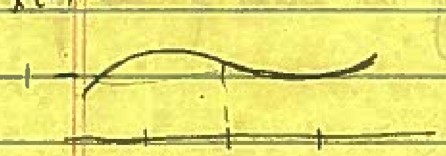
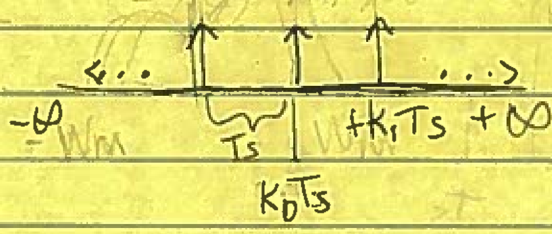
Casey Alvarado

Pset 8

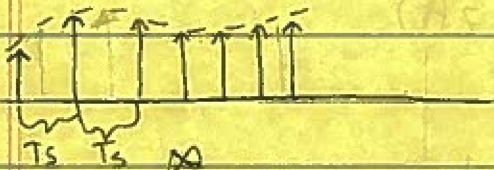
1.a.) $x_p(t) = x(t)p(t)$ $p(t) = \sum_{K=-\infty}^{\infty} \delta(t - KT_s)$

$x(t)$



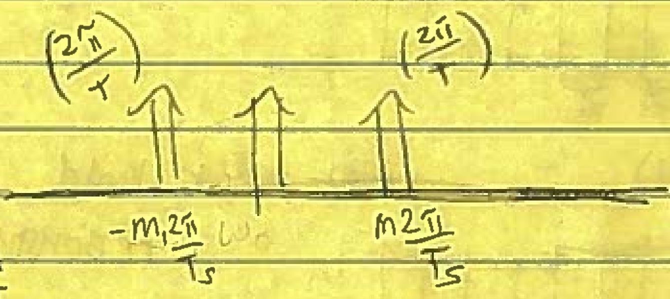


$x_p(t) = x(t)p(t)$



b.) $p(t) = \sum_{K=-\infty}^{\infty} \delta(t - KT_s)$

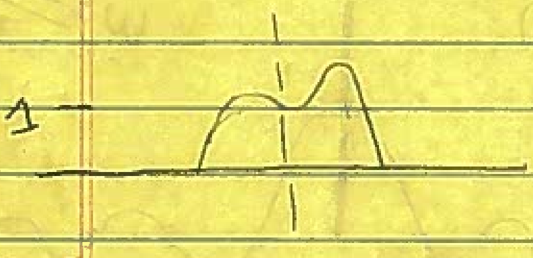
from Pset 6, problem 1d * $P(\omega) = \sum_{m=-\infty}^{\infty} \frac{2\pi}{T_s} \delta(\omega - \omega_0)$ where $\omega = \frac{2\pi}{T_s} m$ ← frequency



c.) $x_p(t) = x(t)p(t)$

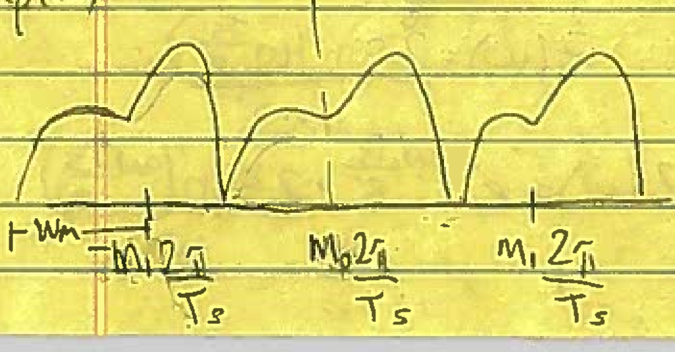
$X_p(\omega) = X(\omega) * P(\omega)$

$X(\omega)$



$*$ $P(\omega) = \sum_{m=-\infty}^{\infty} \frac{2\pi}{T_s} \delta(\omega_m - \omega_0)$ where

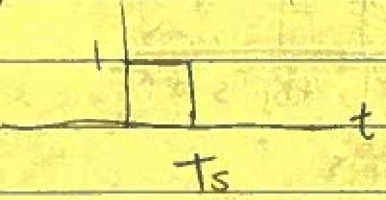
$X_p(\omega)$



d.) $2W_m > T_s$ because if not signals overlap and that means we loose information.

e.) Apply $p(t)$ again? and then apply a low pass filter.

