Summer 2005 IMO camp

PC-TP1

Consider a Cartesian plane. Let H_k be the segment that joins the points $(1, k)$ and $(2005, k)$, and let V_k be the segment that joins the points $(k, 1)$ and $(k, 2005)$. Let G be the grid consisting of all the segments H_k and V_k, where $k = 1, 2, 3, \ldots, 2005$. How many rectangles have their four sides contained in the grid G?

PC-TP2

Find all real numbers x, y and z for which $x + yz = 1$, $y + zx = 1$ and $z + xy = 1$.

PC-TP3

Prove that, for any triangle, if h_a, h_b and h_c denote the three altitudes of the triangle, then $(h_a - h_b)h_c < h_a h_b < (h_a + h_b)h_c$.

PC-TP4

For $n = 1, 2, 3, \ldots,$ let a_n denote the real number closest to . Compute .

PC-TP5

Show that, for any positive numbers a, b, c and d: $a^3 + b^3 + c^3 + d^3 \quad a^2 b + b^2 c + c^2 d + d^2 a$. Describe all the cases in which the equality occur.

PC-TP6

A point P, with coordinates (x, y) in a Cartesian plane is said to be an "knot-point" if and only if both x and y are integer numbers. Is there any circumference that passes through exactly five "knot-points"?