Quadratic Programming Assignment

SC42055 Optimization in Systems and Control

E_1, E_2, and E_3 are parameters changing from 0 to 18 for each group according to the sum of the last three numbers of the student IDs:

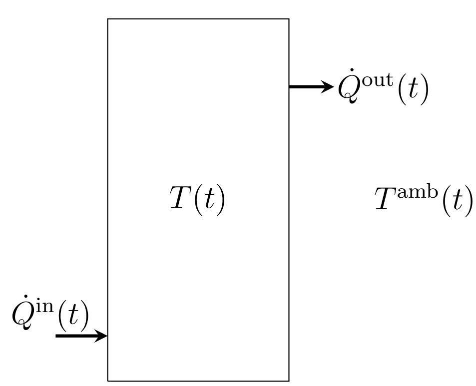
$$E_1 = D_{a,1} + D_{b,1}, \quad E_2 = D_{a,2} + D_{b,2}, \quad E_3 = D_{a,3} + D_{b,3},$$

where $D_{a,3}$ is the right-most digit of one student and $D_{b,3}$ is the right-most digit of the other student.

Traditionally, human beings have relied on fossil fuels to satisfy their energy demands. Fossil fuels produce carbon emissions to the atmosphere that are endangering our environment. In an effort to tackle some of these problems, a revolution has been taking place: the energy transition. In this transformation, to reduce the amount of carbon emissions to the atmosphere, energy systems are transitioning to *renewable energy sources (RES)*. In this process, as the generation of RES is highly intermittent and unpredictable, energy storage has become very important. As nearly 80% of the energy consumption in EU households is destined to water and space heating, heat storage has become paramount to improve the efficiency of RES generation. In particular, by storing electricity as heat energy when there is an excess of RES, and then deploying this heat when there is a shortage of RES, the efficiency of RES systems can be improved.

Heat Tanks is a company that has multiple heat storage tanks and wants to optimize the trade of electricity and heat in order to maximize their profits. They hire you as their lead optimization engineer in order to carry out this task.

After reading the documentation of their systems, you realize that each of these tanks can be modeled by one state $T(t)$ representing the internal temperature in the tank at time instant t, two controls $\dot{Q}^{\text{in}}(t)$ and $\dot{Q}^{\text{out}}(t)$ representing the input and output heat to the tank, and one external disturbance $T^{\text{amb}}(t)$ representing the ambient temperature:



In addition, you derive the equation of the system dynamics as the following ordinary differential equation:

$$\frac{dT(t)}{dt} = a_1\left(T^{\text{amb}}(t) - T(t)\right) + a_2\left(\dot{Q}^{\text{in}}(t) - \dot{Q}^{\text{out}}(t)\right) \tag{1}$$

where a_1 and a_2 are the system parameters, and where $\dot{Q}^{\text{in}}(t), \dot{Q}^{\text{out}}(t) > 0$.

1. As a first task to perform the requested optimization, you are asked to identify the model parameters. To do so, you are provided with real measurements T_k, $\dot{Q}_k^{\text{out}}$, $\dot{Q}_k^{\text{in}}$, T_k^{amb} at discrete time points t_k that are spaced at intervals $\Delta t = t_{k+1} - t_k = 3600\,\text{s}$. To identify the parameters, transform first the continuous-time model defined by (1) into a linear discrete-time version. To do so, use the following approximation:

$$\frac{dT_k}{dt} \approx \frac{T_{k+1} - T_k}{\Delta t}, \tag{2}$$

where T_k represents the internal temperature in the tank at time step t_k. The resulting model has to be of the form of:

$$T_{k+1} = A\,T_k + B\,[\dot{Q}_k^{\text{out}}, \dot{Q}_k^{\text{in}}]^\top + c_k. \tag{3}$$

Provide the values of A, B, and c_k as a function of a_1, a_2, and T_k^{amb}.

2. After deriving the discrete-time model, you are ready to identify the model parameters. To do so, download the file *measurements.csv* from Brightspace containing measurements of $\bar{T}_k$ [K], $\bar{\dot{Q}}_k^{\text{out}}$ [W], $\bar{\dot{Q}}_k^{\text{in}}$ [W], and $\bar{T}_k^{\text{amb}}$ [K]. To read the file, use the MATLAB function `readtable`. Then, formulate the following quadratic optimization problem:

$$\underset{a_1,\,a_2}{\text{minimize}} \quad \sum_{k=1}^{100+E_1} \left(\bar{T}_{k+1} - (A\,\bar{T}_k + B\,[\bar{\dot{Q}}_k^{\text{out}}, \bar{\dot{Q}}_k^{\text{in}}]^\top + c_k)\right)^2 \tag{4}$$

and solve it in `Matlab`. Please note that the number of measurements used for this identification depends on your number of student ID: as defined in (4), the number of measurements used is different for each student and equal to $101 + E1$.