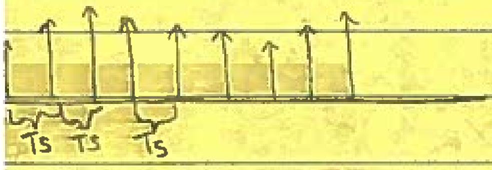
f.) $z(t)$



g.) $x_z(t) = x_p * z(t)$

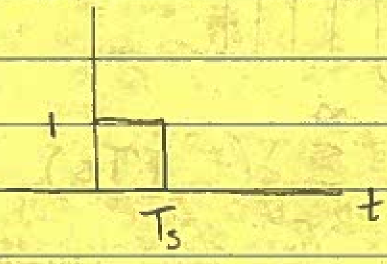
$x_p(t) = x(t)p(t)$

$p(t)$

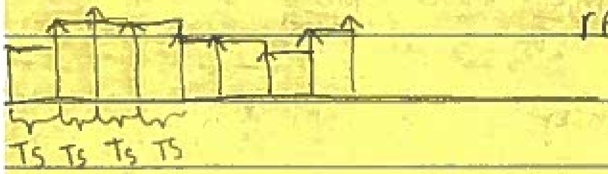


$*$

$z(t)$



$x_z(t)$ zero-order hold reconstruction



$W_m < \frac{\pi}{T_s}$

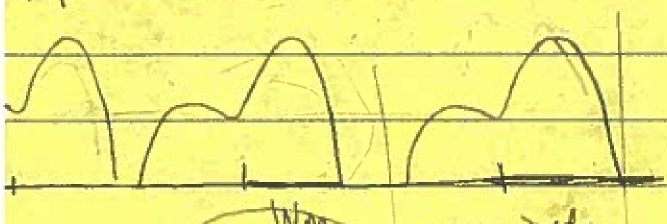
$\sin\left(\omega \frac{T_s}{2}\right) = 0$

$\omega \frac{T_s}{2} = k \cdot \pi$

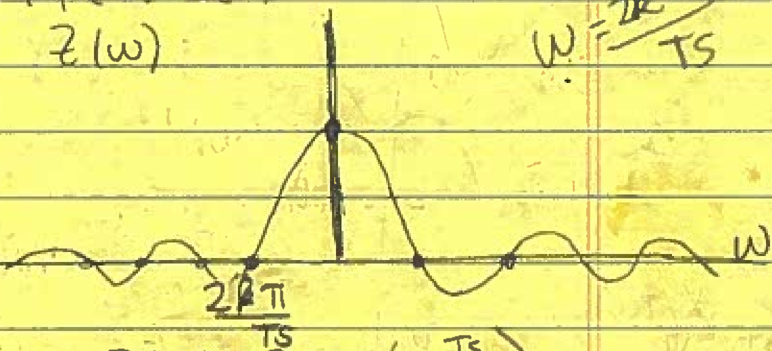
$\omega = \frac{2k\pi}{T_s}$

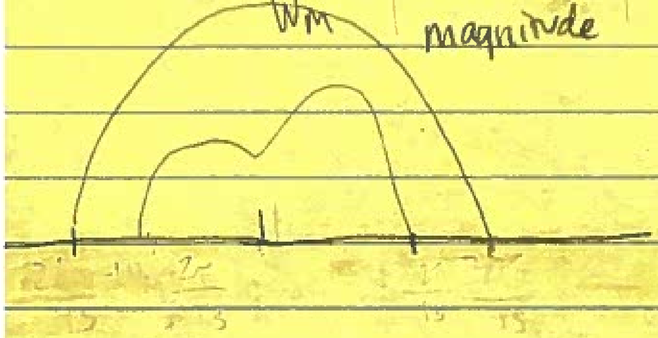
1h.) $X_z = x_p * z(t) \rightarrow X_z(\omega) = X_p(\omega)\, Z(\omega)$

