Thermal anemometry, current state, and future directions

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L. M. Fingerson

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L. M. Fingerson
TSI Incorporated, P.O. Box 64394, St. Paul, Minnesota 55164

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The variation in heat transfer between a fine wire and the surrounding fluid has traditionally been the most accepted method for measuring the detailed movements of a flow fluid. This paper reviews the present understanding of thermal techniques as used in research and recent advancements of the technique. While the basic research sensor is still a fine wire, modern electronics has greatly simplified data analysis as well as established constant temperature operation of the sensor as the control method of choice. Film sensors have extended the use of thermal sensors to new applications. Comparisons with laser Doppler velocimeters help to identify experiments where thermal sensors still represent the best choice. The ability of modern electronics to easily address the typical nonlinear response of thermal sensors has resulted in increased use for commercial applications. While in these applications fast response can be a factor (e.g., fuel flow control in engines), generally only average flows are needed. As a result, calibration shifts due to deposits on the sensor surface when exposed to contaminated fluids can generally be reduced to acceptable levels by using larger sensors.

I. INTRODUCTION

For purposes of this review paper, thermal anemometry is defined as follows: The measurement of fluid flow using the variations in heat transfer from an electrically heated element exposed to the flow. The term "hot-wire anemometry" is often used because the traditional sensor was a fine wire. The above title uses the more general term so film sensors (a thin metal film on a ceramic substrate) and other types of heated elements used to measure flow are included. Industrial applications often use a wire wound on a ceramic substrate and glazed in place or a sensor protected by a metal sheath. Since a heated element is also sensitive to changes in temperature, composition, and pressure, discussion of these parameters necessarily enters the picture. However, they will be discussed only in relation to obtaining valid flow information.

Thermal anemometers are used both in research and for industrial applications. In most applications they are used for essentially point measurements. The small size, high-frequency response (> 100 kHz achievable), and low noise (rms fluctuations <0.01% of the mean value) are important characteristics for fluid mechanics research. As the 1963 review by Corrsin[1] on turbulence experimental methods attests to, the hot-wire anemometer has been the nearly exclusive measuring instrument in turbulence research prior to the introduction of the laser Doppler velocimeter in 1964.[2]

For industrial applications, the ability to measure very low velocities, low cost, and wide velocity range (turn down ratio) are important characteristics. In this paper, "industrial applications" are any applications of thermal anemometry that cannot be classified as fluid mechanics research.

For all applications, perhaps the major problem with thermal anemometers is sensitivity to contamination. Since the measurement is the heat transfer rate between the sensor surface and the environment, contamination on the surface of the sensor will affect this heat transfer, resulting in a shift in calibration. In research, where small sensors are required for fast response and good spatial resolution, this problem is generally solved by keeping the fluid clean. In industrial applications, the use of large sensors substantially reduces the effect of contaminants in the flow as well as being easier to clean and less sensitive to physical damage.

II. BASIC ELEMENTS OF A THERMAL ANEMOMETER

A. Sensors

Figure 1 shows the basic elements of a thermal anemometer. To work, the sensor must have a resistance that varies with temperature. For most sensor materials, the following equation is an adequate expression for this variation:

$$R=R_r[1+\alpha(T_m-T_r)], \tag{1}$$

where R = Sensor resistance at operating temperature, T_m, R_r = Sensor resistance at reference temperature, T_r. α = Temperature coefficient of resistance. While the reference temperature T_r is generally 0 °C, for convenience researchers often use room temperature as a reference. As pointed out later, precise values (such as more coefficients for the resistance-temperature curve) are usually not required since an actual calibration is almost always necessary.

For research applications cylindrical sensors are most common, either a fine wire (typical diameters from 1 to 15 μm) or a cylindrical film (typical diameters from 25 to 150

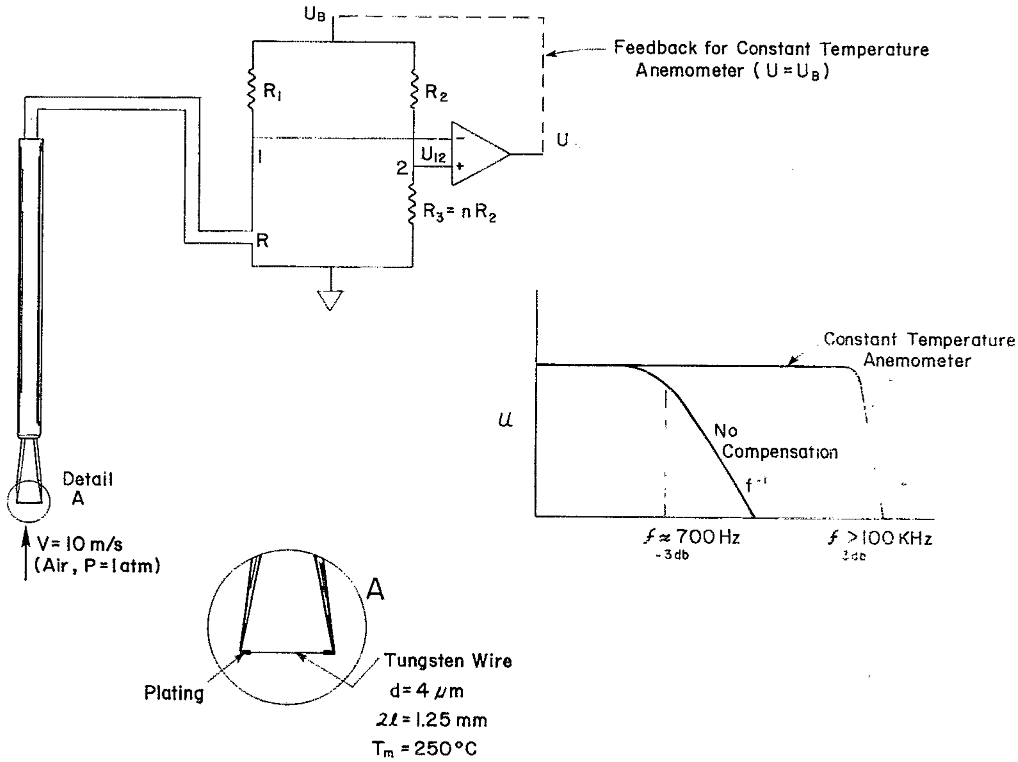


FIG. 1. Basic elements of a hot-wire anemometer.

μm). The low thermal conduction of the substrate gives film sensors some interesting properties compared to hot wires, low conduction losses to the supports for a given diameter being one of them. Various shapes of noncylindrical sensors are also used for some difficult applications or as a surface sensor on a wall.

Industrial sensors are often a resistance wire wrapped around a ceramic substrate. Typical diameters are 0.5–2 mm. In difficult applications such as smokestacks the sensors may be considerably larger.

B. Control circuits

The first control circuits were "constant current" where the current through the sensor was kept nearly constant ($R_1 \gg R$ in Fig. 1) with a constant voltage source, U_B connected to the top of the bridge. The output u of the anemometer was then simply a measure of the off-balance of the bridge due to a change in resistance of the sensor. In other words, with the polarity shown in Fig. 1 for the amplifier, an increase in velocity past the sensor would cool the sensor, causing R to decrease and U to increase. The "no compensation" curve shown in Fig. 1 gives the frequency response of this system for the sensor and conditions given in detail A. In research applications, electronic frequency compensation was used to increase the range where the frequency response was flat.

The "constant temperature" mode adds the feedback line shown in Fig. 1. The amplifier now operates to keep the bridge in balance, resulting in an essentially constant value for R and hence a constant sensor temperature. An increase in velocity past the sensor now causes the current I, through the sensor to increase to maintain the sensor resistance R constant. Again, the value of U will increase.

While the "constant temperature" mode of operation was introduced as early as 1943,[3] its use was rather limited until the introduction of transistorized circuits. Since then the constant temperature mode has replaced constant current systems for both research and industrial applications with the exception of some compressible flow experiments. Advantages of the constant temperature system include: (1) A flat frequency response can be maintained over a wider velocity range without adjustment. (2) Easier to compensate for environment temperature changes either by a temperature sensitive element in the bridge or by measuring temperature and correcting the results.

The signal-to-noise ratio is similar for the two systems. The fact that the constant temperature system is now well understood has also added to researcher's confidence in their results.

C. System response

The basic measurement is convective heat transfer from a sensor, usually cylindrical, to its fluid environment. To understand the response of the system it is necessary to look at the appropriate heat transfer relations. In its simplest form, convective heat transfer can be expressed as

$$Q = hS(T_m - T_a), \tag{2}$$

where Q=heat transfer rate, h=convective heat transfer coefficient between the sensor and the fluid, S=surface area of the sensor, T_a=ambient temperature of the fluid, and T_m=mean sensor temperature.

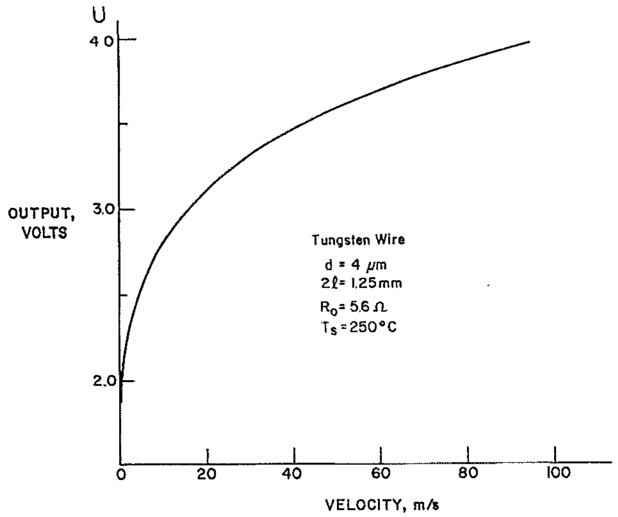


FIG. 2. Typical calibration curve for a hot-wire anemometer.

