Winter Math Camp 2007: Mock Olympiad 2

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1. Find the minimum possible value of $|12^m - 5^n|$ where m, n are positive integers.

2. Find all polynomials f with real coefficients such that

$f(x) = 0, 1, x^n(x-1)^n \quad n \in \mathbb{N}.$

$$f(x^2) = f(x)f(x+1)$$

3. Let ABC be an acute-angled triangle whose inscribed circle touches AB, AC at D, E respectively. Let X and Y be the points of intersection of the bisectors of the angle ACB and ABC with the line DE and Z be the midpoint of BC. Prove that triangle XYZ is equilateral if and only if $\angle BAC = 60^o$.

4. A square 2007×2007 is divided into unit squares in the standard way. Each unit square contains a real number whose absolute value is at most 1, so that the sum of the four entries of every 2×2 square is 0. Determine the maximum possible value of the sum of all of the numbers in this square.

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5. For $0 \leq x, y \leq 1$, let

$\frac{\pi}{4}.$

$$f(x,y) = xy^2\sqrt{1-x^2} - x^2y\sqrt{1-y^2}.$$

Find the minimum constant c so that the following condition holds:

For any integer $n > 1$ and any real numbers $a_1, a_2, \cdots, a_n$ such that $0 \leq a_1 < a_2 < \cdots < a_n \leq 1$, we have

$$f(a_1, a_2) + f(a_2, a_3) + \cdots + f(a_{n-1}, a_n) < c$$