Part2

November 4, 2021

1 Part 2: Fundamental Matrix Estimation, Camera Calibration, Triangulation

1.1 Fundamental Matrix Estimation

[ ]:

```python
from PIL import Image
import numpy as np
import matplotlib.pyplot as plt

##
## load images and match files for the first example
##

def load_data(img1, img2, match_file):
    I1 = Image.open(img1)
    I2 = Image.open(img2)
    matches = np.loadtxt(match_file)

    # this is a N x 4 file where the first two numbers of each row
    # are coordinates of corners in the first image and the last two
    # are coordinates of corresponding corners in the second image:
    # matches(i,1:2) is a point in the first image
    # matches(i,3:4) is a corresponding point in the second image

    N = len(matches)
    ##
    ## display two images side-by-side with matches
    ## this code is to help you visualize the matches, you don't need
    ## to use it to produce the results for the assignment
    ##

    I3 = np.zeros((I1.size[1],I1.size[0]*2,3) )
    I3[:,:I1.size[0],:] = I1
    I3[:,I1.size[0]:,:] = I2
    fig, ax = plt.subplots()
    ax.set_aspect('equal')
    ax.imshow(np.array(I3).astype(np.uint8))
    ax.plot(matches[:,0],matches[:,1],  '+r')
    ax.plot( matches[:,2]+I1.size[0],matches[:,3], '+g')
    ax.plot([matches[:,0], matches[:,2]+I1.size[0]],[matches[:,1], matches[:
↪,3]], 'b')
    plt.show()
    return I1, I2, matches
```

[ ]:

```
##
## display second image with epipolar lines reprojected
## from the first image
##
def image2homo(points):
    # [N, 2] to [N, 3]
    return np.hstack([points, np.ones((len(points),1))])

def homo2image(points):
    # [N, 3] to [N, 2]
    return np.transpose([points[:, 0]/points[:, 2], points[:, 1]/points[:, 2]])

def normalize(pts):
    center = np.mean(pts, axis=0)
    dist = np.linalg.norm(pts - center, axis=1) ** 2
    mean_dist = np.mean(dist)
    scale = np.sqrt(2 / mean_dist)
    translation = np.array([[scale, 0, -scale*center[0]],
                            [0, scale, -scale*center[1]],
                            [0, 0, 1]])
    homo_pts = image2homo(pts)
    trans_pts = np.matmul(translation, homo_pts.T).T
    return translation, homo2image(trans_pts)
```

[ ]:

```
def fit_fundamental(matches, norm=True):
    left_matches = matches[:, :2]
    right_matches = matches[:, 2:]
    if norm:
        T1, left_matches = normalize(left_matches)
        T2, right_matches = normalize(right_matches)
    num_matches = len(left_matches)
    U = np.zeros((num_matches, 9))
    for i in range(num_matches):
        x1, y1 = left_matches[i][0], left_matches[i][1]
        x2, y2 = right_matches[i][0], right_matches[i][1]
        U[i, 0] = x2 * x1
        U[i, 1] = x2 * y1
        U[i, 2] = x2
        U[i, 3] = y2 * x1
        U[i, 4] = y2 * y1
        U[i, 5] = y2
        U[i, 6] = x1
        U[i, 7] = y1
        U[i, 8] = 1
    _, _, V = np.linalg.svd(U)
    F_init = V[len(V)-1].reshape(3,3)
    U, S, V = np.linalg.svd(F_init)
    S[-1] = 0
    S = S * np.identity(3)
    F = U.dot(S).dot(V)
    if norm:
        F = T2.T.dot(F).dot(T1)
    # left_homo = image2homo(left_matches)
    # right_homo = image2homo(right_matches)
    # epl = F.dot(left_homo.T).T
    # dl = epl*left_homo
    # epr = F.T.dot(right_homo.T).T
    # dr = epr*right_homo
    # dist = np.square(np.sum(dl, axis=1)/np.linalg.norm(epl[:, :2])) + np.
↪square(np.sum(dr, axis=1)/np.linalg.norm(epr[:, :2]))
    # residual = np.mean(dist)
    # print('residual is:', residual)
    return F
```

