## Contents

## Sublime Build

```
{
    "cmd" : ["g++ -std=c++14 -DSONIC $file_name -o
        $file_base_name && timeout 4s ./$file_base_name<
        inputf.in>outputf.in"],
    "selector" : "source.cpp",
    "file_regex": "^(..[^:]*):([0-9]+):?([0-9]+)?:? (.*)$
        ",
    "shell": true,
    "working_dir" : "$file_path"
}
```

## vimrc

```
set mouse=a
  set termguicolors
  filetype plugin indent on
  syntax on

" Some useful settings
  set smartindent expandtab ignorecase smartcase
      incsearch relativenumber nowrap autoread splitright
       splitbelow
  set tabstop=4       "the width of a tab
  set shiftwidth=4    "the width for indent
  colorscheme torte

"auto pair curlybraces
  inoremap {<ENTER> {}<LEFT><CR><ESC><S-o>

" mapping jj to esc
  inoremap jj <ESC>

  "compile and run using file input put
  autocmd filetype cpp map <F5> :wa<CR>:!clear && g++ % -
      D LOCAL -std=c++17 -Wall -Wextra -Wconversion -
      Wshadow -Wfloat-equal -o ~/Codes/prog && (timeout 5
       ~/Codes/prog < ~/Codes/in) > ~/Codes/out<CR>
  "copy to input file
  map <F4> :!xclip -o -sel clip > ~/Codes/in <CR><CR>
  map <F6> :vsplit ~/Codes/in<CR>:split ~/Codes/out<CR><C
      -w>=20<C-w><<C-w><C-h>

 " Leader key
  let mapleader=',,'

 " Copy template
  noremap <Leader>t :!cp ~/Codes/temp.cpp %<CR><CR>
  :autocmd BufNewFile *.cpp 0r ~/Codes/temp.cpp
  "note if vim-features +clipboard is not found, it will
      not work
  "for fast check :echo has('clipboard) = 0 if clipboard
      features not present,
  "need vim-gtk / vim-gtk3 package for this
  set clipboard=unnamedplus
```

## 1 All Macros

```
//#pragma GCC optimize("Ofast")
//#pragma GCC optimization ("O3")
//#pragma comment(linker, "/stack:200000000")
//#pragma GCC optimize("unroll-loops")
//#pragma GCC target("sse,sse2,sse3,ssse3,sse4,popcnt,abm
    ,mmx,avx,tune=native")

#include <ext/pb_ds/assoc_container.hpp>
#include <ext/pb_ds/tree_policy.hpp>
using namespace __gnu_pbds;
    //find_by_order(k) --> returns iterator to the kth
        largest element counting from 0
    //order_of_key(val) --> returns the number of items
        in a set that are strictly smaller than our item
template <typename DT>
using ordered_set = tree <DT, null_type, less<DT>,
    rb_tree_tag,tree_order_statistics_node_update>;

/*--- DEBUG TEMPLATE STARTS HERE ---*/
#ifdef LEL
void show(int x) {cerr << x;}
void show(long long x) {cerr << x;}
void show(double x) {cerr << x;}
void show(char x) {cerr << '\'' << x << '\'';}
void show(const string &x) {cerr << '\"' << x << '\"';}
void show(bool x) {cerr << (x ? "true" : "false");}

template<typename T, typename V>
void show(pair<T, V> x) { cerr << '{'; show(x.first);
    cerr << ", "; show(x.second); cerr << '}'; }
template<typename T>
void show(T x) {int f = 0; cerr << "{"; for (auto &i: x)
    cerr << (f++ ? ", " : ""), show(i); cerr << "}";}

void debug_out(string s) {
    cerr << '\n';
}
template <typename T, typename... V>
void debug_out(string s, T t, V... v) {
    s.erase(remove(s.begin(), s.end(), ' '), s.end());
    cerr << "        "; // 8 spaces
    cerr << s.substr(0, s.find(','));
    s = s.substr(s.find(',') + 1);
    cerr << " = ";
    show(t);
    cerr << endl;
    if(sizeof...(v)) debug_out(s, v...);
}

#define debug(x...) cerr << "LINE: " << __LINE__ << endl;
    debug_out(#x, x); cerr << endl;
#else
#define debug(x...)
#endif
```

## 2 DP

### 2.1 1D-1D

```
/// Author: anachor

#include <bits/stdc++.h>
using namespace std;

/// Solves dp[i] = min(dp[j] + cost(j+1, i)) given that
    cost() is QF
long long solve1D(int n, long long cost(int, int)) {
  vector<long long> dp(n + 1), opt(n + 1);
  deque<pair<int, int>> dq;
  dq.push_back({0, 1});
  dp[0] = 0;

  for (int i = 1; i <= n; i++) {
    opt[i] = dq.front().first;
    dp[i] = dp[opt[i]] + cost(opt[i] + 1, i);
    if (i == n) break;

    dq[0].second++;
    if (dq.size() > 1 && dq[0].second == dq[1].second) dq
        .pop_front();

    int en = n;
    while (dq.size()) {
      int o = dq.back().first, st = dq.back().second;
      if (dp[o] + cost(o + 1, st) >= dp[i] + cost(i + 1,
          st))
        dq.pop_back();
      else {
        int lo = st, hi = en;
        while (lo < hi) {
          int mid = (lo + hi + 1) / 2;
          if (dp[o] + cost(o + 1, mid) < dp[i] + cost(i +
              1, mid))
            lo = mid;
          else
            hi = mid - 1;
        }
        if (lo < n) dq.push_back({i, lo + 1});
        break;
      }
      en = st - 1;
    }
    if (dq.empty()) dq.push_back({i, i + 1});
  }
  return dp[n];
}

/// Solves https://open.kattis.com/problems/
    coveredwalkway
const int N = 1e6 + 7;
long long x[N];
int c;
long long cost(int l, int r) { return (x[r] - x[l]) * (x[
    r] - x[l]) + c; }

int main() {
  ios::sync_with_stdio(false);
  cin.tie(0);

  int n;
  cin >> n >> c;
  for (int i = 1; i <= n; i++) cin >> x[i];
  cout << solve1D(n, cost) << endl;
}
```

## 2.2 Convex Hull Trick

```
struct line {
  ll m, c;
  line() {}
  line(ll m, ll c) : m(m), c(c) {}
};
struct convex_hull_trick {
  vector<line> lines;
  int ptr = 0;
  convex_hull_trick() {}
  bool bad(line a, line b, line c) {
    return 1.0 * (c.c - a.c) * (a.m - b.m) < 1.0 * (b.c -
        a.c) * (a.m - c.m);
  }
  void add(line L) {
    int sz = lines.size();
    while (sz >= 2 && bad(lines[sz - 2], lines[sz - 1], L
        )) {
      lines.pop_back();
      sz--;
    }
    lines.pb(L);
  }
  ll get(int idx, int x) { return (1ll * lines[idx].m * x
      + lines[idx].c); }
  ll query(int x) {
    if (lines.empty()) return 0;
    if (ptr >= lines.size()) ptr = lines.size() - 1;
    while (ptr < lines.size() - 1 && get(ptr, x) > get(
        ptr + 1, x)) ptr++;
    return get(ptr, x);
  }
};
ll sum[MAX];
ll dp[MAX];
int arr[MAX];
int main() {
  fastio;
  int t;
  cin >> t;
  while (t--) {
    int n, a, b, c;
    cin >> n >> a >> b >> c;
    for (int i = 1; i <= n; i++) cin >> sum[i];
    for (int i = 1; i <= n; i++) dp[i] = 0, sum[i] += sum
        [i - 1];
    convex_hull_trick cht;
    cht.add(line(0, 0));
    for (int pos = 1; pos <= n; pos++) {
      dp[pos] = cht.query(sum[pos]) - 1ll * a * sqr(sum[
          pos]) - c;
      cht.add(line(2ll * a * sum[pos], dp[pos] - a * sqr(
          sum[pos])));
    }
    ll ans = (-1ll * dp[n]);
    ans += (1ll * sum[n] * b);
    cout << ans << "\n";
  }
}
```

## 2.3 Divide and Conquer dp

```
const int K = 805, N = 4005;
LL dp[2][N], _cost[N][N];
// 1-indexed for convenience
LL cost(int l, int r) {
  return _cost[r][r] - _cost[l - 1][r] - _cost[r][l - 1]
      + _cost[l - 1][l - 1] >> 1;
}
void compute(int cnt, int l, int r, int optl, int optr) {
  if (l > r) return;
  int mid = l + r >> 1;
  LL best = INT_MAX;
  int opt = -1;
  for (int i = optl; i <= min(mid, optr); i++) {
    LL cur = dp[cnt ^ 1][i - 1] + cost(i, mid);
    if (cur < best) best = cur, opt = i;
  }
  dp[cnt][mid] = best;
  compute(cnt, l, mid - 1, optl, opt);
  compute(cnt, mid + 1, r, opt, optr);
}
LL dnc_dp(int k, int n) {
  fill(dp[0] + 1, dp[0] + n + 1, INT_MAX);
  for (int cnt = 1; cnt <= k; cnt++) {
    compute(cnt & 1, 1, n, 1, n);
  }
  return dp[k & 1][n];
}
```

## 2.4 Knuth optimization

```
const int N = 1005;
LL dp[N][N], a[N];
int opt[N][N];
LL cost(int i, int j) { return a[j + 1] - a[i]; }
LL knuth_optimization(int n) {
  for (int i = 0; i < n; i++) {
    dp[i][i] = 0;
    opt[i][i] = i;
  }
  for (int i = n - 2; i >= 0; i--) {
    for (int j = i + 1; j < n; j++) {
      LL mn = LLONG_MAX;
      LL c = cost(i, j);
      for (int k = opt[i][j - 1]; k <= min(j - 1, opt[i +
          1][j]); k++) {
        if (mn > dp[i][k] + dp[k + 1][j] + c) {
          mn = dp[i][k] + dp[k + 1][j] + c;
          opt[i][j] = k;
        }
      }
      dp[i][j] = mn;
    }
  }
  return dp[0][n - 1];
}
```

## 2.5 Li Chao Tree

```
struct line {
  LL m, c;
  line(LL m = 0, LL c = 0) : m(m), c(c) {}
};
LL calc(line L, LL x) { return 1LL * L.m * x + L.c; }
struct node {
  LL m, c;
  line L;
  node *lft, *rt;
  node(LL m = 0, LL c = 0, node *lft = NULL, node *rt =
      NULL)
      : L(line(m, c)), lft(lft), rt(rt) {}
};
struct LiChao {
  node *root;
  LiChao() { root = new node(); }
  void update(node *now, int L, int R, line newline) {
    int mid = L + (R - L) / 2;
    line lo = now->L, hi = newline;
    if (calc(lo, L) > calc(hi, L)) swap(lo, hi);
    if (calc(lo, R) <= calc(hi, R)) {
      now->L = hi;
      return;
    }
    if (calc(lo, mid) < calc(hi, mid)) {
      now->L = hi;
      if (now->rt == NULL) now->rt = new node();
      update(now->rt, mid + 1, R, lo);
    } else {
      now->L = lo;
      if (now->lft == NULL) now->lft = new node();
      update(now->lft, L, mid, hi);
    }
  }
  LL query(node *now, int L, int R, LL x) {
    if (now == NULL) return -inf;
    int mid = L + (R - L) / 2;
    if (x <= mid)
      return max(calc(now->L, x), query(now->lft, L, mid,
          x));
    else
      return max(calc(now->L, x), query(now->rt, mid + 1,
          R, x));
  }
};
```

## 3 Data Structure

### 3.1 BIT-2D

```
const int N = 1008;
int bit[N][N], n, m;
int a[N][N], q;
void update(int x, int y, int val) {
  for (; x < N; x += -x & x)
    for (int j = y; j < N; j += -j & j) bit[x][j] += val;
}
int get(int x, int y) {
  int ans = 0;
  for (; x; x -= x & -x)
    for (int j = y; j; j -= j & -j) ans += bit[x][j];
  return ans;
}
int get(int x1, int y1, int x2, int y2) {
  return get(x2, y2) - get(x1 - 1, y2) - get(x2, y1 - 1)
      + get(x1 - 1, y1 - 1);
}
```

### 3.2 Divide And Conquer Query Offline

```
namespace up {
int l[N], r[N], u[N], v[N], tm;
void push(int _l, int _r, int _u, int _v) {
  l[tm] = _l, r[tm] = _r, u[tm] = _u, v[tm] = _v;
  tm++;
}
}  // namespace up
namespace que {
int node[N], tm;
LL ans[N];
void push(int _node) { node[++tm] = _node; }
}  // namespace que
namespace edge_set {
void push(int i) { dsu ::merge(up ::u[i], up ::v[i]); }
void pop(int t) { dsu ::rollback(t); }
int time() { return dsu ::op.size(); }
LL query(int u) { return a[dsu ::root(u)]; }
}  // namespace edge_set
namespace dncq {
vector<int> tree[4 * N];
void update(int idx, int l = 0, int r = que ::tm, int
    node = 1) {
  int ul = up ::l[idx], ur = up ::r[idx];
  if (r < ul or ur < l) return;
  if (ul <= l and r <= ur) {
    tree[node].push_back(idx);
    return;
  }
  int m = l + r >> 1;
  update(idx, l, m, node << 1);
  update(idx, m + 1, r, node << 1 | 1);
}
void dfs(int l = 0, int r = que ::tm, int node = 1) {
  int cur = edge_set ::time();
  for (int e : tree[node]) edge_set ::push(e);
  if (l == r) {
    que ::ans[l] = edge_set ::query(que ::node[l]);
  } else {
    int m = l + r >> 1;
    dfs(l, m, node << 1);
    dfs(m + 1, r, node << 1 | 1);
  }
  edge_set ::pop(cur);
}
}  // namespace dncq
void push_updates() {
  for (int i = 0; i < up ::tm; i++) dncq ::update(i);
}
```

### 3.3 MO with Update

```
const int N = 1e5 + 5, sz = 2700, bs = 25;
int arr[N], freq[2 * N], cnt[2 * N], id[N], ans[N];
struct query {
  int l, r, t, L, R;
  query(int l = 1, int r = 0, int t = 1, int id = -1)
      : l(l), r(r), t(t), L(l / sz), R(r / sz) {}
  bool operator<(const query &rhs) const {
    return (L < rhs.L) or (L == rhs.L and R < rhs.R) or
        (L == rhs.L and R == rhs.R and t < rhs.t);
  }
} Q[N];
struct update {
  int idx, val, last;
} Up[N];
int qi = 0, ui = 0;
int l = 1, r = 0, t = 0;

void add(int idx) {
  --cnt[freq[arr[idx]]];
  freq[arr[idx]]++;
  cnt[freq[arr[idx]]]++;
}
void remove(int idx) {
  --cnt[freq[arr[idx]]];
  freq[arr[idx]]--;
  cnt[freq[arr[idx]]]++;
}
void apply(int t) {
  const bool f = l <= Up[t].idx and Up[t].idx <= r;
  if (f) remove(Up[t].idx);
  arr[Up[t].idx] = Up[t].val;
  if (f) add(Up[t].idx);
}
void undo(int t) {
  const bool f = l <= Up[t].idx and Up[t].idx <= r;
  if (f) remove(Up[t].idx);
  arr[Up[t].idx] = Up[t].last;
  if (f) add(Up[t].idx);
}
int mex() {
  for (int i = 1; i <= N; i++)
    if (!cnt[i]) return i;
  assert(0);
}
int main() {
  int n, q;
  cin >> n >> q;
  int counter = 0;
  map<int, int> M;
  for (int i = 1; i <= n; i++) {
    cin >> arr[i];
    if (!M[arr[i]]) M[arr[i]] = ++counter;
    arr[i] = M[arr[i]];
  }
  iota(id, id + N, 0);
  while (q--) {
    int tp, x, y;
    cin >> tp >> x >> y;
    if (tp == 1)
      Q[++qi] = query(x, y, ui);
    else {
      if (!M[y]) M[y] = ++counter;
      y = M[y];
      Up[++ui] = {x, y, arr[x]};
      arr[x] = y;
    }
  }
  t = ui;
  cnt[0] = 3 * n;
  sort(id + 1, id + qi + 1, [&](int x, int y) { return Q[
      x] < Q[y]; });
  for (int i = 1; i <= qi; i++) {
    int x = id[i];
    while (Q[x].t > t) apply(++t);
    while (Q[x].t < t) undo(t--);
    while (Q[x].l < l) add(--l);
    while (Q[x].r > r) add(++r);
    while (Q[x].l > l) remove(l++);
    while (Q[x].r < r) remove(r--);
    ans[x] = mex();
  }
  for (int i = 1; i <= qi; i++) cout << ans[i] << '\n';
}
```

