Should Value-Added Models Weight All Students Equally?

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Abstract

Conventional value-added (VA) models estimate teacher quality as a simple average of the difference between students' actual and predicted standardized test scores. These models therefore implicitly assume it is just as important to raise test scores of lower-achieving students as it is to raise test scores of higher-achieving students. I consider whether a weighted average of residuals might be more useful. Using data from North Carolina, I find that teacher VA measures become more predictive of teachers' long-run impacts when the highest-achieving students are weighted more than the median student. Strikingly, even impacts on *low-achieving* students' long-run outcomes are best predicted by increasing the weight on impacts on *high-achieving* students' short-run outcomes. These differences in weights may reflect that either (i) small-sample efficiency (some students are more informative about teachers' true test-score effects than others) or (ii) differences in true effects (e.g. test-score effects for different students might capture different general aspects of teaching). I find empirical evidence supporting both explanations. In particular, the large weights for high-achieving students are partially but not completely explained by the fact that their residuals are less noisy.

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1 Introduction

School systems commonly evaluate teachers using statistical models designed to estimate their impact on student outcomes. These are called value-added (VA) models. Typical construction of VA measures follows a four-step process. First, the researcher uses a regression model to predict students' outcomes using their predetermined characteristics, the most important of which is a student's lagged outcome. In steps 2 and 3, the researcher calculates an average of student residuals (i.e., the gap between actual and predicted outcome) within a class. This is referred to as an unadjusted VA measure. The final step is a shrinkage correction, in which the unadjusted VA is "shrunk" or adjusted towards an average to account for statistical noise in the residuals using an Empirical Bayes framework. These VA measures are reported in units of standard deviations (SD) of student achievement. For instance, a test-score VA of 0.1 SD indicates a teacher is estimated to increase students' test scores on average above predicted levels by 0.1.

Value-added models are popular with researchers and policymakers – currently, 30 states use test-score value-added models to evaluate teachers (National Center on Teacher Quality, 2024) – because they have two main statistical properties which make them useful for evaluating teacher quality. First, there is a large body of evidence that VA models are approximately forecast-unbiased (e.g., Kane and Staiger, 2008; Konstantopoulos, 2009; Hanushek and Rivkin, 2010; Bacher-Hicks, Kane, and Staiger, 2014; Chetty, Friedman, and Rockoff, 2014a; Koedel, Mihaly, and Rockoff, 2015). This means that VA measures are fair in the sense that they do not systematically reward teachers for having better or worse students.

Second, teachers who are estimated to raise test scores also tend to promote student success on long-run measures such as educational attainment, wages as an adult, and teen pregnancy (e.g., Chetty, Friedman, and Rockoff, 2014b; Gilraine and Pope, 2021; Backes et al., 2023; Petek and Pope, 2023; Lavy and Megalokonomou, 2024). If the ideal criteria for evaluating teachers is their ability to raise test scores, conventional test-score VA models by construction estimate this aspect of teacher quality. If, on the other hand, the ideal criteria is based on a teacher's impact on longer-run outcomes, test-score VA proxies for these impacts as well while avoiding the impracticality of waiting until students' long-run outcomes become realized.

While VA models proxy for a teacher's ability to affect long-run outcomes, the variation in a teacher's test-score effects explains little of the variation in a teacher's long-run impacts (e.g., Chetty et al., 2011; Chamberlain, 2013). Adding VA measures using non-cognitive outcomes (such as absences and disciplinary behavior) to predict teachers' long-run impact closes some of this gap (e.g., Jackson, 2012; Chetty, Friedman, and Rockoff, 2014b; Blazar and Kraft, 2017; Jackson, 2018; Mulhern and Opper, 2023). These results imply that a teacher's effects are multidimensional (teachers impact multiple outcomes simultaneously) and that conventional test-score VA measures may miss or leave out important information about teacher quality.

Because test-score VA models use a simple average, these measures equate raising test scores for the lowest-achieving students with raising test scores by the same amount for the highest-achieving students. It is not obvious that a one-point increase in test scores is equally valuable or as important no matter what the baseline achievement is. Nielsen (2019) finds that getting easier questions right on the Armed Forces Qualification Test (AFQT) is more predictive of individuals' long-run outcomes. This suggest there may be room for improvement in using test-score VA measures to evaluate teachers. In particular, weighting students equally in a conventional VA model might be unwise if a teacher has different impacts on different groups of students.

As an example, consider a policymaker whose goal is to select teachers with the highest long-term impact, where the long-run outcome of interest is high school graduation. The policymaker could use a conventional test-score VA measure to proxy for a teacher's impact on high school graduation. But not all students are equally at risk of not graduating high school. Students with high lagged test scores are likely to graduate high school no matter how good or bad a given teacher is. Under such a scenario, a more informative short-run measure of teacher quality might be a VA measure that gives a higher weight to a teacher's impact on lower-achieving students, since those are the students most at risk.

In this paper, I ask the following research questions. Suppose I construct a new test-score value-added model as a weighted (rather than unweighted) average of student residuals. First, if I choose the weights with the objective of maximizing the predictive power of my new VA measure for a teacher's long-run impact (e.g., on high school graduation), how do these estimated weights compare to a conventional VA model in which all students are weighted equally? Second, how much better is this weighted VA at predicting teachers' long-run impacts? Third, to the extent that a weighted VA improves predictive power, what explains the differences in weights across students?

Using data from elementary schools in North Carolina, I find that the estimated optimal weights for my new VA measure are not equal across all students. The data therefore reject a conventional VA model is the most predictive measure of teacher's impact on longer-run outcomes. I find that, for both math and reading, test-score impacts for the lowest-achieving students should receive a lower weight than test-score impacts for the median students. Test-score impacts for the highest-achieving students should receive approximately 1.5 times as much weight as test-score impacts for the median students. Depending on the subject and whether or not I include a teacher's non-cognitive impacts as additional predictors, I find my weighted VA increases the predictive power of test-score VA for high school graduation VA by approximately 10%.

There are three possible explanations for the observed pattern of weights. Explanation 1 is that, in finite samples, these weights reflect differences in the noisiness of residuals. To maximize efficiency, the students with the highest signal to noise ratios receive the highest weights. Like explanation 1, explanation 2 is about efficient use of a small sample. Explanation 2 pertains to the covariance of true teacher test-score effects across different students. I postpone a more in-depth discussion of explanation 2 until Section 5. Explanation 3 is that these weights do not reflect small-sample efficiency but instead reflect true differences in teacher quality. Teacher VA likely reflects many aspects of teaching, such as pacing, clarity of instruction, and classroom management. Test-score impacts for different groups of students might reflect different weights on these general aspects of teaching. If some aspects are more important for promoting high school graduation than others, then students in bins which reflect the most important aspects receive the highest weights.

I find that the observed pattern of weights represent both an efficient use of a small sample and true differences. An efficient use of a small sample approximately completely accounts for the lower weights on the lowest-achieving students. Small-sample efficiency, however, cannot fully explain the high weights on the highest-achieving students. The highest-achieving students receive the highest weight even when I use a teacher's test-score impacts on all students to best predict a teacher's impact on high school graduation measured using only the lowest-achieving students.

I make three contributions to the value-added literature. First, I derive optimal weights for use as an alternative to a conventional VA measure. Second, my results are suggestive that a high test-score VA for the highest-achieving students might reflect teachers who are good at teaching skills especially important for promoting students' long-term outcomes. Third, I provide additional evidence (and confirm the results from Biasi, Fu, and Stromme, 2021 and Eastmond et al., 2024) that teacher's VA for math and reading scores differs across students of different baseline achievement.[1]

The rest of the paper proceeds as follows. In Section 2, I describe the North Carolina data

[1] There is a large literature that teachers have different test-score impacts on different types of students beyond higher-achieving versus lower-achieving students. These demographics include race, gender, and socioeconomic status. For example, see Lavy, Paserman, and Schlosser (2012), Condie, Lefgren, and Sims (2014), Fox (2016), Delgado (2020), Aucejo et al. (2022), Gershenson et al. (2022), and Graham et al. (2023). I describe my methodology for showing that teachers differ in their test-score value-added for higher-achieving versus lower-achieving students in Appendix C. I present results in Table C1.

I use in my analyses and provide student-level summary statistics. In Section 3, I provide additional details regarding conventional VA models. In Section 4, I discuss the details of my procedure for estimating this new VA measure and the estimated weights. I also discuss how much more predictive my new VA measure is compared to a conventional VA. In Section 5, I investigate to what extent each of the three possible reasons for the differences in weights explains my results. In Section 6, I conclude.

2 Data

My data come from the North Carolina Education Research Center (NCERDC) and contain information regarding all North Carolina public school students. I restrict the data to students in grades 3-5 from 1997 to 2011. For students in these grades, it is unambiguous who their teacher is as students have a single teacher for both math and reading. The data include standardized test scores in math and reading, demographic information, and an identifier of the teacher who administered the math and reading test. The data also include information on attendance (available starting in 2006), disciplinary behavior (available starting in 2001) (such as number of days suspended within a given school year), whether each student dropped out in a given year, withdrew, and whether or not each student graduated high school in North Carolina (available starting in 2002).

I drop observations for students which I cannot map to a particular teacher. I limit my data to students in reasonably-sized classrooms, defined as classes with between 10 and 35 students, to ensure data quality.

I construct standardized measures of cognitive and non-cognitive outcomes for use in my analyses. I normalize test scores in math and reading to be mean zero with variance 1 for each testing year and grade. I re-define absences as the negative natural log of days a student is absent plus 1 (to avoid undefined values) following Mulhern and Opper (2023). I replace missing values for absences (as well as days suspended) with values of 0s for years in which absence or disciplinary data is available. I also create a behavioral index for students based on a principal component analysis of suspensions and other disciplinary infractions such that worse behavior receives a lower (more negative) value, which I describe further in Appendix B.

My restricted sample contains approximately 2.5 million student-year observations 1998 to 2011 for students in fourth and fifth grade.[2] My sample contains slightly more males

[2] I exclude 1997 due to the lack of lagged outcomes available. These lagged outcomes would come from the 1996 data, which is not available. Students do not take a standardized test in either math or reading in grade 2. Therefore the earliest grade in which students have a lagged standardized test score for math and

(51%) than females (49%). Most students (74%) are defined as economically disadvantaged, and 80% of my sample graduated high school. The average logged-absences is approximately 8. The average class size in my restricted sample is 23 students. I report summary statistics in Table 1.

3 Conventional Value-Added Models

I begin my discussion of conventional VA models by providing additional details regarding the four-step construction process. The first step is to use a regression model to predict a student's outcome using their predetermined characteristics. The most important characteristic is a student's lagged outcome. Test-score VA models use a student's standardized test scores in math and reading as the outcome, while other VA models (such as non-cognitive VA) use student outcomes such as attendance, disciplinary behavior, high-school graduation, and other longer-term outcomes. Regardless, the construction of VA models is the same for all outcomes. The typical way of estimating these residuals is to estimate models of the form

$$\tilde{Y}^s_{i,t} = \alpha + \gamma \tilde{Y}^s_{i,t-1} + \tilde{\mathbf{X}}_i \beta + \epsilon^s_{i,t} \tag{1}$$

where $Y^s_{i,t}$ represents outcome s for student i in year t. $Y^s_{i,t-1}$ is a function of lagged outcomes, and $\mathbf{X}_i$ represents a vector of student-level demographics.[3] Tilde ($\sim$) indicates a variable demeaned at the classroom level, which is equivalent to including a teacher fixed effect.[4]

Step 2 of constructing a value-added model is to construct a student residual as the difference between a student's actual and predicted outcome. The predicted outcome is calculated using the coefficients from the regression model from step 1. Each student's residual $\epsilon^s_{i,t}$ is calculated as

$$Y^s_{i,t} - \hat{\alpha} - \hat{\gamma} Y^s_{i,t-1} - \mathbf{X}\hat{\beta}. \tag{2}$$

These residuals are then recentered to ensure $\epsilon^s_{i,t}$ is mean 0 by subtracting the mean residual across all students and years. I define the recentered residuals as

$$r^s_{i,t} = \epsilon^s_{i,t} - \bar{\epsilon}^s_t, \tag{3}$$

reading is fourth grade.

