Delft University of Technology

Optimisation in Systems and Control

SC42055

Linear Programming

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1 Formulating the problem

The optimisation can be formulated as a maximisation problem. The function that has to be maximised is related to the profit of the company. Furthermore, there are some limitations that have to be taken into account related to the amount of batteries that can be produced, the amount of available working hours and the amount of storage space available. These requirements can be seen below. Note that the salary is not included in f because it is a constant. Furthermore, because of physical limitations, the amount of cars produced should be greater or equal to 0.

$$f = 55,000 \cdot R + 75,000 \cdot W - 30,000 \cdot R - 45,000 \cdot W \tag{1}$$

$$R \cdot 4 \cdot 10^3 + W \cdot 6 \cdot 10^3 \leq 15,000,000 \tag{2}$$

$$R \cdot 10 + W \cdot 15 \leq 160 \cdot 115 \tag{3}$$

$$R \cdot 10 + W \cdot 12 \leq 26 \cdot 10^3 \tag{4}$$

$$-R \leq 0 \tag{5}$$

$$-W \leq 0 \tag{6}$$

The problem can be written as: find x such that $f^T x$ is minimal with $Ax \leq b$. In this problem the matrices A, b and f are then: A = $\begin{bmatrix} 4 \cdot 10^3 & 6 \cdot 10^3 \\ 10 & 15 \\ 10 & 12 \\ -1 & 0 \\ 0 & -1 \end{bmatrix}$, b = $\begin{bmatrix} 15 \cdot 10^6 \\ 160 \cdot 115 \\ 26 \cdot 10^3 \\ 0 \\ 0 \end{bmatrix}$ and f = $\begin{bmatrix} -25,000 & -30,000 \end{bmatrix}$

2 Optimal Solution

The optimal solution to the problem can be found with Matlab™ by plugging in the equations written in section 1. This optimal solution is found to be $R = 1840$ and $W = 0$. This is logical because the profit per worked hour is larger for R than for W. One can then maximise the profit by producing as much of model R as possible, this also makes the amount of hours worked the limiting factor.

3 Optimal Solution with a Changed Market

When the market situation changes due to a problem with a batch of model R, the linear programming changes because the amount of model R cars that can be sold is limited to 1000 per month. One can just add a row to the matrices A and b to include this requirement. The new optimal solution is then to produce 1000 cars of model R and 560 cars of model W.

4 Further Optimisation of the Manufacturing Process

The manufacturing process can be optimised further by marginal production and some insights. Let M be the amount of workers. The new optimisation problem is then shown below. Note that now that M is not a constant anymore, it has to be added to f.

$$f = 55,000 \cdot R + 75,000 \cdot W - 30,000 \cdot R - 45,000 \cdot W - 3550 \cdot M \tag{7}$$

$$R \cdot 4 \cdot 10^3 + W \cdot 6 \cdot 10^3 \leq 18 \cdot 10^6 \tag{8}$$

$$R \cdot (10 - (M - 115)/12) + W \cdot (15 - (M - 115)/12) \leq 160 \cdot M \tag{9}$$

$$R \cdot 10 + W \cdot 12 \leq 33 \cdot 10^3 \tag{10}$$

$$R \leq 1000 \tag{11}$$

$$M \geq 115 \tag{12}$$

$$M \leq 115 + 72 \tag{13}$$

One can see that if the variable M is introduce, the problem can not be expressed anymore as a LP problem because there is a multiplication of two variables.

5 Optimal Solution of Workers, Models R and Models W

The optimal solution is now not so straightforward to find anymore, one can however note that by fixing one variable (say M), the problem becomes linear again and can be expressed as a LP problem again which can be solved. The answer is thus to go through the 72 possible extra workers and see where the profit is maximised. This is done by using a for loop to find the marginal new employee. The optimal solution is then to produce 1000 cars of model R, 1916 cars of model W and to hire 51 extra employees, for a total of 166 employees.

6 To Build Model V or not to Build Model V, that is the Question

Because of the contract, the success of the model and a lack of skilled workers, the situation changes again. The function one wants to optimise then becomes:

$$f = 25,000 \cdot R + 30,000 \cdot W + 30,000 \cdot V \tag{14}$$

With new requirements:

$$-R \leq -1250 \tag{15}$$

$$-W \leq -1000 \tag{16}$$

$$-V \leq -1500 \tag{17}$$

Taking into account the requirements, one can then see what the optimal solution is for this situation and look at the profit. If it turns out that the profit is greater, the contract requirements are still met and there is a minimum of 1500 cars of model V produced it will turn out to be beneficial to produce the model V. Vice versa, if the contract requirements are not met, if the profit is not higher or if there are less than 1500 cars of model V produced, it is not economically beneficial to produce the model V. Plugging in the numbers and the new constraints, one can quickly see that there is no feasible solution. It is hence not economically beneficial to start producing the model V. If V is not produced, the optimal solution is to produce 2100 cars of model R and 1000 cars of model W. The matlab code actually shows that 999 cars of model W should be produced. However, upon further inspection, this is due to the use of the floor function in matlab in combination with the linprog function. These round numbers off differently. The actual production of model W cars without rounding would be 999.9999999999998. This is sufficiently close enough to 1000 to considerd a rounding error.