Introduction to PET Image Reconstruction

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Fall 2007

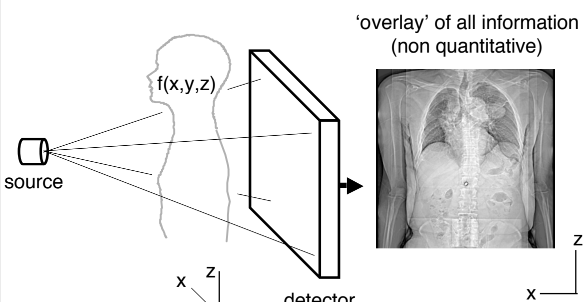
http://depts.washington.edu/nucmed/IRL/education.shtml

Basic Stages of PET

- I. Radionuclide Production
  - Make radio-isotope
- II. Radiochemistry
  - Make radiopharmaceutical - Label a tracer
- III. Imaging
  1. Administer radiotracer
  2. Positron decay - annihilation
  3. Anti-parallel photons travel through patient (some interact)
  4. Photons enter detectors (most interact)
  5. Detected Photons paired into coincident events
  6. Store events in sinogram format (data)
- IV. Data Analysis
  1. Correct data for physical effects
  2. Reconstruction into images and interpret

2

Projection Imaging

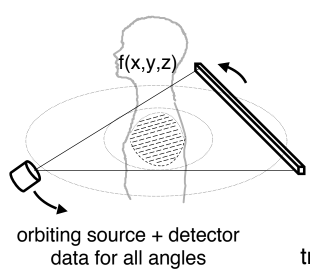


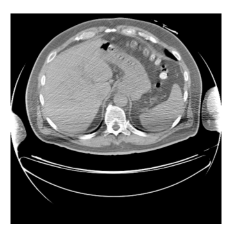
$p(x,z) = \int f(x,y,z)\, dy$

3

Tomographic Imaging

'Tomo' + 'graphy' = Greek: 'slice' + 'picture'

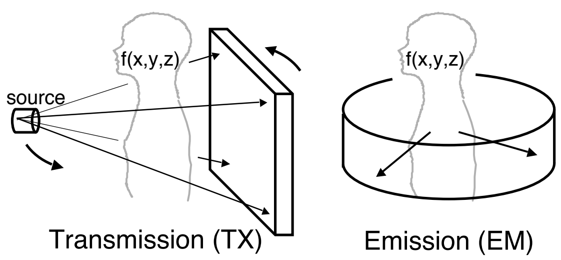




true cross-sectional image

4

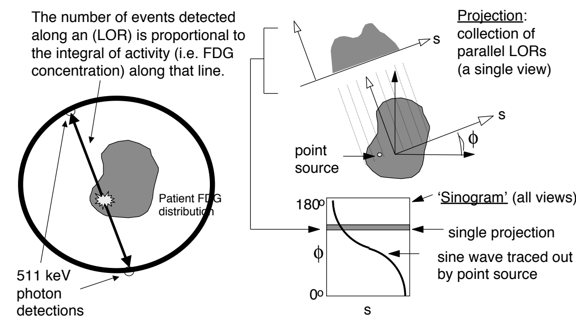
Types of imaging systems



… but same mathematics of tomography

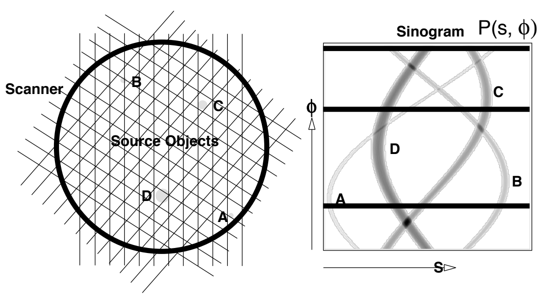
5

From photon detection to data in form of **Sinograms**



6

Sinogram Example



- The sinogram is $p(s,\phi)$ organized as a 2D histogram - Radon Transform of the object

7

IV. Data Analysis

Order of corrections (common application):