$X_p(\omega)$



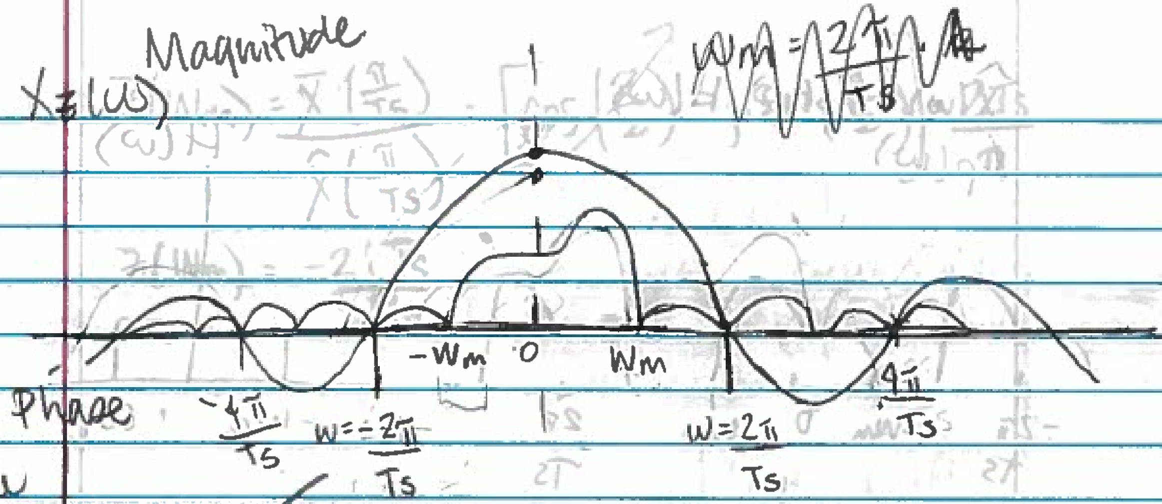
$Z(\omega)$

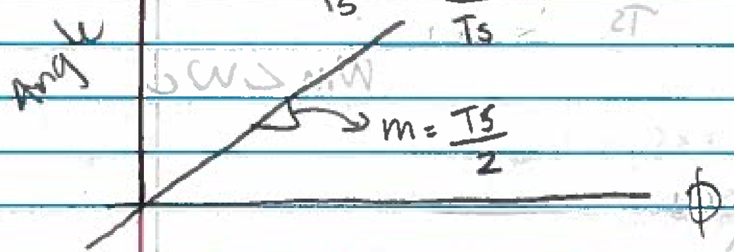




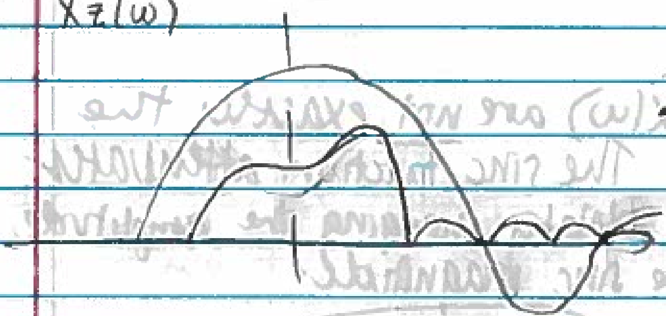
$Z(\omega) = \frac{2\sin\left(\omega \frac{T_s}{2}\right)}{\omega}$

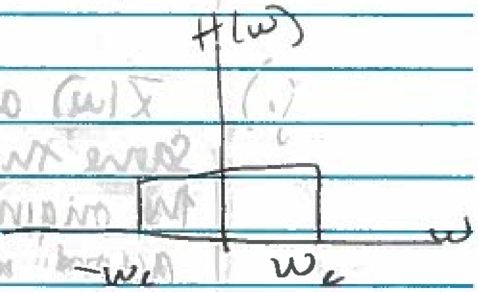
$Z(\omega) = e^{-j\omega \frac{T_s}{2}} \cdot \frac{2\sin\left(\frac{\omega T_s}{2}\right)}{\omega}$





i.) $X(\omega) = X_Z(\omega)H(\omega)$ $\qquad$ $\hat{X}(\omega) = X_p(\omega)H(\omega)$



