Let, $P(x,y) \Longrightarrow f(f(x)+y) = xf(1+xy)$

Now, $P(1,y) \Longrightarrow f(f(1)+y) = f(1+y)$

$\Longrightarrow f(y+nz) = f(y)$, for all $y > \max(1, f(1))$, $n \in \mathbb{N}$ where, $z = |1 - f(1)|$

If $z > 0$, take some $x < 1$, then, there exists some $n \in \mathbb{N}$, such that, $1 + nz > f(x)$

take, $y = \frac{1 + nz - f(x)}{1 - x}$, then, $P(x,y) \Longrightarrow x = 1$, contradiction!

So, $f(1) = 1$

Now, if, $(x-1)(f(x)-1) > 0$ for some $x \in \mathbb{R}^+$, then, $P\left(x, \frac{f(x)-1}{x-1}\right) \Longrightarrow x = 1$

So, $x > 1 \Longleftrightarrow f(x) < 1$ and $x < 1 \Longleftrightarrow f(x) > 1$

So, $P(x,1) \Longrightarrow xf(1+x) = f(f(x)+1) < 1 \Longrightarrow f(1+x) < \frac{1}{x} \Longrightarrow \lim_{x \longrightarrow \infty} f(x) = 0$

Again, $f(1+x) < \frac{1}{x}$, $P(x, y - f(x)) \Longrightarrow f(y) = xf(1 + x\,(y - f(x))) < \frac{x}{x(y - f(x))} = \frac{1}{y - f(x)}$ for all $y > f(x)$

But, we showed, $\forall \varepsilon > 0, \exists x > 0$ such that, $f(x) < \varepsilon$

So, $f(y) < \frac{1}{y - f(x)}$ is equivalent to,

$\forall \varepsilon > 0, \exists x > 0$, such that, $f(x) < \varepsilon \Longrightarrow f(y) < \frac{1}{y - f(x)} < \frac{1}{y - \varepsilon}$ for all $y > \varepsilon > f(x)$

Which is $f(y) \leq \frac{1}{y}$ for all $y > 0$

Now, $P(x, 1 - f(x))_{x > 1} \Longrightarrow f(1 + x(1 - f(x))) = \frac{1}{x} \leq \frac{1}{1 + x(1 - f(x))} \Longrightarrow f(x) \geq \frac{1}{x}$

So, $f(x) = \frac{1}{x}$ for all $x \geq 1$

Now, if $y < 1$, take $x > 1$ such that, $xy > 1$

Then, $P\left(x, y - \frac{1}{x}\right) \Longrightarrow f(y) = xf\left(1 + x\left(y - \frac{1}{x}\right)\right) = xf(xy) = \frac{x}{xy} = \frac{1}{y}$

Hence, $f(x) = \frac{1}{x}$ for all $x > 0$