Δ Classical definition of probability —

☐ Make up your own definition — mutually exclusive, exhaustive, equally likely.

4th Jan 2023

Δ Finite Sample Spaces —

$\Omega = \{\omega_1, \omega_2, \ldots, \omega_n\}$

Since it is finite, we can take the eventspace $\mathcal{E}$ to be,

$\mathcal{E} = 2^{\Omega}$

for $A \in \mathcal{E}$ with $A = \{\omega_1, \ldots, \omega_{n_K}\}$ for $n_1, \ldots, n_K \in \{1, 2, \ldots, n\}$.

$$P(A) = \sum_{i=1}^{K} P(\omega_{n_i}) \quad \text{(They are mutually exclusive and in } \mathcal{E} \text{ as events)}$$

$$= \frac{\#A}{n} \xrightarrow{\circledast} \text{(if the events } \{\omega_{n_1}\}, \ldots, \{\omega_{n_K}\} \text{ are equally likely)}$$

Everything in sample space needs to be equally likely..

So,

$P(\Omega) = 1$

$\Rightarrow \sum_{i=1}^{n} P(\omega_i) = 1$

If $\omega_1, \ldots, \omega_n$ are equally likely, then,

$P(\omega_i) = f \quad \forall i$

$\sum_{i=1}^{n} P(\omega_i) = nf = 1 \Rightarrow f = \frac{1}{n}$

A Classical definition of probability —

~~If~~ Suppose a random experiment results in N mutually exclusive, exhaustive and equally likely outcomes, ~~then~~ let there be $N(A)$ outcomes which are favorable to an event A

Then, the probability of occurrence of the event A is defined as $P(A) = \frac{N(A)}{N}$

This definition of cyclic $\Rightarrow$ as it requires the definition to define 'equally likely'.

Also, we cannot extend from finite sample spaces to countably infinite sample spaces.

$\hookrightarrow \Omega = \{\omega_1, \omega_2, \dots\}$

Suppose $\mathcal{E} = 2^{\Omega}$

if possible, let $P(\omega_i) = P(\omega_j) \ \forall i, j$

(Equally likely)

$$\Omega = \bigcup_{i=1}^{\infty} \{\omega_i\}$$

$$1 = P(\Omega) = \sum_{j=1}^{\infty} P(\omega_j) = \begin{cases} 0, & \text{if } P(\omega_i) = 0 \ \forall i \\ \infty, & \text{otherwise} \end{cases}$$

Suppose, $P(\omega_n) = \frac{1}{2^n}$

$\omega_n :=$ The outcome in which n is chosen

$$P(\Omega) = \sum_{n=1}^{\infty} P(\omega_n) = \frac{1}{2} + \frac{1}{2^2} + \dots + \frac{1}{2^n} + \dots$$

$$= 1$$

$$P(\text{Even No}) = \frac{\frac{1}{4}}{1 - \frac{1}{4}} = \left(\frac{1}{4}\right)\left(\frac{4}{3}\right) = \frac{1}{3}$$

$$P(\text{odd No}) = \frac{\frac{1}{2}}{1 - \frac{1}{4}} = \frac{2}{3}$$

Assign probability to all $\mathbb{N}$ so that probability of choosing even and odd is $\frac{1}{2}$ each.

$$\frac{ar}{1-r^2} = \frac{1}{2}$$

$$\frac{a}{1-r^2} = \frac{1}{2} \qquad \Rightarrow r = 1$$

$$\frac{a}{1-r} = 1 \Rightarrow \boxed{a = 1-r}$$

$$= \frac{1-r}{(1-r)(1+r)} = \frac{1}{2} \Rightarrow 1+r = 2) \Rightarrow r = 1$$

~~A Boole's~~

Δ Relative frequency definition of probability –

If you repeat a random experiment n times and if an event A occurs $f_n(A)$ times then $\frac{f_n(A)}{n}$ is called the relative frequency of A

It says that the $P(A) = \lim_{n \to \infty} \frac{f_n(A)}{n}$

flaw here $\rightarrow$ This limit may not exist

(*) Weak law of large numbers has its definition based on this flawed definition.

Δ Boole's inequality –

~~$A_1, \dots, A_n$ are~~

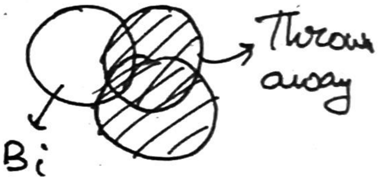
Probability space : $(\Omega, \mathcal{E}, P)$

$A_1, \dots, A_n \in \mathcal{E}$ then

$$P(A_1 \cup \dots \cup A_n) \leq P(A_1) + \dots + P(A_n)$$

Proof $\rightarrow$ Equality holds if they are disjoint $\rightarrow$ just axiom.

Let $B_i = \begin{cases} A_1, & \text{if } i = 1 \\ A_i \setminus \bigcup_{j=1}^{i-1} A_j, & \text{if } i > 1 \end{cases}$



Then, $P(B_1 \cup \dots \cup B_n) = P(A_1 \cup \dots \cup A_j)$

$\downarrow$ Mutually exclusive by def

$\downarrow$ These are not by def.

$\Rightarrow P(A_1 \cup \dots \cup A_n) = P(B_1) + P(B_2) + \dots + P(B_n)$

As ~~B∈~~ $B_j \subseteq A_j \Rightarrow P(B_i) \leq P(A_i)$

$\Rightarrow P(A_1 \cup \cdots \cup A_n) \leq P(A_1) + \cdots + P(A_n)$ $\square$