2001 Winter Camp Mock Olympiad

S1. A square and an equilateral triangle are inscribed in the same circle. All seven vertices are different points. Prove that at least one of the seven arcs does not exceed $\frac{1}{24}$ of the circumference of the circle.

S2. For any positive integer n, let $f(n)$ be the number of positive divisors of n that are congruent to 1 modulo 4, and let $g(n)$ be the number of positive divisors of n that are congruent to 3 modulo 4.

Prove that $f(n) \geq g(n)$ for all positive integers n.

Determine all integers n for which $f(n) = g(n)$.

S3. A binary operation $*$ on real numbers has the property that $(a*b)*c = a+b+c$, for all real a, b, and c.

Prove that $a*b = a+b$ for all real a and b.