3. Finally, you are ready to optimize the energy trade of one of the tanks. To optimize the hourly energy trade over some horizon N, you need to minimize the cost of buying the input energy $\dot{Q}^{\text{in}}$ over that horizon while satisfying the requested heat demand. In this optimization setup, you need to also add a constraint that ensures that the system dynamics are validated throughout the horizon. This optimization problem can be defined as:

$$\begin{aligned}
\underset{\substack{T_2,\ldots,T_{N+1},\\ \dot{Q}_1^{\text{in}},\ldots,\dot{Q}_N^{\text{in}}}}{\text{minimize}} \quad & \sum_{k=1}^{N} \lambda_k^{\text{in}}\, \dot{Q}_k^{\text{in}}\, \Delta t && && \text{(5a)}\\
\text{subject to} \quad & T_{k+1} = A\,T_k + B\,[\dot{Q}_k^{\text{out}}, \dot{Q}_k^{\text{in}}]^\top + c_k, && k = 1,\ldots,N, && \text{(5b)}\\
& 0 \le \dot{Q}_k^{\text{in}} \le \dot{Q}_{\max}^{\text{in}} && k = 1,\ldots,N, && \text{(5c)}\\
& T^{\min} \le T_k && k = 2,\ldots,N+1 && \text{(5d)}
\end{aligned}$$

where λ_k^{in} is the price of buying one unit of input heat at time step k, $\dot{Q}_{\max}^{\text{in}}$ the maximum input power, $T^{\min}$ the minimum internal temperature to ensure the heat demand, and where T_1, T_{amb}, $\dot{Q}_k^{\text{out}}$, and λ_k^{in} are typically given in advance.

For our application, download the $\dot{Q}_k^{\text{out}}$ [W] and λ_k^{in} [€/MWh] values from the *heatDemand.csv* and *inputPrices.csv* files in Brightspace. To read the files, use again the

MATLAB function `readtable`. Then, consider a horizon of 15 days, i.e. $N = 360$, and assume that $\dot{Q}^{\text{in}}_{\text{max}} = 100 + E_2\,\text{kW}$, $T_1 = 330 + E_3\,\text{K}$, $T_{\text{amb}} = 275 + E_1\,\text{K}$, and $T^{\text{min}} = 315\,\text{K}$. In addition, instead of the estimated parameter values in the previous section, use $a_1 = 1.96 \times 10^{-7}$ and $a_2 = 3.80 \times 10^{-9}$. Finally, solve the above problem and answer the following questions:

(a) Is the problem quadratic? Justify.

(b) Transform the prices to the corrects units so that they can be used in the optimization problem. Which factor did you used to multiply them by?

(c) What is the optimal cost of buying the input energy?

4. After solving the optimization problem above, Heat Tanks tells you that the computed solution is not feasible: you have forgotten to constrain the temperature in the tank to be within an upper safety bound. In particular, they indicate that the internal temperature in the tank is to be upper bounded by 368 K. In addition, they tell you that there is a terminal cost if the final internal temperature is above or below a certain reference temperature T^{ref}. This reference temperature represents the lowest possible temperature at the end of the horizon that can still satisfy the heat demand afterwards. As such, the associated terminal cost represents two economic concepts:

   - If the final temperature is lower than the reference temperature, it represents the extra cost to be paid to bring the tank to the temperature needed to satisfy the demand.
   - If the final temperature is higher than the reference temperature, it represents the missing opportunist of not having used the extra added energy for some other task.

   At the current time, this cost is $(0.1 + \frac{E_2}{10})\,\frac{\text{euros}}{\text{K}^2}$ times the squared error between the final temperature T_{N+1} and the reference temperature $T^{\text{ref}} = 323\,\text{K}$.

   (a) Modify (5) to include the temperature constraint and the extra cost. The resulting problem should be a quadratic problem.

   (b) Solve it in `Matlab` using `quadprog`. What is the total optimal cost? How much of that is destined to pay the terminal cost?

The written report on the practical exercise should be uploaded to Brightspace before Wednesday, October 17, 2018 at 17.00 p.m. as one pdf file. The MATLAB code used should also be uploaded to Brightspace as one .m file; please make sure that the code is error free. Please also note that you will lose 0.5 point from your grade on the report for each (started) day of delay in case you exceed the deadline.