1998 IMO Camp

TRAINING PROBLEMS
Problem Set 4

1. Prove that no polyhedron in three-space can have exactly seven edges, while any other integer greater than five is possible.

2. Do there exist two polyhedra, each with 30 faces, bounded by the same faces coinciding pairwise, such that one is convex and the other is not? (Faces of the same figure need not be congruent, but pairwise coinciding faces on each polyhedron must be congruent.)

3. Let all plane sections of a certain surface be circles. Is it true that the surface must be a sphere?

4. Suppose that the shadows cast by a convex planar region perpendicularly onto all lines are line segments of length two. Must the region be a circular disc of unit radius?

5. Suppose that the shadows cast by a convex solid body perpendicularly onto all planes are circular discs. Must the body be a spherical ball?

6. Consider several points lying in the plane. We assume that all interpoint distances are different. We connect each point to the nearest point by means of a straight line segment. Prove that the resulting figure does not contain any closed polygons or intersecting line segments.

7. There are n points lying in the plane, with no three of them lying on the same straight line. Is it always possible to find a closed polygon with n nonintersecting sides whose vertices are these n points?

8. If x, y, z and n are natural numbers, and $n \geq z$ find an elementary proof that $x^n + y^n = z^n$ does not hold.

9. On an elliptical billiard table, ball A is standing by the cushion and ball B on the line segment S connecting the foci of the ellipse. Is it possible to strike A in such a way that it bounces off the cushion and hits B without crossing S before it hits B?