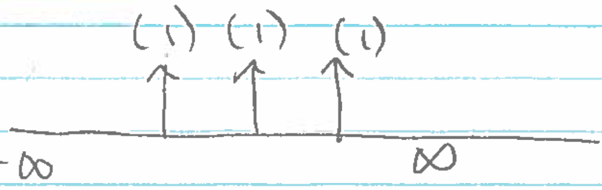
Casey Alvarado

SigSys PS07

1a.) $p(t) = \sum_{k=-\infty}^{\infty} \delta(t-kT)$



1b.) $p(t) = \sum_{k=-\infty}^{\infty} \delta(t-kT)$

$$C_m = \frac{1}{T}\int_{-T/2}^{T/2} p(t)\, e^{-j\frac{2\pi}{T}mt}\, dt = \int_{-T/2}^{T/2} \left[\sum_{k=-\infty}^{\infty} \delta(t-kT)\right] e^{-j\frac{2\pi}{T}mt}$$

$$C_K = \frac{1}{T}\sum_{K=-\infty}^{\infty} \int_{-T/2}^{T/2} \delta(t-kT)\, e^{-j\frac{2\pi}{T}mt}$$

$$\delta(t-kT) = \begin{cases} \infty \ (A=1) & t = kT \\ 0 \ (A=0) & \text{otherwise} \end{cases}$$

$$C_m = \frac{1}{T}\sum_{k=-\infty}^{\infty} \int_{-T/2}^{T/2} \delta(t-kT)\, e^{-j\frac{2\pi}{T}mt}\, dt$$

If $\delta(t-kT)$ at every T, there can only be one impulse in this integration window at $t=0$

when $t \neq 0$

$$C_m = \frac{1}{T}\sum_{K=-\infty}^{\infty} \int_{-T/2}^{T/2} 0 \cdot e^{-j\frac{2\pi}{T}mt}\, dt$$

when $t=0$

$$C_m = \frac{1}{T}\sum_{K=-\infty}^{\infty} \int_{-T/2}^{T/2} \delta(t-kT)\, e^{-j\frac{2\pi}{T}mt}\, dt = \frac{1}{T}\sum_{k=-\infty}^{\infty} e^{-j\frac{2\pi}{T}m(kt)}$$

$$t - kT = 0 \rightarrow t = kT = 0 \rightarrow k = 0$$

$$C_m = \frac{1}{T}\sum_{k=0}^{0} e^{-j\frac{2\pi}{T}m(0)t} = \frac{1}{T} \cdot 1$$

$$P_m(t) = \sum_{k=-M}^{M} C_m e^{j\frac{2\pi}{T}kt}, \quad C_m = \frac{1}{T}$$

$$P_m(t) = \sum_{m=-\infty}^{\infty} \frac{1}{T} \cdot e^{j\frac{2\pi}{T}mt}$$

1c.)

$$x(t) = \sum_{k=-\infty}^{\infty} C_K e^{j\frac{2\pi}{T}kt}, \quad X(\omega) = \int_{-\infty}^{\infty} x(t) e^{-j\omega t} dt$$

$$X(\omega) = \int_{-\infty}^{\infty} \left(\sum_{k=-\infty}^{\infty} C_K e^{j\frac{2\pi}{T}kt} \right) \cdot e^{-j\omega t} dt$$

$$X(\omega) = \sum_{K=-\infty}^{\infty} \int_{-\infty}^{\infty} C_K e^{j\frac{2\pi}{T}kt} \cdot e^{-j\omega t} dt \approx \int_{-\infty}^{\infty} C_K \cdot x(t) e^{-j\omega t} dt$$

$$x(t) = e^{j\frac{2\pi}{T}kt}, \quad \omega = \frac{2\pi}{T}k,$$

$$X(\omega) = \sum_{K=-\infty}^{\infty} C_K \int_{-\infty}^{\infty} e^{j\omega t} \cdot e^{-j\omega t} dt = \sum_{K=-\infty}^{\infty} C_K \cdot 2\pi\delta(\omega - \omega_0) \text{ where } \omega = \frac{2\pi}{T},$$

1d.)

