Formulas used in IPASS-NKepler

Casper Smet

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1 Introduction

This document showcases the equations used in the Python module NKepler. NKepler was written by Casper Smet as his first year's final assignment.

2 Formulas

2.1 Velocity

Newton's law of universal gravitation is the equation the most important equation in NKepler.

$$F_g = G\frac{mM}{r^2} \tag{1}$$

$$F_c = \frac{mv^2}{r} \tag{2}$$

In a circular orbit, F_g and F_c equal each other. From this, the following equation can be derived:

$$\frac{mv^2}{r} = G\frac{mM}{r^2} \tag{3}$$

$$v = \sqrt{\frac{G \times M}{r}} \tag{4}$$

2.2 Period

The period of a satellite is how long it takes to complete one orbit.

$$v = \frac{2\pi r}{T} \tag{5}$$

$$T = \frac{2\pi r}{v} \tag{6}$$

2.3 Angular velocity

Angular velocity is the angle relative to a satellite's focus per second.

$$\omega = \frac{v}{r} \tag{7}$$

2.4 Angular displacement

Angular displacement is the product of angular velocity and time. Δt is only used for the purposes of the Python function.

$$\theta = \omega \cdot t \tag{8}$$

$$\theta = \omega \cdot (t \cdot \Delta t) + \theta 0 \tag{9}$$

2.5 Cartesian coordinates from angular displacement

Using the following two formulas, one can convert angular displacement to a set of Cartesian coordinates.

$$x = \cos\theta \cdot r \tag{10}$$

$$y = \sin\theta \cdot r \tag{11}$$

2.6 Angular displacement from Cartesian coordinates

Using the following formula, one can convert Cartesian coordinates to angular displacement. This is used to set the starting position of a satellite.

$$\theta = \arctan 2(y - cy, x - xy) \tag{12}$$

$$\theta 0 = \theta \tag{13}$$

2.7 Relative to absolute Cartesian coordinates

Converts relative Cartesian coordinates to absolute Cartesian coordinates. In this scenario, the absolute coordinates are not absolutely absolute. They are absolute only to the lowest level focus (E.G. the Sun).

$$x_a = x_f + x_s \tag{14}$$

$$y_a = y_f + y_s \tag{15}$$

3 Symbols

Unless otherwise stated, assume these definitions. In order of appearance.

$$F_g \leftarrow N \qquad \leftarrow \textbf{gravitational force} \tag{16}$$
$$F_c \leftarrow N \qquad \leftarrow \textbf{centripetal force} \tag{17}$$
$$G \leftarrow m^3 \cdot kg^{-1} \cdot s^{-2} \leftarrow \textbf{gravitational constant} \tag{18}$$
$$m, M \leftarrow kg \qquad \leftarrow \textbf{masses of the object} \tag{19}$$
$$r \leftarrow m \qquad \leftarrow \textbf{radius between centres of mass} \tag{20}$$
$$v \leftarrow m \cdot s^{-1} \qquad \leftarrow \textbf{velocity} \tag{21}$$
$$T \leftarrow s \qquad \leftarrow \textbf{period} \tag{22}$$
$$\omega \leftarrow rad \cdot s^{-1} \qquad \leftarrow \textbf{angular velocity} \tag{23}$$
$$\theta, \theta 0 \leftarrow rad \qquad \leftarrow \textbf{angular displacement, known angle} \tag{24}$$
$$t, \Delta t, t_k \leftarrow s \qquad \leftarrow \textbf{time, time interval, known time} \tag{25}$$
$$x \leftarrow m \qquad \leftarrow \textbf{Cartesian x coordinates, various} \tag{26}$$
$$y \leftarrow m \qquad \leftarrow \textbf{Cartesian y coordinates, various} \tag{27}$$

4 Sources

All equation were sourced from the BiNaS v.6 or through personal correspondence with my Product-Owner.