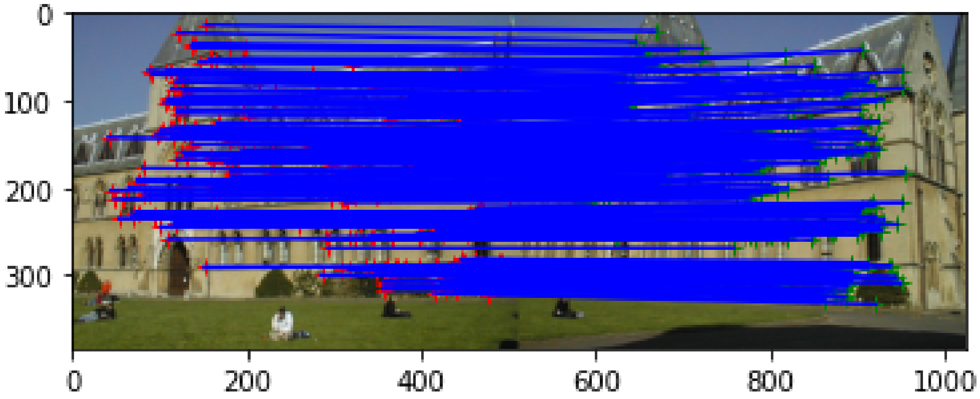
```
[ ]: def epipolar_projection(I2, matches, norm=True):
         N = len(matches)
         # first, fit fundamental matrix to the matches
         F = fit_fundamental(matches, norm=norm); # this is a function that you␣
     ↪should write
         M = np.c_[matches[:,0:2], np.ones((N,1))].transpose()
         L1 = np.matmul(F, M).transpose() # transform points from
         # the first image to get epipolar lines in the second image

         # find points on epipolar lines L closest to matches(:,3:4)
         l = np.sqrt(L1[:,0]**2 + L1[:,1]**2)
         L = np.divide(L1,np.kron(np.ones((3,1)),l).transpose())# rescale the line
         pt_line_dist = np.multiply(L, np.c_[matches[:,2:4], np.ones((N,1))]).
     ↪sum(axis = 1)
         print("residual:", np.mean(np.abs(pt_line_dist)))
         closest_pt = matches[:,2:4] - np.multiply(L[:,0:2],np.kron(np.ones((2,1)),␣
     ↪pt_line_dist).transpose())

         # find endpoints of segment on epipolar line (for display purposes)
         pt1 = closest_pt - np.c_[L[:,1], -L[:,0]]*10# offset from the closest point␣
     ↪is 10 pixels
         pt2 = closest_pt + np.c_[L[:,1], -L[:,0]]*10
```

```
    # display points and segments of corresponding epipolar lines
    fig, ax = plt.subplots()
    ax.set_aspect('equal')
    ax.imshow(np.array(I2).astype(np.uint8))
    ax.plot(matches[:,2],matches[:,3],  '+r')
    ax.plot([matches[:,2], closest_pt[:,0]],[matches[:,3], closest_pt[:,1]],␣
 ↪'r')
    ax.plot([pt1[:,0], pt2[:,0]],[pt1[:,1], pt2[:,1]], 'g')
    plt.show()
```

