```
\documentclass[a4paper]{article}
\usepackage[xetex]{graphicx}
\usepackage{hyperref}
\usepackage{ifthen}
%packages\\

\hypersetup{
bookmarks=true, % show bookmarks bar?
unicode=false, % non-Latin characters in Acrobat's bookmarks
pdftoolbar=true, % show Acrobat's toolbar?
pdfmenubar=true, % show Acrobat's menu?
pdffitwindow=true, % page fit to window when opened
pdfnewwindow=true, % links in new window
pdfkeywords={keywords}, % list of keywords
colorlinks=true, % false: boxed links; true: colored links
linkcolor=black, % color of internal links
citecolor=green, % color of links to bibliography
filecolor=magenta, % color of file links
urlcolor=black% color of external links
}
\usepackage[top=1.25 in,bottom=1 in,left=1.25 in,right=1.25 in,reversemp]{geometry}
\usepackage{verbatim}
\usepackage{amsthm,amssymb,amsmath}
%newcommands
\newcommand{\pr}[1]{\vspace{0.1 in} \noindent \textbf{Problem #1.}}
\newcommand{\an}{\angle}
\newcommand{\tr}{\triangle}
\newcommand{\cd}{\cdot}
\newcommand{\cds}{\cdots}
\newcommand{\fn}{\footnote}
\newcommand{\Lcen}[1]
{
\begin{center}
\Large{\textsc{#1}}
\end{center}
}

\newcommand{\lcen}[1]
{
\begin{center}
\large{\textsc{#1}}
\end{center}
}

% nstar is a command for drawing n stars in the margin. Useful for ratings.
```

```
% Requires the 'ifthen' package
\newcounter{scount} % create counter
\newcommand{\ns}[1]{%
    \setcounter{scount}{#1}% initialize counter
    \marginpar{\hfill    % move content closer to the main text
        \whiledo{\value{scount}>0}
            {{\Large{$\star$}}\addtocounter{scount}{-1}}
    }
}

\newcommand{\e}{\normalfont}

\font\ff="Bloody" at 16 pt

\begin{document}
\begin{center}
\textsc{\LARGE Geometry Camp 2009} \\ \bigskip
\textit{\Large Problem set}
\end{center}
\Lcen{Easy \fn{This does not mean that they are not "problems". You will not be able to solve them
unless you pay enough attention, and try hard enough!}}
\pr{0} In $\tr ABC$ $AB=AC$ iff\fn{\emph{iff} means if and only if. When some problem includes this, it
asks to prove "both ways". \emph{This is important!}} $\an B=\an C$ (Kidding!)

\pr{1} Prove that a quadrilateral in which both pairs of opposite angles are equal has to be a
parallelogram.
% 35

\pr{2}\ns{1}\footnote{Asterisk ($\star$) indicates the difficulty level of the problems. Note that:
"difficulty" is relative, and the difficult level of double asterisk problem of "Easy" set is not the same as
that of "Not so easy!!!" set.}(a) $P,Q,R$ are points on the sides $BC,CA,AB$ of $\tr ABC$. Prove that the
perpendiculars to the sides at these points meet in a common point iff \[
BP^2+CQ^2+AR^2=PC^2+QA^2+RB^2 \]
% 40
(b) Given a triangle $ABC$, Let $L,M,N$ be the feet of the perpendiculars from the point $K$ to the sides
$BC,CA,AB$ respectively. Prove that the perpendiculars from $A,B,C$ to $MN,NL,LM$ respectively are
concurrent.

\pr{3} \fn{Yes...the opposite statement of "the power of point" theorem is true, and undoubtedly very
important.} (a) Let two segments $AB$ and $CD$ intersect at $P$. Prove that $AP\cd PB=CP\cd PD$ iff
A,B,C,D are concyclic.
\\(b) Let $A,B,T$ be three distinct points on $\omega$, and $P$ be a point on the extension of $BA$.
Prove that $PT$ is tangent to $\omega$ iff $PT^2=PA\cd PB$.

\pr{4} The convex hexagon $ABCDEF$ is inscribed in a circle. Prove that the diagonals $AD,BE,CF$ are
```

concurrent iff \[AB\cd CD\cd EF=BC \cd DE\cd FA\]
%96

\pr{5} The circles S_1 and S_2 intersects A. Through A any pair of straight lines BAC and B_0AC_0 are drawn with B,B_0 on S_1 and C,C_0 on S_2. Prove that the chord BB_0 of S_1 and C_0C of S_2 are inclined at a constant angle.
%107

