STAT 206 Homework 4

Due Monday, October 30, 5:00 PM

General instructions for homework: Homework must be completed as an R Markdown file. Be sure to include your name in the file. Give the commands to answer each question in its own code block, which will also produce plots that will be automatically embedded in the output file. Each answer must be supported by written statements as well as any code used. (Examining your various objects in the "Environment" section of RStudio is insufficient -- you must use scripted commands.)

In lecture, we fit a gamma distribution to the weight of cats hearts. We did this by adjusting the parameters so that the theoretical values of the mean and variance matched the observed, sample mean and variance. Since the mean and variance are the first two moments of the distribution, this is an example of the method of moments for estimation.

The method of moments gives a point estimate $\hat{\theta}$ of the parameters θ. To use a point estimate, we need to know how precise it is, i.e., how different it would be if we repeated the experiment with new data from the same population. We often measure imprecision by the standard error, which is the standard deviation of the point estimates $\hat{\theta}$. (You saw the standard error of the mean in your introductory statistics classes, but we are not computing the standard error of the mean here.)

If we actually did the experiment many times, getting many values of $\hat{\theta}$, we could take their standard deviation as the standard error. With only one data set, we need to do something else. There is usually no simple formula for standard errors of most estimates, the way there is for the standard error of the mean. Instead, we will see how to approximate the standard error of for our estimate of the gamma distribution computationally.

We can draw random values from a gamma distribution using the `rgamma()` function. For example, `rgamma(n=35,shape=0.57,scale=15)` would generate a vector of 35 random values, drawn from the gamma distribution with shape parameter a = 0.57 and scale s = 15. By applying the estimator to random samples drawn from the distribution, we can see how much the estimates will change purely due to noise.

Part I - Estimates and standard errors

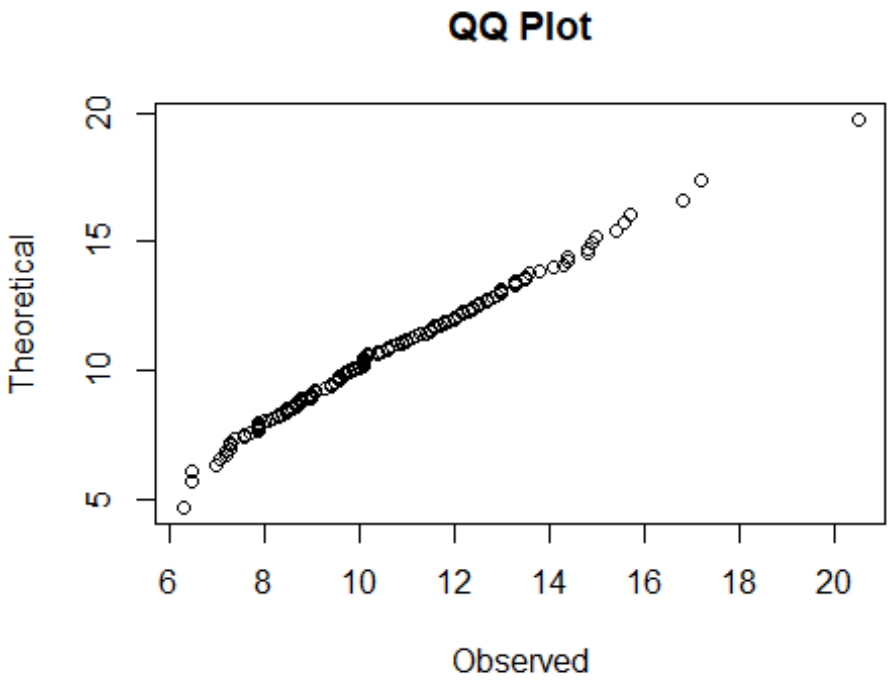
1. Write a function, `gamma.est`, which takes as input a vector of data values, and returns a vector containing the two estimated parameters of the gamma distribution, with components named shape and scale as appropriate.

```
library(MASS)
data("cats", package="MASS")

gamma.est = function(data) {
```

```
  avg = mean(data)
  var = var(data)
  scale = var / avg
  shape = avg / scale
  return(c(shape = shape, scale = scale))

}

gamma.est(cats$Hwt)

##      shape      scale
## 19.0653121  0.5575862
```

2. Verify that your function implements the appropriate formulas by showing that it matches the results from lecture for the cat heart data.

```
#The following code was copied from the leecture notes. This shows that the
values
#are close to the above values. We can also generate a qqplot and look for a
straight line

fitdistr(cats$Hwt, densfun="gamma")

##      shape         rate
##   20.2998092    1.9095724
##  ( 2.3729250) ( 0.2259942)

cats.gamma <- gamma.est(cats$Hwt)
qqplot(cats$Hwt, qgamma(ppoints(500), shape=cats.gamma["shape"],
      scale=cats.gamma["scale"]), xlab="Observed", ylab="Theoretical",
      main="QQ Plot")
```

