Problem-1: Quantized Harmonic Oscillator and Temperature

[Total 12 marks]

The total energy of a harmonic oscillator is given by

$$E = \frac{1}{2}mv^2 + \frac{1}{2}kx^2 = \frac{p^2}{2m} + \frac{1}{2}m\omega^2 x^2$$

where p is the momentum of the oscillator and $\omega = \sqrt{\frac{k}{m}}$ is its angular frequency. Classically the lowest energy of the oscillator is attained when $p = 0, x = 0$ at which $E = 0$.

a) For constant E, draw the momentum vs. position graph with appropriate scales in terms of E, k, m and/or ω.

[2 Mark]

b) What is the area inside this graph?

[1 Mark]

c) Demanding that this area in not arbitrary but multiples of a fundamental constant h, what physical restriction do you get?

[1 Mark]

Quantum mechanics prevents us from doing so as this requires knowing the position and momentum of the oscillator precisely which contradicts uncertainty principle. Assume that at the ground state (lowest energy state) there is a position fluctuation Δx.

d) What is the momentum uncertainty Δp associated with this position?

[1 Mark]

e) What is the energy associated with this position uncertainty?

[1 Mark]

f) What is the minimum value of this energy? What is the associated value of Δx?

[1+1=2 Mark]

g) What are the allowed energy values for the oscillator consistent with your result from the above?

[1 Mark]

h) The atoms in an one dimensional solid can be thought of as a collection of such oscillators. The probability that a system picks up energy E when kept inside a heat bath of temperature T is given by

$$p(E) = Ae^{-E/kT}$$

where A is a constant which can be (but not necessary) fixed by demanding that the sum of all probabilities is 1.

Note that the average value of any function f of a random variable x is given by the formula

$$\bar{f}(x) = \frac{\sum_x f(x)p(x)}{\sum_x p(x)}$$

where the sum is implied over all the possible values of x.

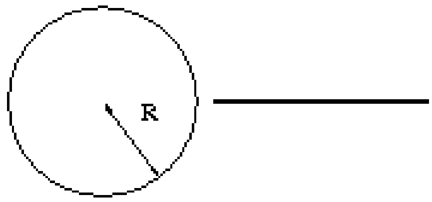
Now using your result for the energies of the oscillator find the average energy of a simple harmonic oscillator as a function of the temeperature T. How does the average energy behave as h is taken vanishingly small , i.e $h \to 0$ and $T \to \infty$? What is the meaning of your result?

[3 Marks]

Problem-2: Reaching the Skies

[Total 4 marks]

One of the interesting creations from the minds of science fiction writers in the "Skyhook" satellite that consists of a long rope placed in orbit at the equator, alligned along a radius from the center of the earth, and moving so that the rope appears suspended in the space above a fixed point on the equator (see figure). The bottom of the rope just reaches the earth's surface



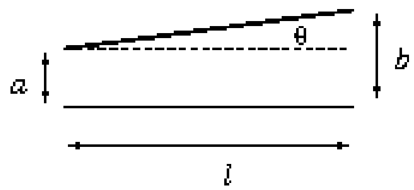
and hangs free. Assume that the rope has a uniform mass per unit length (and it is strong enough to withstand tensile forces), find the length of the rope. (Remember that the meter was defined to be the one-ten thounsandth distance from the north pole to the equator.)

[4 Marks]

Problem-3: Not a parallel plate Capacitor

[Total 6 marks]

Consider two metallic plates each of length l and width D who are used to build a parallel plate capacitor with a separation a between them. But due to a faulty assembly line the capacitor ends being not parallel - the side view of which given below If the angle between the plates is θ,



a) Find the capacitance of this non-parallel capacitor.

[3 Marks]

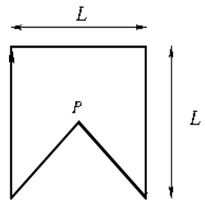
b) If the magnitude of the charge in either of the parallel plate is Q, find the work done to rotate an initially parallel plate capacitor (for which $\theta = 0$) to the final non-parallel capacitor for which $\theta \neq 0$. Your result should be in terms of θ which is assumed to be so small that a taylor series expansion is admissible.

[3 Marks]

Problem-4: Current Flow in F1 Racing

[Total 4 marks]

Consider the following current carrying loop. What is the magnetic field at the point P in terms of current I and the length L? You should provide arguments and working details that you have used to get to your answer.

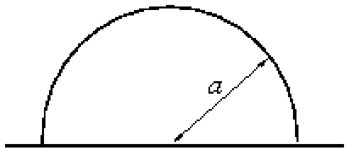


[4 Marks]

Problem-5:Bumped Off a Hill

[Total 4 marks]

A student decides to slide down the surface of a frictionless fixed hemispherical hill of radius a. He starts off his sliding from the top of the hill. After sliding down what vertical height (measured from the top) will he fly off the hill?



[4 Marks]