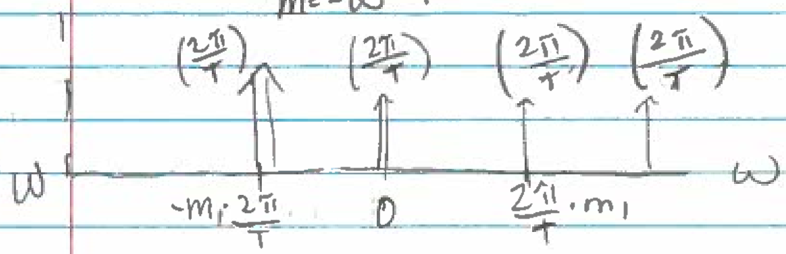
$$P(\omega) = \int_{-\infty}^{\infty} P_m(t) e^{-j\omega t} dt = \cancel{\int_{-\infty}^{\infty} \left(\sum_{k=-\infty}^{\infty} \delta(t - kT) \right) \cdot e^{-j\omega t} dt}$$

$$P(\omega) = \int_{-\infty}^{\infty} \left(\sum_{m=\infty}^{\infty} \frac{1}{T} \cdot e^{j\frac{2\pi}{T}mt} \right) \cdot e^{-j\omega t} dt, \quad \omega = \frac{2\pi}{T}m$$

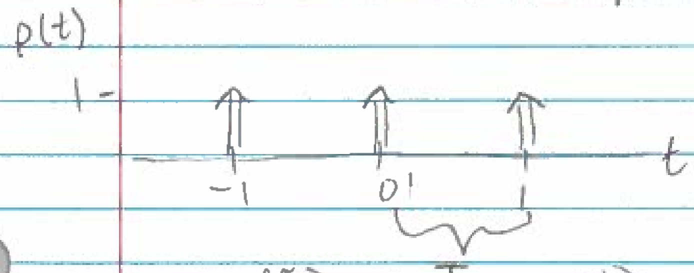
$$P(\omega) = \sum_{m=-\infty}^{\infty} \int_{-\infty}^{\infty} \frac{1}{T} \cdot e^{j\omega t} \cdot e^{-j\omega t} dt = \sum_{m=-\infty}^{\infty} \frac{1}{T} \int_{-\infty}^{\infty} e^{j\omega t} \cdot e^{-j\omega t} dt$$

$$P(\omega) = \sum_{m=-\infty}^{\infty} \frac{1}{T} \cdot 2\pi\delta(\omega - \omega_0) \text{ where } \omega = \frac{2\pi}{T}m$$

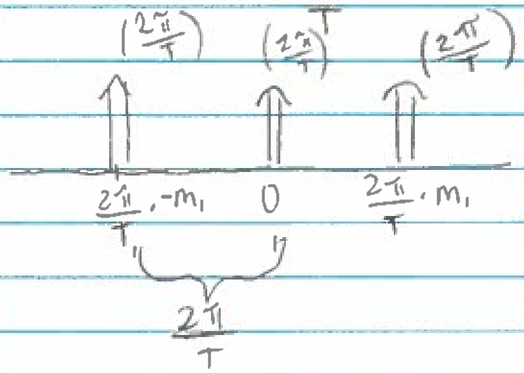
1e.) $P(\omega) = \sum_{m=-\infty}^{\infty} \frac{2\pi}{T} \cdot \delta(\omega - \omega_0)$ where $\omega = \frac{2\pi}{T} m$



Changing T would affect the coefficient of $p(t)$. A $1/T$ relationship means increasing the T would decrease the coefficient and vise versa. This is the same for $P(\omega)$ since $P(\omega)$ depends on $(1/T)$.



In the time domain, each impulse is T apart.



In the frequency domain, each impulse is $\frac{2\pi}{T}$ apart.

As seen here if we increase T in $p(t)$ we can make the spacing between each impulse larger. In the frequency domain, increasing the T has the effect of decreasing the spacing between the impulses. This is what we would expect because this is what we have seen with sampling in previous exercises. The higher the framerate, the better sampling because sampling is done by taking values that are closer together.