The parameter of primary interest is h. Putting h in terms of Nu number ($=hd/k$), where d is sensor diameter and k_f the thermal conductivity of the fluid, as shown in the Appendix, an entirely general dependence would be

$$\mathrm{Nu}=f\left(\mathrm{Re},\mathrm{Pr},\alpha,\mathrm{Gr},\mathrm{Ma},r_h,a_T,\frac{21}{d},\frac{k_f}{k_w}\right). \tag{3}$$

Fortunately, for air at normal room temperatures and flow velocities where air can be assumed incompressible, the following relation[4] represents the heat transfer with quite good accuracy for a cylindrical sensor:

$$\mathrm{Nu}=(A+B\,\mathrm{Re}^n)\left(1+\frac{a_T}{2}\right)^{0.17}, \tag{4}$$

where

$A=0.24$, $B=0.56$, $n=0.45$ for $0.02<\mathrm{Re}<44$,

$A=0$, $B=0.48$, $n=0.51$ for $44<\mathrm{Re}<140$,

$a_T=(T_m-T_a)/T_a=$temperature loading, $\mathrm{Re}=V\rho d/\mu$, $V=$velocity, $\rho=$density, $\mu=$dynamic viscosity, $d=$sensor diameter.

Small modifications of the above constants have been suggested by various authors, but the basic form of the equation has withstood the test of time. Using this equation, Fig. 2 shows a typical calibration curve for a sensor of the type shown in Fig. 1.

In the past the nonlinear response shown in Fig. 2 was a problem. With the availability of microprocessors for linearization it may be more an advantage than a limitation. The shape of the curve permits a percent of reading accuracy specification over a wide velocity range versus percent of full scale that is normal with an inherently linear transducer. Velocities from less than 1 cm/s to over 100 m/s are within the range of a thermal anemometer (10 000 to 1 range). Even higher velocities can be measured but compressibility must be taken into consideration (Appendix).

Heat transfer relations are very helpful for establishing the influence of the various parameters on the sensor response. But for accurate measurements, a calibration against a known reference is a requirement. For most applications, mounting the sensor at the exit of a flow nozzle

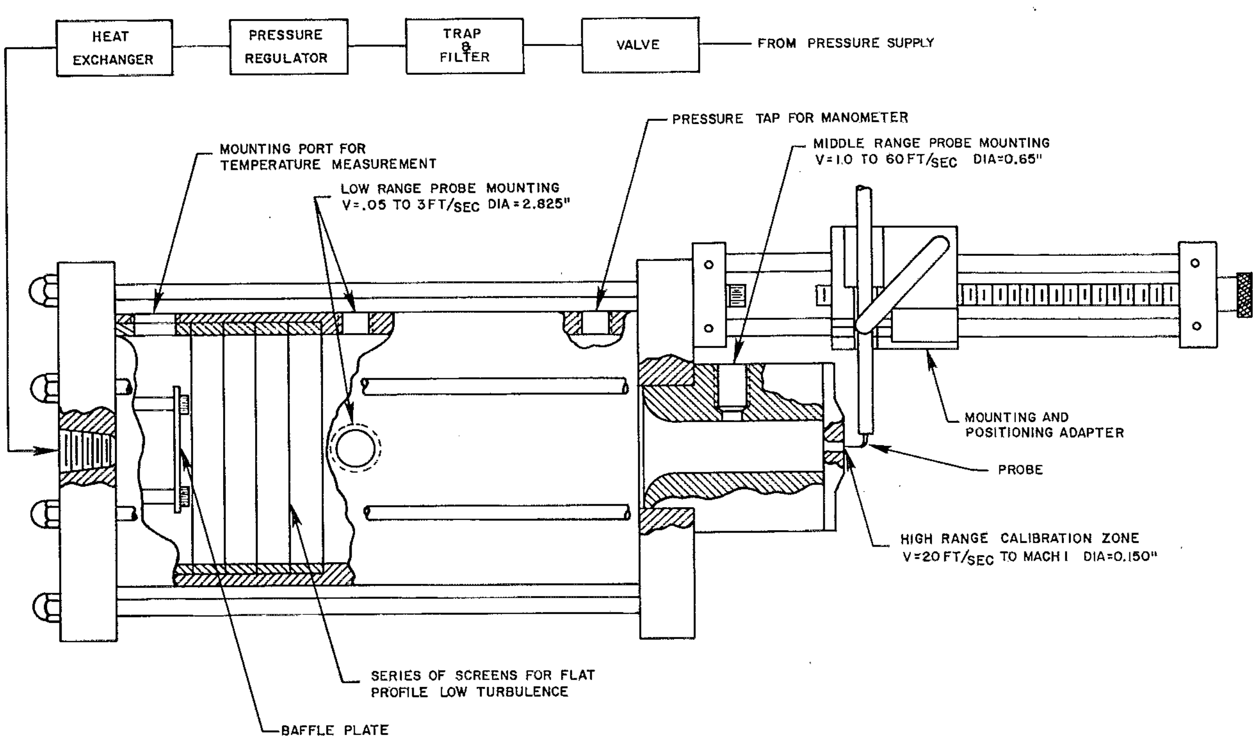


FIG. 3. Calibration system—air.

with pressure drop as the reference measurement is generally adequate (Fig. 3). For very low flows or other special requirements, a laser Doppler velocimeter can give a very accurate reference measurement over a very wide range of conditions.

III. STRENGTHS, LIMITATIONS, AND COMPARISONS WITH LASER VELOCIMETERS

While the following comparison refers primarily to research applications, it is included in this section because both hot wire anemometers and laser anemometers have potential applications outside of research. To measure velocity details in a flowing fluid, the ideal instrument should:

(1) Have high-frequency response to accurately follow transients,

(2) Be small in size for an essentially point measurement,

(3) Measure a wide velocity range,

(4) Measure only velocity, and work in a wide range of temperature, density, and composition,

(5) Measure velocity components and detect flow reversal,

(6) Have high accuracy,

(7) Have high resolution (low noise),

(8) Create minimal flow disturbance,

(9) Be low in cost,

(10) Be easy to use.

For many years, only the hot-wire anemometers satisfied enough of the criteria to be used extensively in turbulence studies. Pitot probes, flow visualization, and other techniques complemented hot-wire data but generally could not get details as well as the hot wire.

For most applications, items 4, 5, 6, and 10 are perhaps the weakest areas of thermal anemometers. To be even more specific, the primary practical limitations are fragility and sensitivity to contamination. From a theoretical point of view, the limitations include the following:

(1) Velocity is not measured directly but deduced from a measurement of convective heat transfer from the sensor.

(2) Normal configurations limit the turbulence intensity that can be measured accurately.

(3) Heat losses from the sensor other than by convection can cause errors.

The importance of these limitations depends on the application. Relative to item 6 (accuracy), thermal-anemometer measurements are very repeatable when the conditions are reproduced exactly. It is the effect of contamination and the effect of variables other than flow on the heat transfer that causes inaccuracies.

Another limitation of thermal anemometers which is common to all devices that aim for point measurements is their inability to fully map velocity and vorticity fields which depend strongly on space coordinates and simultaneously on time. It would require spatial arrays of thousands of probes which have to work without mutual interference. This limitation is strongly felt in emerging areas of research like unsteady separated flows and field mapping of turbulence (in contrast to averaged turbulent fields). Particle image velocimetry[5] has been effective for quantitative evaluation of two dimensional spatial velocity fields with promise for three dimensions and even time. Flow visualization reinforced by computer simulations is also promising with point measuring devices providing a few quantitative reference points.

Since its introduction in 1964,[2] the use of the laser velocimeter to measure velocity details of flowing fluids has expanded rapidly. Since it, too, satisfies many of the "ideal instrument" criteria, a comparison between it and the thermal anemometer follows. It should be emphasized from the start that, rather than replacing thermal anemometers, the laser velocimeter complements their use. As is often the case in making measurements, it is not a question of the best instrument but rather which instrument will perform best for the specific application.

The laser velocimeter uses Doppler-shifted light scattered from particles in the flow to deduce the velocity of those particles. If the particles are small enough to follow the flow, then the flow velocity is measured. A window to the flow must be provided for both the incident light and the scattered light, but no probe needs to enter the flow field. Of course, there must be enough particles of the appropriate size and concentration so the desired statistical data can be determined.

In the following, the thermal anemometer and laser velocimeter are compared on the basis of the above criteria for the ideal instrument.

(1) Frequency response. In current practice, the thermal anemometer is definitely superior. System response to several hundred kilohertz are quite easily attained, with 1 MHz being feasible.

Theoretically, a laser velocimeter could approach the response of a thermal anemometer. Practically, spectra up to only about 30 kHz have been measured. In many applications the problem is adequate size and concentration of the scattering centers (particles). At sufficiently high frequencies, electronic limitations and spatial resolution can also become important.

(2) Spatial resolution. A hot-wire type thermal sensor is typically 5 μm in diameter by about 2 mm long, although wires as small as 1 μm by 0.2 mm have been used.[6]

For a laser velocimeter, measuring volumes of 50 μm by 0.25 mm are common, while a 5 μm by 5 μm measuring volume is achievable in very small test sections or with a fiber optic probe in the flow. If the distance from the focusing lens to the measuring point is long (e.g., over 400 mm), then small measuring volumes are difficult to achieve. They may also be impractical in some flows, owing to movement of the incident beams caused by refractive-index variations in the beam path.

