Never Estimate My Ordering

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Problem 1. Let $n \geq 3$ be an integer. A labelling of the n vertices, the n sides and the interior of a regular n-gon by $2n + 1$ distinct integers is called memorable if the following conditions hold:

(a) Each side has a label that is the arithmetic mean of the labels of its endpoints.

(b) The interior of the n-gon has a label that is the arithmetic mean of the labels of all the vertices.

Determine all integers $n \geq 3$ for which there exists a memorable labelling of a regular n-gon consisting of $2n + 1$ consecutive integers.

Problem 2. On a blackboard there are $n \geq 2, n \in \mathbb{Z}^+$ numbers. In each step we select two numbers from the blackboard and replace both of them by their sum. Determine all numbers n for which it is possible to yield n identical number after a finite number of steps.

Problem 3. Let a, b and c be positive real numbers satisfying $\min(a + b, b + c, c + a) > \sqrt{2}$ and $a^2 + b^2 + c^2 = 3$. Prove that

$$\frac{a}{(b + c - a)^2} + \frac{b}{(c + a - b)^2} + \frac{c}{(a + b - c)^2} \geq \frac{3}{(abc)^2}$$