17th January 2024

We begin by recalling ~~eo~~ eigenvalue equation.

(*) Maxwell's wave equation:

$$\boxed{\nabla^2 \psi = \frac{1}{c^2}\frac{\partial^2 \psi}{\partial t^2}} \qquad \psi \to \vec{E}, \vec{B}$$

$$\psi = \psi(t, \vec{x})$$

Which has the solution of the form,

$$\psi = \psi(t, \vec{x}) = \psi_0 e^{i(\vec{k}\cdot\vec{x} - \omega t)}$$

(general form).

An EM wave with angular frequency ω is represented by a complex valued function,

$$\psi = \psi(t, \vec{x})$$

further, this wave-function ψ also describes a quanta of energy $E = h\nu = \hbar\omega$

Schrodinger tried to formulate a new way to read off the energy of a wave by combining the eigenvalue method and Planck's hypothesis.

He said that we probably are not reading the physics properly.

Consider the action,

$$\boxed{\begin{aligned} i\hbar \frac{\partial \psi}{\partial t} &= (i\hbar)(-i\omega)\psi \\ &= \hbar\omega\psi \\ &= E\psi \\ &= \hat{H}\psi \end{aligned}}$$

This is an eigenvalue equation where eigenvalue is the energy of the light quanta. (*)

Therefore the operator $\boxed{\hat{O} = i\hbar\frac{\partial}{\partial t}}$

represents the energy operator, or usually called the Hamiltonian operator, $\hat{H}$ (say)

$$\Rightarrow \boxed{i\hbar \frac{\partial \Psi}{\partial t} = \hat{H}\Psi}$$

$\boxed{P_x = -i\hbar \frac{\partial}{\partial x}}$ → Momentum operator (You can guess it)

$$\Rightarrow \boxed{\hat{\vec{p}} = -i\hbar \vec{\nabla}}$$

~~As~~ (As $p = \frac{h}{\lambda} = \frac{h}{2\pi}\cdot\frac{2\pi}{\lambda} = \hbar K$)

Nothing is new here, but we invert the question and use this to derive physics (???)

(*) ~~Energy~~ model of Bohr →

Energy of an electron

$$E = \frac{p^2}{2m} + V$$

Hamiltonian operator corresponding to the electron,

$$\boxed{\hat{H} = \frac{\hat{\vec{p}}^{\,2}}{2m} + \hat{V} = -\frac{\hbar^2}{2m}\nabla^2 + V}$$

Like quanta of light, electron should also be described by some wave function Ψ. Such that

$$i\hbar \frac{\partial \Psi}{\partial t} = \hat{H}\Psi$$

Let us say that this came to us in a dream —

(*) Schroedinger's Equation →

$$\boxed{i\hbar \frac{\partial \Psi}{\partial t} = \left(-\frac{\hbar}{2m}\nabla^2 + V\right)\Psi = \hat{H}\Psi}$$

PDEs are not easy to solve in general.

(*) We will employ a trick to ~~con~~ convert this to two ODEs.

⊛ Method of separation of variables:

Ansatz: $\Psi(\vec{x},t) = T(t)\,\psi(\vec{x})$.

Schrödinger equation in (1+1) dimensions →

$$i\hbar \frac{\partial \psi}{\partial t} = -\frac{\hbar^2}{2m}\frac{\partial^2 \psi}{\partial x^2} + V\psi \quad , \psi = \psi(t,x)$$

Plugging in the ansatz,

$$\Rightarrow \frac{1}{T(t)}\, i\hbar \frac{\partial T(t)}{\partial t} = \frac{1}{\psi(x)}\left[-\frac{\hbar^2}{2m}\frac{d^2\psi(x)}{dx^2} + V(x)\psi(x)\right]$$

$$= E = \text{const}$$

As LHS is func of t, and RHS is func of x.

$$\boxed{i\hbar \frac{dT}{dt} = ET}$$ Time dependent part

$$\boxed{-\frac{\hbar^2}{2m}\frac{d^2\psi}{dx^2} + V(x)\psi(x) = E\,\psi(x)}$$

Time independent part.

⊛ We start with particle in a box in the next class.