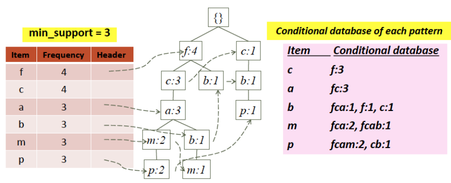
1 Frequent pattern mining

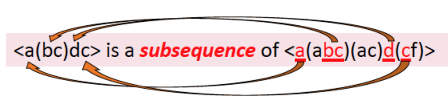
Transaction Database: Absolute support (count): $sup\{X\}$; Relative support (fraction): $s\{X\}$.
Compute the confidence of association rule $X \to Y$:
$x = sup(X, Y)/sup(x)$ (form $X \to Y(s, c)$)
Apriori: Downward closure (any subset of frequent item is frequent). Steps: 1. find the complete set of frequent k-itemsets; 2. derive frequent (k+1)-itemset candidates; 3. Scan DB again to find true frequent (k+1)-itemsets (via self-joining; pruning). It's **breadth-first** search.
FP-growth: depth-first search (subsequent search confined to those with specific itemset). Steps: Order frequent list and insert into FP-trees. Mine (conditional) FP-trees: single path generate all the combinations of its sub-paths.



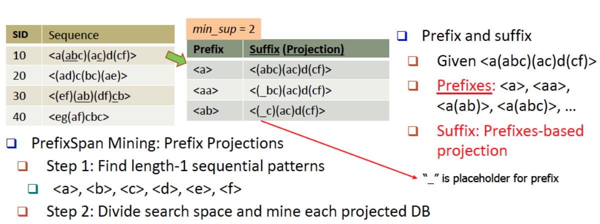
Pattern Evaluation:

Measure	Definition	Range	Null-Invariant?
$\chi^2(A,B)$	$\sum_{i,j}\frac{(e(a_i,b_j)-o(a_i,b_j))^2}{e(a_i,b_j)}$	$[0, \infty]$	No
$Lift(A,B)$	$\frac{s(A\cup B)}{s(A)\times s(B)}$	$[0, \infty]$	No
$Allconf(A,B)$	$\frac{s(A\cup B)}{max\{s(A),s(B)\}}$	$[0, 1]$	Yes
$Jaccard(A,B)$	$\frac{s(A\cup B)}{s(A)+s(B)-s(A\cup B)}$	$[0, 1]$	Yes
$Cosine(A,B)$	$\frac{s(A\cup B)}{\sqrt{s(A)\times s(B)}}$	$[0, 1]$	Yes
$Kulczynski(A,B)$	$\frac{1}{2}(\frac{s(A\cup B)}{s(A)}+\frac{s(A\cup B)}{s(B)})$	$[0, 1]$	Yes
$MaxConf(A,B)$	$max\{\frac{s(A\cup B)}{s(A)}, \frac{s(A\cup B)}{s(B)}\}$	$[0, 1]$	Yes

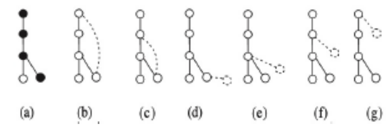
Imbalance ratio: $IR(A, B) = \frac{|s(A)-s(B)|}{s(A)+s(B)-s(A\cup B)}$
Lift and χ^2 are good measures if null transactions are not predominant. Otherwise, Kulczynski + Imbalance Ratio.
Sequential Pattern:
Sequence: e.g.$< (ef)(ab)cb >$; Note: unordered in (..)



GSP: apriori-based sequential pattern mining. Steps: 1. generate initial candidates (singleton); 2. scan and count support; 3. generate length-2 candidates (ordered+unordered: n*n+n(n-1)/2)
SPADE: A sequence database is mapped to: $< SID, EID >$; Grow the subsequences (patterns) one item at a time by Apriori candidate generation
Prefix-span: Pros: No candidate subseqs. to be generated; Projected DBs keep shrinking; Cons: Suffixes largely repeating in recursive projected DBs (solution: pseudo-projection+Physical Projection)



Graph Pattern: Apriori-based method: Breadth-search, Apriori joining two size-k graphs. With one more vertex: AGM; with one more edge: FSG.
Pattern-Growth Approach: Depth-first growth of subgraphs from k-edge to (k+1)-edge. Limits: Generating many duplicate subgraphs; Solutions: Define an order to generate subgraphs; DFS spanning tree: Flatten a graph into a sequence using depth-first search; **gSpan**: Right-most path extension; Backward extension (b-c) vs. forward extension (d-g) DFS code: $(i; j; l_i; l_{(i;j)}; l_j)$, l_i is

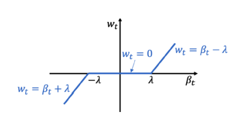


vertex label, and $l_{(i;j)}$ is edge label.

