$Q = Q_1 - Q_2$

We calculate these so that we can calculate η.

(*) If we are given two reservoirs, ~~and~~ with one being at a higher temp than the other, and we want to construct a reversible engine, it can only be done with two isotherms and two adiabats — this is the Carnot engine.

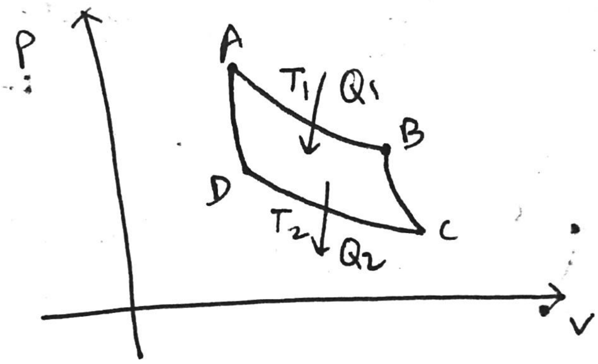
1st Feb 2024

o Heat engine:

→ Two reservoirs $T_1 > T_2$

→ "Reversible" (meaning still not ~~mean~~ discussed)

Can only happen with two isotherms and two adiabats.



(*) All steps are quasistatic

(*) Note: we can also go $D \to C \to B \to A$, that is also a feature.

We begin with first law,

$$\Delta U = \Delta Q + \Delta W \quad ; \quad \Delta W = -\int p\,dV$$

(1) $A \to B$: (Isothermal expansion at temp T_1)

(*) ideal gas.

$\Rightarrow \Delta U = 0$

$\therefore \Delta Q = -\Delta W = \int p\,dV$

$\Rightarrow Q_1 = nRT_1 \ln\left(\frac{V_B}{V_A}\right)$ (+ve, heat goes into system)

(2) $B \rightarrow C$: (Adiabat)

$\Delta Q = 0$

$\Rightarrow \Delta W = \Delta U$

$W_2 = -\int p dV$

We use $pV^\gamma = K$

$$\Rightarrow W_2 = -K \int \frac{dV}{V^\gamma} = -\frac{K}{-\gamma+1} V^{-\gamma+1} \Big|_{V_B}^{V_C}$$

$$= \frac{-K}{1-\gamma} \left(V_C^{1-\gamma} - V_B^{1-\gamma} \right)$$

$$= \frac{K}{\gamma-1} \left(\frac{V_C}{V_C^\gamma} - \frac{V_B}{V_B^\gamma} \right)$$

Now, $P_C V_C^\gamma = P_B V_B^\gamma = K$

Now,

$$W_2 = \frac{K}{\gamma-1} \left(\frac{P_C V_C}{P_C V_C^\gamma} - \frac{P_B V_B}{P_B V_B^\gamma} \right)$$

$$= \frac{1}{\gamma-1} (P_C V_C - P_B V_B)$$

$$= \frac{nR}{\gamma-1} (T_C - T_B)$$

Since $T_B > T_C \Rightarrow W_2 < 0 \Rightarrow$ Work done by system.

(3) $C \rightarrow D$: (Isothermal at T_2)

Similarly,

$Q_3 = nRT_2 \ln\left(\frac{V_D}{V_C}\right)$

$W_3 = -Q_3$

④ $D \to A$: ~~Iso~~ (Adiabatic)

$$W_4 = \frac{nR}{\gamma-1}(T_A - T_D) = \frac{nR}{\gamma-1}(T_B - T_C)$$

$$= -W_2$$

$$\therefore W = W_1 + \cancel{W_2} + W_3 + \cancel{W_4}$$

$$= -nRT_1 \ln\left(\frac{V_B}{V_A}\right) - nRT_2 \ln\left(\frac{V_D}{V_C}\right)$$

$$\left(\begin{array}{l} T_A = T_B = T_1 \\ T_C = T_D = T_2 \end{array}\right)$$

Now, we know that,

$$T_1 V_B^{\gamma-1} = T_2 V_C^{\gamma-1}$$

$$T_1 V_A^{\gamma-1} = T_2 V_D^{\gamma-1}$$

$$\Rightarrow \frac{V_B}{V_A} = \frac{V_C}{V_D}$$

Using this,

$$W = -nR \ln\left(\frac{V_B}{V_A}\right)(T_1 - T_2)$$

(overall -ve, so system does work)

