22nd Jan 2024

Random Monty Hall :

There are 3 cups, you cannot see inside the cups.

A : Your initial guess is correct

L : Your friend lifts a cup which happens to be empty.

We ~~must~~ must compare $P(A|L)$ and $P(A^c|L)$

$$P(L|A) = 1$$

$$P(L|A^c) = \frac{1}{2}$$

Now, $$P(A|L) = \frac{P(A \cap L)}{P(L)} = \cancel{\frac{P(A)P(L|A)}{P(A^c)\;P(A)P(L|A)}}$$

$$= \frac{P(A)P(L|A)}{P(A)P(L|A) + P(A^c)P(L|A^c)}$$

$$= \frac{\left(\frac{1}{3}\right)(1)}{\left(\frac{1}{3}\right)(1) + \left(\frac{2}{3}\right)\left(\frac{1}{2}\right)}$$

$$= \frac{\frac{1}{3}}{\frac{2}{3}} = \frac{1}{2}$$

$\Rightarrow$ There's no advantage in switching.

Base State Fallacy / False positive paradox :

Consider a rare disease. Consider a test gives a true result in 99% cases.

~~Consider~~ The disease affects 0.1% of the population.

Let ~~there be~~ there be a test for this disease with 99% sensitivity (identification of true +ve cases) and 99% specificity (true negative)

Sensitivity of a test ~~means~~ ~~is the probability~~ is 100 times the probability of identification of ~~a~~ the disease in a person. when he or she is actually affected by that disease.

Specificity refers to 100 times the probability of correctly identifying a -ve test

$$\frac{\text{Sensitivity}}{100} = P(P|D) = 0.99$$

$$\frac{\text{Specificity}}{100} = P(N|D^c) = 0.99$$

~~B~~ D: Presence of disease

P: Positive test result

N: Negative test result.

$P(D) = 0.001$

(i) Find the probability that the person has a disease given that the person tested +ve.

Soln : We want to find $P(D|P)$

$$\Rightarrow P(D|P) = \frac{P(D)\,P(P|D)}{P(P)}$$

$$= \cancel{\frac{P(D)\,P(P|D)}{P(N)\,P(P|N)}}$$

$$= \frac{P(D)\,P(P|D)}{P(D)\,P(P|D) + P(D^c)\,P(P|D^c)}$$

$$= 0.09$$

(ii) Given that a person has tested positive once, find the probability that he/she has the disease if he tests positive again.

(iii) Given that a person has tested +ve twice, find the probability that he/she has the disease if he/she tests +ve again.

Solution : (ii) → Our sample space has shrunk.

We replace P(D) with P(D|P)

i.e, the sample space that we are now testing has a probability of 0.09 for being +ve for the disease, not 0.001. Same calculation follows.

(iii) Now, the same procedure carries forward. We should get about 0.999.

23rd Jan 2023

☐ Fill up portions that I missed.