## 3.4 Merge Sort Tree

```
vector<LL> Tree[4 * MAXN];
LL arr[MAXN];

vector<LL> merge(vector<LL> v1, vector<LL> v2) {
  LL i = 0, j = 0;
  vector<LL> ret;

  while (i < v1.size() || j < v2.size()) {
    if (i == v1.size()) {
      ret.push_back(v2[j]);
      j++;
    } else if (j == v2.size()) {
      ret.push_back(v1[i]);
      i++;
    } else {
      if (v1[i] < v2[j]) {
        ret.push_back(v1[i]);
        i++;
      } else {
        ret.push_back(v2[j]);
        j++;
      }
    }
  }

  return ret;
}

void Build(LL node, LL bg, LL ed) {
  if (bg == ed) {
    Tree[node].push_back(arr[bg]);
    return;
  }

  LL leftNode = 2 * node, rightNode = 2 * node + 1;
  LL mid = (bg + ed) / 2;

  Build(leftNode, bg, mid);
  Build(rightNode, mid + 1, ed);

  Tree[node] = merge(Tree[leftNode], Tree[rightNode]);
}

LL query(LL node, LL bg, LL ed, LL l, LL r, LL k) {
  if (ed < l || bg > r) return 0;

  if (l <= bg && ed <= r)
    return upper_bound(Tree[node].begin(), Tree[node].end
        (), k) -
          Tree[node].begin();

  LL leftNode = 2 * node, rightNode = 2 * node + 1;
  LL mid = (bg + ed) / 2;

  return query(leftNode, bg, mid, l, r, k) +
         query(rightNode, mid + 1, ed, l, r, k);
}
```

## 3.5 SparseTable (Rectangle Query)

```
#include <bits/stdc++.h>
using namespace std;

const int MAXN = 505;
const int LOGN = 9;

// O(n^2 (logn)^2
// Supports Rectangular Query
int A[MAXN][MAXN];
int M[MAXN][MAXN][LOGN][LOGN];

void Build2DSparse(int N) {
  for (int i = 1; i <= N; i++) {
    for (int j = 1; j <= N; j++) {
      M[i][j][0][0] = A[i][j];
    }
    for (int q = 1; (1 << q) <= N; q++) {
      int add = 1 << (q - 1);
      for (int j = 1; j + add <= N; j++) {
        M[i][j][0][q] = max(M[i][j][0][q - 1], M[i][j +
            add][0][q - 1]);
      }
    }
  }

  for (int p = 1; (1 << p) <= N; p++) {
    int add = 1 << (p - 1);
    for (int i = 1; i + add <= N; i++) {
      for (int q = 0; (1 << q) <= N; q++) {
        for (int j = 1; j <= N; j++) {
          M[i][j][p][q] = max(M[i][j][p - 1][q], M[i + add
              ][j][p - 1][q]);
        }
      }
    }
  }
}

// returns max of all A[i][j], where x1<=i<=x2 and y1<=j
    <=y2
int Query(int x1, int y1, int x2, int y2) {
  int kX = log2(x2 - x1 + 1);
  int kY = log2(y2 - y1 + 1);
  int addX = 1 << kX;
  int addY = 1 << kY;

  int ret1 = max(M[x1][y1][kX][kY], M[x1][y2 - addY + 1][
      kX][kY]);
  int ret2 = max(M[x2 - addX + 1][y1][kX][kY],
                 M[x2 - addX + 1][y2 - addY + 1][kX][kY]);
  return max(ret1, ret2);
}
```

## 3.6 Segment Tree

```
const int N = 1000006;

using DT = LL;
using LT = LL;
constexpr DT I = 0;
constexpr LT None = 0;
DT val[4 * N];
LT lazy[4 * N];
int L, R;

void pull(int s, int e, int node) {
  val[node] = val[node << 1] + val[node << 1 | 1];
}
void apply(const LT &U, int s, int e, int node) {
  val[node] += (e - s + 1) * U;
  lazy[node] += U;
}
void reset(int node) { lazy[node] = None; }
DT merge(const DT &a, const DT &b) { return a + b; }
DT get(int s, int e, int node) { return val[node]; }
void push(int s, int e, int node) {
  if (s == e) return;
  apply(lazy[node], s, s + e >> 1, node << 1);
  apply(lazy[node], s + e + 2 >> 1, e, node << 1 | 1);
  reset(node);
}
void build(int s, int e, vector<DT> &v, int node = 1) {
  int m = s + e >> 1;
  if (s == e) {
    val[node] = v[s];
    return;
  }
  build(s, m, v, node * 2);
  build(m + 1, e, v, node * 2 + 1);
  pull(s, e, node);
}
void update(int S, int E, LT uval, int s = L, int e = R,
    int node = 1) {
  if (S > E) return;
  if (S == s and E == e) {
    apply(uval, s, e, node);
    return;
  }
  push(s, e, node);
  int m = s + e >> 1;
  update(S, min(m, E), uval, s, m, node * 2);
  update(max(S, m + 1), E, uval, m + 1, e, node * 2 + 1);
  pull(s, e, node);
}
DT query(int S, int E, int s = L, int e = R, int node =
    1) {
  if (S > E) return I;
  if (s == S and e == E) return get(s, e, node);
  push(s, e, node);
  int m = s + e >> 1;
  DT L = query(S, min(m, E), s, m, node * 2);
  DT R = query(max(S, m + 1), E, m + 1, e, node * 2 + 1);
  return merge(L, R);
}
void init(int _L, int _R, vector<DT> &v) {
  L = _L, R = _R;
  build(L, R, v);
}
```

## 3.7 Persistent Segment Tree

```
struct Node {
  Node *l, *r;
  int sum;

  Node(int val) : l(nullptr), r(nullptr), sum(val) {}
  Node(Node* l, Node* r) : l(l), r(r), sum(0) {
    if (l) sum += l->sum;
    if (r) sum += r->sum;
  }
};

int a[MAXN];
Node* root[MAXN];

Node* Build(int bg, int ed) {
  if (bg == ed) return new Node(a[bg]);
  int mid = (bg + ed) / 2;
  return new Node(Build(bg, mid), Build(mid + 1, ed));
}

int Query(Node* v, int bg, int ed, int l, int r) {
  if (l > ed || r < bg) return 0;
  if (l <= bg && ed <= r) return v->sum;
  int mid = (bg + ed) / 2;
  return Query(v->l, bg, mid, l, r) + Query(v->r, mid +
      1, ed, l, r);
}

Node* Update(Node* v, int bg, int ed, int pos, int
    new_val) {
  if (bg == ed) return new Node(v->sum + new_val);
  int mid = (bg + ed) / 2;
  if (pos <= mid)
    return new Node(Update(v->l, bg, mid, pos, new_val),
        v->r);
  else
    return new Node(v->l, Update(v->r, mid + 1, ed, pos,
        new_val));
}
```

## 3.8 Implicit Segment Tree

```
struct node {
  int val;
  node *lft, *rt;
  node() {}
  node(int val = 0) : val(val), lft(NULL), rt(NULL) {}
};

struct implicit_segtree {
  node *root;
  implicit_segtree() {}
  implicit_segtree(int n) {
    root = new node(n);
  }
  void update(node *now, int L, int R, int idx, int val)
      {
    if (L == R) {
      now -> val += val;
      return;
    }
    int mid = L + (R - L) / 2;
    if (now->lft == NULL) now->lft = new node(mid - L +
        1);
    if (now->rt == NULL) now->rt = new node(R - mid);
    if (idx <= mid) update(now->lft, L, mid, idx, val);
    else update(now->rt, mid + 1, R, idx, val);
    now->val = (now->lft)->val + (now->rt)->val;
  }

  int query(node *now, int L, int R, int k) {
    if (L == R) return L;
    int mid = L + (R - L) / 2;
    if (now->lft == NULL) now->lft = new node(mid - L +
        1);
    if (now->rt == NULL) now->rt = new node(R - mid);
    if (k <= (now->lft)->val) return query(now->lft, L,
        mid, k);
    else return query(now->rt, mid + 1, R, k - (now->lft)
        ->val);
  }
};
```

### 3.9 HashTable

```
#include <bits/stdc++.h.>
#include <ext/pb_ds/assoc_container.hpp>
using namespace std;
using namespace __gnu_pbds;

const int RANDOM =
    chrono::high_resolution_clock::now().time_since_epoch
        ().count();
unsigned hash_f(unsigned x) {
  x = ((x >> 16) ^ x) * 0x45d9f3b;
  x = ((x >> 16) ^ x) * 0x45d9f3b;
  x = (x >> 16) ^ x;
  return x;
}

unsigned hash_combine(unsigned a, unsigned b) { return a
    * 31 + b; }
struct chash {
  int operator()(int x) const { return hash_f(x); }
};
typedef gp_hash_table<int, int, chash> gp;
gp table;
```

### 3.10 DSU With Rollbacks

```
struct Rollback_DSU {
  int n;
  vector<int> par, sz;
  vector<pair<int, int>> op;
  Rollback_DSU(int n) : par(n), sz(n, 1) {
    iota(par.begin(), par.end(), 0);
    op.reserve(n);
  }
  int Anc(int node) {
    for (; node != par[node]; node = par[node])
      ; // no path compression
    return node;
  }
  void Unite(int x, int y) {
    if (sz[x = Anc(x)] < sz[y = Anc(y)]) swap(x, y);
    op.emplace_back(x, y);
    par[y] = x;
    sz[x] += sz[y];
  }
  void Undo(int t) {
    for (; op.size() > t; op.pop_back()) {
      par[op.back().second] = op.back().second;
      sz[op.back().first] -= sz[op.back().second];
    }
  }
};
```

### 3.11 Binary Trie

```
const int N = 1e7 + 5, b = 30;
int tc = 1;
struct node {
  int vis = 0;
  int to[2] = {0, 0};
  int val[2] = {0, 0};
  void update() {
    to[0] = to[1] = 0;
    val[0] = val[1] = 0;
    vis = tc;
  }
} T[N + 2];
node *root = T;
int ptr = 0;
node *nxt(node *cur, int x) {
  if (cur->to[x] == 0) cur->to[x] = ++ptr;
  assert(ptr < N);
  int idx = cur->to[x];
  if (T[idx].vis < tc) T[idx].update();
  return T + idx;
}
int query(int j, int aj) {
  int ans = 0, jaj = j ^ aj;
  node *cur = root;
  for (int k = b - 1; ~k; k--) {
    maximize(ans, nxt(cur, (jaj >> k & 1) ^ 1)->val[1 ^ (
        aj >> k & 1)]);
    cur = nxt(cur, (jaj >> k & 1));
  }
  return ans;
}
void insert(int j, int aj, int val) {
  int jaj = j ^ aj;
  node *cur = root;
  for (int k = b - 1; ~k; k--) {
    cur = nxt(cur, (jaj >> k & 1));
    maximize(cur->val[j >> k & 1], val);
  }
}
void clear() {
  tc++;
  ptr = 0;
  root->update();
}
```

# 4 Geometry

### 4.1 Point

```
typedef double Tf;
typedef double Ti; /// use long long for exactness
const Tf PI = acos(-1), EPS = 1e-9;
int dcmp(Tf x) { return abs(x) < EPS ? 0 : (x < 0 ? -1 :
    1); }

struct Point {
  Ti x, y;
  Point(Ti x = 0, Ti y = 0) : x(x), y(y) {}

  Point operator+(const Point& u) const { return Point(x
      + u.x, y + u.y); }
  Point operator-(const Point& u) const { return Point(x
      - u.x, y - u.y); }
  Point operator*(const LL u) const { return Point(x * u,
       y * u); }
  Point operator*(const Tf u) const { return Point(x * u,
       y * u); }
  Point operator/(const Tf u) const { return Point(x / u,
       y / u); }

  bool operator==(const Point& u) const {
    return dcmp(x - u.x) == 0 && dcmp(y - u.y) == 0;
  }
  bool operator!=(const Point& u) const { return !(*this
      == u); }
  bool operator<(const Point& u) const {
    return dcmp(x - u.x) < 0 || (dcmp(x - u.x) == 0 &&
        dcmp(y - u.y) < 0);
  }
};
Ti dot(Point a, Point b) { return a.x * b.x + a.y * b.y;
    }
Ti cross(Point a, Point b) { return a.x * b.y - a.y * b.x
    ; }
Tf length(Point a) { return sqrt(dot(a, a)); }
Ti sqLength(Point a) { return dot(a, a); }
Tf distance(Point a, Point b) { return length(a - b); }
Tf angle(Point u) { return atan2(u.y, u.x); }
```

```
// returns angle between oa, ob in (-PI, PI]
Tf angleBetween(Point a, Point b) {
  Tf ans = angle(b) - angle(a);
  return ans <= -PI ? ans + 2 * PI : (ans > PI ? ans - 2
      * PI : ans);
}
// Rotate a ccw by rad radians, Tf Ti same
Point rotate(Point a, Tf rad) {
  return Point(a.x * cos(rad) - a.y * sin(rad),
               a.x * sin(rad) + a.y * cos(rad));
}
// rotate a ccw by angle th with cos(th) = co && sin(th)
    = si, tf ti same
Point rotatePrecise(Point a, Tf co, Tf si) {
  return Point(a.x * co - a.y * si, a.y * co + a.x * si);
}
Point rotate90(Point a) { return Point(-a.y, a.x); }
// scales vector a by s such that length of a becomes s,
    Tf Ti same
Point scale(Point a, Tf s) { return a / length(a) * s; }
// returns an unit vector perpendicular to vector a, Tf
    Ti same
Point normal(Point a) {
  Tf l = length(a);
  return Point(-a.y / l, a.x / l);
}
// returns 1 if c is left of ab, 0 if on ab && -1 if
    right of ab
int orient(Point a, Point b, Point c) { return dcmp(cross
    (b - a, c - a)); }
/// Use as sort(v.begin(), v.end(), polarComp(O, dir))
/// Polar comparator around O starting at direction dir
struct polarComp {
  Point O, dir;
  polarComp(Point O = Point(0, 0), Point dir = Point(1,
      0)) : O(O), dir(dir) {}
  bool half(Point p) {
    return dcmp(cross(dir, p)) < 0 ||
           (dcmp(cross(dir, p)) == 0 && dcmp(dot(dir, p))
               > 0);
  }
  bool operator()(Point p, Point q) {
    return make_tuple(half(p), 0) < make_tuple(half(q),
        cross(p, q));
  }
};
struct Segment {
  Point a, b;
  Segment(Point aa, Point bb) : a(aa), b(bb) {}
};
typedef Segment Line;
struct Circle {
  Point o;
  Tf r;
  Circle(Point o = Point(0, 0), Tf r = 0) : o(o), r(r) {}
  // returns true if point p is in || on the circle
  bool contains(Point p) { return dcmp(sqLength(p - o) -
      r * r) <= 0; }
  // returns a point on the circle rad radians away from
      +X CCW
  Point point(Tf rad) {
    static_assert(is_same<Tf, Ti>::value);
    return Point(o.x + cos(rad) * r, o.y + sin(rad) * r);
  }
  // area of a circular sector with central angle rad
  Tf area(Tf rad = PI + PI) { return rad * r * r / 2; }
  // area of the circular sector cut by a chord with
      central angle alpha
  Tf sector(Tf alpha) { return r * r * 0.5 * (alpha - sin
      (alpha)); }
};
```