Now,

$$\eta = \frac{|W|}{Q_1} = -nR \ln\left(\frac{V_B}{V_A}\right)\frac{(T_1 - T_2)}{Q}$$

$$\Rightarrow \eta = \frac{-nR\ln\left(\frac{V_B}{V_A}\right)(T_1 - T_2)}{-nR\ln\left(\frac{V_B}{V_A}\right)T_1}$$

$$\Rightarrow \boxed{\eta = 1 - \frac{T_2}{T_1}} < 1$$

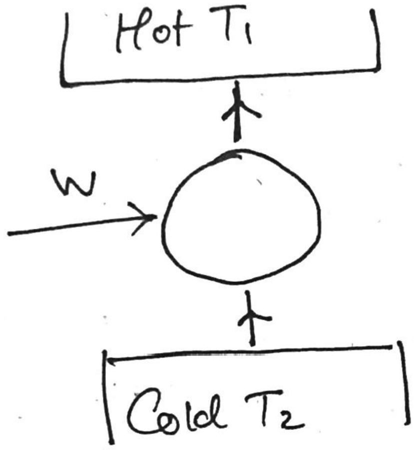
We may also write,

$$\boxed{\eta = 1 - \frac{|Q_3|}{Q_1}}$$

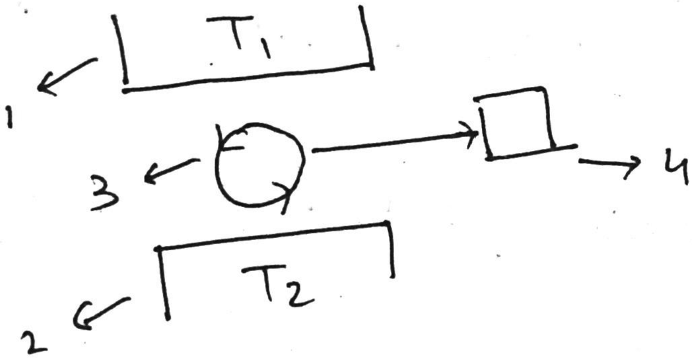
$T_1 > T_2$

$\Rightarrow$ How do you make η large?

o Carnot Cycle as the fridgerator $\longrightarrow$

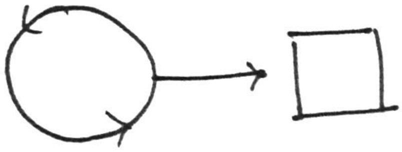


What lead us to the 4 body structure of the engine?



①  $\longrightarrow$ useless.

(2)



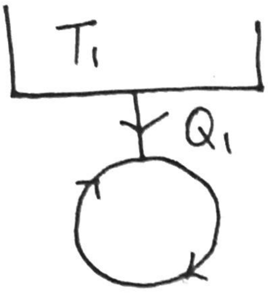
$\Delta U = 0$ (cycle)

$\Delta Q = 0$ here

$\Rightarrow$ $\Delta W = 0$

1st Law

(3)



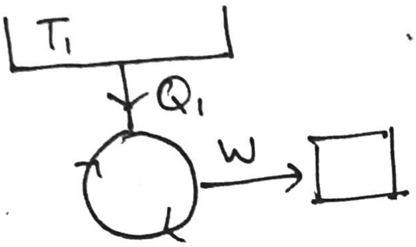
$\Delta U = 0$ (cycle)

$\Delta W = 0$ (Nothing to work on)

$\Rightarrow$ $\Delta Q = 0$

1st Law

(4)



$\Delta U = 0$ (cycle)

$\Rightarrow$ $Q_1 = W$

(*) Second Law is a statement that says that this configuration does not exist

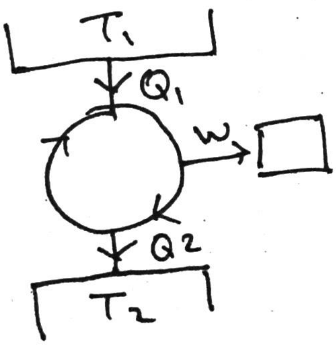
This engine can not exist

How do we say this?

All experiments attempting to make this have failed.

So we add another ingredient to make an engine that works.

(5)



This engine allows you to

(1) Be consistent with 1st Law

(2) Not violate 2nd Law.