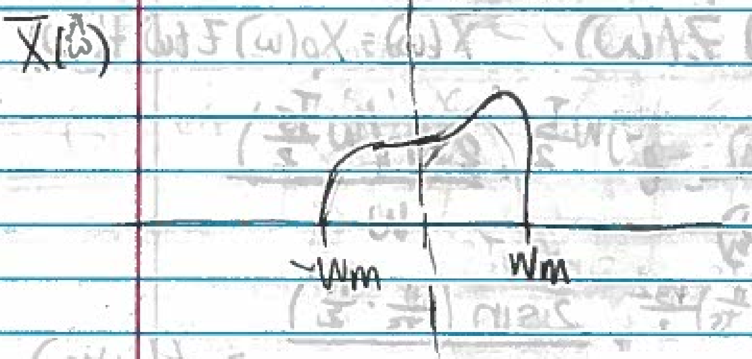


$W_m < \frac{2\pi}{T_S} - W_m$

$2W_m < \frac{2\pi}{T_S}$

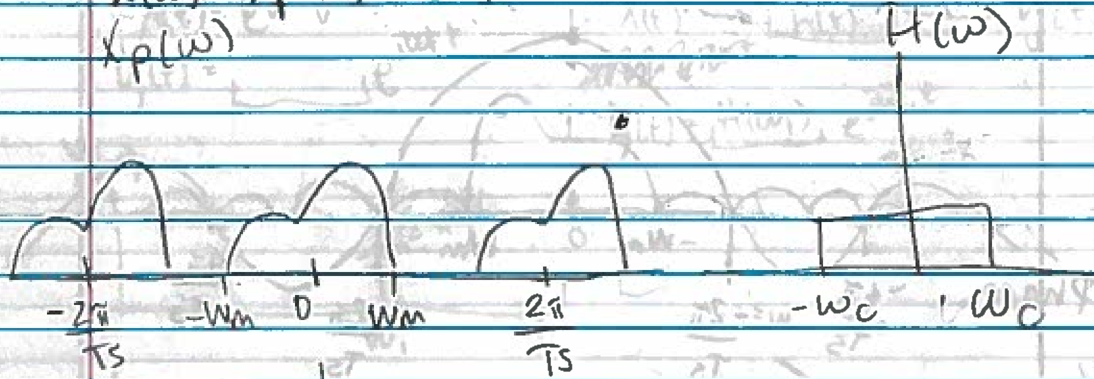
$w_c = \frac{\pi}{T_S}$

$W_m < W_c$

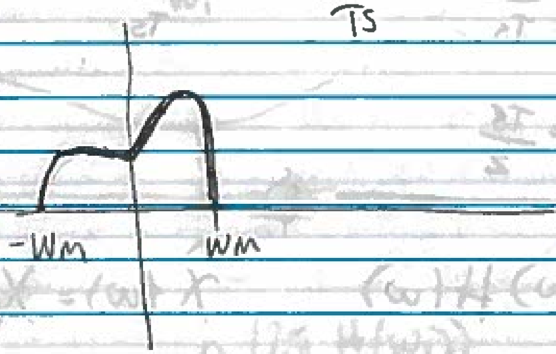


$\hat{X}(\omega) = X_p(\omega) H(\omega)$

$X_p(\omega)$



$W_m < \omega_c$



j.) $\bar{X}(\omega)$ and $\hat{X}(\omega)$ are not exactly the same thing! The sinc function attenuates the original blob by changing the amplitude according to the sinc magnitude.

k.) $\bar{X}(\omega) = \hat{X}(\omega)\, Z(\omega)$ ← $\bar{X}(\omega) = X_p(\omega)\, Z(\omega)\, H(\omega)$

$$Z(\omega) = \frac{\bar{X}(\omega_M)}{\hat{X}(\omega_M)} = e^{-j\omega \frac{T_s}{2}} \cdot \frac{2\sin\left(\omega \frac{T_s}{2}\right)}{\omega}$$

$$\frac{\bar{X}\left(\frac{\pi}{T_s}\right)}{\hat{X}\left(\frac{\pi}{T_s}\right)} = e^{-j\left(\frac{\pi}{T_s}\right)\frac{T_s}{2}} \cdot \frac{2\sin\left(\frac{\pi}{T_s}\cdot\frac{T_s}{2}\right)}{\frac{\pi}{T_s}} = Z(\omega_M)$$

$$Z(\omega_m) = \frac{\bar{X}\left(\frac{\pi}{T_s}\right)}{\hat{X}\left(\frac{\pi}{T_s}\right)} = e^{-j\frac{\pi}{2}} \cdot \frac{2T_s\sin\left(\frac{\pi}{2}\right)}{\pi}$$

$$Z(W_m) = \frac{X\left(\frac{\pi}{T_S}\right)}{\hat{X}\left(\frac{\pi}{T_S}\right)} = \left[\cancel{\cos\left(\frac{\pi}{2}\right)} - j\sin\left(\frac{\pi}{2}\right)\right] \cdot \frac{2T_S}{\pi}$$

$$Z(W_m) = \frac{-2jT_S}{\pi}$$

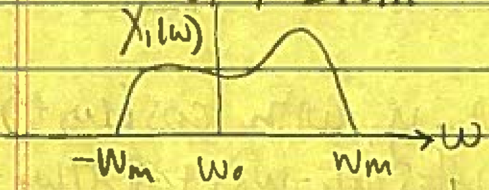
2a.) $y(t) = x_1(t)\cos(w_1 t) + x_2(t)\cos(w_2 t)$