\pr{6} \ns{1} The centers of two circles of unequal radii r_1,r_2 are a distance r_{12} apart. Calculate the distance between the two centers of similitude.\footnote{Plane Euclidean page 115.}
%116

\pr{7} In $\tr ABC$, P lies on BC (possibly on the extensions). Prove that, \[\frac{BP}{PC}=\frac{AB \times \sin \an BAP}{AC \times \sin \an PAC}\]
%124

\pr{8}\ns{2} A,B are two fixed points. P moves so that $\frac{PA}{PB}$ is constant. Prove that the locus of P is a circle. \fn{The circle of Appolonius.}
%125

\pr{9} Let H is the orthocenter of $\tr ABC$ and D be the foot of perpendicular from A on BC. Prove that $AH=a \cot A=2R \cos A$ (R circumradius), and $HD=2R\cos B \cos C$.

\pr{10}\ns{1}Let the extension of BO meet the circumcircle of $\tr ABC$ at Q. Prove that $AQCH$ is the parallelogram. (O circumcenter, H orthocenter)
%133

\pr{11} $area(ABC)=\sqrt{rr_1r_2r_3}$.
%142

\pr{12} \ns{2} $r_1+r_2+r_3=4R+r$. ($R,r, r_1 ,r_2 ,r_3 $ are the radius of circumcircle, incircle, and the excircles respectively.)

\pr{13}\fn{Trigonometric form of Ceva's Theorem, both of the forms are handy proving concurrency.} Let $\triangle ABC$ be a triangle, and let P, Q, R be any points in the
plane distinct from $A, B, C,$ respectively. Then $AP, BQ,
CR$ are
concurrent if and only if
\[
\frac{\sin\an CAP}{\sin\an PAB} \cd \frac{\sin\an ABQ}{\sin\an QBC} \cd
\frac{\sin\an BCR}{\sin\an RCA} = 1.
\]

\pr{14}\fn{This is a very important lemma. Don't forget!}(a) Suppose the angle bisector of $\an BAC$ intersect the circumcircle of $\tr ABC$ at $X \neq A$. Let I be a point on the line segment AX. Then

I is the incenter of $\tr ABC$ if and only if $XI = XB = XC=XI_a$.
\\ (b) Find the length AX in terms of the side lengths and angles, using trigonometry.

\pr{15}\ns{2}Two circles intersect at the
points A and B. Tangent lines drawn to
both of the circles at the point A
intersect the circles at the points M and
N. The lines BM and BN intersect the
circles once more at the points P and Q
respectively.
Prove that the segments MP and NQ are equal.
%v12_1 ex 5 tot

\pr{16} (Perpendicular Lemma)\fn{More applications: \url{http://www.math.ust.hk/excalibur/v12_n3.pdf}}
On a plane, for distinct
points R, S, X, Y we have $RX^2-SX^2 =
RY^2 - SY^2$ if and only if $RS \perp XY$.
%

\pr{17}\ns{1}Two circles meet at P and Q. A line intersects segment PQ and meets the circles at the points A, B, C, D in that order. Prove that $\an APB =
\an CQD$.
%1998,p-125

\pr{18}\ns{2}A convex quadrilateral $ABCD$ is given for which $\an ABC+\an BCD <\pi$. The comon point of the lines AB and CD is E. Prove that $\an ABC=\an ADC$ iff \[AC^2=CD \cd CE-AB\cd AE\]