2a) $h(t) = \frac{1}{2\pi}\int_{-\infty}^{\infty} H(\omega)\, e^{j\omega t}\, d\omega$

$H\omega = \begin{cases} 0 & -\infty < \omega < -W_c \\ 1 & -W_c < \omega < W_c \\ 0 & W_c < \omega < \infty \end{cases}$

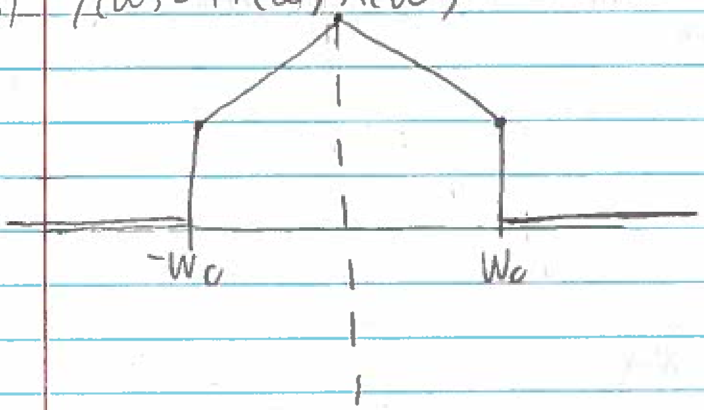
$h(t) = \frac{1}{2\pi}\cancel{\int_{-\infty}^{-W_c} 0 \cdot e^{j\omega t}\, d\omega} +$

$\frac{1}{2\pi}\int_{-W_c}^{W_c} 1 \cdot e^{j\omega t}\, d\omega + \frac{1}{2\pi}\cancel{\int_{W_c}^{\infty} 0 \cdot e^{j\omega t}\, d\omega}$

$h(t) = \frac{1}{2\pi}\int_{-W_c}^{W_c} 1\, e^{j\omega t}\, d\omega = \frac{1}{2\pi}\frac{1}{jt} \cdot e^{j\omega t}\Big|_{-W_c}^{W_c}$

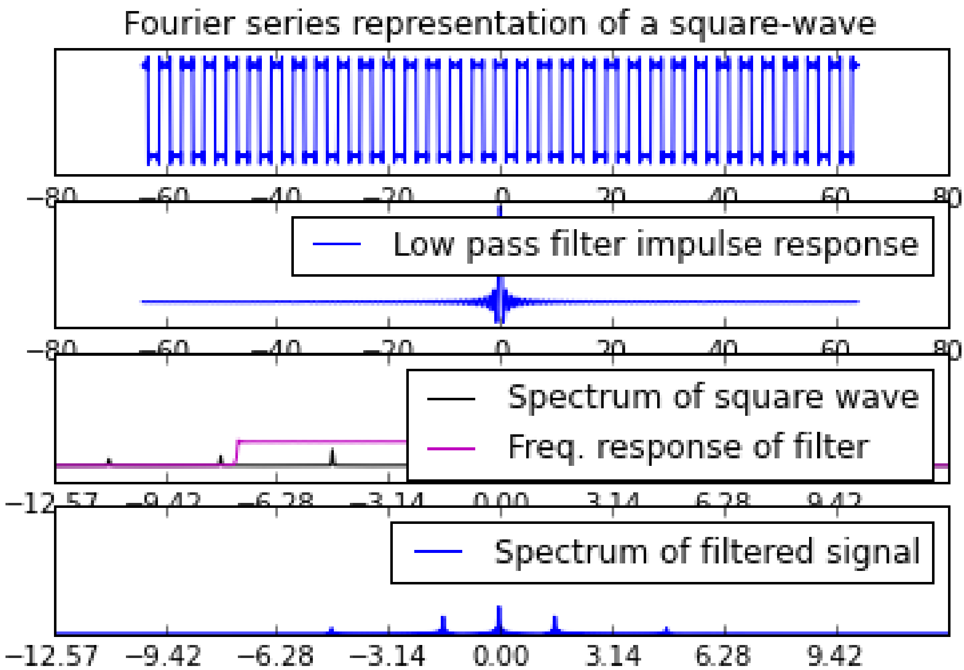
$h(t) = \frac{1}{2\pi j t}\left(e^{jW_c t} - e^{j(-W_c)t}\right) = \frac{\sin(W_c t)}{\pi t}$

