2001 Winter Camp

A Comprehensive Suitcase Of Famous And Practical Inequalities™

(Note: Most of these inequalities only work if the a_i (or x_i and y_i) are positive)

Weighted AM–GM

$$w_1x_1 + w_2x_2 + \ldots + w_nx_n \geq x_1^{w_1}x_2^{w_2}\cdots x_n^{w_n}$$

if the w_i are weights such that $w_1 + w_2 + \ldots + w_n = 1$.

QM–AM–GM–HM (Quadratic, Arithmetic, Geometric and Harmonic Means)

$$\sqrt{\frac{a_1^2 + a_2^2 + \ldots + a_n^2}{n}} \geq \frac{a_1 + a_2 + \ldots + a_n}{n} \geq \sqrt[n]{a_1a_2\cdots a_n} \geq \frac{n}{\frac{1}{a_1} + \frac{1}{a_2} + \ldots + \frac{1}{a_n}}$$

Power

$$\left(\frac{a_1^p + a_2^p + \ldots + a_n^p}{n}\right)^{\frac{1}{p}} \geq \left(\frac{a_1^q + a_2^q + \ldots + a_n^q}{n}\right)^{\frac{1}{q}}$$

where p and q are real numbers, with $p > q$

Hölder

$$\left(\sum_{i=1}^n x_i^p\right)^{\frac{1}{p}}\left(\sum_{i=1}^n y_i^q\right)^{\frac{1}{q}} \geq \sum_{i=1}^n x_iy_i$$

where p and q are real, with $p + q = 1$

Minkowski

$$\left(\sum_{i=1}^n x_i^p\right)^{\frac{1}{p}} + \left(\sum_{i=1}^n y_i^p\right)^{\frac{1}{p}} \geq \left(\sum_{i=1}^n (x_i + y_i)^p\right)^{\frac{1}{p}}$$

where p is any real number ≥ 1

(when $p = 2$, the Minkowski Inequality turns into the Triangle Inequality)

Chebycheff

$$\left(\sum_{i=1}^n a_i\right)\left(\sum_{i=1}^n b_i\right) \leq n\sum_{i=1}^n a_ib_i$$

if a_i and b_i are both non–decreasing sequences or non–increasing sequences

Jensen

$$\frac{f(x_1) + f(x_2) + \ldots + f(x_n)}{n} \geq f\left(\frac{x_1 + x_2 + \ldots + x_n}{n}\right)$$

if f(x) is a convex function.
The direction of inequality is reversed if f(x) is concave.

Cauchy–Schwarz

$$\sqrt{\left(\sum_{i=1}^n x_i^2\right)\left(\sum_{i=1}^n y_i^2\right)} \geq \sum_{i=1}^n x_iy_i$$

The Hölder Inequality for $p = q = 1/2$

Triangle

$$\sqrt{\sum_{i=1}^n x_i^2} + \sqrt{\sum_{i=1}^n y_i^2} \geq \sqrt{\sum_{i=1}^n (x_i + y_i)^2}$$

Bernoulli

$$\prod_{i=1}^n (1 + x_i) \geq 1 + \sum_{i=1}^n x_i$$

if the x_i are non–zero reals which have the same sign and satisfy $x_i \geq -2$ for all i.

Schur's Inequality

$x, y, z \geqslant 0$

$x^n(x-y)(x-z) + y^n(y-x)(y-z)$

$+ z^n(z-x)(z-y) \geqslant 0$