[3] For test-score VA and high school graduation VA measures, I use a cubic of lagged test scores for both math and reading as well as whether a student is economically disadvantaged, gender, disability status, and english learner status. To estimate non-cognitive VA measures, I use a cubic of the lagged non-cognitive outcome in addition to whether a student is economically disadvantaged, gender, disability status, and english learner status.

[4] For example, see Mulhern and Opper (2023). This is necessary for feasibility of estimation given the number of teachers in my sample.

where $\bar{\epsilon}_t^s$ represents the average residual across all students and years.

In step 3, a teacher j's unadjusted VA in year t is calculated as the average of student residuals within a classroom. This unadjusted VA measure, defined as

$$\hat{VA}_{j,t}^{s} = \sum_i r_{i,t}^{s}, \tag{4}$$

contains a mixture of a teacher's true VA in year t and statistical noise. In step 4, this unadjusted VA is adjusted, or "shrunk", to account for sampling error.[5]

Value-added models are a popular way to evaluate teacher quality and have desirable statistical properties. In 2023, 30 states used VA measures as part of teacher evaluations. Perhaps the most desirable statistical property of VA models is that they are approximately forecast-unbiased. This means that for all teachers who are estimated to raise test scores by $\hat{\theta}$, the average of those teacher's true effects on test scores is $\hat{\theta}$. This property has been confirmed in the literature using both experimental methods (e.g., Kane and Staiger, 2008; Bacher-Hicks, Kane, and Staiger, 2014) and quasi-experimental methods (e.g., Chetty, Friedman, and Rockoff, 2014a; Rivkin, Hanushek, and Kain, 2005). In Appendix E, I provide evidence that high school graduation VA measures are also approximately forecast unbiased.

Another desirable property of VA models is that test-score VA measures in particular have been found to be predictive of a teacher's impacts on long-run outcomes. This enables test-score VA measures to act as a short-run proxy for teacher quality on long-run impacts. For example, (Chetty, Friedman, and Rockoff, 2014b) find that teachers with higher test-score VA are also teachers who improve students' long-term outcomes such as earnings, college attendance, and reduce teen pregnancy among their students.

Test-score VA measures, however, do not perfectly predict a teacher's impacts on these long-run outcomes. For example, Chetty et al. (2011) analyze data from Project STAR and finds that the actual variation in teacher impacts on long-run outcomes is five times larger than the variation implied by test-score VA measures. Similarly, Chamberlain (2013) finds variation in teacher test-score effects in elementary school explain 1% of the variation in whether or not a student attends college. Including a teacher's impact on short-run non-cognitive outcomes as additional predictors closes some of this gap (e.g., Jackson, 2012; Chetty, Friedman, and Rockoff, 2014b; Mulhern and Opper, 2023). For example, (Jackson, 2012) finds adding 9^{th}-grade teachers' impacts on short-run non-cognitive outcomes as

[5] This is similar to a penalized regression, such a lasso or ridge regression. For a more detailed explanation of shrinkage, see Chetty, Friedman, and Rockoff (2014a). In this paper, I follow the shrinkage methodology from Mulhern and Opper (2023). My results are not sensitive to whether or not I include shrinkage. I include summary statistics of my VA estimates in Appendix A.

additional predictors increases the predictive power of teachers' test-score impacts for high school graduation by 20% for reading and 200% for math.

4 Main Analysis: Estimating Weighted-Average Value-Added

In this section I discuss my econometric strategy for estimating my weighted VA measure. I then estimate optimal weights using the data. Using these weights, I ask how much more predictive this weighted VA measure is, compared to a conventional VA, in predicting a teacher's impact on high school graduation.

4.1 Weighted Average VA: Econometrics

To construct weights, I first divide a classroom into 5 bins, or quintiles, based on the distribution of lagged test scores in subject s within a given school and grade. Then I construct VA^{*s} as

$$VA^{*s}_{j,t} = \sum_i \Big(\frac{\beta_k \mathbf{1}\{i \in k\}\mathbf{1}\{i \in j\} r^s_{i,t}}{\sum_i \sum_k \beta_k \mathbf{1}\{i \in k\}\mathbf{1}\{i \in j\}} \Big), \tag{5}$$

where $r^s_{i,t}$ represents student i's residual in subject s in year t, and student i is in teacher j's class. An alternative and equivalent formulation of VA^* is given by

$$VA^{*s}_{j,t} = \frac{1}{\sum_i \sum_k \beta_k \mathbf{1}\{i \in k\}\mathbf{1}\{i \in j\}} \sum_i \beta_k \mathbf{1}\{i \in k\}\mathbf{1}\{i \in j\} r^s_{i,t}, \tag{6}$$

where $\sum_i \sum_k \beta_k \mathbf{1}\{i \in k\}\mathbf{1}\{i \in j\}$ represents the sum of the bin weight times the number of students in each bin, summed over all bins within class j.

VA^* is a weighted average of student residuals. Students in the same bin receive the same weight. Students in different bins receive different weights if the estimated weight β_k for bin k is different from the estimated weight $\beta_{k'}$ for bin k'. The denominator ensures the weights sum to 1 within a particular class.[6] I report the estimated weights relative to the estimated weight on bin 3, or the weight for the median students. Bin weights smaller than 1 indicate bins of students which should be weighted less heavily than the median group of students. Bin weights larger than 1 indicate bins of students which should be weighted more than the median student.

[6] For classes with zero students in a particular bin, I set the sum of residuals and the number of students in these bins to 0 rather than dropping such a class from my analysis.

To demonstrate how this weighted VA measure works, I show that a conventional VA model is a special case of my weighted average measure. A simple average of student residuals is equivalent to setting $\beta_k = 1$ for all five bins (or setting $\beta_k = 3$ or any other number, as long as the number is the same for all bins). Under such a scenario (using the example where all $\beta_k = 1$), my value-added estimator becomes

$$VA^{*s}_{j,t} = \frac{1}{N^s_{j,t}} \sum_i r^s_{i,t}, \tag{7}$$

where $\sum_i \sum_k \beta_k \mathbf{1}\{i \in k\}\mathbf{1}\{i \in j\}$ becomes the number of students in class j, or $N_{j,s,t}$, and $\beta_k \mathbf{1}\{i \in k\}\mathbf{1}\{i \in j\}$ collapses to 1 for all i, leaving $\sum_i r_{i,j,s,t}$. I allow for such a set of weights to occur if weighting all students equally is empirically the most predictive of a teacher's high school graduation VA.

As another example, consider a set of estimated weights using 3 bins. Suppose there are 20 students in the class, with 5 students in bin 1, 15 students in bin 2, and 0 students in bin 3. Also suppose the estimated bin-weights (using all classrooms) for each bin are $\beta_1 = 2$, $\beta_2 = 1$, and $\beta_3 = 0.5$ The denominator of $VA^* = \sum_i \sum_k \beta_k \mathbf{1}\{i \in k\}\mathbf{1}\{i \in j\}$ is equal to $2 \cdot 15 + 1 \cdot 5 + 0.5 \cdot 0 = 35$ for this particular class. The estimated weight on each student's residuals in bin k is $\frac{\beta_k}{35}$, or $\frac{2}{35}$ on each student's residual in bin 1 and $\frac{1}{35}$ on each student's residual in bin 2. There are no students in bin 3 for this class, but the student-level weights still sum to 1.

4.2 Weighted Average VA: Estimation and Results

To estimate my weighted VA model on the actual data, I choose the weights β_k to maximize the predictive power of this new VA measure for a teacher's long-run impacts. The longest-run outcome I observe in my data is a student's high school graduation. Therefore I seek to maximize the predictive power of this new VA measure on a teacher's high school graduation VA. I estimate the weights to minimize the sum of squared errors in the predicted high school graduation VA of teacher j in year t using the test-score residuals of teacher j's students. I obtain these residuals using step 1 of the conventional VA methodology as described in Equation 1.

Of course, using the same students to calculate both the weights for this new VA measure and a teacher's high school graduation VA would introduce mechanical bias in my estimates (as noted in Jackson, 2018). Therefore, to construct a teacher's high school graduation VA, I combine Jackson's leave-one-year-out (LOYO) methodology with the methodology for calculating a shrunken VA measure using multiple years for a given teacher as described in Mulhern and Opper (2023). I estimate a teacher's pooled high school graduation VA by estimating Equation 1 through Equation 4 with high school graduation as the outcome variable. I then calculate a teacher j's adjusted high school graduation VA using all years other than year t, which I denote as $\tilde{VA}^{grad}_{j,-t}$.

I then use non-linear least squares to minimize the squared prediction error of a teacher's LOYO high school graduation VA. I define my optimization problem as

$$\min_{\beta_k} \left[\tilde{VA}^{grad}_{j,-t} - \beta_0 - VA^{*s}_{j,t}\right]^2, \tag{8}$$

where the choice of β_k influences the minimization problem because β_k determines the value of $VA^{*s}_{j,t}$. I choose an intercept β_0 and weights on each bin 1 through 5. I report weights relative to the estimated weight on bin 3, or the weight on test-score impacts for the median students in each subject.

Figure 1 presents estimated weights for math and reading. The data reject a conventional VA model is most predictive of a teacher's long-run impact. In particular, for both subjects, the higher-achieving students (bins 4 and 5) receive a higher weight than the median student. For reading, lower-achieving students (bins 1 and 2) receive a lower weight. The estimated weight on the lowest-achieving students based on lagged reading scores is approximately two-thirds as large as the weight on the median students. The estimated weight on the highest-achieving students is approximately twice as high as the weights placed on the other students.

The result that the highest-achieving students receive the highest weights might seem counterintuitive. In Appendix F, I construct a toy model in which I pretend the only factor which impacts a student's probability of graduating high school is the teacher's impact on test scores for a student's particular bin. For this toy model, I also suppose a teacher's impact on test scores in each bin is independent of a teacher's test-score impact in all other bins. This is a supposition of the toy model and not an assumption I believe to hold in reality. (In fact, I show in Appendix C that teacher's test-score impacts are correlated between lower-achieving and higher-achieving students.) Under these non-realistic conditions, the estimated weights I obtain from the toy model (reported in Figure F1) are highest for the lowest-achieving students and lowest for the highest-achieving students.

Suppose instead of estimating weights to predict a teacher's high school graduation VA for all students, the goal is to predict a teacher's high school graduation VA specifically for the lowest-achieving students using a teacher's test-score impacts for all students. If what matters is a teacher's true test-score impacts for a student's particular bin, I would expect to see higher weights for the lower-achieving students and lower weights for the highest-achieving students.

