IMO Mock 005

Day 1

Problem 1: Integers $a_0, a_1, a_2, \ldots a_n$ are greater than or equal to -1 and are all non-zeros. If $a_0 + 2a_1 + \ldots + 2^n a_n = 0$, then prove that $a_0 + a_1 + a_2 + \ldots + a_n > 0$

Problem 2: Determine (with proof) all functions $f\colon [0, +\infty) \rightarrow [0, +\infty)$ such that for every $x \geq 0$, we have $4f(x) \geq 3x$ and $f(4f(x) - 3x) = x$

Problem 3: Let O and H be the circumcenter and orthocenter of acute $\triangle ABC$. The bisector of $\angle BAC$ meets the circumcircle τ of $\triangle ABC$ at D. Let E be the mirror image of D with respect to line BC. Let F be on τ such that DF is a diameter. Assume that lines AE and FH meet at G. Let M be the mid-point of side BC. Prove that $GM \perp AF$.

Day 2

Problem 4: In how many ways can one choose $n-3$ diagonals of a regular n-gon, so that no two have an intersection strictly inside that n-gon, and no three form a triangle?

Problem 5: In acute $\triangle ABC$, $AB > AC$, Let M be the mid-point of BC. The exterior angle bisector of $\angle BAC$ meets ray BC at P. Points K and F lie on line PA such that $MF \perp BC$ and $MK \perp PA$. Prove that $BC^2 = 4PF \cdot AK$.

Problem 6: Let a, b, c and d be real numbers, and at least one of c or d is not zero. Let $f\colon \mathbb{R} \rightarrow \mathbb{R}$ be the function defined by $f(x) = \dfrac{ax+b}{cx+d}$. Assume that $f(x) \neq x$ for every $x \in \mathbb{R}$. Prove that if there exists at least one p such that $f^{1387}(p) = p$, then for every x, for which $f^{1387}(x)$ is defined, we have $f^{1387}(x) = x$.

All problems are collected by Sourav Das

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