Not Every Math is Obvious

3rd NEMO, September 29 2016

Problem 1. Let ABC be an acute-angled triangle with circumcircle ω and incentre I. Suppose D is the midpoint of arc BAC. Let DI intersect BC at E and ω for a second time at F. Let P be a point on line AF such that PE is parallel to AI. Prove that PE bisects angle BPC.

Problem 2. For positive reals a, b and c with $ab + bc + ca = 1$, show that

$$\sqrt{3}(\sqrt{a} + \sqrt{b} + \sqrt{c}) \leq \frac{a\sqrt{a}}{bc} + \frac{b\sqrt{b}}{ca} + \frac{c\sqrt{c}}{ab}$$

Problem 3. The sequence a_i satisfies $a_{pk+1} = pa_k - 3a_p + 13$ for all positive integers k and primes p. Find a_{2016}

Problem 4. When a Knight moves in chess, it can move to a square that is away two squares horizontally and one square vertically, or two squares vertically and one square horizontally. A closed tour for a Knight is a sequence of moves, which have the Knight start and end in the same square and visit each other square exactly once. On a 4×3 board, is there a possible closed tour for a Knight?