I empirically test whether this is true in the data. I define each teacher's LOYO high school graduation VA for students in the bottom b percentile of the achievement distribution as $\bar{VA}^{grad,b}_{j,t}$. I define a student to be in the lowest-achieving group based on a student's combined lagged math and reading standardized test scores, relative to all students within the same year and grade. I normalize this total test score to be a standard normal for each grade and year. I use four alternative definitions of lowest-achieving students, (i) bottom 50% (ii) bottom 30%, (iii) bottom 25%, and (iv) bottom 20%. For each definition, I estimate

$$\min_{\lambda_k} \left[\tilde{VA}^{grad,b}_{j,-t} - \lambda_0 - VA^{*s}_{j,t}\right]^2, \tag{9}$$

where $VA^{*s}_{j,t}$ is defined as described above in Equation 5, and $\tilde{VA}^{grad,b}_{j,-t}$ is teacher j's high school graduation VA measured using only students in the bottom b percentile of the achievement distribution.

I report estimated bin-weights predicting these low-achieving high school graduation VA measures below in Figure 2. I also include the Figure 1 estimates (using all students within a classroom) as hollow gray boxes. The weights on the lower-achieving and higher-achieving students do not differ when I change which students I use to calculate a teacher's high school graduation VA. The implication of this result is that even if I am using a teacher's test-score impacts for all students to best predict a teacher's impact on high school graduation for the lowest-achieving students, it is a teacher's impact for the *highest*-achieving students that receives the highest weight.

4.3 Estimating Weights Jointly vs Separately

I also estimate optimal weights using a teacher's impacts on math and reading scores to jointly predict a teacher's high school graduation VA. This joint-estimation is more consistent with the literature finding that a teacher's impact is multidimensional. In particular, it follows the idea from Mulhern and Opper (2023) that a teacher's math VA should incorporate information about a teacher's estimate reading VA, and vice versa.

The way I estimate weights for math and reading jointly follows directly from how I initially estimated weights in Equation 8. I define teacher j's weighted VA for subject s in year t as $VA^{*s}_{j,t}$ based on Equation 5. A teacher's out-of-sample high school graduation VA constructed in the usual way and denoted $\tilde{VA}^{grad}_{j,-t}$. I then estimate

$$\min_{\kappa_k} \left[\tilde{VA}^{grad}_{j,-t} - \kappa_0 - VA^{*,read}_{j,t} - VA^{*,math}_{j,t}\right]^2, \tag{10}$$

where the only difference from Equation 8 is that I include VA^* for both subjects.

I report these results below as Figure 3, and I include the Figure 1 weights as hollow gray squares. For math, adding in a teacher's impacts on reading scores increases the weight on the lowest-achieving student from below to above 1. The weight on the highest-achieving students also increases. For reading, the weight on the lowest-achieving students decreases when I include a teacher's impacts on math scores. The weight on the highest-achieving students also increases. The weights for bins 2 and 4 are almost identical whether I estimate weights for math and reading separately or jointly.

4.4 How Much More Predictive is a Weighted VA Compared to a Conventional Value-Added?

I now ask how much more predictive a teacher's test-score impacts are for a teacher's impact on high school graduation when I use a weighted, compared to unweighted, average of student test-score residuals. For each subject, I construct a teacher's conventional VA in year t for subject s as the average of student residuals ($VA_{j,s,t}$). I calculate that same teacher's weighted VA ($VA^{*}_{j,s,t}$) using the weights I estimated in Figure 1.

I define the increase in predictive power of a weighted VA measure (compared to a conventional VA measure) using the percentage of explained variation in teachers' high school graduation VA. I perform the following steps. First, I regress a teacher's LOYO high school graduation VA on a teacher's conventional VA for subject s. I calculate the R-squared from this regression. I then regress a teacher's LOYO high school graduation VA on a teacher's *weighted* VA measure, and again calculate the R-squared. I then calculate the percentage increase in the R-squared when I use the weighted VA compared to the R-squared when I use the conventional VA as my main independent variable. I limit my sample to teachers with both a non-missing conventional and weighted VA for a particular subject. This procedure runs the potential risk of overfitting. My results are robust to cross-validation. I postpone a more complete description of how I attempt to address overfitting concerns until after I present results

I begin by showing these results without controls in Table 2. I report results for math in columns 1 and 2, and results for reading in columns 3 and 4. I cluster standard errors at the teacher level. The R-squared is higher when I use a weighted VA to predict high school graduation VA for both math ($\sim 20\%$) and reading ($\sim 8\%$) compared to a conventional VA in each subject. Both a weighted and conventional VA, however, explain less than 1% of the variation in teachers' high school graduation VA. My R-squared values are low in large part due to a small sample size for each individual teacher. Even if I used a teacher's true impacts on test scores as my independent variable, the R-squared value would likely be modest.

I next explore whether including a teacher's impact on short-run non-cognitive outcomes absorbs some of the increase in predictive power due to my weighted test-score VA measure.

Further, I ask if my weighted value-added measure offers additional information about a teacher's high school graduation VA beyond what can be explained by using a conventional VA and non-cognitive VA measures. Therefore I estimate additional specifications in which I vary whether or not I include these non-cognitive VA measures as additional predictors.

I report results for these additional specifications in Table 3. Columns 1 through 4 show results for math, while columns 5 through 8 show results for reading. The first two columns for each subject show results without non-cognitive VA controls. My sample is limited to teachers with non-missing test-score VA and non-missing non-cognitive VA measures. I therefore have a smaller number of observations compared to Table 2. Including the baseline specification ensures comparability between specifications with and without controls.

My results are consistent with the evidence that a teacher's impacts on non-cognitive outcomes matter more for predicting a teacher's high school graduation VA than a teacher's test-score impacts. For both subjects, the R-squared roughly doubles when I include non-cognitive VA measures, regardless of which test-score VA I use. The percentage increase in explained variation due to a weighted test-score VA is also much lower when I include these non-cognitive VA measures. For math, the percentage increase in explained variation decreases from $\sim$ 27% to $\sim$ 7.5% when I include non-cognitive VA measures. For reading, the percentage increase in explained variation decreases from $\sim$ 9% to $\sim$ 4%.

My qualitative findings hold when I use a teacher's test-score impacts on math and reading to jointly predict a teacher's high school graduation VA. Table 4 presents results. I obtain the same pattern of R-squared values, in which (i) the R-squared is much higher when I include a teacher's non-cognitive VA measures and (ii) the increase in percentage of explained variation when I use weighted VA compared to conventional VA decreases (from $\sim$ 13% to $\sim$ 6%) when I include non-cognitive VA measures as controls.

4.5 Addressing Overfitting Concerns

I perform a series of analysis to show that my results presented above are not due to overfitting concerns. First, I randomly assign each teacher into either the training half of my data or the test half of my data. I estimate weights using the training half of the data. I use these weights to construct VA^* for both the test and training data. The mean-squared prediction error in both the test and training sample are not statistically different. This qualitative result holds when I perform 10-fold cross validation. I randomly assign each teacher to one of 10 different groups. For each group, I calculate the mean-squared prediction error using a weighted VA measure where the weights are estimated using the other nine groups. The average out-of-sample mean-squared error is not statistically different from the in-sample mean-squared error.

5 Why These Weights

I now seek to understand to what extent each of the three possible explanations might explain my estimated weights shown in Figure 1. Recall there are three potential explanations for the pattern of weights. The first explanation is that the noisiness in student residuals differ for higher- versus lower-achieving students. In particular, the literature has found that lower-achieving students often have more noisy test score residuals than higher-achieving students (e.g., Kane and Staiger, 2008; Koedel, Mihaly, and Rockoff, 2015). The lower weights on the lowest-achieving students for reading, for example, might be due to relatively high variance of residuals for these students relative to the variance of residuals in the middle bin.

The second explanation is that a teacher's VA for certain bins may be more predictive about a teacher's impact on other bins. This is a small sample efficiency explanation rather than a true effects interpretation. It could be that teachers do truly differ in their effects across different bins, but the bins that receive the highest weight receive the most information about a teacher's impacts on the largest number of students, both within and outside of a particular bin. For example, suppose a teacher's true test-score impact for the lowest-achieving students (bin 1) is independent of a teacher's true test-score impacts for the other four bins. If true, the students in bin 1 receive the lowest weight. This is driven by small-sample efficiency. In this scenario, a teacher's impacts on bin-1 students are not informative about a teacher's impacts on any other students, even if teachers do differ in their test-score impacts on the lowest-achieving students compared to other students in the class.

The third possible explanation is a "true effect" interpretation. A teacher's VA measure likely captures a combination of many aspects of teaching, such as pacing, classroom management, inspiring critical thinking and creativity, and clarity of instruction. Perhaps being able to increase test scores for particular bins weights particular aspects of teaching more heavily. The bins that capture the most generally useful aspects of teaching receive the highest weight.

5.1 Weighted VA: Evaluating Explanation 1

I begin by asking to what extent relative differences in the noisiness of student residuals between bins might explain my estimated weights. A straightforward way to assess this is to estimate a set of weights entirely determined by the relative variance of student residuals within each bin. I estimate this variance-only set of weights for math and reading as follows.

First, I estimate the variance of student residuals in a particular subject across all classrooms and across all years. I then normalize these variances such that the variance of student residuals in the middle bin is 1. The variance of student residuals in the other bins therefore becomes the relative variance of student residuals compared to the other bin. I then calculate the weight for each bin as the inverse of the relative variance of student residuals. For example, suppose the variance of student residuals for reading scores in bin 1 (the lowest bin) is twice the variance of student residuals for reading scores in bin 3. I define the variance-only reading weight for bin 1 as 0.5.

I report these variance-only weights as gray squares (alongside the estimated weights from Figure 1 as blue dots with red standard error bars) below as Figure 4. These variance-only weights confirm that, for both subjects, residuals for lower-achieving students are noisier than for higher-achieving students. This relative noisiness explains some but not all of my results. In particular, this explanation seems to explain the lower weight on lower-achieving students, but does not fully account for the high weights on higher-achieving students. This relative noise story does a relatively better job of explaining the weights for the middle bins (bins 2 and 4) for both subjects.

I conclude from these alternative sets of weights that the pattern of weights I observe in Figure 1 cannot be fully explained by differences in the noisiness of residuals for lower-achieving versus higher-achieving students. In particular, the weights implied by such a difference in noisiness story for the highest-achieving students are larger than those I obtain in Figure 1. This explanation does account for the lower weights on lower-achieving students.

5.2 Disentangling Explanations 1 & 2 From Explanation 3

I now propose a set of estimated bin-weights to evaluate to what extent an efficient use of a small sample can explain the observed weights. I attribute any remaining differences between the observed weights and weights explained by an efficient use of a small sample to potentially differences in teacher quality.

Suppose an efficient use of a small sample perfectly explained the estimated weights in Figure 1. If so, then I would expect to see identical weights when I estimate weights to predict a teacher's out-of-sample impact on test scores. I estimate weights that best predict the impact of teacher j on test scores for the next cohort of students in year $t+1$. By definition, the students used to calculate the outcome I am trying to predict on the right-hand side of the regression (a teacher's impact on test scores in the next year $t+1$) and the student residuals I am using as my left-hand side variables (from period t) are not the same set of students. Therefore I simply estimate a teacher's test-score VA separately for each year and estimate

$$\min_{(\gamma_k)s} \left[VA^{s}_{j,t+1} - \gamma_0 - VA^{*}_{j,s,t} \right]^2, \tag{11}$$

where $VA^{s}_{j,t+1}$ is teacher j's test-score VA in year $t+1$ for subject s.

I report results from estimating Equation 11 as Figure 5 below. I include the initial weights from Figure 1 as hollow gray squares. Note that the weights estimated using a teacher's out-of-sample test-score impacts (dots) are different than the hollow gray squares. This is more true for math than reading, and more true for the highest-achieving students compared to the lowest-achieving students. These results indicate an efficient use of a small sample cannot fully explain my results.