## 4.2 Linear

```
// **** LINE LINE INTERSECTION START ****
// returns true if point p is on segment s
bool onSegment(Point p, Segment s) {
  return dcmp(cross(s.a - p, s.b - p)) == 0 && dcmp(dot(s
      .a - p, s.b - p)) <= 0;
}
// returns true if segment p && q touch or intersect
bool segmentsIntersect(Segment p, Segment q) {
  if (onSegment(p.a, q) || onSegment(p.b, q)) return true
      ;
  if (onSegment(q.a, p) || onSegment(q.b, p)) return true
      ;

  Ti c1 = cross(p.b - p.a, q.a - p.a);
  Ti c2 = cross(p.b - p.a, q.b - p.a);
  Ti c3 = cross(q.b - q.a, p.a - q.a);
  Ti c4 = cross(q.b - q.a, p.b - q.a);
  return dcmp(c1) * dcmp(c2) < 0 && dcmp(c3) * dcmp(c4) <
       0;
}
bool linesParallel(Line p, Line q) {
  return dcmp(cross(p.b - p.a, q.b - q.a)) == 0;
}
// lines are represented as a ray from a point: (point,
    vector)
// returns false if two lines (p, v) && (q, w) are
    parallel or collinear
// true otherwise, intersection point is stored at o via
    reference, Tf Ti Same
bool lineLineIntersection(Point p, Point v, Point q,
    Point w, Point& o) {
  if (dcmp(cross(v, w)) == 0) return false;
  Point u = p - q;
  o = p + v * (cross(w, u) / cross(v, w));
  return true;
}
// returns false if two lines p && q are parallel or
    collinear
// true otherwise, intersection point is stored at o via
    reference
bool lineLineIntersection(Line p, Line q, Point& o) {
  return lineLineIntersection(p.a, p.b - p.a, q.a, q.b -
      q.a, o);
}
// returns the distance from point a to line l
// **** LINE LINE INTERSECTION FINISH ****
Tf distancePointLine(Point p, Line l) {
  return abs(cross(l.b - l.a, p - l.a) / length(l.b - l.a
      ));
}
// returns the shortest distance from point a to segment
    s
Tf distancePointSegment(Point p, Segment s) {
  if (s.a == s.b) return length(p - s.a);
  Point v1 = s.b - s.a, v2 = p - s.a, v3 = p - s.b;
  if (dcmp(dot(v1, v2)) < 0)
    return length(v2);
  else if (dcmp(dot(v1, v3)) > 0)
    return length(v3);
  else
    return abs(cross(v1, v2) / length(v1));
}
// returns the shortest distance from segment p to
    segment q
Tf distanceSegmentSegment(Segment p, Segment q) {
  if (segmentsIntersect(p, q)) return 0;
  Tf ans = distancePointSegment(p.a, q);
  ans = min(ans, distancePointSegment(p.b, q));
  ans = min(ans, distancePointSegment(q.a, p));
  ans = min(ans, distancePointSegment(q.b, p));
  return ans;
}
// returns the projection of point p on line l, Tf Ti
    Same
Point projectPointLine(Point p, Line l) {
  Point v = l.b - l.a;
  return l.a + v * ((Tf)dot(v, p - l.a) / dot(v, v));
}
```

### 4.3 Circular

```
// Extremely inaccurate for finding near touches
// compute intersection of line l with circle c
// The intersections are given in order of the ray (l.a,
    l.b), Tf Ti same
vector<Point> circleLineIntersection(Circle c, Line l) {
  vector<Point> ret;
  Point b = l.b - l.a, a = l.a - c.o;
  Tf A = dot(b, b), B = dot(a, b);
  Tf C = dot(a, a) - c.r * c.r, D = B * B - A * C;
  if (D < -EPS) return ret;
  ret.push_back(l.a + b * (-B - sqrt(D + EPS)) / A);
  if (D > EPS) ret.push_back(l.a + b * (-B + sqrt(D)) / A
      );
  return ret;
}
// signed area of intersection of circle(c.o, c.r) &&
// triangle(c.o, s.a, s.b) [cross(a-o, b-o)/2]
Tf circleTriangleIntersectionArea(Circle c, Segment s) {
  using Linear::distancePointSegment;
  Tf OA = length(c.o - s.a);
  Tf OB = length(c.o - s.b);
  // sector
  if (dcmp(distancePointSegment(c.o, s) - c.r) >= 0)
    return angleBetween(s.a - c.o, s.b - c.o) * (c.r * c.
        r) / 2.0;
  // triangle
  if (dcmp(OA - c.r) <= 0 && dcmp(OB - c.r) <= 0)
    return cross(c.o - s.b, s.a - s.b) / 2.0;
  // three part: (A, a) (a, b) (b, B)
  vector<Point> Sect = circleLineIntersection(c, s);
  return circleTriangleIntersectionArea(c, Segment(s.a,
      Sect[0])) +
         circleTriangleIntersectionArea(c, Segment(Sect
             [0], Sect[1])) +
         circleTriangleIntersectionArea(c, Segment(Sect
             [1], s.b));
}
// area of intersecion of circle(c.o, c.r) && simple
    polyson(p[])
Tf circlePolyIntersectionArea(Circle c, Polygon p) {
  Tf res = 0;
  int n = p.size();
  for (int i = 0; i < n; ++i)
    res += circleTriangleIntersectionArea(c, Segment(p[i
        ], p[(i + 1) % n]));
  return abs(res);
}
// locates circle c2 relative to c1
// interior          (d < R - r)       ----> -2
// interior tangents (d = R - r)      ----> -1
// concentric       (d = 0)
// secants            (R - r < d < R + r) ----> 0
// exterior tangents (d = R + r)      ----> 1
// exterior          (d > R + r)       ----> 2
int circleCirclePosition(Circle c1, Circle c2) {
  Tf d = length(c1.o - c2.o);
  int in = dcmp(d - abs(c1.r - c2.r)), ex = dcmp(d - (c1.
      r + c2.r));
  return in < 0 ? -2 : in == 0 ? -1 : ex == 0 ? 1 : ex >
      0 ? 2 : 0;
}
// compute the intersection points between two circles c1
    && c2, Tf Ti same
vector<Point> circleCircleIntersection(Circle c1, Circle
    c2) {
  vector<Point> ret;
  Tf d = length(c1.o - c2.o);
  if (dcmp(d) == 0) return ret;
  if (dcmp(c1.r + c2.r - d) < 0) return ret;
  if (dcmp(abs(c1.r - c2.r) - d) > 0) return ret;

  Point v = c2.o - c1.o;
  Tf co = (c1.r * c1.r + sqLength(v) - c2.r * c2.r) / (2
      * c1.r * length(v));
  Tf si = sqrt(abs(1.0 - co * co));
  Point p1 = scale(rotatePrecise(v, co, -si), c1.r) + c1.
      o;
  Point p2 = scale(rotatePrecise(v, co, si), c1.r) + c1.o
      ;

  ret.push_back(p1);
  if (p1 != p2) ret.push_back(p2);
  return ret;
}
// intersection area between two circles c1, c2
Tf circleCircleIntersectionArea(Circle c1, Circle c2) {
  Point AB = c2.o - c1.o;
  Tf d = length(AB);
  if (d >= c1.r + c2.r) return 0;
  if (d + c1.r <= c2.r) return PI * c1.r * c1.r;
  if (d + c2.r <= c1.r) return PI * c2.r * c2.r;

  Tf alpha1 = acos((c1.r * c1.r + d * d - c2.r * c2.r) /
      (2.0 * c1.r * d));
  Tf alpha2 = acos((c2.r * c2.r + d * d - c1.r * c1.r) /
      (2.0 * c2.r * d));
  return c1.sector(2 * alpha1) + c2.sector(2 * alpha2);
}
// returns tangents from a point p to circle c, Tf Ti
    same
vector<Point> pointCircleTangents(Point p, Circle c) {
  vector<Point> ret;
  Point u = c.o - p;
  Tf d = length(u);
  if (d < c.r)
    ;
  else if (dcmp(d - c.r) == 0) {
    ret = {rotate(u, PI / 2)};
  } else {
    Tf ang = asin(c.r / d);
    ret = {rotate(u, -ang), rotate(u, ang)};
  }
  return ret;
}
// returns the points on tangents that touches the circle
    , Tf Ti Same
vector<Point> pointCircleTangencyPoints(Point p, Circle c
    ) {
  Point u = p - c.o;
  Tf d = length(u);
  if (d < c.r)
    return {};
  else if (dcmp(d - c.r) == 0)
    return {c.o + u};
  else {
    Tf ang = acos(c.r / d);
    u = u / length(u) * c.r;
    return {c.o + rotate(u, -ang), c.o + rotate(u, ang)};
  }
}
// for two circles c1 && c2, returns two list of points a
    && b
// such that a[i] is on c1 && b[i] is c2 && for every i
// Line(a[i], b[i]) is a tangent to both circles
// CAUTION: a[i] = b[i] in case they touch | -1 for c1 =
    c2
int circleCircleTangencyPoints(Circle c1, Circle c2,
    vector<Point> &a,
                                vector<Point> &b) {
  a.clear(), b.clear();
  int cnt = 0;
  if (dcmp(c1.r - c2.r) < 0) {
    swap(c1, c2);
    swap(a, b);
  }
  Tf d2 = sqLength(c1.o - c2.o);
  Tf rdif = c1.r - c2.r, rsum = c1.r + c2.r;
  if (dcmp(d2 - rdif * rdif) < 0) return 0;
  if (dcmp(d2) == 0 && dcmp(c1.r - c2.r) == 0) return -1;

  Tf base = angle(c2.o - c1.o);
  if (dcmp(d2 - rdif * rdif) == 0) {
    a.push_back(c1.point(base));
    b.push_back(c2.point(base));
    cnt++;
    return cnt;
  }

  Tf ang = acos((c1.r - c2.r) / sqrt(d2));
  a.push_back(c1.point(base + ang));
  b.push_back(c2.point(base + ang));
  cnt++;
  a.push_back(c1.point(base - ang));
  b.push_back(c2.point(base - ang));
  cnt++;

  if (dcmp(d2 - rsum * rsum) == 0) {
    a.push_back(c1.point(base));
    b.push_back(c2.point(PI + base));
    cnt++;
  } else if (dcmp(d2 - rsum * rsum) > 0) {
    Tf ang = acos((c1.r + c2.r) / sqrt(d2));
    a.push_back(c1.point(base + ang));
    b.push_back(c2.point(PI + base + ang));
    cnt++;
    a.push_back(c1.point(base - ang));
    b.push_back(c2.point(PI + base - ang));
    cnt++;
  }
  return cnt;
}
```