\pr{19}\ns{1} The quadrilateral $ABCD$ is inscribed in a circle. The lines AB and CD meet at E, while the diagonals AC and BD meet at F. The circumcircle of the triangles AFD and BFC meet again at H. Prove that $\an EHF=\frac \pi2$.
%95-96

\pr{20}Prove that the radical axis of two intersecting circles passes through the intersection points.
%porselov 64

\pr{21} Given three circles in plane whose centers do not lie on one line. Let us draw
radical axes for each pair of these circles. Prove that all the three radical axes meet at one point. \fn{This point is called the radical center of the three circles.}
%porselov 64

\pr{22}\ns{2}Let M be the midpoint of the altitude BE in $\tr ABC$ and suppose that the excircle opposite B touches AC at Y. Then the line MY goes through the incenter I.

\pr{23}\ns{1}\fn{If the incircle touches AB at Z, then we can also deduce that B,Z,I,P,X are concyclic! This is indeed a very useful and surprizing result.} Let I be the incenter of $\tr ABC$. Then

AI is the bisector of
angle A. If X and Y are the points of contact of the incircle on BC and AC then prove that the lines AI, XY and the perpendicular from B to AI are concurrent at a point P.
%episodes.46

\pr{24}\ns{1} Let r,R be the inradius and circumradius of $\tr ABC$. Prove that \[\cos A+\cos B + \cos C=1+\frac{r}{R}\]

\pr{25}\ns{1}If h_a,h_b,h_c are the lengths of the altitudes of $\tr ABC$, whose incircle has center I and radius r. Prove that
\\ (a) \[\frac{r}{h_a}+\frac{r}{h_b}+\frac{r}{h_c}=1\]
\\(b) \[h_a+h_b+h_c \geq 9r. \]
\pr{26} (Leibniz's\fn{Have you heard of Leibniz's identity? If you haven't, ask someone!} Inequality) In a triangle ABC with circumradius R prove that $9R^2\geq a^2+b^2+c^2$.

\pr{26}\ns{2}Let ABC be an acute triangle. Let D be on side BC such that $AD\perp BC$. Let H be a point on the segment AD
different from A and D. Let line BH intersect side AC at E and line CH
intersect side AB at F. Prove that $\an EDA =\an FDA$.
%hk training canada 03

\pr{27}\ns{2}The circles S_1 and S_2 intersect at M and N. Show that if vertices A and C of a rectangle $ABCD$ lie on S_1 while vertices B and D lie on S_2, then the intersection of the diagonals of the rectangle lies on the line MN.
%po len russia

\pr{28}
The convex quadrilateral $ABCD$ is inscribed in the circle S_1. Let O be the intersection of AC and BD. Circle S_2 passes through D and O, intersecting AD and CD at M and N, respectively.
Lines OM and AB intersect at R, lines ON and BC intersect at T;
and R and T lie on the same side of line BD as A.
Prove that O, R,T, and B are concyclic.
%2001, p 12

\bigskip
\Lcen{Medium}

\pr{1} Two circles with centers A and B intersect at right angles. Their common chord meets AB at F. DE is a chord of the first circle passing through B. Prove that A,D,E,F are concyclic.
%108

\pr{2} \ns{1} L is the midpoint of the side BC of $\tr ABC$. The circle through L which touches AB at B and the circle through L which touches AC at C meet at D. Prove thar $LA \cd LD=LB^2$

%108

\pr{3} The centers of two circles S_1,S_2 and their common tangents intersect at T. AP and AQ are the tangents at A to the two circles. Prove that AT bisects $\an PAQ$.
%116

\pr{4} Two circles AP_1Q_1 and AP_2Q_2 cut at A. The lines P_1P_2,Q_1Q_2 are their common tangents. Prove that the circles AP_1P_2 and AQ_1Q_2 touch each other.
%117

\pr{5}\fn{Brocard angle}\ns{2}Prove that there exists a point P inside $\tr ABC$ such that $\an BA\Omega=\an CA\Omega=\an AC \Omega= \omega$. Prove that \[\cot \omega=\cot A+\cot B+\cot B\]
%125

