Inversion Problems

(Hint: Use inversion)

1998 IMO Camp

1) Three congruent circles are concurrent at H. They also intersect in pairs at A, B, C. Show that H is the orthocenter of ΔA

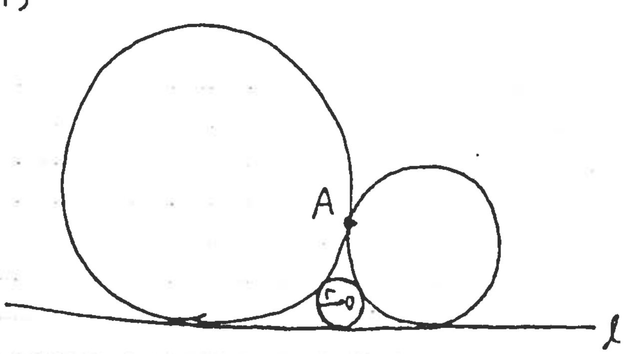
2) The circles S_1 and S_2 are tangent at the point A. A straight line l through A intersects S_1 and S_2 at the points C_1 and C_2 respectively. A circle S, which contains C_1 and C_2, meets S_1 and S_2 at points B_1 and B_2 respectively. Let χ be the circumscribed circle of the triangle AB_1B_2. The circle k tangent to χ at the point A meets C_1 and C_2 at the points D_1 and D_2 respectively. Prove that:

   a) The points C_1, C_2, D_1, D_2 are concircular or collinear.

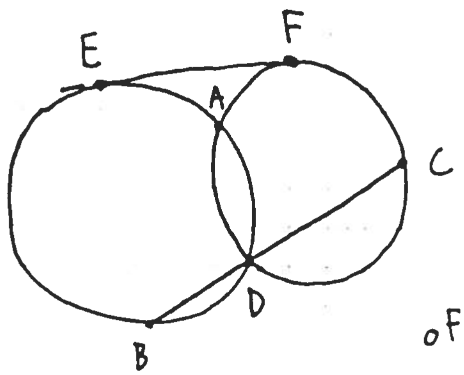
   b) The points B_1, B_2, D_1, D_2 are concircular if and only if AC_1 and AC_2 are diameters of C_1 and C_2.

3) Two circles intersect at A and O. The tangents to the circles at A are drawn, and they intersect the circles again at B and C. Prove that $BO \cdot AC = AO \cdot AB$

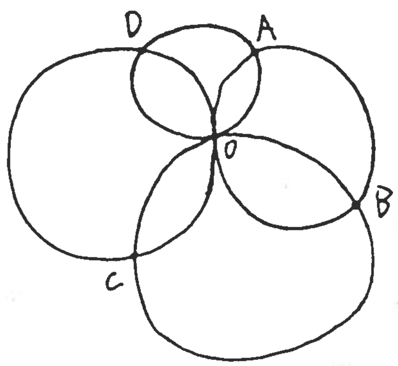
4)



Two circles C_1 and C_2 are externally tangent at A, and the line l is tangent to both of them as shown. A circle with center O and radius r is tangent to l, C_1 and C_2. Prove that $AO \leq 3r$

5) 

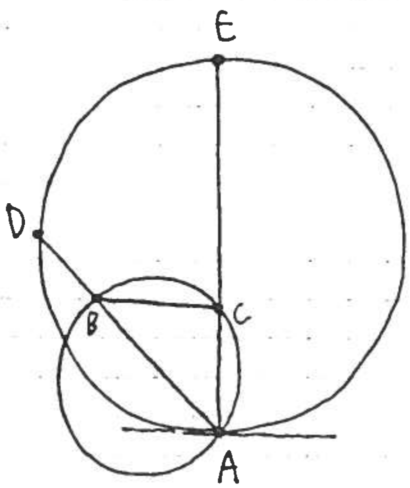
Two circles intersect at A and D. As shown, EF is a common tangent and a line drawn through $D \parallel$ to EF intersects the circles at B, C. Show that the circumcirc of $\triangle BDF$ and $\triangle CDE$ intersect again on AD.

6) 

Four circles C_1, C_2, C_3 and C_4 are concurrent at a point O, with pairs C_1, C_3 and C_2, C_4 externally tangent at O. The circles intersect again at A, B, C, D as shown. Show that $AB \cdot OD \cdot OC = CD \cdot OA \cdot OB$

7) Given a convex quadrilateral $ABCD$, Show that there exists a circle which is tangent to AB extended, AD extended, and the circumcircles of $\triangle ABC$ and $\triangle ACD$ if and only if

$$AC \cdot AD + AB \cdot CD = AB \cdot AC + BC \cdot AD$$

8) 

Two circles C_1, C_2 intersect at A. A chord BC of C_1 is parallel to the tangent at A of C_2. AB and AC intersect C_2 at D and E. Prove that $BCED$ is cyclic.