## 4.4 Convex

```
/// minkowski sum of two polygons in O(n)
Polygon minkowskiSum(Polygon A, Polygon B) {
  int n = A.size(), m = B.size();
  rotate(A.begin(), min_element(A.begin(), A.end()), A.
      end());
  rotate(B.begin(), min_element(B.begin(), B.end()), B.
      end());

  A.push_back(A[0]);
  B.push_back(B[0]);
  for (int i = 0; i < n; i++) A[i] = A[i + 1] - A[i];
  for (int i = 0; i < m; i++) B[i] = B[i + 1] - B[i];

  Polygon C(n + m + 1);
  C[0] = A.back() + B.back();
  merge(A.begin(), A.end() - 1, B.begin(), B.end() - 1, C
      .begin() + 1,
       polarComp(Point(0, 0), Point(0, -1)));
  for (int i = 1; i < C.size(); i++) C[i] = C[i] + C[i -
      1];
  C.pop_back();
  return C;
}
// finds the rectangle with minimum area enclosing a
    convex polygon and
// the rectangle with minimum perimeter enclosing a
    convex polygon
// Tf Ti Same
pair<Tf, Tf> rotatingCalipersBoundingBox(const Polygon &p
    ) {
  using Linear::distancePointLine;
  int n = p.size();
  int l = 1, r = 1, j = 1;
  Tf area = 1e100;
  Tf perimeter = 1e100;
  for (int i = 0; i < n; i++) {
    Point v = (p[(i + 1) % n] - p[i]) / length(p[(i + 1)
        % n] - p[i]);
    while (dcmp(dot(v, p[r % n] - p[i]) - dot(v, p[(r +
        1) % n] - p[i])) < 0)
      r++;
    while (j < r || dcmp(cross(v, p[j % n] - p[i]) -
                      cross(v, p[(j + 1) % n] - p[i])) <
                          0)
      j++;
    while (l < j ||
          dcmp(dot(v, p[l % n] - p[i]) - dot(v, p[(l + 1)
                % n] - p[i])) > 0)
      l++;
    Tf w = dot(v, p[r % n] - p[i]) - dot(v, p[l % n] - p[
        i]);
    Tf h = distancePointLine(p[j % n], Line(p[i], p[(i +
        1) % n]));
    area = min(area, w * h);
    perimeter = min(perimeter, 2 * w + 2 * h);
  }
  return make_pair(area, perimeter);
}
// returns the left side of polygon u after cutting it by
     ray a->b
Polygon cutPolygon(Polygon u, Point a, Point b) {
  using Linear::lineLineIntersection;
  using Linear::onSegment;

  Polygon ret;
  int n = u.size();
  for (int i = 0; i < n; i++) {
    Point c = u[i], d = u[(i + 1) % n];
    if (dcmp(cross(b - a, c - a)) >= 0) ret.push_back(c);
    if (dcmp(cross(b - a, d - c)) != 0) {
      Point t;
      lineLineIntersection(a, b - a, c, d - c, t);
      if (onSegment(t, Segment(c, d))) ret.push_back(t);
    }
  }
  return ret;
}
// returns true if point p is in or on triangle abc
bool pointInTriangle(Point a, Point b, Point c, Point p)
    {
  return dcmp(cross(b - a, p - a)) >= 0 && dcmp(cross(c -
       b, p - b)) >= 0 &&
        dcmp(cross(a - c, p - c)) >= 0;
}
// pt must be in ccw order with no three collinear points
// returns inside = -1, on = 0, outside = 1
int pointInConvexPolygon(const Polygon &pt, Point p) {
  int n = pt.size();
  assert(n >= 3);

  int lo = 1, hi = n - 1;
  while (hi - lo > 1) {
    int mid = (lo + hi) / 2;
    if (dcmp(cross(pt[mid] - pt[0], p - pt[0])) > 0)
      lo = mid;
    else
      hi = mid;
  }

  bool in = pointInTriangle(pt[0], pt[lo], pt[hi], p);
  if (!in) return 1;

  if (dcmp(cross(pt[lo] - pt[lo - 1], p - pt[lo - 1])) ==
       0) return 0;
  if (dcmp(cross(pt[hi] - pt[lo], p - pt[lo])) == 0)
      return 0;
  if (dcmp(cross(pt[hi] - pt[(hi + 1) % n], p - pt[(hi +
      1) % n])) == 0)
    return 0;
  return -1;
}
```

```cpp
// Extreme Point for a direction is the farthest point in
     that direction
// u is the direction for extremeness
int extremePoint(const Polygon &poly, Point u) {
  int n = (int)poly.size();
  int a = 0, b = n;
  while (b - a > 1) {
    int c = (a + b) / 2;
    if (dcmp(dot(poly[c] - poly[(c + 1) % n], u)) >= 0 &&
        dcmp(dot(poly[c] - poly[(c - 1 + n) % n], u)) >=
             0) {
      return c;
    }

    bool a_up = dcmp(dot(poly[(a + 1) % n] - poly[a], u))
        >= 0;
    bool c_up = dcmp(dot(poly[(c + 1) % n] - poly[c], u))
        >= 0;
    bool a_above_c = dcmp(dot(poly[a] - poly[c], u)) > 0;

    if (a_up && !c_up)
      b = c;
    else if (!a_up && c_up)
      a = c;
    else if (a_up && c_up) {
      if (a_above_c)
        b = c;
      else
        a = c;
    } else {
      if (!a_above_c)
        b = c;
      else
        a = c;
    }
  }

  if (dcmp(dot(poly[a] - poly[(a + 1) % n], u)) > 0 &&
      dcmp(dot(poly[a] - poly[(a - 1 + n) % n], u)) > 0)
    return a;
  return b % n;
}
// For a convex polygon p and a line l, returns a list of
     segments
// of p that touch or intersect line l.
// the i'th segment is considered (p[i], p[(i + 1) modulo
     |p|])
// #1 If a segment is collinear with the line, only that
     is returned
// #2 Else if l goes through i'th point, the i'th segment
     is added
// Complexity: O(lg |p|)
vector<int> lineConvexPolyIntersection(const Polygon &p,
     Line l) {
  assert((int)p.size() >= 3);
  assert(l.a != l.b);

  int n = p.size();
  vector<int> ret;

  Point v = l.b - l.a;
  int lf = extremePoint(p, rotate90(v));
  int rt = extremePoint(p, rotate90(v) * Ti(-1));
  int olf = orient(l.a, l.b, p[lf]);
  int ort = orient(l.a, l.b, p[rt]);

  if (!olf || !ort) {
    int idx = (!olf ? lf : rt);
    if (orient(l.a, l.b, p[(idx - 1 + n) % n]) == 0)
      ret.push_back((idx - 1 + n) % n);
    else
      ret.push_back(idx);
    return ret;
  }
  if (olf == ort) return ret;

  for (int i = 0; i < 2; ++i) {
    int lo = i ? rt : lf;
    int hi = i ? lf : rt;
    int olo = i ? ort : olf;

    while (true) {
      int gap = (hi - lo + n) % n;
      if (gap < 2) break;

      int mid = (lo + gap / 2) % n;
      int omid = orient(l.a, l.b, p[mid]);
      if (!omid) {
        lo = mid;
        break;
      }
      if (omid == olo)
        lo = mid;
      else
        hi = mid;
    }
    ret.push_back(lo);
  }
  return ret;
}
// Calculate [ACW, CW] tangent pair from an external
     point
constexpr int CW = -1, ACW = 1;
bool isGood(Point u, Point v, Point Q, int dir) {
  return orient(Q, u, v) != -dir;
}
Point better(Point u, Point v, Point Q, int dir) {
  return orient(Q, u, v) == dir ? u : v;
}
Point pointPolyTangent(const Polygon &pt, Point Q, int
     dir, int lo, int hi) {
  while (hi - lo > 1) {
    int mid = (lo + hi) / 2;
    bool pvs = isGood(pt[mid], pt[mid - 1], Q, dir);
    bool nxt = isGood(pt[mid], pt[mid + 1], Q, dir);

    if (pvs && nxt) return pt[mid];
    if (!(pvs || nxt)) {
      Point p1 = pointPolyTangent(pt, Q, dir, mid + 1, hi
          );
      Point p2 = pointPolyTangent(pt, Q, dir, lo, mid -
          1);
      return better(p1, p2, Q, dir);
    }

    if (!pvs) {
      if (orient(Q, pt[mid], pt[lo]) == dir)
        hi = mid - 1;
      else if (better(pt[lo], pt[hi], Q, dir) == pt[lo])
        hi = mid - 1;
      else
        lo = mid + 1;
    }
    if (!nxt) {
      if (orient(Q, pt[mid], pt[lo]) == dir)
        lo = mid + 1;
      else if (better(pt[lo], pt[hi], Q, dir) == pt[lo])
        hi = mid - 1;
      else
        lo = mid + 1;
    }
  }

  Point ret = pt[lo];
  for (int i = lo + 1; i <= hi; i++) ret = better(ret, pt
      [i], Q, dir);
  return ret;
}
// [ACW, CW] Tangent
```

```
pair<Point, Point> pointPolyTangents(const Polygon &pt,
    Point Q) {
  int n = pt.size();
  Point acw_tan = pointPolyTangent(pt, Q, ACW, 0, n - 1);
  Point cw_tan = pointPolyTangent(pt, Q, CW, 0, n - 1);
  return make_pair(acw_tan, cw_tan);
}
```

### 4.5 Polygon

```
typedef vector<Point> Polygon;
// removes redundant colinear points
// polygon can't be all colinear points
Polygon RemoveCollinear(const Polygon &poly) {
  Polygon ret;
  int n = poly.size();
  for (int i = 0; i < n; i++) {
    Point a = poly[i];
    Point b = poly[(i + 1) % n];
    Point c = poly[(i + 2) % n];
    if (dcmp(cross(b - a, c - b)) != 0 && (ret.empty() ||
         b != ret.back()))
      ret.push_back(b);
  }
  return ret;
}
// returns the signed area of polygon p of n vertices
Tf signedPolygonArea(const Polygon &p) {
  Tf ret = 0;
  for (int i = 0; i < (int)p.size() - 1; i++)
    ret += cross(p[i] - p[0], p[i + 1] - p[0]);
  return ret / 2;
}
// given a polygon p of n vertices, generates the convex
    hull in in CCW
// Tested on https://acm.timus.ru/problem.aspx?space=1&
    num=1185
// Caution: when all points are colinear AND
    removeRedundant == false
// output will be contain duplicate points (from upper
    hull) at back
Polygon convexHull(Polygon p, bool removeRedundant) {
  int check = removeRedundant ? 0 : -1;
  sort(p.begin(), p.end());
  p.erase(unique(p.begin(), p.end()), p.end());

  int n = p.size();
  Polygon ch(n + n);
  int m = 0; // preparing lower hull
  for (int i = 0; i < n; i++) {
    while (m > 1 &&
           dcmp(cross(ch[m - 1] - ch[m - 2], p[i] - ch[m -
              1])) <= check)
      m--;
    ch[m++] = p[i];
  }
  int k = m; // preparing upper hull
  for (int i = n - 2; i >= 0; i--) {
    while (m > k &&
           dcmp(cross(ch[m - 1] - ch[m - 2], p[i] - ch[m -
              2])) <= check)
      m--;
    ch[m++] = p[i];
  }
  if (n > 1) m--;
  ch.resize(m);
  return ch;
}
// returns inside = -1, on = 0, outside = 1
int pointInPolygon(const Polygon &p, Point o) {
  using Linear::onSegment;
  int wn = 0, n = p.size();
  for (int i = 0; i < n; i++) {
    int j = (i + 1) % n;
    if (onSegment(o, Segment(p[i], p[j])) || o == p[i])
      return 0;
    int k = dcmp(cross(p[j] - p[i], o - p[i]));
    int d1 = dcmp(p[i].y - o.y);
    int d2 = dcmp(p[j].y - o.y);
    if (k > 0 && d1 <= 0 && d2 > 0) wn++;
    if (k < 0 && d2 <= 0 && d1 > 0) wn--;
  }
  return wn ? -1 : 1;
}
// Given a simple polygon p, and a line l, returns (x, y)
// x = longest segment of l in p, y = total length of l
    in p.
pair<Tf, Tf> linePolygonIntersection(Line l, const
    Polygon &p) {
  using Linear::lineLineIntersection;
  int n = p.size();
  vector<pair<Tf, int>> ev;
  for (int i = 0; i < n; ++i) {
    Point a = p[i], b = p[(i + 1) % n], z = p[(i - 1 + n)
        % n];
    int ora = orient(l.a, l.b, a), orb = orient(l.a, l.b,
        b),
        orz = orient(l.a, l.b, z);
    if (!ora) {
      Tf d = dot(a - l.a, l.b - l.a);
      if (orz && orb) {
        if (orz != orb) ev.emplace_back(d, 0);
        // else // Point Touch
      } else if (orz)
        ev.emplace_back(d, orz);
      else if (orb)
        ev.emplace_back(d, orb);
    } else if (ora == -orb) {
      Point ins;
      lineLineIntersection(l, Line(a, b), ins);
      ev.emplace_back(dot(ins - l.a, l.b - l.a), 0);
    }
  }
  sort(ev.begin(), ev.end());

  Tf ans = 0, len = 0, last = 0, tot = 0;
  bool active = false;
  int sign = 0;
  for (auto &qq : ev) {
    int tp = qq.second;
    Tf d = qq.first; /// current Segment is (last, d)
    if (sign) {      /// On Border
      len += d - last;
      tot += d - last;
      ans = max(ans, len);
      if (tp != sign) active = !active;
      sign = 0;
    } else {
      if (active) { /// Strictly Inside
        len += d - last;
        tot += d - last;
        ans = max(ans, len);
      }
      if (tp == 0)
        active = !active;
      else
        sign = tp;
    }
    last = d;
    if (!active) len = 0;
  }
  ans /= length(l.b - l.a);
  tot /= length(l.b - l.a);
  return {ans, tot};
}
```

# 5 Graph

### 5.1 Graph Template

```
struct edge {
  int u, v;
  edge(int u = 0, int v = 0) : u(u), v(v) {}
  int to(int node) { return u ^ v ^ node; }
};
struct graph {
  int n;
  vector<vector<int>> adj;
  vector<edge> edges;
  graph(int n = 0) : n(n), adj(n) {}
  void addEdge(int u, int v, bool dir = 1) {
    adj[u].push_back(edges.size());
    if (dir) adj[v].push_back(edges.size());
    edges.emplace_back(u, v);
  }
  int addNode() {
    adj.emplace_back();
    return n++;
  }
  edge &operator()(int idx) { return edges[idx]; }
  vector<int> &operator[](int u) { return adj[u]; }
};
```

## 5.2 LCA, ETT, VT

```
struct lca_table {
  tree &T;
  int n, LOG = 20;
  vector<vector<int>> anc;
  vector<int> level;

  void setupLifting(int node, int par) {
    for (int v : T[node])
      if (v != par) {
        anc[v][0] = node, level[v] = level[node] + 1;
        for (int k = 1; k < LOG; k++) anc[v][k] = anc[anc
            [v][k - 1]][k - 1];
        setupLifting(v, node);
      }
  }
  lca_table(tree &T, int root = 0) : T(T), n(T.n) {
    LOG = 33 - __builtin_clz(n);
    anc.assign(n, vector<int>(LOG, root));
    level.resize(n);
    setupLifting(root, root);
  }
  int lca(int u, int v) {
    if (level[u] > level[v]) swap(u, v);
    for (int k = LOG - 1; ~k; k--)
      if (level[u] + (1 << k) <= level[v]) v = anc[v][k];
    if (u == v) return u;
    for (int k = LOG - 1; ~k; k--)
      if (anc[u][k] != anc[v][k]) u = anc[u][k], v = anc[
          v][k];
    return anc[u][0];
  }
  int getAncestor(int node, int ht) {
    for (int k = 0; k < LOG; k++)
      if (ht & (1 << k)) node = anc[node][k];
    return node;
  }
  int distance(int u, int v) {
    int g = lca(u, v);
    return level[u] + level[v] - 2 * level[g];
  }
};
struct euler_tour {
  int time = 0;
  tree &T;
  int n;
  vector<int> start, finish, level, par;
  euler_tour(tree &T, int root = 0)
      : T(T), n(T.n), start(n), finish(n), level(n), par(
          n) {
    time = 0;
    call(root);
  }
  void call(int node, int p = -1) {
    if (p != -1) level[node] = level[p] + 1;
    start[node] = time++;
    for (int e : T[node])
      if (e != p) call(e, node);
    par[node] = p;
    finish[node] = time++;
  }
  bool isAncestor(int node, int par) {
    return start[par] <= start[node] and finish[par] >=
        finish[node];
  }
  int subtreeSize(int node) { return finish[node] - start
      [node] + 1 >> 1; }
};
tree virtual_tree(vector<int> &nodes, lca_table &table,
    euler_tour &tour) {
  sort(nodes.begin(), nodes.end(),
       [&](int x, int y) { return tour.start[x] < tour.
           start[y]; });
  int n = nodes.size();
  for (int i = 0; i + 1 < n; i++)
    nodes.push_back(table.lca(nodes[i], nodes[i + 1]));
  sort(nodes.begin(), nodes.end());
  nodes.erase(unique(nodes.begin(), nodes.end()), nodes.
      end());
  sort(nodes.begin(), nodes.end(),
       [&](int x, int y) { return tour.start[x] < tour.
           start[y]; });
  n = nodes.size();
  stack<int> st;
  st.push(0);
  tree ans(n);
  for (int i = 1; i < n; i++) {
    while (!tour.isAncestor(nodes[i], nodes[st.top()]))
        st.pop();
    ans.addEdge(st.top(), i);
    st.push(i);
  }
  return ans;
}
set<int> getCenters(tree &T) {
  int n = T.n;
  vector<int> deg(n), q;
  set<int> s;
  for (int i = 0; i < n; i++) {
    deg[i] = T[i].size();
    if (deg[i] == 1) q.push_back(i);
    s.insert(i);
  }
  for (vector<int> t; s.size() > 2; q = t) {
    for (auto x : q) {
      for (auto e : T[x])
        if (--deg[e] == 1) t.push_back(e);
      s.erase(x);
    }
  }
  return s;
}
```

## 5.3 SCC

```
typedef long long LL;
const LL N = 1e6 + 7;

bool vis[N];
vector<int> adj[N], adjr[N];
vector<int> order, component;
// tp = 0 ,finding topo order, tp = 1 , reverse edge
    traversal

void dfs(int u, int tp = 0) {
  vis[u] = true;
  if (tp) component.push_back(u);
  auto& ad = (tp ? adjr : adj);
  for (int v : ad[u])
    if (!vis[v]) dfs(v, tp);
  if (!tp) order.push_back(u);
}
int main() {
  for (int i = 1; i <= n; i++) {
    if (!vis[i]) dfs(i);
  }
  memset(vis, 0, sizeof vis);
  reverse(order.begin(), order.end());
  for (int i : order) {
    if (!vis[i]) {
      // one component is found
      dfs(i, 1), component.clear();
    }
  }
}
```

## 5.4 Euler Tour on Edge

```
// for simplicity, G[idx] contains the adjacency list of
    a node
// while G(e) is a reference to the e-th edge.
const int N = 2e5 + 5;
int in[N], out[N], fwd[N], bck[N];
int t = 0;
void dfs(graph &G, int node, int par) {
  out[node] = t;
  for (int e : G[node]) {
    int v = G(e).to(node);
    if (v == par) continue;
    fwd[e] = t++;
    dfs(G, v, node);
    bck[e] = t++;
  }
  in[node] = t - 1;
}
void init(graph &G, int node) {
  t = 0;
  dfs(G, node, node);
}
```