5.3 Additional Weight Estimations

I include additional sets of estimated weights in Appendix D. I observe a similar pattern of weights when I estimate weights separately for larger versus smaller classrooms (see Figure D1 and Figure D2). Weights are also similar when I swap the years used to estimate a teacher's high school graduation VA and a teacher's impact on test scores (see Figure D3). Weights are similar when I try to predict a teacher's leave-one-year-out VA for (i) test scores in subsequent grades (see Figure D4), (ii) bin-specific test scores (see Figure D5) and (iii) bin-specific test scores for students in the next cohort (see Figure D6).

6 Discussion

A weighted average of student test-score residuals increases the variation explained of a teacher's impacts on high school graduation by about 10%. The exact increase in predictive power depends on which subject I use to calculate residuals and whether or not I include a teacher's impacts on non-cognitive outcomes as additional predictors.

My estimated optimal way to weight student residuals in a value-added model is to place a lower weight on lower-achieving students (based on baseline achievement), and a higher weight on higher-achieving students. On average, I estimate the highest-achieving students should be weighted about 1.5 times as much as the median student. For reading, the lowest-achieving students should be weighted about 0.5 times as much as the median student. I observe the same pattern of weights even when I use a teacher's test-score impacts for all students to best predict a teacher's long-run impacts for only the lowest-achieving students.

Because I observe this same pattern of weights when I use test-score impacts to predict other outcomes besides high school graduation, such as a teacher's impact on future test scores, my results provide suggestive evidence standardized tests may be directly testing students' general and specific skills. By specific skills, I mean the math or reading concepts (e.g., long-addition, reading comprehension, etc.) required to answer a given test question correctly. General skills might include non-cognitive outcomes such as the ability to stay focused during a long test. If standardized tests only evaluated students on their specific skills, I would have expected to see a different pattern of weights when I predict a teacher's impacts on future test scores or test scores for specific students compared to when I predict a teacher's impact on high school graduation.

I posit two potential reasons as to how a standardized test might directly measure a teacher's impact on general skills. First, a teacher's impact on the higher-achieving versus lower-achieving students might weight general aspects of teaching differently if the skills which are important for raising test scores are different for students with different levels of baseline achievement. Such aspects might include pacing, clarity of instruction, promoting critical thinking, or encouraging students to take pride in their work.

Second, the easy questions on standardized tests might be a more direct test of general skills. This might be especially true for the highest-achieving students. As an example, consider the fourth-grade standardized math test. The highest-achieving students likely know the specific-skills required to answer every question correctly. For these students, whether or not they actually answer every question correctly might not be a matter of knowledge, but rather a matter of whether or not the students are able to stay focused for the entire duration of the test and/or take the time to check their answers. In such a scenario, the standardized math test is directly measuring general (i.e., non-cognitive) skills for the highest-achieving students. For the lowest-achieving students, the same standardized test may more directly test whether or not students know the material being tested.

This explanation might account for why small-sample efficiency cannot fully explain the high weight placed on the highest achieving students using a teacher's impacts on student math scores. For reading, small-sample efficiency does a better job explaining the high weights placed on the highest-achieving students. This suggests that perhaps the explanation based on the scaling of the standardized test is more applicable for math as opposed to reading. For reading, it might be the case that even if the highest-achieving students know the basic skills on the test (e.g., how to read paragraphs or use context clues to derive definitions of new words), they still might come across a word they have not encountered before or a passage about a subject they are unfamiliar with. There might be a ceiling for a student's math-specific skills that does not exist in the same way for a student's reading-specific skills.

While suggestive, the idea that standardized tests may directly test general skills (especially for the highest-achieving students) aligns with other evidence from the literature.

In particular, my results would align with the finding from Nielsen (2019) that answering the easier questions correctly on standardized tests is more predictive of students' long-run outcomes. My findings suggest the reason why getting the easier questions correct is more predictive is because easier questions might test students' ability to execute more than their knowledge. A particularly important skill for students' long-run outcomes might be this underlying ability to ensure tasks are done correctly, even if the student is tired or believes the task to be easy. If a teacher's true test-score impacts on the highest-achieving students more heavily weights a teacher's impact on this underlying skill, that might explain why my weighted value-added measure places the highest weight on test-score impacts for the highest-achieving students.

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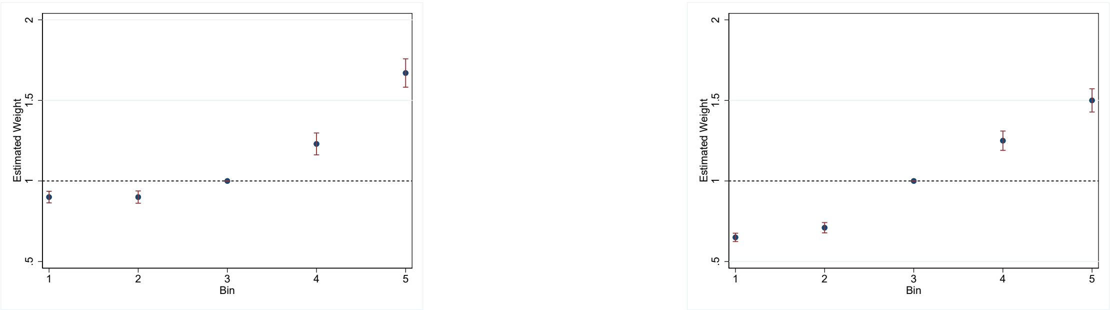
7 Tables and Figures

Figure 1: Empirical Bin-Weight Estimates for Math and Reading


(a) Math

(b) Reading

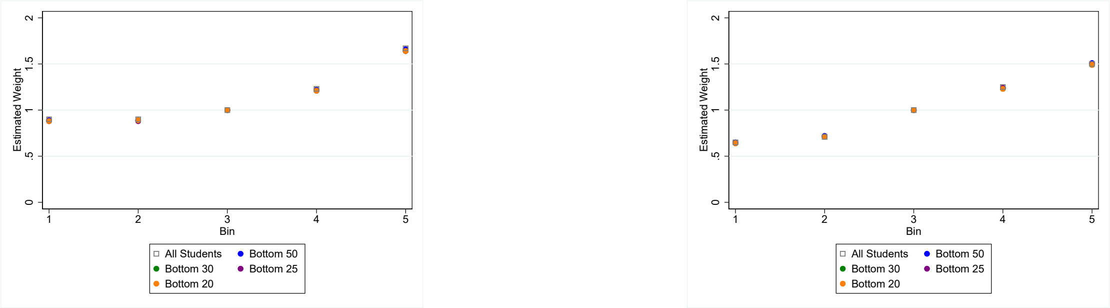
Notes: Each dot represents the estimated weight on each bin from estimating Equation 8. Bars indicate the 95% confidence interval around each estimated weight. The weight on bin 3 is defined to be 1. Figure 1a shows weights for student math residuals and Figure 1b shows weights for student reading residuals. Student residuals are calculated based on Equation 1. Limited to fourth and fifth-grade classes with between 10 and 35 students.

Table 1: Summary Statistics of Student Data

Variable	Mean	SD	Min	Max
Female	0.510	(0.500)		
Black	0.235	(0.424)		
Hispanic	0.053	(0.225)		
White	0.483	(0.500)		
Asian	0.0159	(0.125)		
Economically Disadvantaged	0.838	(0.368)		
Student With Disabilities	0.157	(0.364)		
Academically Gifted	0.0282	(0.166)		
English Language Learner	0.125	(0.111)		
Ever Suspended	0.36	(0.48)		
Graduated High School	0.805	(0.396)		
-Ln(1+Absences)	-0.0973	(0.437)	-4.95	0
-Days Suspended	-0.0880	(1.314)	-447	0
Classroom Size	22.869	(3.73)	10	35
Student-Year Observations		2,587,625		
Students		1,633,504		

Notes: Table reports summary statistics for student demographic, non-cognitive outcomes, and high school graduation. I restrict my sample to students in classes with between 10 and 35 students. I do not report min and max values for indicator variables that take values of either 0 or 1. I calculate absences as the inverse of the natural log of 1 plus the number of days a student is absent (consistent with Mulhern and Opper, 2023). I multiple days suspended by negative 1 such that fewer days suspended is a better student outcome. My analysis sample is limited to 1998-2011 (due to the lack of lagged test score information in 1997) for students in fourth and fifth grade. Lagged test-scores are required for inclusion in my analyses, and third-graders do not take a standardized test in second grade.

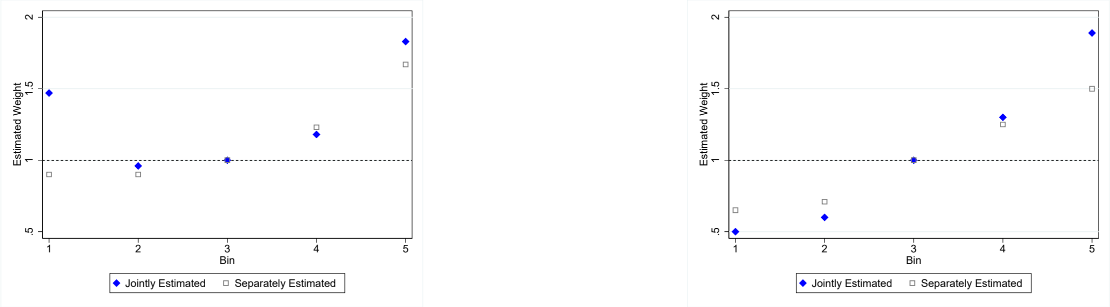
Figure 2: Estimated Bin-Weights Predicting Low-Achieving HS Grad VA


(a) Math

(b) Reading

Notes: Each dot represents an estimated weight. For each outcome, the weight on bin 3 is normalized to 1. I report initial weights from Figure 1 as hollow gray squares. The legend indicates which students I used to calculate a teacher's out-of-sample high school graduation VA. For example, "Bottom 50" indicates I calculated a teacher's leave-one-year-out high school graduation VA only for students in the bottom 50th percentile of combined baseline math and reading achievement. This baseline measure is a normalized sum of lagged math and reading standardized test scores at the school, grade, and year level. Figure 2a shows results for math, and Figure 2b shows shows results for reading. Limited to fourth and fifth-grade classes with between 10 and 35 students.

Figure 3: Jointly Estimated Bin-Weights for Math and Reading


(a) Math

(b) Reading

Notes: Each dot represents an estimated weight. For each subject, the weight on bin 3 is normalized to 1. I report initial weights from Figure 1 as hollow gray squares. The weights (shown in blue) are estimated to maximize the predictive power of predicting a teacher's high school graduation VA using both math and reading as shown in Equation 10 discussed in Section 4.3. Figure 3a shows results for math, and Figure 3b shows shows results for reading. Limited to fourth and fifth-grade classes with between 10 and 35 students.

