Nice Easy Maths :O

5th NEMO, 13 October 2016

Problem 1. Let n be an integer greater than 1. For a positive integer m, let $S_m = \{1, 2, \ldots, mn\}$. Suppose that there exists a $2n$-element set T such that

1. each element of T is an m-element subset of S_m;
2. each pair of elements of T share at most one common element;
3. each element of S_m is contained in exactly two elements of T.

Determine the maximum value of m in terms of n.

Problem 2. We define:

- One straight edge construction to be the process of selecting two points, and drawing the line joining both points.
- One circle construction to be the process of selecting a point A and another point B, and drawing the circle with centre A which has B on its circumeference.
- A point definition can either define an arbitrary point on some portion of a figure, or the intersection of two or more figures.

Given a point P and a line l passing through it, find a construction for the line perpendicular to l and passing through l. You may use as many point definitions as you want, but only a total of 3 straight edge or circle constructions.

Problem 3. Find all prime numbers p such that $p^2 - p - 1$ is the cube of some integer.

Problem 4. For a given integer $n \geq 3$, let P be the regular polygon having n sides. Any set of $n - 3$ diagonals of P which don't intersect in the interior of P is called a triangulation of P into $n - 2$ triangles. For which values of n does there exist a triangulation of P consisting entirely of isosceles triangles?