2 Classification

Feature selection: remove irrelevant and redundant features. How to construct? Domain knowledge or deep learning.
Methods: filter methods (based on goodness measure, independent of classification model): fisher scores: $s = \sum_{j=1}^{c} n_j(\mu_j - \mu)^2 / \sum_{j=1}^{c} n_j \sigma_j^2$; χ^2 test; information gain; mutual information.
wrapper methods (combine feature selection and classification model iteratively): exhaustive search ($2^p - 1$); stepwise forward selection; stepwise backward elimination; hybrid method
Embedded methods (simultaneously construct classification model and select features): **LASSO**
Model $\hat{L}(w) = \frac{1}{2}\sum_{i=1}^{n}(y_i - \hat{y}_i)^2 + \lambda\|w\_1\| = \frac{1}{2}\sum_{i=1}^{n}(y_i - w^T x_i)^2 + \lambda\sum_{j=0}^{d}|w_j|$ (goodness of prediction+(convex) approximation of # of selected features). Training: coordinate descent (for $f(x) = g(x) + \sum_i h_i(x_i)$, if f(x) is convex but not differential, but separable (g(x) is convex and smooth; $h_i(x)$ is convex) $\to$ global minima).
Training: update on w_t while fixing others.
$\beta_t = argmin_{w_t}\frac{1}{2}\sum_{i=1}^{n}(r_i - w_t x_{i,t})^2$ and $r_i = y_i - \sum_{j=0, j\neq t}^{d} w_j x_{i,j}$
Then solve the optimization problem: $L(w_t) = \frac{1}{2}(w_t - \beta_t)^2 + \lambda|w_t|$, assume each feature is normalized: $x_t^T x_t = \|x_t\|^2 = 1$. **Solution**: soft-thresholding (intuition: push β_t toward 0)

$$w_t = \begin{cases} \beta_t - \lambda & \text{if } \beta_t \geq \lambda \\ \beta_t + \lambda & \text{if } \beta_t \leq -\lambda \\ 0 & \text{otherwise} \end{cases}$$



SVM
Reasons of distorted boundaries: irregular distribution, imbalanced training size, outliers.
Maximum margin classification: margin $m = \frac{2c}{\|w\|}$. Optimization: $max_{w,b}\frac{1}{\|w\|}$ $s.t$ $y_i(w^T x_i + b) \geq 1, \forall i \Leftrightarrow min_{w,b}\frac{1}{2}w^T w$ $s.t$ $1 - y_i(w^T x_i + b) \leq 0, \forall$. Then write Lagrangian: $L(w, b, \alpha) = \frac{1}{2}w^T w - \sum_{i=1}^{m}\alpha_i[y_i(w^T x_i + b) - 1]$ for $max_{\alpha_i \geq 0} min_{w,b} L(w, b, a)$.
Dual opt problem: $max_\alpha L(\alpha) = \sum_{i=1}^{m}\alpha_i - \frac{1}{2}\sum_{i,j=1}^{m}\alpha_i\alpha_j y_i y_j(\mathbf{x}_i^T\mathbf{x}_j)$ $s.t$ $\alpha_i \geq 0, \sum_{i=1}^{m}\alpha_i y_i = 0$
After solving it, $w = \sum_{i=1}^{m}\alpha_i y_i \mathbf{x}_i$: linear combination of small number of data points (sparsity). To compute α_i, specify inner products between examples $(\mathbf{x}_i^T\mathbf{x}_j)$. Prediction: $y^* = sign(\sum \alpha_i y_i(\mathbf{x}_i^T z) + b)$
Kernels for non-linear classifier: $K(x, x') = \phi(x_i)^T\phi(x_j)$. Linear kernel: $x^T x'$; polynomial kernel: $(1 + x^T x')^n$; radial basis kernel: $exp(-\frac{1}{2}\|x - x'\|^2)$
Soft-margin SVM:
$min_{w,b}\frac{1}{2}w^T w + C\sum_{i=1}^{m}\xi_i,$ $s.t$ $y_i(w^T x_i + b) \geq 1 - \xi_i, and$ $\xi_i \geq$