Table 2: Predictive Power of Weighted vs Unweighted VA: Baseline

	Math		**Reading**	
	(1)	(2)	(3)	(4)
Conventional (Unweighted) VA	0.000566***		0.000967***	
	(0.000114)		(0.000127)	
Weighted VA		0.000581***		0.000984***
		(0.000106)		(0.000123)
Observations	84,704	84,704	84,678	94,678
R^2	0.0010	0.0012	0.0017	0.0019
% Increase in Explained Variation	–	19.56	–	8.15

Notes: Table 2 reports results from regressing a teacher's leave-one-year-out high school graduation value-added (VA) on a teacher's test-score VA in a particular subject. There are no other controls in the regression. I report clustered-standard errors at the teacher level in parentheses. Conventional (Unweighted) VA indicates test-score VA is calculated as the unweighted average of student residuals in a the subject indicated by the column heading. Weighted VA indicates test-score VA is calculated using the estimated weights shown in Figure 1. The percentage (%) increase in explained variation for each subject is calculated as the percentage change in the R-squared value of the regression when I use the weighted VA as the regressor compared to when I use the conventional VA for the same subject as the regressor. Limited to fourth and fifth grade classes to between 10 and 35 students. $p < 0.001^{***}, p < 0.05^{**}, p < 0.1^{*}$.

Table 3: Predictive Power of Weighted vs Unweighted VA: Non-Cognitive

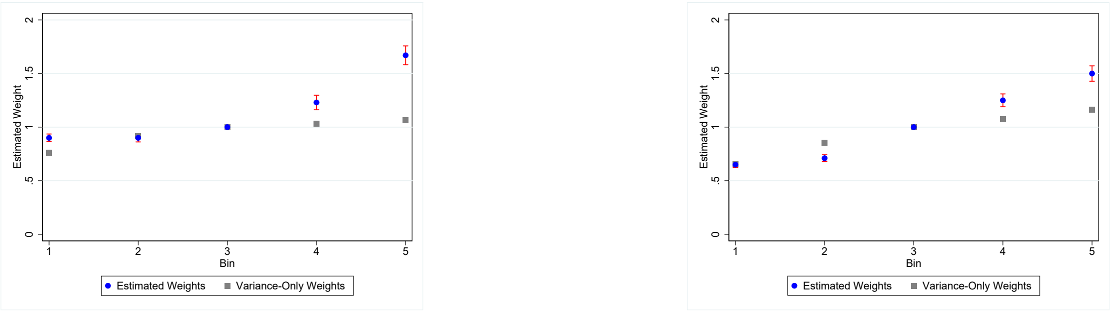
	Math				**Reading**			
	Baseline		**Non-Cognitive**		**Baseline**		**Non-Cognitive**	
	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)
Conventional (Unweighted) VA	0.000508***		0.000439***		0.00984***		0.000898***	
	(0.000134)		(0.000133)		(0.000150)		(0.000147)	
Weighted VA		0.000538***		0.00463***		0.00100***		0.000921***
		(0.000125)		(0.000123)		(0.00146)		(0.00143)
Suspension VA			0.0112***	0.0112**			0.0112**	0.0112**
			(0.00421)	(0.00421)			(0.00420)	(0.00420)
Behavioral Index VA			0.00400*	0.00388*			0.00371*	0.00367**
			(0.00208)	(0.00208)			(0.00207)	(0.00207)
Observations	67,595	67,595	67,595	67,595	67,571	67,571	67,571	67,571
R^2	0.0008	0.0010	0.0023	0.0025	0.0016	0.0018	0.0031	0.0033
% Increase in Explained Variation	–	26.73	–	7.52	–	8.96	–	4.25

Notes: Table 3 reports results from regressing a teacher's leave-one-year-out high school graduation value-added (VA) on a teacher's test-score VA in a particular subject. I report clustered standard errors at the teacher level in parentheses. Conventional (Unweighted) VA indicates test-score VA is calculated as the unweighted average of student residuals in a the subject indicated by the column heading. Weighted VA indicates test-score VA is calculated using the estimated weights shown in Figure 1. Suspension VA is defined as a teacher's out-of-sample VA for reducing number of days suspended. Behavioral Index VA is defined as a teacher's out-of-sample VA for increasing a student's behavioral index score (described in Appendix B). Columns 1,2, 4, and 5 represent analogous results from Table 2 restricted to teacher-years with non-missing non-cognitive VA measures. Columns 3,4,7, and 8 show results include the non-cognitive VA measures as controls. The percentage (%) increase in explained variation for each subject and whether or not I include non-cognitive VA measures as controls is calculated as the percentage change in the R-squared value of the regression when I use the weighted VA as the main regressor compared to when I use the conventional VA for the same subject as the main regressor. Limited to fourth and fifth grade classes to between 10 and 35 students. $p < 0.001^{***}, p < 0.05^{**}, p < 0.1^{*}$.

Figure 4: Estimated and Variance-Based Bin-Weights by Subject

(a) Math

(b) Reading



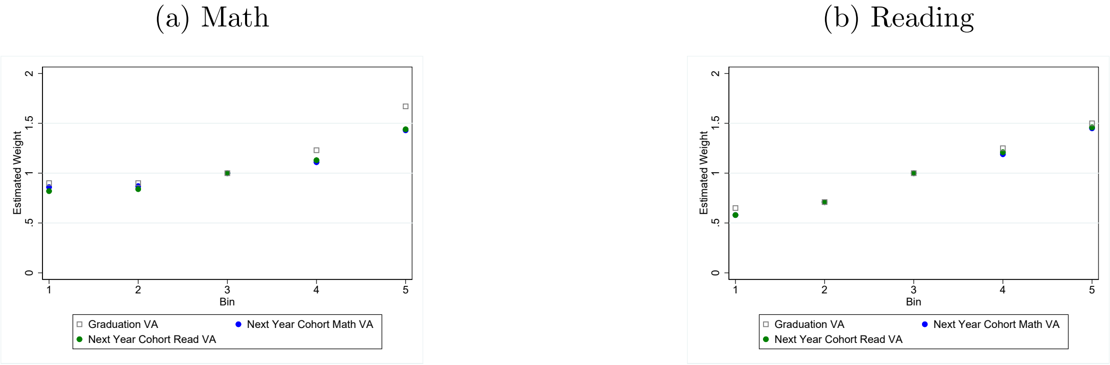
Notes: I report variance-only weights as gray squares in the above graph. The blue dots with error bars represent the initial estimated weights and their 95% confidence intervals from Figure 1. Figure 4a shows results for math, and Figure 4b shows shows results for reading. Limited to fourth and fifth-grade classes with between 10 and 35 students.

Table 4: Predictive Power of Weighted vs Unweighted VA: Joint Estimation

	Baseline		**Non-Cognitive**	
	(1)	(2)	(3)	(4)
Conventional Math VA	0.000143		0.000100	
	(0.000147)		(0.000146)	
Conventional Reading VA	0.000879***		0.000826***	
	(0.000158)		(0.000157)	
Weighted Math VA		0.000175		0.000135
		(0.000128)		(0.000128)
Weighted Reading VA		0.000827***		0.000779***
		(0.000138)		(0.000123)
Suspension VA			0.0111***	0.0111**
			(0.00420)	(0.00420)
Behavioral Index VA			0.00366*	0.00359*
			(0.00207)	(0.00208)
Observations	67,549	67,549	67,549	67,549
R^2	0.0017	0.0019	0.0031	0.0033
% Increase in Explained Variation	–	13.27	–	6.36

Notes: Table 4 reports results from regressing a teacher's leave-one-year-out high school graduation value-added (VA) on a teacher's test-score VA for both math and reading. I report clustered standard errors at the teacher level in parentheses. Conventional (Unweighted) VA indicates test-score VA is calculated as the unweighted average of student residuals for each subject. Weighted VA indicates test-score VA is calculated using the estimated weights predicting a teacher's impact on graduation using both math and reading test-score residuals. These weights are shown in Figure 3. Suspension VA is defined as a teacher's out-of-sample VA for reducing number of days suspended. Behavioral Index VA is defined as a teacher's out-of-sample VA for increasing a student's behavioral index score (described in Appendix B). Columns 1 and 2 report results without controlling for non-cognitive outcomes. In columns 3 and 4 I also control for a teacher's non-cognitive VA measures. The percentage (%) increase in explained variation for each subject and whether or not I include non-cognitive VA measures as controls is calculated as the percentage change in the R-squared value of the regression when I use the weighted VA as the main regressor compared to when I use the conventional VA for the same subject as the main regressor. Limited to fourth and fifth grade classes to between 10 and 35 students. $p < 0.001^{***}, p < 0.05^{**}, p < 0.1^{*}$.

Figure 5: Estimated Bin-Weights Predicting Subsequent Cohort VA


(a) Math

(b) Reading

Notes: Each dot represents an estimated weight. For each outcome, the weight on bin 3 is normalized to 1. I report initial weights from Figure 1 as hollow gray squares. Next Year Cohort indicates a teacher's estimated VA using math or reading scores for students in the next year. Figure 5a shows results for math, and Figure 5b shows shows results for reading. Limited to fourth and fifth-grade classes with between 10 and 35 students.

A Appendix A: Additional Tables and Figures

I include summary stats for my estimated value-added (VA) measures by subject and portion of the class below in Table A1. Each row indicates both which students are included in estimating a teacher's VA and whether the VA uses math or reading test scores. The mean of all VA measures is appropriately 0. I report the standard deviation (Std. Dev) of each VA measure. Column 1 indicates the students used in the calculation of each VA. Whole-Class, for example, indicates a teacher's VA is calculated using test-score impacts for all students, while Top 50% indicates a teacher's VA is calculated using only test-score impacts for students above the median of lagged achievement in a particular subject (relative to a student's year and grade) are included. Column 3 indicates the raw, or unadjusted VA measures calculated based on Equation 1 and Equation 4. To calculate an adjusted, or shrunk VA measure, I follow the procedure for estimating VA using all years according to Mulhern and Opper (2023). I assume there is no drift in teacher VA. I calculate the Empirical Bayes estimate for each teacher and subject, and report the Std. Dev of these measures in Column 4. This is equivalent to assuming that a teacher's impact on math or reading scores is independent of the same teacher's impact on the other subject's test scores. I denote this as 1D shrinkage. I relax this assumption in Column 5, which I denote 2D shrinkage. Finally, I report the number of teachers for which I have non-missing data for Column 5 in Column 6.

Table A1: Summary of Value-Added Measures by Dimensionality (Grades 3-5) Pooled Years

Group	Subject	Unadjusted Std. Dev	1D Std. Dev	2D Std. Dev	2D Teachers
(1)	(2)	(3)	(4)	(5)	(6)
Whole-Class (Typical VA)	Math	0.209	0.108	0.149	45,221
	Reading	0.155	0.0557	0.0711	45,221
Top 50%	Math	0.216	0.0851	0.132	35,763
	Reading	0.157	0.0282	0.0371	35,763
Bottom 50%	Math	0.230	0.102	0.146	41,209
	Reading	0.191	0.0541	0.0622	41,211
Top 30%	Math	0.224	0.0834	0.122	30,245
	Reading	0.169	0.0254	0.0298	30,246
Bottom 30%	Math	0.246	0.0969	0.140	38,988
	Reading	0.222	0.0529	0.0590	39,005
Top 25%	Math	0.230	0.0839	0.120	29,156
	Reading	0.177	0.0252	0.0296	29,156
Bottom 25%	Math	0.253	0.0948	0.135	38,026
	Reading	0.234	0.0529	0.0558	38,032

Notes: This table reports the student-weighted standard deviation of estimated teacher value-added (VA) measures for the students and subject indicated in columns 1 and 2. Top/Bottom XX% represent the split used to determine value-added (VA) for top and bottom students. For example, Top/Bottom 50% Math indicates I define top math students as students with a lagged math score above the median lagged math score within a particular school, grade, and year. I define Bottom 50% math students as students with a lagged math score below or equal to this same median. Column 3 represents the standard deviation (Std. Dev) of student residuals within a split and subject across teachers, or each teacher's unadjusted VA measure. For columns 4 and 5 I apply the methodology detailed in Mulhern and Opper (2023). In column 4 I assume a teacher's VA within a particular split is independent across math and reading. In column 5 I relax this assumption (which most aligns with Mulhern and Opper, 2023). I include the number of teachers included in my Column 5 estimates in Column 6. These estimates use all years available for a given teacher. I restrict to classrooms with between 10 and 35 students and to student test-scores in grades 4 and 5. Note that my effective sample is limited to 1998-2011 as there are no lagged test scores in the 1997 data, the earliest year I have available.