Start with Raw Data:

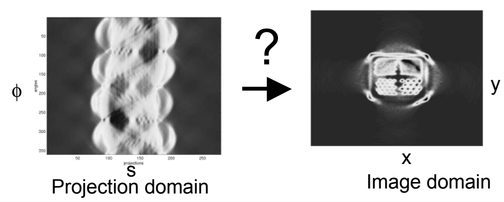
Prompt Events = Trues + Randoms + Scatter
Delayed Events = Approximation of Randoms

1. Randoms correction (Yr = Prompt-Delayed)
2. Detection efficiency normalization (Yn = Yr * Norm)
3. Deadtime (Yd = Yn * Dead)
4. Scatter (Ys = Yd - Scat)
5. Attenuation (Ya = Ys * ACF) attenuation correction factors
6. **Image Reconstruction**

8

IV. Data Analysis

Sinograms reconstructed into images



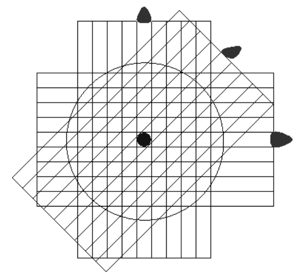
Reconstruction Problem

All modalities that collect line-integral data (sinograms) can employ the same methods for reconstruction:

1. Analytical Methods - most popular: FBP- Filtered Backprojection
2. Iterative Methods - most popular: OSEM - ordered subset expectation maximization

9

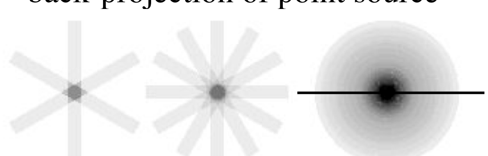
Analytical Methods: Data Formation



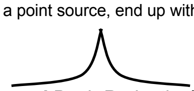
- Point Source - Forward Projection to 3 projections… Can get estimate of point source with Back-projection

10

Analytical Methods: simple back-projection of point source



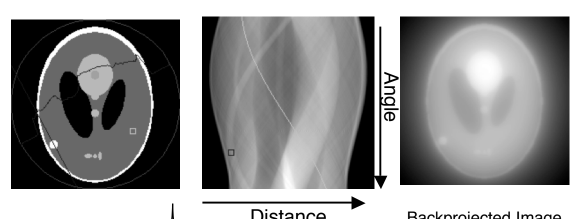
- Instead of getting a point source, end up with:

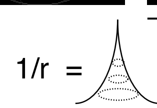


- Need to do "**Filtered** Back-Projection" FBP

11

Image & Sinogram Space





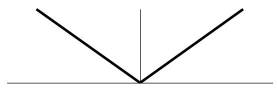
Backprojected Image
No filtering ==>
Result = true⊗1/r

To undo this, filter projections with ramp filter before backprojecting…

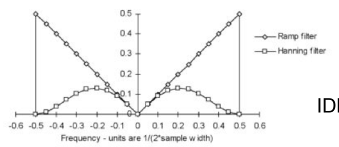
12

Analytical Method: Filtered back-projection

Ramp filter accentuates high frequency - Not good for noise



Either clip ramp filter or often use filters to clip ramp to reduce noise: ex: Hanning filter



IDL Demo

13

FBP Characteristics

- **PROS:**
  - Analytic method ("inverse Radon transform")
  - FBP is "exact" IF:
    - No noise
    - No attenuation
    - Complete, continuously sampled data
    - Uniform spatial resolution
  - Easy to implement
  - Computationally Fast
  - Linear, other properties well understood (2x uptake = 2x intensity in image)
  - Can Adjust filter window to trade off bias vs. variance
- **CONS:**
  - cannot model noise in data,
  - cannot model non-idealities of system, (resolution recovery methods)
  - does not easily work with unusual geometries,
  - cannot include knowledge about the image (like non-negative activity)

