

Scale-space and Image Restoration

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Scale-space operators

1. Consider a Gaussian kernel

$$G(x, y, \sigma) = \frac{1}{2\pi\sigma^2} e^{-\frac{x^2+y^2}{2\sigma^2}}. \quad (1)$$

The convolution of a Gaussian with itself is also a Gaussian, i.e.,

$$G(x, y, \sigma) * G(x, y, \tau) = G(x, y, \sqrt{\sigma^2 + \tau^2}) \quad (2)$$

Make an image,

$$I(x, y) = G(x, y, \sigma), \quad (3)$$

for some fixed σ , and confirm the above by calculating images from its scale-space,

$$I(x, y, \tau) = I(x, y) * G(x, y, \tau), \quad (4)$$

using Matlab and, e.g., using the `scale` function written previously.

2. Consider the 2-dimensional scale normalized derivatives at scale τ ,

$$I_{x^i y^j}(x, y, \tau) = \tau^{\gamma(i+j)} \frac{\partial^{i+j} I(x, y, \tau)}{\partial x^i \partial y^j}, \quad (5)$$

where $\gamma \in \mathbb{R}$ is a parameter of the scale normalization. Now consider the scale normalized image of the Laplacian,

$$H(x, y, \tau) = I_{xx}(x, y, \tau) + I_{yy}(x, y, \tau), \quad (6)$$

Using $\gamma = 1$, solve the following:

- (a) Calculate the analytical expression for $H(x, y, \tau)$.
 - (b) Consider the point $(x, y) = (0, 0)$ and derive analytically the scale(s), τ , for which $H(0, 0, \tau)$ is extremal. Maple (or Mathematica) may be helpful. Characterize these extremal point(s) in terms of maximum, saddle, and minimum in (x, y, τ) .
 - (c) Confirm your result in Matlab.
 - (d) The maxima in (x, y, τ) of (6) is called blob detection. Detect the 20 largest maxima in the `sunflower.tiff` image, and indicate each detected scale τ with a circle centered on the point of detection and with a radius of τ .
3. Consider a soft edge,

$$J(x, y) = \int_{-\infty}^x G(x', y, \sigma) dx' \quad (7)$$

for some constant σ . Consider also its scale-space,

$$J(x, y, \tau) = J(x, y) * G(x, y, \tau), \quad (8)$$

and the scale-normalized spatial squared gradient magnitude operator

$$\|\nabla J(x, y, \tau)\|^2 = J_x^2(x, y, \tau) + J_y^2(x, y, \tau). \quad (9)$$

Using $\gamma = \frac{1}{2}$, solve the following:

- (a) Write the closed form expression for $\|\nabla J\|^2$.
- (b) Derive analytically the scale, τ , for which $\|\nabla J\|^2$ is maximal in the point $(x, y) = (0, 0)$. Is this a maximum in (x, y, τ) ?
- (c) Confirm your result in Matlab.
- (d) The maxima in (x, y, τ) of (9) is edge detection with scale-selection. Detect the 100 largest maxima in the `hand.tif` image, and indicate the point of detection and scale by circles.

Inverse filtering

- 4. Write a program that takes an image, a kernel and a realisation of a noise source, and which returns the linear, shift invariantly (LSI) degraded result.
- 5. Implement inverse filtering together with a band-pass filter to avoid numerical instabilities, and apply it to a LSI degraded image of `lena.tif`. Discuss this approach's ability to recover the original image.
- 6. Repeat the above exercise for the Wiener filter.
- 7. Consider the Laplacian as the linear operator and implement constrained deconvolution without using `deconvreg`. Evaluate the result on suitable test images.