**Inventory Management**

**EBM026A05**

**Group assignment part 1**

**Group 03**
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## 1. EOQ and variants

**Optimal order quantity**

In order to calculate the optimal policy, we use the formula for optimal order quantity and optimal average costs:

$$Q^* = \sqrt{\frac{2KD}{h}} \qquad C(Q^*) = \frac{hQ}{2} + \frac{KD}{Q} + cD$$

In the table below, the values can be found for each particular variable and the optimal order quantity and associated costs.

| | | A | B | C | D | E |
|---|---|---|---|---|---|---|
| Demand per month | D | 1200 | 1200 | 1200 | 1200 | 1200 |
| Fixed replenishment cost per order | K | 11500 | 16000 | 11000 | 9500 | 15000 |
| Procurement cost per unit | c | 29 | 28 | 33 | 30 | 29 |
| Holding costs per month per unit | h[1] | 3,82 | 3,69 | 4,35 | 3,95 | 3,82 |
| Lead time (months) | L | 0.5 | 2 | 1.5 | 1.5 | 2.5 |
| Further characteristics | | Qmax=2000 | Qmin=4500 | | | Batches of 800 |
| | | | | | | |
| Optimal order quantity | Q* | 2687,9 | 3226,5 | 4000 | 2401 | 3069,7 |
| Order quantity after correction | | 2000 | 4500 | | | 3200 |
| Costs | C(Q) | 45520 | 46166 | 49179 | 45492 | 46538 |
| Re-order level (L*D) | | 600 | 2400 | 1800 | 1800 | 3000 |

For supplier A, the optimal order quantity can be calculated with the function above. The optimal order quantity is 2687,87, which is higher than the maximum order quantity. Therefore the order quantity will become the highest possible value of 2000, the associated costs will be 45520.

For supplier B, the optimal order quantity can also be calculated with the function above and is 3226,52, which is lower than the minimum order quantity. Therefore the order quantity will become the lowest possible value of 4500 with costs of 46166.

For supplier C, we have to take into account the all-unit discount. The optimal order quantity formula and the cost formula changes to:

Average cost per time:

$$C(Q) = \begin{cases} C_0(Q) = \frac{\alpha c_0 Q}{2} + \frac{KD}{Q} + c_0 D & \text{if } 0 \leq Q < Q_1 \\ C_1(Q) = \frac{\alpha c_1 Q}{2} + \frac{KD}{Q} + c_1 D & \text{if } Q_1 \leq Q < Q_2 \\ C_2(Q) = \frac{\alpha c_2 Q}{2} + \frac{KD}{Q} + c_2 D & \text{if } Q_2 \leq Q \end{cases}$$

Optimal order quantity:

$$Q_0^* = \sqrt{\frac{2KD}{\alpha c_0}} \quad Q_1^* = \sqrt{\frac{2KD}{\alpha c_1}} \quad Q_2^* = \sqrt{\frac{2KD}{\alpha c_2}}$$

[1] h is calculated as (1.004)^31 - 1 * c

For supplier C the optimal order quantity is 4000 with a cost of 49179.

For supplier D, we have incremental quantity discounts. The formulas become the following:

Average cost per time:

$$C(Q) = \begin{cases} C_0(Q) = \frac{\alpha v_0}{2} + \frac{\alpha c_0 Q}{2} + \frac{(K+v_0)D}{Q} + c_0 D & \text{if } 0 \le Q < Q_1 \\ C_1(Q) = \frac{\alpha v_1}{2} + \frac{\alpha c_1 Q}{2} + \frac{(K+v_1)D}{Q} + c_1 D & \text{if } Q_1 \le Q < Q_2 \\ C_2(Q) = \frac{\alpha v_2}{2} + \frac{\alpha c_2 Q}{2} + \frac{(K+v_2)D}{Q} + c_2 D & \text{if } Q_2 \le Q \end{cases}$$

Optimal order quantity:

$$Q_0^* = \sqrt{\frac{2(K+v_0)D}{\alpha c_0}} \quad Q_1^* = \sqrt{\frac{2(K+v_1)D}{\alpha c_1}} \quad Q_2^* = \sqrt{\frac{2(K+v_2)D}{\alpha c_2}}$$

The lowest costs are 45492, with an optimal quantity of 2401.

