Nice Endings, or Mortifying Overcomplications?

4th NEMO, 6 October 2016

Problem 1. Prove that the sum of any n entries from the following table situated in different rows and different columns is not less than 1.

$$\begin{array}{ccccc}
1 & \frac{1}{2} & \frac{1}{3} & \cdots & \frac{1}{n} \\
\frac{1}{2} & \frac{1}{3} & \frac{1}{4} & \cdots & \frac{1}{n} \\
. & & & & . \\
. & & & & . \\
. & & & & . \\
\frac{1}{n} & \frac{1}{n+1} & \frac{1}{n+2} & \cdots & \frac{1}{2n-1}
\end{array}$$

Problem 2. $P(x), Q(x)$ are two polynomials such that $P(x) = Q(x)$ has no real solution, and $P(Q(x)) \equiv Q(P(x)) \forall x \in R$. Prove that $P(P(x)) = Q(Q(x))$ has no real solution.

Problem 3. 2016 nonnegative integers are written on a board. In each step, you can erase two of the numbers and replace them with their sum and their difference. Is it possible, in a finite number of steps, to reach a state where applying the operation to any pair of numbers does not change any of the numbers?

Problem 4. We denote the circumcircle of a triangle XYZ by (XYZ). Let ABC be an acute angled triangle and P a point inside the triangle such that $\angle BPC = 180^\circ - \angle A$. BP, CP intersect CA, AB at E, F. Circle (AEF) intersects (ABC) again at G. The circle with diameter PG intersects (ABC) again at K. D is projection of P on BC and M is the midpoint of BC. Prove that the circle (KPG) and circle (KDM) are tangent.