Various physical constants

Electron's charge $e = 1.609 \times 10^{-19}$ C
Planck constant $\hbar = 6.62 \times 10^{-34}$ J s
Boltzmann Constant $k_B = 1.38 \times 10^{-19}$ J K^{-1}
$\frac{1}{4\pi\epsilon_0} = 9 \times 10^9$ SI Units

1. Consider an equilateral triangle whose sides are of length L. Identical charges $+Q$ are now placed at each of its vertex.

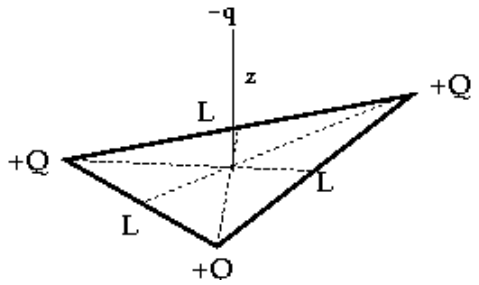


Figure 1: Triangular Charge arrangement

(a) Find the electric field at a vertical distance z above the plane containing the triangle, measured along the symmetry axis of the equilateral triangle. (Note that any point on the axis is equidistant from all the corners of the triangle). (2 points)

(b) If now another charge -q with mass m is placed at the center of the triangle and nudged very gently along the symmetry axis so that its initial kinetic energy is K, find the maximum distance D that the central charged particle will travel along the symmetry axis. (2 points)

(c) Will the particle return to the center? How can you deduce that? If so find the time taken for it return to the center under the approximation $D << L$. (2 points)

2. Consider the Lenard-Jones potential which describes the interaction of molecules in a solid:

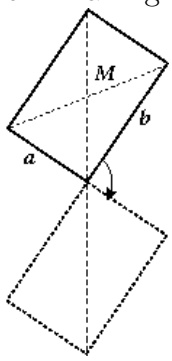
$$V(r) = \frac{a}{r^6} - \frac{b}{r^{12}}$$

(a) Find the equilibrium distance between the molecules. (1 point)

(b) What is the energy required to separate two such molecules from this equilibrium position to infinite separation against the above interaction. (1 point)

3. Take a solid uniform rectangular plate whose sides are of length a and b and the total mass is M. The plate is hinged at one of its corner so that it can swing freely. The plate is now held in a way so that its center of mass lies exactly above the hinge point and then allowed to"fall".

Figure 2: Falling Plate



What would be the velocity of its center of mass when the plates center of mass passes exactly below the hinge? (3 points)

4. Consider a gas of mono-atomic ideas gas which is kept at a temperature of 47^o C. Now if a dust particle of mass .0001 gm is introduced inside the gas what will be its speed after 1 year? How should you proceed to calculate this? (2 points)

5. An astronomical object (called PhO-2013) of mass m revolves in a circular orbit around the center of a gaseous nebulae of whose density is given by $\rho(r) = \beta r$ with $r < R$, where R is the radius of the nebula.

   (a) Find the period of revolution of the object if its distance from the center of the nebula is r_0 with $r_0 < R$. (2 points)

(b) Find the total energy of PhO-2013 if its mass is m. (1 point)

(c) Now a tiny asteroid falling radially towards the center of the nebula accidentally hits Ph0-2013. Find the frequency of the radial oscillation of Ph0-2013. (2 points)

6. A scientist tries to measure the dependence of the Force F between two objects as a function of the separation x between them and asks you to fit his results is terms of a formula $F = Ax^m$. Find the best fit for the values A and m, using the given graph in the next page. Label your graph properly and record your findings in THIS PAGE(2 point)

data no	x	F
1	.1	0.505326
2	.3	0.362315
3	.5	0.313016
4	.7	0.283513
5	.9	0.265762
6	1.1	0.239462
7	1.3	0.237042
8	1.5	0.210459
9	1.7	0.200616
10	1.9	0.198840