B Appendix B: Creating Students' Behavioral Index

In this section, I discuss my methodology for constructing a student's behavioral index. I first conduct a principal component analysis (PCA) using the number of (i) short-term suspension days, (ii) long-term suspension days, (iii) number of incidents leading to short-term suspension, (iv) number of incidents leading to long-term suspension, (v) number of incidents leading to expulsion, (vi) number of incidents leading to detention, (vii) number of incidents leading to privileges being revoked, (viii) number of incidents leading to other consequences, (ix) number of incidents leading to a 1-year suspension, and (x) number of days in which a student is sent home from school. I report summary statistics for these measures below as Table B1.

Table B1: Summary Statistics of Student Data: Behavior

Variable	Mean	SD	Min	Max
Short-term Suspension Days	0.152	1.000	0	58
Long-term Suspension Days	0.003	0.351	0	90
Short-term Suspensions	0.229	0.721	0	15
Long-term Suspensions	0.000	0.021	0	1
Times Expelled	0.000	0.005	0	1
Times Detention	0.001	0.032	0	6
Times Privileges Revoked	0.001	0.048	0	9
Times Other Consequences	0.001	0.030	0	4
Times Received 1-year Suspension	0.000	0.004	0	1
Times Sent Home	0.008	0.145	0	13

Notes: Table reports summary statistics for student behavioral outcomes used to construct my behavioral index. These summary statistics are for students who appear in the suspension data. I restrict my sample to students in classes with between 10 and 35 students. Data is available from 2001 to 2011. I exclude 2001 due to a lack of available lagged-behavioral information for students.

I report the results from the PCA estimation using one principal component below in Table B2. I generate a behavioral index as the predicted value from this PCA regression. I flip the sign of this index so that a more negative (i.e., lower) index represents worse behavior. Values closer to 0 represent better behavior or fewer disciplinary consequences.

When I merge this behavioral index onto the test-score data there are some students who do not appear in the suspension/disciplinary data. I interpret this as these students did not have any disciplinary incidents in a given year. I therefore assign these students as having a higher behavioral index than the best-behaving students with a non-missing behavioral index value. I then transform this behavioral index variable into a standard normal variable within each grade and year.

Table B2: Principal Component Analysis Loadings: Component 1

Variable	Component 1 Loading
Short-term Suspension Days	0.5317
Long-term Suspension Days	0.4345
Short-term Suspensions	0.5408
Long-term Suspensions	0.4686
Times Expelled	0.0459
Times Detention	0.0315
Times Privileges Revoked	0.0308
Times Other Consequences	0.0623
Times Received 1-year Suspension	0.0887
Times Sent Home	-0.0259
Eigenvalue	1.68
Variance Explained (%)	16.81

Notes: Table B2 shows results from a principal component analysis (PCA) on the variables included in the table. I include summary statistics for these outcomes above in Table B1. I use the predicted value from this PCA to generate an initial behavioral index for students. I multiply this inverse by -1 such that a behavioral index closer to 0 (smaller in absolute value) represent better-behaved students. Students not in the suspension data receive a behavioral index score higher than the best-behaved students in the suspension data.

C Appendix C: Documenting Differences in Teacher VA for High Versus Low Achieving Students

I now describe my methodology for documenting that teachers have different underlying, or true, test-score impacts for higher-achieving versus lower-achieving students. I first discuss how I define higher-achieving and lower-achieving students. I then describe the log-likelihood function I combine with a maximum likelihood estimator to estimate the correlation between a teacher's underlying VA for higher-achieving students and a teacher's underlying VA for lower-achieving students. I discuss my assumptions and then present results below in Table C1.

I divide students within a class into top and bottom students based on their lagged test-score in a given subject. I first calculate the distribution of lagged math and reading scores for all students within a given year, school, and grade. I then classify students into "top-" or "bottom-" performing students based on how their lagged achievement compares to this broader distribution. I use three different definitions of top vs. bottom students for each subject. First, I define a "Top/Bottom 50" split in which I classify a top student as having a lagged test score above the median, and a bottom student as having a lagged test score below the median. Second, I define a "Top/Bottom 30" split in which top students have a lagged achievement in the top 30th percentile, and bottom students have a lagged achievement in the bottom 30th percentile. Third, I define a "Top/bottom 25" split using the top and bottom 25th percentile to classify top and bottom students.

For each definition of top- and bottom-performing students, I estimate a teacher's VA for math and reading separately for top and bottom students according to equations 1 - 4. For example, I estimate a teacher's unadjusted math VA for top students using the Top/Bottom 50 split by estimating equations 1 - 4 restricting my sample to only students whose lagged math score is above the median grade-level score in a particular year. I estimate a standard error for this unadjusted VA measure calculated as the standard deviation of student residuals within a class and "group" (top versus bottom) divided by the square root of the number of students within that class and group. As an example, if there are 16 top students and the standard deviation of these 16 students' math residuals is 1, I calculate the standard error of a teacher's top unadjusted VA as 0.25.

I assume that the variance of each of these estimated VA measures is the variance of a teacher's true test-score impacts plus noise. I also assume that both the estimated VA and the noise in these estimated values are normally distributed. These assumptions imply that I can use a maximum-likelihood estimator (MLE) to estimate the correlation between teachers' true VA for top and bottom students by minimizing

$$v_{j,t}^{s} \cdot \Sigma_{j,t}^{s} \cdot v_{j,t}^{s}, \tag{12}$$

where $v_{j,t}^{s}$ is a vector of a teacher j's estimated VA in year t for subject s. This matrix $v_{j,t}^{s}$ is defined as

$$v = \begin{bmatrix} \hat{VA}_{j,t}^{top,s} \\ \hat{VA}_{j,t}^{bot,s} \end{bmatrix},$$

where $\hat{VA}_{j,t}^{p,s}$ represents the unadjusted VA for teacher j in subject s in year t for either top-performing students ($\hat{VA}_{j,t}^{top,s}$) or bottom-performing students ($\hat{VA}_{j,t}^{bot,s}$).

I define $\Sigma_{j,t}^{s}$ as the variance-covariance matrix between a teacher's true VA measures for top and bottom students, denoted as

$$\Sigma^{s} = \begin{bmatrix} \sigma_{VA_{top,s}}^{2} & \sigma_{VA_{top}VA_{bot,s}} \\ \sigma_{VA_{top}VA_{bot,s}} & \sigma_{VA_{bot,s}}^{2} \end{bmatrix}.$$

I present the results of these MLE estimates below in Table C1. Columns 1 and 2 represent the estimates from a baseline specification. In columns 3 and 4 I control for the average classroom achievement and variance in classroom achievement in order to account for differences in signal to noise ratio of student residuals due to class size. For both specifications the estimated correlation in true VA measures for top and bottom students is less than 1. This correlation becomes weaker as the definition of top versus bottom students becomes stricter. These results providence additional evidence teachers differ in their true test-score effects for students with different baseline levels of achievement.

Table C1: Estimated Latent Correlation Among Top/Bottom Value-Added

		Baseline		Classroom Moments	
Split	Subject	Correlation	Std. Error	Correlation	Std. Error
		(1)	(2)	(3)	(4)
Top/Bottom 50%	Math	0.836	0.00227	0.811	0.0024
	Reading	0.682	0.00431	0.645	0.00436
Top/Bottom 30%	Math	0.590	0.00463	0.590	0.00470
	Reading	0.439	0.00672	0.313	0.00743
Top/Bottom 25%	Math	0.509	0.00531	0.531	0.00524
	Reading	0.362	0.00722	0.212	0.00795

Notes: Top/Bottom XX% represent the split used to determine value-added (VA) for top and bottom students. For example, Top/Bottom 50% Math indicates I define top math students as students with a lagged math score above the median lagged math score within a particular school, grade, and year. I define bottom 50 math students as students with a lagged math score below or equal to this same median. Correlation (columns 1 and 3) and Std. Error (columns 2 and 4) represent the estimated correlation coefficient and standard error from a maximum likelihood estimator as described in Appendix C. VA is defined as the average of unadjusted residuals defined in Equation 1 restricted to either top or bottom students. Columns 1 and 2 do not include any additional controls. In columns 3 and 4 I include the class-level mean and variance of lagged achievement as additional controls. Restricted to fourth and fifth grade classes of between 10 and 35 students from 1998 to 2011.

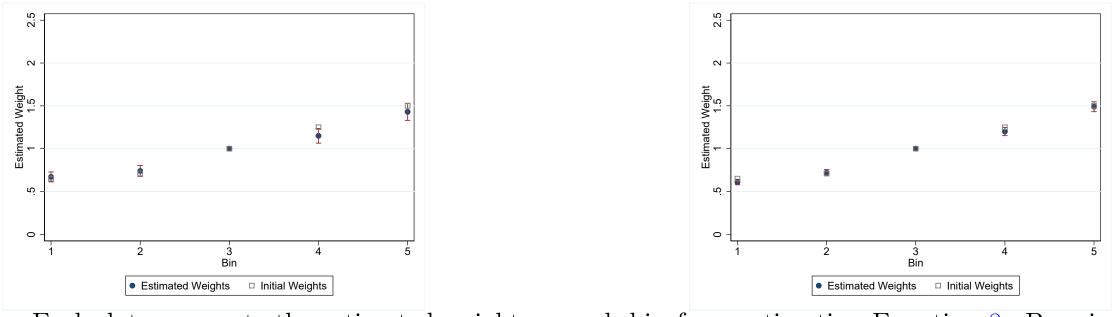
D Appendix D: Additional Weighted VA Estimates

In this section I discuss alternative bin estimates to investigate (i) the stability of my estimated bin-weights across outcomes and (ii) which of the three explanations could explain the observed pattern of weights. I begin by ruling out that differences in class-sizes is driving my results by estimating Equation 8 separately for smaller (10-19 students) and larger (20-35 students) classes.[7] I find no significant qualitative nor quantitative difference in my estimated weights for reading and math scores. I include those results below as Figure D1 (reading) and Figure D2 (math).

I present my estimated weights for smaller (10-19 students) and larger (20-35 students) classes below as Figure D1 and Figure D2. I do not observe differences in estimated weights for differently-sized classes, and these weights are similar to the estimated weights I report in Figure 1.

[7] Recall that I restrict all analyses to classes with between 10 and 35 students.

Figure D1: Reading Bin-Weight Estimates by Class Size



(a) Small (10-19 students) Classes

(b) Large (20-35 students) Classes

Notes: Each dot represents the estimated weight on each bin from estimating Equation 8. Bars indicate the 95% confidence interval around each estimated weight. The weight on bin 3 is defined to be 1. Figure 1a shows weights for student math residuals and Figure 1b shows weights for student reading residuals. Student residuals are calculated based on Equation 1. Limited to fourth and fifth-grade classes with between 20 and 35 students.

