2004 Winter Camp (buffet contest)

A set A of positive integers is called "uniform" if after any of its elements is removed, then the remaining ones can be disjointed into two subsets with equal sums of their elements. Find the least positive integer $n > 1$ such that there exists a "uniform" set A with n elements.

Compute the maximum area of a triangle having a median of length 1 and a median of length 2.

Find positive integers a, b if for every $x, y \in [a, b]$, $\frac{1}{x} + \frac{1}{y} \in [a, b]$.

Circles S_1 and S_2 with centers at O_1 and O_2, respectively, intersect at points A and B. Tangents to S_1 and S_2 at point A intersect segments BO_2 and BO_1 at points K and L, respectively. Prove that $KL \parallel O_1O_2$.

Consider an integer $n > 1$. One want to colour all natural numbers red and blue so that the following conditions are simultaneously satisfied:
i/ every number is coloured red or blue, there are an infinite number of numbers coloured red and an infinite number of numbers coloured blue;
ii/ the sum of n distinct numbers coloured red is coloured red and the sum of n distinct numbers coloured blue is coloured blue.
Is it possible to colour in a such manner, if:
1/ $n = 2002$?
2/ $n = 2003$?

The set of numbers M, containing 2003 various numbers, is such that for any two different elements a, b from M number $a^2 + b\sqrt{2}$ is rational. Prove that for each a from M number $a\sqrt{2}$ is rational.

Find all polynomials $P(x)$ with real coefficients, satisfying the relation

$$(x^3 + 3x^2 + 3x + 2)P(x - 1) = (x^3 - 3x^2 + 3x - 2)P(x)$$

for every real number x.

Prove that the midpoints of the altitudes of a triangle are collinear if and only if the triangle is right.

Let a, b, c be positive numbers, the sum of which is 1. Prove the inequality

$$\frac{1}{1-a} + \frac{1}{1-b} + \frac{1}{1-c} \geqslant \frac{2}{1+a} + \frac{2}{1+b} + \frac{2}{1+c}.$$

In the triangle ABC centers of circumcircle and incircle are O and I, respectively. Excircle ω_a touches the continuation of sides AB and AC at points K and M, respectively, and of side BC at point N. It is known that the midpoint P of the segment KM lies on the circumcircle of triangle ABC. Prove that points O, N, and I are collinear.

Find all solutions in positive integers a, b, c to the equation

$$a!\,b! = a! + b! + c!$$