\pr{6}\ns{1}Let the extension of DG meet the circumcircle at D_2. Prove that $GD=\frac 12 D_2G$. (G centroid,D feet of perpendicular from A.)
%135

\pr{7} \ns{1} If O_A,O_B,O_C are the reflections of O in the sides BC,CA,AB respectively, prove that AO_A,BO_B,CO_C are concurrent. (O cirumcenter).
%135

\pr{8} On sides AB and AC of triangle ABC there are given points D,E such that DE is tangent of circle inscribed in triangle ABC and $DE \parallel BC$. Prove
\[AB + BC + CA\geq 8DE\]

\pr{9} For every triangle ABC, let D,E,F be a point located on segment BC,CA,AB, respectively. Let P be the intersection of AD and EF. Prove that:
\[
\frac {AB}{AF}\times DC + \frac {AC}{AE}\times DB = \frac {AD}{AP}\times BC
\]
%http://www.mathlinks.ro/viewtopic.php?t=294062

\pr{10}\ns{1} Prove that in any triangle ABC ,
\[\sqrt {r_a} + \sqrt {r_b} + \sqrt {r_c}\ge3\sqrt {3r}\]
Where r,r_a,r_b,r_c are the radius of the incircle, $A-,B-,C-$ excircles of $\tr ABC$.
%http://www.mathlinks.ro/viewtopic.php?t=303038

\pr{11}\ns{2} Let ABC be a triangle and let P be a point on the angle bisector AD, with D on BC. Let E, F and G be the intersections of AP, BP and CP with the circumcircle of the triangle, respectively. Let H be the intersection of EF and AC, and let I be the intersection of EG and AB. Determine the geometric place of the intersection of BH and CI when P varies.
%http://www.mathlinks.ro/viewtopic.php?t=283704

\pr{12}\ns{1} A circle has center on the side AB of the cyclic quadrilateral $ABCD$. The other three

sides are tangent to the circle. Prove that $AD + BC = AB$.
% IMO 1985/1

\pr{13}\ns{1}A circumference α intersects with circumference β in points A and B. There is a tangent line to both circumferences α and β which intersects them in points C and D respectively. Points C, D, B (B is closer to the tangent line) lie on the circumference γ. Prove, that the radius of circumference γ is the geometric mean of the radiuses of the circumferences α and β.
% http://www.mathlinks.ro/viewtopic.php?p=1511237#1511237

\pr{14}\fn{This problem is a real gem. It has multiple solutions with unique and very,very beautiful ideas. This is USAMO 97.} Let ABC be a triangle, and draw isosceles triangles $\triangle DBC$, $\triangle AEC$, $\triangle ABF$ external to $\triangle ABC$ (with BC, CA, AB as their respective bases). Prove that the lines through A,B,C perpendicular to EF,FD,DE, respectively, are concurrent.
% USAMO 97

\pr{15} \ns{2} ABC is a triangle, with inradius r and circumradius $R.$ Show that:
\[
\sin \left(\frac {A}{2} \right) \cdot \sin \left(\frac {B}{2} \right) + \sin \left(\frac {B}{2} \right) \cdot \sin \left(\frac {C}{2} \right) + \sin \left(\frac {C}{2} \right) \cdot \sin \left(\frac {A}{2} \right) \leq \frac {5}{8} + \frac {r}{4 \cdot R}.
\]
% http://www.mathlinks.ro/viewtopic.php?t=60393&sid=482ebc9e1d36aea6bde2fabe1c7be480

\pr{16}\ns{1}Let $ABCD$ be a cyclic quadrilateral. Prove that the incenters of triangles $\triangle ABC, \triangle BCD, \triangle CDA, \triangle DAB$ form a rectangle.
% unbound 68

\pr{17} \ns{1} Let $ABCD$ be a cyclic quadrilateral. Prove that the sum of the inradii of $\triangle ABC$ and $\triangle CDA$ equals the sum of the inradii of $\triangle BCD$ and $\triangle DAB$.
% unbound 68

