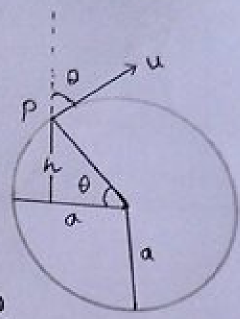
Solutions

Prob-1 Consider a lump jumping of from point P. It attains max hight,



$$h_0 = \frac{u_y^2}{2g} = \frac{u^2\cos^2\theta}{2g}$$

$\therefore$ Total hight from ground,

$$H = a + h + h_0 = a + a\sin\theta + \frac{u^2\cos^2\theta}{2g}$$

$$\therefore \frac{dH}{d\theta} = 0 = a\cos\theta - \frac{u^2}{g}\cos\theta\sin\theta$$

it has two solutions: $\cos\theta = 0$; $\theta = \frac{\pi}{2}$; but this gives $H = 2a$ and implies $u^2 \leq ga$ [check 2nd derivatie*]. So we go for other solution,

$$\sin\theta = \frac{ga}{u^2}$$

So,

$$\boxed{H = a + \frac{u^2}{2g} + \frac{ga^2}{2u^2}}$$

* $\theta = \frac{\pi}{2}$ gives a maximum if one assumes $u^2 \leq ga$ in 2nd derivative test. Also, ~~H = 2R~~ $H = 2a$ solution is not somewhat physical.

Prob-2. At the instant the spring compresses at maximum, the projectile m and the target M moves with the same velocity V_e. Conservation of energy:

$$\frac{mV^2}{2} = \frac{mV_e^2}{2} + \frac{MV_e^2}{2} + \frac{k(\Delta x)^2}{2}$$

Conservation of momentum:

$$mV = (m+M)V_e$$

$$\boxed{\therefore \Delta x = \sqrt{\frac{mM}{k(m+M)}}\,V}$$

Prob-3 Let ω, ω', J and J' be the angular velocity of the ball with respect to its center of mass and its angular momentum about point of impact A before and after the collision ~~respectly~~ respectively. We have,

$$J = mv(a-h) + \frac{2}{5} ma^2\omega = \frac{7}{5} mva - mvh$$

as $v = a\omega$ for rolling without sliping and,

$$J' = \frac{2}{5} m a^2 \omega' + m\cdot(\omega' a)\cdot a = \frac{7}{5} m a^2 \omega'.$$

as the centre of mass of the ball is momentarily at rest after the collision. Conservation of angular momentum gives,

$$\frac{7}{5} ma^2\omega' = \frac{7}{5} m va - mvh$$

$$\therefore \omega' = \left(1 - \frac{5h}{7a}\right)\frac{v}{a}$$

In order the ball can just tip over the step, its kinetic energy must be sufficient to provide the increase in potential energy.

$$\frac{1}{2} I'\omega'^2 = mgh$$

where, $I' = \frac{2}{5} ma^2 + ma^2 = \frac{7}{5} ma^2$

So,

$$\frac{7}{5} ma^2 \left(1 - \frac{5h}{7a}\right)^2 \left(\frac{v}{a}\right)^2 = mgh$$

$$\boxed{\therefore v = \frac{a\sqrt{70gh}}{7a - 5h}}$$

Prob - 4

(a) Let the velocity while passing equilibrium be v.

$$\therefore N - mg = \frac{mv^2}{R}$$

Energy conservation gives,

$$mgR(1-\cos\theta_0) = \frac{1}{2}mv^2 + \frac{1}{2}I\cdot\left(\frac{v}{r}\right)^2 \quad \left[I = \frac{1}{2}mr^2\right]$$

$$\therefore v^2 = \frac{4}{3}\,mgR(1-\cos\theta_0)$$

$$\therefore N = mg + \frac{4}{3}mg(1-\cos\theta_0)$$

$$\boxed{\therefore N = \frac{1}{3}(7 - 4\cos\theta_0)}$$

(b) $$(x, y) = \left((R-r)\sin\theta,\ -(R-r)\cos\theta\right)$$

(c) $$(R-r)\theta = r\phi$$

(d) Lagrangian,

$$\mathcal{L} = T - V$$

$$= \frac{1}{2}m(R-r)^2\dot{\theta}^2 + \frac{1}{4}mr^2\dot{\varphi}^2 + mg(R-r)\cos\theta$$

$$= \frac{3}{4}m(R-r)^2\dot{\theta}^2 + mg(R-r)\cos\theta$$

[use result (c) to eliminate $\dot{\varphi}$]

So,

$$\therefore \frac{d}{dt}\left(\frac{\partial\mathcal{L}}{\partial\dot{\theta}}\right) = \frac{\partial\mathcal{L}}{\partial\theta}$$

$$\Rightarrow \boxed{\ddot{\theta} + \frac{2}{3}\left(\frac{g}{R-r}\right)\sin\theta = 0}$$

(e) Put $\sin\theta \simeq \theta$ for small angle

$$\therefore T = \frac{2\pi}{\omega} = \boxed{\pi\sqrt{\frac{6(R-r)}{g}}}$$