40 Functional Equations

1 Favorites

1. $f:\mathbb{N}\longrightarrow\mathbb{N},\ f(n+1)>f(f(n))$ for all $n\in\mathbb{N}$
2. Find a function $f:\mathbb{N}\longrightarrow\mathbb{N}$ such that, $f(f(n))=2n$ for all $n\in\mathbb{N}$
3. $f:\mathbb{N}\longrightarrow\mathbb{N},\ f(m)^2+f(n)\mid(m^2+n)^2$ for all $m,\ n\in\mathbb{N}$
4. $f:\mathbb{N}\longrightarrow\mathbb{N},\ f(f(a)+f(b))=a+b-1$ for all $a,b\in\mathbb{N}$
5. $f:\mathbb{N}\longrightarrow\mathbb{N},$ for all $a,b\in\mathbb{N}$ there exists a non-degenerate triangle with side length a, $f(b)$, $f(b+f(a)-1)$.
6. Let $g:\ \mathbb{R}\longrightarrow\mathbb{R}$, such that $g(x)=x-[x]$.
   Find all functions $f:\ \mathbb{R}\longrightarrow\mathbb{R}$ such that, $g(f(x+y))=g(f(x))+g(f(y))$ for all $x,y\in\mathbb{R}$.
7. $f:\mathbb{R}\longrightarrow\mathbb{R},\ (f(x)+f(z))(f(y)+f(t))=f(xy-zt)+f(xt-yz)$ for all $x,y,z,t\in\mathbb{R}$.
8. $f:\mathbb{R}_+\longrightarrow\mathbb{R}_+,\ f$ continuous, $f(x)+f(y)=f\left(\frac{x+y}{2}\right)+f\left(\frac{2xy}{x+y}\right)$ for all $x,y\in\mathbb{R}_+$.
9. f: $\mathbb{R}\longrightarrow\mathbb{R},\ f(x+y)=\max\{f(x),y\}+\min\{f(y),x\}$ for all $x,y\in\mathbb{R}$.
10. f: $\mathbb{R}\longrightarrow\mathbb{R},\ f(x-f(y))=f(f(y))+xf(y)+f(x)-1$ for all $x,y\in\mathbb{R}$.
11. f: $\mathbb{R}\longrightarrow\mathbb{R},\ f(x)\geq e^x$ for all $x\in\mathbb{R}$ and $f(x+y)\geq f(x)f(y)$ for all $x,y\in\mathbb{R}$.
12. Prove that there are no functions f,g such that, $f(g(x))=x^2$ and $g(f(x))=x^3$ for all $x\in\mathbb{R}$.
13. Does there exist any continuous function $f:\mathbb{R}\longrightarrow\mathbb{R}$ such that, $f(x)\in\mathbb{Q}\Longleftrightarrow x\notin\mathbb{Q}$?
14. $f:\mathbb{R}_+\longrightarrow\mathbb{R}_+,\ f\left(yf\left(\frac{x}{y}\right)\right)=\frac{x^4}{f(y)}$ for all $x,y\in\mathbb{R}_+$.
15. $f:\mathbb{R}\longrightarrow\mathbb{R},\ |f(x)-f(y)|\leq(x-y)^2$ for all $x,y\in\mathbb{R}$.
16. $f:\mathbb{R}\longrightarrow\mathbb{R},\ f(x+f(y))=y+f(x)$ for all $x,y\in\mathbb{R}$ and the set $\left\{\frac{x}{f(x)}\ \middle|\ x\in\mathbb{R}\right\}$ is finite.
17. $f:\mathbb{R}_+\longrightarrow\mathbb{R},\ f(x)f(y)=y^{\alpha}f\left(\frac{x}{2}\right)+x^{\beta}f\left(\frac{y}{2}\right)$ for some constant $\alpha,\beta\in R$ and for all $x,y\in\mathbb{R}_+$
18. $f:\mathbb{R}_+\longrightarrow\mathbb{R},\ xf(y)-yf(x)=f\left(\frac{x}{y}\right)$ for all $x,y\in\mathbb{R}_+$
19. $f:\mathbb{R}\longrightarrow\mathbb{R},\ f(f(x)-y^2)=f(x)^2-2f(x)y^2+f(f(y))$ for all $x,y\in\mathbb{R}$
20. $f:\mathbb{R}\longrightarrow\mathbb{R},\ f(f(x)+y)=xf(1+xy)$ for all $x,y\in\mathbb{R}$
21. $f:\mathbb{R}\longrightarrow\mathbb{R},\ f\left(\frac{x+f(x)}{2}+y+f(2z)\right)=2x-f(x)+f(f(y))+2f(z)$ for all $x,y,z\in\mathbb{R}$
22. $f:\mathbb{R}\longrightarrow\mathbb{R},\ f$ surjective and strictly increasing, $f(f(x))=f(x)+12x$ for all $x\in\mathbb{R}$
23. $f:\mathbb{R}\longrightarrow\mathbb{R},\ f(x+y^2)\geq(y+1)f(x)^2$ for all $x,y\in\mathbb{R}$
24. $f:\mathbb{R}\longrightarrow\mathbb{R},\ f(y)f(xf(y))=f(xy)$ for all $x,y\in\mathbb{R}$
25. $f:\mathbb{R}\longrightarrow\mathbb{R},\ f(x)f(y)\leq|x-y|$ for all $x\in\mathbb{Q}$ and $y\notin\mathbb{Q}$
26. $f:\mathbb{R}\longrightarrow\mathbb{R},\ f(x+y)\leq yf(x)+f(f(x))$, Prove that $f(x)=0$ for all $x\leq 0$
27. $f:\mathbb{R}\longrightarrow\mathbb{R},\ f((x+1)f(y))=y(f(x)+1)$ for all $x,y\in\mathbb{R}$
28. $f:\mathbb{R}_+\longrightarrow\mathbb{R}_+,\ f(x+y)\geq f(x)+yf(f(x))$ for all $x,y\in\mathbb{R}_+$
29. $f:\mathbb{R}_+\times\mathbb{R}_+\longrightarrow\mathbb{R}_+,\ xf(x,y)f\left(y,\frac{1}{x}\right)=yf(y,x)$ for all $x,y\in\mathbb{R}_+$
30. $f:\mathbb{R}_+\longrightarrow\mathbb{R}_+,\ f(x)^2\geq f(x+y)(f(x)+y)$ for all $x,y\in\mathbb{R}_+$
31. $f:\mathbb{R}\longrightarrow\mathbb{R},\ f(xy)(f(x)-f(y))=f(x)f(y)(x-y)$ for all $x,y\in\mathbb{R}$

