2008 Winter Camp (Mock)

① Let $a, b, c > 0$ and $abc \leq 1$. Prove that

$$a/c + b/a + c/b \geq a + b + c.$$

② Let $P(x)$ be a polynomial with real coefficients such that $P(x) \geq 0$ for $0 \leq x \leq 1$. Show that there are polynomials $A(x)$, $B(x)$, $C(x)$ with real coefficients such that

(a) $A(x) \geq 0$, $B(x) \geq 0$, $C(x) \geq 0$ for all real x, and

(b) $P(x) = A(x) + x B(x) + (1-x) C(x)$

③ If A, B, C, D are four distinct points such that every circle through A and B intersects every circle through C and D, prove that A, B, C, D are collinear or concyclic