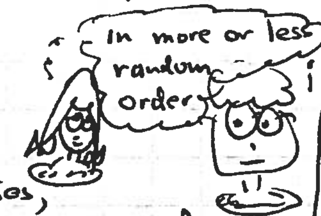
1998 IMO Camp

11/7/95.

SOME VERY USEFUL STUFF!



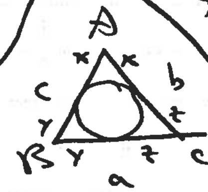
1. In Locus Questions, think lines (half-)lines, segments,... or circles (arcs) – show $\Leftrightarrow$!

2. Think about other bases – especially base 2.

3. Try Complex Numbers!

4. Look for recurrence relations.

5. Use R.S. $\left.\begin{array}{l} x = s-a \\ y = s-b \\ z = s-c \end{array}\right\} \Rightarrow \begin{array}{l} a = y+z \\ b = x+z \\ c = x+y \end{array}$ for $x, y, z \geq 0$



[This ensures Δ-inequality & makes Heron's Formula nice: $(\Delta ABC) = \sqrt{xyz(x+y+z)}$]

6. Split up inequalities into parts, prove each, then recombine.

7. Look for & use symmetry.

8. For ΔABC: When $A+B+C=\pi$: $\tan\frac{A}{2}\tan\frac{B}{2}+\tan\frac{B}{2}\tan\frac{C}{2}+\tan\frac{C}{2}\tan\frac{A}{2}=1$
$\tan A+\tan B+\tan C=\tan A\tan B\tan C$

9. Pick's Theorem: The area of any simple polygon whose vertices are lattice points is given by $K = a+\frac{1}{2}b-1$ $\left[\begin{array}{l} a = \text{interior pts} \\ b = \text{bdry pts} \end{array}\right]$.

10. Special Factorizations

$$x^4+4 = (x^2+2x+2)(x^2-2x+2)$$
$$a^3+b^3+c^3-3abc = (a+b+c)(a^2+b^2+c^2-ab-bc-ac)$$
$$(a+b+c)^3-a^3-b^3-c^3 = 3(b+c)(c+a)(a+b)$$
$$a^4+b^4+c^4-2a^2b^2-2b^2c^2-2c^2a^2 = (a+b+c)(a+b-c)(a-b+c)(-a+b+c)$$

RAVI's ROCK-CRUSHER

$\cos 45° = \sqrt{2}$.

11. Various formulas for the area of a Δ:

$$K = \tfrac{1}{2}bh = \tfrac{1}{2}bc\sin A = \sqrt{s(s-a)(s-b)(s-c)} = \sqrt{xyz(x+y+z)} = sr = \frac{abc}{4R}$$
$$= 2R^2\sin A\sin B\sin C$$

12. In ΔABC: $\tan\frac{A}{2}\tan\frac{B}{2}\tan\frac{C}{2} = \frac{K}{s^2} = \frac{r}{s} = \frac{r^2}{K}$

$$4\cos\tfrac{A}{2}\cos\tfrac{B}{2}\cos\tfrac{C}{2} = \frac{s}{R} = \sin A+\sin B+\sin C$$
$$4\sin\tfrac{A}{2}\sin\tfrac{B}{2}\sin\tfrac{C}{2} = \frac{r}{R} = \cos A+\cos B+\cos C-1$$
$$\cos^2 A+\cos^2 B+\cos^2 C+2\cos A\cos B\cos C = 1$$
$$(1+\tan A)(1+\tan B) = 2 \Leftrightarrow A+B = \frac{\pi}{4}.$$

13. The equation of the line through (x_1, y_1), and (x_2, y_2) is given by

$$\det\begin{vmatrix} x & y & 1 \\ x_1 & y_1 & 1 \\ x_2 & y_2 & 1 \end{vmatrix} = 0$$

14. The pt. of intersection of the lines $A_1x+B_1y+C_1=0$ and $A_2x+B_2y+C_2=0$ is found by writing

$$\det\begin{vmatrix} x & y & 1 \\ A_1 & B_1 & C_1 \\ A_2 & B_2 & C_2 \end{vmatrix}$$

in the form $(a)(hx+ky+1)$. Then (h,k) is pt.

Useful words: "clearly", "it is now obvious", "it is easy", "as any idiot can see"... etc.

I.P.'s first Principle: When in doubt – bluff!

15.