I next discuss additional sets of estimated weights showing noisiness of student residuals does not completely explain the weights from Figure 1. Perhaps the variance-only weights I report in Figure 4 do not fully explain my results because while I estimate variances across all years and classrooms, Equation 8 estimates average weights using classroom-year observations where the number of students in any given classroom and year is by definition limited to between 10 and 35 students. If I had larger classrooms, perhaps the weights I would obtain by estimating Equation 8 would more closely mirror the variance-only weights I show in Figure 4.

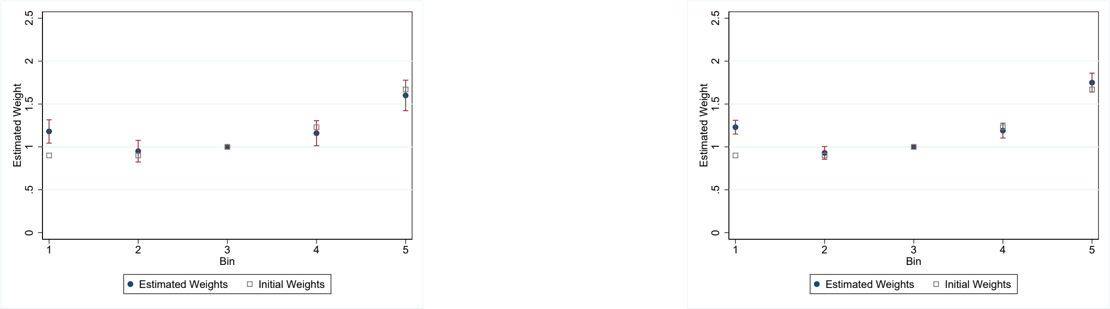
To test this, I artificially increase the number of students defined as being in the same classroom by estimating weights where I switch the years used to calculate a teacher's high school graduation VA and a teacher's test-score impacts. I use a teacher's estimated high school graduation VA using only students in year t. Instead of calculating VA^* using students in year t, I treat the data as if all students taught by teacher j not in year t were in one large class taught by teacher j in year t. I then estimate

$$\min_{\alpha_k} \left[VA^{grad}_{j,t} - \alpha_0 - VA^{*s}_{j,-t} \right]^2, \tag{13}$$

where $VA^{grad}_{j,t}$ teacher j's high school graduation VA calculated only for students in year t. $VA^{*s}_{j,-t}$ is defined as

$$VA^{*s}_{j,t} = \frac{1}{\sum_i \sum_k \beta_k \mathbf{1}\{i \in k\} \mathbf{1}\{i \in j\}} \sum_i \beta_k \mathbf{1}\{i \in k\} \mathbf{1}\{i \in j\} r^s_{i,-t}, \tag{14}$$

Figure D2: Math Bin-Weight Estimates by Class Size



(a) Small (10-19 students) Classes

(b) Large (20-35 students) Classes

Notes: Each dot represents the estimated weight on each bin from estimating Equation 8. Bars indicate the 95% confidence interval around each estimated weight. The weight on bin 3 is defined to be 1. Figure 1a shows weights for student math residuals and Figure 1b shows weights for student reading residuals. Student residuals are calculated based on Equation 1. Limited to fourth and fifth-grade classes with between 10 and 19 students.

or the weighted sum of residuals for all students who had teacher j in any year other than year t, which I denote $r^s_{i,-t}$.

I report estimated weights for this reverse leave-one-year-out approach below as Figure D3. I include the initial weights from Figure 1 as hollow gray squares for comparison. The bin weights for the first two bins are qualitatively similar for both math and reading. For math (Figure D3a), there is no statistical difference between these weights and the initial weight for bin 4. For math and reading, the weight on the highest-achieving students (bin 5) is higher than the corresponding weight from Figure 1.

D.1 Additional Estimates Regarding Efficient Use of Small Sample Explanations

In this section I present additional estimates concerning whether explanation 2 can explain my results. Recall that explanation two is also a story about small-sample efficiency. A teacher's impact on a particular bin may be more informative about a teacher's impact on students in other bins. The high weight on the highest bins may simply reflect that a teacher's impact on the highest-achieving students is also more predictive about a teacher's impact on other students within the class in other bins. This explanation does not rule out that a teacher may have different impacts on higher versus lower-achieving students, which is explanation 3.

If the weights shown in Figure 1 are driven by such an explanation, then the outcome measure I am trying to best predict using student residuals should not change the observed pattern of the weights. Suppose this explanation is true. The highest-achieving students receive a higher weight because a teacher's impact on their high school graduation is more informative about a teacher's impact on high school graduation for other students in the same class. If I estimate weights to predict, say, a teacher's impact on students' test-scores in the subsequent grade, I would still expect that a teacher's impact on the highest achieving students to be more informative of a teacher's impact on other students (to the extent that a teacher's high school graduation VA and future test-score VA are correlated). I would expect to observe a lower weight on lower-achieving students and a higher weight on the highest-achieving students for any outcome for which I can estimate a teacher's out of sample value-added.

I use a teacher's test-score VA for the next two years as my alternative outcomes. That is to say I estimate Equation 8 replacing a teacher's leave-one-year-out (LOYO) high school graduation VA with a teacher's LOYO VA on math and reading scores in both the next grade and the next next grade. For example, I estimate a third grade teacher's VA on her students' math and reading scores in fourth and fifth grade. I define a teacher's VA on her students' test scores in the next grade as a "1-year post" VA and scores in the next next grade as a "2-year post" VA. More formally, I define teacher j's pooled LOYO VA for test scores in subject s in future period $t+\tau$ as $\tilde{VA}^{s}_{j,t+\tau}$. I define $VA^{*}_{j,s,t}$ using student residuals for subject s in the current year (using Equation 5) and then estimate

$$\min_{(\Delta_k)s} \left[\tilde{VA}^{s}_{j,t+1} - \Delta_0 - VA^{*s}_{j,t}\right]^2, \tag{15}$$

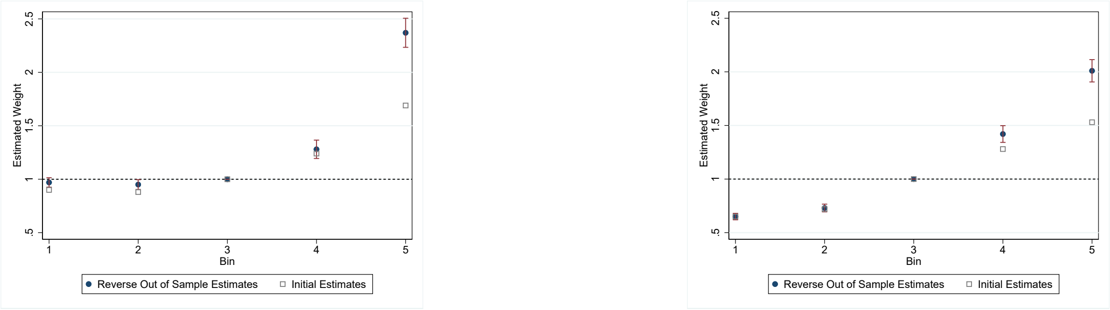
separately for each subject and each future time period.

I report results from estimating Equation 15 below in Figure D4. I include the initial weights from Figure 1 as hollow gray boxes. The results provide empirical support that this small-sample story may have a lot of explanatory power. I observe almost no differences in weights regardless of which outcome I estimate weights to try and predict.

The finding that the highest-achieving students are most predictive of outcomes for the lowest-achieving students warrants additional investigation. If this is a real result, I would also expect the highest-achieving students to receive the highest weight when I predict a teacher's impacts on test scores for any one particular group of students. To test this, I estimate weights that best predict a teacher's out-of-sample test-score VA for students in one particular bin. I repeat this for each of the five bins for both math and reading.

I calculate five bin-specific VA measures for each subject, one for each bin. For example, I calculate a teacher's bin 1 reading VA as a teacher's reading VA only for students who are

Figure D3: Reverse LOYO Bin-Weight Estimates by Subject


(a) Math

(b) Reading

Notes: The blue dots with error bars represent estimated leave-one-year-out weights and their 95% confidence intervals. I report the initial weights from Figure 1 as hollow gray squares. Figure D3a shows results for math, and Figure D3b shows shows results for reading. Limited to fourth and fifth-grade classes with between 10 and 35 students.

in the bottom 20th percentile of lagged achievement in reading. I then estimate

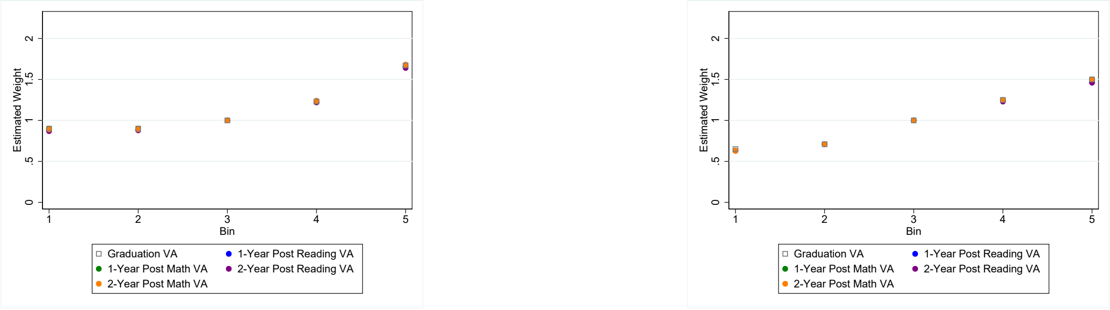
$$\min_{\Lambda_k} \left[\tilde{VA}^{s}_{j,b,-t} - \Lambda_0 - VA^{*s}_{j,t} \right]^2, \tag{16}$$

where $\tilde{VA}^{s}_{j,b,-t}$ is the LOYO bin-specific VA for teacher j in year t limited to students in bin b for subject s. $VA^{*}_{j,s,t}$ is the weighted average of student residuals in year t for subject s as defined in Equation 5.

I report estimation results for Equation 16 for each subject below as Figure D5. I include the initial Figure 1 weights as hollow gray squares. I observe the same pattern of weights for both subjects *regardless* of which bin-specific VA measure I am trying to predict.

I can again estimate an alternative set of weights using a teacher's impact on the next cohort of students (similar to my approach described in Section D.1). Here, I estimate each teacher's bin-specific VA for math and reading scores in the next year t+1. I use the residuals of all students in year t to estimate the weights that best predict each of the five possible bin-specific VA measures in $t+1$. I report results below as Figure D6. I find similar results to those shown in Figure D5. I table a more detailed discussion of the implications of these results to Section 6.

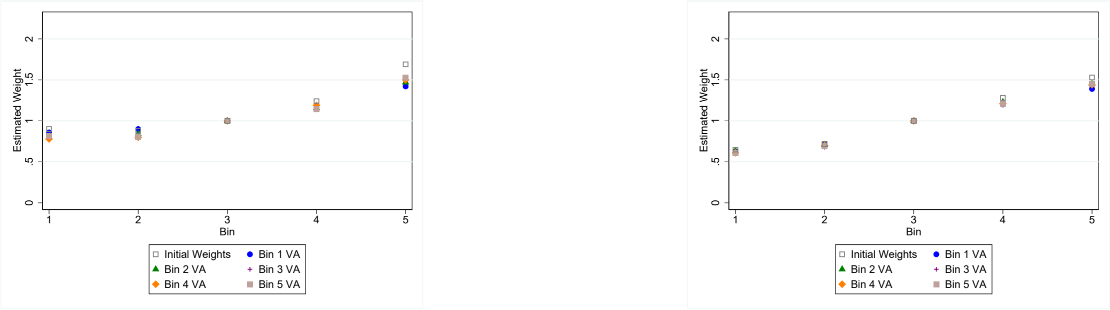
Figure D4: Estimated Bin-Weights Predicting Future Test-Score VA


(a) Math

(b) Reading

Notes: Each dot represents an estimated weight. For each outcome, the weight on bin 3 is normalized to 1. I report initial weights from Figure 1 as hollow gray squares. Figure 4a shows results for math, and Figure 4b shows shows results for reading. Limited to fourth and fifth-grade classes with between 10 and 35 students.

