1998 IMO Camp

Problems Sheet

1. Prove that the line joining one vertex of a parallelogram to the midpoint of the opposite side trisects a diagonal of the parallelogram.

2. For an n-gon, label the vertices in any order $A_0, A_1, \ldots, A_{n-1}$. Starting at A_0, move to A_1, then halfway from A_1 to A_2, then one-third of the way to A_3, then one-fourth of the way to A_4, and so on until you travel one-n-th of the way from the point so reached back to A_0. Prove that no matter how the points are labelled, you alwayp arrive at the same point.

3. Let ABC and $A'BC$ be triangles inscribed in the same circle, with orthocentres H and H', respectively. Show that $AA'H'H$ is a parallelogram.

4. In $\triangle ABC$, we have $AB = AC$. Further, D is the midpoint of BC, E is the foot of the perpendicular drawn from D to AC and F is the midpoint of DE. Prove that $AF \perp BE$.

5. In the quadrilateral $ABCD$, the points P, Q, R and S are the midpoints of AB, BC, CD and DA. Prove that PR and QS bisect each other.

6. On the sides of an arbitrary parallelogram, squares are constructed exterior to it. Prove that the centres of these squares are themselves the vertices of a square.

7. A convex hexagon $ABCDEF$ is inscribed in a circle of radius r. $AB = CD = EF = r$. Let P, Q, R respectively be the midpoints of BC, DE and FA.
Prove that PQR is an equilateral triangle.

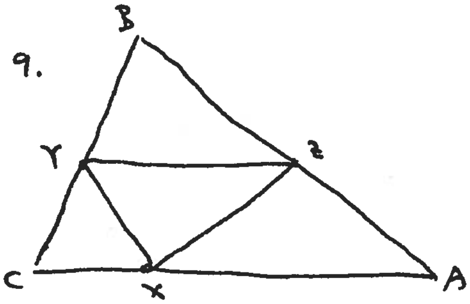
8. In $\triangle ABC$, prove that the areal coordinates of points I, O and H are as given:

$$I = \left(\frac{a}{a+b+c}, \frac{b}{a+b+c}, \frac{c}{a+b+c}\right)$$

$$O = (\sin 2A, \sin 2B, \sin 2C)$$

$$H = (\cot B \cot C, \cot C \cot A, \cot A \cot B) = (\tan A, \tan B, \tan C)$$

(for O, H must normalize)

9. 

Let X, Y, Z be a triangle whose vertices lie on AC, CB and BA, respectively.

Prove that among the triangles AXZ, BZY, CYZ and XYZ, triangle XYZ cannot have smallest area except when X, Y, Z are midpoints.

10. In $\triangle ABC$, let I be the incentre and let Q be the point on BC where the excircle that lies opposite A touches BC. Let A', B' and C' be the points where the incircle touches sides BC, CA and AB, respectively. Let M be the midpoint of AC.

Prove that the points Q, I and M are collinear if and only if $CA \times CB = 2\, AC' \times BC'$.

11. Let I be the incentre of $\triangle ABC$ and let A', B', C' be the points where the incircle is tangent to the sides BC, CA and AB, respectively. For any circle centred at I, let X, Y, Z be the points where this circle cuts the lines IA', IB' and IC', respectively.

Show that the lines AX, BY and CZ are concurrent.

12. See 13.

13. See 12.

14. There is no 14.