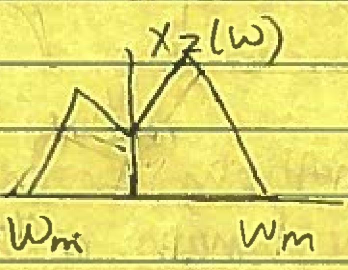
$X_1(w) = 0$ $\quad$ $X_2(w) = 0$

if $|w| > W_m$

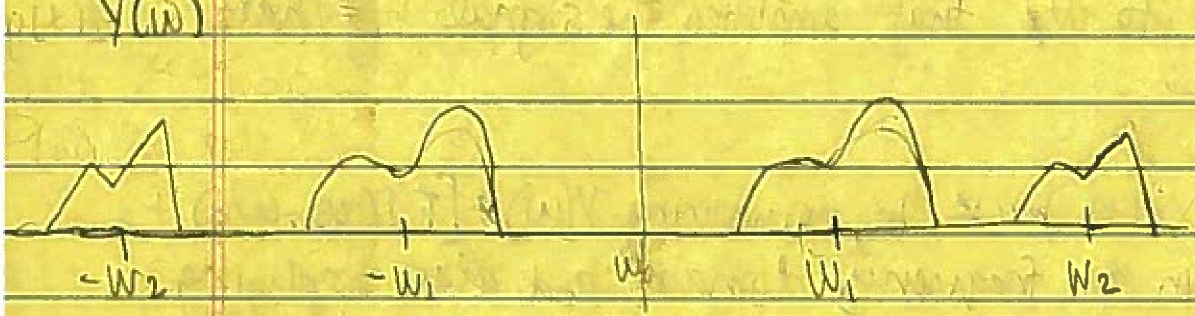
$w_1 >> W_m$ and $w_2 >> W_m$ and

$w_1 + 2W_m < w_2$



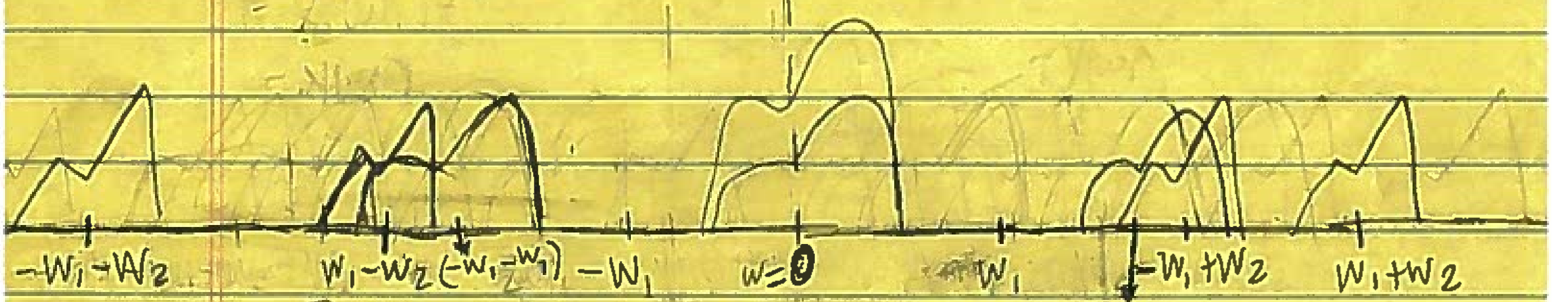


$$Y(w) = X_1(w_1) * [\pi\delta(W_1 - W_0) + \pi\delta(W_1 + W_0)] + X_2(w_2) * [\pi\delta(W_2 - W_0) + \pi\delta(W_2 + W_0)]$$

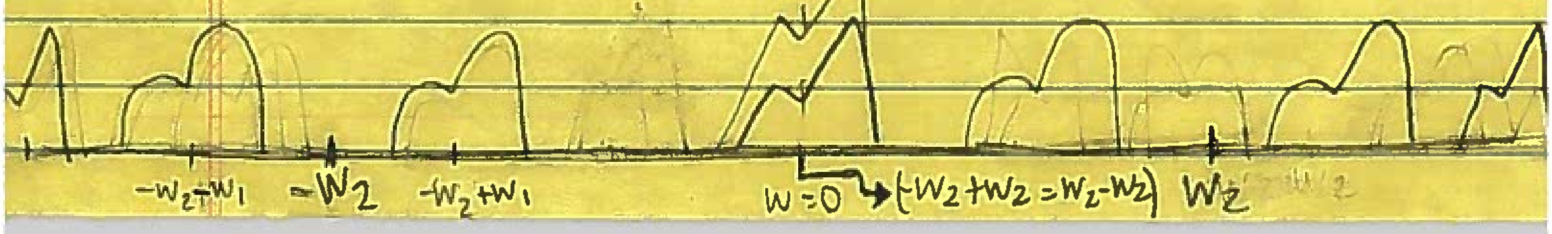


b.) $y(t)\cos(w_1 t) = [x_1(t)\cos(w_1 t) + x_2(t)\cos(w_2 t)]$

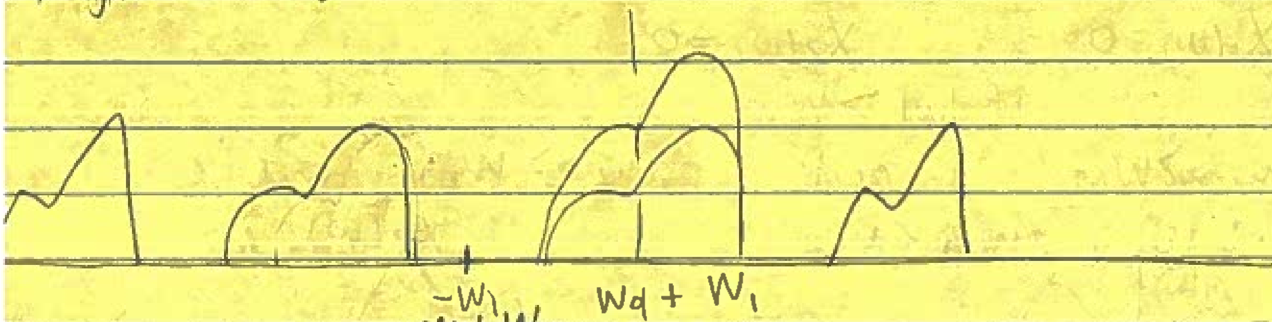
$F\{y(t)\cos(w_1 t)\} = Y(w) * [\pi\delta(W_1 - W_0) + \pi\delta(W_1 + W_0)]$