(3) Velocity range. Both techniques have a very wide velocity range. The laser has the advantage at very low velocities because the "free convection" effects that affect hot-wire readings are usually not a problem.

Although both measure high-speed (compressible) flows, laser data are easier to interpret because they are sensitive only to particle velocity and no calibration is required. At the same time, providing particles that follow the flow, scatter enough light, and have high enough concentration for spectral measurements can be difficult.

The thermal anemometer is sensitive to recovery temperature, Mach number, and Reynolds number in compressible flows. Measurements in transonic flows require considerable calibration while, above $M=1.5$, Mach-number independence makes measurements more feasible.

(4) Measure only velocity over wide temperature, density, and composition ranges. The laser velocimeter measures only the velocity of the scattering center (particle), and it measures it in a known direction (pure cosine response). The hot-wire sensor measures heat transfer to the environment. This can be a plus since, for example, by using two sensors, both temperature and velocity fluctuations can be measured. Generally, though, it is preferable to be sensitive to velocity only.

Both instruments will operate over wide temperature, density, and composition ranges. However, even though the cooled-film probe (a thermal sensor with operating principle similar to that of a hot wire) can be used at high temperatures, its application is limited by sensitivity to other variables. At low density, measurements become more difficult for both. Conduction losses become excessive, and slip flow effects complicate hot-wire anemometry, while in laser velocimetry there are problems in finding particles that both follow the flow and scatter enough light.

The laser must "see" into the flow. In liquid metals, Hg for example, thermal anemometers can be used (with difficulty) but generally not laser velocimeters. Of course, laser velocimeters can be used to measure the surface velocity of opaque liquids if that is of interest.

(5) Component resolution. The hot wire can be used to resolve one, two, or all three components of a flow field by using one, two, or three sensors respectively. However, it appears limited to rather low turbulence intensities even with rather sophisticated data-reduction procedures.

The laser velocimeter can resolve components and, with frequency shifting, can detect flow reversals. While it is more difficult to obtain the third component, systems are available.

(6) Accuracy. Hot-wire results can be very repeatable, so accuracy is really a function of how closely the calibration conditions are reproduced in the flow to be measured. In practice, contamination, temperature changes, and other factors generally limit accuracy to a few percent.

The laser velocimeter can give very high accuracy (0.1%) in carefully controlled experiments. In many practical measurements, refractive-index variations, limited accuracy on beam-crossing angle, and signal-processor limitations make a value of 1% more realistic.

(7) Resolution. the hot wire is clearly superior, since it can have a very low-noise level. Resolution of 1 part in 10 000 is easily accomplished, while with a laser velocimeter 1 part in 1000 is difficult with present technology.

(8) Flow disturbance. Since only light needs to enter the flow, the laser is clearly better. The size and concentration of particles normally required does not measurably alter the flow field.

(9) Cost. The hot wire is lower in cost by a factor of 3–10 for most applications. Of course this can change as technology changes.

(10) Ease of use. At present, the laser velocimeter is probably more difficult to set up and start getting valid data with, although new technology and designs with fiber optics and/or laser diodes are rapidly changing this. Once it is set up, the laser velocimeter may be easier to use, since there are no fragile sensors to get dirty, break, or shift calibration. A complexity in data interpretation with the laser velocimeter is the fact that discrete measurements are made (on each measurable particle), which gives a discontinuous output.

As a general rule, if other instruments (such as a Pitot tube or pressure transducer) cannot give the detailed measurements required, one should consider a hot-wire (or hot-film) anemometer. If high temperatures, moving objects in the flow, proximity to walls, dirt in the flow, high turbulence intensities, or some other problem makes a thermal anemometer difficult or impossible to use, then a laser velocimeter should be considered.

In summary, thermal anemometers can theoretically be used in almost any fluid-flow situation. However, sensor fragility, calibration shift due to contamination, or difficulties in separating out variables make many potential applications difficult. The most common and easiest measurements with thermal anemometers are in constant-temperature gases near atmospheric pressure, at relatively low turbulence intensities, and at flow velocities low enough so that the assumption of incompressibility is adequate. But when the need is sufficient, good measurements can be made over a much wider range of conditions.

IV. THERMAL ANEMOMETRY IN RESEARCH

A. Some developments of thermal anemometry in research

In research papers, the first reference normally cited is that of King[7] in 1914, where he suggested a relationship between the heat transfer from a fine wire exposed to moving air and the velocity of the air surrounding the wire. However, according to the "Bibliography of Thermal Anemometry" assembled by Professor Freymuth[8] there were 25 earlier papers related to thermal anemometry with the first being published in 1817.[9] In other words, the technique is far from new. Also it is a subject that has received considerable attention, with the bibliography listing more than 2500 papers.

Significant events in thermal anemometry instrumentation include the introduction of constant temperature control circuits,[3] the introduction of film sensors,[10,11] and the introduction of transistorized constant temperature control circuits.[12,13] The use of orthogonal sensors,[14,15] removed some of the limitations in measuring high turbulence intensities. The present availability of computers for data collection, storage, and analysis has had an impact on the ease of use of thermal anemometers.

A great many authors have contributed to the theory of thermal anemometers. This review represents what this

TABLE I. Some properties of common hot-wire materials.

	Tungsten	Platinum	80% Platinum 20% Iridium
Temperature coefficient of resistance (°C^{-1})	0.0045	0.0039	0.0008
Resistivity (Ω cm)	5.5×10^{-6}	10×10^{-6}	31×10^{-6}
Ultimate tensile Strength (kg/mm^2)	420	24.6	100
(lbs/in)	(60×10^4)	(3.5×10^4)	(14.22×10^4)
Thermal conductivity (cal/cm °C)	0.47	0.1664	0.042

author feels are some conclusions from this literature in terms of instrument operation. Also, no attempt will be made here to address the many applications where thermal anemometry has been used. A book[16] on the use of hot wires in research is expected to be published in the summer of 1993.

B. Sensors for research

Fine hot wires such as that shown in Fig. 1 still provide the optimum sensitivity and frequency response. Film sensors are used either because of their rigidity, the fact they can be coated (for abrasion resistance or electrical insulation), or because of the lower conduction losses for a given diameter (the larger diameter reducing contamination effects). Noncylindrical films sensors are used for certain special applications, especially in liquids.

Typical wire material and properties are given in Table I. At temperatures where oxidation is not a problem (<300 °C) tungsten is the most common because of its high-temperature coefficient and high strength. At high temperatures, platinum is preferred with platinum-iridium being substituted when strength is a problem.

In the following two sections, a hot wire similar to that in Fig. 1 is used as an example. The differences between hot wires and cylindrical film sensors are quite minor and explained in a separate section.

C. Frequency response

In turbulence measurements, it is important that the amplitude response of the instrument being used is constant over the frequency range of interest. While an actual calibration would be ideal, generating a known variation in velocity over the frequency range of interest is difficult. Therefore, an electrical test is normally used.

In Fig. 4 a variation in current is being applied to one leg of the bridge through resistor R_4 generated by varying voltage U_t. If U_t is a square wave at a frequency such that the system stabilizes between step changes, a properly "trimmed" anemometer will give an output waveform, U, as shown on Fig. 5. The frequency response (3 dB point) is calculated from the equation[17]

$$f_{\mathrm{cut}}=1/1.3\tau, \tag{5}$$

where τ is defined as shown on Fig. 5.

While the electrical test is certainly not identical to a change in velocity past the sensor, Freymuth[17] defined the

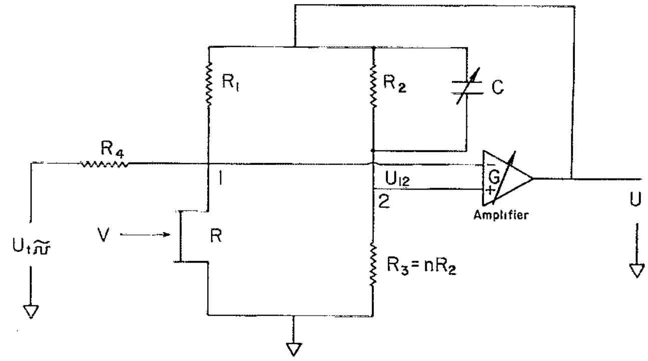


FIG. 4. Constant temperature anemometer control circuit showing electrical test signal U introduced through resistor R.

relationship using a third-order differential equation to represent the constant temperature anemometer system. A sine wave input gives the results shown in Fig. 6 (along with photographs of some actual square wave results) where the 6 dB/octave amplitude increase represents the effective amplitude gain required to maintain a flat frequency response. The horizontal portion of the curves is the range where the sensor needs no compensation to provide a flat response.

The electrical test does not, of course, directly test possible lags in the boundary layer around the sensor. While this can be a problem in water and other fluids when large sensors are used,[18] it is generally not a problem for gases. For the conditions of Fig. 1, estimates[19] give a boundary layer response of 400 kHz.

A more serious problem in three-dimensional flows is the length of the sensor compared with the wavelength of the maximum frequencies of interest. Estimates for the sensor of Fig. 1 give errors as high as 20% at just over 3 kHz for a one-dimensional spectra of turbulence.[20,21] In two-dimensional flows where the relevant dimension is the sensor diameter, this is not a problem.

