Delft University of Technology

Optimisation in Systems and Control
SC42055

Quadratic Programming

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1 Building the Model

Before we can optimise the energy trade of the tanks, it is important to first have an accurate model that describes the system well enough. This is done by the model given in Equation 1 with Q being a vector containing the heat input and output power. By using a linearisation with a time step dt, one can rewrite the model with A, B and c_k being expressed in terms of a1, a2 and T_{amb}. The resulting expressions for them are listed below.

$$T_{k+1} = AT_k + BQ + c_k \tag{1}$$

$$A = -a1 \cdot dt \tag{2}$$
$$B = [-a2 \cdot dt, a2 \cdot dt] \tag{3}$$
$$c_k = T_k + a1 \cdot dt \cdot T_{amb} \tag{4}$$

Note that in the equations listed, a1 and a2 are still undefined. Since the model fit needs to be as good as possible, optimisation is needed. This is done by solving a quadratic programming method and computing the a1 and a2 for which the error squared is as small as possible. By filling in the expressions obtained for A, B and c_k and squaring the term, one can derive a Hessian matrix and an f matrix to solve the problem numerically. The numerical representation of the matrices H and f are shown below H $= \begin{bmatrix} -1.7025 & -0.0001 \\ -0.0001 & 1.3210 \end{bmatrix} \cdot 10^{17}$ and f $= \begin{bmatrix} 1.1226 \\ 0 \end{bmatrix} \cdot 10^{13}$
Optimising for the least squares solution then yields values $a1 = -3.77 \cdot 10^{-12}$ and $a2 = 3.77 \cdot 10^{-9}$. A comparison of the model with the actual measured data is shown in Figure 1.

