Group 2: Assignment 1.5

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1 E-Level Problem (1.5)

Let it be so that:

$$k = \text{Number of infected computers}$$

$$X = \text{Number of computers infected out of a pool of 10 randomly selected computers}$$

The boss claims that $P(X \geq 1 | k \geq 0.1n) \geq 9/10$. Thus, to be able to either prove or refute the boss's claim, the probability $P(X = 0 | k \geq 0.1n)$ of not selecting an infected computer when choosing 10 at random (provided that at least 10 percent of the computers are infected) must be calculated. The probability of $P(X = 0 | k \geq 0.1n)$ is given as

$$P(X = 0 | k \geq 0.1n) = \prod_{i=0}^{9} \frac{(n-i) - k}{n-i}$$

That is due to the fact that we are choosing 10 computers without replacement. Therefore, the number of possible options decreases with every selection (given by $n-i$) but the number of infected computers (given by k) remains constant. It allows for the following estimation

$$\begin{aligned} P(X = 0 | k \geq 0.1n) &= \prod_{i=0}^{9} \frac{(n-i) - k}{n-i} \overbrace{\geq}^{k \geq 0.1n} \left(\frac{0.9n - 9}{n} \right)^{10} \\ &= \left(0.9 - \frac{9}{n} \right)^{10} \underbrace{>}_{n > 1000} \left(0.9 - \frac{9}{1000} \right)^{10} > 0.3153 \end{aligned}$$

It follows that

$$P(X \geq 1 | k \geq 0.1n) = 1 - \underbrace{P(X = 0 | k \geq 0.1n)}_{> 0.3153} < 0.6847 \overset{!}{<} \frac{9}{10}$$

That refutes the claim.