Natural Evolution of MOd

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Problem 1. For a positive integer a, a' is the integer obtained by the following method: the decimal writing of a' is the inverse of the decimal writing of a (the decimal writing of a' can begin by zeros, but not the one of a); for instance if $a = 2370$, $a' = 0732$, that is 732.

Let a_1 be a positive integer, and $(a_n)_{n\geq 1}$ the sequence defined by a_1 and the following formula for $n \geq 1$:

$$a_{n+1} = a_n + a'_n.$$

Can a_7 be prime?

Problem 2. Let $\{R_i\}_{1\leq i\leq n}$ be a family of disjoint closed rectangular surfaces with total area 4 such that their projections of the Ox axis is an interval. Prove that there exist a triangle with vertices in $\bigcup_{i=1}^{n} R_i$ which has an area of at least 1.

Problem 3. Find all functions $f : \mathbb{Q} \to \mathbb{R}$ such that $f(x)f(y)f(x+y) = f(xy)(f(x)+f(y))$ for all $x, y \in \mathbb{Q}$.