O First Law of Thermodynamics → We continue. 18th January 2024

$$du = dQ + dW$$

→ U is a state function, Q and W are not
To distinguish state functions and path functions,
we write du for state, and $đQ$ and $đW$ for not state func

Exact differential Inexact differential.

Example : System described by (x, y)
and $df = y\,dx + x\,dy$
We want to calculate,

$$\Delta f = \int_{(0,0)}^{(1,1)} df = \int_{(0,0)}^{(1,1)} d(xy) = (xy)\Big|_{(0,0)}^{(1,1)}$$

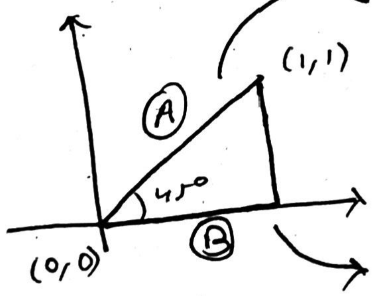
$$= (1)(1) - (0)(0) = 1$$

(*) Note that we do not need to care about the path taken in doing the integral.
The change of the function does not depend upon the path taken.

(2) A quantity g s.t $dg = y\,dx$
We want to calculate,

$$\Delta g = \int_{(0,0)}^{(1,1)} y\,dx$$ → We cannot proceed unless we know the path.

Say the path,



$y = x \Rightarrow \Delta g = \int_{(0,0)}^{(1,1)} x\,dx = \frac{1}{2}$ (path A)

$$\Delta g = \int_{(0,0)}^{(1,0)} (0)\,dx + \int_{(1,0)}^{(1,1)} y \cdot (0) = 0$$ (path B)

$\Rightarrow$ The quantity depends on the path.

Putting it mathematically,

$F_1(x,y)\,dx + F_2(x,y)\,dy$ is exact if it can be written as some $d(f)$

$$\therefore df = \left(\frac{\partial f}{\partial x}\right)_y dx + \left(\frac{\partial f}{\partial y}\right)_x dy$$

By comparison,

$$F_1 = \frac{\partial f}{\partial x}, \quad F_2 = \frac{\partial f}{\partial y}$$

Now, say,

$$\vec{F} = \vec{\nabla} f$$

$$\Rightarrow (F_1 \hat{i} + F_2 \hat{j}) = \left(\frac{\partial f}{\partial x}\hat{i} + \frac{\partial f}{\partial y}\hat{j}\right)$$

So most forces that are expressed as gradient of a scalar have exact differential structure.

So,

$$\int_1^2 F_1(x,y)\,dx + F_2(x,y)\,dy$$

$$= \int_1^2 \vec{F}\cdot d\vec{r}$$

$$= \int_1^2 (\vec{\nabla}\cdot f)\,d\vec{r}$$

$$= \int_1^2 df$$

$$= f(2) - f(1)$$

f only at end points.

□ Use Stokes theorem for a closed loop to show,

$$\frac{\partial^2 f}{\partial y \partial x} = \frac{\partial^2 f}{\partial y \partial x} \quad \text{or} \quad \frac{\partial F_2}{\partial x} = \frac{\partial F_1}{\partial y} \quad \text{then it is } \underline{\underline{\text{exact}}}$$

$$đQ = \left(\frac{\partial u}{\partial T}\right)_V dT + \left(p + \left(\frac{\partial u}{\partial V}\right)_T\right) dV$$

Again, true in <u>general</u>.

This again shows that $đQ$ is inexact.

$$\Rightarrow \boxed{đQ = C_V dT + \left(p + \left(\frac{\partial u}{\partial V}\right)_T\right) dV} \quad \text{(Try to remember)}$$

$$C_V \equiv \left(\frac{\partial Q}{\partial T}\right)_V = \left(\frac{\partial u}{\partial T}\right)_V$$

In the special case of ideal gas,

$$\boxed{đQ = C_V dT + p dV}$$

We calculated $đQ$ for several processes,

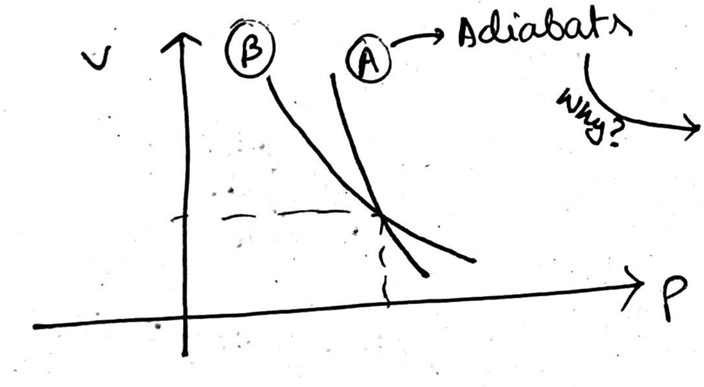
$đQ = 0$ for adiabatic process.

Isothermal,

$pV = \text{constant}$

Adiabatic,

$pV^{\gamma} = \text{Const}, \quad \gamma = \frac{C_p}{C_V}$



Isotherms,

$p dV + V dp = 0$

$\Rightarrow \frac{dp}{dV} = -\frac{p}{V}$

Adiabats,

$\gamma p V^{\gamma-1} dV + V^{\gamma} dp = 0$

$\Rightarrow \frac{dp}{dV} = -\gamma \frac{p}{V}$

$\frac{dp}{dV}$ is slope of graph. $\gamma > 1$ (as $C_p > C_V$)

$\Rightarrow$ Adiabats have <u>steeper slope</u> as compared to isotherms.

○ Adiabatic lapse rate →

⊛ Air is bad thermal conductor.

⊛ Thus, we can take approximation that expansion of air is adiabatic.

We want to find out how temp changes as we go up the atmosphere.

⊛ Ideal gas assumption.

So we ~~now~~ know that, $pV^{\gamma} = \text{const}$

Using ideal gas eqn,

$$\Rightarrow p^{1-\gamma} T^{\gamma} = \text{const}$$

Now taking differential (Standard Strategy)

$$\Rightarrow d\left(p^{1-\gamma} T^{\gamma}\right) = 0$$

$$\Rightarrow \frac{dp}{p} = \frac{\gamma}{\gamma - 1} \frac{dT}{T}$$

→ Easiest to take log before differentiation,

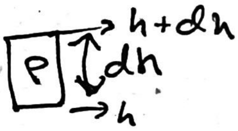
$$\ln\left(p^{1-\gamma} T^{\gamma}\right) = \ln(\text{const}) = \text{const}$$

$$\Rightarrow (1-\gamma) \ln p + \gamma \ln(T) = \text{const}$$

$$\Rightarrow (1-\gamma) \frac{dp}{p} + (\gamma) \frac{dT}{T} = 0$$

$$\Rightarrow \frac{dp}{p} = \frac{\gamma}{\gamma - 1} \frac{dT}{T}$$

Now, $dp = -\rho g dh$



$$\Rightarrow \frac{dp}{p} = -\frac{\rho g dh}{p}$$

Mass of air of 1 mol gas

Now, $pV = RT \Rightarrow p\left(\frac{m}{\rho}\right) = RT$

There fore,

$$\frac{dP}{P} = -\frac{gM}{RT}\,dh$$

$$\therefore \boxed{\frac{dT}{dh} = -\frac{\gamma-1}{\gamma}\frac{gM}{R}}$$

If we plugin the numbers, $\frac{dT}{dh} = 9.7°C/Km$

Observational $\rightarrow$ $7°C/Km$.

We can apply this to many other systems.