Q4.1:

(a)

$$\int_{-\infty}^{\infty} dt\ y(t)(\frac{d}{dt}step(t-t_0)) = \int_{-\infty}^{\infty} dt\ y(t)\delta(t-t_0) = y(t_0)$$

(b)

$$\int_{0}^{\infty} dt\ y(t) - \int_{-\infty}^{0} dt\ y(t) = \int_{0}^{\infty} dt\ 1 * y(t) + \int_{-\infty}^{0} dt\ (-1) * y(t)$$

To make $\displaystyle\int_{-\infty}^{\infty} dt\ y(t)\psi(t)$ equal previous term, $\psi(t) = \begin{cases} 1 & t \geq 0 \\ -1 & t < 0 \end{cases}$

Q4.2:

(a). $\psi_{gg}(t) = \int_{-\infty}^{\infty} dt'\ g(t')g(t'-t) = \int_{-\infty}^{\infty} dt'\ e^{-\frac{t'^2}{2\sigma^2}} * e^{-\frac{(t'-t)^2}{2\sigma^2}}$

$= \int_{-\infty}^{\infty} dt'\ e^{-\frac{1}{\sigma^2}t'^2+\frac{t}{\sigma^2}t'} * e^{-\frac{t^2}{2\sigma^2}} = e^{-\frac{t^2}{2\sigma^2}} \int_{-\infty}^{\infty} dt'\ e^{-\frac{1}{\sigma^2}t'^2+\frac{t}{\sigma^2}t'}$

So $a = \frac{1}{\sigma}, b = \frac{t}{\sigma^2}$, then

$\psi_{gg}(t) = e^{-\frac{t^2}{2\sigma^2}} * \sqrt{\pi\sigma^2} e^{\frac{t^2}{4\sigma^2}} = \sigma\sqrt{\pi} e^{-\frac{t^2}{4\sigma^2}}$

(b). Let $t - \tau = \tau_0$, then $t = \tau + \tau_0$ and $\tau = t - \tau_0$

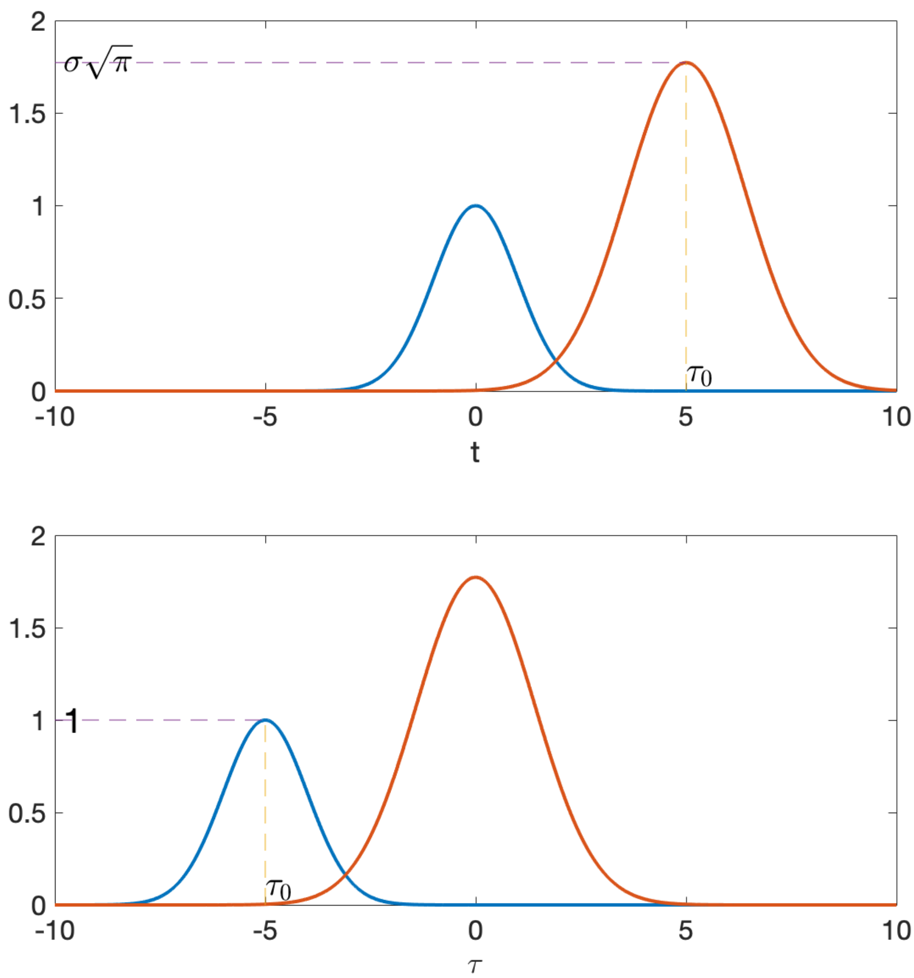

Figure 1: The upper image is puting $\psi(\tau)$ onto $g(t)$ and the lower one is $g(t)$ on $\psi(\tau)$

(c). For $g(t) = e^{-\frac{t^2}{2\sigma^2}}$, it has maximumm value at $t = 0$: $max\ g(t) = g(0) = 1$. So its FWHM value is the solution of $e^{-\frac{t^2}{2\sigma^2}} = 1/2 \longrightarrow t_{FWHM} = \sigma\sqrt{2ln2}$.