For supplier E, the initial formulas can be used. The optimal order quantity is 3069. However, this supplier only accepts orders with a multiple of 800. The costs are thus computed for an order quantity of 2400 and 3200, whereafter, the quantity with the lowest cost is chosen. In this case the order quantity while taking into account the batch requirement is thus 3200.

$C(2400) = 46884$

$C(3200) = 46538$

Overall, we can rank the costs for all suppliers and the results are shown in the table below. In this case supplier D is the least expensive, whereas, supplier C is the most expensive supplier.

| Ranking (based on average costs) | Costs from low to high |
|---|---|
| Supplier D | C = 45492 |
| Supplier A | C = 45520 |
| Supplier B | C = 46166 |
| Supplier E | C = 46538 |
| Supplier C | C = 49179 |

## CO2 emissions

For each of the suppliers the CO2 emissions are calculated. Where necessary the optimal order quantity is adjusted to meet the monthly carbon limit of 3 tons. To calculate the emission per month we used the following function:

$$AverageEmissionsPerPeriod\,(Q) = K_{emission} * \frac{D}{Q} + c_{emission} * D$$

In the table below, the

| | | A | B | C | D | E |
|---|---|---|---|---|---|---|
| Demand per month | D | 1200 | 1200 | 1200 | 1200 | 1200 |

| | | | | | | |
|---|---|---|---|---|---|---|
| CO2 emissions per unit | $c_{emission}$ | 1 | 1.9 | 2 | 1.8 | 1.7 |
| Fixed CO2 emission per order | $K_{emission}$ | 1000 | 2900 | 2600 | 2500 | 3000 |
| Optimal policy: | | | | | | |
| | Q * | 2000 | 4500 | 4000 | 2401 | 3200 |
| Total emission per order | | 3000 | 11450 | 10600 | 6821.8 | 8440 |
| Average emission per month | $E_{, monthly}$ | 1800 | 3053.33 | 3180 | 3409.48 | 3165 |

As can be seen from the table, suppliers B, C, D, and E exceed the limit of 3000. To calculate the new optimal order quantity Q* while respecting the carbon limit, we used the formula for average emission per month and rewrote it to solve for Q. We know that the maximum emission is 3000 and can thus solve what the maximum order quantity is while respecting the carbon limit.

$$Q = (D * K_{emission}) / (3000 - D * c_{emission})$$

For the suppliers that exceed the carbon limit when using the EOQ(suppliers B,C,D&E), the order quantity is recalculated using the above formula. The associated costs with the new quantity is calculated according to the same cost formula as before.

| Supplier A | | |
|---|---|---|
| D = 1200 | $K_{emission}$ = 1000 | $c_{emission}$ = 1 |
| Q =2000 | C = 45520.34 | E = 1800 |

| Supplier B | | |
|---|---|---|
| D = 1200 | $K_{emission}$ = 2900 | $c_{emission}$ = 1.9 |
| Q = 4833.33 | C = 46486.54 | E = 3000 |

| Supplier C | | |
|---|---|---|
| D = 1200 | $K_{emission}$ = 2600 | $c_{emission}$ = 2 |
| Q = 5200 | C = 45728.84 | E = 3000 |

| Supplier D | | |
|---|---|---|
| D = 1200 | $K_{emission}$ = 2500 | $c_{emission}$ = 1.8 |

| | | |
|---|---|---|
| Q = 3571.43 | C = 43378.80 | E = 3000 |

| | | |
|---|---|---|
| Supplier E | | |
| D = 1200 | $K_{emission}$ = 3000 | $c_{emission}$ = 1.7 |
| Q = 4000 [1] | C = 46940.68 | E = 3000 |

1: As the optimal order quantity of E is not meeting the batch size restriction, either Q should be 3200 or 4000. Based on the associated costs, an order quantity of 4000 is meeting both the batch size and emission restriction.