Another element in amplitude response is the dynamic effects of conduction from the heated sensor to the cooler and more massive supports. Again, for the sensor and conditions shown in Fig. 1 (constant temperature environ-

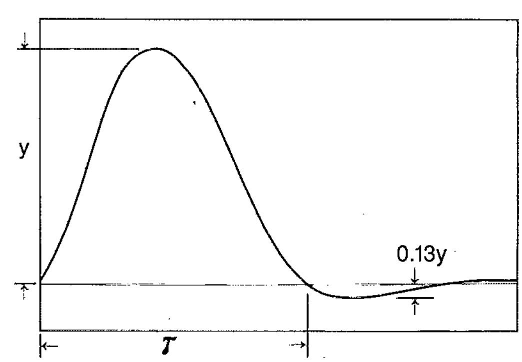


FIG. 5. Output U in response to a step current input for a properly adjusted constant temperature anemometer with a wire sensor.

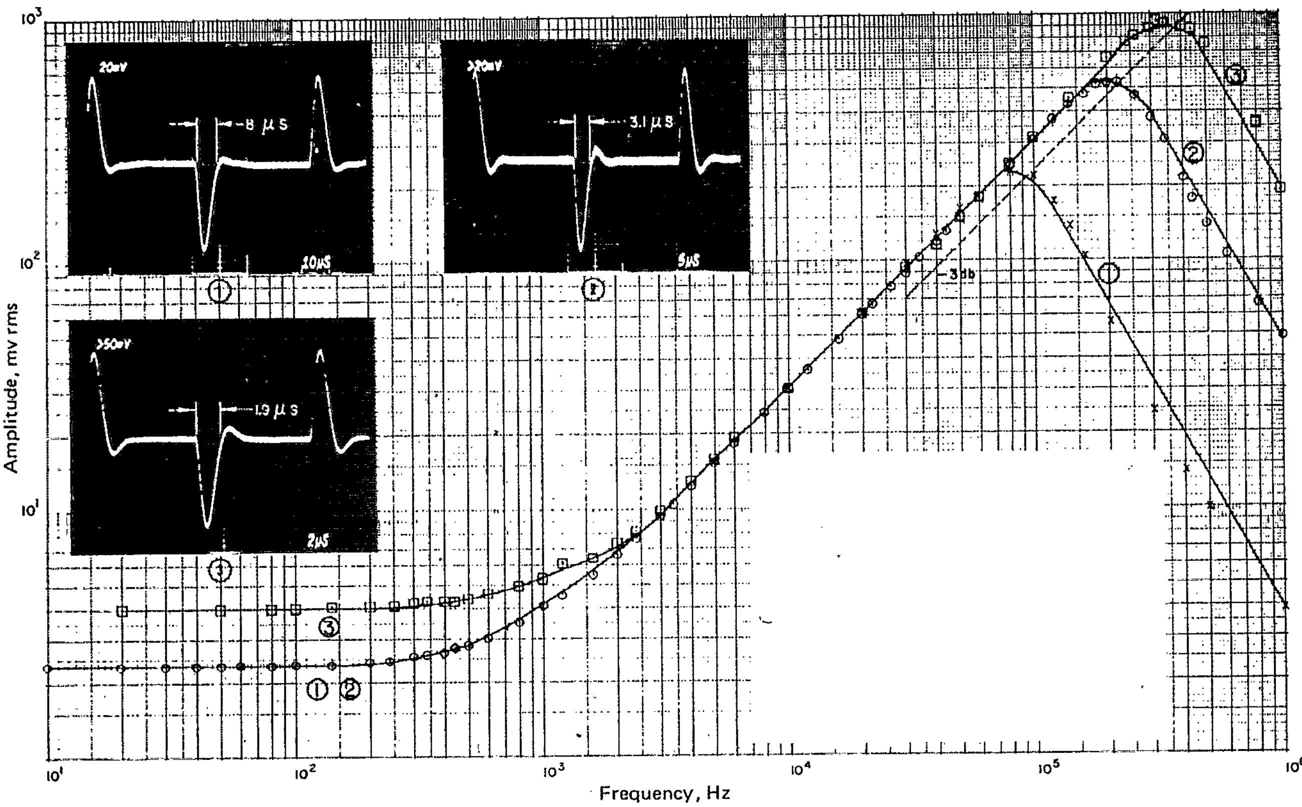


FIG. 6. Sine- and square-wave tests on a 4 μm tungsten hot wire in air. (1) $v=30$ ft/s, $R_3/R=5$ (Fig. 1), (2) $v=30$ ft/s, $R_3/R=1$, (3) $v=300$ ft/s, $R_3/R=1$.

ment, varying velocity), the error is around 5% and occurs above a frequency of about 80 Hz.[19]

D. Noise

One of the strengths of thermal anemometry versus laser anemometry is its ability to measure very low levels of turbulence. Just how low depends on the background noise of the anemometer itself. Of course the first step is to eliminate any extraneous background noise from power lines, radio or television stations, and stray magnetic fields. Proper shielding and grounding should eliminate these sources. However, Johnson noise from the bridge resistors (including the sensor) and electronic noise generated by the bridge amplifier cannot be eliminated.

Of course, with a commercial anemometer the bridge resistors and amplifier are predetermined. When there is a choice, it should be noted that with modern amplifier technology the equivalent input noise of the amplifier can approach that of the bridge resistors. While sensors generally have a low impedance ($\sim$5–15 Ω), and the resistor R_1 (Fig. 1) should be large for maximum signal, the resistor across from the sensor (R_3) should be kept small. Typically, a ratio across the bridge (R_2/R_1) of one is ideal. This ratio also helps optimize frequency response.

It is the ratio of signal amplitude to noise amplitude (N_W) at the output that is of most interest. The following equation[22] is helpful:

$$N_W=\frac{\alpha R_a}{n_b+1}\,\frac{B^{3/2}V^{3/4}}{2\pi f c}\,\frac{(T_m-T_a)^{3/2}}{2R^{1/2}}\,\frac{v}{V}\,\frac{1}{\sqrt{\overline{u^2_{12,n}}}},\qquad(6)$$

where f=signal frequency, c=thermal capacity of the sensor, $n_b=R_3/R_2$, $\overline{u^2_{12,n}}$=equivalent noise at amplifier input, and v=variation in velocity.

For a given equivalent input noise, to maximize signal-to-noise ratio:

(1) Operate at high overheat to maximize T_m-T_a.

(2) Use a wire material with a high-temperature coefficient of resistance, α.

(3) Use a thin wire to minimize thermal capacity c.

While the above equation is only weakly dependent on velocity ($V^{-1/4}$), it is at high velocities where high frequency response is required. The signal-to-noise ratio is inversely proportional to the frequency f_c and $\overline{u^2_{12,n}}$ is proportional to bandwidth Δf. Therefore, noise is very dependent on the mean frequency (as expected from the rising curve in Fig. 6) and on the bandwidth being observed.

E. Film sensors

1. Cylindrical sensors

Cylindrical film sensors have many of the same characteristics as hot-wire sensors.

Advantages of film sensors

(1) Because the substrate (often quartz) has a low conductivity compared to metals, the length to diameter

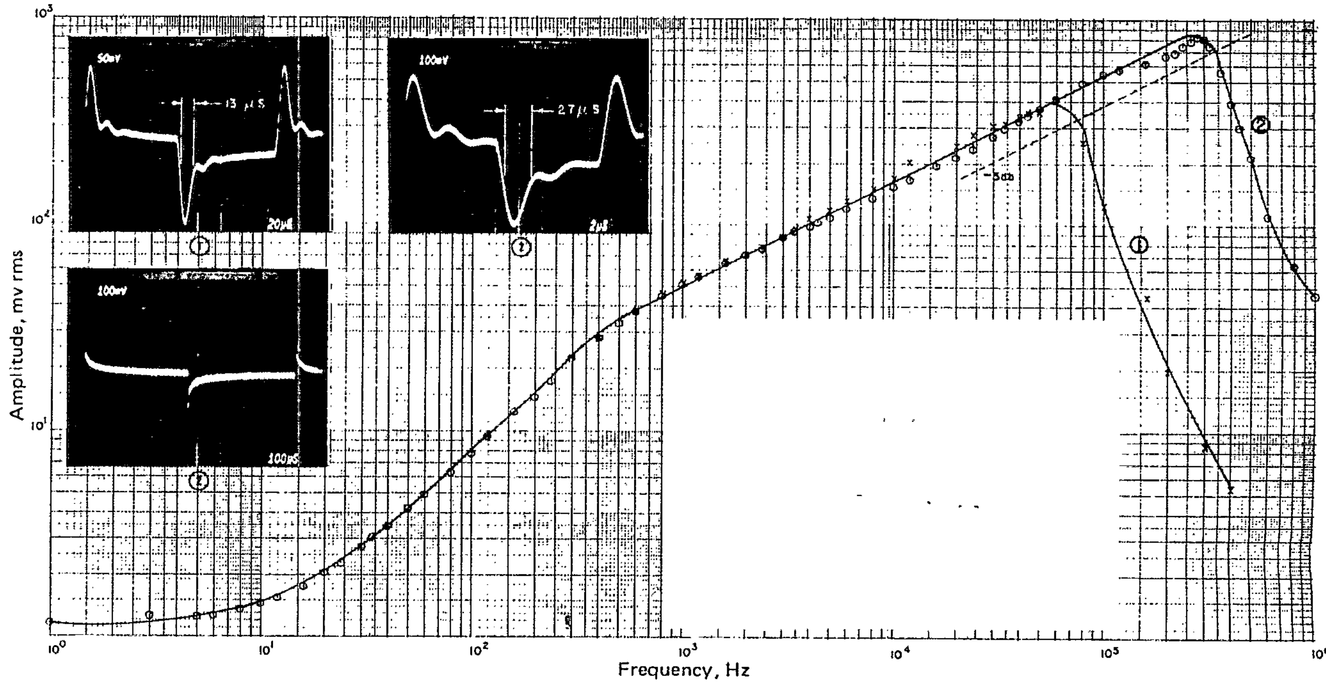


FIG. 7. Sine- and square-wave tests on a 50 μm hot film sensor in air with v=30 ft/s. (1) $R_3/R=5$, (2) $R_3/R=1$.

ratio can be much smaller with equivalent end losses.[19] For example, a length to diameter ratio of 20 on a film sensor can give approximately the same end losses as the wire of Fig. 1 with a length to diameter ratio of over 300. This larger diameter: (a) reduces the effects of contaminants in the fluid on the sensor calibration, (b) provides a much more rigid sensor, improving calibration repeatability, especially for velocities that are not normal to the sensor axis.