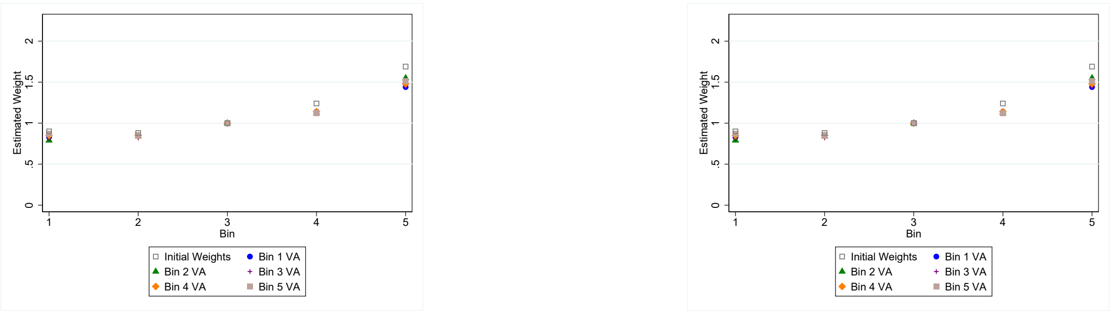
Figure D5: Estimated Bin-Weights Bin-Specific VA


(a) Math

(b) Reading

Notes: Each dot represents an estimated weight. For each outcome, the weight on bin 3 is normalized to 1. I report initial weights from Figure 1 as hollow gray squares. The legend indicates which students I used to calculate a teacher's out-of-sample test-score VA in each subject. For example, "Bin 1" in Figure D5a indicates I calculated a teacher's leave-one-year-out math VA using only the lowest-achieving students, or the students with a baseline standardized math test score in the bottom 20th percentile. This baseline measure is relative to standardized test scores in each subject at the school, grade, and year level. Figure D5a shows results for math, and Figure D5b shows shows results for reading. Limited to fourth and fifth-grade classes with between 10 and 35 students.

Figure D6: Estimated Bin-Weights Next Cohort Bin-Specific VA


(a) Math

(b) Reading

Notes: Each dot represents an estimated weight. For each outcome, the weight on bin 3 is normalized to 1. I report initial weights from Figure 1 as hollow gray squares. The legend indicates which students I used to calculate a teacher's test-score VA in the next year in each subject. For example, "Bin 1" in Figure D6a indicates I calculated a teacher's math VA in the next year using only the lowest-achieving students, or the students with a baseline standardized math test score in the bottom 20th percentile. This baseline measure is relative to standardized test scores in each subject at the school, grade, and year level. Figure D6a shows results for math, and Figure D6b shows shows results for reading. Limited to fourth and fifth-grade classes with between 10 and 35 students.

E Appendix E: Are High School Graduation Value-Added Measures Unbiased?

In this section, I investigate whether graduation value-added measures are approximately forecast unbiased. Previous literature has shown that test-score VA measures are approximately forecast unbiased, but it is not obvious a priori this statistical property extends to value-added measures using a long-run outcome nor a binary outcome such as high school graduation. To do so, I follow the methodology described in Chetty, Friedman, and Rockoff (2014a) for showing that test-score VA measures are approximately forecast unbiased.

I begin by showing that, on average, changes in the average high school graduation value-added within a school and grade are strongly correlated with changes in actual graduation rates for students in the same school and grade. I first estimate each teacher's high school graduation value-added for year t using students in all years except for current year t and previous year $t-1$. I then estimate the graduation rate for all students within a given school and grade for each year. I then estimate

$$\Delta \hat{VA}^{grad}_{s,g,-\{t,t-1\}} = \delta + \Delta Grad_{s,g,t} + \epsilon_{s,g,t}, \tag{17}$$

where $\Delta \hat{VA}_{s,g,-\{t,t-1\}}$ is the change (defined as year t minus year $t-1$) from year $t-1$ to t in the average high school graduation value-added for all grade g teachers in school s (leaving out students in years t and $t-1$). $\Delta Grad_{s,g,t}$ is the change in graduation rates for students in grade g and school s from year $t-1$ to year t. I weight this regression by the number of students.

I report results below in Table E1. I report baseline estimates in Column 1, and include year fixed effects in Column 2. I include year by school fixed effects in Column 3. My baseline results mirror results from Chetty, Friedman, and Rockoff (2014a)'s Table 4. The standard errors do not rule out that high school graduation VA is forecast unbiased at the 5% level. The bias implied by the coefficient estimate is (1-0.970) 3%. This bias is higher ($\sim$ 8%) when I include year fixed effects.

Table E1: Evaluating Forecast Unbiasedness of High School Graduation VA

	(1)	(2)	(3)
Change in Mean Teacher HS Grad VA	0.970***	0.917***	0.921***
	(0.0772)	(0.0744)	(0.0786)
Year Fixed Effects		X	
School x Year Fixed Effects			X
Observations	41,810	41,810	41,810
R^2	0.0012	0.0038	0.110

Notes:. Dependent variable in all columns is the change in the high school graduation rate for students in a given school and grade between year t and year $t-1$. The independent variable, Change in Mean Teacher HS Grad VA, is the change in the average teacher high school graduation within a school, grade, and year. I calculate this change as follows. First, I calculate each teacher's high school graduation value-added in each year excluding the current and previous year. I then calculate a student-weighted average high school graduation value-added for each school, grade, and year. I compute the difference in this average high school graduation between year t and year $t-1$. Column 1 does not include controls. In Column 2 I add year fixed effects. Column 3 includes school by year fixed effects. Limited to fourth and fifth grade classes to between 10 and 35 students. $p < 0.001^{***}, p < 0.05^{**}, p < 0.1^{*}$.

F Appendix F: Toy Model

F.1 Toy Example Estimation and Results

I step back from reality and construct a toy example in order to provide additional intuition as to why I might expect a weighted value-added measure to weight higher-achieving and lower-achieving students differently. In this toy example, suppose the long-run outcome of interest to the policy maker is whether or not a student graduates high school. Also suppose a student's latent, or underlying, propensity to graduate high school follows a normal distribution according to the model

$$PR(Graduated_{i,j,t}|\tilde{Y}_{i,s,t-1}) = \Phi(\delta_0 + \delta_1 \tilde{Y}_{i,s,t-1} + \delta_2 \tilde{Y}^2_{i,s,t-1} + \delta_3 \tilde{Y}^3_{i,s,t-1} + \tilde{X}_i \beta + \omega_{i,s,t}), \quad (18)$$

where $\tilde{Y}$ represents the lagged test score of student i from year $t-1$ in subject s. The vector of covariates $\mathbf{X}$ contains the same covariates as included in Equation 1.

From this data generating process, there are some students who are more at risk of not graduating high school than other students. Suppose that 80% of students graduate high school, as is the case in my North Carolina sample. The students who are more at risk of not graduating high school are more likely to be lower-achieving students. In a value-added setting, if the policy maker's goal is to maximize high school graduation, the optimal weighting scheme might place a higher weight on students more at risk of not graduating than on students who are high achieving. Higher-achieving students are more likely to graduate high school irrespective of having a teacher who has a high impact on high school graduation, and may receive a lower weight according to this framework.

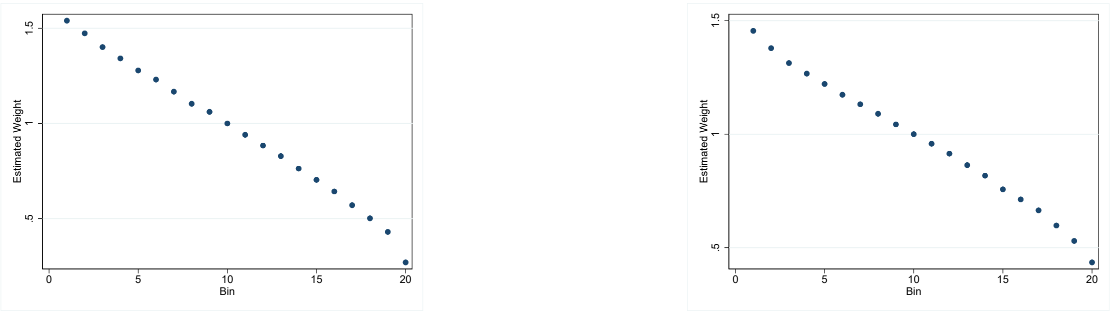
I derive a set of empirical weights according to this framework by directly estimating the relationship between lagged achievement and a student's marginal propensity to graduate high school. First, I estimate Equation 18 in order to obtain each student's propensity to graduate. I then determine the slope of the normal distribution at each student's estimated latent propensity to graduate. This slope gives an approximation of how "at risk" a particular student is of not graduating high school. A steeper slope indicates a slightly better teacher might make the difference between a student graduating high school or not. Student's with the highest marginal propensity to graduate receive the highest weight. I normalize the weights within each classroom such that the sum of weights within a classroom sum to 1.

I estimate marginal propensity to graduate, and therefore student weights, separately using lagged math and reading scores. I divide students into vintiles (20 bins) of lagged achievement in each subject and report the average estimated weight within each bin. I then re-scale the weights such that the weight on the middle bin (bin 10) is equal to 1. Weights higher than 1 therefore represent students who are more at-risk, and therefore should receive a higher weight, than the median student. Weights lower than 1 similarly represent students less at-risk and students who should receive a lower weight than the median student. I present these results below as Figure F1.

I make the following suppositions in this toy example. First, I suppose a teacher's value-added (VA) in each bin is independent of a teacher's VA in every other bin. Second, that a teacher's VA for a student's particular bin is the only factor that affects that particular student's probability of graduating high school. I present results of this toy example using both math and reading lagged test scores as Figure F1.

These toy example weights illustrate that, in a scenario in which students' risk of not graduating are correlated with lagged achievement, a weighted VA may increase the explanatory power of a teacher's long-run impact compared to an unweighted, conventional VA measure. The weights for both math and reading in Figure F1 imply that students with the lowest lagged achievement, those at the highest risk of not graduating, receive the highest weight. These weights decrease in a non-linear fashion as lagged achievement increases. The weights are lowest for students with the highest lagged-achievement. The weights imply that highest-achieving students should be weighted about 25%-33% as much as the median student, which suggests that even the best students are slightly at risk of not graduating high school. I now turn to the data to empirically decide how different students should be weighted within a VA model.

Figure F1: Toy Model Estimated Weights By Vintiles of Lagged Achievement



(a) Math Weights

(b) Reading Weights

Notes: Each dot represents the average weight on students with a lagged achievement within a particular vintile of lagged achievement. Figure F1a shows weights based on lagged math achievement, and Figure F1b shows weights based on lagged reading achievement. Weights are calculated as the estimated slope in a student's predicted latent, or underlying, propensity to graduate according to the model given by Equation 18. Weights are normalized within each classroom such that the sum of student-level weights sum to one within each classroom. I rescale weights such that the weight on the bin 10 is equal to 1. I restrict my analysis to fourth and fifth grades in classrooms with between 10 and 35 students between 1998 and 2011.