5th Jan 2024

Recall: Classical Physics

↳ Deterministic predictions.

⊛ Inbuilt in all theories in classical physics.

⊛ 3 key failures of classical physics (Led the the birth of QM)

→ ① Black body radiation (Max Planck 1900AD)

→ ② Photoelectric effect (Albert Einstein, 1905)

→ ③ Spectral lines in photo-emission (Niels Bohr 1913)

All three are connected by light.

What is light?

In vacuum, $\vec{J}=0$, $\rho=0$

Then, Maxwell equations become simpler —

① $\vec{\nabla}\cdot\vec{E}=0$ ② $\vec{\nabla}\times\vec{E}=-\frac{\partial\vec{B}}{\partial t}$

③ $\vec{\nabla}\cdot\vec{B}=0$ ④ $\vec{\nabla}\times\vec{B}=\mu_0\epsilon_0\frac{\partial\vec{E}}{\partial t}$

Usual derivation (as waves) —

$$\vec{\nabla}\times(\vec{\nabla}\times\vec{E})=\vec{\nabla}\times\left(-\frac{\partial\vec{B}}{\partial t}\right)$$

$$\Rightarrow \vec{\nabla}\times(\vec{\nabla}\times\vec{E})=-\frac{\partial}{\partial t}(\vec{\nabla}\times\vec{B})$$

$$\Rightarrow \vec{\nabla}\times(\vec{\nabla}\times\vec{E})=-\frac{\partial}{\partial t}\left(\mu_0\epsilon_0\frac{\partial\vec{E}}{\partial t}\right)$$

$$\Rightarrow \vec{\nabla}(\vec{\nabla}\cdot\vec{E})-\vec{\nabla}^2\vec{E}=-\frac{\partial}{\partial t}\left(\mu_0\epsilon_0\frac{\partial\vec{E}}{\partial t}\right)$$

$$\Rightarrow \boxed{\nabla^2\vec{E}=\mu_0\epsilon_0\frac{\partial^2\vec{E}}{\partial t^2}}$$

(wave equation)

⊛ Assignment: Similarly we can show $\nabla^2\vec{B}=\mu_0\epsilon_0\frac{\partial^2\vec{B}}{\partial t^2}$

A1/Q1

This is a kind of propagating solution.

Both electric field and magnetic field satisfy wave equation of the form,

$$\nabla^2 f = \frac{1}{v^2}\frac{\partial^2 f}{\partial t^2}$$

$v \rightarrow$ Speed of propagation of wave.

Speed of electromagnetic wave:

$$v = \frac{1}{\sqrt{\mu_0 \epsilon_0}}$$

$\epsilon_0 \rightarrow$ Permittivity of vacuum (Value known from Coulomb's Law)

$\mu_0 \rightarrow$ Permeability of vacuum (Value known from Biot-Savart law)

Plugging in the values,

$$v \approx 3 \times 10^8 \ m \cdot s^{-1}$$

* Speed of light (say c):
as measured through astronomical observation first by Ole Roemer in 1676 AD

* Maxwell (1864): "perhaps light is an electromagnetic wave"

as $\boxed{v \approx c}$

1. Blackbody radiation:

→ Every physical body spontaneously emits electromag radiation

Ex: Heated iron rod, Human body.

→ Such radiation depends on the temperature T of the body.

* Wilhelm ~~Wien~~ Wien (1896):

Spectral density energy: (Energy density of radiation having frequency ν to $\nu + d\nu$)

$$u_\nu = \frac{8\pi h}{c^3}\nu^3 e^{-h\nu/k_B T}$$

$K_B \rightarrow$ Boltzmann constant

$h \rightarrow$ A constant needed to make $\frac{h\nu}{K_B T}$ dimensionless.

$\rightarrow$ It describes observations accurately for high frequency (short wave length)

$$\boxed{h\nu >> K_B T}$$

It does not work for low frequency, i.e,

$$h\nu << K_B T$$

(*) Rayleigh - Jeans law: (1900)

$$\boxed{U_\nu = \frac{8\pi K_B T}{c^3} \nu^2}$$

Based on classical consideration.

$\rightarrow$ It works for low frequency, i.e,

$$h\nu << K_B T$$

But fails for,

$$h\nu >> K_B T$$

(*) UV catastrophe

☐ Max Planck guessed the formula as an interpolation b/w these two. Guess the formula.