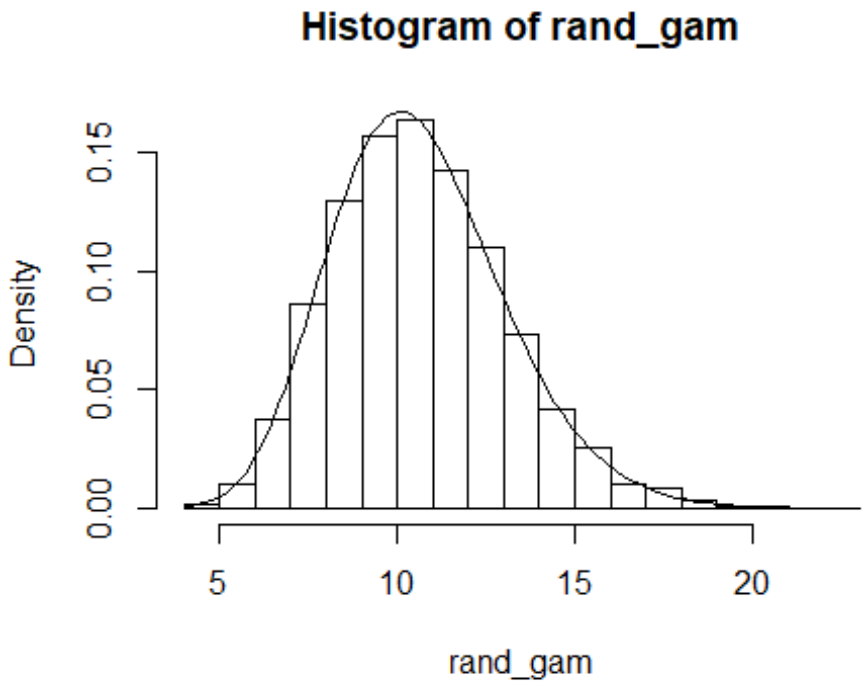


3. Generate a vector containing ten thousand random values from the gamma distribution with $a = 19$ and $s = 0.56$. What are the theoretical values of the mean and of the variance? What are their sample values?

```
#to create a vector of 100000 random values from the gamma distribution we
use rgamma
#it is saved to a variable to we can use mean and var on that variable
# avg = shape * scale and var = shape * scale^2
#we can see the random sample values and derived values are very close

rand_gam = rgamma(seq(1,10000,1), shape = 19, scale =0.56)
mean(rand_gam)

## [1] 10.6569

var(rand_gam)

## [1] 5.935505

derived_avg = 19 * .56
derived_avg

## [1] 10.64

derived_var = 19*(.56^2)
derived_var

## [1] 5.9584
```

4. Plot the histogram of the random values, and add the curve of the theoretical probability density function.

```
#To make this overlay work we need to pass the probability = TRUE argument to
hist
#if not, we will not have the correct scaling along the y axis
#to make the curve go into the same windows as the hist we must pass the add
= TRUE

hist(rand_gam, probability = TRUE)
curve(dgamma(x, shape= 19, scale=.56), add = TRUE)
```



5. Apply your `gamma.est` function to your random sample. Report the estimated parameters and how far they are from the true values.

```
#the values are quite close to the actual values. First we take a look at the
shape and scale
#from the random values, then we subtract out the actual values to see how
close they are

gamma.est(rand_gam)

##      shape      scale
## 19.1339113  0.5569638

gamma.est(rand_gam)[1] - gamma.est(cats$Hwt)[1]

##      shape
## 0.06859928
```

```
gamma.est(rand_gam)[2] - gamma.est(cats$Hwt)[2]
```

```
##         scale
## -0.0006224296
```

6. Write a function, `gamma.est.se`, to calculate the standard error of your estimates of the gamma parameters, on simulated data drawn from the gamma distribution. It should take the following arguments: true shape parameter `shape` (or a), true scale parameter `scale` (or s), size of each sample `n`, and number of repetitions at that sample size `B`. It should return two standard errors, one for the shape parameter a and one for the scale parameter s. (These can be either in a vector or in a list, but should be named clearly.) It should call a function `gamma.est.sim` which takes the same arguments as `gamma.est.se`, and returns an array with two rows and B columns, one row holding shape estimates and the other row scale estimates. Your `gamma.est.se` function should not, itself, estimate any parameters or generate any random values

```
#first I want to write the gamma.est.sim function that will be called upon by
the gamma.est.se function
#then I can wrap this function with another function that will use those
values to calculate
#standard error of each row


#set.seed(10)
row_1 = c()
row_2 = c()

gamma.est.sim= function(shape, scale, n, B) {
  raw_data = rgamma(seq(1,B,1), shape = shape, scale = scale)
  for(mean in raw_data) {
    value_1 = mean / scale
    row_1 = c(row_1, value_1)
    value_2 = mean / shape
    row_2 = c(row_2, value_2)
  }

  return(rbind(row_1,row_2))

}

gamma.est.se = function(shape, scale, n, B) {
  shape_error = sd(gamma.est.sim(shape, scale, n, B)[1,])
    scale_error = sd(gamma.est.sim(shape, scale, n, B)[2,])
    return(c(shape_error, scale_error))
}

gamma.est.se(19,.56,100,10000)
```

```
## [1] 4.3803847 0.1288364
```