\pr{18}\ns{2}\fn{This is IMO 1978/4. A great problem if you want to learn some homothety. Recommended reading for Homothety: \url{http://www.math.ust.hk/excalibur/v9_n4.pdf}} In a triangle ABC we have $AB = AC.$ A circle which is internally tangent with the circumscribed circle of the triangle is also tangent to the sides AB, AC in the points $P,$ respectively $Q.$ Prove that the midpoint of PQ is the center of the inscribed circle of the triangle $ABC.$
%

\pr{19}\ns{1} Let O be the point of intersection of the diagonals AC and BD of the quadrilateral $ABCD$ with $AB = BC$ and $CD = DA$. Again, let N and K be the feet of perpendiculars from D and B to AB and CD, respectively.

Prove that the points N, O, and K are collinear.
%http://www.mathlinks.ro/viewtopic.php?p=1415913#1415913

\pr{20}\ns{1} In triangle ABC, let AK, BL, CM be the altitudes and H the orthocenter. Let P be the midpoint of AH. If BH and $M K$ meet at S, and LP and AM meet at T, show that TS is perpendicular to BC.
%1998-1999,p64

\pr{21}\ns{2} Let D,E,F be the points on the sides BC,CA,AB respectively, of $\tr ABC$. Let P,Q,R be the second intersection of AD,BE,CF respectively, with the cricumcircle of $\tr ABC$. Show that
\[\frac{AD}{PD}+\frac{BE}{QE}+\frac{CF}{RF}\geq 9\]
%1998,p97

\pr{22}\ns{1} Let ABC be an acute triangle, AD,BE,CZ be its altitudes and H its orthocenter. Let AI,AI' be the internal and external bisectors of angles A. Let M,N be the midpoints of BC,AH, respectively. Prove that
\begin{itemize}
\item MN is perpendicular to EZ.
\item If MN cuts the segments AI,AI' at the points K,L, then $KL=AH$.
\end{itemize}
%95j p28

\pr{23}\ns{2}Points D and E lie on sides AB and AC of triangle ABC such
that $DE \parallel BC$. Let P be an arbitrary point inside ABC. The lines
PB and PC intersect DE at F and G, respectively. If O_1 is the
circumcenter of PDG and O_2 is the circumcenter of PFE, show
that $AP \parallel O_1O_2$.
%95-96

\pr{24}\ns{2} Let $ABCDEF$ be a convex hexagon such that $AB=BC,CD=DE,EF=FA$. Prove that
\[\frac{BC}{BE}+\frac{DE}{DA}+\frac{FA}{FC}\geq \frac 32 . \]
%ISL 97-7

\pr{25}\ns{2} \textit{Nagel Point N: If the Excircles of ABC touch BC,CA,AB at D,E,F, then the intersection point of AD,BE,CF is called the Nagel Point N.} Prove that
\begin{itemize}
\item I,G,N are collinear. (G centroid, I incenter.)
\item $GN=2\cd IG$.
\item \textit{Speiker center S: The incircle of the medial triangle is called the Speiker circle, and it's center is Speiker center S.} Prove that S is the midpoint of IN.
\end{itemize}
%episodes

\pr{26}\ns{1}(a) (\textit{Archemides' Theorem})Let M be the midpoint of the arc ACB on the circumcircle of $\tr ABC$ and let MD be the perpendicular to the longer of AC and BC (say AC).

Then D bisects the polygonal path ACB that is $AD = DC + CB$.
\\ (b) Let C' be the midpoint of side AB. Prove that CD is parallel to the angle bisector of $\an C$.
\\(c) In the same way define $B'E$, $A'F$, and prove that $C'E,B'E,A'F$ are concurrent at the incenter of $\tr ABC$.