$F\{y(t)\cos(w_2 t)\} = Y(w) * [\pi\delta(W_2 - W_0) + \pi\delta(W_2 + W_0)]$



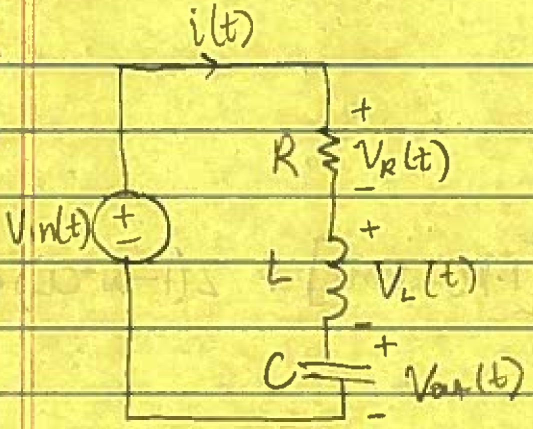
2.7 $y_1 \cos(w_1 t) =$



If we multiply the original signal, y, with $\cos(w_1 t)$ in the time domain or convolve $Y(\omega) * [\pi\delta(W_1 - W_0) + \pi\delta(W_2 - W_0)]$ in the frequency, Then we can see that the original $X_1(t)$ multiply in amplitude but are centered around $w = 0$. We can simply apply a bandpass filter ~~to~~ from $-W_1$ to W_1 that amplifies the signal by half the amplitude.

We can get $X_2(t)$ back by convolving $Y(\omega) * [\pi\delta(W_2 - W_0) + \pi\delta[W_2 - W_0)]$ in the frequency domain and also applying a bandpass from $-W_2$ to W_2.

3.



$$i(t) = C\frac{\partial}{\partial t}V_{out}(t)$$

$$V_L(t) = L\frac{\partial}{\partial t}i(t)$$

$V_{out}(t)$ and $V_{in}(t)$

$$V_{in}(t) = V_R(t) + V_L(t) + V_{out}(t)$$

$$V_{in}(t) = R\,i(t) + L\frac{\partial}{\partial t}i(t) + V_{out}(t)$$

$$V_{in}(t) = RC\frac{\partial}{\partial t}V_{out}(t) + L\frac{\partial}{\partial t}\left[C\frac{\partial}{\partial t}V_{out}(t)\right] + V_{out}(t)$$

$$\boxed{V_{in}(t) = RC\frac{\partial}{\partial t}V_{out}(t) + CL\frac{\partial^2}{\partial t^2}V_{out}(t) + V_{out}(t)}$$

b.) $V_{in}(\omega) = RC(j\omega)V_{out}(\omega) + CL(j\omega)^2V_{out}(\omega) + V_{out}(\omega)$

$$H(\omega) = \frac{V_{out}(\omega)}{V_{in}(\omega)}$$

$$V_{in}(\omega) = V_{out}(\omega)\left(j\omega RC + (j\omega)^2CL + 1\right)$$

$$\boxed{\frac{V_{out}(\omega)}{V_{in}(\omega)} = \frac{1}{j\omega RC + (j\omega)^2CL + 1} = H(\omega) = \frac{1}{j\omega RC - 1\cdot\omega^2CL + 1}}$$

c.) $|H(\omega)| = \dfrac{1}{|j\omega RC - 1\cdot\omega^2CL + 1|} = \dfrac{1}{\sqrt{(j\omega RC)^2 + (-1\cdot\omega^2CL + 1)^2}}$

$$|H(\omega)| = \frac{1}{\sqrt{(\omega RC)^2 + (1-\omega^2CL)^2}}$$

d.) $|H(w)| = \dfrac{1}{\sqrt{(wRC)^2 + (1-w^2CL)^2}}$

To maximize

$$\frac{\partial}{\partial w}\left((wRC)^2 + (1-w^2CL)^2\right) = 2(wRC)\cdot[1\cdot RC - w\cdot 0] + 2(1-w^2CL)\cdot[0 - 2wCL - \cancel{w^2 0}]$$

$$\frac{\partial}{\partial w}\left((wRC)^2 + (1-w^2CL)^2\right) = 2wRC\cdot RC + 2(1-w^2CL)\cdot(-2wCL)$$

$$0 = 2w(RC)^2 - 4wCL - 2w^2CL\cdot -2wCL$$

$$0 = \cancel{2w}(RC)^2 - \cancel{4w}CL + \cancel{4}w^{\cancel{3}2}(CL)^2 = (RC)^2 - 2CL + 2w^2(CL)^2$$

$$2w^2(CL)^2 = -(RC)^2 + 2CL$$

$$w^2 = \frac{-(RC)^2 + 2CL}{2(CL)^2} \rightarrow w = \sqrt{\frac{-(RC)^2 + 2CL}{2(CL)^2}} = \sqrt{\frac{-\cancel{C}(CR^2 + 2L)}{\cancel{C}(2CL^2)}}$$