Part II - Testing with a *stub*

To check that `gamma.est.se` works properly, write a *stub* or *dummy* version of `gamma.est.sim`, which takes the correct arguments and returns an array of the proper size, but whose entries are fixed so that itâ□□s easy for us to calculate what `gamma.est.se` ought to do.

7. Write `gamma.est.sim` so that the entries in the first row of the returned array alternate between `shape` and `shape+1`, and those in the second row alternate between `scale` and `scale+n`. For example `gamma.est.sim(2,1,10,10)` should return (row names are optional)

```
##        [,1] [,2] [,3] [,4] [,5] [,6] [,7] [,8] [,9] [,10]
## shapes    2    3    2    3    2    3    2    3    2     3
## scales    1   11    1   11    1   11    1   11    1    11
```

```
#we need to write some code that does different opetations for alternating
columns

gamma.est.sim = function(shape, scale, n, B) {
  row_1 = c()
  row_2 = c()

  for(value in 1:B) {
    value_1 = value / scale
    value_2 = value / shape
    if (value %% 2 == 0) {
      row_1 = c(row_1, shape +1)
      row_2 = c(row_2, scale + n)

    }
    else {
      row_1 = c(row_1, shape)
      row_2 = c(row_2, scale)
    }
  }

return(rbind(row_1, row_2))

}

gamma.est.sim(2,1,10,10)
```

```
##       [,1] [,2] [,3] [,4] [,5] [,6] [,7] [,8] [,9] [,10]
## row_1    2    3    2    3    2    3    2    3    2     3
## row_2    1   11    1   11    1   11    1   11    1    11
```

```
gamma.est.sim(2,8,5,7)
```

```
##       [,1] [,2] [,3] [,4] [,5] [,6] [,7]
## row_1    2    3    2    3    2    3    2
## row_2    8   13    8   13    8   13    8
```

and `gamma.est.sim(2,8,5,7)` should return

```
##      [,1] [,2] [,3] [,4] [,5] [,6] [,7]
## [1,]    2    3    2    3    2    3    2
## [2,]    8   13    8   13    8   13    8
```

8. Calculate the standard deviations of each *row* in the two arrays above.

```
#I choose to use the built-in function of sd to accomplish this and using [] to extract the
#desired rows

sd(gamma.est.sim(2,1,10,10)[1,])

## [1] 0.5270463

sd(gamma.est.sim(2,1,10,10)[2,])

## [1] 5.270463

sd(gamma.est.sim(2,8,5,7)[1,])

## [1] 0.5345225

sd(gamma.est.sim(2,8,5,7)[2,])

## [1] 2.672612
```

9. Run your `gamma.est.se`, with this version of `gamma.est.sim`. Do its standard errors match the standard deviations you just calculated? Should they?

```
#The values from my function gamma.est.se match the standard deviations from the above

gamma.est.se(2,1,10,10)

## [1] 0.5270463 5.2704628

gamma.est.se(2,8,5,7)

## [1] 0.5345225 2.6726124
```

Part III - Replacing the stub

10. Write the actual `gamma.est.sim`. Each of the B columns in its output should be the result of applying `gamma.est` to a vector of `n` random numbers generated by a different call to `rgamma`, all with the same shape and scale parameters.

```
#The replicate function is ideal for this because its arguments, are the number of replications
```

```r
#which in this case we want to do this for the number of columns, then and
expression
#which we use our gamma estimate and finally we can use the rgamma to
generate the random numbers

gamma.est.sim = function(shape, scale, n, B) {
  adjusted = replicate(n, gamma.est(rgamma(n, shape = shape, scale = scale)))
  return(adjusted)
}

#gamma.est.sim(19,.56,100,10000)
```

11. Run `gamma.est.se`, calling your new `gamma.est.sim`, with `shape=2`, `scale=1`, `n=10` and `B=1e5`. Check that the standard error for `shape` is approximately 1.6 and that for `scale` approximately 0.54. Explain why your answers are not exactly 1.6 and 0.54.

```r
#Note: everytime we run this function the output is being generated off of
"random" numbers generated
#by the rgamma function. This means that each time we run this chunk we will
get different numbers
#sometimes that are close to 1.6 and .54 and sometimes not.

gamma.est.se(shape=2, scale=1, n=10, B=1e5)
```

```
## [1] 1.0545956 0.5139997
```