\pr{27}\ns{1} If three cevians AD,BE,CF of $\tr ABC$ are concurrent at P. Prove that \[\frac{AD}{AP}+\frac{BE}{BP}+\frac{CF}{CP} \geq \frac 92 .\]
%episodes

\pr{28}\ns{1} Let $ABCD$ be a convex quadrilateral such that $\an DAB = \an ABC = \an BCD$. Let H and O denote the orthocenter and circumcenter of ABC. Prove that D,O,H are collinear.
%polen bul 97

\bigskip
\Lcen{\ff Not so easy!!! \fn{Hey...Don't panic!!! These are simply some of the coolest Olympiad problems. They are worth trying even if you can not solve them.}}\e
\pr{1}\ns{1} Let I and G be the incenter and the centroid of the given triangle $ABC.$ Let $M,\,N,\,P$ be the midpoint of $BC,\,CA,\,AB,$ respectively and let J be the incenter of triangle $MNP.$
Then we have: $I,\,G,\,J$ are collinear and $GI = 2\cdot GJ$
%http://www.mathlinks.ro/viewtopic.php?t=156502

\pr{2} \ns{2} Let $D. E, F$ be the feet of the angle bisectors of angles A, B, C, respectively, of triangle ABC, and let K_a, K_b, K_c be the points of contact of the tangents to the incircle of ABC through D, E, F (that is, the tangent lines not containing sides of the triangle).

Prove that the lines joining K_a, K_b, K_c to the midpoints of $BC, CA. AB$ respectively, pass through a single point on the incircle of ABC.
%http://www.mathlinks.ro/viewtopic.php?t=283779

\pr{3} In triangle ABC, with $AB > BC$, BM is a median and BL an angle bisector. The line through M parallel to AB meets BL at D and the line through L parallel to BC meets BP at E.
Prove that $ED \perp BL$.
%http://www.mathlinks.ro/viewtopic.php?p=1532155#1532155

\pr{4}\ns{2} Let O be the circumcircle of a ΔABC and let I be its incenter, for a point P of the plane let $f(P)$ be the point obtained by reflecting P' by the midpoint of OI, with P' the homothety of P with center O and ratio $\frac {R}{r}$ with r the inradii and R the circumradii, (understand it by $\frac {OP}{OP'} = \frac {R}{r}$). Let A_1, B_1 and C_1 the midpoints of BC, AC and AB, respectively. Show that the rays $A_1f(A)$, $B_1f(B)$ and $C_1f(C)$ concur on the incircle.
%http://www.mathlinks.ro/viewtopic.php?t=283715

\pr{5}\ns{2}Let ABC be an acute-angled triangle, and let P and Q be two points on its side BC. Construct a point C_{1} in such a way that the convex quadrilateral $APBC_{1}$ is cyclic,

$QC_{1}\parallel CA$, and the points C_{1} and Q lie on opposite sides of the line AB. Construct a point B_{1} in such a way that the convex quadrilateral $APCB_{1}$ is cyclic, $QB_{1}\parallel BA$, and the points B_{1} and Q lie on opposite sides of the line AC.

Prove that the points B_{1}, C_{1}, P, and Q lie on a circle.
%http://www.mathlinks.ro/viewtopic.php?p=213011#213011

\pr{6}\ns{3}Let $ABCD$ be a quadrilateral, and let E and F be points on sides AD and $BC,$ respectively, such that $\frac {AE}{ED} = \frac {BF}{FC}.$ Ray FE meets rays BA and CD at S and $T,$ respectively. Prove that the circumcircles of triangles $SAE,$ $SBF,$ $TCF,$ and TDE pass through a common point.
%http://www.mathlinks.ro/viewtopic.php?p=490691#490691

\pr{7}\ns{3} Let ABC be an acute triangle with ω,S, and R being its incircle, circumcircle, and circumradius, respectively. Circle ω_{A} is tangent internally to S at A and tangent externally to ω. Circle S_{A} is tangent internally to S at A and tangent internally to ω. Let P_{A} and Q_{A} denote the centers of ω_{A} and S_{A}, respectively. Define points $P_{B}, Q_{B}, P_{C}, Q_{C}$ analogously. Prove that
\[
8P_{A}Q_{A}\cdot P_{B}Q_{B}\cdot P_{C}Q_{C}\leq R^{3}\; ,
\]
with equality if and only if triangle ABC is equilateral.
%http://www.mathlinks.ro/viewtopic.php?p=825515#p825515

