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WINTER CAMP 2004
GEOMETRY

EXERCISES.

1. Given a point P, a line L such that $P \notin L$ and an angle α such that $0<\alpha<\pi$. Find a point O such that $P \in r(L)$ and $r(P) \in L$, where r is the rotation with centre O and angle α.

2. Let $ABCDEFG$ be a regular heptagon. Compute $\dfrac{\frac{1}{AB}}{\frac{1}{AC}+\frac{1}{AD}+\frac{1}{AE}+\frac{1}{AF}+\frac{1}{AG}}$.

3. Let ABC be an equilateral triangle. Consider three parallel lines each of them passing through one the vertices of the triangle. Let p and q denote the distances from the inside parallel to the outside parallels. Express the area of the triangle ABC in terms of p and q.

4. Let M denote the midpoint of the segment AB. Describe the set consisting of all points P for which $AP \times BP = AB \times MP$.

5. Let 3, $2-x$ and $\sqrt{x^2+8x+7}$ be the lengths of the sides of a triangle.
   a) Find all the possible values of A, the area of this triangle..
   b) Find all the possible values of P, the perimeter of this triangle.

6. Let ABC be a triangle inscribed in a circumference of centre O. Let BB' and CC' be two of its altitudes. Let l denote any straight line parallel to OB. Let M be the intersection of the lines l and AB and let K be the intersection of l and the circumference $MB'C'$. Show that $B'K$ is perpendicular to AB.

7. Let ABC be a triangle in which the length of the side BC is the average of the lengths of the sides AB and AC. Show that the line that bisects the angle A is perpendicular to the line that joins the incenter and the circumcenter of this triangle.

8. Let $ABCD$ be a tetrahedron such that $AB \times CD = AC \times BD = AD \times BC$. Show that the lines that join each of the vertices of the tetrahedron with the incenter of the opposite face are concurrent.

9. Let r denote the radius of the incircle of a triangle and let s, t and u the radii of the circles which are tangent, respectively, to two of the sides of the triangle and to its incircle. Express r in terms of s, t and u.

Side 1

10. Let $ABCD$ be a rhombus for which there exist points M and N on the sides BC and CD, respectively, such that $CM+CN+MN=2$ and $\angle BAD = 2\angle MAN$. Find the area of the rhombus.