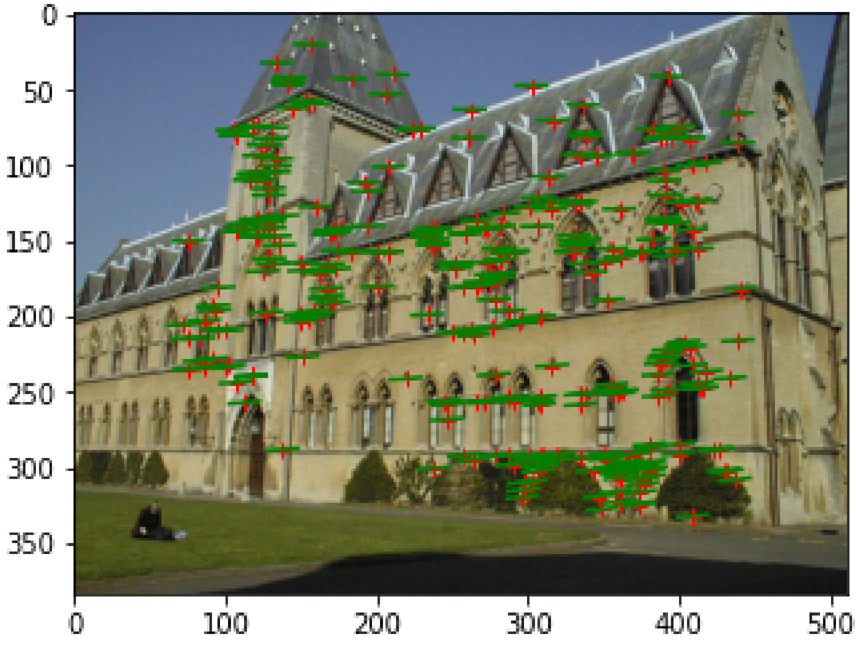
[ ]:

```
import cv2
from scipy.spatial import distance

def sift_descriptor(image):
    sift = cv2.xfeatures2d.SIFT_create()
    # compute descriptors
    kp, dsp = sift.detectAndCompute(image, None)
    pt = cv2.KeyPoint_convert(kp)
    pt = np.array(pt)
    return pt, dsp

def get_matches(dsp1, dsp2, topk=100):
    dist = distance.cdist(dsp1, dsp2, metric="sqeuclidean")
    sort_idx = np.argsort(dist, axis=None)
    match_idx = np.unravel_index(sort_idx,(dist.shape[0], dist.shape[1]))
    match_idx1 = match_idx[0][:topk]
    match_idx2 = match_idx[1][:topk]
    return np.array([match_idx1, match_idx2]).T

def RANSAC(matches, img2, iteration=100, inlier_thres=2):
    max_inlier = 0
    num_matches = matches.shape[0]
    for i in range(iteration):
        select_idx = np.random.choice(range(num_matches), 8)
        select_matches = matches[select_idx]
        F  = fit_fundamental(select_matches, norm=True);
        M = image2homo(matches[:, :2]).T
        L1 = np.matmul(F, M).T

        l = np.sqrt(L1[:,0]**2 + L1[:,1]**2)
        L = np.divide(L1,np.kron(np.ones((3,1)),l).transpose())    # rescale␣
 ↪the line
        pt_line_dist = np.multiply(L, np.c_[matches[:,2:4], np.
 ↪ones((num_matches,1))]).sum(axis = 1)

        inlier_idx = pt_line_dist < inlier_thres
        inliers = matches[inlier_idx]
        num_inliers = len(inliers)
        if num_inliers > max_inlier:
            max_inlier = num_inliers
            best_inliers = inliers
    print("max inliers: ", max_inlier)

    epipolar_projection(img2, best_inliers, True)
    return
```

```
[ ]: img1 = 'MP3_part2_data/library1.jpg'
     img2 = 'MP3_part2_data/library2.jpg'
     match_file = 'MP3_part2_data/library_matches.txt'

     I1, I2, matches = load_data(img1, img2, match_file)
     epipolar_projection(I2, matches, norm=False)
```

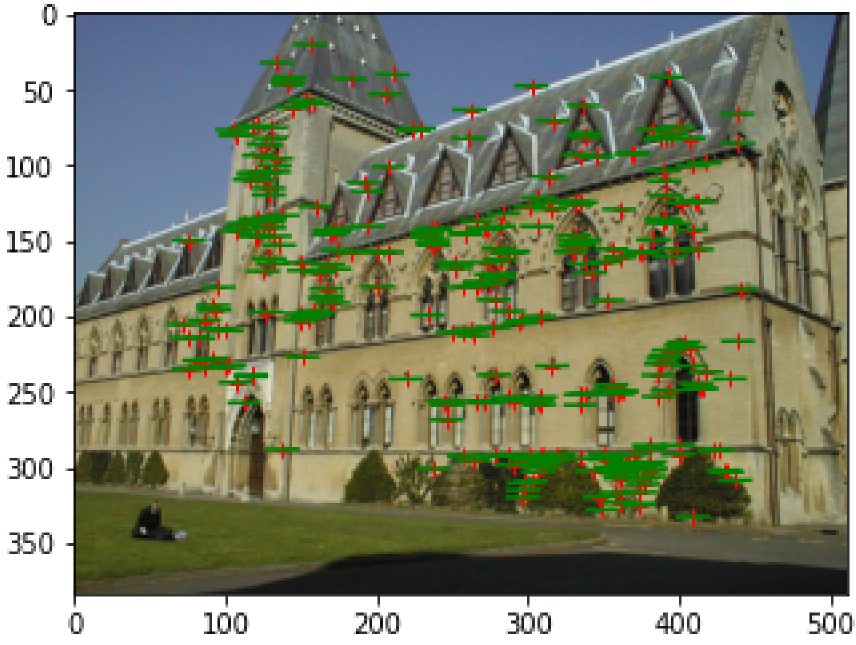


```
residual: 0.3384950960799586
```



```
[ ]: # normalize
     epipolar_projection(I2, matches, norm=True)
```