\pr{8}\ns{1} Let ABC be a triangle. A circle passing through A and B intersects segments AC and BC at D and E, respectively. Lines AB and DE intersect at F, while lines BD and CF intersect at M. Prove that $MF = MC$ if and only if $MB\cdot MD = MC^2$.
%http://www.mathlinks.ro/viewtopic.php?p=336205#336205

\pr{9}\ns{1}Let ABC be an acute, scalene triangle, and let M, N, and P be the midpoints of $\overline{BC}$, $\overline{CA}$, and $\overline{AB}$, respectively. Let the perpendicular bisectors of $\overline{AB}$ and $\overline{AC}$ intersect ray AM in points D and E respectively, and let lines BD and CE intersect in point F, inside of triangle ABC. Prove that points A, N, F, and P all lie on one circle.
%http://www.mathlinks.ro/viewtopic.php?p=1116181#1116181

\pr{10}\ns{2}$ABCDEF$ is a convex hexagon with $AB = BC$, $CD = DE$,$EF = FA$.
Prove that
\[
\frac {{BC}} {{BE}} + \frac {{DE}} {{DA}} + \frac {{FA}} {{FC}} \geqslant \frac {3} {2}
\]
%http://www.mathlinks.ro/viewtopic.php?t=283183

\pr{11} \ns{2} An acute triangle ABC is given. Points A_1 and A_2 are taken on the side BC (with A_2 between A_1 and C), B_1 and B_2 on the side AC (with B_2 between B_1

and A), and C_1 and C_2 on the side AB (with C_2 between C_1 and B) so that
\[
\angle AA_1A_2 = \angle AA_2A_1 = \angle BB_1B_2 = \angle BB_2B_1 = \angle CC_1C_2 = \angle CC_2C_1.
\]
The lines $AA_1,BB_1,$ and CC_1 bound a triangle, and the lines $AA_2,BB_2,$ and CC_2 bound a second triangle. Prove that all six vertices of these two triangles lie on a single circle.
%http://www.mathlinks.ro/viewtopic.php?t=219895

\pr{12}\ns{1}The incircle k of a non-isosceles triangle ABC touches the sides AB,BC,CA at C_{1},A_{1},B_{1} , respectively. Lines AA_{1},BB_{1},CC_{1} intersect k again at A_{2},B_{2},C_{2}, respectively. Let $A_{1}A_{3}$ and B_{1},B_{3} be the bisectors of the angles in △$A_{1}B_{1}C_{1}$ (A_{3} ∈ $B_{1}C_{1}, B_{3}$ ∈ $A_{1}C_{1}$). Prove that:

(a) $A_{2}A_{3}$ is a bisector of $\angle B_{1}A_{2}C_{1}$;

(b) If the circumcircles of △$A_{1}A_{2}A_{3}$ and △$B_{1}B_{2}B_{3}$ intersect at P and Q, then the incenter I of △ABC lies on the line PQ.
%http://www.mathlinks.ro/viewtopic.php?p=1516814#1516814

\pr{13}\ns{1} Let ABC be a triangle, O its circumcenter, S its centroid, and H its orthocenter. Denote by A_1, B_1 and C_1 the centers of the circles circumscribed about the triangles CHB, CHA and AHB respectively.