For $\psi(\tau) = \sqrt{\pi\sigma^2} e^{-\frac{t^2}{4\sigma^2}}$, $t_{FWHM} = 2\sigma\sqrt{ln2}$

So their ratio is: $\frac{\sigma\sqrt{2ln2}}{2\sigma\sqrt{ln2}} = \frac{1}{\sqrt{2}}$

Q4.4

(a). $SNR = \frac{var(s(t))}{var(e(t))} - 1 = \frac{var(gg)}{var(nois)} = 0.144$

(b).

1. Suppress noise with gaussian matched filter
   design 2D gaussian filter as $K(x, y) = exp(\frac{-x^2}{2\sigma^2})\ for\ y \leq L/2$ where L is the filter kernel size. Then rotate the kernel to get rotated filter with every 30 degree, in order to fit cell walls oriented at any angle. Then use *conv* to apply the all the filters to the raw image with noise and retain the maximum activation signal. By setting the $\sigma = 1, L = 9$, the SNR increases to 13.0478.

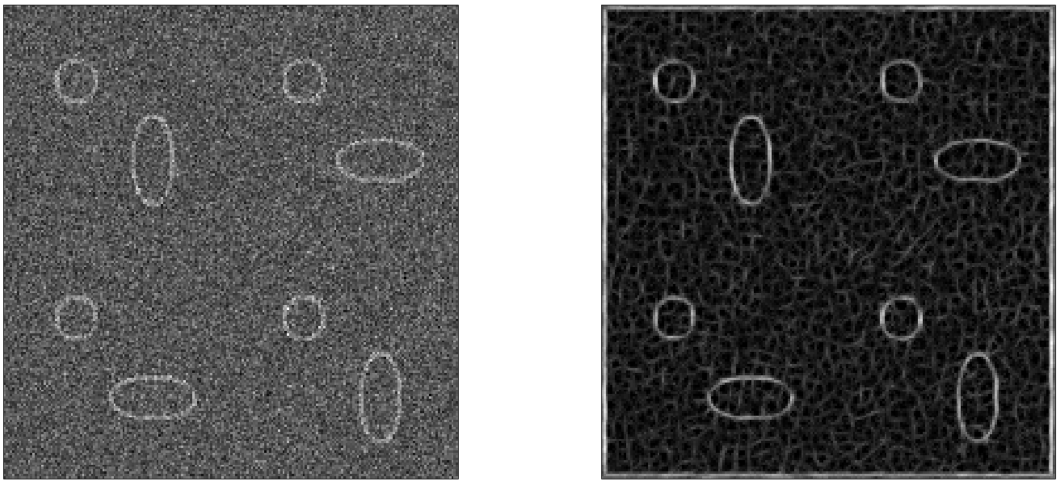

Figure 2: Raw image with noise VS image after applying gaussian filter

Code:

```
out=zeros(size(gg));
s=1;L=9;
m=(L-1)/2;
[x,y]=meshgrid(-m:m,-m:m);
% from 0 to 150 with 30 as interval
theta=0:30:150;
for t=theta
    % rotate
    u = cos(t)*x - sin(t)*y;
    v = sin(t)*x + cos(t)*y;
    N = (abs(u)<=m) & (abs(v)<=m);
    k = exp(-u.^2/(2*s.^2)); % filter
    k = k - mean(k(N));
    k(~N) = 0;
    res = conv2(gg, k, 'same');
    out = max(out, res);
end
```

2. Detect cells with circle filter
   Because the shape of cells is circle or ellipse, so we could design circle filter by: `fspecial('disk', r)`, where **r** is the radius of the circle filter. Like what we did in step, we range **r** from 3 to 12 and iteratively apply the obtained circle filter to denoised image via gaussian filter, then keep the maximum activation outputs. Clustering the locations with maximum activation value should be consistent with the cell locations.

