

# IMO Mock 004

*Time: 4:30 hours*

*Total Marks: 21*

**Problem 1:** Find (with proof) all odd integers  $n > 1$  such that if  $ab|n$  and  $a$  and  $b$  are relatively primes, then  $a + b - 1|n$ .

**Problem 2:** For  $a, b, c > 0$  and  $abc \geq 1$ , prove that

$$\frac{a^2bc}{\sqrt{bc}+1} + \frac{b^2ca}{\sqrt{ca}+1} + \frac{c^2ab}{\sqrt{ab}+1} \geq \frac{3}{2}$$

**Problem 3:**  $O$  is the circumcenter of  $\triangle ABC$  and  $T$  is that of  $\triangle AOC$ .  $M$  is the mid-point of side  $AC$ . On sides  $AB$  and  $BC$ , there are points  $D$  and  $E$ , respectively, such that  $\angle BDM = \angle BEM = \angle ABC$ . Prove that  $BT \perp DE$ .

*All problems collected by Adib Hasan (<http://www.facebook.com/phlembac.hidden>)*