Non-Existential Mathematical Oxymoron

team_7

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Problem 1 $2N$ students take a quiz in which the possible scores are $0, 1 \ldots 10$. It is given that each of these scores appeared at least once, and the average of their scores is 7.4. Prove that the students can be divided into two sets of N student with both sets having an average score of 7.4.

Problem 2 Triangle ABC circumscribed (O) has A-excircle (J) that touches $AB,\ BC,\ AC$ at $F,\ D,\ E$, resp.

1. L is the midpoint of BC. Circle with diameter LJ cuts $DE,\ DF$ at $K,\ H$. Prove that (BDK), (CDH) has an intersecting point on (J).
2. Let $EF \cap BC = \{G\}$ and GJ cuts $AB,\ AC$ at $M,\ N$, resp. $P \in JB$ and $Q \in JC$ such that

$$\angle PAB = \angle QAC = 90^\circ.$$

$PM \cap QN = \{T\}$ and S is the midpoint of the larger BC-arc of (O). (I) is the incircle of ABC. Prove that $SI \cap AT \in (O)$.

Problem 3 Let p_n be the n^{th} prime counting from the smallest prime 2 in increasing order. For example, $p_1 = 2, p_2 = 3, p_3 = 5, \cdots$

1. For a given $n \geq 10$, let r be the smallest integer satisfying

$$2 \leq r \leq n-2, \quad n-r+1 < p_r$$

and define $N_s = (sp_1p_2 \cdots p_{r-1}) - 1$ for $s = 1, 2, \ldots, p_r$. Prove that there exists $j, 1 \leq j \leq p_r$, such that none of $p_1, p_2, \cdots, p_n$ divides N_j.

2. Using the result of (3.1), find all positive integers m for which

$$p_{m+1}^2 < p_1p_2 \cdots p_m$$