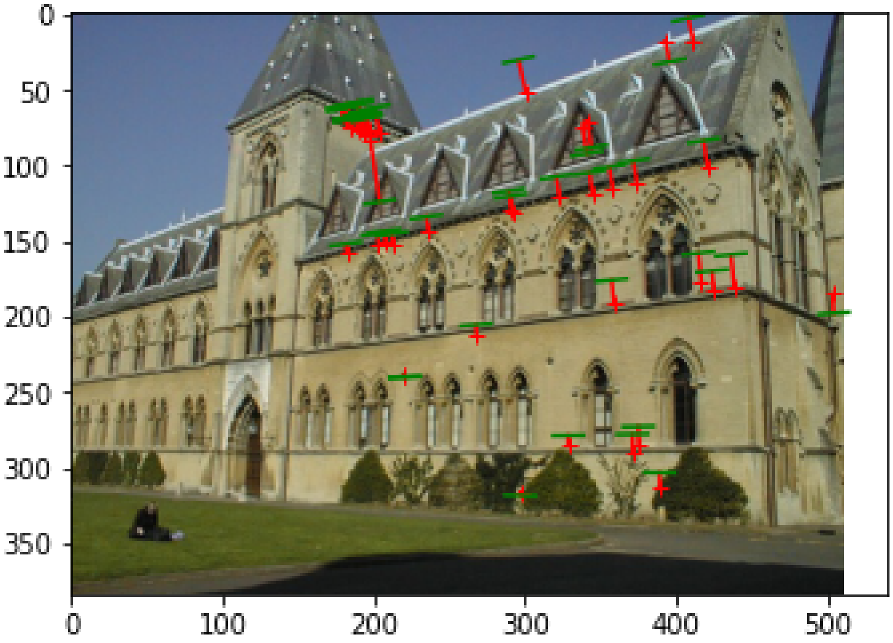
```
residual: 0.18359661716193226
```



[ ]:

```
topk = 50
iteration = 50
inlier_thres = 1.5

# ransac to find F
I1_gray = I1.convert("L")
I2_gray = I2.convert("L")
I1_gray = np.asarray(I1_gray)
I2_gray = np.asarray(I2_gray)

filtered_coords1, dsp1 = sift_descriptor(I1_gray)
filtered_coords2, dsp2 = sift_descriptor(I2_gray)
matches = get_matches(dsp1, dsp2, topk)
matched_pt1 = [filtered_coords1[i[0]] for i in matches]
matched_pt2 = [filtered_coords2[i[1]] for i in matches]
matched_pt = np.hstack([matched_pt1, matched_pt2])

RANSAC(matched_pt, I2, iteration, inlier_thres)
```

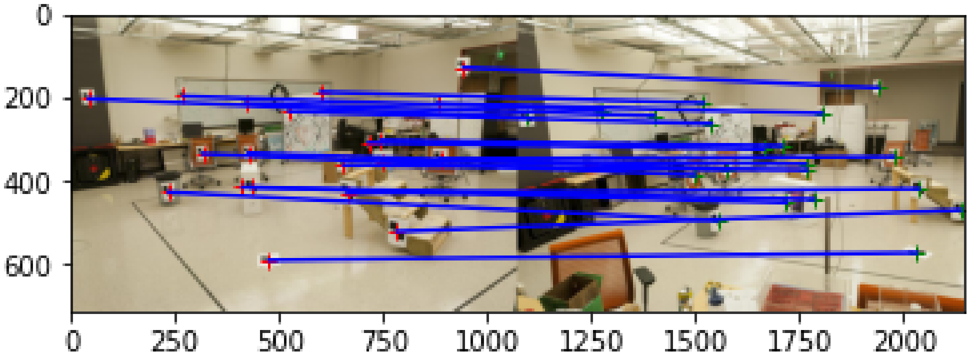
```
max inliers:  47
residual: 12.212905545149148
```