$$W = \sqrt{\frac{-(CR^2 + 2L)}{2CL^2}}$$

e.) $H(w) = \dfrac{1}{\sqrt{(wRC)^2 + (1-w^2CL)^2}}$ Magnitude $= (w, |H(w)|)$

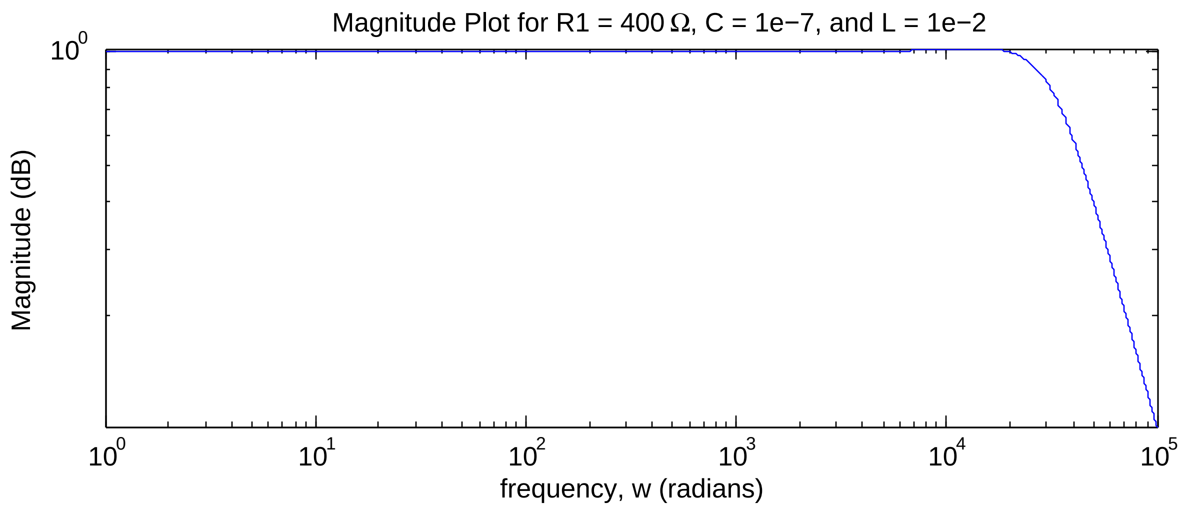
Plot attached.