- ~~2nd law of thermodynamics~~ → (Kelvin - Planck)

No cyclic process is possible whose sole result is Complete conversion of heat to work.

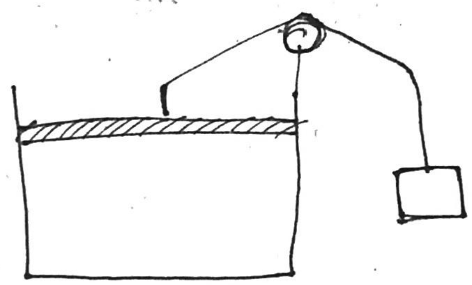
(*) Complete conversion happens in isothermal process — but that is not cyclic.

- 2nd law of thermodynamics (Clausius - Claypeyron)

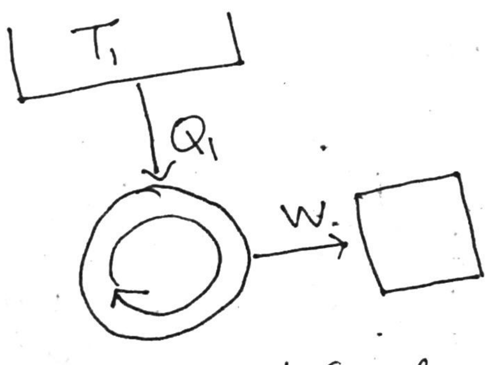
It is impossible to transfer heat from cold ~~to hot~~ body to hot body by means of a cyclic process without any effect to the surroundings.

3rd February 2023

Modelling → (the heat engine)

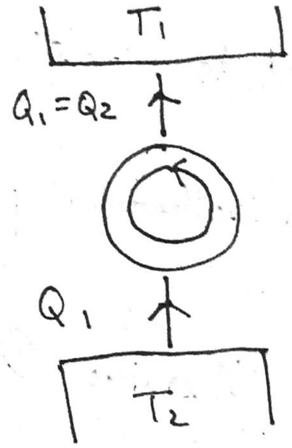


- Kelvin:



Does not exist (2nd law)

o Clausius statement:



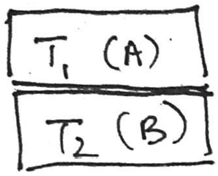
$T_1 > T_2$

Doesnotexist (2nd Law)

These statements are equivalent.

o Reversibility →

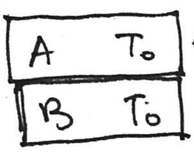
Example 1:



$T_1 > T_2$

Is the heat flow from T_1 to T_2 reversible?
No, it is irreversible.

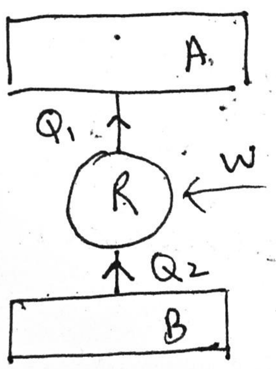
After some time,



$T_1 > T_0 > T_2$ (Q_2 heat moved $A \to B$)

These two bodies can ~~not~~ be put back to original state.

o Use a refridgerator.



$Q_1 = Q_2 + W$

→ State of B is now exactly what you started with.

→ But A ~~has~~ now gets more heat than it lost.

A has W ~~extra~~ energy extra.

- Take A with another cold body C such that W energy is taken out.

→ Now A is back to the original state.

So the statement of reversibility does not state that we cannot put them back.

But what else has happened?

→ Refridgerator worked (W)

→ C got some ~~heat~~ heat (Q)

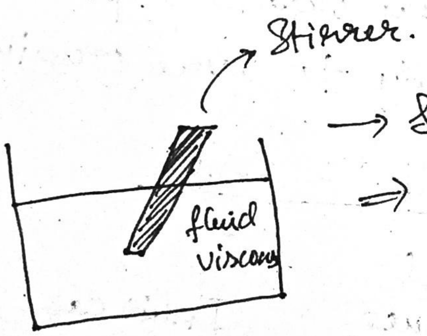
By conservation, $Q = W$.

To bring the surroundings back to original condition, we would need to convert Q to W completely — but 2nd law prohibits this.

(*) You can put the system to same state, but you cannot ~~convert~~ put system + surroundings to the same original state.

