2080

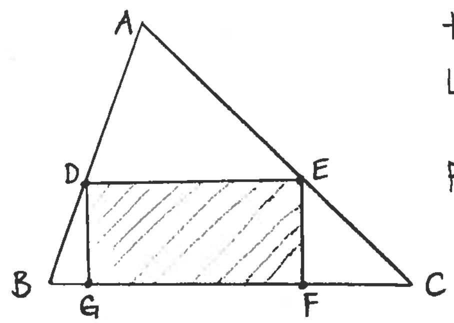
Fun Problems

1. Let A and B be four-digit palindromes and let C be a five-digit palindrome. If $A+B=C$, determine all possible values of C.

2. Suppose that A, B, and C are positive integers in arithmetic progression with $A<B<C<180$. If $\sin A^\circ + \sin B^\circ = \sin C^\circ$ and $\cos A^\circ = \cos B^\circ + \cos C^\circ$, determine the triplet (A, B, C).

3. In ΔABC, $AB=\sqrt{c}$, $BC=\sqrt{a}$, and $CA=\sqrt{b}$, where a, b, c are positive integers. If the area of the triangle is 1, show that $ab-4$, $ac-4$, and $bc-4$ are all perfect squares.

4. In ΔABC with area T, we inscribe a rectangle DEFG so that F and G are on the line BC. Let R be the area of the triangle.



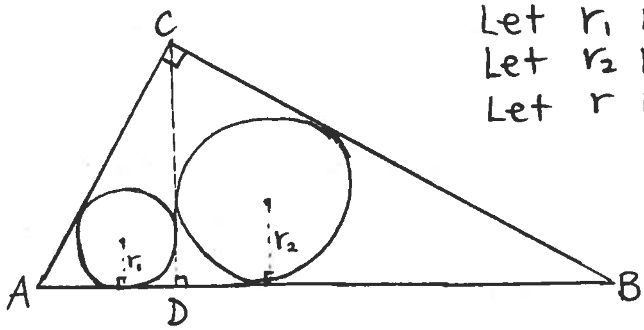
Prove that $T \geq 2R$. When does $T = 2R$?

5. For each integer n, we let $\bar{n}$ be the integer n written backwards. (For example, if $n=1234$, then $\bar{n}=4321$). We say that a four digit integer n is <u>magical</u> if both $n+\bar{n}$ and $n-\bar{n}$ are palindromes. For example, 2001 is magical.

   If n is magical, determine all possible values of $n-\bar{n}$.

6. a, b, c are three consecutive terms of a geometric sequence, where a, b, and c are all integers. If $a+b+c=7$, determine all possible values of b.

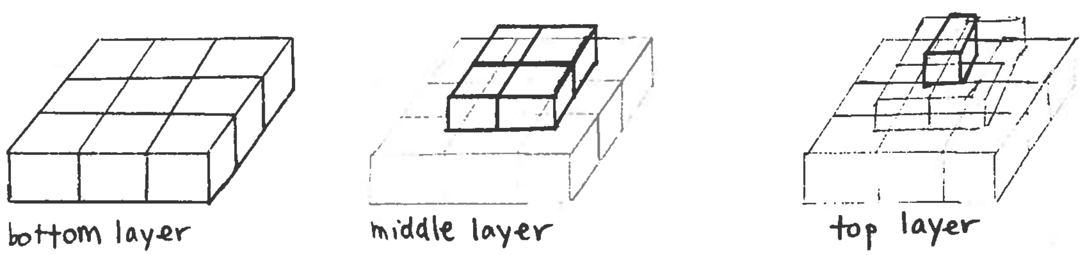
7. Let ΔABC be a right-angled triangle with $\angle C = 90^\circ$. Let CD be the altitude from C to AB.



Let r_1 be the inradius of ΔADC.
Let r_2 be the inradius of ΔBDC.
Let r be the inradius of ΔABC.

Prove that $r_1 + r_2 \le \sqrt{2}\, r$.
When does equality occur?

8. There are 14 cubes, arranged in a pyramid, as shown. A number is assigned to each cube.



The nine cubes on the bottom layer are each assigned a unique digit (1 to 9 inclusive). Each cube on the middle and top layers is assigned the number that is the <u>average</u> of the four cubes directly below it. Suppose our pyramid is numbered as follows:

1	5	P_1
P_2	P_3	P_4
P_5	P_6	P_7

(bottom)

$\longrightarrow$

a	b
c	d

(middle)

$\longrightarrow$

e

(top)

Given that $a, b, c, d,$ and e are <u>distinct integers</u>, determine the values of $P_1, P_2, \ldots, P_7$.