2004

If you're bored...

- 1. ABC is a triangle. A' is the midpoint of the arc BC of the circumcircle not containing A. B' and C' are defined similarly. The segments A'B', B'C', C'A' intersect the sides of the triangle in six points, two on each side. These points divide each side of the triangle into three parts. Show that the three middle parts are equal iff ABC is equilateral.
- 2. C and C' are circles with centres O and O', respectively. X and X' are points on C and C' respectively such that the lines OX and O'X' intersect. M and M' are variable points on C and C' respectively, such that $\angle XOM = \angle X'O'M'$ (both measured clockwise). Find the locus of the midpoint of MM'. Let OM and O'M' meet at Q. Show that the circumcircle of QMM' passes through a fixed point.
- 3. A circle centre O meets a circle centre O' at A and B. The line TT' touches the first circle at T and the second circle at T'. The perpendiculars from T and T' meet the line OO' at S and S'. The ray AS meets the first circle again at R and the ray AS' meets the second circle again at R'. Show that R, R and R' are collinear.
- 4. ABC is an isosceles triangle with AB = AC. O is a variable point on the line BC such that the circle with centre O and radius OA does not have lines AB or AC as tangents. The lines AB and AC meet the circle again at M and N respectively. Find the locus of the orthocentre of the triangle AMN.