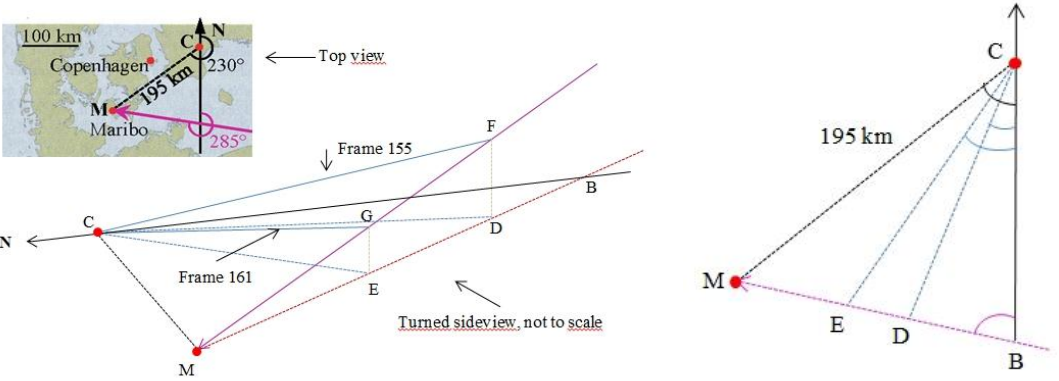


Solutions

1.1	 <p>Top view: Triangle MCB: $CM = 195 \text{ km}$, $\angle MCB = 230^\circ - 180^\circ = 50^\circ$, and $\angle MBC = 75^\circ$, so $\angle CMB = 180^\circ - 75^\circ - 50^\circ = 55^\circ$.</p> <p>Then $CB = \frac{ CM \sin(\angle CMB)}{\sin(\angle MBC)} = 165.4 \text{ km}$.</p> <p>Triangle DCB: $CB = 165.4 \text{ km}$, $\angle DCB = 215^\circ - 180^\circ = 35^\circ$, and $\angle DBC = 75^\circ$, so $\angle CDB = 180^\circ - 75^\circ - 35^\circ = 70^\circ$.</p> <p>Then $CD = \frac{ CB \sin(\angle DBC)}{\sin(\angle CDB)} = 170.0 \text{ km}$.</p> <p>Triangle ECB: $CB = 165.4 \text{ km}$, $\angle ECB = 221^\circ - 180^\circ = 41^\circ$, and $\angle EBC = 75^\circ$, so $\angle CEB = 180^\circ - 75^\circ - 41^\circ = 64^\circ$.</p> <p>Then $CE = \frac{ CB \sin(\angle EBC)}{\sin(\angle CEB)} = 177.7 \text{ km}$.</p> <p>Triangle ECD: $\angle ECD = 41^\circ - 35^\circ = 6^\circ$. Horizontal distance travelled by Maribo: $DE = \frac{ DC \sin(\angle ECD)}{\sin(\angle CED)} = 19.77 \text{ km}$</p> <p>Side view: Triangle CFD: $FD = CD \tan(\angle FCD) = 59.20 \text{ km}$</p> <p>Triangle CGE: $GE = CE \tan(\angle GCE) = 46.62 \text{ km}$</p> <p>Thus vertical distance travelled by Maribo: $FD - GE = 12.57 \text{ km}$.</p> <p>Total distance travelled by Maribo from frame 155 to 161:</p> <p>$FG = \sqrt{ DE ^2 + (FD - GE)^2} = 23.43 \text{ km}$.</p> <p>The speed of Maribo is $v = \frac{23.43 \text{ km}}{2.28 \text{ s} - 1.46 \text{ s}} = 28.6 \text{ km/s}$</p>	1.2
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1.2a	<p>Newton's second law: $m_M \frac{dv}{dt} = -k\rho_{\text{atm}}\pi R_M^2 v^2$ yields $\frac{1}{v^2} dv = -\frac{k\rho_{\text{atm}}\pi R_M^2}{m_M} dt$.</p> <p>By integration $t = \frac{m_M}{k\rho_{\text{atm}}\pi R_M^2} \left(\frac{1}{0.9} - 1 \right) \frac{1}{v_M} = 0.88 \text{ s}$.</p>	0.7
1.2b	<p>$\frac{E_{\text{kin}}}{E_{\text{melt}}} = \frac{\frac{1}{2} v_M^2}{c_{\text{sm}}(T_{\text{sm}} - T_0) + L_{\text{sm}}} = \frac{4.2 \times 10^8}{2.1 \times 10^6} = 2.1 \times 10^2 \gg 1$.</p>	0.3

