Glossary

When writing the book, I have assumed you are familiar with terminology such as 'orthocentre' and 'geometric mean'. As this may not necessarily be the case, some common terms are explained here.

- **abscissa:** the x-coordinate of a point on the plane. Compare with *ordinate*.
- **altitude:** a line from a vertex of a triangle, which is perpendicular to the opposite side. The three altitudes intersect at the *orthocentre*.
- **AM-GM inequality:** for n non-negative real numbers, the *arithmetic mean* is greater than or equal to the *geometric mean*, with equality if and only if all variables are equal.
- **Apollonius' theorem:** in a triangle ABC, where M is the centre of BC, we have $AM^2 = \frac{1}{2}b^2 + \frac{1}{2}c^2 - \frac{1}{4}a^2$.
- **areal coordinates:** a system of projective homogeneous coordinates where each point is considered to be the weighted barycentre of three variable masses, each of which is positioned at a vertex of a fixed 'reference triangle'.
- **Argand plane:** the idea of representing the real and imaginary parts of a complex number as the Cartesian coordinates of a point on the Euclidean plane.
- **arithmetic mean:** for n variables $\{x_1, \ldots, x_n\}$, the arithmetic mean is $\frac{1}{n}(x_1 + \ldots + x_n)$.
- **barycentre:** the centre of mass of a set of masses positioned at points (on the plane).
- **barycentric coordinates:** a synonym of *areal coordinates*.
- **Bezout's theorem:** two algebraic curves of degrees m and n intersect in precisely mn points on the complex projective plane, when counted with the appropriate multiplicity.
- **Brahmagupta's formula:** for a cyclic quadrilateral with side lengths a, b, c, d and semiperimeter s, the area is given by $\sqrt{(s-a)(s-b)(s-c)(s-d)}$. This is a generalisation of *Heron's formula*.
- **Brianchon's theorem:** if a hexagon $ABCDEF$ is circumscribed about a circle (or, more generally, a *conic*), its three major diagonals (AD, BE and CF) are concurrent.
- **Cardano's formula:** the general solution to a cubic equation.
- **Catalan sequence:** a sequence of integers that counts the number of valid strings of $2n$ parentheses.
- **Cauchy-Schwarz inequality:** for two vectors u and v, $u \cdot v \leq |u|\ |v|$, with equality if and only if u and v have the same direction.
- **Cayley-Bacharach theorem:** if two cubics intersect in nine points and a third cubic passes through eight of those points, then it also passes through the ninth.
- **Cayley-Menger determinant:** a formula for the square of the volume of a simplex in terms of the squares of the side lengths.
- **centroid:** the intersection of the three *medians* of a triangle. More generally, it is synonymous with *barycentre*.
- **Ceva's theorem:** if D, E and F are points on the (possibly extended) sides BC, CA and AB, respectively, then AD, BE and CF are concurrent if and only if $\frac{\overrightarrow{BD}}{\overrightarrow{DC}} \cdot \frac{\overrightarrow{CE}}{\overrightarrow{EA}} \cdot \frac{\overrightarrow{AF}}{\overrightarrow{FB}} = 1$.
- **circular points at infinity:** a pair of points on the complex projective plane through which all circles pass.
- **circumcentre:** the point O equidistant from the three vertices of a triangle.
- **circumradius:** the radius R of the circumscribing circle of a triangle or cyclic polygon.
- **collinear:** points lying on the same straight line.
- **concentric:** objects sharing the same centre. This is usually applied to circles, but is equally applicable to conics.
- **conconic:** points lying on the same conic section.

