Sample Combinatorics Problems

Irish Mathematical Olympiad Training, 2011

26 March 2011

Problem 1:

How many "words" can be written using exactly five letters A and no more than three letters B (and no other letters)?

Problem 2:

Mother has two apples, three pears and four oranges. Every morning, for nine days, she gives one fruit to her son for breakfast. How many ways are there to do this?

Problem 3:

How many nine-digit numbers have an even sum of their digits?

Problem 4:

n distinct points on the circumference of a circle are chosen. Adjacent points on the circle are connected by line segments to form an n-sided polygon. The diagonals of this polygon are drawn.

(a) How many diagonals has this polygon?

(b) Assume that no three diagonals meet in a single point. Into how many regions is the circle divided by the sides and diagonals of the polygon?

Problem 5:

How many subsets of $\{1, 2, 3, \ldots, n\}$ contain no pair of consecutive numbers?

Problem 6: (IrMO 1994)

A sequence $\{x_n\}$ is defined by the rules

$$x_1 = 2$$

and

$$nx_n = 2(2n-1)x_{n-1}\ ; \quad n = 2, 3, \ldots$$

Prove that x_n is an integer for every positive integer n.

Problem 7:

How many solutions are there to the equation

$$x_1 + x_2 + x_3 + x_4 + x_5 + x_6 + x_7 = 22$$

(a) If $x_1, x_2, \ldots, x_7$ are non-negative integers?

(b) If $x_1, x_2, \ldots, x_7$ are non-negative integers and $x_i \leq 7$ for each i?

Problem 8:

A *north-east lattice path* is a path starting at the origin $(0, 0)$ and taking steps *north* $(0, +1)$ or *east* $(+1, 0)$.

(a) How many north-east lattice paths end at the point $(9, 9)$?

(b) How many of the paths in part **(a)** do not pass through the point $(5, 5)$?