Mock IMO 2

1. Let n be an integer greater than 1. For a positive integer m, let $S_m = \{1, 2, \ldots, mn\}$. Suppose that there exists a $2n$-element set T such that

   (a) each element of T is an m-element subset of S_m;

   (b) each pair of elements of T shares at most one common element; and

   (c) each element of S_m is contained in exactly two elements of T.

   Determine the maximum possible value of m in terms of n.

2. Given positive integers a and c and integer b, prove that there exists a positive integer x such that $a^x + x \equiv b \pmod{c}$.

3. Let $\mathcal{P}$ be a convex polygon. A segment connecting two points on (distinct) sides of $\mathcal{P}$ is called a *chord*. Suppose that if a chord bisects the area of the polygonal region, then the chord has length at most 1. Prove that the area of $\mathcal{P}$ is at most $\frac{\pi}{4}$.