Nearly Every Mistake $\rightarrow 0$

2nd NEMO, September 22 2016

Problem 1. The numbers from 1 to 81 are written on each square of a 9×9 board, so that each number appears only once. Prove that there exists two squares separated by one side, whose numbers differ by at least 6.

Problem 2. Let $ABCD$ be a trapezium with $AB \parallel CD$. Let X be the intersection of diagonals AC and BD. Let α be the line through X parallel to AB. Let P be a points on α, and let P' be the reflection of P across X. We define the vanishing points of P as the intersection points of AD and BC; PA and $P'C$; PD and $P'B$. Prove that the three vanishing points of P are collinear and one is the midpoint of the other two.

Problem 3. Find all positive real solutions $(x_1, x_2, x_3, x_4, x_5, a)$ to the following set of equations:

$$\sum_{k=1}^{5} kx_k = a$$

$$\sum_{k=1}^{5} kx_k^3 = a^2$$

$$\sum_{k=1}^{5} kx_k^5 = a^3$$

Problem 4. We have n bowls $b_1, b_2, \ldots, b_n$ distributed evenly on the edge of a circular table. For $k > n$, $b_k = b_r$ if and only if $r \equiv k \pmod{n}$. Sans walks clockwise around the table from bowl to bowl and places a marble in them according to a rule: if the kth marble is placed in the bowl b_i, the next $(k+1)$th marble is placed in the bowl b_{i+k}. Sans comes to a stop if each bowl has at least one marble in it. For a positive integer n, how many marbles will Sans drop before coming to a stop?