## 5.5 LCA In O(1)

```
/* LCA in O(1)
 * depth calculates weighted distance
 * level calculates distance by number of edges
 * Preprocessing in NlongN */
LL depth[N];
int level[N];

int st[N], en[N], LOG[N], par[N];
int a[N], id[N], table[L][N];

vector<PII> adj[N];
int n, root, Time, cur;

void init(int nodes, int root_) {
  n = nodes, root = root_, LOG[0] = LOG[1] = 0;
  for (int i = 2; i <= n; i++) LOG[i] = LOG[i >> 1] + 1;
  for (int i = 0; i <= n; i++) adj[i].clear();
}

void addEdge(int u, int v, int w) {
  adj[u].push_back(PII(v, w));
  adj[v].push_back(PII(u, w));
}

int lca(int u, int v) {
  if (en[u] > en[v]) swap(u, v);
  if (st[v] <= st[u] && en[u] <= en[v]) return v;

  int l = LOG[id[v] - id[u] + 1];
  int p1 = id[u], p2 = id[v] - (1 << l) + 1;
  int d1 = level[table[l][p1]], d2 = level[table[l][p2]];

  if (d1 < d2)
    return par[table[l][p1]];
  else
    return par[table[l][p2]];
}

LL dist(int u, int v) {
  int l = lca(u, v);
  return (depth[u] + depth[v] - (depth[l] * 2));
}

/* Euler tour */
void dfs(int u, int p) {
  st[u] = ++Time, par[u] = p;

  for (auto [v, w] : adj[u]) {
    if (v == p) continue;
    depth[v] = depth[u] + w;
    level[v] = level[u] + 1;
    dfs(v, u);
  }

  en[u] = ++Time;
  a[++cur] = u, id[u] = cur;
}

/* RMQ */
void pre() {
  cur = Time = 0, dfs(root, root);
  for (int i = 1; i <= n; i++) table[0][i] = a[i];

  for (int l = 0; l < L - 1; l++) {
    for (int i = 1; i <= n; i++) {
      table[l + 1][i] = table[l][i];

      bool C1 = (1 << l) + i <= n;
      bool C2 = level[table[l][i + (1 << l)]] < level[
          table[l][i]];

      if (C1 && C2) table[l + 1][i] = table[l][i + (1 <<
          l)];
    }
  }
}
```

## 5.6 HLD

```
const int N = 1e6 + 7;
template <typename DT>
struct Segtree {
  // write lazy segtree here
};
Segtree<int> tree(N);
vector<int> adj[N];
int depth[N], par[N], pos[N];
int head[N], heavy[N], cnt;

int dfs(int u, int p) {
  int SZ = 1, mxsz = 0, heavyc;
  depth[u] = depth[p] + 1;

  for (auto v : adj[u]) {
    if (v == p) continue;
    par[v] = u;
    int subsz = dfs(v, u);
    if (subsz > mxsz) heavy[u] = v, mxsz = subsz;
    SZ += subsz;
  }
  return SZ;
}
void decompose(int u, int h) {
  head[u] = h, pos[u] = ++cnt;
  if (heavy[u] != -1) decompose(heavy[u], h);

  for (int v : adj[u]) {
    if (v == par[u]) continue;
    if (v != heavy[u]) decompose(v, v);
  }
}
int query(int a, int b) {
  int ret = 0;
  for (; head[a] != head[b]; b = par[head[b]]) {
    if (depth[head[a]] > depth[head[b]]) swap(a, b);
    ret += tree.query(1, 0, cnt, pos[head[b]], pos[b]);
  }

  if (depth[a] > depth[b]) swap(a, b);
  ret += tree.query(1, 0, cnt, pos[a], pos[b]);
  return ret;
}
```

## 5.7 Centroid Decomposition

```
class Centroid_Decomposition {
  vector<bool> blocked;
  vector<int> CompSize;
  int CompDFS(tree &T, int node, int par) {
    CompSize[node] = 1;
    for (int &e : T[node])
      if (e != par and !blocked[e]) CompSize[node] +=
          CompDFS(T, e, node);
    return CompSize[node];
  }
  int FindCentroid(tree &T, int node, int par, int sz) {
    for (int &e : T[node])
      if (e != par and !blocked[e])
        if (CompSize[e] > sz / 2) return FindCentroid(T,
            e, node, sz);
    return node;
  }
  pair<int, int> GetCentroid(tree &T, int entry) {
    int sz = CompDFS(T, entry, entry);
    return {FindCentroid(T, entry, entry, sz), sz};
  }
  c_vector<LL> left[2], right[2];
  int GMin, GMax;
  void dfs(tree &T, int node, int par, int Min, int Max,
      int sum) {
    if (blocked[node]) return;
    right[Max < sum or Min > sum][sum]++;
    Max = max(Max, sum), Min = min(Min, sum);
    GMin = min(GMin, sum), GMax = max(GMax, sum);
    for (int i = 0; i < T[node].size(); i++)
      if (T[node][i] != par) {
        dfs(T, T[node][i], node, Min, Max, sum + T.col[
            node][i]);
      }
  }
  LL solve(tree &T, int c, int sz) {
    LL ans = 0;
    left[0].clear(-sz, sz), left[1].clear(-sz, sz);
    for (int i = 0; i < T[c].size(); i++) {
      GMin = 1, GMax = -1;
      dfs(T, T[c][i], c, GMin, GMax, T.col[c][i]);
      ans += right[0][0] + left[1][0] * right[1][0];
      for (int j : {0, 1})
        for (int k = GMin; k <= GMax; k++) {
          ans += right[j][k] * (left[0][-k] + (j == 0) *
              left[1][-k]);
        }
      for (int j : {0, 1})
        for (int k = GMin; k <= GMax; k++) {
          left[j][k] += right[j][k];
          right[j][k] = 0;
        }
    }
    return ans;
  }

 public:
  LL operator()(tree &T, int entry) {
    blocked.resize(T.n);
    CompSize.resize(T.n);
    for (int i : {0, 1})
      left[i].resize(2 * T.n + 5), right[i].resize(2 * T.
          n + 5);
    auto [c, sz] = GetCentroid(T, entry);
    LL ans = solve(T, c, sz);
    blocked[c] = true;
    for (int e : T[c])
      if (!blocked[e]) ans += (*this)(T, e);
    return ans;
  }
};
```

## 5.8 Dinic Max Flow

```
/// flow with demand(lower bound) only for DAG
// create new src and sink
// add_edge(new src, u, sum(in_demand[u]))
// add_edge(u, new sink, sum(out_demand[u]))
// add_edge(old sink, old src, inf)
//  if (sum of lower bound == flow) then demand satisfied
// flow in every edge i = demand[i] + e.flow

using Ti = long long;
const Ti INF = 1LL << 60;
struct edge {
  int v, u;
  Ti cap, flow = 0;
  edge(int v, int u, Ti cap) : v(v), u(u), cap(cap) {}
};
const int N = 1e5 + 50;
vector<edge> edges;
vector<int> adj[N];
int m = 0, n;
int level[N], ptr[N];
queue<int> q;
bool bfs(int s, int t) {
  for (q.push(s), level[s] = 0; !q.empty(); q.pop()) {
    for (int id : adj[q.front()]) {
      auto &ed = edges[id];
      if (ed.cap - ed.flow > 0 and level[ed.u] == -1)
        level[ed.u] = level[ed.v] + 1, q.push(ed.u);
    }
  }
  return level[t] != -1;
}
Ti dfs(int v, Ti pushed, int t) {
  if (pushed == 0) return 0;
  if (v == t) return pushed;
  for (int &cid = ptr[v]; cid < adj[v].size(); cid++) {
    int id = adj[v][cid];
    auto &ed = edges[id];
    if (level[v] + 1 != level[ed.u] || ed.cap - ed.flow <
        1) continue;
    Ti tr = dfs(ed.u, min(pushed, ed.cap - ed.flow), t);
    if (tr == 0) continue;
    ed.flow += tr;
    edges[id ^ 1].flow -= tr;
    return tr;
  }
  return 0;
}
void init(int nodes) {
  m = 0, n = nodes;
  for (int i = 0; i < n; i++) level[i] = -1, ptr[i] = 0,
      adj[i].clear();
}
void addEdge(int v, int u, Ti cap) {
  edges.emplace_back(v, u, cap), adj[v].push_back(m++);
  edges.emplace_back(u, v, 0), adj[u].push_back(m++);
}
Ti maxFlow(int s, int t) {
  Ti f = 0;
  for (auto &ed : edges) ed.flow = 0;
  for (; bfs(s, t); memset(level, -1, n * 4)) {
    for (memset(ptr, 0, n * 4); Ti pushed = dfs(s, INF, t
        ); f += pushed)
      ;
  }
  return f;
}
```

## 5.9 Min Cost Max Flow

```
mt19937 rnd(chrono::steady_clock::now().time_since_epoch
    ().count());
const LL inf = 1e9;
struct edge {
  int v, rev;
  LL cap, cost, flow;
  edge() {}
  edge(int v, int rev, LL cap, LL cost)
      : v(v), rev(rev), cap(cap), cost(cost), flow(0) {}
};
struct mcmf {
  int src, sink, n;
  vector<int> par, idx, Q;
  vector<bool> inq;
  vector<LL> dis;
  vector<vector<edge>> g;
  mcmf() {}
  mcmf(int src, int sink, int n)
      : src(src),
        sink(sink),
        n(n),
        par(n),
        idx(n),
        inq(n),
        dis(n),
        g(n),
        Q(10000005) {} // use Q(n) if not using random
  void add_edge(int u, int v, LL cap, LL cost, bool
      directed = true) {
    edge _u = edge(v, g[v].size(), cap, cost);
    edge _v = edge(u, g[u].size(), 0, -cost);
    g[u].pb(_u);
    g[v].pb(_v);
    if (!directed) add_edge(v, u, cap, cost, true);
  }
  bool spfa() {
    for (int i = 0; i < n; i++) {
      dis[i] = inf, inq[i] = false;
    }
    int f = 0, l = 0;
    dis[src] = 0, par[src] = -1, Q[l++] = src, inq[src] =
        true;
    while (f < l) {
      int u = Q[f++];
      for (int i = 0; i < g[u].size(); i++) {
        edge &e = g[u][i];
        if (e.cap <= e.flow) continue;
        if (dis[e.v] > dis[u] + e.cost) {
          dis[e.v] = dis[u] + e.cost;
          par[e.v] = u, idx[e.v] = i;
          if (!inq[e.v]) inq[e.v] = true, Q[l++] = e.v;
          // if (!inq[e.v]) {
          //   inq[e.v] = true;
          //   if (f && rnd() & 7) Q[--f] = e.v;
          //   else Q[l++] = e.v;
          // }
        }
      }
      inq[u] = false;
    }
    return (dis[sink] != inf);
  }
  pair<LL, LL> solve() {
    LL mincost = 0, maxflow = 0;
    while (spfa()) {
      LL bottleneck = inf;
      for (int u = par[sink], v = idx[sink]; u != -1; v =
          idx[u], u = par[u]) {
        edge &e = g[u][v];
        bottleneck = min(bottleneck, e.cap - e.flow);
      }
      for (int u = par[sink], v = idx[sink]; u != -1; v =
          idx[u], u = par[u]) {
        edge &e = g[u][v];
        e.flow += bottleneck;
        g[e.v][e.rev].flow -= bottleneck;
      }
      mincost += bottleneck * dis[sink], maxflow +=
          bottleneck;
    }
    return make_pair(mincost, maxflow);
  }
};
// want to minimize cost and don't care about flow
// add edge from sink to dummy sink (cap = inf, cost = 0)
// add edge from source to sink (cap = inf, cost = 0)
// run mcmf, cost returned is the minimum cost
```

## 5.10 Block Cut Tree

```
vector<vector<int> > components;
vector<int> cutpoints, start, low;
vector<bool> is_cutpoint;
stack<int> st;
void find_cutpoints(int node, graph &G, int par = -1, int
    d = 0) {
  low[node] = start[node] = d++;
  st.push(node);
  int cnt = 0;
  for (int e : G[node])
    if (int to = G(e).to(node); to != par) {
      if (start[to] == -1) {
        find_cutpoints(to, G, node, d + 1);
        cnt++;
        if (low[to] >= start[node]) {
          is_cutpoint[node] = par != -1 or cnt > 1;
          components.push_back({node}); // starting a new
              block with the point
          while (st.top() != node)
            components.back().push_back(st.top()), st.pop
                ();
        }
      }
      low[node] = min(low[node], low[to]);
    }
}
graph tree;
vector<int> id;
void init(graph &G) {
  int n = G.n;
  start.assign(n, -1), low.resize(n), is_cutpoint.resize(
      n), id.assign(n, -1);
  find_cutpoints(0, G);
  for (int u = 0; u < n; ++u)
    if (is_cutpoint[u]) id[u] = tree.addNode();
  for (auto &comp : components) {
    int node = tree.addNode();
    for (int u : comp)
      if (!is_cutpoint[u])
        id[u] = node;
      else
        tree.addEdge(node, id[u]);
  }
  if (id[0] == -1) // corner - 1
    id[0] = tree.addNode();
}
```

## 5.11 Bridge Tree

```
vector<vector<int>> components;
vector<int> depth, low;
stack<int> st;
vector<int> id;
vector<edge> bridges;
graph tree;
void find_bridges(int node, graph &G, int par = -1, int d
    = 0) {
  low[node] = depth[node] = d;
  st.push(node);
  for (int id : G[node]) {
    int to = G(id).to(node);
    if (par != to) {
      if (depth[to] == -1) {
       find_bridges(to, G, node, d + 1);
       if (low[to] > depth[node]) {
         bridges.emplace_back(node, to);
         components.push_back({});
         for (int x = -1; x != to; x = st.top(), st.pop()
             )
           components.back().push_back(st.top());
       }
      }
      low[node] = min(low[node], low[to]);
    }
  }
  if (par == -1) {
    components.push_back({});
    while (!st.empty()) components.back().push_back(st.
        top()), st.pop();
  }
}
graph &create_tree() {
  for (auto &comp : components) {
    int idx = tree.addNode();
    for (auto &e : comp) id[e] = idx;
  }
  for (auto &[l, r] : bridges) tree.addEdge(id[l], id[r])
      ;
  return tree;
}
void init(graph &G) {
  int n = G.n;
  depth.assign(n, -1), id.assign(n, -1), low.resize(n);
  for (int i = 0; i < n; i++)
    if (depth[i] == -1) find_bridges(i, G);
}
```

### 5.12 Tree Isomorphism

```
mp["01"] = 1;
ind = 1;
int dfs(int u, int p) {
  int cnt = 0;
  vector<int> vs;
  for (auto v : g1[u]) {
    if (v != p) {
      int got = dfs(v, u);
      vs.pb(got);
      cnt++;
    }
  }
  if (!cnt) return 1;

  sort(vs.begin(), vs.end());
  string s = "0";
  for (auto i : vs) s += to_string(i);
  vs.clear();
  s.pb('1');
  if (mp.find(s) == mp.end()) mp[s] = ++ind;
  int ret = mp[s];
  return ret;
}
```

# 6 Math

## 6.1 Adaptive Simpsons

```
/*
    For finding the length of an arc in a range
    L = integrate(ds) from start to end of range
    where ds = sqrt(1+(d/dy(x))^2)dy
*/
const double SIMPSON_TERMINAL_EPS = 1e-12;
/// Function whose integration is to be calculated
double F(double x);
double simpson(double minx, double maxx) {
  return (maxx - minx) / 6 * (F(minx) + 4 * F((minx +
      maxx) / 2.) + F(maxx));
}
double adaptive_simpson(double minx, double maxx, double
    c, double EPS) {
  //   if(maxx - minx < SIMPSON_TERMINAL_EPS) return 0;

  double midx = (minx + maxx) / 2;
  double a = simpson(minx, midx);
  double b = simpson(midx, maxx);

  if (fabs(a + b - c) < 15 * EPS) return a + b + (a + b -
      c) / 15.0;

  return adaptive_simpson(minx, midx, a, EPS / 2.) +
       adaptive_simpson(midx, maxx, b, EPS / 2.);
}
double adaptive_simpson(double minx, double maxx, double
    EPS) {
  return adaptive_simpson(minx, maxx, simpson(minx, maxx,
      i), EPS);
}
```