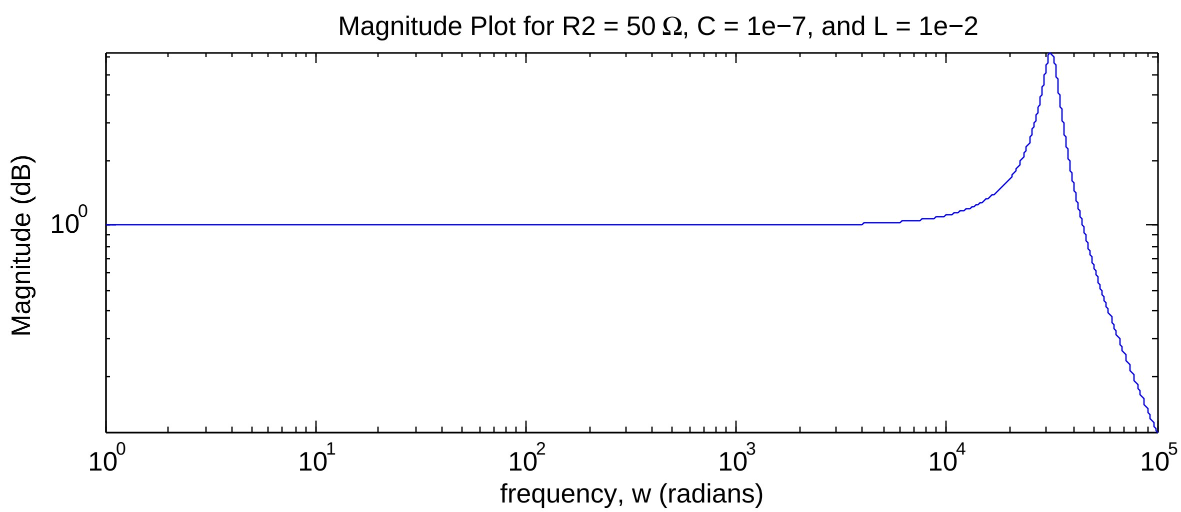
finding the phase

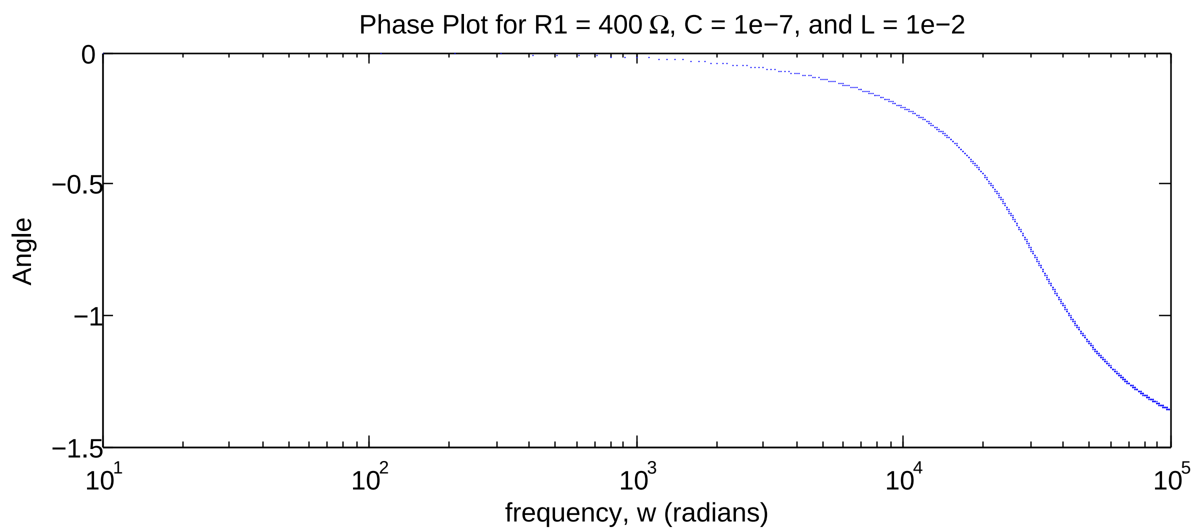
$$H(w) = \frac{1}{\sqrt{(wRC)^2 + (1-w^2CL)^2}} \cdot \frac{\sqrt{(wRC)^2 - (1-w^2CL)^2}}{\sqrt{(wRC)^2 - (1-w^2CL)^2}} = \frac{\sqrt{(wRC)^2 - (1-w^2CL)^2}}{(wRC)^4 - (1-w^2CL)^4}$$

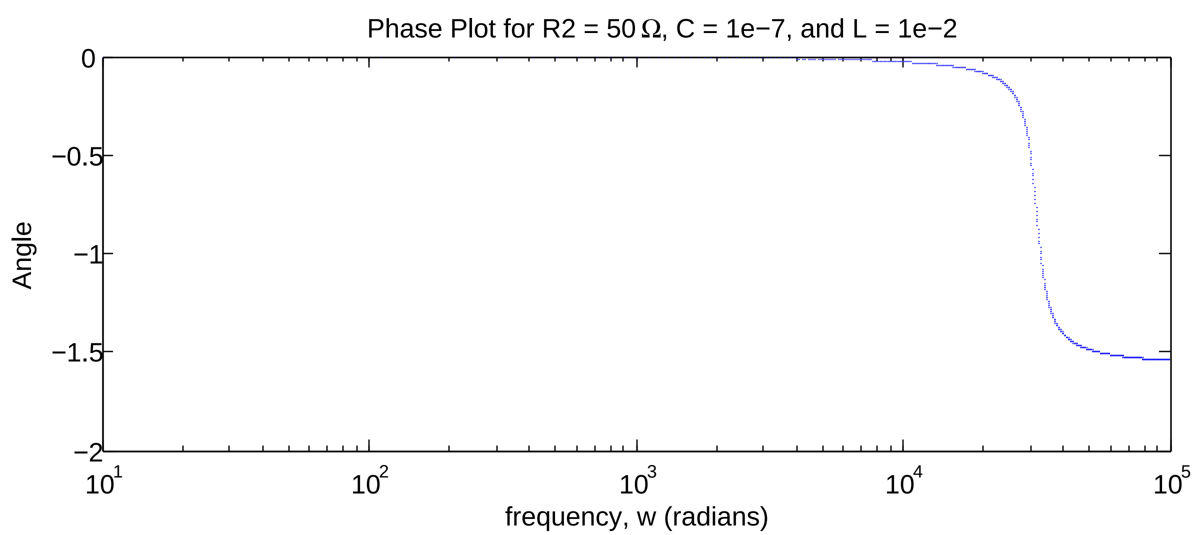
Phase $= H\measuredangle = \measuredangle 1 - \measuredangle[-w^2LC + RCjw + 1]$

$\measuredangle H(w) = 0 - \tan^{-1}\left(\dfrac{RCw}{1-LCw^2}\right)$ plot in MATLAB included.


Magnitude Plot for R1 = 400 Ω, C = 1e−7, and L = 1e−2
Magnitude (dB)
10^0
10^0
10^1
10^2
10^3
10^4
10^5
frequency, w (radians)


Magnitude Plot for R2 = 50 Ω, C = 1e−7, and L = 1e−2
Magnitude (dB)
10^0
10^0
10^1
10^2
10^3
10^4
10^5
frequency, w (radians)


Phase Plot for R1 = 400 Ω, C = 1e−7, and L = 1e−2
Angle
0
−0.5
−1
−1.5
10^1
10^2
10^3
10^4
10^5
frequency, w (radians)


Phase Plot for R2 = 50 Ω, C = 1e−7, and L = 1e−2
Angle
0
−0.5
−1
−1.5
−2
10^1
10^2
10^3
10^4
10^5
frequency, w (radians)