

The Final Mock Contest

1. At the Banff Dim Sum restaurant, all dishes come in three sizes: small, medium, and large. Small dishes cost $\$x$, medium dishes cost $\$y$, and large dishes cost $\$z$, where x, y, z are positive integers with $x < y < z$. At this restaurant, there is no tax on any dish and the prices haven't changed for a long time.

Elena, Terry and Bill had dinner there last night, and together, they ordered 9 small dishes, 6 medium dishes and 8 large dishes. When the bill came, the following conversation ensued:

Elena: This bill is exactly twice as much as when I last came here.

Terry: This bill is exactly three times as much as when I last came here.

Bill: Oh, that was a delicious meal, and very reasonably priced too. Even if we give the waiter a ten percent tip, the total is still less than \$100."

Determine the values of x , y , and z , and prove that your answer is unique.

2. Q is the midpoint of side AB of a cyclic quadrilateral. AC and BD meet at point S . P and R are the respective feet of the perpendiculars from S to AD and BC . Prove that $PQ = QR$.
3. Let a_n denote the number of elements in the n^{th} row of Pascal's Triangle that are congruent to 1 mod 3. Let b_n denote the number of elements in the n^{th} row of Pascal's Triangle that are congruent to 2 mod 3.

Prove that for all n , $a_n - b_n$ is a power of 2.

4. In triangle ABC with incentre I , $AB > BC$. K and M are the respective midpoints of AB and AC . The extension of MK and CI meet at P . The line through P perpendicular to KM meets the line through I perpendicular to AC at point Q . Prove that QM is parallel to BI .
5. Consider the complete graph on n vertices, and colour each edge either red or blue. (A complete graph has an edge joining every pair of vertices).

Determine the smallest positive integer n for which any colouring of the edges has the property that there must either exist three vertices, all of whose connecting edges are red, or four vertices, all of whose connecting edges are blue.