IMO Training 2007
July 17, 2007

Mock IMO 4

1. Suppose H and O are orthocentre and circumcentre of triangle ABC. ω is the circumcircle of ABC. AO intersects ω at A_1. A_1H intersects ω at A' and A'' is the intersection point of ω and AH. We define B', B'', C' and C'' similarly. Prove that $A'A'', B'B''$ and $C'C''$ are concurrent at a point on the Euler line of triangle ABC.

   (The Euler line is the line that passes through the circumcentre, the centroid and the orthocentre of ABC)

2. Let P be a polynomial with real coefficients such that $P(\sin x) = P(\cos x)$ for all reals x. Prove that there exists a polynomial Q with real coefficients such that $P(x) = Q(x^4 - x^2)$ for all reals x.

3. Each edge of a convex polyhedron is oriented with an arrow in such a way that for every vertex v, there is at least one arrow entering v and at least one arrow leaving v. Prove that there exists a face of the polyhedron such that its incident edges form an oriented cycle.