1.3a	$[x] = [t]^\alpha [\rho_{\text{sm}}]^\beta [c_{\text{sm}}]^\gamma [k_{\text{sm}}]^\delta = [s]^\alpha [\text{kg m}^{-3}]^\beta [\text{m}^2 \text{s}^{-2} \text{K}^{-1}]^\gamma [\text{kg m s}^{-3} \text{K}^{-1}]^\delta$, so $[m] = [\text{kg}]^{\beta+\delta} [m]^{-3\beta+2\gamma+\delta} [s]^{\alpha-2\gamma-3\delta} [\text{K}]^{-\gamma-\delta}$. Thus $\beta + \delta = 0$, $-3\beta + 2\gamma + \delta = 1$, $\alpha - 2\gamma - 3\delta = 0$, and $-\gamma - \delta = 0$. From which $(\alpha, \beta, \gamma, \delta) = \left(+\frac{1}{2}, -\frac{1}{2}, -\frac{1}{2}, +\frac{1}{2}\right)$ and $x(t) \approx \sqrt{\frac{k_{\text{sm}} t}{\rho_{\text{sm}} c_{\text{sm}}}}$.	0.6
1.3b	$x(5 \text{ s}) = 1.6 \text{ mm}$ $x/R_M = 1.6 \text{ mm}/130 \text{ mm} = 0.012$.	0.4
1.4a	Rb-Sr decay scheme: ${}^{87}_{37}\text{Rb} \rightarrow {}^{87}_{38}\text{Sr} + {}^0_{-1}\text{e} + \bar{\nu}_e$	0.3
1.4b	$N_{87\text{Rb}}(t) = N_{87\text{Rb}}(0)e^{-\lambda t}$ and Rb→Sr: $N_{87\text{Sr}}(t) = N_{87\text{Sr}}(0) + [N_{87\text{Rb}}(0) - N_{87\text{Rb}}(t)]$. Thus $N_{87\text{Sr}}(t) = N_{87\text{Sr}}(0) + (e^{\lambda t} - 1)N_{87\text{Rb}}(t)$, and dividing by $N_{86\text{Sr}}$ we obtain the equation of a straight line: $\frac{N_{87\text{Sr}}(t)}{N_{86\text{Sr}}} = \frac{N_{87\text{Sr}}(0)}{N_{86\text{Sr}}} + (e^{\lambda t} - 1) \frac{N_{87\text{Rb}}(t)}{N_{86\text{Sr}}}.$	0.7
1.4c	Slope: $e^{\lambda t} - 1 = a = \frac{0.712-0.700}{0.25} = 0.050$ and $T_{1/2} = \frac{\ln(2)}{\lambda} = 4.9 \times 10^{10} \text{ year}$. So $\tau_M = \ln(1+a) \frac{1}{\lambda} = \frac{\ln(1+a)}{\ln(2)} T_{1/2} = 3.4 \times 10^9 \text{ year}$.	0.4
1.5	Kepler's 3rd law on comet Encke and Earth, with the orbital semi-major axis of Encke given by $a = \frac{1}{2}(a_{\text{min}} + a_{\text{max}})$. Thus $t_{\text{Encke}} = \left(\frac{a}{a_E}\right)^{\frac{3}{2}} t_E = 3.30 \text{ year} = 1.04 \times 10^8 \text{ s}$.	0.6