## 6.2 Berlekamp Massey

```
struct berlekamp_massey { // for linear recursion
  typedef long long LL;
  static const int SZ = 2e5 + 5;
  static const int MOD = 1e9 + 7; /// mod must be a prime
  LL m , a[SZ] , h[SZ] , t_[SZ] , s[SZ] , t[SZ];
  // bigmod goes here
  inline vector <LL> BM( vector <LL> &x ) {
    LL lf , ld;
    vector <LL> ls , cur;
    for ( int i = 0; i < int(x.size()); ++i ) {
      LL t = 0;
      for ( int j = 0; j < int(cur.size()); ++j ) t = (t
          + x[i - j - 1] * cur[j]) % MOD;
      if ( (t - x[i]) % MOD == 0 ) continue;
      if ( !cur.size() ) {
        cur.resize( i + 1 );
        lf = i; ld = (t - x[i]) % MOD;
        continue;
      }
      LL k = -(x[i] - t) * bigmod( ld , MOD - 2 , MOD ) %
          MOD;
      vector <LL> c(i - lf - 1);
      c.push_back( k );
      for ( int j = 0; j < int(ls.size()); ++j ) c.
          push_back(-ls[j] * k % MOD);
      if ( c.size() < cur.size() ) c.resize( cur.size() )
          ;
      for ( int j = 0; j < int(cur.size()); ++j ) c[j] =
          (c[j] + cur[j]) % MOD;
      if (i - lf + (int)ls.size() >= (int)cur.size() ) ls
          = cur, lf = i, ld = (t - x[i]) % MOD;
      cur = c;
    }
    for ( int i = 0; i < int(cur.size()); ++i ) cur[i] =
        (cur[i] % MOD + MOD) % MOD;
    return cur;
  }
  inline void mull( LL *p , LL *q ) {
    for ( int i = 0; i < m + m; ++i ) t_[i] = 0;
    for ( int i = 0; i < m; ++i ) if ( p[i] )
      for ( int j = 0; j < m; ++j ) t_[i + j] = (t_[i +
          j] + p[i] * q[j]) % MOD;
    for ( int i = m + m - 1; i >= m; --i ) if ( t_[i] )
      for ( int j = m - 1; ~j; --j ) t_[i - j - 1] = (
          t_[i - j - 1] + t_[i] * h[j]) % MOD;
    for ( int i = 0; i < m; ++i ) p[i] = t_[i];
  }
  inline LL calc( LL K ) {
    for ( int i = m; ~i; --i ) s[i] = t[i] = 0;
    s[0] = 1; if ( m != 1 ) t[1] = 1; else t[0] = h[0];
    while ( K ) {
      if ( K & 1 ) mull( s , t );
      mull( t , t ); K >>= 1;
    }
    LL su = 0;
    for ( int i = 0; i < m; ++i ) su = (su + s[i] * a[i]) % MOD;
    return (su % MOD + MOD) % MOD;
  }
  /// already calculated upto k , now calculate upto n.
  inline vector <LL> process( vector <LL> &x , int n , int k ) {
    auto re = BM( x );
    x.resize( n + 1 );
    for ( int i = k + 1; i <= n; i++ ) {
      for ( int j = 0; j < re.size(); j++ ) {
        x[i] += 1LL * x[i - j - 1] % MOD * re[j] % MOD; x[i] %= MOD;
      }
    }
    return x;
  }
  inline LL work( vector <LL> &x , LL n ) {
    if ( n < int(x.size()) ) return x[n] % MOD;
    vector <LL> v = BM( x ); m = v.size(); if ( !m ) return 0;
    for ( int i = 0; i < m; ++i ) h[i] = v[i], a[i] = x[i];
    return calc( n ) % MOD;
  }
} rec;
vector <LL> v;
void solve() {
  int n;
  cin >> n;
  cout << rec.work(v, n - 1) << endl;
}
```

## 6.3 Fractional Binary Search

```
/**
Given a function f and n, finds the smallest fraction p / q in [0, 1] or [0,n]
such that f(p / q) is true, and p, q <= n.
Time: O(log(n))
**/
struct frac { long long p, q; };
bool f(frac x) {
 return 6 + 8 * x.p >= 17 * x.q + 12;
}
frac fracBS(long long n) {
  bool dir = 1, A = 1, B = 1;
  frac lo{0, 1}, hi{1, 0}; // Set hi to 1/0 to search within [0, n] and {1, 1} to search within [0, 1]
  if (f(lo)) return lo;
  assert(f(hi)); //checking if any solution exists or not
  while (A || B) {
    long long adv = 0, step = 1; // move hi if dir, else lo
    for (int si = 0; step; (step *= 2) >>= si) {
      adv += step;
      frac mid{lo.p * adv + hi.p, lo.q * adv + hi.q};
      if (abs(mid.p) > n || mid.q > n || dir == !f(mid)) {
        adv -= step; si = 2;
      }
    }
    hi.p += lo.p * adv;
    hi.q += lo.q * adv;
    dir = !dir;
    swap(lo, hi);
    A = B; B = !!adv;
  }
  return dir ? hi : lo;
}
```

## 6.4 Lagrange

```
// p is a polynomial with n points.
// p(0), p(1), p(2), ... p(n-1) are given.
// Find p(x).
LL Lagrange(vector<LL> &p, LL x) {
  LL n = p.size(), L, i, ret;
  if (x < n) return p[x];
  L = 1;
  for (i = 1; i < n; i++) {
    L = (L * (x - i)) % MOD;
    L = (L * bigmod(MOD - i, MOD - 2)) % MOD;
  }
  ret = (L * p[0]) % MOD;
  for (i = 1; i < n; i++) {
    L = (L * (x - i + 1)) % MOD;
    L = (L * bigmod(x - i, MOD - 2)) % MOD;
    L = (L * bigmod(i, MOD - 2)) % MOD;
    L = (L * (MOD + i - n)) % MOD;
    ret = (ret + L * p[i]) % MOD;
  }
  return ret;
}
```

## 6.5 Pollard Rho

```
LL mul(LL a, LL b, LL mod) {
  return (__int128)a * b % mod;
  // LL ans = a * b - mod * (LL) (1.L / mod * a * b);
  // return ans + mod * (ans < 0) - mod * (ans >= (LL) mod);
}
LL bigmod(LL num, LL pow, LL mod) {
  LL ans = 1;
  for (; pow > 0; pow >>= 1, num = mul(num, num, mod))
    if (pow & 1) ans = mul(ans, num, mod);
  return ans;
}
bool is_prime(LL n) {
  if (n < 2 or n % 6 % 4 != 1) return (n | 1) == 3;
  LL a[] = {2, 325, 9375, 28178, 450775, 9780504, 1795265022};
  LL s = __builtin_ctzll(n - 1), d = n >> s;
  for (LL x : a) {
    LL p = bigmod(x % n, d, n), i = s;
    for (; p != 1 and p != n - 1 and x % n and i--; p = mul(p, p, n))
      ;
    if (p != n - 1 and i != s) return false;
  }
  return true;
}
LL get_factor(LL n) {
  auto f = [&](LL x) { return mul(x, x, n) + 1; };
  LL x = 0, y = 0, t = 0, prod = 2, i = 2, q;
  for (; t++ % 40 or gcd(prod, n) == 1; x = f(x), y = f(f(y))) {
    (x == y) ? x = i++, y = f(x) : 0;
    prod = (q = mul(prod, max(x, y) - min(x, y), n)) ? q : prod;
  }
  return gcd(prod, n);
}
map<LL, int> factorize(LL n) {
  map<LL, int> res;
  if (n < 2) return res;
  LL small_primes[] = {2, 3, 5, 7, 11, 13, 17, 19, 23, 29, 31, 37, 41,
                       43, 47, 53, 59, 61, 67, 71, 73, 79, 83, 89, 97};
  for (LL p : small_primes)
    for (; n % p == 0; n /= p, res[p]++)
      ;

  auto _factor = [&](LL n, auto &_factor) {
    if (n == 1) return;
    if (is_prime(n))
      res[n]++;
    else {
      LL x = get_factor(n);
      _factor(x, _factor);
      _factor(n / x, _factor);
    }
  };
  _factor(n, _factor);
  return res;
}
```

## 6.6 Shanks' Baby Step, Giant Step

```
// Finds a^x = b (mod p)

LL bigmod(LL b, LL p, LL m) {}

LL babyStepGiantStep(LL a, LL b, LL p) {
  LL i, j, c, sq = sqrt(p);
  map<LL, LL> babyTable;

  for (j = 0, c = 1; j <= sq; j++, c = (c * a) % p)
      babyTable[c] = j;

  LL giant = bigmod(a, sq * (p - 2), p);

  for (i = 0, c = 1; i <= sq; i++, c = (c * giant) % p) {
    if (babyTable.find((c * b) % p) != babyTable.end())
      return i * sq + babyTable[(c * b) % p];
  }

  return -1;
}
```

## 6.7 Xor Basis

```
struct XorBasis {
  static const int sz = 64;
  array<ULL, sz> base = {0}, back;
  array<int, sz> pos;
  void insert(ULL x, int p) {
    ULL cur = 0;
    for (int i = sz - 1; ~i; i--)
      if (x >> i & 1) {
        if (!base[i]) {
          base[i] = x, back[i] = cur, pos[i] = p;
          break;
        } else x ^= base[i], cur |= 1ULL << i;
      }
  }
  pair<ULL, vector<int>> construct(ULL mask) {
    ULL ok = 0, x = mask;
    for (int i = sz - 1; ~i; i--)
      if (mask >> i & 1 and base[i]) mask ^= base[i], ok
           |= 1ULL << i;
    vector<int> ans;
    for (int i = 0; i < sz; i++)
      if (ok >> i & 1) {
        ans.push_back(pos[i]);
        ok ^= back[i];
      }
    return {x ^ mask, ans};
  }
};
```

## 6.8 Combi

```
array<int, N + 1> fact, inv, inv_fact;
void init() {
  fact[0] = inv_fact[0] = 1;
  for (int i = 1; i <= N; i++) {
    inv[i] = i == 1 ? 1 : (LL)inv[i - mod % i] * (mod / i
          + 1) % mod;
    fact[i] = (LL)fact[i - 1] * i % mod;
    inv_fact[i] = (LL)inv_fact[i - 1] * inv[i] % mod;
  }
}
LL C(int n, int r) {
  return (r < 0 or r > n) ? 0 : (LL)fact[n] * inv_fact[r]
        % mod * inv_fact[n - r] % mod;
}
```

## 6.9 Linear Sieve

```
const int N = 1e7;
vector<int> primes;
int spf[N + 5], phi[N + 5], NOD[N + 5], cnt[N + 5], POW[N
      + 5];
bool prime[N + 5];
int SOD[N + 5];
void init() {
  fill(prime + 2, prime + N + 1, 1);
  SOD[1] = NOD[1] = phi[1] = spf[1] = 1;
  for (LL i = 2; i <= N; i++) {
    if (prime[i]) {
      primes.push_back(i), spf[i] = i;
      phi[i] = i - 1;
      NOD[i] = 2, cnt[i] = 1;
      SOD[i] = i + 1, POW[i] = i;
    }
    for (auto p : primes) {
      if (p * i > N or p > spf[i]) break;
      prime[p * i] = false, spf[p * i] = p;
      if (i % p == 0) {
        phi[p * i] = p * phi[i];
        NOD[p * i] = NOD[i] / (cnt[i] + 1) * (cnt[i] + 2)
             ,
                cnt[p * i] = cnt[i] + 1;
        SOD[p * i] = SOD[i] / SOD[POW[i]] * (SOD[POW[i]]
             + p * POW[i]),
                POW[p * i] = p * POW[i];
        break;
      } else {
        phi[p * i] = phi[p] * phi[i];
        NOD[p * i] = NOD[p] * NOD[i], cnt[p * i] = 1;
        SOD[p * i] = SOD[p] * SOD[i], POW[p * i] = p;
      }
    }
  }
}
```

## 6.10 Chinese Remainder Theorem

```
// given a, b will find solutions for
// ax + by = 1
tuple<LL, LL, LL> EGCD(LL a, LL b) {
  if (b == 0)
    return {1, 0, a};
  else {
    auto [x, y, g] = EGCD(b, a % b);
    return {y, x - a / b * y, g};
  }
}
// given modulo equations, will apply CRT
PLL CRT(vector<PLL> &v) {
  LL V = 0, M = 1;
  for (auto &[v, m] : v) { // value % mod
    auto [x, y, g] = EGCD(M, m);
    if ((v - V) % g != 0) return {-1, 0};
    V += x * (v - V) / g % (m / g) * M, M *= m / g;
    V = (V % M + M) % M;
  }
  return make_pair(V, M);
}
```

## 6.11 Mobius Function

```
const int N = 1e6 + 5;
int mob[N];
void mobius() {
  memset(mob, -1, sizeof mob);
  mob[1] = 1;
  for (int i = 2; i < N; i++)
    if (mob[i]) {
      for (int j = i + i; j < N; j += i) mob[j] -= mob[i
           ];
    }
}
```

### 6.12 FFT

```
using CD = complex<double>;
typedef long long LL;
const double PI = acos(-1.0L);

int N;
vector<int> perm;
vector<CD> wp[2];
void precalculate(int n) {
  assert((n & (n - 1)) == 0), N = n;
  perm = vector<int>(N, 0);
  for (int k = 1; k < N; k <<= 1) {
    for (int i = 0; i < k; i++) {
      perm[i] <<= 1;
      perm[i + k] = 1 + perm[i];
    }
  }
  wp[0] = wp[1] = vector<CD>(N);
  for (int i = 0; i < N; i++) {
    wp[0][i] = CD(cos(2 * PI * i / N), sin(2 * PI * i / N
        ));
    wp[1][i] = CD(cos(2 * PI * i / N), -sin(2 * PI * i /
        N));
  }
}
void fft(vector<CD> &v, bool invert = false) {
  if (v.size() != perm.size()) precalculate(v.size());
  for (int i = 0; i < N; i++)
    if (i < perm[i]) swap(v[i], v[perm[i]]);
  for (int len = 2; len <= N; len *= 2) {
    for (int i = 0, d = N / len; i < N; i += len) {
      for (int j = 0, idx = 0; j < len / 2; j++, idx += d
          ) {
        CD x = v[i + j];
        CD y = wp[invert][idx] * v[i + j + len / 2];
        v[i + j] = x + y;
        v[i + j + len / 2] = x - y;
      }
    }
  }
  if (invert) {
    for (int i = 0; i < N; i++) v[i] /= N;
  }
}
void pairfft(vector<CD> &a, vector<CD> &b, bool invert =
    false) {
  int N = a.size();
  vector<CD> p(N);
  for (int i = 0; i < N; i++) p[i] = a[i] + b[i] * CD(0,
      1);
  fft(p, invert);
  p.push_back(p[0]);
  for (int i = 0; i < N; i++) {
    if (invert) {
      a[i] = CD(p[i].real(), 0);
      b[i] = CD(p[i].imag(), 0);
    } else {
      a[i] = (p[i] + conj(p[N - i])) * CD(0.5, 0);
      b[i] = (p[i] - conj(p[N - i])) * CD(0, -0.5);
    }
  }
}
vector<LL> multiply(const vector<LL> &a, const vector<LL>
    &b) {
  int n = 1;
  while (n < a.size() + b.size()) n <<= 1;
  vector<CD> fa(a.begin(), a.end()), fb(b.begin(), b.end
      ());
  fa.resize(n);
  fb.resize(n);
  //       fft(fa); fft(fb);
  pairfft(fa, fb);
  for (int i = 0; i < n; i++) fa[i] = fa[i] * fb[i];
  fft(fa, true);
  vector<LL> ans(n);
  for (int i = 0; i < n; i++) ans[i] = round(fa[i].real()
      );
  return ans;
}
const int M = 1e9 + 7, B = sqrt(M) + 1;
vector<LL> anyMod(const vector<LL> &a, const vector<LL> &
    b) {
  int n = 1;
  while (n < a.size() + b.size()) n <<= 1;
  vector<CD> al(n), ar(n), bl(n), br(n);
  for (int i = 0; i < a.size(); i++) al[i] = a[i] % M / B
      , ar[i] = a[i] % M % B;
  for (int i = 0; i < b.size(); i++) bl[i] = b[i] % M / B
      , br[i] = b[i] % M % B;
  pairfft(al, ar);
  pairfft(bl, br);
  //       fft(al); fft(ar); fft(bl); fft(br);
  for (int i = 0; i < n; i++) {
    CD ll = (al[i] * bl[i]), lr = (al[i] * br[i]);
    CD rl = (ar[i] * bl[i]), rr = (ar[i] * br[i]);
    al[i] = ll;
    ar[i] = lr;
    bl[i] = rl;
    br[i] = rr;
  }
  pairfft(al, ar, true);
  pairfft(bl, br, true);
  //       fft(al, true); fft(ar, true); fft(bl, true);
      fft(br, true);
  vector<LL> ans(n);
  for (int i = 0; i < n; i++) {
    LL right = round(br[i].real()), left = round(al[i].
        real());
    ;
    LL mid = round(round(bl[i].real()) + round(ar[i].real
        ()));
    ans[i] = ((left % M) * B * B + (mid % M) * B + right)
         % M;
  }
  return ans;
}
```

