Online Number Theory Camp: Exam 1

August 24, 2015

1. PROBLEMS

*Problems are not in increasing order of difficulty. But these problems are not that hard. Think strategically. Only the definitions may seem scary but the solutions are not.

Problem 1.1. Let m and n be positive integers such that $5m + n$ is a divisor of $5n + m$. Prove that m is a divisor of n.

Problem 1.2. Let a_0 be a positive integer and $a_n = a_{n-1} + d(a_{n-1})$ where $d(n)$ can be any divisor of n greater than 1. Find all k so that there is an index n for which $a_n = k$.

Problem 1.3. Positive integers a, b and prime p satisfy $a^2 + p^2 = b^2$. Prove that $2(b + p)$ is a square.

Problem 1.4. Find all n so that $6^n - 1$ divides $7^n - 1$.

Problem 1.5. A quadruple (p, a, b, c) of positive integers is called a *Leiden quadruple* if

- p is an odd prime number,

- a, b, and c are distinct and

- $ab + 1, bc + 1$ and $ca + 1$ are divisible by p.

i Prove that for every Leiden quadruple (p, a, b, c) we have $p + 2 \leq \dfrac{a + b + c}{3}$.

ii Determine all numbers p for which a Leiden quadruple (p, a, b, c) exists with $p + 2 = \dfrac{a + b + c}{3}$.