2005 IMO-TRAINING SUMMER CAMP

TRAINING TEST #3

Question #1

Find all pairs of real numbers (a, b) for which it is possible to find real numbers x and y such that $\frac{x+y}{x^2+y^2} = a$ and $\frac{x^3+y^3}{x^2+y^2} = b$.

Question #2

Two circles in a plane intersect. Let A be one of the points of intersection. Starting simultaneously from A two points move with constant speed, each travelling along its own circle in the same sense. The two points return to A simultaneously after one revolution. Prove that there is a fixed point P in the plane such that, at any time, the distances from P to the moving points are equal.

Question #3

Let a, b, c and d be odd integers such that $0 < a < b < c < d$, $ad = bc$, $a + d = 2^m$ and $b + c = 2^n$ for some integers m and n. Find the value of a.