IMO Mock 001

Time: 4:30 hours *Total Marks: 21*

Problem 1: Find all functions $f\colon \mathbb{R} \longrightarrow \mathbb{R}$ such that for all $x, y \in \mathbb{R}$, the following equation holds:

$$f(xy + f(x)) = xf(y) + f(x)$$

Problem 2: The points A_1, B_1, C_1 lie on the sides BC, CA and AB of $\triangle ABC$ respectively.Suppose that $AB_1 - AC_1 = CA_1 - CB_1 = BC_1 - BA_1$. Let O_A, O_B and O_C be the circumcentres of triangles AB_1C_1, A_1BC_1 and A_1B_1C respectively.Prove that $\triangle ABC$ and $\triangle O_AO_BO_C$ have same incentre.

Problem 3: Fine all positive integers $m, n \geq 2$, such that

(i) $m+1$ is a prime number of type $4k-1$

(ii) there is a (positive) prime number p and non-negative integer a, such that

$$\frac{m^{2^n-1}-1}{m-1} = m^n + p^a.$$

Problems collected by Adib Hasan (http://www.facebook.com/phlembac.hidden)