- **concurrent:** three (or more) lines are said to be concurrent if they all intersect at a single point or are all mutually parallel.
- **concyclic:** points lying on the same circle.
- **conic:** a curve in the plane described by a quadratic equation in Cartesian coordinates.
- **coplanar:** points (or curves) lying on the same flat plane.
- **coprime:** two integers with a greatest common divisor of 1.
- **cosine rule:** for a generic triangle $A\,B\,C$, $a^2 = b^2 + c^2 - 2\,b\,c \cos A$.
- **cross-ratio:** for four collinear points, the ratio $\frac{\overrightarrow{A\,B}\cdot\overrightarrow{C\,D}}{\overrightarrow{B\,C}\cdot\overrightarrow{D\,A}}$. If the cross-ratio is -1, the points form a *harmonic range*.
- **cube roots of unity:** the three roots of the polynomial $z^3 - 1$. We often use ω to represent the 'north-west' complex cube root of unity $\frac{\sqrt{3}}{2}\,i - \frac{1}{2}$.
- **Desargues' theorem:** two triangles are in perspective about a point if and only if they are in perspective about a line.
- **difference of two squares:** the polynomial $a^2 - b^2 = (a - b)\,(a + b)$.
- **difference of three cubes:** the polynomial $a^3 + b^3 + c^3 - 3\,a\,b\,c = (a + b + c)\left(a + b\,\omega + c\,\omega^2\right)\left(a + b\,\omega^2 + c\,\omega\right)$, where ω is a primitive *cube root of unity*.
- **Euclid's algorithm:** the greatest common divisor of a and b can be obtained by subtracting the smaller from the larger and repeating until one of the numbers is zero. For example, $(26,\ 10) \to (16,\ 10) \to (6,\ 10) \to (6,\ 4) \to (2,\ 4) \to (2,\ 2) \to (2,\ 0)$, so the greatest common divisor of 26 and 10 is 2.
- **Euler-Apollonius lollipop:** the disc on diameter $G\,H$, which contains the *incentre*, *symmedian point* and *Gergonne point*.
- **Euler-Fermat theorem:** if a and n are coprime, then $a^{\varphi(n)} \equiv 1 \pmod{n}$, where $\varphi(n)$ is the number of positive integers $\leq n$ which are coprime to n.
- **Euler line:** the circumcentre, centroid, nine-point centre and orthocentre are collinear in the ratio $O\,G : G\,T : T\,H = 2 : 1 : 3$.
- **Euler's inequality:** $O\,I^2 = R^2 - 2\,R\,r$, where the *circumcircle* has centre O and radius R, and the *incircle* has centre I and radius r.
- **excentre:** the centre of an *excircle*.
- **excircle:** one of three circles (other than the *incircle*) tangent to (the extensions of) the three sides of a triangle.
- **Feuerbach's theorem:** the *nine-point circle* is tangent to the *incircle* and three *excircles*.
- **Fibonacci sequence:** the sequence defined with $F_0 = 0$, $F_1 = 1$ and $F_{n+2} = F_{n+1} + F_n$. If you extrapolate it backwards, you obtain the 'nega-Fibonacci numbers'.
- **fundamental theorem of algebra:** a degree-d polynomial can be factorised into d linear factors over the complex numbers.
- **fundamental theorem of arithmetic:** every positive integer has a unique prime factorisation.
- **geometric mean:** for n variables $\{x_1,\ \ldots,\ x_n\}$, the geometric mean is $\sqrt[n]{x_1\,x_2\,\ldots\,x_n}$.
- **Gergonne point:** the intersection of the lines joining each vertex of a triangle to the point of tangency of the incircle with the opposite side.
- **glide-reflection:** the composition of a reflection in a line and a translation parallel to the line.
- **harmonic mean:** for n variables $\{x_1,\ \ldots,\ x_n\}$, the harmonic mean is $\frac{n}{\frac{1}{x_1}+\frac{1}{x_2}+\ldots+\frac{1}{x_n}}$.
- **harmonic quadrilateral:** a cyclic quadrilateral where the products of opposite side lengths are equal.