1.6a	For Earth around its rotation axis: Angular velocity $\omega_E = \frac{2\pi}{24 \text{ h}} = 7.27 \times 10^{-5} \text{ s}^{-1}$. Moment of inertia $I_E = 0.83 \frac{2}{5} m_E R_E^2 = 8.07 \times 10^{37} \text{ kg m}^2$. Angular momentum $L_E = I_E \omega_E = 5.87 \times 10^{33} \text{ kg m}^2 \text{s}^{-1}$. Astroid $m_{\text{ast}} = \frac{4\pi}{3} R_{\text{ast}}^3 \rho_{\text{ast}} = 1.57 \times 10^{15} \text{ kg}$ and angular momentum $L_{\text{ast}} = m_{\text{ast}} v_{\text{ast}} R_E = 2.51 \times 10^{26} \text{ kg m}^2 \text{s}^{-1}$. L_{ast} is perpendicular to L_E , so by conservation angular momentum: $\tan(\Delta\theta) = L_{\text{ast}}/L_E = 4.27 \times 10^{-8}$. The axis tilt $\Delta\theta = 4.27 \times 10^{-8} \text{ rad}$ (so the north pole move $R_E \Delta\theta = 0.27 \text{ m}$).	0.7
1.6b	At vertical impact $\Delta L_E = 0$ so $\Delta(I_E \omega_E) = 0$. Thus $\Delta\omega_E = -\omega_E(\Delta I_E)/I_E$, and since $\Delta I_E/I_E = m_{\text{ast}} R_E^2/I_E = 7.9 \times 10^{-10}$ we obtain $\Delta\omega_E = -5.76 \times 10^{-14} \text{ s}^{-1}$. The change in rotation period is $\Delta T_E = 2\pi \left(\frac{1}{\omega_E + \Delta\omega_E} - \frac{1}{\omega_E} \right) \approx -2\pi \frac{\Delta\omega_E}{\omega_E^2} = 6.84 \times 10^{-5} \text{ s}$.	0.7
1.6c	At tangential impact L_{ast} is parallel to L_E so $L_E + L_{\text{ast}} = (I_E + \Delta I_E)(\omega_E + \Delta\omega_E)$ and thus $\Delta T_E = 2\pi \left(\frac{1}{\omega_E + \Delta\omega_E} - \frac{1}{\omega_E} \right) = 2\pi \left(\frac{I_E + \Delta I_E}{L_E + L_{\text{ast}}} - \frac{1}{\omega_E} \right) = -3.62 \times 10^{-3} \text{ s}$.	0.7

1.7a	Minimum impact speed is the escape velocity from Earth: $v_{\text{imp}}^{\text{min}} = \sqrt{\frac{2Gm_E}{R_E}} = 11.2 \text{ km/s}$	0.5
1.7b	<p>Maximum impact speed $v_{\text{imp}}^{\text{max}}$ arises from three contributions:</p> <p>(I) The velocity v_b of the body at distance a_E (Earth orbit radius) from the Sun, $v_b = \sqrt{\frac{2Gm_S}{a_E}} = 42.1 \text{ km/s}$.</p> <p>(II) The orbital velocity of the Earth, $v_E = \frac{2\pi a_E}{1 \text{ year}} = 29.8 \text{ km/s}$.</p> <p>(III) Gravitational attraction from the Earth and kinetic energy seen from the Earth: $\frac{1}{2}(v_b + v_E)^2 = -\frac{Gm_E}{R_E} + \frac{1}{2}(v_{\text{imp}}^{\text{max}})^2$.</p> <p>In conclusion: $v_{\text{imp}}^{\text{max}} = \sqrt{(v_b + v_E)^2 + \frac{2Gm_E}{R_E}} = 72.8 \text{ km/s}$.</p>	1.2
	Total	9.0