2003 Winter Camp

IMO Geometrical Inequality Problems

1. P is a point inside $\triangle ABC$. D, E, and F are the feet of the perpendiculars from P to the lines BC, CA, and AB, respectively. Find all P which minimize $\frac{BC}{PD}+\frac{CA}{PE}+\frac{AB}{PF}$.

   (1981 IMO, Question 1)

2. ABC is a triangle, right-angled at A, and D is the foot of the altitude from A. The straight line joining the incentres of the triangles ABD and ACD intersects the sides AB, AC at K and L respectively. Show that the area of the triangle ABC is at least twice the area of the triangle AKL.

   (1988 IMO, Question 5)

3. Let $ABCD$ be a convex quadrilateral such that the sides AB, AD, BC satisfy $AB = AD + BC$. There exists a point P inside the quadrilateral at a distance h from the line CD such that $AP = h + AD$ and $BP = h + BC$. Prove that

   $$\frac{1}{\sqrt{h}} \geq \frac{1}{\sqrt{AD}} + \frac{1}{\sqrt{BC}}.$$

   (1989 IMO, Question 4)

4. Given a triangle ABC, let I be the incenter. The internal bisectors of angles A, B, and C meet the opposite sides in A', B', and C', respectively. Prove that:

   $$\frac{1}{4} < \frac{AI}{AA'} \cdot \frac{BI}{BB'} \cdot \frac{CI}{CC'} \leq \frac{8}{27}.$$

   (1991 IMO, Question 1)

5. For three points P, Q, R in the plane define $m(PQR)$ as the minimum length of the three altitudes of the triangle PQR (or zero if the points are collinear). Prove that for any points A, B, C, X, we have $m(ABC) \leq m(ABX) + m(AXC) + m(XBC)$.

   (1993 IMO, Question 4)

6. Let $ABCDEF$ be a convex hexagon with $AB = BC = CD$ and $DE = EF = FA$, such that $\angle BCD = \angle EFA = 60°$. Suppose that G and H are points in the interior of the hexagon such that $\angle AGB = \angle DHE = 120°$. Prove that

   $$AG + GB + GH + DH + HE \geq CF.$$

   (1995 IMO, Question 5)

7. Let $ABCDEF$ be a convex hexagon such that AB is parallel to DE, BC is parallel to EF, and CD is parallel to FA. Let R_A, R_C, R_E denote the circumradii of triangles FAB, BCD, DEF respectively, and let p denote the perimeter of the hexagon. Prove that $R_A + R_C + R_E \geq \frac{p}{2}$. *(Hint: extend sides BC and FE and take lines perpendicular to them through A and D, forming a rectangle).*

   (1996 IMO, Question 5)