$0, \forall i$, where ξ_i is slack variable which approximiates the numebr of misclassified samples. C: balance error and margin. The dual form: $max_\alpha L(\alpha) = \sum_{i=1}^{m} \alpha_i - \frac{1}{2}\sum_{i,j=1}^{m} \alpha_i\alpha_j y_i y_j (\mathbf{x}_i^T \mathbf{x}_j), s.t\ 0 \leq \alpha_i \leq C\ and \sum_{i=1}^{m} \alpha_i y_i = 0$

SMO algorithm: coordinate ascend. 1. Select some pair $\alpha_i$and α_j to update next; 2. reoptimize $J(\alpha)$ w.r.t α_i and α_j while holding others fixed.

LR as unconstraint opt problem: $argmin_{w,b} w^t w + \lambda \sum_{i=1}^{m}(ln(1 + exp(-w^T x_i)) + (1 - y_i)w^T x_i)$.

Weakly-supervised learning

SSL: self-training: use classifier to label unlabeled data; co-training: f1 and f2 on two separate feature sets; classify unlabeled data; add most confident $(x, f_1(x))$ to labeled dataset of f2, etc.

SSL assumption: cluster assumption: Data tuples from same cluster are likely to share same label; manifold assumption: A pair of close tuples are likely to share the same class label

Active learning: find best unlabeled data to ask oricle (human involved). Key: how to choose? Uncertainty sampling; query-by-committee; version space; decision-theoretic approach.

Transfer learning: transfer most relevant/similar data from source to target. Challenge: negative transfer (quantify difference between source and target; transfer margin, divergence metric)

Distance supervision: challenge: noisy but large in volume

Zero-shot learning: predict test tuple not observed in training. Semantic attribute classifier: train and use semantic attribute classifier to infer semantic attributes and use them to predict novel class.

Stream data classification: challenge: high arrival speed (use most recent chunk); infinite length; one-pass constraint (Each incoming data is accessed once to train classifier and update weights of each classifier); concept drifting (adjust weights to focus on most relevant chunks). method: VFDT

Sequence classification: convert sequence to vector. Via Distance/kernel function

Graph data classification: proximity measure (bad proximity: shortest path (pizza delivery guy problem); max netflow (no punishment on long path)). Random walk with restart (RWR): $r_i = c \times A \times r_i + (1-c) \times e_i = argmin\ cr_i'(1-A)r_i + (1-c) \times \|r_i - e_i\|^2$, network smoothness + query preference. Why RWR is good? High proximity→ many, short, heavy-weighted paths. The benefit of restart: task is to find the value such that further simulations won't affect the probabilities.

3 Cluster

Cluster analysis: unsupervised learning. Partition data into a set of groups which are as similar as possible (high intra-class and low inter-class similarity). **Applications**: outlier detection, compression, recommendation, etc. **Consideration**: partition criteria (single/hierarchical); separation (exclusive?); similarity measure (Distance or connectivity-based); cluster space (full space or sub). **Challenge**: quality (different attribute, arbitrary shape, noisy data); scalability (cluster beyond samples; high dim; incremental, insensitive to input order, constraint-based clustering (domain knowledge, user queries, etc), interpretability)

K-means: Select K points as initial centroids; form K clusters by assigning samples to centroids; recompute centroids; repeat until convergence.

objective function $SSE(C) = \sum_{j=1}^{K}\sum_{i=1}^{n} m_{i,j}\|x_i - C_j\|^2$. Membership $m_{i,j}$ and centers are correlated. Given C_j, $m_{i,j} = 1, if\ j = argmin_k (x_i - C_j)^2; 0, otherwise$; Given membership $m_{i,j}$, $C_j = \sum_{i=1}^{n} m_{i,j} x_i / \sum_{i=1}^{n} m_{i,j}$.