14

Iterative Reconstruction Characteristics

- **Pros**
  - Reduce variance (noise) for a given level of accuracy (bias, resolution, etc.)
  - Reduce or eliminate streaks
  - Incorporate (model) physical effects
    - Counting statistics (noise)
    - Confidence weighting
    - Distance dependent resolution
    - Scatter, attenuation, detector efficiency, deadtime, randoms
    - *a priori* information (non-negativity, anatomical information, *etc.*)
- **Cons**
  - Slow (computationally intensive)
  - Non-linear -- hard to analyze
  - A lot of "knobs" to adjust: smoothing parameters, number iterations, etc.
  - Streaks replaced with different noise character (e.g. "blobs")

15

The Reconstruction Problem: An Inverse Problem

Unknown image

$$y = \mathrm{P}x + n$$

Observed PET data — system matrix — Error in observations

- x is N x 1 image vector (typically N ~ 128 x 128)
- y is M x 1 data vector (typically M ~ 280 x 336)
- P is M x N system matrix (provides probability entry j from x will be placed in entry i of y)

16

Iterative Reconstruction: Basic Components

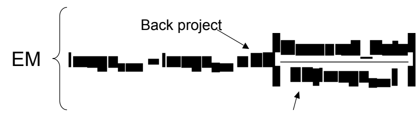
1. Description of the form of the image
   (pixels, voxels, blobs…)
2. System model relating unknown image to each detector measurement : relates image to data
   (Can include detector response, corrections for attenuation, efficiencies, etc…)
3. Statistical Model describing how each measurement behaves around its mean
   (Poisson, Gaussian,…)
4. Objective Function defining the “best” image estimate
   (Log-likelihood, WLS,MAP…)
5. Method for maximizing the objective function (EM)

Main Point: Lots of options, Not all “EM” algorithms the same

17

Overview of common choices for components

5. Method for maximizing the objective function
EM (or some minor variation) is most common approach
Expectation Maximization is a general method for solving all kinds of statistical estimation problems, in tomography results in easy maximization step



Tomography EM: Method for maximizing the Poisson Likelihood Function (ML-EM)

18

Iterative Recon Algorithm

Initialize Activity & Attenuation

y=Px + noise

1. Image Estimate: x^n

2. Model System: P

Iterate

Sinogram Data: y

3,4.Objective Function

5. Locate “Downhill”

5. Minimizer: x^{n+1}

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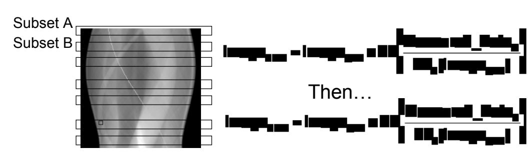
19

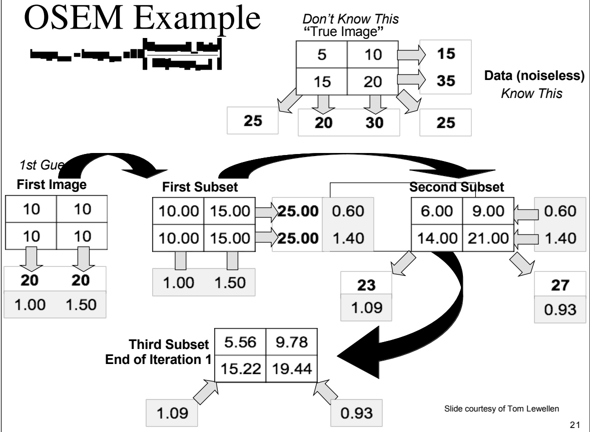
Common Methods

1. **EM** (ML-EM - maximum likelihood method)
2. **OSEM** (Ordered Subset Expectation Maximization)
   - Variant of EM (still Maximum Likelihood) - Pro: Fast | Con: Does not converge
   - Uses subsets of the data to compute each estimate…Subset A then B then …then repeat



20

OSEM Example



Slide courtesy of Tom Lewellen

21

Common Methods

3. Regularized Methods

– MAP(maximum *a posteriori)*, PWLS(penalized weighted least squares), GEM (generalized EM)
– All consist of variation in objective function…