Based on the new order quantities that respect the carbon limit, the costs were recalculated. All suppliers were ranked based on these costs and ranked as can be found in the table below. Supplier D is the cheapest supplier and supplier E the most expensive.

| Ranking (based on average costs) | Costs from low to high |
|---|---|
| Supplier D | C = 43378.80 |
| Supplier A | C = 45520.34 |
| Supplier C | C = 45728.84 |
| Supplier B | C = 46486.54 |
| Supplier E | C = 46940.68 |

**Production**

Instead of purchasing, the components can also be produced in house. The specific parameters associated with producing are stated below:

- cost of production per unit; c 35
- fixed cost; K 700
- production rate per month; P 1500
- holding cost per unit per month; h (1 + 0.004)^31 - 1
- emission per unit; $c_{emission}$ 3.6

To calculated the optimal production quantity and cost the following formulas are used:

$$C(Q) = h'\frac{Q}{2} + K\frac{D}{Q} + cD \qquad \text{with } h' = h * (1 - \frac{D}{P})$$

Optimal production quantity $= \sqrt{\frac{2*K*D}{h*(1-D/P)}}$

Applying the optimal production quantity formula, an optimal production quantity of 7985.23 per cycle is obtained. The associated costs for this production quantity is 43018.98.

By multiplying the optimal production quantity with the emission per unit, we get the total value of emissions, which is 28746.8361. Since the machine can produce a maximum of 1500 products per month, this means that it takes 7985.23/1500=5.32 months. To calculate the monthly emission:

28746/5.32=5400. As can be seen, this value is far more than the maximum of 3000. In order to meet the carbon limit, the company can produce a maximum of 3000/3.6 = 833.33 units. However, as 833.33 < 1200, when producing the components in house and respecting the carbon limit, it is not possible to meet demand. If there would have been no monthly carbon limit, the policy could have been better based on cost, however, in this case producing the critical component in-house is not optimal.

**DLS and variants**

**MIP for each substance**

First, based on the demand of substance A, we calculate the demands for substance B and C:

Substance A = [60, 115, 90, 80, 110, 200, 40 , 20]

Substance B = [20, 38.33, 30, 26.67, 66.67, 13.33, 6.66]

Substance C = [120, 230, 180, 160, 220, 400, 80, 40]

Furthermore, the unit cost per liter for each substance is cA = \$5, cB = \$8 and cC = \$7 respectively. The fixed replenishment cost K = \$200 is the same for all substances. The holding cost per liter per period is 10% of the procurement cost of the substance. The maximum capacity (in liters) of the storage tank of each of the substances is $M_a$,= 400, $M_b$= 700 and $M_c$= 500 respectively. These assumptions hold for all MIP, unless stated otherwise.

First, we will design the MIP to solve the dynamic lot sizing problem for each substance separately. Meaning that for each substance the optimal policy is calculated. This gives the following MIP for each substance:

$$\min \sum_{N=1}^{T} (K * y_N + H * I_N)$$

s.t.

(1) $I_N = I_{N-1} + Q_N - D_N$ $\quad\forall$ N = 1, …, T

(2) $Q_N \leq M * y_N$ $\quad\forall$ N = 1,…, T

(3) $I_0 = 0$

(4) $I_{N-1} + Q_N \leq \text{Capacity}$ $\quad\forall$ N = 1,…, T

(5) $y_N \in \{0,1\}$ $\quad\forall$ N = 1,…, T

(6) $Q_N, I_N \geq 0$ $\quad\forall$ N = 1,…, T

* Note: in the MIP we refer to Capacity; in the problem this is stated as $M_a$, $M_b$, and $M_c$, respectively. Not to refuse with big M shown in constraint (2).

In the tables below the demand, optimal order quantity, inventory level, and the costs per period for substance A, B and C can be found after solving the MIP described above.

| **A.** Period *n* | Demand | Optimal order quantity | Inventory level | Costs per period |
|---|---|---|---|---|
| 0 | 0 | 0 | 0 | 0 |
| 1 | 60 | 345 | 285 | 2067.50 |
| 2 | 115 | 0 | 170 | 85 |
| 3 | 90 | 0 | 80 | 40 |
| 4 | 80 | 0 | 0 | 0 |
| 5 | 110 | 370 | 260 | 2180 |
| 6 | 200 | 0 | 60 | 30 |
| 7 | 40 | 0 | 20 | 10 |
| 8 | 20 | 0 | 0 | 0 |

