

NE, Moja Over

8th NEMO, January 21, 2017

Problem 1. WE are given 40 balloons, the air pressure inside each of which is unknown and may differ from balloon to balloon. It is permitted to choose upto k of the balloons and equalize the pressure in them (to the arithmetic mean of their original pressures). What is the smallest k for which it is possible to equalize the pressure of all the balloons?

Problem 2. Anna and Barney play a game. We start with a polynomial $x^3 + x + 2015$. They alternate moves with Anna going first. On her turn, Anna can raise or lower the constant term by 1. On his turn, Barney can raise or lower the coefficient of x by 1. Barney wins if the polynomial acquires an integer root at any point. Prove that Barney has a winning strategy.

Problem 3. For a positive integer n , define the set A_n to consist of all positive integers x such that $x \leq n$ and $n|x^n + 1$. Does there exist n such that A_n contains precisely 130 elements?