### 6.13 NTT

```
const LL N = 1 << 18;
const LL MOD = 786433;

vector<LL> P[N];
LL rev[N], w[N | 1], a[N], b[N], inv_n, g;
LL Pow(LL b, LL p) {
  LL ret = 1;
  while (p) {
    if (p & 1) ret = (ret * b) % MOD;
    b = (b * b) % MOD;
    p >>= 1;
  }
  return ret;
}
LL primitive_root(LL p) {
  vector<LL> factor;
  LL phi = p - 1, n = phi;
  for (LL i = 2; i * i <= n; i++) {
    if (n % i) continue;
    factor.emplace_back(i);
    while (n % i == 0) n /= i;
  }
  if (n > 1) factor.emplace_back(n);
  for (LL res = 2; res <= p; res++) {
    bool ok = true;
    for (LL i = 0; i < factor.size() && ok; i++)
      ok &= Pow(res, phi / factor[i]) != 1;
    if (ok) return res;
  }
  return -1;
}
void prepare(LL n) {
  LL sz = abs(31 - __builtin_clz(n));
  LL r = Pow(g, (MOD - 1) / n);
  inv_n = Pow(n, MOD - 2);
  w[0] = w[n] = 1;
  for (LL i = 1; i < n; i++) w[i] = (w[i - 1] * r) % MOD;
  for (LL i = 1; i < n; i++)
    rev[i] = (rev[i >> 1] >> 1) | ((i & 1) << (sz - 1));
}
void NTT(LL *a, LL n, LL dir = 0) {
  for (LL i = 1; i < n - 1; i++)
    if (i < rev[i]) swap(a[i], a[rev[i]]);
  for (LL m = 2; m <= n; m <<= 1) {
    for (LL i = 0; i < n; i += m) {
      for (LL j = 0; j < (m >> 1); j++) {
        LL &u = a[i + j], &v = a[i + j + (m >> 1)];
        LL t = v * w[dir ? n - n / m * j : n / m * j] %
            MOD;
        v = u - t < 0 ? u - t + MOD : u - t;
        u = u + t >= MOD ? u + t - MOD : u + t;
      }
    }
  }
  if (dir)
    for (LL i = 0; i < n; i++) a[i] = (inv_n * a[i]) %
        MOD;
}
vector<LL> mul(vector<LL> p, vector<LL> q) {
  LL n = p.size(), m = q.size();
  LL t = n + m - 1, sz = 1;
  while (sz < t) sz <<= 1;
  prepare(sz);

  for (LL i = 0; i < n; i++) a[i] = p[i];
  for (LL i = 0; i < m; i++) b[i] = q[i];
  for (LL i = n; i < sz; i++) a[i] = 0;
  for (LL i = m; i < sz; i++) b[i] = 0;

  NTT(a, sz);
  NTT(b, sz);
  for (LL i = 0; i < sz; i++) a[i] = (a[i] * b[i]) % MOD;
  NTT(a, sz, 1);

  vector<LL> c(a, a + sz);
  while (c.size() && c.back() == 0) c.pop_back();
  return c;
}
```

### 6.14 WalshHadamard

```
#include <bits/stdc++.h>
using namespace std;
typedef long long LL;
#define bitwiseXOR 1
// #define bitwiseAND 2
// #define bitwiseOR 3
const LL MOD = 30011;

LL BigMod(LL b, LL p) {
  LL ret = 1;
  while (p > 0) {
    if (p % 2 == 1) {
      ret = (ret * b) % MOD;
    }
    p = p / 2;
    b = (b * b) % MOD;
  }
  return ret % MOD;
}

void FWHT(vector<LL>& p, bool inverse) {
  LL n = p.size();
  assert((n & (n - 1)) == 0);

  for (LL len = 1; 2 * len <= n; len <<= 1) {
    for (LL i = 0; i < n; i += len + len) {
      for (LL j = 0; j < len; j++) {
        LL u = p[i + j];
        LL v = p[i + len + j];

#ifdef bitwiseXOR
        p[i + j] = (u + v) % MOD;
        p[i + len + j] = (u - v + MOD) % MOD;
#endif // bitwiseXOR

#ifdef bitwiseAND
        if (!inverse) {
          p[i + j] = v % MOD;
          p[i + len + j] = (u + v) % MOD;
        } else {
          p[i + j] = (-u + v) % MOD;
          p[i + len + j] = u % MOD;
        }
#endif // bitwiseAND

#ifdef bitwiseOR
        if (!inverse) {
          p[i + j] = u + v;
          p[i + len + j] = u;
        } else {
          p[i + j] = v;
          p[i + len + j] = u - v;
        }
#endif // bitwiseOR
      }
    }
  }

#ifdef bitwiseXOR
  if (inverse) {
    LL val = BigMod(n, MOD - 2); // Option 2: Exclude
    for (LL i = 0; i < n; i++) {
      // assert(p[i]%n==0); //Option 2: Include
      p[i] = (p[i] * val) % MOD; // Option 2: p[i]/=n;
    }
  }
#endif // bitwiseXOR
}
```

# 7 String

## 7.1 Aho Corasick

```
const int sg = 26, N = 1e3 + 9;
struct aho_corasick {
  struct node {
    node *link, *out, *par;
    bool leaf;
    LL val;
    int cnt, last, len;
    char p_ch;
    array<node*, sg> to;
    node(node* par = NULL, char p_ch = '$', int len = 0)
        : par(par), p_ch(p_ch), len(len) {
      val = leaf = cnt = last = 0;
      link = out = NULL;
    }
  };
  vector<node> trie;
  node* root;
  aho_corasick() {
    trie.reserve(N), trie.emplace_back();
    root = &trie[0];
    root->link = root->out = root;
  }
  inline int f(char c) { return c - 'a'; }
  inline node* add_node(node* par = NULL, char p_ch = '$'
      , int len = 0) {
    trie.emplace_back(par, p_ch, len);
    return &trie.back();
  }
  void add_str(const string& s, LL val = 1) {
    node* now = root;
    for (char c : s) {
      int i = f(c);
      if (!now->to[i]) now->to[i] = add_node(now, c, now
          ->len + 1);
      now = now->to[i];
    }
    now->leaf = true, now->val++;
  }
  void push_links() {
    queue<node*> q;
    for (q.push(root); q.empty(); q.pop()) {
      node *cur = q.front(), *link = cur->link;
      cur->out = link->leaf ? link : link->out;
      int idx = 0;
      for (auto& next : cur->to) {
        if (next != NULL) {
          next->link = cur != root ? link->to[idx++] :
              root;
          q.push(next);
        } else
          next = link->to[idx++];
      }
    }
    cur->val += link->val;
  }
};
```

## 7.2 Double hash

```
ostream& operator<<(ostream& os, PLL hash) {
  return os << "(" << hash.ff << ", " << hash.ss << ")";
}

PLL operator+(PLL a, LL x) { return PLL(a.ff + x, a.ss +
    x); }
PLL operator-(PLL a, LL x) { return PLL(a.ff - x, a.ss -
    x); }
PLL operator*(PLL a, LL x) { return PLL(a.ff * x, a.ss *
    x); }
PLL operator+(PLL a, PLL x) { return PLL(a.ff + x.ff, a.
    ss + x.ss); }
PLL operator-(PLL a, PLL x) { return PLL(a.ff - x.ff, a.
    ss - x.ss); }
PLL operator*(PLL a, PLL x) { return PLL(a.ff * x.ff, a.
    ss * x.ss); }
PLL operator%(PLL a, PLL m) { return PLL(a.ff % m.ff, a.
    ss % m.ss); }

PLL base(1949313259, 1997293877);
PLL mod(2091573227, 2117566807);

PLL power(PLL a, LL p) {
  if (!p) return PLL(1, 1);
  PLL ans = power(a, p / 2);
  ans = (ans * ans) % mod;
  if (p % 2) ans = (ans * a) % mod;
  return ans;
}

PLL inverse(PLL a) { return power(a, (mod.ff - 1) * (mod.
    ss - 1) - 1); }
PLL inv_base = inverse(base);

PLL val;
vector<PLL> P;

void hash_init(int n) {
  P.resize(n + 1);
  P[0] = PLL(1, 1);
  for (int i = 1; i <= n; i++) P[i] = (P[i - 1] * base) %
      mod;
}

/// appends c to string
PLL append(PLL cur, char c) { return (cur * base + c) %
    mod; }

/// prepends c to string with size k
PLL prepend(PLL cur, int k, char c) { return (P[k] * c +
    cur) % mod; }

/// replaces the i-th (0-indexed) character from right
    from a to b;
PLL replace(PLL cur, int i, char a, char b) {
  cur = (cur + P[i] * (b - a)) % mod;
  return (cur + mod) % mod;
}

/// Erases c from the back of the string
PLL pop_back(PLL hash, char c) {
  return (((hash - c) * inv_base) % mod + mod) % mod;
}

/// Erases c from front of the string with size len
PLL pop_front(PLL hash, int len, char c) {
  return ((hash - P[len - 1] * c) % mod + mod) % mod;
}
/// concatenates two strings where length of the right is
     k
PLL concat(PLL left, PLL right, int k) { return (left * P
    [k] + right) % mod; }
/// Calculates hash of string with size len repeated cnt
    times
/// This is O(log n). For O(1), pre-calculate inverses
PLL repeat(PLL hash, int len, LL cnt) {
  PLL mul = (P[len * cnt] - 1) * inverse(P[len] - 1);
  mul = (mul % mod + mod) % mod;
  PLL ret = (hash * mul) % mod;

  if (P[len].ff == 1) ret.ff = hash.ff * cnt;
  if (P[len].ss == 1) ret.ss = hash.ss * cnt;
  return ret;
}
LL get(PLL hash) { return ((hash.ff << 32) ^ hash.ss); }
struct hashlist {
  int len;
  vector<PLL> H, R;

  hashlist() {}
  hashlist(string& s) {
    len = (int)s.size();
    hash_init(len);
    H.resize(len + 1, PLL(0, 0)), R.resize(len + 2, PLL
        (0, 0));
    for (int i = 1; i <= len; i++) H[i] = append(H[i -
        1], s[i - 1]);
    for (int i = len; i >= 1; i--) R[i] = append(R[i +
        1], s[i - 1]);
  }

  /// 1-indexed
  inline PLL range_hash(int l, int r) {
    int len = r - l + 1;
    return ((H[r] - H[l - 1] * P[len]) % mod + mod) % mod
        ;
  }

  inline PLL reverse_hash(int l, int r) {
    int len = r - l + 1;
    return ((R[l] - R[r + 1] * P[len]) % mod + mod) % mod
        ;
  }

  inline PLL concat_range_hash(int l1, int r1, int l2,
      int r2) {
    int len_2 = r2 - l2 + 1;
    return concat(range_hash(l1, r1), range_hash(l2, r2),
         len_2);
  }
  inline PLL concat_reverse_hash(int l1, int r1, int l2,
      int r2) {
    int len_1 = r1 - l1 + 1;
    return concat(reverse_hash(l2, r2), reverse_hash(l1,
        r1), len_1);
  }
};
```

## 7.3 Manacher's

```
vector<int> d1(n);
// d[i] = number of palindromes taking s[i] as center
for (int i = 0, l = 0, r = -1; i < n; i++) {
  int k = (i > r) ? 1 : min(d1[l + r - i], r - i + 1);
  while (0 <= i - k && i + k < n && s[i - k] == s[i + k])
      k++;
  d1[i] = k--;
  if (i + k > r) l = i - k, r = i + k;
}

vector<int> d2(n);
// d[i] = number of palindromes taking s[i-1] and s[i] as
    center
for (int i = 0, l = 0, r = -1; i < n; i++) {
  int k = (i > r) ? 0 : min(d2[l + r - i + 1], r - i + 1)
      ;
  while (0 <= i - k - 1 && i + k < n && s[i - k - 1] == s
      [i + k]) k++;
  d2[i] = k--;
  if (i + k > r) l = i - k - 1, r = i + k;
}
```

## 7.4 Palindromic Tree

```
struct state {
  int len, link;
  map<char, int> next;
};
state st[MAX];
int id, last;
string s;
ll ans[MAX];
void init() {
  for (int i = 0; i <= id; i++) {
    st[i].len = 0; st[i].link = 0;
    st[i].next.clear(); ans[i] = 0;
  }
  st[1].len = -1; st[1].link = 1;
  st[2].len = 0; st[2].link = 1;
  id = 2; last = 2;
}
void extend(int pos) {
  while (s[pos - st[last].len - 1] != s[pos]) last = st[
      last].link;
  int newlink = st[last].link;
  char c = s[pos];
  while (s[pos - st[newlink].len - 1] != s[pos]) newlink
      = st[newlink].link;
  if (!st[last].next.count(c)) {
    st[last].next[c] = ++id;
    st[id].len = st[last].len + 2;
    st[id].link = (st[id].len == 1 ? 2 : st[newlink].next
        [c]);
    ans[id] += ans[st[id].link];
    if (st[id].len > 2) {
      int l = st[id].len / 2 + (st[id].len % 2 ? 1 : 0);
      if (h.range_hash(pos - st[id].len + 1, pos - st[id
          ].len + l) == h.reverse_hash(pos - st[id].len +
           1, pos - st[id].len + l)) ans[id]++;
    }
  }
  last = st[last].next[c];
}
```

## 7.5 String Match FFT

```
//find occurrences of t in s where '?'s are automatically
    matched with any character
//res[i + m - 1] = sum_j=0 to m - 1_{s[i + j] * t[j] * (s
    [i + j] - t[j])
vector<int> string_matching(string &s, string &t) {
  int n = s.size(), m = t.size();
  vector<int> s1(n), s2(n), s3(n);
  for(int i = 0; i < n; i++) s1[i] = s[i] == '?' ? 0 : s[
      i] - 'a' + 1; //assign any non zero number for non
      '?'s
  for(int i = 0; i < n; i++) s2[i] = s1[i] * s1[i];
  for(int i = 0; i < n; i++) s3[i] = s1[i] * s2[i];
  vector<int> t1(m), t2(m), t3(m);
  for(int i = 0; i < m; i++) t1[i] = t[i] == '?' ? 0 : t[
      i] - 'a' + 1;
  for(int i = 0; i < m; i++) t2[i] = t1[i] * t1[i];
  for(int i = 0; i < m; i++) t3[i] = t1[i] * t2[i];
  reverse(t1.begin(), t1.end());
  reverse(t2.begin(), t2.end());
  reverse(t3.begin(), t3.end());
  vector<int> s1t3 = multiply(s1, t3);
  vector<int> s2t2 = multiply(s2, t2);
  vector<int> s3t1 = multiply(s3, t1);
  vector<int> res(n);
  for(int i = 0; i < n; i++) res[i] = s1t3[i] - s2t2[i] *
       2 + s3t1[i];
  vector<int> oc;
  for(int i = m - 1; i < n; i++) if(res[i] == 0) oc.
      push_back(i - m + 1);
  return oc;
}
```

