A.C.M.O.

7/8/98

✓3. A sequence of real numbers $x_0, x_1, \ldots$ is defined as follows: $x_0 = 1989$ and for each $n \geq 1$

$$x_n = -\frac{1989}{n}\sum_{k=0}^{n-1} x_k$$

Calculate the value of $\displaystyle\sum_{k=0}^{1989} 2^k x_k$

✓5. Let $f(x)$ satisfy, for all $x \geq 1$, the equation

$$f(x)^2 = 1 + x f(x+1)$$

and the inequalities

$$\frac{x+1}{2} \leq f(x) \leq 2(x+1)$$

Prove that $f(x) = x + 1$

10. If $T(x) = x^3 + 14x^2 - 2x + 1$, show that there exists an $n > 0$ such that 101 divides $T^n(x) - x$ for all integers x.

$$\left(T^k(x) = \underbrace{T(T(T(\cdots(T(x))\cdots)))}_{k \text{ times}} \right)$$