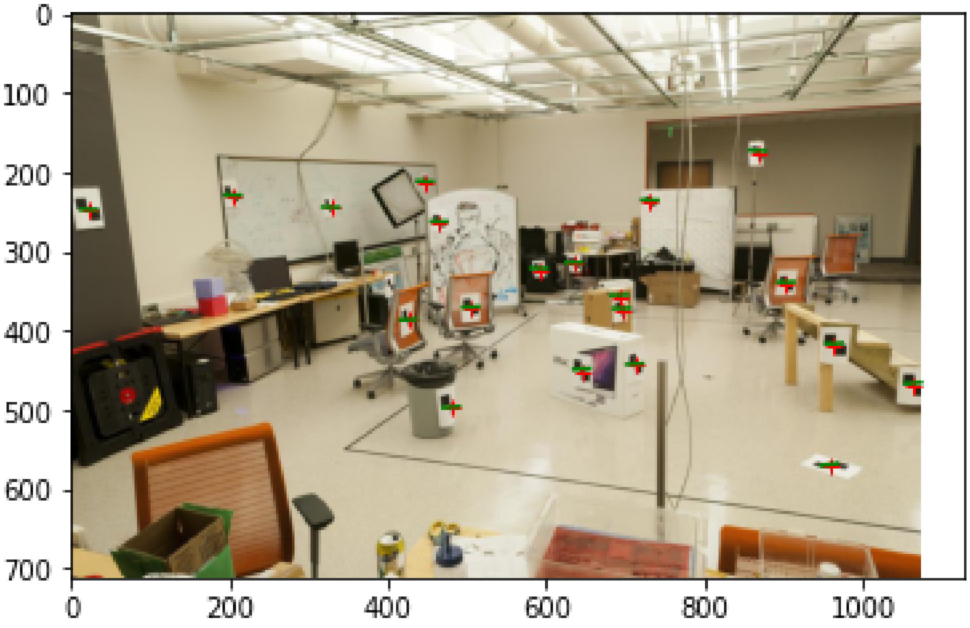
[ ]:

```
img1 = 'MP3_part2_data/lab1.jpg'
img2 = 'MP3_part2_data/lab2.jpg'
match_file = 'MP3_part2_data/lab_matches.txt'
```

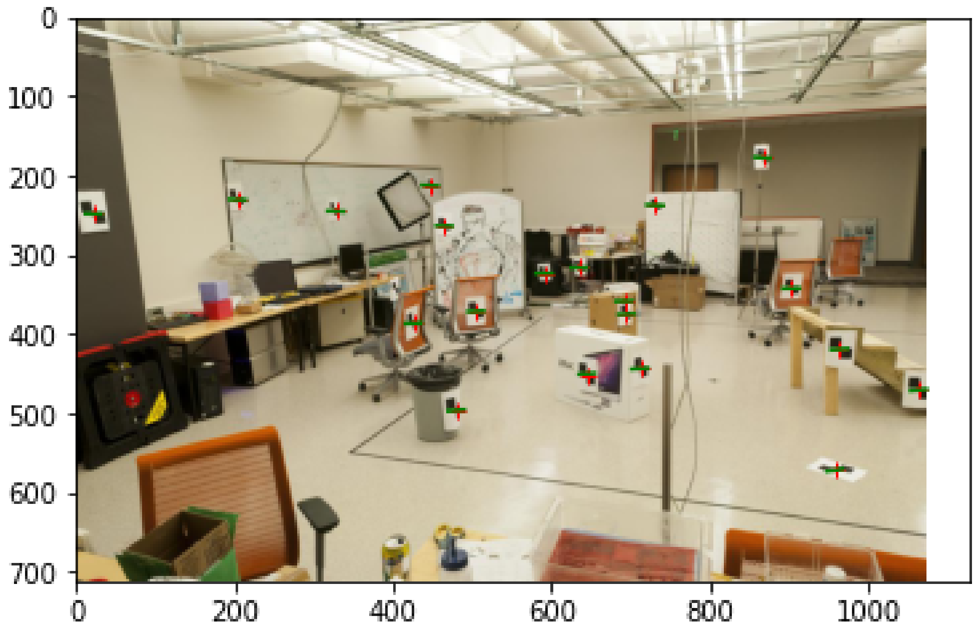
```
I1, I2, matches = load_data(img1, img2, match_file)
epipolar_projection(I2, matches, norm=False)
epipolar_projection(I2, matches, norm=True)
```



```
residual: 2.237977425348351
```



```
residual: 0.6172517559032379
```



1.2 Camera Calibration

[ ]:

```python
def evaluate_points(M, points_2d, points_3d):
    """
    Visualize the actual 2D points and the projected 2D points calculated from
    the projection matrix
    You do not need to modify anything in this function, although you can if you
    want to
    :param M: projection matrix 3 x 4
    :param points_2d: 2D points N x 2
    :param points_3d: 3D points N x 3
    :return:
    """
    N = len(points_3d)
    points_3d = np.hstack((points_3d, np.ones((N, 1))))
    points_3d_proj = np.dot(M, points_3d.T).T
    u = points_3d_proj[:, 0] / points_3d_proj[:, 2]
    v = points_3d_proj[:, 1] / points_3d_proj[:, 2]
    residual = np.sum(np.hypot(u-points_2d[:, 0], v-points_2d[:, 1]))
    points_3d_proj = np.hstack((u[:, np.newaxis], v[:, np.newaxis]))
    return points_3d_proj, residual
```

