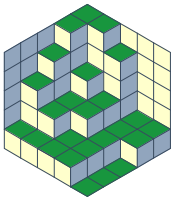


Art of Problem Solving

WOOT 2010–11

Practice Olympiad 3

Instructions

- You should take the test under "olympiad conditions," meaning that the test should be completed in one sitting, with handwritten solutions (just like on an actual olympiad exam). Take the test using only the resources that would be available to you on an actual olympiad, meaning you should use scrap paper, a ruler, a compass, etc., but no calculators and no reference materials.
- You should allot 3 hours to take the test.
- Completely fill out the cover sheet and make sure it is the first page of your solutions.
- On the WOOT Home Page, there is a **WOOT Practice Olympiad Answer Sheet**. Print out (or copy) several blank copies of the answer sheets, and write all of your work on these sheets. Use only black pen or very dark pencil. Make sure that the top of every answer sheet page is completely filled out. Each problem's solution should start on a new page, along with new page numbering.
- Do not discuss the problems on or before the due date of Wednesday, November 24, 2010.

How to submit your solutions

You can submit solutions by upload, email, or by fax. DO NOT MAIL YOUR SOLUTIONS!

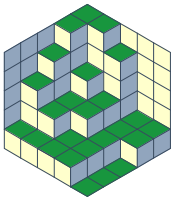
By Upload: Scan your solutions as a single PDF file. **Check to make sure that the file is legible before submitting it!** In your "My Classes" area, follow the link that says "Submit."

By email: Scan your solutions as a single PDF file. **Check to make sure that the file is legible before emailing it!** Email to `woot@artofproblemsolving.com`. Put "WOOT Practice Olympiad 3" in the subject line, attach your solutions as a single PDF file, and write something in the message body - if you leave the message body blank, it will get blocked by our spam filters.

By fax: Fax to (619)659-8146.

Solutions are due by Wednesday, November 24, 2010

Late submissions will not be accepted except under extraordinary circumstances



Art of Problem Solving

WOOT 2010–11

Practice Olympiad 3

WOOT Practice Olympiad Cover Sheet

Username: _____

Class ID: _____

User ID: _____
(Your Class ID and User ID can be found in the "My Classes" section of the website)

Practice Olympiad Number: 3

Beginning	
Intermediate	
Advanced	

Number of pages (including cover sheet): _____

WOOT 2010–11
Practice Olympiad 3

1. A square of side n is divided into n^2 unit squares, each colored red, yellow, or green. Find the minimum value of n such that for any such coloring, we can find a row or a column containing at least three squares of the same color.

2. The function $f : \mathbb{R} \to \mathbb{R}$ satisfies
$$f(x) \le x$$
and
$$f(x+y) \le f(x) + f(y)$$
for all real numbers x and y. Prove that $f(x) = x$ for all real numbers x.

3. Determine the largest positive integer that is a factor of
$$n^4(n-1)^3(n-2)^2(n-3)$$
for all positive integers n.

4. Let n be a positive integer. For $1 \le k \le n$, let a_k denote the number of pairs (x, y) of nonnegative integers satisfying $kx + (k+1)y = n - k + 1$. Show that $a_1 + a_2 + \cdots + a_n = n$.

5. A semicircle has center O and diameter AB. Let M be a point on AB extended past B. A line through M intersects the semicircle at C and D, so that D is closer to M than C. The circumcircles of triangles AOC and DOB intersect at O and K. Show that $\angle MKO = 90^\circ$.

6. Let $\{A_1, A_2, \ldots, A_{2010}\}$ and $\{B_1, B_2, \ldots, B_{2010}\}$ be two partitions of the set $\{1, 2, \ldots, n\}$, such that for all $1 \le i, j \le 2010$, if $A_i \cap B_j = \emptyset$, then $|A_i| + |B_j| \ge 2010$. Find the minimum value of n for which such partitions exist.

   Note: We say that $\{S_1, S_2, \ldots, S_k\}$ is a partition of the set S if $S_i \cap S_j = \emptyset$ for all $i \ne j$, and $S_1 \cup S_2 \cup \cdots \cup S_k = S$.

7. The sequence a_0, a_1, a_2, $\ldots$ is defined by $a_0 = 1$ and
$$a_{n+1} = \frac{1}{2}\left(a_n + \frac{1}{3a_n}\right).$$
Let
$$A_n = \frac{3}{3a_n^2 - 1}.$$
Prove that A_n is a perfect square and that it has at least n distinct prime divisors for all $n \ge 1$.

Remember . . .

- Download the WOOT Practice Olympiad Answer Sheet (in your "My Classes" area), and write all your solutions on copies of this sheet.
- Complete the WOOT Practice Olympiad Cover Sheet (included in this document), and make it the first page of your solutions. We will not accept your solutions without this cover sheet.
- Do not mail your solutions. Use only upload, e-mail, or fax. If you use e-mail, send your solutions to

  woot@artofproblemsolving.com,

  NOT **classes**@artofproblemsolving.com. If you use e-mail, we will only accept solutions in PDF format. In particular, solutions in JPG or Word format will not be accepted.