Next Evening, Message'em 0s

6th NEMO, 6 October 2016

Problem 1. There are n pieces of candy in a pile. One is allowed to separate a pile into two piles, and add the product of the sizes of the two new piles to a running total. The process terminates when each piece of candy is in its own pile. Show that the final sum is independent of the order of the operations performed.

Problem 2. Prove that for any positive intger n, $n!$ is a divisor of:

$$\prod_{i=0}^{n-1}(2^n - 2^i)$$

Problem 3. Let $ABCD$ be a cyclic quadrilateral. Lines AB, CD meet at E and AD, BC meets at F. Suppose M, N and O are the midpoints of AC, BD and EF respectively. Prove that, $OE \times OF = OM \times ON$.

Problem 4. Prove that, $a, b, c > 0$ can be the sides of a triangle if and only if for any p, q such that $p + q = 1$, the following inequality is true:

$$a^2p + b^2q > c^2pq$$