

2nd January 2025 (Thursday) →

*Ankiified

- Probability Theory
- Classical Statistical Mechanics } Upto midsem
- Quantum Statistical Mechanics

- ④ Follows Kardar
(Particles)
- ④ For Quantum → Huang

Grading →

2 CT, midsem, endsem → No Assignments

④ Notutorials in first few weeks

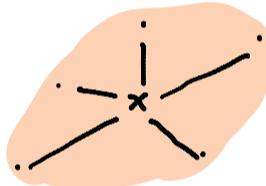
Next Monday we start at 9:00, then next monday 8:30

→ Tutorial may be shifted to Wednesday.

Statistical Mechanic is coarse-grained properties of very large systems.

Ex: Gas (lots of molecules) → dilute (we want to avoid interactions)

OR, we can look at the particles as things that sees the avg behaviour of all the other particles.



Coarse-grained: Like temp cannot be defined for one particle, but rather for a large number.

Thermodynamics → Phenomenological description (Empirical observation)

↪ Equilibrium (State variables do not change with time)

④ Note: Steady state $\not\Rightarrow$ Equilibrium

Ex: Bar heated on one side reaches steady state temp gradient, but it is not equilibrium.

Stat mech will be giving us a justification of the phenomena using microscopic properties. — like $\{\vec{x}, \vec{p}\}$

Something like entropy (monotonically increasing state variable) was difficult to construct
— H-theorem was used to resolve this.

Systems we will deal with →

- ① gas → density
- ② Magnets → Spins / Dipole moments

We will NOT be studying phase transitions in this course — that in Adv Stat Mech.

- ③ Biological Systems

④ Social Systems



An example is Boltzmann Entropy →

$$S = k_B \ln [S_2(E, V, N)]$$

↳ Some Const ↳ No. of microstates corresponding to a macrostate

Macroscopic Entropy (Clausius) Microscopic (Boltzmann)

There used to be (?) many ways to see entropy —

$$S = -k_B \sum_i P_i \ln(P_i)$$

↳ Probability of one state existing out of several

Shannon

④ Another challenge is to see how the reversibility of Newtonian Dynamics reflects on the irreversibility of the macroscopic phenomena justified.

⑤ Crash course on Thermodynamics →

① Zeroth Law: A and B are in equilibrium, B and C are in eq ⇒ A and C are in equilibrium.

Say, A has some state variables associated with it,

$$A = \{A_1, A_2, \dots\}$$

same,
 $B = \{B_1, B_2, \dots\}$

} Since they are in eq, we have constraint,
 $f_{AB}(A_1, A_2, \dots, B_1, B_2, \dots) = 0$

Similarly,

$$C = \{C_1, C_2, \dots\}$$

} Same as above, we have →
 $f_{BC}(B_1, B_2, \dots, C_1, C_2, \dots) = 0$

(Not a proof, justification)

Thus A and C
are in eq

We may eliminate
 $\{B_1, B_2, \dots\}$ from
these to get constraint f_{AC}

* 1st Law: Conservation.

$$dU = \delta Q - \delta W$$

↓ ↗ work done
Internal energy Heat exchanged

(Reminder: δ means inexact differential - dependent on path)

We may write , → General displacement → Things like length, vol
 $\delta W = \sum_i F_i dx_i$ (Extensive)
 ↗ General force . → May be thing like surface tension, pressure, etc.
 (Intensive)

Mostly we have checked the response of this thing by changing some parameters (like temperature)

* 2nd Law: We define some efficiency ,

$$\eta = \frac{W}{Q} \xrightarrow{\text{Work done}} \xrightarrow{\text{Heat exchanged}}$$

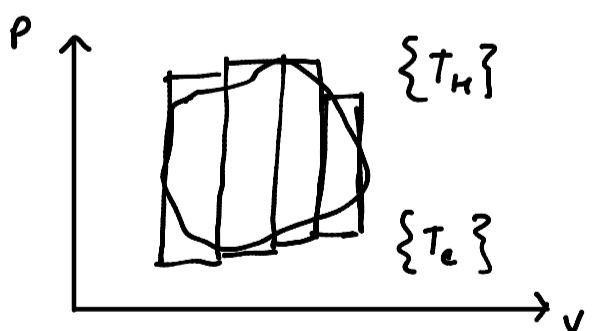
$$= \frac{Q_H - Q_C}{Q_H} = 1 - \frac{Q_C}{Q_H} \leq 1$$