## 7.6 Suffix Array

```
/** Suffix Array Construction: O(NlogN)
    LCP Array Construction: O(NlogN)
    Suffix LCP: O(logN) **/
typedef pair<int, int> PII;
typedef vector<int> VI;

/// Equivalence Class INFO
vector<VI> c;
VI sort_cyclic_shifts(const string &s) {
  int n = s.size();
  const int alphabet = 256;
  VI p(n), cnt(alphabet, 0);

  c.clear();
  c.emplace_back();
  c[0].resize(n);

  for (int i = 0; i < n; i++) cnt[s[i]]++;
  for (int i = 1; i < alphabet; i++) cnt[i] += cnt[i -
      1];
  for (int i = 0; i < n; i++) p[--cnt[s[i]]] = i;

  c[0][p[0]] = 0;
  int classes = 1;

  for (int i = 1; i < n; i++) {
    if (s[p[i]] != s[p[i - 1]]) classes++;
    c[0][p[i]] = classes - 1;
  }

  VI pn(n), cn(n);
  cnt.resize(n);

  for (int h = 0; (1 << h) < n; h++) {
    for (int i = 0; i < n; i++) {
      pn[i] = p[i] - (1 << h);
      if (pn[i] < 0) pn[i] += n;
    }
    fill(cnt.begin(), cnt.end(), 0);

    /// radix sort
    for (int i = 0; i < n; i++) cnt[c[h][pn[i]]]++;
    for (int i = 1; i < classes; i++) cnt[i] += cnt[i -
        1];
    for (int i = n - 1; i >= 0; i--) p[--cnt[c[h][pn[i
        ]]]] = pn[i];

    cn[p[0]] = 0;
    classes = 1;

    for (int i = 1; i < n; i++) {
      PII cur = {c[h][p[i]], c[h][(p[i] + (1 << h)) % n
          ]};
      PII prev = {c[h][p[i - 1]], c[h][(p[i - 1] + (1 <<
          h)) % n]};
      if (cur != prev) ++classes;
      cn[p[i]] = classes - 1;
    }
    c.push_back(cn);
  }
  return p;
}

VI suffix_array_construction(string s) {
  s += "!";
  VI sorted_shifts = sort_cyclic_shifts(s);
  sorted_shifts.erase(sorted_shifts.begin());
  return sorted_shifts;
}

/// LCP between the ith and jth (i != j) suffix of the
    STRING
int suffixLCP(int i, int j) {
  assert(i != j);
  int log_n = c.size() - 1;

  int ans = 0;
  for (int k = log_n; k >= 0; k--) {
    if (c[k][i] == c[k][j]) {
      ans += 1 << k;
      i += 1 << k;
      j += 1 << k;
    }
  }
  return ans;
}

VI lcp_construction(const string &s, const VI &sa) {
  int n = s.size();
  VI rank(n, 0);
  VI lcp(n - 1, 0);
  for (int i = 0; i < n; i++) rank[sa[i]] = i;

  for (int i = 0, k = 0; i < n; i++) {
    if (rank[i] == n - 1) {
      k = 0;
      continue;
    }

    int j = sa[rank[i] + 1];
    while (i + k < n && j + k < n && s[i + k] == s[j + k
        ]) k++;
    lcp[rank[i]] = k;
    if (k) k--;
  }
  return lcp;
}

const int MX = 1e6 + 7, K = 20;
int lg[MX];

void pre() {
  lg[1] = 0;
  for (int i = 2; i < MX; i++) lg[i] = lg[i / 2] + 1;
}

struct RMQ {
  int N;
  VI v[K];
  RMQ(const VI &a) {
    N = a.size();
    v[0] = a;

    for (int k = 0; (1 << (k + 1)) <= N; k++) {
      v[k + 1].resize(N);
      for (int i = 0; i - 1 + (1 << (k + 1)) < N; i++) {
        v[k + 1][i] = min(v[k][i], v[k][i + (1 << k)]);
      }
    }
  }

  int findMin(int i, int j) {
    int k = lg[j - i + 1];
    return min(v[k][i], v[k][j + 1 - (1 << k)]);
  }
};
```

## 7.7 Suffix Automata

```
/**
   Linear Time Suffix Automata contruction.
   Build Complexity: O(n * alphabet)
   To achieve better build complexity and linear space,
   use map for transitions.
**/

#include<bits/stdc++.h>
using namespace std;

const int MAXN = 1e5+7, ALPHA = 26;
int len[2*MAXN], link[2*MAXN], nxt[2*MAXN][ALPHA];
int sz;
int last;

void sa_init() {
    memset(nxt, -1, sizeof nxt);

    len[0] = 0;
    link[0] = -1;
    sz = 1;
    last = 0;
}

void add(char ch) {
    int c = ch-'a';

    int cur = sz++;                         //create new
        node
    len[cur] = len[last]+1;

    int u = last;
    while (u != -1 && nxt[u][c] == -1) {
        nxt[u][c] = cur;
        u = link[u];
    }

    if (u == -1) {
        link[cur] = 0;
    }
    else {
        int v = nxt[u][c];
        if (len[v] == len[u]+1) {
            link[cur] = v;
        }
        else {
            int clone = sz++;               //create node
                by cloning
            len[clone] = 1 + len[u];
            link[clone] = link[v];

            for (int i=0; i<ALPHA; i++)
```

```
                nxt[clone][i] = nxt[v][i];

            while (u != -1 && nxt[u][c] == v) {
                nxt[u][c] = clone;
                u = link[u];
            }

            link[v] = link[cur] = clone;
        }
    }
    last = cur;
}

vector<int> edge[2*MAXN];
///Optional, Call after adding all characters
void makeEdge() {
    for (int i=0; i<sz; i++) {
        edge[i].clear();
        for (int j=0; j<ALPHA; j++)
            if (nxt[i][j]!=-1)
                edge[i].push_back(j);
    }
}

// The following code solves SPOJ SUBLEX
// Given a string S, you have to answer some queries:
// If all distinct substrings of string S were sorted
// lexicographically, which one will be the K-th smallest
    ?

long long dp[2*MAXN];
bool vis[2*MAXN];

void dfs(int u) {
    if (vis[u]) return;
    vis[u] = 1;
    dp[u] = 1;
    for (int i: edge[u]) {
        if (nxt[u][i] == -1) continue;
        dfs(nxt[u][i]);
        dp[u] += dp[nxt[u][i]];
    }
}

void go(int u, long long rem, string &s) {
    if (rem == 1) return;
    long long sum = 1;
    for (int i: edge[u]) {
        if (nxt[u][i] == -1) continue;
        if (sum + dp[nxt[u][i]] < rem) {
            sum += dp[nxt[u][i]];
        }
        else {
            s += ('a' + i);
            go(nxt[u][i], rem-sum, s);
            return;
        }
    }
}

int main() {
    ios::sync_with_stdio(0);
    cin.tie(0);

    string s;
    cin>>s;

    sa_init();
    for (char c: s) add(c);
    makeEdge();


    dfs(0);
    int q;
    cin>>q;

    while (q--) {
        long long x;
        cin>>x;
        x++;
        string s;
        go(0, x, s);
        cout<<s<<"\n";
    }
}
```

### 7.8 Z Algo

```
vector<int> calcz(string s) {
    int n = s.size();
    vector<int> z(n);
    int l, r; l = r = 0;
    for (int i = 1; i < n; i++) {
        if (i > r) {
            l = r = i;
            while (r < n && s[r] == s[r - l]) r++;
            z[i] = r - l; r--;
        } else {
            int k = i - l;
            if (z[k] < r - i + 1) z[i] = z[k];
            else {
                l = i;
                while (r < n && s[r] == s[r - l]) r++;
                z[i] = r - l; r--;
            }
        }
    }
    return z;
}
```

# 8 Equations and Formulas

## 8.1 Catalan Numbers

$C_n = \frac{1}{n+1}\binom{2n}{n}$ $C_0 = 1, C_1 = 1$ and $C_n = \sum_{k=0}^{n-1} C_k C_{n-1-k}$

The number of ways to completely parenthesize $n$+1 factors.
The number of triangulations of a convex polygon with $n$+2 sides (i.e. the number of partitions of polygon into disjoint triangles by using the diagonals).
The number of ways to connect the $2n$ points on a circle to form $n$ disjoint i.e. non-intersecting chords.
The number of rooted full binary trees with $n$+1 leaves (vertices are not numbered). A rooted binary tree is full if every vertex has either two children or no children.
Number of permutations of $1, \ldots, n$ that avoid the pattern 123 (or any of the other patterns of length 3); that is, the number of permutations with no three-term increasing sub-sequence. For $n = 3$, these permutations are 132, 213, 231, 312 and 321.

## 8.2 Stirling Numbers First Kind

The Stirling numbers of the first kind count permutations according to their number of cycles (counting fixed points as cycles of length one).
$S(n,k)$ counts the number of permutations of $n$ elements with $k$ disjoint cycles.
$S(n,k) = (n-1) \cdot S(n-1,k) + S(n-1,k-1),\ where,\ S(0,0) = 1, S(n,0) = S(0,n) = 0$ $\sum_{k=0}^{n} S(n,k) = n!$
The unsigned Stirling numbers may also be defined algebraically, as the coefficient of the rising factorial:

$$x^{\bar{n}} = x(x+1)...(x+n-1) = \sum_{k=0}^{n} S(n,k)x^k$$

Lets $[n,k]$ be the stirling number of the first kind, then

$$\left[n \overset{n}{-} k\right] = \sum_{0 \leq i_1 < i_2 < i_k < n} i_1 i_2 .... i_k.$$

## 8.3 Stirling Numbers Second Kind

Stirling number of the second kind is the number of ways to partition a set of n objects into k non-empty subsets.
$S(n,k) = k \cdot S(n-1,k) + S(n-1,k-1),\ where\ S(0,0) = 1, S(n,0) = S(0,n) = 0$ $S(n,2) = 2^{n-1} - 1$ $S(n,k) \cdot k! =$ number of ways to color $n$ nodes using colors from 1 to $k$ such that each color is used at least once.
An $r$-associated Stirling number of the second kind is the number of ways to partition a set of $n$ objects into $k$ subsets, with each subset containing at least $r$ elements. It is denoted by $S_r(n,k)$ and obeys the recurrence relation. $S_r(n+1,k) = kS_r(n,k) + \binom{n}{r-1} S_r(n-r+1,k-1)$
Denote the n objects to partition by the integers $1, 2, \ldots, n$. Define the reduced Stirling numbers of the second kind, denoted $S^d(n,k)$, to be the number of ways to partition the integers $1, 2, \ldots, n$ into k nonempty subsets such that all elements in each subset have pairwise distance at least d. That is, for any integers i and j in a given subset, it is required that $|i - j| \geq d$. It has been shown that these numbers satisfy, $S^d(n,k) = S(n-d+1, k-d+1), n \geq k \geq d$

## 8.4 Other Combinatorial Identities

$\binom{n}{k} = \frac{n}{k}\binom{n-1}{k-1}$

$\sum_{i=0}^{k} \binom{n+i}{i} = \sum_{i=0}^{k} \binom{n+i}{n} = \binom{n+k+1}{k}$

$n, r \in N, n > r, \sum_{i=r}^{n} \binom{i}{r} = \binom{n+1}{r+1}$

If $P(n) = \sum_{k=0}^{n} \binom{n}{k} \cdot Q(k)$, then,

$$Q(n) = \sum_{k=0}^{n} (-1)^{n-k} \binom{n}{k} \cdot P(k)$$

If $P(n) = \sum_{k=0}^{n} (-1)^k \binom{n}{k} \cdot Q(k)$ , then,

$$Q(n) = \sum_{k=0}^{n} (-1)^k \binom{n}{k} \cdot P(k)$$

## 8.5 Different Math Formulas

**Picks Theorem :** $A = i + b/2 - 1$
**Deragements :** $d(i) = (i-1) \times (d(i-1) + d(i-2))$

$$\frac{n}{ab} \quad - \quad \left\{\frac{b'n}{a}\right\} \quad - \quad \left\{\frac{a'n}{b}\right\} \quad + \quad 1$$

## 8.6 GCD and LCM

if $m$ is any integer, then $\gcd(a + m \cdot b, b) = \gcd(a,b)$
The gcd is a multiplicative function in the following sense: if $a_1$ and $a_2$ are relatively prime, then $\gcd(a_1 \cdot a_2, b) = \gcd(a_1, b) \cdot \gcd(a_2, b)$.
$\gcd(a, \operatorname{lcm}(b,c)) = \operatorname{lcm}(\gcd(a,b), \gcd(a,c))$.
$\operatorname{lcm}(a, \gcd(b,c)) = \gcd(\operatorname{lcm}(a,b), \operatorname{lcm}(a,c))$.
For non-negative integers $a$ and $b$, where $a$ and $b$ are not both zero, $\gcd(n^a - 1, n^b - 1) = n^{\gcd(a,b)} - 1$

$\gcd(a,b) = \sum_{k|a \text{ and } k|b} \phi(k)$

$\sum_{i=1}^{n} [\gcd(i,n) = k] = \phi\left(\frac{n}{k}\right)$

$\sum_{k=1}^{n} \gcd(k,n) = \sum_{d|n} d \cdot \phi\left(\frac{n}{d}\right)$

$\sum_{k=1}^{n} x^{\gcd(k,n)} = \sum_{d|n} x^d \cdot \phi\left(\frac{n}{d}\right)$

$\sum_{k=1}^{n} \frac{1}{\gcd(k,n)} = \sum_{d|n} \frac{1}{d} \cdot \phi\left(\frac{n}{d}\right) = \frac{1}{n} \sum_{d|n} d \cdot \phi(d)$

$\sum_{k=1}^{n} \frac{k}{\gcd(k,n)} = \frac{n}{2} \cdot \sum_{d|n} \frac{1}{d} \cdot \phi\left(\frac{n}{d}\right) = \frac{n}{2} \cdot \frac{1}{n} \cdot \sum_{d|n} d \cdot \phi(d)$

$\sum_{k=1}^{n} \frac{n}{\gcd(k,n)} = 2 * \sum_{k=1}^{n} \frac{k}{\gcd(k,n)} - 1$, for $n > 1$

$\sum_{i=1}^{n} \sum_{j=1}^{n} [\gcd(i,j) = 1] = \sum_{d=1}^{n} \mu(d) \lfloor \frac{n}{d} \rfloor^2$

$\sum_{i=1}^{n} \sum_{j=1}^{n} \gcd(i,j) = \sum_{d=1}^{n} \phi(d) \lfloor \frac{n}{d} \rfloor^2$

$\sum_{i=1}^{n} \sum_{j=1}^{n} i \cdot j [\gcd(i,j) = 1] = \sum_{i=1}^{n} \phi(i) i^2$

$F(n) = \sum_{i=1}^{n} \sum_{j=1}^{n} \operatorname{lcm}(i,j) = \sum_{l=1}^{n} \left( \frac{\left(1 + \lfloor \frac{n}{l} \rfloor\right)\left(\lfloor \frac{n}{l} \rfloor\right)}{2} \right)^2 \sum_{d|l} \mu(d) l d$