Art of Problem Solving

WOOT 2010–11
Practice Olympiad 5

Instructions

- You should take the test under "olympiad conditions," meaning that the test should be completed in one sitting, with handwritten solutions (just like on an actual olympiad exam). Take the test using only the resources that would be available to you on an actual olympiad, meaning you should use scrap paper, a ruler, a compass, etc., but no calculators and no reference materials.

- You should allot 3 hours to take the test.

- Completely fill out the cover sheet and make sure it is the first page of your solutions.

- On the WOOT Home Page, there is a **WOOT Practice Olympiad Answer Sheet**. Print out (or copy) several blank copies of the answer sheets, and write all of your work on these sheets. Use only black pen or very dark pencil. Make sure that the top of every answer sheet page is completely filled out. Each problem's solution should start on a new page, along with new page numbering.

- Do not discuss the problems on or before the due date of Wednesday, February 23, 2011.

How to submit your solutions

You can submit solutions by upload, email, or by fax. DO NOT MAIL YOUR SOLUTIONS!

By Upload: Scan your solutions as a single PDF file. **Check to make sure that the file is legible before submitting it!** In your "My Classes" area, follow the link that says "Submit."

By email: Scan your solutions as a single PDF file. **Check to make sure that the file is legible before emailing it!** Email to woot@artofproblemsolving.com. Put "WOOT Practice Olympiad 5" in the subject line, attach your solutions as a single PDF file, and write something in the message body - if you leave the message body blank, it will get blocked by our spam filters.
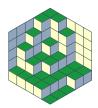
By fax: Fax to (619)659-8146.

Solutions are due by Wednesday, February 23, 2011

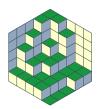Late submissions will not be accepted except under extraordinary circumstances

D E Shaw & Co

TWO SIGMA

Art of Problem Solving

WOOT 2010–11
Practice Olympiad 5

WOOT Practice Olympiad Cover Sheet

Username: _____

Class ID: _____

User ID: _____
(Your Class ID and User ID can be found in the "My Classes" section of the website)

Practice Olympiad Number: 5

Beginning	
Intermediate	
Advanced	

Number of pages (including cover sheet): _____

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1. For a positive integer n, let

$$S_n = 5.5 + 5.55 + 5.555 + \cdots + 5.\underbrace{555\ldots5}_{n \text{ 5's}}.$$
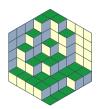
   Show that $555 < S_{100} < 556$.

2. Let $ABCD$ be a convex quadrilateral. Prove that there exists exactly one point O such that the circumcenters of the triangles AOB, BOC, COD, and DOA form the vertices of a (nondegenerate) parallelogram.

3. Renee writes down a list of 50 positive integers, all between 1 and 50 inclusive, such that the sum of the integers is 100. Show that the list can be partitioned into two lists, such that the the sum of the numbers in each list is 50.

4. Determine all triples of nonnegative integers (x, y, z) satisfying $8^x + 15^y = 17^z$.

5. Let $ABCD$ be a convex quadrilateral such that $AB + CD = \sqrt{2}AC$ and $BC + DA = \sqrt{2}BD$. Prove that $ABCD$ is a parallelogram.

6. On a circle are marked 999 points. How many ways are there to assign to each point one of the letters A, B, or C, so that on the arc between any two points marked with the same letter, there are an even number of letters differing from these two?

7. Jerry and Donna are playing the following game: start with the polynomial

$$x^4 + \_x^3 + \_x^2 + \_x + 1,$$

   where the coefficients of x^3, x^2, and x are missing. Jerry goes first, and in turn they choose an empty coefficient and fill in any real number. Donna wins if the resulting polynomial has a real root; otherwise Jerry wins. Prove that Donna has a winning strategy.

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Remember . . .

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- Complete the WOOT Practice Olympiad Cover Sheet (included in this document), and make it the first page of your solutions. We will <u>not</u> accept your solutions without this cover sheet.

- Do not mail your solutions. Use only upload, e-mail, or fax. If you use e-mail, send your solutions to

<div align="center">

<u>woot</u>@artofproblemsolving.com,

</div>

NOT **<u>classes</u>**@artofproblemsolving.com. If you use e-mail, we will only accept solutions in PDF format. In particular, solutions in JPG or Word format will not be accepted.

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