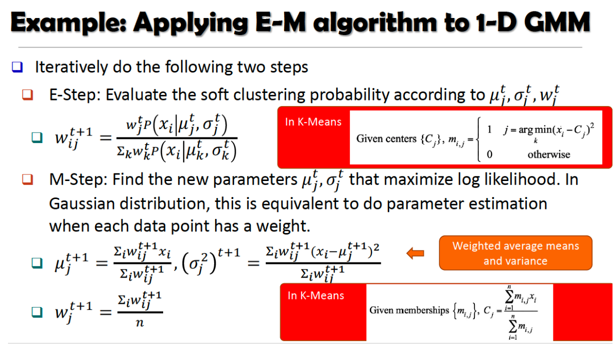
Kmeans as matrix factorization: $X \sim F \times G$, where X is data, $F : n \times kclustermembershipmatrix(0/1); G : k \times dcluster - descriptionmatrix$.

Indexing methods: d is small: B-tree, Quad-tree, K-d-tree; if d is large: LSH.

GMM: assumption: Each data point from a class; cluster prior distribution w_j is unknown; Each class follows a Gaussian. Distribution: $p(x|c_j, \theta_j) = \frac{1}{\sqrt{2\pi\sigma_j^2}} exp(-\frac{(x-\mu_j)^2}{2\sigma_j^2})$ where μ_j, σ_j to be learned. The prob of x_i is sum over all classes: $P(x_i|\theta) = \sum_{j=1}^{K} P(x_i|c_j, \theta_j)P(c_j)$

Soft-clustering with GMM: for each object i: $P(z_i = j) \in [0, 1]$ and $\sum_j P(z_i = j) = 1$, where z_i shows which cluster x_i is. After learning, $P(z_i = c_j|x_i) \propto P(x_i, z_i = c_j) = w_j P(x_i|z_j = c_j)$, where w_j is cluster prior prob and P is prob density function of each function.

E-M algorithm: E-step: Assigns objects to clusters. $w_{ij}^{t+1} = P(z_i = j|x_i, \theta_j^t) \propto w_j P(x_i|z_i = j, \theta_j^t)$. M-step: find new parameters to maximize expected likelihood: $\theta_{t+1} = argmax_\theta \sum_i \sum_j w_{ij}^{t+1} log(L(x_i, z_j|\theta))$



Pros: more general than partitioning (different densities and cluster sizes); cluster can be characterized by a small number of parameters; results satisfy statistical assumptions **Cons**: local minimal (solve by repeat running and random initialize); computationally expensive, hard to estimate # of clusters; only deal with spherical cluster

Mixture model for doc clustering: a set of language model $\Theta = \theta_1, \ldots, \theta_k$, where $\theta_i = \{p(w_1|\theta_i), \ldots, p(w_v|\theta_i)\}$. Prob $p(d = d_i) = \sum_{\theta_j} p(\theta = \theta_j)p(d = d_i|\theta = \theta_j) \propto \sum_{\theta_j} p(\theta = \theta_j)\prod_{k=1}^{V}[p(w_k|\theta_j)]^{tf(w_k, d_i)}$.

E-step: $E[z_{ij}] = p(\theta = \theta_j|d = d_i) = \frac{p(d=d_i|\theta=\theta_j)p(\theta=\theta_j)}{\sum_{n=1}^{k} p(d=d_i|\theta=\theta_n)p(\theta=\theta_n)} = \frac{\prod_{k=1}^{V}[p(w_k|\theta_j)]^{tf(w_k,d_i)}p(\theta=\theta_j)}{\sum_{\theta_j}\prod_{k=1}^{V}[p(w_k|\theta_n)]^{tf(w_k,d_i)}p(\theta=\theta_n)}$, where K: # of clusters, V: # of terms; N:# of docs.

M-step: $p(w_i|\theta_j) = \frac{\sum_{k=1}^{N} E[z_{kj}]tf(w_i, d_k)}{\sum_{k=1}^{N} E[z_{kj}]|d_k|}$ and $p(\theta = \theta_j) = \frac{1}{N}\sum_{i=1}^{N} E[z_{ij}]$