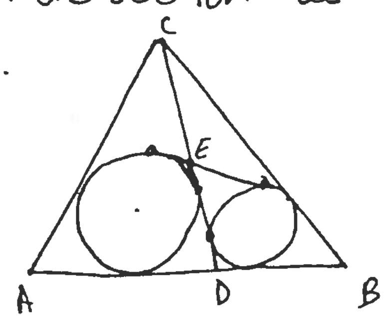
More Locus Hokus Pokus 1998 IMO Camp

1. Let H be the orthocenter of ΔABC. Let p be any line through H. Let q, r, s be the reflections of p in BC, CA and AB respectively. Prove that the lines q, r, s are concurrent, and find the locus of their point of intersection as p rotates about H. (Hard!)

2. 

Prove that as D varies along AB, E traces the arc of a circle.

3. A segment EF of constant length slides along the diameter AB of a semi-circle. The perpendiculars to AB at E and F meet the semi-circle at M and N. Let C, D be fixed points on AB. Show that the points of intersection P and Q of the circles passing through M and N respectively with centres at C and D are on a circle concentric with the semi-circle.

4. An arbitrary point M is selected in the interior of the segment AB. The squares $AMCD$ and $MBEF$ are constructed on the same side of AB, with the segments AM and MB as their respective bases. The circles circumscribed about these squares, with centers P and Q, intersect at M and also at another point N. Let N' denote the point of intersection of the straight lines AF and BC.

   a) Prove that N and N' coincide.

   b) Prove that the straight lines MN pass through a fixed point S independent of the choice of M.

c) Find the locus of the midpoint of PQ as M varies between A and B.

5. Consider the cube $ABCDA'B'C'D'$ (with face $ABCD$ directly above face $A'B'C'D'$).

   a) Find the locus of the midpoints of segments XY, where X is any point of AC and Y is any point of $B'D'$.

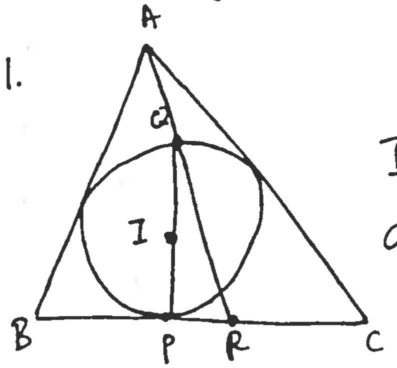
   b) Find the locus of points Z which lie on the segments XY of part (a) with $ZY = 2XZ$.

6. In the same situation as #5, the point X moves at constant speed along the perimeter of the square $ABCD$ in the direction $ABCDA$, and the point Y moves at the same rate along the perimeter of the square $B'C'CB$ in the direction $B'C'CBB'$. Points X and Y begin their motion at the same instant from the starting positions A and B'. Determine the locus of the midpoints of the segments XY.

7. P is a given point inside a given sphere. Three mutually perpendicular rays from P intersect the sphere at points U, V, W. Q denotes the vertex diagonally opposite to P in the parallelopiped determined by PU, PV, and PW. Find the locus of Q for all such triads of rays from P.

Homothety and Locus Questions.

1\.



Given triangle ABC, incentre I, incircle $\mathcal{C}$. I is tangent to BC at P. PI intersects $\mathcal{C}$ again at Q. AQ intersects BC at R. Prove that the midpoint of BC is the midpoint of PR.

2\. If PQ is a variable diameter of a given circle, and A, B two fixed points collinear with the center O of the circle, find the locus of the point $M = AP \cap BQ$.

3\. A variable point P moves on a fixed circle, center C, and A is a fixed point. Find the locus of the point of intersection of the line AP with the internal bisector of the angle ACP.

4\. Three congruent circles have a common point O and lie inside a given triangle. Each circle touches a pair of sides of the triangle. Prove that the incenter and the circumcenter of the triangle and the point O are collinear.

~~5. O_1 is the circumcenter of the medial triangle, and O_2 is the circumcenter of the original triangle. Prove that~~

5\. Let A' be the circumcenter of BCH, and similarly for B', C'. Prove that AA', BB' and CC' are concurrent.

6\. If A and B are fixed points on a given circle and XY is a variable diameter of the same circle, determine the locus of the points of intersection of lines AX and BY. (You may assume that AB is not a diameter.)