

Programmer som data

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1 Exercise 6.4

$$\frac{}{\rho \vdash i : \mathbf{int}} p_1$$

$$\frac{}{\rho \vdash b : \mathbf{bool}} p_2$$

$$\frac{\rho(f) = \forall \alpha_1, \dots, \alpha_n. t}{\rho \vdash f : [t_1/\alpha_1, \dots, t_n/\alpha_n]t} p_3$$

$$\frac{\rho \vdash e_1 : \mathbf{int} \quad \rho \vdash e_2 : \mathbf{int}}{\rho \vdash e_1 + e_2 : \mathbf{int}} p_4$$

$$\frac{\rho \vdash e_1 : \mathbf{int} \quad \rho \vdash e_2 : \mathbf{int}}{\rho \vdash e_1 < e_2 : \mathbf{bool}} p_5$$

$$\frac{\rho \vdash e_r : t_r \quad \rho[x \mapsto \forall \alpha_1 \dots \alpha_n. t_r] \vdash e_b : t \quad \alpha_1 \dots \alpha_n \text{ not free in } \rho}{\rho \vdash \mathbf{let } x = e_r \text{ in } e_b \text{ end} : t} p_6$$

$$\frac{\rho \vdash e_1 : \mathbf{bool} \quad \rho \vdash e_2 : t \quad \rho \vdash e_3 : t}{\rho \vdash \mathbf{if } x = e_1 \text{ then } e_2 \text{ else } e_3 : t} p_7$$

$$\frac{\rho[x \mapsto t_x, f \mapsto t_x \rightarrow t_r] \vdash e_r : t_r \quad \rho[f \mapsto \forall \alpha_1 \dots \alpha_n. t_x \rightarrow t_r] \vdash e_b : t \quad \alpha_1 \dots \alpha_n \text{ not free in } \rho}{\rho \vdash \mathbf{let } f x = e_r \text{ in } e_b \text{ end} : t} p_8$$

$$\frac{\rho \vdash e_1 : t_x \rightarrow t_r \quad \rho \vdash e_2 : t_x}{\rho \vdash e_1 e_2 : t_r} p_9$$

let f x = 1 in f f end

$$\begin{array}{c}
 \begin{array}{c}
 p_1 \frac{}{\rho[x \mapsto \alpha, f \mapsto \alpha \rightarrow \text{int}] \vdash 1 : \text{int}} \\
 \hline
 \end{array}
 \quad
 \begin{array}{c}
 \begin{array}{c}
 \frac{\rho(f) = \forall \alpha \alpha \rightarrow \text{int}}{\rho \vdash f : [a \rightarrow \text{int} / \alpha](\alpha \rightarrow \text{int}) \rightarrow \text{int}} \quad p_3
 \qquad
 \frac{\rho(f) = \forall \alpha \alpha \rightarrow \text{int}}{\rho \vdash f : [a / \alpha]'a \rightarrow \text{int}} \quad p_3 \\
 \hline
 \rho[f \mapsto \forall \alpha. \alpha \rightarrow \text{int}] \vdash f f : \text{int} \quad p_9
 \end{array} \\
 \hline
 [] \vdash \text{let } f \ x = 1 \text{ in } f f \text{ end} : \text{int} \quad p_8
 \end{array}$$

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let f x = if x < 10 then 42 else f(x+1)
in f 20 end

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$$\begin{array}{c}
\frac{p_3 \frac{\rho(x) = \forall \alpha. \alpha \rightarrow \text{int}}{\rho \vdash x : \text{int}} \quad \frac{}{\rho \vdash 10 : \text{int}} p_1}{p_5 \frac{}{\rho \vdash x < 10 : \text{bool}}} \quad p_1 \frac{}{\rho \vdash 42 : \text{int}} \quad \frac{\frac{\rho(f) = \forall \alpha. \alpha \rightarrow \text{int}}{\rho \vdash f : [\text{int}/\alpha] \alpha \rightarrow \text{int} = \text{int} \rightarrow \text{int}} p_3 \quad \frac{\frac{\frac{\rho(x) = \forall \alpha. \alpha \rightarrow \text{int}}{\rho \vdash x : \text{int}} p_3 \quad \frac{}{\rho \vdash 1 : \text{int}} p_1}{\rho \vdash x + 1 : \text{int}} p_4}{\rho \vdash f(x+1) : \text{int}} p_9}{p_7 \frac{}{\rho = [x \mapsto \text{int}, f \mapsto \text{int} \rightarrow \text{int}] \vdash \text{if } x < 10 \text{ then } 42 \text{ else } f(x+1) : \text{int}}} p_8 \\
\rho = [] \vdash \text{let } f x = \text{if } x < 10 \text{ then } 42 \text{ else } f(x+1) \text{ in } f 20 \text{ end} : \text{int}
\end{array}$$

because of latex formatting and fuckery the tree above continued from rule 8.

$$\frac{p_3 \frac{\rho(f) = \forall \alpha \alpha \rightarrow \text{int}}{\rho \vdash f : \text{int}} \quad p_1 \frac{}{\rho \vdash 20 : \text{int}}}{\rho \vdash f 20 : \text{int}} p_9$$