Also,

$$\eta_{\text{Carnot}} > \eta_{\text{non.Carnot}}$$

This has some more consequences —

Suppose we have a process that we break in Carnot cycles (approx)



Here, we define efficiency →

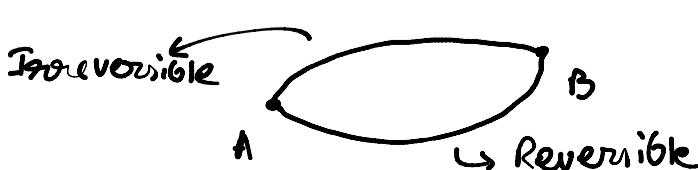
$$\eta = 1 - \frac{\Delta Q_C}{\Delta Q_H} = 1 - \frac{T_C}{T_H}$$

$$\Rightarrow \frac{\Delta Q_C}{T_C} + \frac{\Delta Q_H}{T_H} = 0 \quad \xrightarrow{\text{Continuous limit}}$$

$$\Rightarrow \oint \frac{\delta Q}{T} = 0 \quad (\text{Remember, this is for Carnot})$$

$$\Rightarrow \oint dS = 0 \quad \xrightarrow{\text{Reversible path}} \text{Entropy}$$

Say I have two points in phase space , A and B



$$\Rightarrow \int_A^B \frac{\delta Q}{T} + \int_B^A \frac{\delta Q_{\text{rev}}}{T} \leq 0$$

$$\Rightarrow \int_A^B \frac{dQ_{\text{inn}}}{T} \leq \int_B^A \frac{dQ_{\text{rev}}}{T} \Rightarrow \int_A^B \frac{dQ_{\text{inn}}}{T} \leq S(B) - S(A)$$

$\Rightarrow dS \geq \frac{dQ}{T}$

We can claim that if two states approach the same temp equilibrium, $dS \rightarrow 0$

$A, B \rightarrow$ Thermally isolated from surrounding
 ↳ weakly interacting with each other.

So define,

$$U_T = U_A + U_B$$

$$S = S_A + S_B$$

Now, using chain rule (uses the weakly interacting condition in saying $\frac{\partial S_B}{\partial U_A} \approx \frac{\partial S_A}{\partial U_B} \approx 0$)

$$\frac{dS}{dt} \approx \frac{dU_A}{dt} \left(\frac{\partial S_A}{\partial U_A} \right) + \frac{dU_B}{dt} \left(\frac{\partial S_B}{\partial U_B} \right)$$

We get from conservation,

$$\frac{dU_B}{dt} = - \frac{dU_A}{dt}$$

Putting this in,

$$\frac{dS}{dt} \approx \frac{dU_A}{dt} \left(\frac{\partial S_A}{\partial U_A} - \frac{\partial S_B}{\partial U_B} \right) = \frac{dU_A}{dt} \left(\frac{1}{T_A} - \frac{1}{T_B} \right)$$

Note that if T_A and T_B approach same equilibrium temp, $\frac{dS}{dt} \rightarrow 0$

We will not study this dynamical part now, though — this is non-equilibrium stat mech

⊗ 3rd Law: $\lim_{T \rightarrow 0} \Delta S(x, T) \rightarrow 0$

↳ State variable

We cannot use classical mech to argue this, due to low temperature.

Feynman's defn: As $T \rightarrow 0$, S approaches some universal number

Another equivalent statement is that $T=0$ cannot be achieved.

Look at it using HUP \rightarrow

$$\Delta E \Delta t \geq \hbar \Rightarrow \Delta E \geq \frac{\hbar}{\Delta t} \quad \text{As } \Delta E \rightarrow 0, \text{ we get nonsensical result.}$$