[ ]:

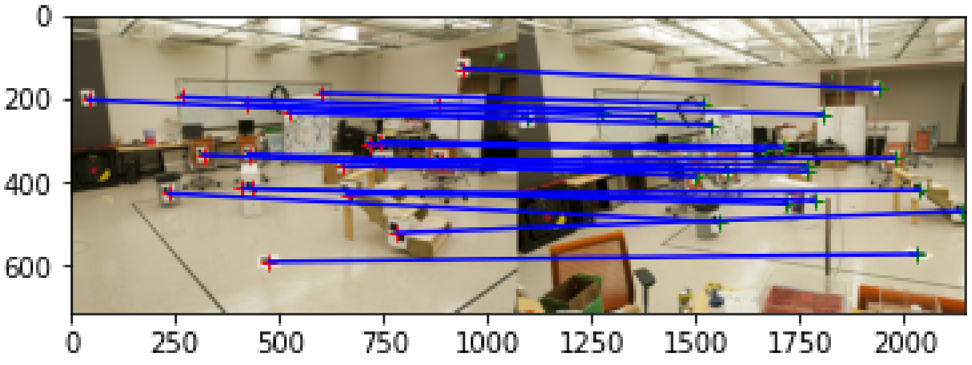
```python
def get_projection(point3d, point2d):
    n = point3d.shape[0]
    A = np.zeros((n*2, 12))
    pt3d_homo = image2homo(point3d)
    for i in range(n):
        X = pt3d_homo[i]
        x, y = point2d[i][0], point2d[i][1]
        A[i*2, 4:8] = X
        A[i*2, 8:] = -y * X
        A[i*2+1, :4] = X
        A[i*2+1, 8:] = -x * X
    _, _, V = np.linalg.svd(A)
    M = V[len(V)-1].reshape(3, 4)
    return M
```

[ ]:

```
def camera_calib(matches, point3d):
    left_matches = matches[:, :2]
    right_matches = matches[:, 2:]
    left_M = get_projection(point3d, left_matches)
    right_M = get_projection(point3d, right_matches)
    pt3d_left, resi_left = evaluate_points(left_M, left_matches, point3d)
    print('Left image camera calibration redisual: ', resi_left)
    pt3d_right, resi_right = evaluate_points(right_M, right_matches, point3d)
    print('Right image camera calibration redisual: ', resi_right)
    return left_M, right_M
```

[ ]:

```
img1 = 'MP3_part2_data/lab1.jpg'
img2 = 'MP3_part2_data/lab2.jpg'
match_file = 'MP3_part2_data/lab_matches.txt'
pt3d_file = "MP3_part2_data/lab_3d.txt"

I1, I2, matches = load_data(img1, img2, match_file)
point3d = np.loadtxt(pt3d_file)
lab_left_M, lab_right_M = camera_calib(matches, point3d)
```



```
Left image camera calibration redisual:  13.545832900248698
Right image camera calibration redisual:  15.544953457947923
```

1.3 Camera Centers

[ ]:
```python
from scipy.linalg import null_space

center_left = null_space(lab_left_M)
center_left_lab = center_left/center_left[-1] # homo 2 image
print('Left lab cam center:', center_left_lab)
center_right = null_space(lab_right_M)
center_right_lab = center_right/center_right[-1]
print('Right lab cam center:', center_right_lab)
```

```
Left lab cam center: [[305.83276769]
 [304.20103826]
 [ 30.13699243]
 [  1.        ]]
Right lab cam center: [[303.10003925]
 [307.18428016]
 [ 30.42166874]
 [  1.        ]]
```

[ ]:
```python
library_left_M = np.loadtxt('MP3_part2_data/library1_camera.txt')
library_right_M = np.loadtxt('MP3_part2_data/library2_camera.txt')
center_left = null_space(library_left_M)
center_left_library = center_left/center_left[-1]
print('Left library cam center:', center_left_lab)
center_right = null_space(library_right_M)
center_right_library = center_right/center_right[-1]
print('Right libray cam center:', center_right_library)
```

```
Left library cam center: [[305.83276769]
 [304.20103826]
 [ 30.13699243]
 [  1.        ]]
Right libray cam center: [[  6.89405488]
 [-15.39232716]
 [ 23.41498687]
 [  1.        ]]
```

