IMO Winter Camp 2006

Mock Olympiad 2 – January 7, 2006

1. Find all positive integers x and y such that $(x+y)(xy+1)$ is a power of 2.

2. Let $ABCD$ be a convex quadrilateral with $\angle ADC = \angle BCD > 90°$. Let E be the point at which the line AC intersects the line parallel to AD through B, and let F be the point at which the line BD intersects the line parallel to BC through A. Prove that $EF \| CD$.

3. Determine the number of different arrangement $a_1, a_2, \ldots, a_{10}$ of the integers $1, 2, \ldots, 10$ such that

$$a_i > a_{2i} \quad \text{for } 1 \leq i \leq 5,$$

and

$$a_i > a_{2i+1} \quad \text{for } 1 \leq i \leq 4.$$