$$W = (I - VP)^{-1}V$$

with

$$V = \begin{pmatrix} \tilde{V} & V \\ V & \tilde{V} \end{pmatrix} \text{ and } P = \begin{pmatrix} P^{\uparrow} & 0 \\ 0 & P^{\downarrow} \end{pmatrix}$$

So we have:

$$I - VP = \begin{pmatrix} I - \tilde{V}P^{\uparrow} & -VP^{\downarrow} \\ -VP^{\uparrow} & I - \tilde{V}P^{\downarrow} \end{pmatrix}$$

The inverse is then given by:

$$(I - VP)^{-1} = \begin{pmatrix} (I - \tilde{V}P^{\uparrow})^{-1} + (I - \tilde{V}P^{\uparrow})^{-1}VP^{\downarrow}SVP^{\uparrow}(I - \tilde{V}P^{\uparrow})^{-1} & (I - \tilde{V}P^{\uparrow})^{-1}VP^{\downarrow}S \\ SVP^{\uparrow}(I - \tilde{V}P^{\uparrow})^{-1} & S \end{pmatrix}$$

With $S = (I - \tilde{V}P^{\downarrow} - VP^{\uparrow}(I - \tilde{V}P^{\uparrow})^{-1}VP^{\downarrow})^{-1}$ So we have:

$$\begin{aligned} W^{\uparrow} &= ((I - \tilde{V}P^{\uparrow})^{-1} + (I - \tilde{V}P^{\uparrow})^{-1}VP^{\downarrow}SVP^{\uparrow}(I - \tilde{V}P^{\uparrow})^{-1})\tilde{V} + (I - \tilde{V}P^{\uparrow})^{-1}VP^{\downarrow}SV \\ W^{\downarrow} &= SVP^{\uparrow}(I - \tilde{V}P^{\uparrow})^{-1}V + S\tilde{V} \end{aligned}$$