1.4 Triangulation

[ ]:
```python
def triangulate(P1, P2, matches):
    num_match = matches.shape[0]
    x1 = image2homo(matches[:, :2])
    x2 = image2homo(matches[:,2:])

    X = np.zeros((num_match, 4))
    for i in range(num_match):
        x1x = np.array([[0, -x1[i, 2], x1[i, 1]],
                        [x1[i, 2], 0, -x1[i, 0]],
                        [-x1[i, 1], x1[i, 0], 0]])
        x2x = np.array([[0, -x2[i, 2], x2[i, 1]],
                        [x2[i, 2], 0, -x2[i, 0]],
                        [-x2[i, 1], x2[i, 0], 0]])

        A1 = np.matmul(x1x, P1)
        A2 = np.matmul(x2x, P2)
        A = np.vstack((A1, A2))

        _, _, V = np.linalg.svd(A)
        Xi = V[len(V)-1]
        X[i] = Xi / Xi[-1]
    return X

def get_residual(P1, P2, X, matches):
    left_proj = np.matmul(P1, X.T).T
    right_proj = np.matmul(P2, X.T).T
    left_x = homo2image(left_proj)
    right_x = homo2image(right_proj)
    left_resi = np.linalg.norm(left_x - matches[:, :2]) ** 2
    right_resi = np.linalg.norm(right_x-matches[:, 2:]) ** 2
    resi = (left_resi + right_resi) / (2 * len(matches))
    return resi
```

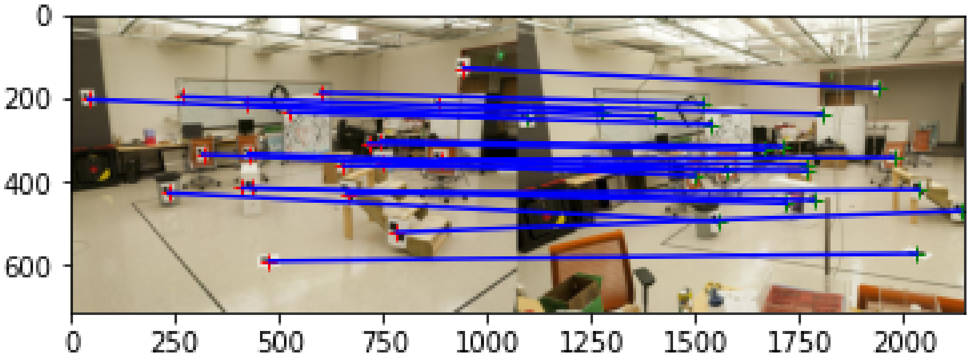
[ ]:

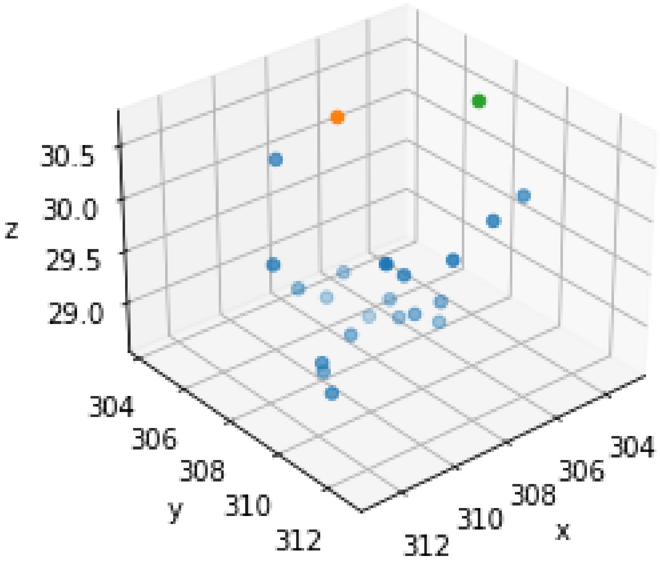
```
def plot3d(X, center_left, center_right):
    fig = plt.figure()
    ax = fig.add_subplot(111, projection='3d')
    ax.view_init(30, 50)
    ax.scatter(X[:, 0], X[:, 1], X[:, 2])
    ax.scatter(center_left[0], center_left[1], center_left[2], '+r')
    ax.scatter(center_right[0], center_right[1], center_right[2], '+g')

    ax.set_xlabel('x')
    ax.set_ylabel('y')
    ax.set_zlabel('z')

    plt.show()
```

