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Math Camp
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Practice Problems

(a) Find integers a and b such that

$$(a + b\sqrt{3})(2 + \sqrt{3}) = 3.$$

(b) Let a_1, a_2, a_3, a_4 be integers. If

$$(2 + a_1 + a_2 + a_3 + a_4)(a_1 - a_2 + a_3 - a_4) \neq 0$$

and r is a root of

$$x^5 + a_1x^4 + a_2x^3 + a_3x^2 + a_4x + 1 = 0$$

then r is irrational.

15 16 17

(c) The polynomial $1 - x + x^2 - x^3 + \ldots - x^{15} + x^{16} - x^{17}$ can be written as a polynomial in $y = x + 1$. Find the coefficient of y^2.

(d) Let $\alpha = 2 + \sqrt{17}$. Find rational numbers a_0, a_1, a_2 such that

$$a_0\alpha^2 + a_1\alpha + a_2 = 0.$$

(e) Determine the rational roots of

$$x^4 - 4x^3 - 8x^2 + 13x + 10 = 0.$$

(f) If $r^3 = 17$ then r is not rational.

(g) Given that

$$\begin{aligned} x + y + z &= a \\ x^2 + y^2 + z^2 &= b \\ x^3 + y^3 + z^3 &= c \end{aligned}$$

determine xyz in terms of a, b and c.

(h) Find the sum of the solutions to $x^{1/4} = 12/(7 - x^{1/4})$.

(i) Find the product of the real roots of the equation $x^2 + 18x + 30 = 2\sqrt{(x^2 + 18x + 45)}$.

(j) Let

$$f(x) = x^4 + x^3 + x^2 + x + 1.$$

Find the remainder when $f(x^5)$ is divided by $f(x)$.

(d′) $\alpha = 2 + \sqrt[3]{17}$ find rat'l cubic poly with α as a root

(k) If $x^3 + x^2 - 20 = 0$ and $y^2 + y - 48 = 0$

show $x+y$ is a root of a polynomial with integer coefficients.

(d″) $\alpha = 2 + \sqrt[3]{17} + 4 \cdot (\sqrt[3]{17})^2 \ldots$