- **harmonic range:** a set of collinear points with a cross-ratio of -1.
- **Heron's formula:** if a triangle has side lengths a, b, c and semiperimeter s, the area is given by $\sqrt{s(s-a)(s-b)(s-c)}$. It is a special case of *Brahmagupta's formula*.
- **heterochromatic:** differently-coloured.
- **homothety:** a synonym of enlargement, homothecy, scaling, dilation or dilatation.
- **incentre:** the centre of the *incircle* of a triangle (or, more generally, inscribable polygon).
- **incircle:** the circle tangent to the three sides of a triangle and contained within it.
- **inradius:** the radius r of the *incircle*.
- **intersecting chords theorem:** if there is a point P in the plane of a circle Γ, and a line l passing through P and meeting Γ at A and B, then the value of $PA \cdot PB$ is independent of l and equal to the *power of the point* P.
- **median:** a straight line joining a vertex of a triangle to the midpoint of its opposite side.
- **Menelaus' theorem:** if D, E and F are points on the (possibly extended) sides BC, CA and AB, respectively, then D, E and F are collinear if and only if $\frac{\overrightarrow{BD}}{\overrightarrow{DC}} \cdot \frac{\overrightarrow{CE}}{\overrightarrow{EA}} \cdot \frac{\overrightarrow{AF}}{\overrightarrow{FB}} = -1$.
- **monic polynomial:** a polynomial of degree n where the coefficient of x^n is 1. Every polynomial is a scalar multiple of a monic polynomial.
- **monochromatic:** everything is the same colour.
- **Nagel point:** the intersection of the lines joining each vertex of a triangle to the point of tangency of the opposite excircle with its corresponding side.
- **nine-point circle:** the circle passing through the midpoints of the sides, the feet of the *altitudes* and the midpoints of AH, BH and CH, where H is the *orthocentre*.
- ***n*th roots of unity:** the n roots of the complex polynomial $z^n - 1$. If it cannot be expressed as a mth root of unity for some $m < n$, then it is known as 'primitive'. The *monic polynomial* whose roots are the $\varphi(n)$ primitive nth roots of unity is known as a 'cyclotomic polynomial'.
- **ordinate:** the y-coordinate of a point on the plane. Compare with abscissa.
- **orthocentre:** the intersection point H of the three *altitudes* of a triangle.
- **Pappus' theorem:** the special case of *Pascal's theorem* when the conic is a pair of straight lines.
- **parallelepiped:** a three-dimensional version of a parallelogram, obtained by applying a generic affine transformation to a cube. The n-dimensional generalisation is called a parallelotope.
- **Pascal's theorem:** if a hexagon is inscribed in a circle (or, more generally, a *conic*), the three pairwise intersections of opposite sides are collinear.
- **power of a point:** for a point P in the plane of a circle with centre O and radius R, the value of $OP^2 - R^2$ is known as its 'power'.
- **projective plane:** an extension of the Euclidean plane where parallel lines are considered to meet on a line at infinity.
- **Ptolemy's inequality:** if A, B, C and D are four points in space, the inequality $AB \cdot CD + BC \cdot DA \geq AC \cdot BD$ holds, with equality if and only if $ABCD$ is a (non-self-intersecting) cyclic quadrilateral.
- **Pythagoras' theorem:** for a right-angled triangle ABC, where $C = \frac{\pi}{2}$, the identity $a^2 + b^2 = c^2$ applies. It is a special case of the *cosine rule*.
- **quadratic mean (RMS):** for n variables $\{x_1, \ldots, x_n\}$, the quadratic mean is $\sqrt{\frac{1}{n}\left(x_1{}^2 + \ldots + x_n{}^2\right)}$.
- **radical axis:** the locus of points of equal power with respect to two circles Γ_1 and Γ_2. This is necessarily a straight line.
- **semiperimeter:** half of the perimeter of a polygon.

- **semiprime:** the product of two distinct primes, *e.g.* $23 \times 89 = 2047$.
- **sine rule:** For every triangle ABC, $\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C} = 2R$, where R is the *circumradius*.
- **Stewart's theorem:** If D is a point on the line BC, then $BD \cdot DC \cdot BC + AD^2 \cdot BC = AC^2 \cdot BD + AB^2 \cdot DC$.
- **symmedian:** the reflection of a *median* of a triangle in the corresponding interior angle bisector.
- **symmedian point:** the intersection of the three *symmedians* of a triangle. It has unnormalised areal coordinates $\left(a^2, b^2, c^2\right)$.
- **triangle inequality:** each side of a triangle is smaller than the sum of the other two sides. In terms of vectors, this is $|\underline{a} + \underline{b}| \leq |\underline{a}| + |\underline{b}|$.