Sample Questions

2007?

1. If $\cos\alpha = \dfrac{p}{q}$ (p and $q \neq 0$ are whole numbers), show that $q^n\cos(n\alpha)$ is a whole number.

2. If real numbers x, y and z satisfy $x^2+y^2+z^2 = xy+yz+xz$, find $\dfrac{\sqrt{x}}{\sqrt{x}+\sqrt{y}+\sqrt{z}}$

3. Find all polynomials $P(x)$ such that $xP(x-1) = (x-12)P(x)$.

4. Solve $(m-9)^2m! + (n-8)^2n! + 50p! + 49q! = \sqrt{\overline{mnpq}}$. Note that m, n, p and q are digits and $\overline{mnpq} = 1000m + 100n + 10p + q$.

5. Show that at least six points with rational coordinates exist on the curve $y^2 = x^3 + x + 1370^{1370}$.

6. Find all whole numbers m and n that satisfy $\dfrac{1}{m} + \dfrac{1}{n} - \dfrac{1}{mn^2} = \dfrac{3}{4}$.

7. Find all odd prime numbers p such that $\dfrac{2^{p-1}-1}{p}$ becomes a complete square.

8. Show
   that for all natural numbers t, $A = 1^t + 2^t + \ldots + 9^t - 3(1+6^t+8^t)$ is a multiple of 18.

9. Assume that $x_1, x_2, \ldots, x_{12}$ are real numbers, $|x_i| \geq 1$ and $I = [a,b]$ such that $b - a \leq 2$. Show that there are fewer than 1000 numbers of the form $t = \sum_{i=1}^{12} r_i x_i$ with $r_i = \pm 1$ on the interval I.

10. Find all real numbers $a_1, a_2, \ldots, a_{1375}$ such that for $n = 1, 2, \ldots, 1374$, $2\sqrt{a_n - (n-1)} \geq a_{n+1} - (n-1)$, and $2\sqrt{a_{1375} - 1374} \geq a_1 + 1$.

11. Find all real non-negative numbers x, y and z that satisfy $2^x + 3^y = z^2$.