Q4.5

$g(t) = [h * f](t) = \int_{-\infty}^{\infty} dt' h(t-t') f(t')$

$= \int_{-\infty}^{\infty} dt' \frac{1}{\tau} exp(\frac{t'-t}{\tau}) step(t-t') rect(\frac{t'}{2T_0})$

where $step(t-t') = \begin{cases} 1; & t' \leq t \\ 0; & t' > 0 \end{cases}$, and $rect(\frac{t'}{2T_0}) = \begin{cases} 1; & -T_0 \leq t' \leq T_0 \\ 0; & t' > T_0 \; or \; t' < -T_0 \end{cases}$

Therefore, $g(t)$ has 3 cases due to the relationship between t and T_0:

1. when $t \geq T_0$, the rect is entirely covered by $step = 1$ area, so
   $g(t) = \int_{-T_0}^{T_0} dt' \frac{1}{\tau} exp(\frac{t'-t}{\tau}) = exp(\frac{t'-t}{\tau})\Big|_{-T_0}^{T_0} = exp(\frac{T_0-t}{\tau}) - exp(\frac{-T_0-t}{\tau})$
2. when $-T_0 \leq t < T_0$, the rect is partially covered by $step = 1$ area, so
   $g(t) = \int_{-T_0}^{t} dt' \frac{1}{\tau} exp(\frac{t'-t}{\tau}) = exp(\frac{t'-t}{\tau})\Big|_{-T_0}^{t} = 1 - exp(\frac{-T_0-t}{\tau})$
3. when $t < -T_0$, the rect has no overlap with $step = 1$ area, which means their multiplication value is all 0.
   $g(t) = 0$

Q4.6

Let $\mathbf{H} = \frac{100}{m} rect(\frac{t-0.01m}{0.02m})$, where $m = 1, 2, ..., 1000$.

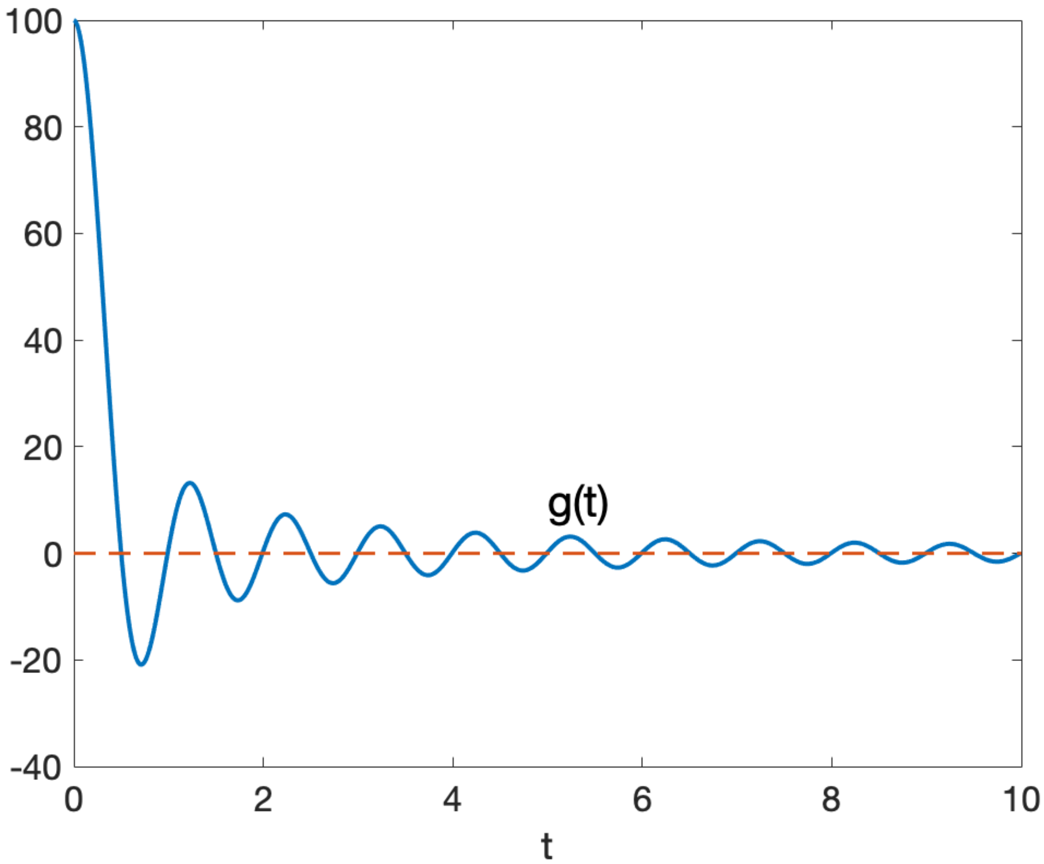

Figure 3: $g(t)$ when applying LTV response $\mathbf{H}$ to $f[n]$

Code:

```
t=0:0.01:9.99;
fn=cos(2*pi.*t);
N = length(t);
h1 = zeros(N, 1);
h1(1) = 100;
H(1, :) = h1;
for j=2:N
    h = zeros(N, 1);
    h(1:j) = 1/(j*0.01);
    H(j, :) = h;
end
g = H*fn';
```