| **B.** Period *n* | Demand | Optimal order quantity | Inventory level | Costs per period |
|---|---|---|---|---|
| 0 | 0 | 0 | 0 | 0 |
| 1 | 20 | 115 | 95 | 1196 |
| 2 | 38.33 | 0 | 56.67 | 45.33 |
| 3 | 30 | 0 | 26.67 | 21.33 |
| 4 | 26.67 | 0 | 0 | 0 |
| 5 | 36.67 | 123.33 | 86.67 | 1256 |
| 6 | 66.67 | 0 | 20 | 16 |
| 7 | 13.33 | 0 | 6.67 | 5.33 |
| 8 | 6.67 | 0 | 0 | 0 |

| **C.** Period *n* | Demand | Optimal order quantity | Inventory level | Costs per period |
|---|---|---|---|---|
| 0 | 0 | 0 | 0 | 0 |
| 1 | 120 | 350 | 230 | 2811 |
| 2 | 230 | 0 | 0 | 0 |

| | | | | |
|---|---|---|---|---|
| 3 | 180 | 340 | 160 | 2692 |
| 4 | 160 | 0 | 0 | 0 |
| 5 | 220 | 220 | 0 | 1740 |
| 6 | 400 | 400 | 0 | 3000 |
| 7 | 80 | 120 | 40 | 1068 |
| 8 | 40 | 0 | 0 | 0 |

**Safety regulations**

Following the previous MIP, for this MIP we need to take into account a specific safety requirement. This safety requirement states that replenishment orders for different substances cannot take place in the same period — this holds for all periods except the first one.

Taking this into account the following MIP is constructed:

S = {a, b, c} : set of substances

$$\min \sum_{N=1}^{T} \sum_{i \in S} (K * y_N^i + h^i * I_N^i)$$

s.t.

(1) $I_N^i = I_{N-1}^i + Q_N^i - D_N^i$ $\quad \forall$ N = 1, …, T , i ∈ S

(2) $I_0^i = 0$ $\quad \forall$ i ∈ S

(3) $I_{N-1}^i + Q_N^i \leq Capacity^i$ $\quad \forall$ N = 1, …, T , i ∈ S

(4) $Q_N^i \leq M * y_N^i$ $\quad \forall$ N = 1, …, T , i ∈ S

(5) $\sum_{i \in S} y_N^i \leq 1$ $\quad \forall$ N = 2, …, T

(6) $y_N^i \in \{0,1\}$ $\quad \forall$ N = 1, …, T , i ∈ S

(7) $Q_N^i, I_N^i \geq 0$ $\quad \forall$ N = 1, …, T , i ∈ S

In the tables below the demand, optimal order quantity, inventory level and the costs per period of substance A, B, and C can be found, taking into account the condition where an order can be placed at only one at a time, except for the first period.

| **A.** Period *n* | Demand | Optimal order quantity | Inventory level | Costs per period |
|---|---|---|---|---|
| 0 | 0 | 0 | 0 | 0 |
| 1 | 60 | 345 | 285 | 2067.50 |
| 2 | 115 | 0 | 170 | 85 |
| 3 | 90 | 0 | 80 | 40 |
| 4 | 80 | 0 | 0 | 0 |
| 5 | 110 | 370 | 260 | 2180 |
| 6 | 200 | 0 | 60 | 30 |
| 7 | 40 | 0 | 20 | 10 |
| 8 | 20 | 0 | 0 | 0 |

| **B.** Period *n* | Demand | Optimal order quantity | Inventory level | Costs per period |
|---|---|---|---|---|
| 0 | 0 | 0 | 0 | 0 |
| 1 | 20 | 58.33 | 38.33 | 697.33 |
| 2 | 38.33 | 0 | 0 | 0 |
| 3 | 30 | 180 | 150 | 1760 |
| 4 | 26.67 | 0 | 123.33 | 98.67 |
| 5 | 36.67 | 0 | 86.67 | 69.33 |
| 6 | 66.67 | 0 | 20 | 16 |
| 7 | 13.33 | 0 | 6.67 | 5.33 |
| 8 | 6.67 | 0 | 0 | 0 |

| **C.** Period *n* | Demand | Optimal order quantity | Inventory level | Costs per period |
|---|---|---|---|---|
| 0 | 0 | 0 | 0 | 0 |
| 1 | 120 | 120 | 0 | 1040 |
| 2 | 230 | 410 | 180 | 3196 |
| 3 | 180 | 0 | 0 | 0 |

| | | | | |
|---|---|---|---|---|
| 4 | 160 | 380 | 220 | 3014 |
| 5 | 220 | 0 | 0 | 0 |
| 6 | 400 | 400 | 0 | 3000 |
| 7 | 80 | 120 | 40 | 1068 |
| 8 | 40 | 0 | 0 | 0 |

From the tables it can be observed that an order is placed for all substances in the first period. Thereafter, an order for substance A is placed in period 5, for substance B in period 3 and for substance C in period 2,4,6 and 7. This confirms that the safety requirements are obeyed by the model.