(2) Since only the surface temperature is "sensed," film sensors can maintain response at high frequencies even though the diameter is larger.

Disadvantages of film sensors

(1) At Re > 150 (V> 50 m/s for a 50-μm-diam sensor in air), self-generated "turbulence" due to real flow effects around the sensor can limit the performance when measuring low turbulence intensities.

(2) The temperature coefficient of resistance of the film is generally less than that of wires.

(3) Maximum operating temperatures for generally available film sensors is 350 °C with recommended operating temperatures of 250 °C.

(4) The cost of film sensors is generally higher than for wires.

Figure 7 shows the sine wave response of a cylindrical film sensor to electronic testing with the configuration of Fig. 4 and the flow conditions of Fig. 1. It can be noted that the sensor itself, which is over 13 times larger in diameter than the hot wire used for Fig. 6, has a much lower inherent response (3 dB point at about 22 vs 700 Hz for the wire). At the same time, the electronically compensated 3 dB point is very close at about 75 vs 93 kHz for the wire.

It should be apparent that for research work in clean environments, especially at low turbulence intensities, fine hot wires are best. But film sensors can provide excellent results in environments where calibration shifts in wires due to strain or contamination make them impractical. Also, for resolving components in high turbulent intensity flows, film sensors are generally superior because of their rigidity.

2. Noncylindrical sensors

The primary application of noncylindrical sensors is in liquids, especially water. There are two reasons for this:

(1) When used in gases, the effective "end losses" of noncylindrical film sensors are high, causing substantial attenuation of fluctuating signals. The high thermal conductivity of water keeps this attenuation in the 5%–10% range instead of the 30%–70% range that can occur in air.[23]

(2) Except under very controlled conditions, it is difficult to keep cylindrical sensors from shifting calibration due to sensor contamination in liquid flows. Noncylindrical sensors are generally designed specifically to reduce this contamination effect.

Typical noncylindrical sensors are shown in Fig. 8. The wedge, cone, and surface sensor are quite common. The "patch" sensor can be very rugged for severe environments or where careful handling is difficult to ensure. Film sensors have been built in a wide variety of configurations to meet specific requirements.

F. Angle sensitivity and support interference

For an infinitely long wire, the angle sensitivity of the hot wire is expressed (Fig. 9) as

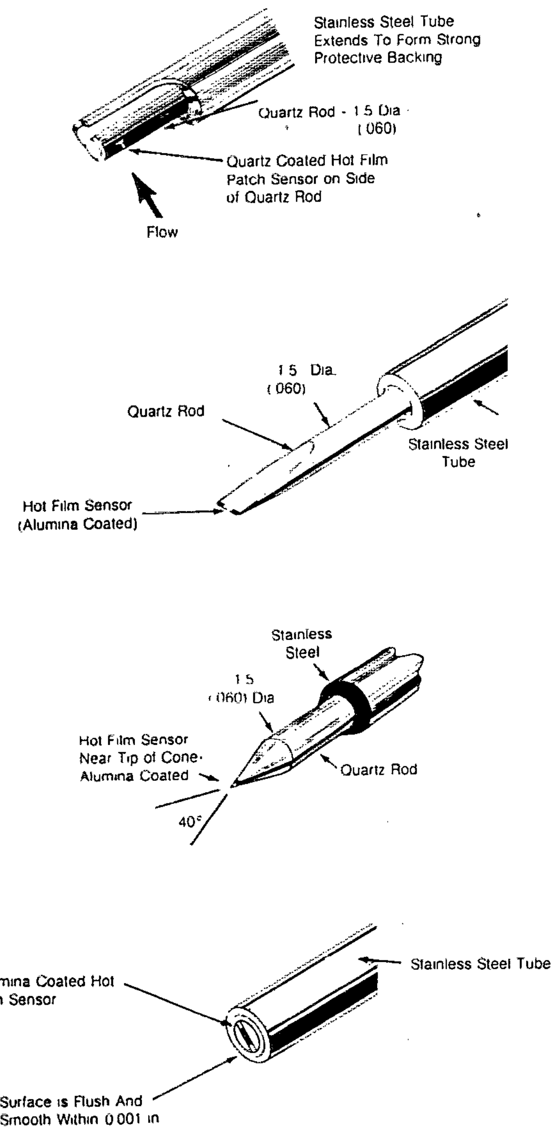


FIG. 8. Typical noncylindrical film sensors.

$$V_{\text{eff}} = V \cos \alpha_1, \tag{7}$$

where V_{eff} is the effective cooling velocity past the sensor. Equation (7) essentially states that the velocity along the sensor has no cooling effect on the sensor and that the sensor is rotationally symmetrical in both construction and response. In many calculations and experiments, Eq. (7) is adequate, maintains simplicity, and, depending on probe and sensor design, can be quite accurate.[24]

Because the sensor has a finite length, there is heat transfer due to the flow parallel to the sensor (V_T). To account for this, a second term is added[24,25] to give

$$V_{\text{eff}} = V \sqrt{\cos^2 \alpha_1 + k_t^2 \sin^2 \alpha_1}, \tag{8}$$

where k_T is an empirically determined factor. Although k_T is not truly a constant for all velocities and values of α_1, for a limited velocity range and angles from 0° to 60° a fixed value works quite well. Champagne[26] found that k_T decreases nearly linearly with $2l/d$ from a value of $k_T=0.2$ at $2l/d=200$ to zero at $2l/d=600$–800. Similar results were obtained with both platinum and tungsten wires. Other equations for yaw sensitivity[27–29] have been suggested and may prove more accurate in certain applications. For the sensor of Fig. 1, $2l/d=312$ and $k_T=0.15$.

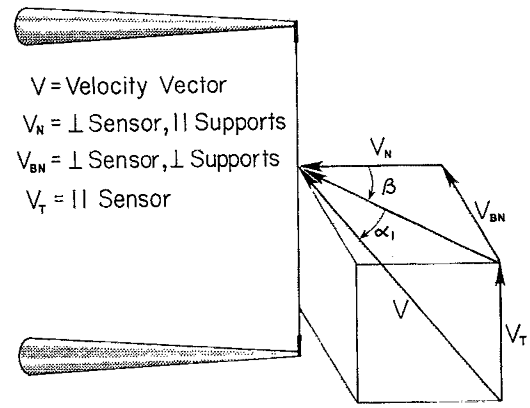


FIG. 9. Velocity components at sensor.

Equation (8) assumes that the response of the sensor is rotationally symmetric. It has been shown[24,30] that aerodynamic effects from both the support needles and the probe body affected the readings, with the minimum reading occurring with the probe parallel with the flow, and the maximum with the probe perpendicular to the flow. To account for the support interference, an equation of the following form has been suggested[31]

$$V_{\text{eff}} = \sqrt{V_N^2 + k_T^2 V_T^2 + k_N^2 V_{BN}^2}, \tag{9}$$

where V_{BN} is the velocity vector perpendicular to both the sensor and the support prongs. The value of k_N can range from 1.0 to 1.2, depending on the design of the probe support and needles.[29] As is true of k_T, k_N is not constant for all angles and velocities, but careful use can improve accuracy as compared with assuming that $k_N=1$ (theoretical value). It was found[31] that plating the wire ends as shown in Figs. 1 and 8 reduced the values of k_T and k_N.

Equations (8) and (9) are given here to provide concrete examples. There is no intent to imply that they always represent the best functional relationships. It should be emphasized that, in using a single calibrated sensor in turbulence intensities under 20%, good accuracy can be obtained without the above equations. It is when multisensor probes are used in large turbulence intensities, or when the sensor orientations during calibration and use are different, that the above considerations become important.

G. Measuring mean velocity, velocity components, and temperature

The most common measurement is the use of a single hot-wire probe, perpendicular to the flow, to measure mean velocity $\bar{V}$ and fluctuations in the mean flow direction $\overline{v_t^2}$. Two components are often measured with an X probe, while three components can be measured by adding a third sensor or by rotating the X probe. In addition, both temperature and velocity can be obtained by operating two

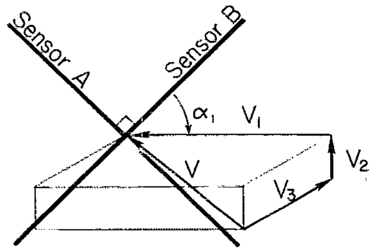


FIG. 10. Configuration of X probe.

parallel sensors at different temperatures. In all these measurements there are limitations that must be observed.

