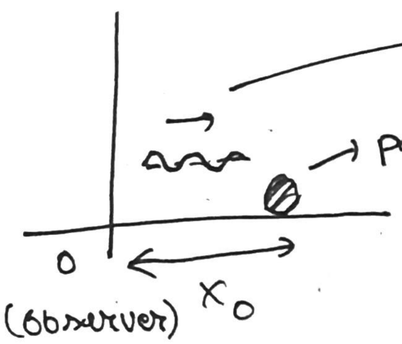
15th Jan 2024

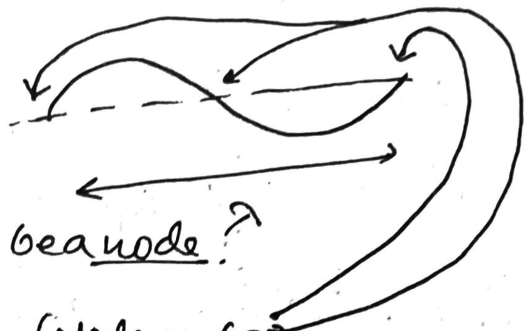
We start with the determinism question.

→ How do we measure the position (say $x_0$) of a particle?




→ Send photon to particle & count time it takes to come back, and calculate position

But there is a ~~pb~~ problem –
Light has finite extent



When it reflects, there must be a node?

~~$\Delta x$~~ $\Delta x \sim \frac{\lambda}{2}$ (Node can be)

→ Send a light, measure the delay in arrival time, say $t_0$, of the reflected wave,

$$\boxed{x_0 = \frac{c t_0}{2}}$$

→ There is an inherent error $\boxed{\Delta x \sim \frac{\lambda}{2}}$

→ We should use smaller $\lambda$ ~~for~~ for measuring position. (to minimise the error)

(*) How do we measure $v_0$?

→ Measure the position again, say $x_0'$ after an interval at say $t_0'$

$$\Rightarrow \boxed{v_0 = \frac{x_0' - x_0}{\frac{(t_0' - t)}{2}}}$$

photon has momentum $= \frac{h}{\lambda}$

But since photon comes back, the particle whose position is being measured gets momentum.

→ In order to reflect the photon the particle's original momentum ($P_0 = mv_0$) changes!

→ Inherent error in momentum measurement

$$\frac{h}{\lambda_3} + P_1 = -\frac{h}{\lambda} + P_2$$

$$\Rightarrow \boxed{\Delta P \sim \frac{2h}{\lambda}}$$

→ To minimize $\Delta P$, we should use larger $\lambda$

So, $\Delta x$ and $\Delta p$ counter each other.

(*) We note, $\boxed{\Delta x \Delta p \sim h}$

(First primitive form of the Heisenberg Uncertainty Principle)

⇒ $\Delta x \ \Delta p$ is independent of measurement

(*) We will come back to this.

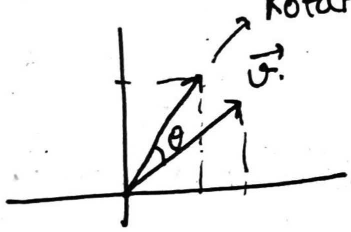
(*) Determinism in classical mechanics depends on the condition that cannot be provided.

→ x and p are not good variables to describe the quanta.

(*) Vector Calculus →

An element of a set, known as linear vector space, is called a vector

$v \in V$.



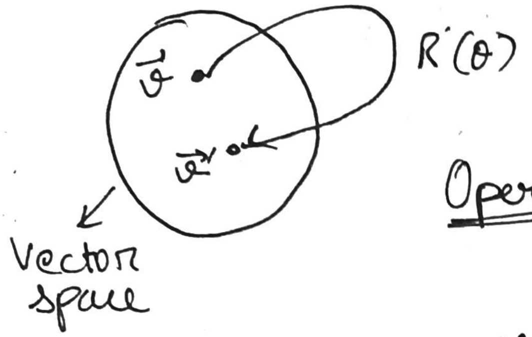

Rotation, $v_x' = v_x \cos\theta + v_y \sin\theta$

& $v_y' = -v_x \sin\theta + v_y \cos\theta$

Written in matrix form,

$$\begin{pmatrix} v_x \\ v_y \end{pmatrix}' = \begin{pmatrix} \cos\theta & \sin\theta \\ -\sin\theta & \cos\theta \end{pmatrix} \begin{pmatrix} v_x \\ v_y \end{pmatrix}$$

$$\Rightarrow \vec{v}' = R(\theta)\, \vec{v}$$




$\longrightarrow$ $\boxed{R(\theta) : V \to V}$

<u>Operator:</u> $\hat{O}$ is a map a linear vector space to itself i.e.,

$v' = \hat{O}\, v$ such that $\forall\, v, v' \in V$

(*) Linear : The addition operation is <u>linear</u>.

Here, $R(\theta)$, rotation by ~~ang~~ angle $\theta$ is an example of an <u>operator</u>.

(*) Consider operator $\hat{O} = R(\theta = \pi)$

$$v = R_{\theta=\pi} v = -v$$

$$\Rightarrow \boxed{R_{\theta=\pi} v = \lambda v}, \quad \lambda = -1$$

An operator equation of the form,

$$\boxed{\hat{O}\,\psi = \lambda\,\psi}$$

is called an eigenvalue equation, where the vector $\psi$ is called an <u>eigen vector</u> and the complex number $\lambda$ (in general) is called the <u>eigenvalue</u>.