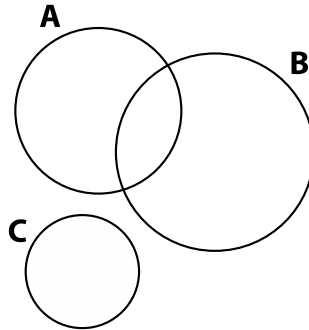


## Homework #1

1. (10 points) If  $P(A) = 0.2$ ,  $P(B) = 0.2$ , and  $A$  and  $B$  are mutually exclusive, are they independent?

**Answer:** A and B are not independent.

2. (10 points) Three events are shown on the Venn diagram in the following figure:



Reproduce the figure and shade the region that corresponds to each of the following events.

- (a)  $A'$       (b)  $(A \cap B) \cup (A \cap B')$       (c)  $(A \cap B) \cup C$       (d)  $(B \cup C)'$       (e)  $(A \cap B)' \cup C$

**Answers are not shown here**

3. (10 points) Consider the hospital emergency department data in the following table. Let  $A$  denote the event that a visit is to Hospital 1 and let  $B$  denote the event that a visit results in admittance to any hospital. Determine the number of persons in each of the following events.

Hospital	1	2	3	4	total
Total	5292	6991	5640	4329	22,252
LWBS	195	270	246	242	953
Admitted	1277	1558	666	984	4485
Not admitted	3820	5163	4728	3103	16,814

LWBS: People leave without being seen by a physician.

- (a)  $A \cap B$       hos1 and admited:1277  
 (b)  $A'$       total-hos1=22252-5292=16960  
 (c)  $A \cup B$       hos1+total adm-hos1 and adm=5292+4485-1277=8500  
 (d)  $A \cup B'$       =hos1+(total-adm)-hos1 and lwbs - hos1 and adm=5292+22252-4485-195-3820=19044  
 (e)  $A' \cap B'$       =hos2 and lwbs+hos3 and lwbs + hos4 and lwbs+hos2 and not ad +hos3 and not ad + hos4 and not ad=270+246+242+5163+4728+3103= 13752

**Answers:**

- a) 1277  
 b) 16960  
 c) 8500  
 d) 19044  
 e) 13752

4. (10 points) There are 4 red balls and 6 white balls in a box. One draws two balls simultaneously. What is the probability that they are the same color?  $(C_{4,2} + C_{6,2}) / C_{10,2} = 7/15$

Answer: 7/15

5. (10 points) George has asked a professor for a recommendation for graduate school. He estimates that the probability that the letter will be strong is 0.5, the probability that the letter will be weak is 0.2, and mediocre is 0.3. he also estimates that if the letter is strong, the probability that he will get the job is 0.8; if it is weak, 0.0; and if it is mediocre, then 0.4. Given that he did get the job, what is the probability that (a) the letter was strong and (b) the letter was weak?

$$=0.4*0.8/(0.4*0.8+0.3*0.4)=0.769$$

S for strong, G for get the job,

$$P(S|G)=P(G|S)P(S)/P(G)$$

$$P(G)=P(G|S)+P(G|S')=0.5*0.8+0.0*0.2+0.3*0.4=0.52$$

$$P(S)=0.5$$

$$P(S|G)=0.8*0.5/0.52=10/13$$

Answer: 0.769

6. (10 points) Suppose that a bag contains ten coins, three of which are fair, the remaining seven having probability 0.6 of giving heads when flipped. A coin is taken at random from the bag and flipped five times. All five flips give heads. What's the probability that a coin is fair given the five coin flips?

$$0.3*0.5^5/(0.3*0.5^5+0.7*0.6^5)=0.147$$

F for fair, D for 5 head

$$P(F|D)=P(D|F)P(F)/P(D)$$

$$P(D)=0.3*0.5^5+0.7*0.6^5=0.0638$$

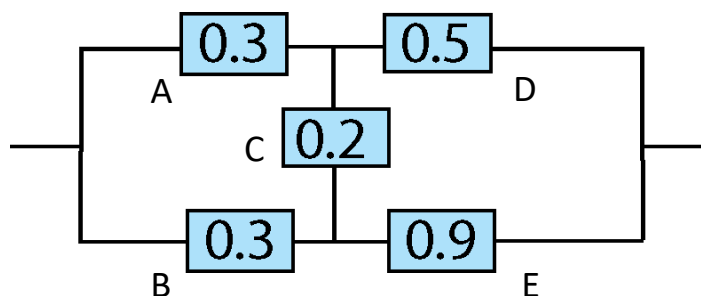
$$P(F)=0.3$$

$$P(D|F)=0.5^5=0.03125$$

$$P(F|D)=0.03125*0.3/0.0638=0.147$$

Answer: 0.147

7. (10 points) The following circuit operates if and only if there is a path of functional devices from left to right. The probability that each device functions is as shown. Assume that the probability that a device is functional does not depend on whether or not other devices are functional. What is the probability that the circuit operates?



$$0.8 * (1-0.3*0.5) * (1-0.3*0.9) + 0.2 * (1-(1-0.3)*(1-0.3)) * (1-(1-0.5)*(1-0.9))=0.4005$$

C is off

$$P = 0.8 * (1 - (1-0.3*0.5)*(1-0.3*0.9)) = 0.3036$$

C is on

$$P=0.2*(1-(1-(1-0.3)*(1-0.3)))*(1-(1-0.5)*(1-0.9))=0.0969$$

$$p=0.0969+0.3036=0.4005$$

Answer: 0.4005