One component using a single hot wire

In Fig. 9, assume the sensor is oriented in the flow stream so that $V_N = V_1$, $V_T = V_2$, and $V_{BN} = V_3$, where v_1, v_2, v_3 are the desired orthogonal velocity components of the velocity vector V. From Eq. (9), the effective cooling velocity past the sensor is

$$V_{\text{eff}} = \sqrt{V_1^2 + k_T^2 V_2^2 + k_N^2 V_3^2}. \tag{10}$$

If, further, the mean flow is in the V_1 direction, then $\overline{V_2} = \overline{V_3} = 0$. If the fluctuations are v_1, v_2, and v_3 then

$$V_{\text{eff}} = \sqrt{(\overline{V_1} + v_1)^2 + k_T^2 v_2^2 + k_N^2 v_3^2}. \tag{11}$$

Since k_T is small and $k_N \approx 1$, this can be approximated by

$$V_{\text{eff}} = \sqrt{(\overline{V_1} + v_1)^2 + v_3^2}. \tag{12}$$

If we neglect v_3, then

$$\begin{aligned} \overline{V_1} &= \overline{V} = \overline{V_{\text{eff}}}, \\ \sqrt{\overline{v_1^2}} &= \sqrt{\overline{v^2}}. \end{aligned} \tag{13}$$

Traditionally, the value of $\overline{V_1}$ was obtained with a mean value (averaging) meter and $\sqrt{\overline{U_1^2}}$ was obtained with an ac coupled true rms meter. These instruments have been largely replaced with high speed analog to digital converters and an appropriate computer and software.

When $\sqrt{\overline{v_1^2}}/\overline{V} = 0.2$, the error due to ignoring v_3 is about 2% for isotropic, normally distributed, and normally correlated turbulence.[29] The mean velocity error is also about 2%.

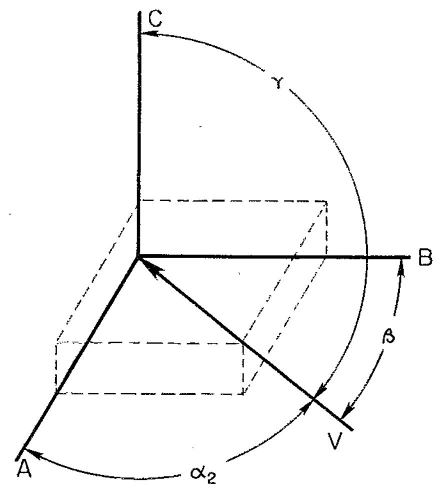
H. Two components using an X probe

The cross wire or X probe (Fig. 10) is used to measure the two velocity components in the plane of the two sensors. With the sensors oriented at ±45° to the mean velocity and assuming $\overline{V_3} = 0$, v_3 small so it can be ignored, $k_T = 0$ and $k_N = 1$, the mean flow velocity will be

$$V_1 = 2^{-1/2}(V_{A,\text{eff}} + V_{B,\text{eff}}) \tag{14}$$

and the velocity perpendicular to the mean flow and in the plane of the sensors will be

$$V_2 = 2^{-1/2}(V_{A,\text{eff}} - V_{B,\text{eff}}). \tag{15}$$



$$V^2_{A,\text{eff}} = V^2 (\sin^2 \alpha_2 + k^2 \cos^2 \alpha_2)$$

$$V^2_{B,\text{eff}} = V^2 (\sin^2 \beta + k^2 \cos^2 \beta)$$

$$V^2_{C,\text{eff}} = V^2 (\sin^2 \gamma + k^2 \cos^2 \gamma)$$

$$V^2 = \frac{V_A^2 + V_B^2 + V_C^2}{2 + k^2}$$

FIG. 11. Direction sensitivity using three mutually perpendicular sensors.

If we further assume that $\overline{V_2} = 0$, then the results give the usual equations for the X probe:

$$\begin{aligned} \overline{V} &= 2^{-1/2}\overline{(V_{A,\text{eff}} + V_{B,\text{eff}})}, \\ \overline{v_1^2} &= \overline{(v_{A,\text{eff}} + v_{B,\text{eff}})^2}/2, \\ \overline{v_2^2} &= \overline{(v_{A,\text{eff}} - v_{B,\text{eff}})^2}/2, \\ \overline{v_1 v_2} &= \overline{(v_{A,\text{eff}} + v_{B,\text{eff}})(v_{A,\text{eff}} - v_{B,\text{eff}})}/2. \end{aligned} \tag{16}$$

In other words, mean velocity, two components of turbulence intensity, and the cross correlation can be measured using an X probe. Neglecting v_3 gives an error of about 8% when the turbulence intensity is 20%, with the same flow field as discussed for the single wire.[32]

Adding a third wire at 45° to the mean flow and in a plane perpendicular to the X probe permits measuring the third component. With the above simplifying assumptions the equations are similar to those above. In this case more complex relations could be developed, for example, to correct the above equations for a nonzero v_3 since it is being measured.

I. Orthogonal sensors for three components

An alternative approach for obtaining all three components is to use three sensors in an orthogonal configuration.[14] From Fig. 11 it can be seen that the only assumption required is that Eq. (7) hold, $k_N = 1$, and that the velocity vector stays within the octant where the sensors represent the corners. Figure 12 shows a probe designed for three orthogonal sensors. Lekakis[15] has taken the basic orthogonal configuration of Fig. 12 and characterized it with detailed calibrations. As expected, it was

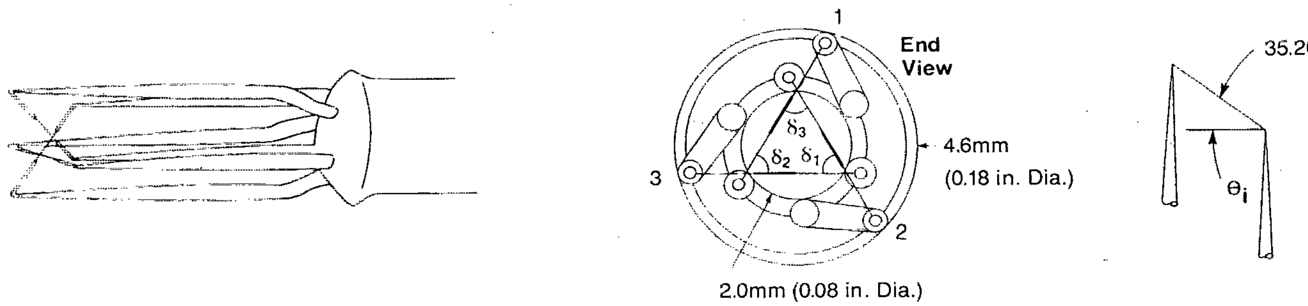


FIG. 12. Probe with three mutually perpendicular sensors.

very difficult to obtain reproducible results with hot-wire sensors so most of the work was done with film sensors which are much more rigid.

While orthogonal sensors improve performance at high turbulence intensities, the maximum turbulence intensity is still limited. As soon as the velocity vector crosses a plane defined by two of the sensors, the results are "rectified" and the turbulence intensity measured will be low. A number of techniques have been used to solve this problem including using "split" films[33] to detect octant. Another technique is to add a mean velocity by moving the sensor through the flow.[34] This is equivalent to frequency shifting in laser velocimetry. However, in high turbulence intensities the laser velocimeter is almost always the preferred instrument since with frequency shifting there is no theoretical limit on the turbulence intensity that can be measured.

J. Nonisothermal flows

All of the above equations assume a constant temperature, constant composition fluid. In most applications constant composition is a valid assumption but often the temperature is not constant. The most common situation is a slow variation in the mean temperature of the fluid since few experimental facilities have temperature control.

To be independent of temperature, the heat transfer from the sensor to the environment must be independent of temperature. From Eqs. (2) and (4)

$$Q=2l\pi k_f(A+B\,\mathrm{Re}^n)\left(1+\frac{a_T}{2}\right)^{0.17}(T_m-T_a), \qquad (17)$$

where $2l$=length of sensitive area of sensor. This equation can be compared with

$$Q=H_1(V)(T_m-T_a). \qquad (18)$$

It was found[35] that, for example, with $T_m=230\,°\mathrm{C}$ and $T_a=23\,°\mathrm{C}$, for a 50 °C increase in T_a, the velocity difference calculated with the two equations was $\pm 3\%$ for the range of 6–100 m/s. Therefore, simply maintaining the temperature difference T_m-T_a constant or a linear correction for changes in this temperature difference is adequate for many applications. This temperature difference can be adjusted manually in some research applications. A temperature sensitive element used for R_3 in Fig. 1 and exposed to the flow is one method of automatically compensating (allowing for the change in voltage reading due to a change in resistance).

With digital data acquisition, the preferred method is to simply measure the temperature of the flow and use that data to correct the velocity readings. Certainly in this case a more complex relation than Eq. (18) could be used if desired.

For rapid temperature changes, the temperature sensor used must have an adequate response. While very rapid temperature changes could be measured using two anemometers with the sensors at two different temperatures, this is sufficiently difficult that it has rarely been used outside of high-speed flows applications.

V. INDUSTRIAL APPLICATIONS

As mentioned earlier, the sensitivity to low velocities, the wide dynamic range, and the potentially low cost make thermal sensors applicable in a number of industrial applications. Generally in industrial applications frequency response and spatial resolution, at least on the order of that required for research measurements, are not important. The following are examples where thermal techniques have been used to measure flow in applications other than fluid mechanics research.

A. Portable air velocity meters

Industrial hygienists and air conditioning engineers often need to measure the air movement in areas where people live and work. These air movements cover a wide range of velocities, including very low velocities. The wide range and sensitivity to low velocities, as well as the low cost, make thermal sensor technology applicable.

