

Matrix

Part 1 Concepts

Definition 0.1. Matrix

$$A = (a_{ij})_{m \times n} = \begin{pmatrix} a_{11} & a_{12} & \cdots & a_{1n} \\ a_{21} & a_{22} & \cdots & a_{2n} \\ \vdots & \vdots & \cdots & \vdots \\ a_{m1} & a_{m2} & \cdots & a_{mn} \end{pmatrix} \quad 1 \leq i \leq m, 1 \leq j \leq n.$$

Operations

- $A = (a_{ij})_{m \times n}$ and $B = (b_{ij})_{m \times n}$

$$A \pm B = (a_{ij} \pm b_{ij})_{m \times n}$$

Example:

$$\begin{pmatrix} 1 & 2 \\ 3 & 4 \end{pmatrix} + \begin{pmatrix} 5 & 6 \\ 7 & 8 \end{pmatrix} = \begin{pmatrix} 6 & 8 \\ 10 & 12 \end{pmatrix}$$

- $A = (a_{ij})_{m \times n}$ and $B = (b_{ij})_{n \times l}$

$$AB = \left(\sum_{k=1}^n a_{ik} b_{kj} \right)_{m \times l}$$

Example:

$$\begin{pmatrix} 1 & 2 \\ 3 & 4 \end{pmatrix} \begin{pmatrix} 5 \\ 6 \end{pmatrix} = \begin{pmatrix} 1 \times 5 + 2 \times 6 \\ 3 \times 5 + 4 \times 6 \end{pmatrix} = \begin{pmatrix} 17 \\ 39 \end{pmatrix}$$

Definition 0.2. The identity matrix $I_n = (\delta_{ij})_{n \times n}$ of size n is a square matrix that all the main diagonal elements are 1, otherwise 0, i.e.:

$$\delta_{ij} = \begin{cases} 1 & i = j \\ 0 & i \neq j \end{cases}$$

For example, the identity matrix of size 2 I_2 is:

$$I_2 = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$$

If A is a $m \times n$ matrix,

$$I_m A = A I_n = A$$

Definition 0.3. A square matrix of size n , $A = (a_{ij})_{n \times n}$, is said to be invertible if its inverse, denoted by A^{-1} , exists and satisfies:

$$AA^{-1} = A^{-1}A = I.$$

Definition 0.4. Dot product. If \mathbf{a} and \mathbf{b} are vectors, $\mathbf{a} = (a_1, \dots, a_n)$, $\mathbf{b} = (b_1, \dots, b_n)$, then their dot product is defined by

$$\mathbf{a} \cdot \mathbf{b} = a_1 b_1 + \cdots + a_n b_n.$$