Ireland 2001

2005 Winter Camp

Functional Equations [M]

Determine, with proof, all functions f from the set of positive integers to itself which satisfy

$$f(x+f(y)) = f(x) + y$$

for all positive integers x, y.

Ukrain 1999

Geometry [M]

Let AA_1, BB_1, CC_1 be the altitudes of an acute triangle ABC, O an arbitrary point inside $A_1B_1C_1$. Denote the bases of the perpendiculars from O to the lines AA_1 and BC respectively, by M and N respectively; the ones from O to the lines BB_1 and CA resp., by P and Q resp., and the ones from O to CC_1 and AB resp. by R and S resp. Prove that the lines MN, PQ, and RS are concurrent.

Czech Slovak 2001 [M]

F.E.

Find all functions $f: \mathbb{R} \to \mathbb{R}$ such that for all real numbers x and y,

$$f(x^2+f(y)) = (x-y)^2 \cdot f(x+y)$$

CS 1999 Number Theory [M]

Show that for every natural number n the product

$$\left(4-\frac{2}{1}\right)\left(4-\frac{2}{2}\right)\left(4-\frac{2}{3}\right)\cdots\left(4-\frac{2}{n}\right)$$

is an integer.

2005 Winter Camp

A1 Find, with reasons, all integers $a, b,$ and c, such that
$$\frac{1}{2}(a+b)(b+c)(c+a)+(a+b+c)^3 = 1-abc.$$

G3 Let AB be a diameter of a circle O.
Suppose that l_a, l_b are tangent lines to O at A, B respectively. C is an arbitrary point on O. BC meets l_a at the point K. The bisector of $\angle CAK$ meets CK at H. M is the midpoint of the arc CAB and S is the ~~another~~ other intersection point of HM with O. T is the intersection of l_b and the tangent line to O at M.
Show that S, T, and K are collinear.

F2 Find all functions from the real numbers to the real numbers such that
$$f(xf(x)+f(y)) = (f(x))^2+y$$
for all real numbers x and y.

[illegible]

F4 Find all functions $f: \mathbb{R} \to \mathbb{R}$ satisfying
$$f(xy)(f(x)-f(y)) = (x-y)f(x)f(y).$$

IE4 Let $x_1, x_2, \ldots, x_n$ be arbitrary real numbers. Prove the inequality
$$\frac{x_1}{1+x_1^2}+\frac{x_2}{1+x_1^2+x_2^2}+\cdots+\frac{x_n}{1+x_1^2+x_2^2+\cdots+x_n^2} < \sqrt{n}.$$

N3 Let $a>b>c>d$ be positive integers and suppose
$$ac+bd = (b+d+a-c)(b+d-a+c).$$
Prove that $ab+cd$ is not prime.

2006 Winter Camp

C1. Let S be a set of n points, numbered from 1 to n, lying on a circle (in the order of the numbering).
Let $S_{n,k}$ be the family of subsets A of S with k elements satisfying the following property:

for any point i in A, there are at least 3 points in S between i and the nearest j which is in A.

For $n, k \geq 2$, find the number of elements of $S_{n,k}$.

j2. Let R and r be the circumradius and the inradius of $\triangle ABC$, respectively. Suppose that $\angle A < 90°$ is the largest angle of $\triangle ABC$. Let m be the midpoint of BC and X the intersection of the tangents to the circumcircle of $\triangle ABC$ at B and C. Show that

$$\frac{r}{R} \geq \frac{AM}{AX}$$

Find all primes p for which there exist integers m, n satisfying the conditions $p = m^2+n^2$ and $p \mid m^3+n^3-4$.

Let $ABCDEF$ be a regular hexagon of side length 1, and let O be the center of the hexagon. In addition to the sides of the hexagon, line segments are drawn from O to each vertex, making a total of 12 unit line segments. Find the number of paths of length 2003 along these line segments that start at O and end at O.

2003 Winter Camp

IMO Prelim Select Contest (1988–1997)
93-94 #10 ~~(Hard)~~ (Medium)
Given 12 rods of lengths 1, 2, 3, .., 12, in how many ways can we chose 3 rods to form a triangle?

43rd Nat Math Olymp. Slovenia
1999 #1 (Medium) Number Th.

Find all integers x and y that satisfy the equation $x^3+9xy+127=y^3$
