the amount of energy produced has been estimated crudely from the previous year statistics. However, you will need to produce a more complex model that takes into account the weather data - some of which can be found here to help you start.

2.4 Cost estimation

Similarly, you will need to calculate the cost of renewable per region b_{ij}. In the first model given to you, this has been done using various data such as the pre-development, construction and maintenance costs, weighting with the load factor and the operating period. Additional information can be found here, but again, this is a first approximation that you can complexify.

3 Qiskit

You have at your disposal a notebook to run the QAOA algorithm on of one IBM's chip using Qiskit, that solves the problem above with the values in the example spreadsheet. You should be familiar with Qiksit by now, but otherwise you can find useful tutorials here.

Your task is now to improve the model we created here, and to find the optimal location of renewable energy sources in the UK. Good luck!