CANADA
1995

GEOMETRY PROBLEMS
Winter Training Camp - 200,

BUL 1

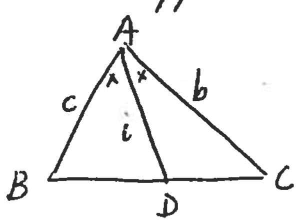
---

1. Let $\mathcal{C}$ be a circle with centre O and radius 1 and let $\mathcal{P}$ be a closed convex region inside $\mathcal{C}$.

Suppose from each point of $\mathcal{C}$ we can draw two rays tangent to $\mathcal{P}$ meeting at an angle of $60°$. Describe $\mathcal{P}$.

2. The centres of the circumscribed and inscribed spheres of a given tetrahedron coincide. Prove that the four triangular faces are congruent.

3. Suppose lengths ~~a, b~~ b, c, i are given. Construct a triangle ABC for which $|AC| = b$, $|AB| = c$ and the length of the bisector AD of angle A is i.



4. Let ABC be an acute-angled triangle, with a point H in side. Let U, V, W be, respectively, the reflected image of H with respect to axes BC, AC, AB.

Prove that H is the orthocentre of ΔABC if and only if U, V, W lie on the circumcircle of triangle ABC.

5. Given a parallelogram $ABCD$, inscribe in the angle $\angle BAD$ a circle that lies entirely inside the parallelogram. Similarly, inscribe a circle in the angle $\angle BCD$ that lies entirely inside the parallelogram and such that the two circles are tangent. Find the locus of the point of tangency of the two circles, as the two circles vary.

6. Let $ABCD$ be a trapezoid with $AB \parallel CD$ and let F lie on the segment AB such that $DF = CF$. Let E be the intersection of AC and BD, and let O_1, O_2 be the respective circumcentres of ADF, BCF. Prove that the lines EF and O_1O_2 are perpendicular.