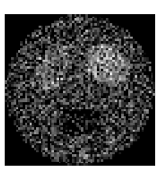
– Assume have some knowledge about the image before we even get the data (*a priori* knowledge)
– **PROS:** Can enforce noise/resolution properties in final image (don't need to post-smooth), Can include anatomical information (PET/CT?), Enforce non-negativity, Methods converge to final solution, More accurate model of data
– **CONS:** Usually takes longer, has more variables to set and understand (more knobs…), can impose odd noise structures

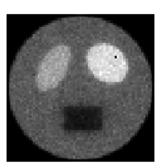
22

Examples of EM vs. FBP: Simulation

- Simulation with Poisson noise based on an average of 100 counts per detector channel


Original Image

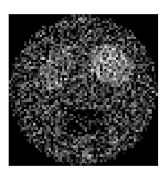

Filtered Backprojection (ramp filter)

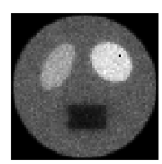

Maximum-Likelihood (ML-EM)

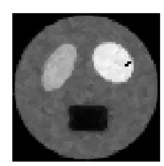
23

FBP vs. EM vs. MAP

(not exhaustive comparison)


Filtered Backprojection (ramp filter)

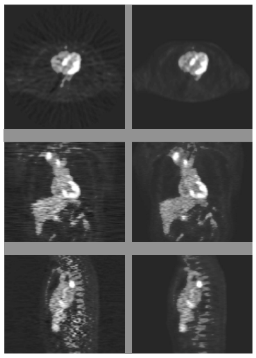

Maximum-Likelihood (ML-EM)


Maximum a Posteriori (MAP)

24

Some More Examples:

FBP, 12 mm Hann, no axial smoothing

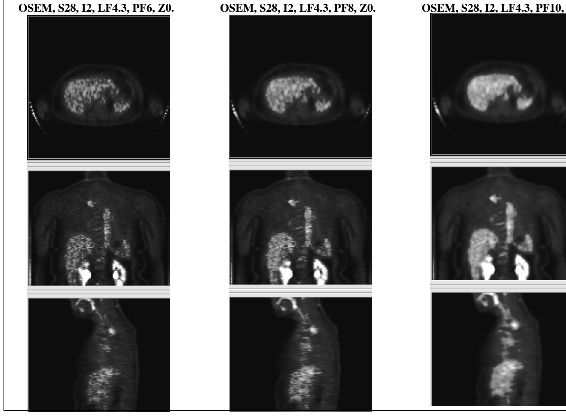


OSEM, 10 mm Gaussian, no axial smoothing

25

Example of different post filters

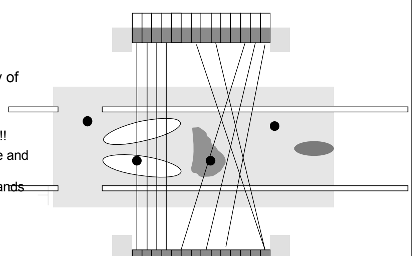
OSEM, S28, I2, LF4.3, PF6, Z0. | OSEM, S28, I2, LF4.3, PF8, Z0. | OSEM, S28, I2, LF4.3, PF10, Z1.



26

Imaging - Fully 3D vs. 2D

- Fully 3D PET data increases sensitivity of scanner (~ 8x)
- Drawbacks:
  - Increased scatter !!
  - Significant storage and **reconstruction** computation demands



27

Fully 3D Reconstruction

- Direct Analytic Approach
  - 3DRP: 3D reprojection (Kinahan and Rogers 1988)
- Iterative Approach
  - Simple conceptual extension: Just need system model that relates voxel to fully 3D data (as opposed to a pixel to 2D data)
  - System model becomes 100-2000x larger (big computational challenge. (2D:1.5 Billion entries to 3D: 1000 Billion entries)
- Rebinning Approach
  - Reduce Fully 3D data to decoupled sets of 2D data, then do normal 2D reconstruction
  - FORE (Fourier Rebinning) most common form

28