- Reversibility ≡ (System + Surroundings)

(*) Example 2:



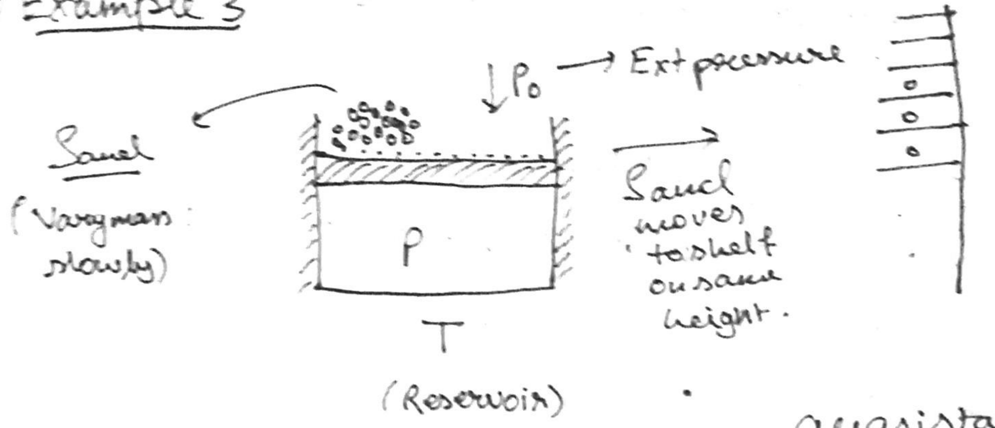
→ Stir the fluid quasistatically.

⇒ fluid will heat up.

To bring system back to original state, the heat has to be extracted and converted completely to work — prohibited ~~to~~ by 2nd law

→ Quasistatic process does not necessarily mean reversible.

Example 3

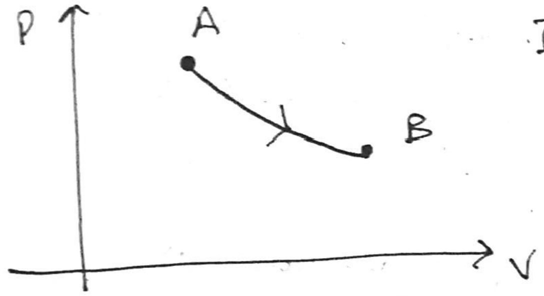


→ If we take sand off (gradually) → quasistatically, the ~~sys~~ piston moves up (taking heat from reservoir)

Work done $= \int p\,dv$

$\Delta U = 0$

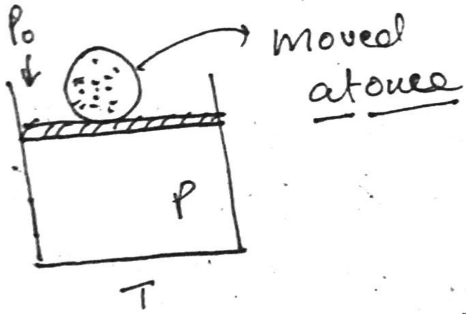
$Q = \int p\,dv$



Isothermal

→ If we move sand from shelf to on piston, the system goes back quasistatically to same original state (A)

Quasistatic reversible



In this case, the gas will go to state B non-quasistatically..

But system goes A → B.

$$\text{Work done by gas} = \int F \cdot dx$$
$$= P_0 A\,(x_f - x_i)$$
$$= P_0\,(V_f - V_i)$$

Heat taken from reservoir, $Q_1 = P_0(V_f - V_i)$

Now put the weight (mass) at top of the piston.

$$W = (mg + p \cdot A)\Delta x = \text{heat that goes to reservoir.}$$

So, extra heat goes to the reservoir $= mg\Delta x$

But notice that we had to take the mass from x_i to x_f before we can put it on the piston at state B.

This is extra heat is precisely ~~what~~ the amount of work we did against the gravitational force.

To put the system back to original state, we would have to extract $mg\Delta x$ heat from reservoir and convert it completely to work — prohibited by 2nd law.

So,

Reversible →

1. Must be quasistatic
2. No dissipative forces.

Typically all irreversible processes, one of the following ~~must~~ must be true

→ Mechanical / thermal / chemical equilibrium NOT satisfied

→ Dissipative force.

HW: Convince yourself that Joule's free expansion is irreversible.