2b.) $Y(\omega) = H(\omega)\, X(\omega)$

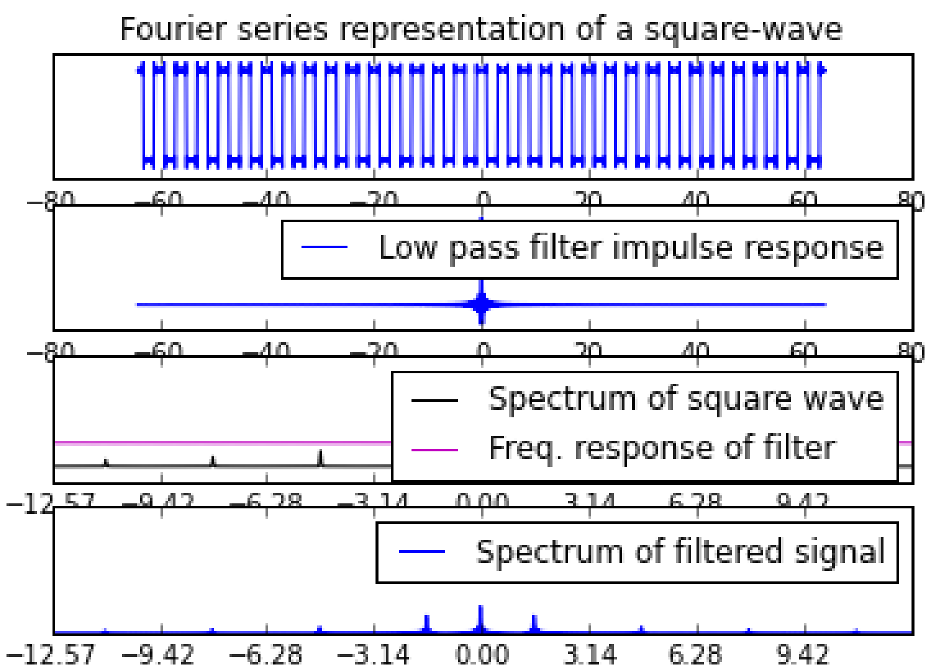


2c.) This LTI system clobbers all frequencies that are outside of the range, W_c.

This is the fourier series representation for w = 0.75*pi



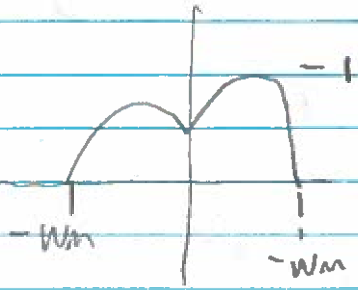
This is the fourier series representation for w = 1.75*pi

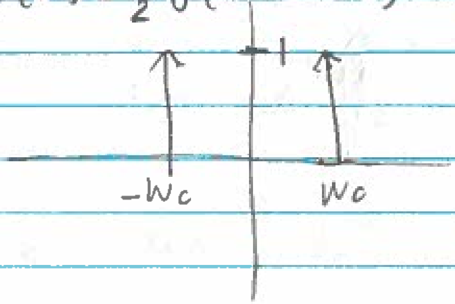


3.) $Y(\omega) = H(\omega) * X = X(\omega) * H$

$Y(\omega) = \frac{1}{2\pi} H(\omega) * X = \frac{1}{2\pi}\left[\pi\delta(W_m - W_c) + \pi\delta(W_m - W_c)\right]$

$Y(\omega) = \frac{1}{2}\delta(W_m - W_c) + \frac{1}{2}\delta(W_m - W_c)$





$Y(\omega) = H(\omega) * X$ $\quad W_m = 0$