8th January 2024

○ Recall:

Blackbody radiation —

Wien's Law (1896)

Spectral energy density

$$u_\nu = \frac{8\pi h}{c^3} \nu^3 e^{-h\nu/k_B T}$$

(works for $h\nu >> k_B T$).

Rayleigh-Jeans Law (1900)

$$u_\nu = \frac{8\pi \, (k_B T)}{c^3} \nu^2$$

(Works for $h\nu << k_B T$)

Blows up as $\nu \to \infty$ (UV catastrophe)

Same year as the RJ formula,

○ Max Planck (1900):

→ Empirical formula — He did not know then how it worked.
It only fits the experiment.

→ Matches Wien's formula at $h\nu >> k_B T$, and RJ at $h\nu << k_B T$.

$$\boxed{u_\nu = \frac{8\pi h \nu^3}{c^3} \cdot \frac{1}{e^{h\nu/k_B T} - 1}}$$

⊛ High frequency → $h\nu >> k_B T \Rightarrow e^{h\nu/k_B T} - 1 \approx e^{h\nu/k_B T}$

Thus it becomes Wien's formula,

$$u_\nu = \frac{8\pi h \nu^3}{c^3} e^{-h\nu/k_B T}$$

(But a power of ν is off?).

⊛ Low frequency → $h\nu << k_B T \Rightarrow e^x \approx 1 + x$

$$\Rightarrow e^{h\nu/k_B T} \approx 1 + \frac{h\nu}{k_B T}$$

Then, it becomes, $u_\nu = \frac{8\pi (k_B T)}{c^3} \nu^2$

* In 1901, Planck proposed the notion of hypothetical oscillator to describe black body radiation of frequency ν and having energy as integer multiples of $h\nu$

→ The idea of 'quanta' as a 'mathematical device' that leads to a single formula and 'need not really exist somewhere in nature'

This was a hand-wavy mathematical derivation.

* In Summary →

Planck proposed that the energy of a monochromatic beam of radiation with frequency ν should be of the form

$$E = \underbrace{N}_{\text{Integer} \geq 0} \underbrace{h}_{\text{Constant (Planck constant)}} \nu$$

ν → freq of monochromatic radiation.

Also,

$$E = N \underbrace{\left(\frac{h}{2\pi}\right)}_{\hbar} (2\pi\nu)$$

(2πν) → Angular freq

$$\Rightarrow E = N\hbar(2\pi\nu) = N\hbar\omega$$

* We will redefine h as ħ as convention, generally.

* Energy flux: $S = n h\nu$

Energy flux → 'Spectral' form

n → Number of light quanta passing through per unit area, per unit time

But we ~~alway~~ already have the expression for the poynting vector — from Maxwell's electrodynamics

(*) Maxwell's electrodynamics —

$$\nabla^2 \vec{E} = \mu_0 \epsilon_0 \frac{\partial^2 \vec{E}}{\partial t^2} \quad , \quad \nabla^2 \vec{B} = \mu_0 \epsilon_0 \frac{\partial^2 \vec{B}}{\partial t^2}$$

Solutions that describe a monochromatic light beam with frequency ν along z direction.

The solutions of this are,

$$\boxed{\begin{aligned} \vec{E} &= E_0 \cos(kz - \omega t)\, \hat{i} \\ \vec{B} &= \underbrace{B_0}_{E_0/c} \cos(kz - \omega t)\, \hat{j} \end{aligned}}$$

→ Reason why ω is easier to use in $\hbar$ notation.

Energy flux given by Maxwell →

$$\vec{S} = \frac{1}{\mu_0}(\vec{E} \times \vec{B}) = \frac{E_0^2}{\mu_0} \cos^2(kz - \omega t)\, \hat{k}$$

The one we should compare ~~average~~ with Planck's new formula is the time averaged version of this.

Max Planck

$$\boxed{S = nh\nu}$$

Time average of flux :

$$\langle \vec{S} \rangle = \frac{E_0^2 \hat{k}}{2\mu_0 c} = \frac{\epsilon_0 E_0^2 c}{2}$$ → No freq dependence

It says that the flux depends on amplitude, not the frequency as Planck's formula suggests.

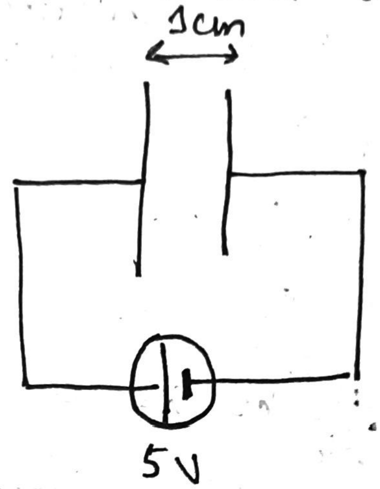
So there is a problem.

There is a Conceptual problem with classical physics.

Now we calculate value of 'n' — To see why there is a problem in certain domains and no problem in others.

Ex 1 : Compute the electric field between two parallel plates that are separated by 1cm and connected to a 5 V battery.

Soln : Assume that the electric field is uniform.



$$V = \int \vec{E} \cdot d\vec{l} = EL \Rightarrow E = \frac{V}{L}$$

$$\therefore \boxed{E = \frac{5V}{1cm} = 500 V/m}$$

E2

Q1 / A2 Compute the peak electric field due to a light beam generated by a 5 watt LED bulb (assume 1 conversion factor — no heat loss) and passing through a square of side 10 cm

E3

Q2/A2 In the two configurations (of E1 and E2), which one has a stronger peak of electric field.