Figure 13 shows an example of an air velocity meter with a thermal sensor. While the most simple ones measure velocity only, these meters often have other sensors such as temperature, humidity, and even pressure. Temperature is the most common added measurement since the sensors used for temperature compensation of the velocity measurement can also be used for a temperature measurement. Depending on the instrument, temperature compensation is done either with the temperature sensor as part of the bridge circuit or with a separate temperature measurement and subsequent correction of the velocity data by computation. In the past, air velocity meters provided a non-

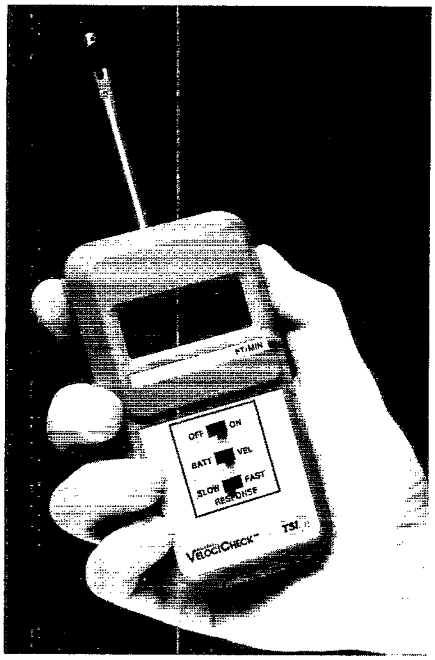


FIG. 13. Air velocity meter using thermal sensor.

FIG. 14. Air velocity transducer with power supply and readout.

linear scale on an analog meter for readout. With present microprocessor technology, most meters now provide a linearized, digital readout.

Air velocity meters represent one of the most common uses of thermal anemometers and are available from a number of companies in a wide variety of configurations.

B. Air velocity transducers

These are basically similar to the above portable air velocity meters but are designed to be mounted into air conditioning ducts, industrial dryers, or other locations where continuous monitoring and/or control is desired. The basic probe generally requires dc power (e.g., 12 V) and has an electrical output (0–5 V dc or 4–20 ma) proportional to the air velocity it is exposed to. A separate unit provides the dc power and readout (Fig. 14). Thermal sensors are used again because of their sensitivity to low velocities, wide range and low cost. As mentioned before, sensitivity to contamination is perhaps the major problem. This is at least partially overcome by using relatively large sensors.

In ducts and other confined flows, it is generally the total flow that is of interest, not simply the flow at a point. One technique to reduce the effect of a varying flow profile is to use a multisensor probe (Fig. 15). This can provide a more accurate average velocity as well as give better results if one of the sensors become contaminated. The unit pictured uses the temperature difference between two sensors, one heated by a constant power heater, to measure the flow at each point.

One of the more difficult environments for thermal sensors is the measurement of flow out of smokestacks. The 1990 clean air act has added urgency to this difficult measurement. Thermal sensors have been used—again reducing the contamination problem using large sensors and the flow profile problem using multiple sensors on a probe.

C. Fume hood and room pressure measurements

To protect workers using fume hoods from potentially toxic materials, it is important that the flow into the fume hood be maintained. While a pressure measurement could be considered, the actual pressure drop at normal face velocities in fume hoods (100 ft/min) is very low (20×10^{-12} psi). However, a thermal anemometer can measure 100 ft/min easily. By using a thermal anemometer to measure the flow velocity in an installed "leak" in the side of the fume hood, the face velocity is measured. The output of the anemometer can be used to simply monitor the flow and sound an alarm if too low, or it can be used in a feedback loop to control the flow at a setpoint (Fig. 16).

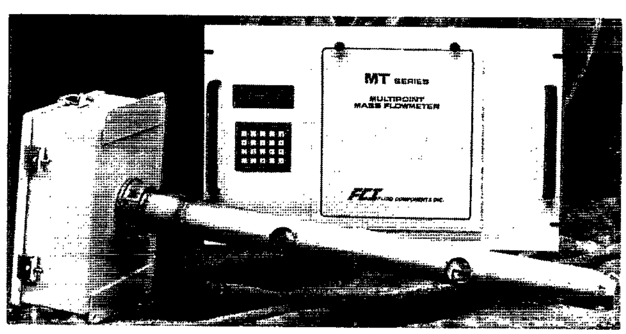


FIG. 15. Multisensor probe for obtaining average flow in large flow channels (courtesy of Fluid Components, Inc.).

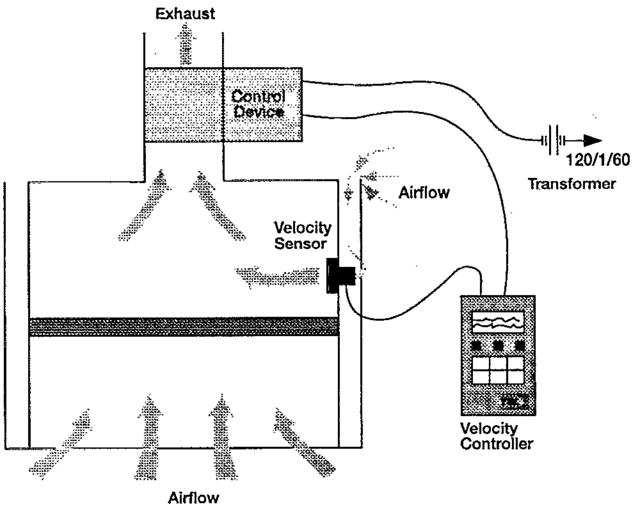


FIG. 16. System using thermal sensor to measure flow through a "leak" for fume hood control.

A similar technique can be used to measure the pressure in a room relative to its surroundings (Fig. 17). Again, a "leak" is installed with a thermal anemometer used to measure the flow through the "leak." In clean areas, the system can be used for monitoring or to control the room pressure above that of the surroundings. In chemical laboratories, it would be used to maintain the room pressure below that of the surroundings. Another application is hospital rooms. Since the room pressure control needs to know flow direction, two sensors are used rather than just one as in the unit for fume hoods.

D. Total flow sensors

One way of turning a point sensor into a total flow sensor is to locate it in the center of a nozzle that gives a repeatable flow profile. With this technique a line of flowmeters with different size nozzles can be developed. An example application is test stands to verify the flow versus

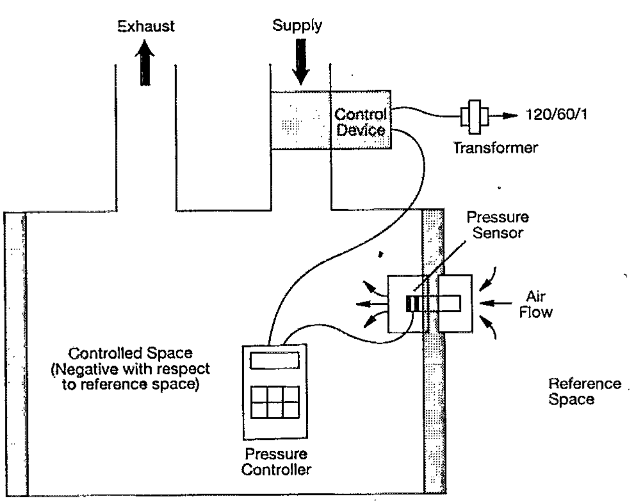


FIG. 17. System using thermal sensor to measure flow and direction through a "leak" for room pressure control.

pressure through small orifices. In this kind of controlled environment, thermal flowmeters with film sensors can be very repeatable with the accuracy dependent primarily on the accuracy of the original calibration. Thermal sensors are used because of their wide dynamic range and, in some production applications, their fast response.

A specific application of thermal total flow sensors is in ventilators used in hospitals. In this application, the very low pressure drop and rapid response are the primary advantages of thermal sensors. They have been used for the incoming gas streams as well as the exhaled breath.

Perhaps the highest volume application of thermal sensors is to measure air flow in engines with fuel injection. The sensor provides an output proportional to the total flow of air into the engine. This signal, along with other parameters, is then used in a microprocessor controller to adjust the fuel flow to the engine.

All of the devices mentioned so far use essentially a "point" sensor, either alone or in combination with a nozzle or some other device to control the flow profile. However, there is another class of flow measuring and/or controlling devices where the thermal sensor essentially "surrounds" the flow by being located on the outside of the passage that contains the flow (Fig. 18). The primary application for these devices is the control of gases flowing to diffusion furnaces used in the semiconductor industry. The major advantage in this application is the fact that no sensor is exposed to the flowing fluid, which may be very corrosive. Also, very low flows can be measured. This technique is generally applied to flows in the range of 1 cc/min to 350 ℓ/min. The larger flows are measured using the bypass arrangement shown.

VI. FUTURE DIRECTIONS OF THERMAL ANEMOMETRY

A. Research

The thermal anemometer remains the preferred tool for measuring low turbulence intensities in fluid mechanics research. It is also the instrument of choice for detailed measurements when the maximum turbulence intensities are below 20% and the fluid is a gas at normal temperatures.

Materials used for hot-wire sensors are still the same ones that were used 50 years ago. Certainly a material that would not oxidize at high temperatures, had good high-temperature strength, a high coefficient of resistance, high resistivity, and low thermal conductivity would be helpful. Of course, it also has to be formed into a fine wire.

While microprocessors can continue to make the actual control circuits easier to use, the primary advantage of modern electronics is the ability to rapidly digitize the data and then operate on this digital data using computers. Faster and more cost effective A/D converters and fast memories to acquire the data will all be forthcoming. Continuing improvement in data analysis software should also be expected.

