Online Number Theory Camp: Day 1

August 23, 2015

There are all sorts of problems. You don't have to send solutions for practice problems. And start reading **Problem Solving Strategies** as stated in this post already.

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1. Practice Problems

Problem 1. If a, b, c are positive integers so that a divides b and c prove that a divides $\gcd(a, b)$.

Problem 2. Prove that, $\gcd(a, bc) = \gcd(a, \gcd(a, b)c)$.

Problem 3. If n divides $a^x - b^x$ and $a^y - b^y$, then n divides $a^{\gcd(x,y)} - b^{\gcd(x,y)}$.

Problem 4. If 30 divides $a + b + c$, then prove that 30 divides $a + b + c$

Problem 5. Find all positive integer k so that $\sigma(n) = n + k$ has an infinite number of solutions where $\sigma(n)$ is the sum of divisors of n.

Problem 6. Let $\mathcal{S} = \{1, 2, ..., 2n\}$. Prove that $\mathcal{S}$ can be partitioned into n pairs of positive integers (a_i, b_i) so that $a_i + b_i$ is a prime for all i.

Problem 7. If $p_1, p_2, ..., p_n$ are all primes, prove that $2^{p_1 \cdots p_n} + 1$ has at least $2^{2^n - 1}$ divisors.

Problem 8. Let $m, n, a_1, a_2, ..., a_n$ be positive integers so that $\forall i, a_i + m$ is a prime. Let $N = \prod_{i=1}^{n} p_i^{a_i}$ and $S(N, m)$ be the number of ways to write N as a product of m positive integers. Calculate the remainder of $S(N, m)$ when divided by m^n.