32. Prove that there doesn't exist any function $f:\mathbb{R}\longrightarrow\mathbb{R}$, such that,
    - $f(1)=1$
    - $\exists M\in\mathbb{R}_+$ s.t. $|f(x)|\leq M\ \forall x\in\mathbb{R}$
    - $f\left(x+\frac{1}{x^2}\right)=f(x)+f\left(\frac{1}{x}\right)^2$ for all $x\in\mathbb{R}$
33. $f:\mathbb{R}_0\longrightarrow\mathbb{R}$ and $f(x)\leq\int_0^x f(t)\,\mathrm{d}t$ for all $x\geq 0$. Prove that, $f(x)=0$ for all $x\geq 0$
34. $f:\mathbb{R}_+\longrightarrow\mathbb{R}_+$, $f(x+y^n+f(x))=f(x)$, $\frac{f(x)+x^n}{f(y)+y^n}\in\mathbb{Q}$ for all $x,y\in\mathbb{R}_+$
35. $f:\mathbb{R}_+\longrightarrow\mathbb{R}_+$, $f(x+y^n+f(x))=f(x)$ for all $x,y\in\mathbb{R}_+$

2 Most Favorites

1. Find all functions $f:\mathbb{R}_+\longrightarrow\mathbb{R}_+$, such that,
   $f(x-f(y))=f(x+y^n)+f(y+f(y))$ for all $x,y\in\mathbb{R}_+$ and a fixed positive integer $n\geq 2$.
2. Find all continuous functions $f:\mathbb{R}_+\longrightarrow\mathbb{R}_+$, such that, $f(xf(y)+yf(x))=f(f(xy))$ for all $x,y\in\mathbb{R}_+$
3. f is a function such that $f'(x)=\frac{x^2-f(x)^2}{x^2(f(x)^2+1)}$ for all $x>1$. Prove that, $\lim_{x\longrightarrow\infty}f(x)=\infty$
4. Is there any strictly increasing function $f:\mathbb{R}_+\longrightarrow\mathbb{R}_+$, such that, $f'(x)=f(f(x))$?
5. $f:\mathbb{R}_+\longrightarrow\mathbb{R}_+$ such that,
    - $f(x)=x$ if $x\leq e$
    - $f(x)=xf(\ln x)$ if $x>e$

   Prove that $\sum_{n=1}^{\infty}\frac{1}{f(x)}$ diverges.

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