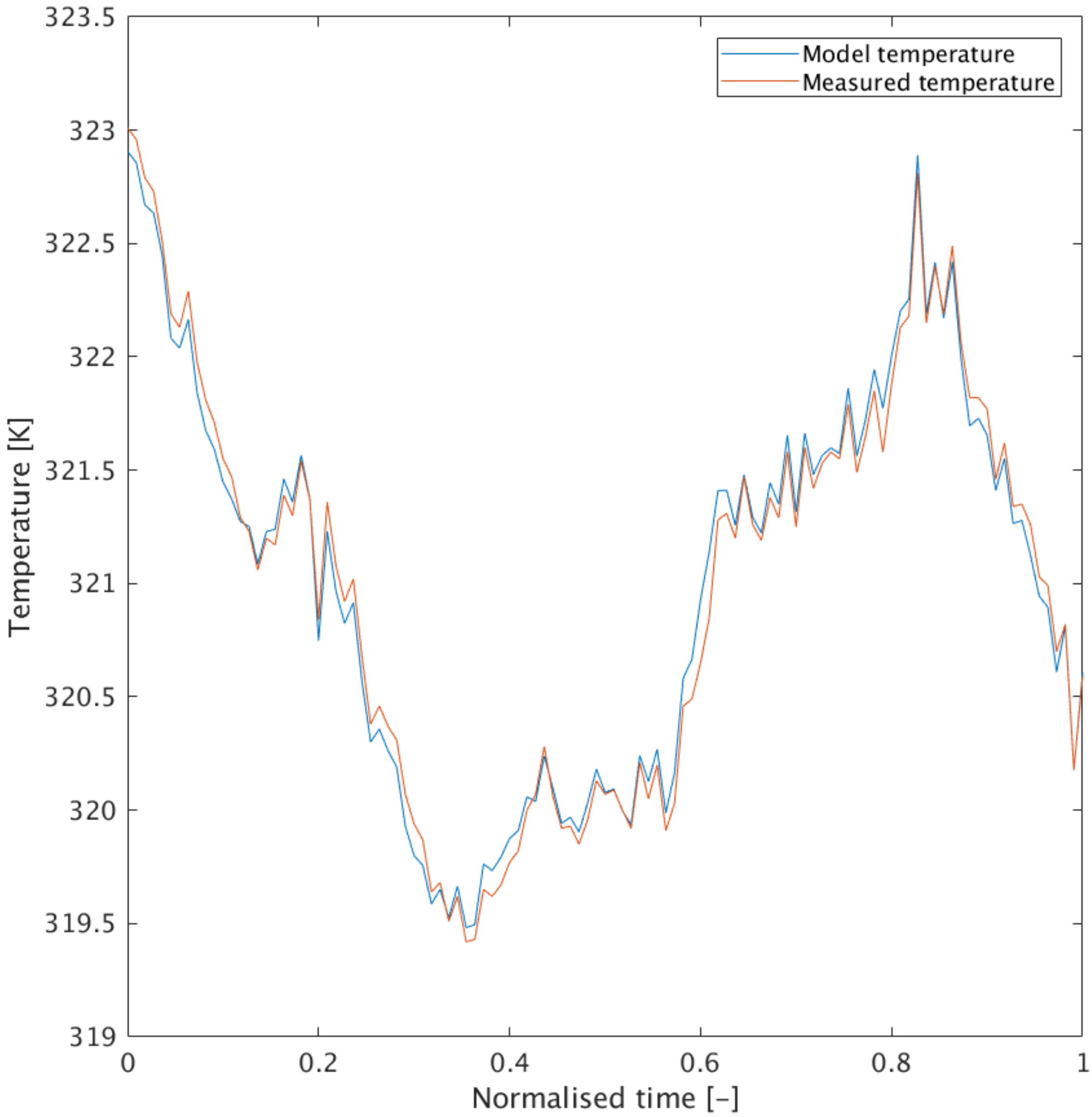

Figure 1: This figure shows a comparison of the temperature that is measured and the temperature that is predicted at each time step by the model. Note the relatively small scale on the temperature axis.

2 Optimising the Energy Trade

Now that the model has been built, it can be used to optimise the energy trade that is done by the company. Since the total cost has to be minimised and that is just a sum of products, the problem is linear and not quadratic. Hence it can be solved by using the Matlab^{TM} linprog function when $Ax \leq b$ and f are known. Thinking of and constructing the needed matrices is the hardest part of this optimisation. Because both Q_k^{in} and the temperature T_k are variables, they are both put in the x matrix. This matrix will have 720 rows (360 for both Q_k^{in} and T_k). Constructing the A matrix depends on the constraints of the problem. Using the constraints given in the exercise. The A matrix is a 1080×721 matrix consisting of four matrix blocks. The first block will check if the $T_{k+1} \leq T_{min}$, it will therefore consist of a block of $(N) \times (N+1)$ zeros and an $N \times N$ identity matrix. The next two blocks will check on the energy input. They will check if Q_k^{in} is bounded by 0 and Q_{max}^{in}. This is done by using an $N \times N$ identity matrix followed by an $(N) \times (N+1)$ block of zeros to check the maximum energy input. The minimum energy input has to be greater than or equal to 0 and therefore the next block will consist of a negative $N \times N$ identity matrix followed by an $(N) \times (N + 1)$ block of zeros. The corresponding b vector is an 1080×1 vector consisting of $N \times 1$ entries of $-T_{min}$, $N \times 1$ entries of Q^{in}_{max} and $N \times 1$ zeros. To optimise for the total cost, the prices have to be scaled down such that they can be used. The scaling factor used for this is $1/(3600 \cdot 10^6)$. That is, the prices are converted from EUR/MWh to EUR/J. Optimising for the cost of buying then yields a buying cost of 119.92 EUR.

3 Optimising the Energy Trade Revisited

To improve the previous optimisation, a new constraint has to be added to the set of constraints. This constraint is shown below.

$$T_k \leq 368K \tag{5}$$

To check if this constraint is met, a new block will be added to the A matrix. This block consists of $(N)\times(N+1)$ zeros and an $N \times N$ identity matrix. A new block of $N \times 1$ entries of T^{max}_k has to be added at the end of the b vector.

Furthermore, a new minimisation problem is obtained because we want to minimise the terminal cost at the end of the time horizon.

$$min(\sum_{k=1}^{N} \lambda^{in}_k \cdot Q^{in}_k \cdot \Delta t) + (T_{N+1} - T_{ref})^2 \cdot 1.6 \tag{6}$$

Solving this quadratic programming problem with Matlab then yields a total optimal cost of 135.02 EUR, of which 0.67 EUR is the terminal cost. This terminal cost corresponds to a very small temperature difference of 0.65 K.