[ ]:

```
img1 = 'MP3_part2_data/lab1.jpg'
img2 = 'MP3_part2_data/lab2.jpg'
match_file = 'MP3_part2_data/lab_matches.txt'

I1, I2, matches = load_data(img1, img2, match_file)
X = triangulate(lab_left_M, lab_right_M, matches)
plot3d(X, center_left_lab, center_right_lab)
resi = get_residual(lab_left_M, lab_right_M, X, matches)
```

```
print("residual: ", resi)
print(lab_left_M)
print(lab_right_M)
```

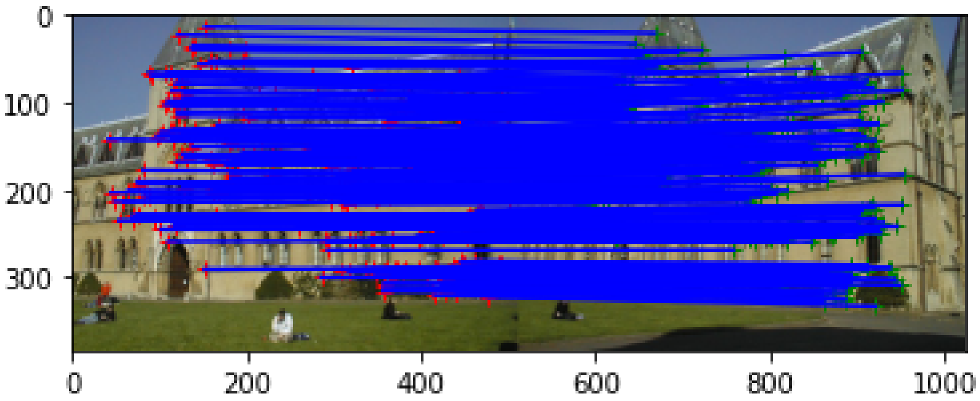


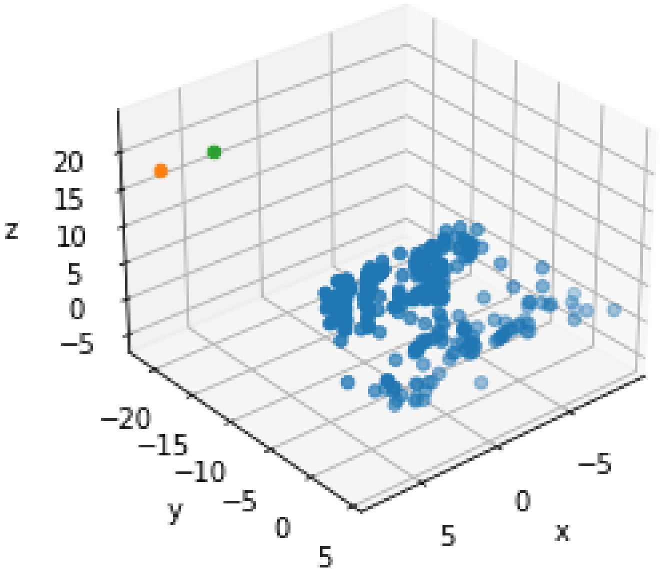


```
residual:  1.0318213201651718
[[-3.09963996e-03 -1.46204548e-04  4.48497465e-04  9.78930678e-01]
 [-3.07018252e-04 -6.37193664e-04  2.77356178e-03  2.04144405e-01]
 [-1.67933533e-06 -2.74767684e-06  6.83964827e-07  1.32882928e-03]]
[[ 6.93154686e-03 -4.01684470e-03 -1.32602928e-03 -8.26700554e-01]
 [ 1.54768732e-03  1.02452760e-03 -7.27440714e-03 -5.62523256e-01]
 [ 7.60946050e-06  3.70953989e-06 -1.90203244e-06 -3.38807712e-03]]
```

[ ]:

```
img1 = 'MP3_part2_data/library1.jpg'
img2 = 'MP3_part2_data/library2.jpg'
match_file = 'MP3_part2_data/library_matches.txt'

I1, I2, matches = load_data(img1, img2, match_file)
X = triangulate(library_left_M, library_right_M, matches)
plot3d(X, center_left_library, center_right_library)
resi = get_residual(library_left_M, library_right_M, X, matches)
print("residual: ", resi)
```





```
residual:  0.17040373843022627
```