Nervous Experiences May Occur

October 2016

Problem 1. Suppose that the set $\{1, 2, ..., 1998\}$ has been partitioned into disjoint pairs $\{a_j, b_j\}, j = 1, 2, ..., 999$, so that for all $j, |a_j - b_j|$ is either 1 or 6. Prove that the sum

$$|a_1 - b_1| + |a_2 - b_2| + ... + |a_{999} - b_{999}|$$

ends in the digit 9 .

Problem 2. A fly and k spiders are placed in some vertices of $n \times n$ lattice. One move consists of following: firstly, fly goes to some adjacent vertex or stays where it is and then every spider goes to some adjacent vertex or stays where it is (more than one spider can be in the same vertex). Spiders and fly knows where the others are all the time. Find the smallest k so that the spiders can catch the fly in a finite number of moves, regardless of their initial position.
(Vertices in lattice are adjacent if exactly one coordinate of one vertex is different from the same coordinate of the other vertex, and their difference is equal to 1. Spider catches a fly if they are in the same vertex.)

Problem 3. Let E be a set of $2n - 1$ points on a circle, with $n > 2$. Suppose that precisely k of the points are coloured black. We say that this solouring is admissible if there is at least one pair of black points such that the interior of one of the arcs they determine contains exactly n points of E. What is the smallest k such that any colouring of k points is admissible ?

Problem 4. Let us culture a dragon on the co-ordinate plane. We do this by drawing dragon curves d_i, each having their head at the point $O(0, 0)$ and their tail at a point P_i. Let the baby dragon curve d_0 be the a line segment of unit length and head/endpoint $O(0, 0)$ and tail/endpoint $P_0(0, -1)$. To obtain d_{k+1}, you have to draw d_k and the figure obtained by rotating d_k about P_k at an angle of $\frac{\pi}{2}$ clockwise.
(a) For which i, j are O, P_i, P_j collinear? If they are collinear, what ratio do they divide their segment in?
(b) Prove that the dragon curve never overlaps (or otherwise the dragon would die :P). i.e. prove that the intersection d_k and the figure obtained by rotating d_k about P_k at an angle of $\frac{\pi}{2}$ clockwise is a set of disjoint points.