Compared to the previous case, the optimal policy for supplier A did not change, meaning that the corresponding cost is the same. For both supplier B and C the optimal policy did change as can be seen in the displayed results. Since in the second case the model became more restrictive, the resulting optimal policy must be more expensive than before. This can be verified using the shown results above.

## Re-assignation of storage tanks

In the following case, a MIP is constructed that finds the optimal inventory policy over time period T. However, contrary to the previous cases the capacities of the tanks can now be optimally assigned to a substance by the model and are no longer bound to a specific substance. Furthermore, in this case it is assumed that the safety requirements of the previous case are no longer needed.

Model:

S = {a, b, c} : set of substances

J = {400, 700, 500}: set of capacities

$x_{ij}$ = 1 if substance i uses capacity j : otherwise 0

$$\min \sum_{N=1}^{T} \sum_{i \in S} (K * y_N^i + h^i + I_N^i)$$

s.t.

(1) $I_N^i = I_{N-1}^i + Q_N^i - D_N^i$ $\quad \forall\, i \in S,\ N = 1, \ldots, T$

(2) $I_0^i = 0$ $\quad \forall\, i \in S$

(3) $x_{ij}[\, I_{N-1}^i + Q_N^i] \le Capacity_j$ $\quad \forall\, i \in S,\ j \in J,\ N = 1, \ldots, T$

(4) $Q_N^i \le M * y_N^i$ $\quad \forall\, N = 1, \ldots, T\ ,\ i \in S$

(5) $\sum_{i \in S} x_{ij} = 1$ $\forall j \in J$

(6) $\sum_{j \in J} x_{ij} = 1$ $\forall i \in S$

(7) $y^i_N \in \{0,1\}$ $\forall N = 1, \dots, T \ , \ i \in S$

(8) $Q^i_N, I^i_N \geq 0$ $\forall N = 1, \dots, T \ , \ i \in S$

(9) $x_{ij} \in \{0,1\}$ $\forall i \in S, j \in J$

The model returns the optimal assignment of the storage tanks and these are displayed in the table below. We recommend to the management of using this specific assignment. This assignment is the optimal assignment given the constraints and will thus result in the lowest possible costs.

| Substance | Capacity of storage tank |
|---|---|
| A | 500 |
| B | 400 |
| C | 700 |

In the tables below the optimal policy for each substance can be found, taking into account the re-assignation of the storage tanks as stated above.

| **A.** Period *n* | Demand | Optimal order quantity | Inventory level | Costs per period |
|---|---|---|---|---|
| 0 | 0 | 0 | 0 | 0 |
| 1 | 60 | 345 | 285 | 2067.50 |
| 2 | 115 | 0 | 170 | 85 |
| 3 | 90 | 0 | 80 | 40 |
| 4 | 80 | 0 | 0 | 0 |
| 5 | 110 | 370 | 260 | 2180 |
| 6 | 200 | 0 | 60 | 30 |
| 7 | 40 | 0 | 20 | 10 |
| 8 | 20 | 0 | 0 | 0 |

| **B.** Period *n* | Demand | Optimal order | Inventory level | Costs per period |
|---|---|---|---|---|

| | | | | |
|---|---|---|---|---|
| | | quantity | | |
| 0 | | 0 | 0 | 0 |
| 1 | 20 | 115 | 95 | 1196 |
| 2 | 38.33 | 0 | 56.67 | 45.33 |
| 3 | 30 | 0 | 26.67 | 21.33 |
| 4 | 26.67 | 0 | 0 | 0 |
| 5 | 36.67 | 123.33 | 86.67 | 1256 |
| 6 | 66.67 | 0 | 20 | 16 |
| 7 | 13.33 | 0 | 6.67 | 5.33 |
| 8 | 6.67 | 0 | 0 | 0 |

