Prob 1.

(A) $P(\bar{X} < 70) = P(Z = \frac{\bar{X} - u}{\sigma / \sqrt{n}} < \frac{70-80}{12/\sqrt{16}})$

$= P(Z < -3.33) = 1 - 0.999566 = 0.000434$

(B). $P(\bar{X}_1 - \bar{X}_2 > 6) = P(Z = \frac{\bar{X}_1 - \bar{X}_2 - (u_1 - u_2)}{\sqrt{\frac{\sigma_1^2}{n_1} + \frac{\sigma_2^2}{n_2}}})$

$= P(Z > \frac{6 - (80-70)}{\sqrt{\frac{12^2}{16} + \frac{10^2}{25}}})$

$= P(Z > -1.11)$

$= 0.8665$

Prob 2:

$H_0 =$ mean level of sodium in the serum is 140ppm.

$H_1 =$ mean level of sodium is different from 140ppm.

$Z = \frac{\bar{X} - u}{s/\sqrt{n}} = \frac{140.55 - 140}{9.445/\sqrt{20}} = 0.26$.

So the two-tail test rejection area is.

$[-\infty, Z_{-\alpha/2}], [Z_{\alpha/2}, +\infty)$, where $\alpha = 0.05$.

$\Rightarrow (-\infty, -1.96], [1.96, +\infty)$.

Z is not in the rejection area, we cannot reject H_0

Prob 3:

(A): ~~Ho: new treatment has same effect with old one.~~

H_0: new treatment is no better than or identical to the old one.

H_1: new treatment is better than the old one.

Survived population proportion of new method: $P = \frac{92}{200} = 0.46$, $s = \sqrt{\frac{P(1-P)}{n}} = 0.035$.

$$Z = \frac{P - \bar{P}}{S} = \frac{0.46 - 0.4}{0.035} = 1.71$$

For the single-tail test, reject area is $[Z_\alpha, +\infty)$.

which is $[1.65, +\infty)$.

So: $Z > Z_{0.05}$, we can reject the null hypothesis.

when means the new method is more effective

(B). For two-tail confidence interval,

$$\bar{p} - Z_{\alpha/2}\sqrt{\frac{\bar{p}(1-\bar{p})}{n}} \leq P \leq \bar{p} + Z_{\alpha/2}\sqrt{\frac{\bar{p}(1-\bar{p})}{n}}, \text{ where } \alpha = 0.05$$

$$\Rightarrow 0.46 - 1.96 \times 0.035 \leq P \leq 0.46 + 1.96 \times 0.035.$$

~~$P \in [0.4264, 0.5188]$~~

$\Rightarrow P \in [0.3914, 0.5286]$ for 95% CI

Prob 4.

(A): $P(x>5) = \int_5^{+\infty} \frac{1}{4} \cdot e^{-\frac{1}{4}x}\, dx$

$= e^{-\frac{1}{4} \cdot 5} = 0.2865$

(B). It's a poisson process. with constant $r = 1/4$.

$E(N_x) = r \cdot x = \frac{1}{4} \times 12 = 3$.

(C). It's a memoryless distribution.

$P(X>9 \mid X>4) = P(X>9-4=5) = e^{-\frac{5}{4}} = 0.2865$.

Prob 5: (A): parallel fashion,

~~P(success) = (1 - P_fail)~~ $P_{fail\ total} = (P_{fail\ single})^n$

~~P_total = 1 - (1 - P_success)^n~~, which could increase overall success probability.

(B). $P_{fail} = (0.2)^n < 10^{-6}$.

$\Rightarrow n \cdot \log_{10} 0.2 < \log_{10} 10^{-6} = -6$.

$\Rightarrow n > \frac{6}{0.699} = 8.58$.

we need at lest 9 components to achieve desired reliability.

Prob 6:

(A): PMF of x and y is:

y \ x	0	1	2	3	margin
0	0.1	0	0	0	0.1
1	0	0.1	0.3	0	0.4
2	0	0.3	0.1	0	0.4
3	0	0	0	0.1	0.1
margin	0.1	0.4	0.4	0.1	1

$E(X) = 0\times0.1 + 1\times0.4 + 2\times0.4 + 3\times0.1 = 1.5$

$E(Y) = 1.5$.

$$Cov(x,y) = E(xy) - u_x u_y$$

$$= 0\times0\times0.1 + 1\times1\times0.1 + 1\times2\times0.3 + 2\times1\times0.3 + 2\times2\times0.1 + 3\times3\times0.1 - 1.5\times1.5$$

$$= 2.6 - 2.25$$

$$= 0.35$$

(B): $Pearson = \dfrac{\sigma_{xy}}{\sigma_x \sigma_y}$

$V(x) = \sigma_x^2 = E(x^2) - E(x)^2 = 1^2\times0.4 + 2^2\times0.4 + 3^2\times0.1 - 2.25 = 0.65$

$\Rightarrow \sigma_x = 0.81$

$V(y) = \sigma_y^2 \Rightarrow \sigma_y = 0.81$

$$\rho_{xy} = \frac{0.35}{0.81\times0.81} = 0.538.$$

Prob 7:

(A): D: get defective BGMD.

V: from Vampire Inc, A: from Acme,

~~P(D)~~ T: from Theranos. Company:

$$P(D) = P(D|A) + P(D|V) + P(D|T)$$

$$= 0.15 \times 0.06 + 0.8 \times 0.04 + 0.05 \times 0.09$$

$$= 0.0455.$$

(B). $$P(T|D) = \frac{P(D|T) \cdot P(T)}{P(D)}$$

$$= \frac{0.09 \times 0.05}{0.0455}$$

$$= 0.099$$