Solution-

1 a. $F = i\,l \times B$, direction of F is right.
Bar will move to the right. [1 mark]

b. Induced voltage on the bar moving with velocity v, $e_{ind} = uBl$ [1mark]
$-V + iR + e_{ind} = 0$
or, $uBl = V - iR$
or, $u = \frac{V-iR}{Bl}$ [1 marks]
$F = ma$, as v increases, so does e_{ind}.
$i = \frac{V-\text{eind}}{R}$
when $e_{ind} = V$, $i = 0$. $u_{max} = \frac{V}{Bl}$ [1 marks]

c. $t=0 \quad i(0) = \frac{20}{10} = 2A$

$F = iLB$, $e_{ind} = uBL$

$\therefore a = \frac{F}{m} = \frac{iLB}{m}$

$du = a\,dt = \frac{iLB}{m}dt$ or, $i = \frac{m\,du}{LB\,dt}$

$i = \frac{V - e_{ind}}{R}$

$\Rightarrow i = \frac{V - uBL}{R}$

$\Rightarrow \frac{m\,du}{LB\,dt} = \frac{V - uBl}{R}$

$\Rightarrow \frac{du}{V - uBl} = \frac{LB\,dt}{mR}$

$\Rightarrow \int_0^u \frac{du}{V - uBL} = \frac{LB}{mR}\int_0^t dt$

$\Rightarrow \Big[\ln|V - uBL|\Big]_0^u = -\frac{LBt}{mR}$

$\Rightarrow \ln|V - uBl| - \ln V = -\frac{LBt}{mR}$

$\Rightarrow \frac{V - uBL}{V} = e^{-\frac{LBt}{mR}}$

$\Rightarrow u = \frac{V}{Bl}\left(1 - e^{-\frac{LBt}{mR}}\right)$

now $t = 5\,s$

$$u = \frac{20}{.5 \times 1}\left(1 - e^{-\frac{1 \times .5 \times 5}{.25 \times 10}}\right) ms^{-1}$$

$$= 25.29\ ms^{-1}$$ [2marks]

Now for next 5 seconds

$V = -20$ Volts

at $t = 5$ $\quad u = 25.29$

$$\int_{25.29}^{u} \frac{du}{V - uBL} = \frac{LB}{mR}\int_{5}^{10} dt$$

$$\Rightarrow \left[\ln|V - uBL|\right]_{25.29}^{u} = -\frac{LB}{mR}(10-5)$$

$$\Rightarrow \ln|V - uBL| - \ln|V - 25.29\,BL| = -1$$

$$\Rightarrow \frac{-20 - uBL}{-20 - 12.645} = e^{-1}$$

$$\Rightarrow u = \frac{1}{BL}\left(32.645\,e^{-1} - 20\right)$$

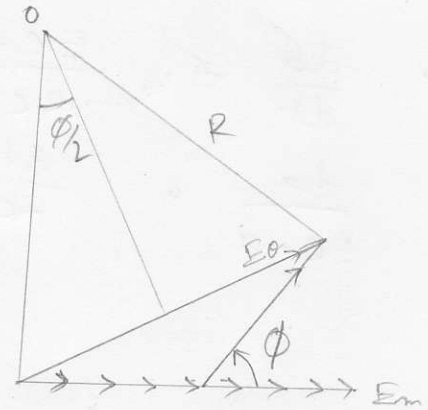
$$= \frac{1}{.5} \times -8$$

$$= -16\ ms^{-1}$$ [2marks]

2 at point P

phase difference of two rays at two edges

$$\phi = \frac{2\pi}{\lambda} d \sin\theta$$



$$E_\theta = 2R \sin \phi/2$$

$$\phi = \frac{E_m}{R}$$

$$\therefore E_\theta = \frac{2E_m}{\phi} \sin\frac{\phi}{2}$$

$$\Rightarrow E_\theta = E_m \frac{\sin \phi/2}{\phi/2}$$

$$\text{now } I = \frac{1}{2\mu_0 c} E^2$$

$$\therefore I \propto E^2$$

$$\text{So } \frac{I_\theta}{I_0} = \frac{E_\theta^2}{E_m^2} = \left(\frac{\sin \phi/2}{\phi/2}\right)^2$$

$$\therefore I_\theta = I_0 \left(\frac{\sin \phi/2}{\phi/2}\right)^2 \text{ where } \phi = \frac{2\pi}{\lambda} d \sin\theta$$

3. constructive interference happens on oil film

$$\lambda_0 = \mu_k \lambda_k$$

$$2d = n\lambda_k$$

$$\Rightarrow \lambda_k = \frac{2d}{n}$$

$$\Rightarrow \lambda_0 = \frac{2d\mu_k}{n} = 1116\,nm,\ 558\,nm,\ 372\,nm$$

$n=1$, $n=2$, $n=3$

in visible light range $\lambda_0 = 558$ nm

[Ans]

[2 marks]

4. $Ri + \frac{q}{c} = E$

$\Rightarrow R\frac{dq}{dt} + \frac{q}{c} = E$

$\Rightarrow \frac{dq}{dt} = \frac{EC-q}{RC}$

$\Rightarrow \frac{dq}{EC-q} = \frac{dt}{RC}$

$\Rightarrow \int_0^q \frac{dq}{EC-q} = \int_0^t \frac{dt}{RC}$

$\Rightarrow \left[\ln|EC-q|\right]_0^q = -\frac{t}{RC}$

$\Rightarrow \frac{EC-q}{EC} = e^{-t/RC}$

$\Rightarrow q = EC\left(1-e^{-t/RC}\right)$

[3 marks]

5. (a) $z = 3 + j\,\omega L$

$= 3 + j4$ [1 mark]

(b) $L\frac{di}{dt} + Ri = E_0 \sin\omega t$

$\Rightarrow \frac{di}{dt} + \frac{R}{L}i = \frac{E}{L}\sin\omega t$

$\Rightarrow e^{\frac{Rt}{L}}\frac{di}{dt} + \frac{R}{L}e^{Rt/L}\,i = \frac{E}{L}e^{Rt/L}\sin\omega t$

$\Rightarrow d\left(e^{Rt/L}\,i\right) = \frac{E}{L}e^{Rt/L}\sin\omega t\,dt$

$\Rightarrow e^{Rt/L}\,i = \frac{E}{L}\int e^{Rt/L}\sin\omega t\,dt$

$\Rightarrow i = \frac{E}{\sqrt{R^2+\omega^2L^2}}\sin(\omega t - \theta) + c_1 e^{-Rt/L}$

[Here $\theta = \tan^{-1}\frac{\omega L}{R}$
$= 53.13°$]

$t=0$,

$0 = \frac{110}{5}\sin(-53.13°) + c_1$

or, $c_1 = 17.6$ A

So $i = 22\sin(400t - 53.13°) + 17.6\,e^{-300t}$ A

[3 marks]

c. voltage leads, $53.13°$ [1 mark]