Matrix

Definition 0.1. Matrix

$$A = \big(a_{ij}\big)_{m\times n} = \begin{pmatrix} a_{11} & a_{12} & \cdots & a_{1n} \\ a_{21} & a_{22} & \cdots & a_{2n} \\ \vdots & \vdots & \cdots & \vdots \\ a_{m1} & a_{m2} & \cdots & a_{mn} \end{pmatrix} \quad 1 \le i \le m,\ 1 \le j \le n.$$

Operations

- $A = \big(a_{ij}\big)_{m\times n}$ and $B = \big(b_{ij}\big)_{m\times n}$

$$A \pm B = \big(a_{ij} \pm b_{ij}\big)_{m\times n}$$

  Example:

$$\begin{pmatrix} 1 & 2 \\ 3 & 4 \end{pmatrix} + \begin{pmatrix} 5 & 6 \\ 7 & 8 \end{pmatrix} = \begin{pmatrix} 6 & 8 \\ 10 & 12 \end{pmatrix}$$

- $A = \big(a_{ij}\big)_{m\times n}$ and $B = \big(b_{ij}\big)_{n\times l}$

$$AB = \big(\sum_{k=1}^{n} a_{ik}b_{kj}\big)_{m\times l}$$

  Example:

$$\begin{pmatrix} 1 & 2 \\ 3 & 4 \end{pmatrix}\begin{pmatrix} 5 \\ 6 \end{pmatrix} = \begin{pmatrix} 1\times 5 + 2\times 6 \\ 3\times 5 + 4\times 6 \end{pmatrix} = \begin{pmatrix} 17 \\ 39 \end{pmatrix}$$

Definition 0.2. The identity matrix $I_n = \big(\delta_{ij}\big)_{n\times n}$ of size n is a square matrix that all the main diagonal elements are 1, otherwise 0, i.e.:

$$\delta_{ij} = \begin{cases} 1 & i = j \\ 0 & i \neq j \end{cases}$$

For example, the identity matrix of size 2 I_2 is:

$$I_2 = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$$

If A is a $m \times n$ matrix,

$$I_m A = A I_n = A$$

Definition 0.3. A square matrix of size n, $A = \big(a_{ij}\big)_{n\times n}$, is said to be invertible if its inverse, denoted by A^{-1}, exists and satisfies:

$$AA^{-1} = A^{-1}A = I.$$

Definition 0.4. Dot product. If $\mathbf{a}$ and $\mathbf{b}$ are vectors, $\mathbf{a} = (a_1, \cdots, a_n)$, $\mathbf{b} = (b_1, \cdots, .b_n)$, then their dot product is defined by

$$\mathbf{a}\cdot\mathbf{b} = a_1b_1 + \cdots + a_nb_n.$$

For example, if $\mathbf{a} = (1, 2, 3)$ and $\mathbf{b} = (4, 5, 6)$, then:

$$\mathbf{a}\cdot\mathbf{b} = 1\times 4 + 2\times 5 + 3\times 6 = 32$$