MOLLOY PROBLEMS 1999

1. For a certain collection of closed discs in the plane, every set of 3 has a point (circumferential or interior) in common. Prove that the entire collection has a point in common.

2. For a certain collection of (infinite) lines, no three have a common point. Prove that the colours Red (R), Blue (B) and Green (G) can be assigned to the intersections points of pairs of lines so that no two consecutive points on a line are of the same colour.

3. On a one-way street there are n parking ~~lots~~ places in a straight line along the curb designated $1, 2, 3, \ldots, n$. Now n drivers appear in sequence, each picking a number x with $1 \le x \le n$ at random. If the space x is occupied, the driver takes the first available (if any) of spaces $x+1, x+2, \ldots, n$. If none is available, the driver departs in frustration.

   What is the probability that all n drivers successfully park?

4. The n^2 squares of an $n \times n$ grid are ~~labeled~~ coloured black and white (B and W), not necessarily in the standard checkerboard fashion. No row nor column has more than n white squares. Prove that we can assign to each white square an integer from $\{1, 2, 3, \ldots, n\}$ so that no integer appears twice in the same row nor twice in the same column.