

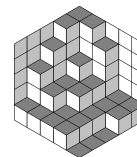
## Algebra

- A5 Let  $n \geq 2$  be a positive integer and  $\lambda$  a positive real number. Initially there are  $n$  fleas on a horizontal line, not all at the same point. We define a move as choosing two fleas at some points  $A$  and  $B$ , with  $A$  to the left of  $B$ , and letting the flea from  $A$  jump over the flea from  $B$  to the point  $C$  so that  $\frac{BC}{AB} = \lambda$ .

Determine all values of  $\lambda$  such that, for any point  $M$  on the line and for any initial position of the  $n$  fleas, there exists a sequence of moves that will take them all to the position right of  $M$ .

- A1 Let  $a, b, c$  be positive real numbers so that  $abc = 1$ . Prove that

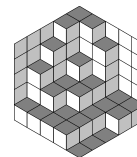
$$\left(a - 1 + \frac{1}{b}\right) \left(b - 1 + \frac{1}{c}\right) \left(c - 1 + \frac{1}{a}\right) \leq 1.$$



## Combinatorics

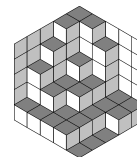
- C5 A number of  $n$  rectangles are drawn in the plane. Each rectangle has parallel sides and the sides of distinct rectangles lie on distinct lines. The rectangles divide the plane into a number of regions. For each region  $R$  let  $v(R)$  be the number of vertices. Take the sum  $\sum v(R)$  over the regions which have one or more vertices of the rectangles in their boundary. Show that this sum is less than  $40n$ .
- C1 A magician has one hundred cards numbered 1 to 100. He puts them into three boxes, a red one, a white one and a blue one, so that each box contains at least one card. A member of the audience draws two cards from two different boxes and announces the sum of numbers on those cards. Given this information, the magician locates the box from which no card has been drawn.

How many ways are there to put the cards in the three boxes so that the trick works?



## Geometry

- G1** In the plane we are given two circles intersecting at  $X$  and  $Y$ . Prove that there exist four points with the following property:
- (P) For every circle touching the two given circles at  $A$  and  $B$ , and meeting the line  $XY$  at  $C$  and  $D$ , each of the lines  $AC$ ,  $AD$ ,  $BC$ ,  $BD$  passes through one of these points.
- G2** Two circles  $G_1$  and  $G_2$  intersect at two points  $M$  and  $N$ . Let  $AB$  be the line tangent to these circles at  $A$  and  $B$ , respectively, so that  $M$  lies closer to  $AB$  than  $N$ . Let  $CD$  be the line parallel to  $AB$  and passing through the point  $M$ , with  $C$  on  $G_1$  and  $D$  on  $G_2$ . Lines  $AC$  and  $BD$  meet at  $E$ ; lines  $AN$  and  $CD$  meet at  $P$ ; lines  $BN$  and  $CD$  meet at  $Q$ . Show that  $EP = EQ$ .
- G5** Let  $ABC$  be an acute-angled triangle, and let  $w$  be the circumcircle of triangle  $ABC$ . The tangent to the circle  $w$  at the point  $A$  meets the tangent to the circle  $w$  at  $C$  at the point  $B'$ . The line  $BB'$  intersects the line  $AC$  at  $E$ , and  $N$  is the midpoint of the segment  $BE$ . Similarly, the tangent to the circle  $w$  at the point  $B$  meets the tangent to the circle  $w$  at the point  $C$  at the point  $A'$ . The line  $AA'$  intersects the line  $BC$  at  $D$ , and  $M$  is the midpoint of the segment  $AD$ .
- a)** Show that  $\angle ABM = \angle BAN$ . **b)** If  $AB = 1$ , determine the values of  $BC$  and  $AC$  for the triangles  $ABC$  which maximise  $\angle ABM$ .
- G6** Let  $ABCD$  be a convex quadrilateral. The perpendicular bisectors of its sides  $AB$  and  $CD$  meet at  $Y$ . Denote by  $X$  a point inside the quadrilateral  $ABCD$  such that  $\angle ADX = \angle BCX < 90^\circ$  and  $\angle DAX = \angle CBX < 90^\circ$ . Show that  $\angle AYB = 2 \cdot \angle ADX$ .
- G8** Let  $AH_1, BH_2, CH_3$  be the altitudes of an acute angled triangle  $ABC$ . Its incircle touches the sides  $BC, AC$  and  $AB$  at  $T_1, T_2$  and  $T_3$  respectively. Consider the symmetric images of the lines  $H_1H_2, H_2H_3$  and  $H_3H_1$  with respect to the lines  $T_1T_2, T_2T_3$  and  $T_3T_1$ . Prove that these images form a triangle whose vertices lie on the incircle of  $ABC$ .



## Number Theory

- N3 Does there exist a positive integer  $n$  such that  $n$  has exactly 2000 prime divisors and  $n$  divides  $2^n + 1$ ?
- N4 Find all triplets of positive integers  $(a, m, n)$  such that  $a^m + 1 \mid (a + 1)^n$ .
- N2 For every positive integers  $n$  let  $d(n)$  the number of all positive integers of  $n$ . Determine all positive integers  $n$  with the property:  $d^3(n) = 4n$ .