| **C.** Period *n* | Demand | Optimal order quantity | Inventory level | Costs per period |
|---|---|---|---|---|
| 0 | | 0 | 0 | 0 |
| 1 | 120 | 350 | 230 | 2811 |
| 2 | 230 | 0 | 0 | 0 |
| 3 | 180 | 340 | 160 | 2692 |
| 4 | 160 | 0 | 0 | 0 |
| 5 | 220 | 220 | 0 | 1740 |
| 6 | 400 | 520 | 120 | 3924 |
| 7 | 80 | 0 | 40 | 28 |
| 8 | 40 | 0 | 0 | 0 |

**Substance D**

In the last case, the optimal policy for substance D is found. Substance D suffers from quality deterioration and has two suppliers. The quality deterioration from supplier X is 2 periods and from supplier Y is 1 period. Furthermore, all replenishment orders in horizon T would be placed exclusively to the supplier selected. Both suppliers offer the substance at purchasing cost of cD = \$5 per liter and the holding cost is h = \$0.5 per liter per period. The fixed replenishment cost of Supplier X is \$500, and that of Supplier Y is \$400 and the storage tank capacity is 350 liters. Lastly, demand for substance D is:

Demand substance D = [60, 115, 90, 80, 110, 200, 40 , 20]

To solve this MIP two separate models are constructed and solved, one for supplier X and one for supplier Y. Again in this case it is assumed that the safety requirements are not needed anymore.

Model Supplier X:

$$\min \sum_{N=1}^{T} (K * y_N + H * I_N)$$

s.t.

(1) $I_0 = 0$

(2) $I_1 = I_0 + Q_1 - D_1$

(3) $I_N = Q_{N-1} - D_{N-1} + Q_N - D_N$ $\quad \forall$ N = 2, …, T

(4) $I_{N-1} + Q_N \leq$ Capacity $\quad \forall$ N = 1,…, T

(5) $Q_N \leq M * y_N$ $\quad \forall$ N = 1,…, T

(6) $y_N \in \{0,1\}$ $\quad \forall$ N = 1,…, T

(7) $Q_N, I_N \geq 0$ $\quad \forall$ N = 1,…, T

Model Supplier Y:

$$\min \sum_{N=1}^{T} (K * y_N + H * I_N)$$

s.t.

(1) $I_0 = 0$

(2) $I_N = Q_N - D_N$ $\quad \forall$ N = 1, …, T

(3) $I_{N-1} + Q_N \leq$ Capacity $\quad \forall$ N = 1,…, T

(4) $Q_N \leq M * y_N$ $\quad \forall$ N = 1,…, T

(5) $y_N \in \{0,1\}$ $\quad \forall$ N = 1,…, T

(6) $Q_N, I_N \geq 0$ $\quad \forall$ N = 1,…, T

In the table below the optimal policy for supplier X and Y can be found. The total cost of the optimal policy of supplier X is 6875, and for supplier Y it is 6775. This implies that supplier Y is the best option in terms of costs. Furthermore, for supplier Y it can be observed that no inventory is held. This is the case because any purchases at the beginning of the period, that are not consumed, must be thrown at the beginning of the next period.

| **D.** Period *n* | Demand | Supplier X | | | Supplier Y | | |
|---|---|---|---|---|---|---|---|
| | | Optimal order quantity | Inventory level | Costs per period | Optimal order quantity | Inventory level | Costs per period |
| 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| 1 | 60 | 175 | 115 | 1432.50 | 60 | 0 | 700 |
| 2 | 115 | 0 | 0 | 0 | 115 | 0 | 975 |
| 3 | 90 | 205 | 0 | 1525 | 90 | 0 | 850 |
| 4 | 80 | 0 | 35 | 17.50 | 80 | 0 | 800 |
| 5 | 110 | 310 | 120 | 2110 | 110 | 0 | 950 |
| 6 | 200 | 0 | 0 | 0 | 200 | 0 | 1400 |
| 7 | 40 | 240 | 0 | 1700 | 40 | 0 | 600 |
| 8 | 20 | 0 | 180 | 90 | 20 | 0 | 500 |