The primary competition presently is the laser anemometer. While in most cases the ability to make the mea-

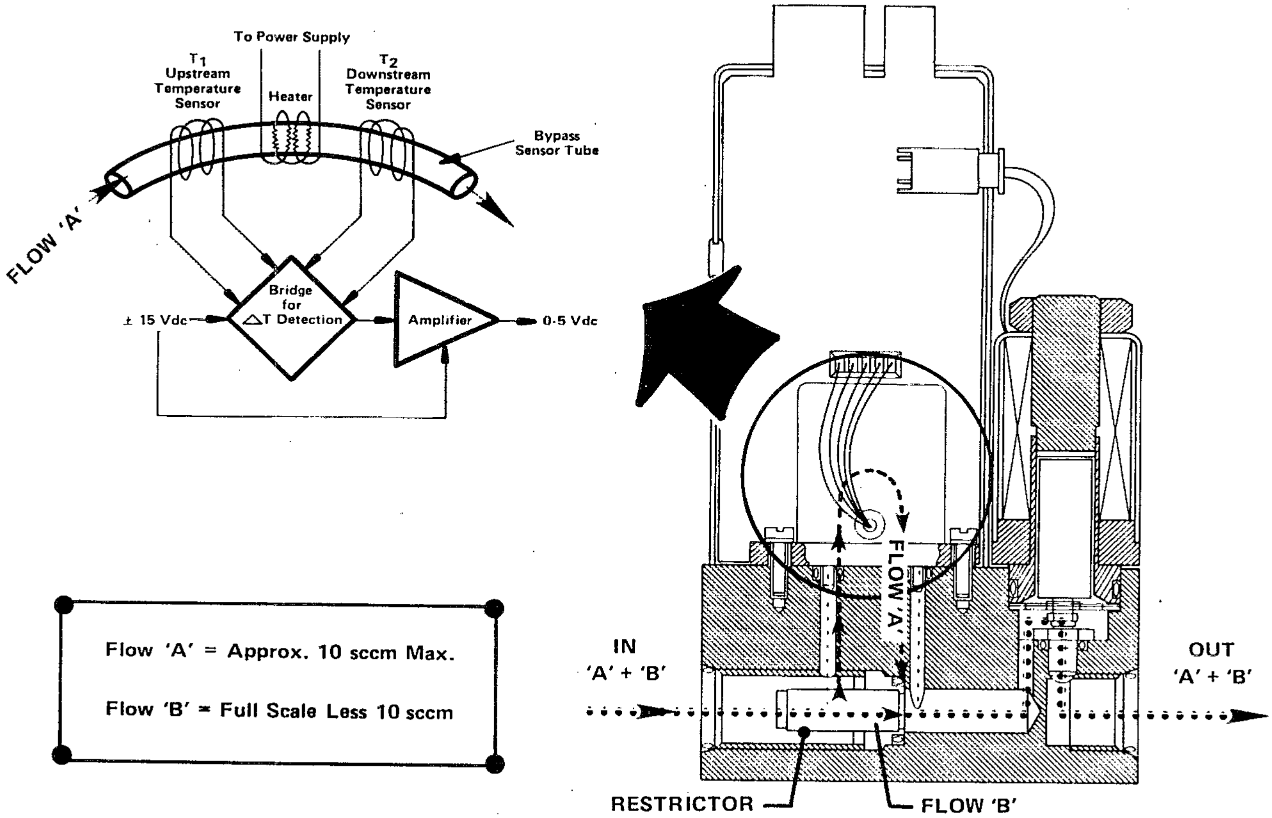


FIG. 18. Total flow sensor utilizing thermal techniques (courtesy of Brooks Instrument Division, Emerson Electric Co.).

surement well has been the primary criteria, in the past cost has often been a factor in selecting a thermal anemometer over a laser anemometer. That will likely be less of a consideration in the future as costs come down with developments in laser diodes, photodiodes, and lower cost electronics and optics. Even so, the discontinuous output of a laser anemometer and the difficulty in measuring low turbulence intensities will continue to give the thermal anemometer advantages in many applications in fluid mechanics research.

B. Industrial applications

These applications could increase for the following reasons:

(1) The introduction of microprocessor technology. The nonlinear response of a thermal anemometer was a major disadvantage. Now it is a plus because it is easily linearized and the fundamental nonlinear response provides a sensor with a very large dynamic range.

(2) The development of highly stable sensors that are large enough to resist contamination and, when contaminated, are easily cleaned.

(3) Environmental concerns, which may increase the needs for low cost sensors that can measure very low velocities.

Sensitivity to contamination continues to be a problem in many potential applications. There seems to be no solutions to the problem other than making the sensor larger. At the same time, the fact thermal sensors have been permanently installed in air conditioning ducts and even used in smokestacks would indicate that large sensors can have a major effect in reducing contamination sensitivity. Of course, they also require more power so may not be practical for portable instruments.

VII. DISCUSSION

Thermal anemometers are well understood and will continue to be used in many applications by fluid mechanics researchers. They are especially effective for measuring low turbulence intensities and for high frequencies. The continuous output also gives thermal anemometers a definite advantage over laser anemometers in many applications. However, new developments in solid state lasers, electronics, and optics will make laser anemometers more price competitive in some applications.

It is expected that the natural advantages of sensitivity and dynamic range of thermal sensors will generate increased industrial applications with the advent of stable, rugged sensors, low cost linearization, and increased emphasis on environmental measurements.

ACKNOWLEDGMENT

The author is indebted to the advice of Professor Peter Freymuth, University of Colorado. His knowledge of both the theory and the literature of thermal anemometry complements the author's own experience.

APPENDIX. HEAT TRANSFER FROM THERMAL SENSORS

The heat transfer from a cylindrical sensor (e.g., a wire) oriented perpendicular to a flowing fluid can be written as follows:

$$Q=\mathrm{Nu}\,\pi 2lk_f(T_m-T_a),$$

where $\pi=3.14159$, $2l$=length of sensor, T_m=mean temperature of sensor, and T_a=temperature of fluid. The variables affecting Nu can be listed as follows:

$$\mathrm{Nu}=f\left(\mathrm{Re},\mathrm{Pr},\alpha,\mathrm{Gr},\mathrm{Ma},\gamma_h,a_T,\frac{2l}{d},\frac{k_f}{k_w}\right),$$

where Pr=Prantl number$=c_p\mu/k_f$, α=angle between fluid velocity and a plane perpendicular to the sensor, Gr=Grashof Number$=d^3\rho^2\beta_\gamma(T_m-T_a)/\mu^3$, $\beta_v=1/T_a$ =volume coefficient of expansion, Ma=Mach number $=v/\sqrt{\gamma_h RT}$, γ_h=ratio of specific heat at constant volume and constant pressure of the fluid, k_w=Thermal conductivity of sensor material.

The following is a brief explanation of each of the parameters and when they are important.

Reynolds number (Re) and the "thermal overload" a_T are included in the simplified Eq. (4) in the text. The term a_T basically provides for a temperature dependence beyond the basic temperature difference between the sensor and its environment included in Eq. (2) of the text. The effect is minimized by defining the fluid thermal conductivity at the "film" temperature or the average temperature between the sensor and the fluid.

The Prantl number (Pr) is the nondimensional parameter primarily associated with the composition of the fluid. A nondimensional equation that was valid for more than one fluid would have to include the Prantl number as a variable.

The angle a between the sensor and the velocity vector must be considered in any use of thermal sensors. Heat transfer equations for a cylinder are generally written for flow perpendicular to the sensor. The angle is often taken into account by defining an "effective" velocity v_{eff} as is done in the text of this paper.

The Grashof number (Gr) represents the "free convection." It only becomes important at very low velocities where flow generated by the density difference between the heated fluid surrounding the sensor and the bulk fluid becomes significant in comparison to the forced convection. Wills[36] suggests that Buoyancy effects are important in air only if

$$\mathrm{Gr}^{1/3}>\mathrm{Re}.$$

For high velocity or low density flows, the Mach number and ratio of specific heats must be considered variables. For low densities, the most relevant parameter is the Knudsen number, which is

$$\mathrm{Kn}=\frac{\lambda}{d}=\left(\frac{\pi\gamma_h}{2}\right)^{0.5}\left(\frac{\mathrm{Ma}}{\mathrm{Re}}\right),$$

where λ is the molecular mean-free path. Heat transfer equations generally assume continuum flow, defined as $\mathrm{Kn}<0.01$. It turns out that fine hot wires are often operating in the slip flow range where $0.01<\mathrm{Kn}<1$. In this range, the usual continuum assumption works as long as the density changes are small. In free molecular flow, ($\mathrm{Kn}>1$) Nu becomes proportional to Re (rather than $\mathrm{Re}^{1/2}$) and heat transfer becomes more sensitive to density and rather insensitive to velocity—explaining the application of thermal sensors to measure pressure in vacuum systems.

High-speed flows present a number of special problems and the reader is referred to publications on the subject.[37–40] The ratio of length to diameter enters the equation both because of conduction losses to the probe and the possible flow effects of the sensor supports. Finally, the ratio of thermal conductivities will effect the temperature distribution along and around the sensor. Especially in film sensors, this temperature distribution could be important but it has not, to this authors knowledge, been studied.

Even the above considerations are not comprehensive. For example, if a sensor is close to a solid surface, conduction to the surface or the change in velocity profile around the sensor due to the presence of the surface can change the heat transfer. In most applications of thermal sensors, radiation is not a factor. Of course this could change with a large sensor, low fluid density, and an intense source of radiation. The important point is that anything that impacts the heat transfer from the sensor to the environment is going to be sensed. However, Eq. (4) does represent this transfer quite well for most applications in air.

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