New Zealand Mathematical Olympiad Committee

2011 Squad Assignment Two

Geometry

Due: Monday 28th February 2011

1. Find all possible values of the quotient
$$\frac{r+\rho}{a+b}$$
where r and ρ are respectively the radii of the circumcircle and incircle of the right triangle with legs a and b.

2. Let ABC be an isosceles triangle with $|AB| = |AC|$ and P, Q are interior points of AB and AC respectively. Prove that the circumcircle of $\triangle APQ$ passes through the circumcentre of $\triangle ABC$ if and only if $|AP| = |CQ|$

3. Let $ABCD$ be a rhombus and let a tangent of its incircle cut the interior of the sides BC and CD, and denote R, S the intersections of the tangent with the lines AB, AD respectively. Prove that the value of $|BR| \cdot |DS|$ is independent of the choice of the tangent.

4. In an acute-angled triangle ABC, M is the midpoint of side BC, and D, E and F the feet of the altitudes from A, B and C, respectively. Let H be the orthocentre of triangle ABC, S the midpoint of AH, and G the intersection of FE and HA. If N is the intersection of the line segment AM and the circumcircle of triangle BCH, prove that $\angle HMA = \angle GNS$.

5. Points C, D, E and F lie on a circle with centre O. The two chords CD and EF intersect at a point N. The tangents at C and D intersect at A, and the tangents at E and F intersect at B. Prove that $ON \perp AB$.

6. Can the four incentres of the four faces of a tetrahedron be coplanar?

7. Let O be the circumcentre of an acute-angled triangle ABC. A line through O intersects the sides CA and CB at points D and E respectively, and meets the circumcircle of triangle ABO again at point $P \neq O$ inside the triangle. A point Q on side AB is such that
$$\frac{AQ}{QB} = \frac{DP}{PE}.$$
Prove that $\angle APQ = 2\angle CAP$.

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