2018 Special Camp - IMO Mock

The IMO Moshari

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Problem 1: A communications network consisting of some terminals is called a 3-connector if among any three terminals, some two of them can directly communicate with each other. A communications network contains a windmill with n blades if there exist n pairs of terminals $\{x_1, y_1\}, \{x_2, y_2\}, \ldots, \{x_n, y_n\}$ such that each x_i can directly communicate with the corresponding y_i and there is a hub terminal that can directly communicate with each of the $2n$ terminals $x_1, y_1, \ldots, x_n, y_n$. What is the minimum number of terminals for a 3-connector to ensure that it contains a windmill with n blades?

Problem 2: Find all functions $f : \mathbb{R} \to \mathbb{R}$ satisfying

$$f(x + f(y)) - f(x) = (x + f(y))^4 - x^4$$

for all $x, y \in \mathbb{R}$.

Problem 3: Let P be a point inside the quadrilateral $ABCD$. Prove that the isogonal conjugate of P with respect to $ABCD$ exists if and only if $\angle APB + \angle CPD = 180°$.