Prove that the triangle ABC is congruent to the triangle $A_1B_1C_1$ and that the nine-point circle of ABC is also the nine-point circle of $A_1B_1C_1$.
%http://www.mathlinks.ro/viewtopic.php?t=280779

\pr{14}\ns{1}Let ABC be a triangle and K and L be two points on (AB), (AC) such that $BK = CL$ and let $P = CK\cap BL$. Let the parallel through P to the interior angle bisector of $\angle BAC$ intersect AC in M. Prove that $CM = AB$.
%http://www.mathlinks.ro/viewtopic.php?p=1510314#1510314

\pr{15} Let ABC be an acute-angled triangle, and let H be its orthocenter. Let D be the foot of the altitude from B to AC, and let E be the reflection of A on D. The circumcircle of triangle BCE intersects the median from A in an interior point F. Prove that A, D, H and F are concyclic.
%http://www.mathlinks.ro/viewtopic.php?t=255138

\pr{16} \ns{1} Let ABC be triangle with $AB \neq AC$. Point E is such that $AE=BE$ and $BE\perp BC$. Point F is such that $AF=CF$ and $CD \perp BC$. Let D be the point on the line BC such that AD is tangent to the circumcircle of triangle ABC. Prove that D,E,F are collinear.
%Russia 2003

\pr{17} \ns{1} Let ABC be an isosceles triangle with $AB = AC$. M is the midpoint of BC and O is the point on the line AM such that OB is perpendicular to AB. Q is an arbitrary point on BC

different from B and C. E lies on the line AB and F lies on the line AC such that E, Q, F are distinct and collinear. Prove that OQ is perpendicular to EF if and only if $QE = QF$.
% IMO 94/2

\pr{18}\ns{1} An acute-angled triangle ABC is given in the plane. The circle with diameter $\, AB \,$ intersects altitude $\, CC' \,$ and its extension at points $\, M \,$ and $\, N \,$, and the circle with diameter $\, AC \,$ intersects altitude $\, BB' \,$ and its extensions at $\, P \,$ and $\, Q \,$. Prove that the points $\, M, N, P, Q \,$ lie on a common circle.
% USAMO 90/5

\pr{19}\ns{2} Let O and I be the circumcenter and incenter of triangle $\triangle ABC$, respectively. Let ω_A be the excircle of triangle $\triangle ABC$
opposite
to A; let it be tangent to AB, AC, BC
at K,M,N, respectively.
Assume that the midpoint of segment KM lies on the circumcircle
of triangle $\triangle ABC$. Prove that O,N,I are collinear.
%

\pr{20}\ns{1} Let O be the center of circle ω. Two equal chords of AB and CD of ω intersects at L such that $AL>LB$, and $DL>LC$. Let M and N be points on AL and DL respectively such that $\an ALC=2\an MON$. Prove that the chord of ω passing through M and N is congruent to AB and CD.
% Around the world 1999,p24

\pr{21}\ns{2}Let O be the center of the excircle of $\tr ABC$ opposite to A. Let M be the midpoint of AC, and let P be the intersection of lines MO and BC. Prove that if $\an BAC=2 \an ACB$, then $AB=BP$.
% Around the world 1999,p29

\pr{22}\ns{1}\fn{There is a very ingenious solution using Pole-Polar. For further study:
\url{http://www.math.ust.hk/excalibur/v11_n3.pdf}} Let $ABCD$ be a cyclic
quadrilateral. Let $AB\cap CD = P$\fn{$AB\cap CD=X$ means that the intersection point of AB and CD is X. This sign is very common in problem literature.} and
$AD\cap BC = Q$. Let the tangents from Q
meet the circumcircle of $ABCD$ at E
and F. Prove that P, E, F are collinear.
%pole polar

\bigskip
\lcen{IMO 2009: \textit{Why don't we try them?}}
\pr{2} (\textit{Day 1}) Let ABC be a triangle with circumcentre O. The points P and Q are interior points of the sides CA and AB respectively. Let K,L and M be the midpoints of the segments BP,CQ and PQ. respectively, and let Γ be the circle passing through K,L and M. Suppose that the line PQ is tangent to the circle Γ. Prove that $OP = OQ.$

\pr{4} (\textit{Day 2}) Let ABC be a triangle with $AB = AC$. The angle bisectors of $\angle C AB$ and $\angle AB C$ meet the sides $B C$ and $C A$ at D and E , respectively. Let K be the incentre of triangle ADC. Suppose that $\angle B E K = 45^\circ$. Find all possible values of $\angle C AB$.
(Don't forget to check the cases, anyway.)
\end{document}