Homework #3

1. **(20 points)** The joint probability mass function of discrete random variables X and Y taking values x = 1, 2, 3 and y = 1, 2, 3, respectively, is given by $f_{XY}(x, y) = c*(x + y)$. Determine the following:

a) **(2 points)** Find c ∑fxy(x,y)=1 => c=1/36

b) **(2 points)** Find probability of the event, where X = 1 and Y < 3 P(Y<3|X=1)=3+2/36=5/36

c) **(2 points)** Find marginal probability $P_Y(Y = 2)$ Py(Y=2)=12/36

d) **(2 points)** Find marginal probability distribution of the random variable X Px(x=1)=9/36;Px(x=2)=12/36 Px(x=3)=15/36

e) **(2 points)** Find E(X), E(Y), V(X), and V(Y) E(X)=1*fx(x=1)+2*fx(x=2)+3*fx(x=3)=78/36=E(Y). V(X)=E(X^2)-E(X)^2=192/36-(13/6)^2=23/36=V(Y)

f) **(2 points)** Find conditional probability distribution of Y given that X = 1 Py|x(y)=Pxy(x=1,y)/P(x=1)=2/9;3/9;4/9

g) **(2 points)** Conditional probability distribution of X given that Y = 2 Px|y(x)=Pxy(x,y=2)/P(y=2)=3/12;4/12;5/12

h) **(2 points)** Are X and Y independent? No, fxy(1,1)≠fx(1)fy(1)

i) **(2 points)** What is the covariance for X and Y? cov(x,y)=E(XY)-E(X)E(Y)=1*2+2*3+3*4+2*3+4*4+4*6+3*4+6*5+9*6/36-(13/6)^2=14/3-2.167^2=-0.0278

j) **(2 points)** What is the correlation for X and Y? corr=cov(x,y)/sd(x)sd(y)=-0.0278/0.8^2

2. **(8 points)** A random variable X has density function $f(X = x) = c(x + x^3)$ for $x \in [0,1]$ and 0 otherwise.

a) **(2 points)** Determine c. ∫c(x+x^3)dx=1=>c(x^2/2+x^4/4)=c(0.5+0.25)=1=>c=4/3

b) **(2 points)** Compute E(1/X) E(x)=∫f(x)xdx= ∫ 4/3(1/X+1/X^3)*1/Xdx=4/3(0-(-1-1/3))=16/9

c) **(4 points)** Determine the probability density function of $Y = X^2$ P(Y=y)=dP(X<sqrt(y))/dy=d4/3(0.5y+0.25y^2)/dy=2/3(1+y)

3. **(10 points)** An unfair coin is tossed 4 times. The probability of heads is 0.6 and that of tails is 0.4. Let X be the total number of heads among the first two tosses and Y the total number of heads among the last two tosses.

a) **(4 points)** Write down the joint probability mass function of X and Y. Pxy(X,Y)

b) **(2 points)** Are X and Y independent? Please explain. Pxy(X,Y)=Px(X)Py(Y), independent

c) **(4 points)** Compute the conditional probability $P(X \geq Y | X \geq 1)$ P(X>=Y|X>=1)=P(X>=Y,X>=1)/P(X>=1)

4. **(6 points)** A random variable X is the average of p independent random variables X_k, i.e., $X = \frac{1}{p}\sum_{k=0}^{p} X_k$,

Calculate the mean and the variance of X for three different cases:

E(X)=E(Xk)=0.5,V(X)=V(Xk)/p=1/12p

a) **(2 points)** When all X_k are independent uniform random variables in the interval (0,1)

b) **(2 points)** When all X_k are independent exponential random variables with PDF $f_k(x) = \lambda_k e^{-\lambda_k x}$

E(X)=∑1/lambda_k/p,V(X)=∑1/lambda_k^2*1/p^2

c) **(2 points)** When all X_k are Independent normal random variables but each one has its own mean μ_k and its own standard deviation σ_k E(X)=1/p∑mu_k;V(X)=1/p^2∑sigma^2

5. **(4 points)** Random variables X, Y have standard derivations, $\sigma_X = 2$ and $\sigma_Y = 6$, respectively, and their correlation is given by corr(X, Y) = −1/3.

(a) **(2 points)** Find cov(X, Y). corr(x,y)=cov(x,y)/sigma_x*sigma_y=>cov(x,y)=2*6*-1/3=-4

(b) **(2 points)** Find Var(4X − 2Y). Var(4X-2Y)=Var(4X)+Var(2Y)+2cov(4X,-2Y)=4^2Var(X)+(-2)^2Var(Y)-16Cov(x,y)=16*4+4*36+64=272

6. **(12 points)** Time interval separating subsequent bus arrivals at a stop is an exponential random variable with mean 20 minutes. Steve and Andrew work at the same place and each will be late to work unless they board a bus on or before 8:40am. Steve comes to the bus stop exactly at 8am. Andrew also comes to the same bus stop but at a random time, uniformly distributed between 8am and 8:30am. Both of them take the first bus that arrives.

exponential distribution: r=1/20
x is the time of andrew arrive
t is the bus come

(a) **(4 points)** What is the probability that Steve will be late for work tomorrow? P(x>40)=exp(-1/20*40)=1/e^2

(b) **(4 points)** What is the probability that Andrew will be late for work tomorrow? P(x+t>40)=∫t in [0,30] 1/30 exp(-r*(40-t))dt=20e^-2/30(e^1.5-1)

(c) **(4 points)** What is the probability that Steve and Andrew will ride the same bus

P(x>t)=∫t in [0,30] 1/30*exp(-r*t)dt=20/30(1-e^-1.5)