

MIXING BY STIRRING OF A STRATIFIED FLUID

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PURPOSE

- (1) Explore the dependence of the turbulent tracer diffusivity on the stratification of the fluid
- (2) Quantify the mixing efficiency for different stratifications

INTRODUCTION

Upper ocean mixing affects many processes, including horizontal circulation, meridional overturning circulation, and heat transport (Gregg *et al.*, 2018). When stably stratified geophysical flows are mixed, turbulence is dissipated, and mixing is generally inefficient. In ocean models, turbulent fluxes are parameterized by diascalar diffusivities, such as K_ρ , which is the turbulent diffusivity for mass and is perpendicular to isopycnals. Estimating K_ρ is challenging. One method to estimate K_ρ is to observe shear variance ϵ . However, obtaining K_ρ from ϵ depends on mixing efficiency. In Gregg *et al.* (2018), Mixing efficiency is defined as:

$$\text{Mixing efficiency} = \frac{\text{Change in the background potential energy due to mixing}}{\text{Energy expended}}$$

The balance of terms is between turbulence production (work), dissipation, and buoyancy loss.

$$\text{Work done by stirring} = (\text{irreversible mixing}) + (\text{dissipation into heat})$$

Osborn (1980) predicted K_ρ upper bound, where N^2 is squared buoyancy frequency:

$$K_\rho \leq 0.2 \frac{\epsilon}{N^2}$$

The upper bound of the mixing efficiency $\Gamma \approx 0.2$. Moreover, $\Gamma \sim 0.2$ as the upper bound over a range of turbulent processes and Reynolds numbers but not all (Barry *et al.*, 2001; Osborn, 1980). $\Gamma = 0.2$ holds true when $10 < \frac{\epsilon}{N^2} < 1000$ (Barry *et al.*, 2001). $\frac{\epsilon}{N^2} > 1000$, the mixing efficiency diverges from the findings from Osborn (1980). These results were criticized, and generally previous studies estimate Γ to be between 0.1 and 0.35 (Gregg *et al.*, 2018).

Our goal is to quantify the dynamics of density-stratified fluids subjected to turbulent stirring and test whether the mixing efficiency Γ is ~ 0.2 .

METHODS

Laboratory set-up

The experimental set-up consisted of 4 beakers, each filled initially with 250 mL dense salty dyed water layer at the bottom and 250 mL less dense distilled water at the top. The salty dyed water density water layers varied in density in each beaker (1020 kg/m³, 1060 kg/m³, 1130 kg/m³ and 1160 kg/m³). First, each beaker was filled with distilled water. Then, salty dyed water was slowly injected at the bottom of the beaker using a syringe. This created two distinct water layers.

We expect the density profile then looks like a Heaviside step function. The fluid in each beaker was manually stirred using a mixing rod contraption. The duration of stirring was recorded using a phone camera to be analyzed by frame computationally.

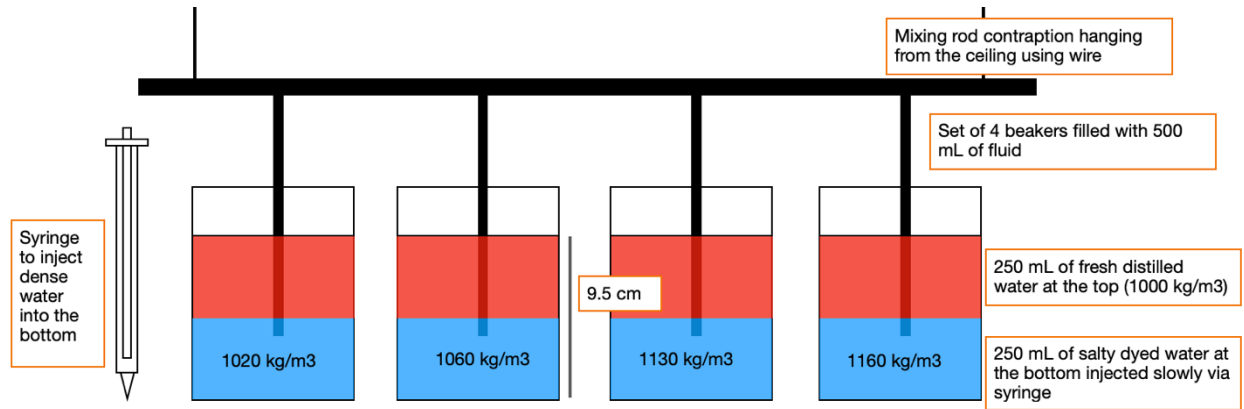


Figure 1. Experimental set-up

Computation set-up

The video frames were straightened, converted to grayscale, and cropped to avoid the glass glare. Pixel intensity profiles with depth were created, allowing for pixel intensity to be a function of time.

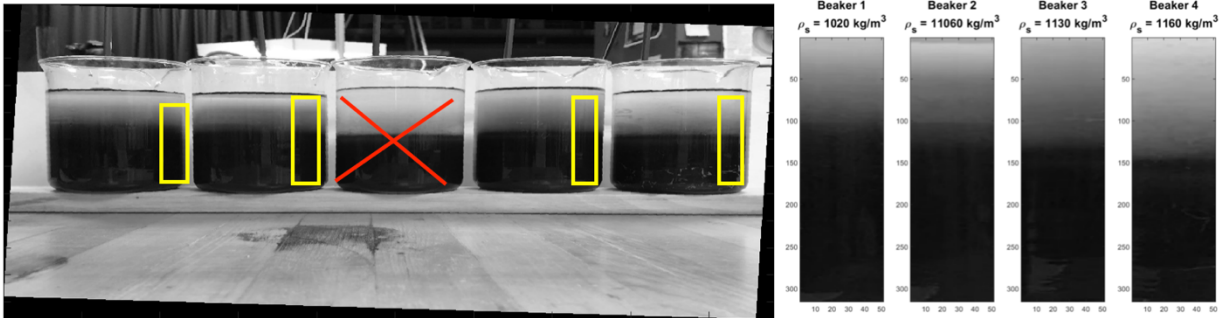


Figure 2. Computation set-up.

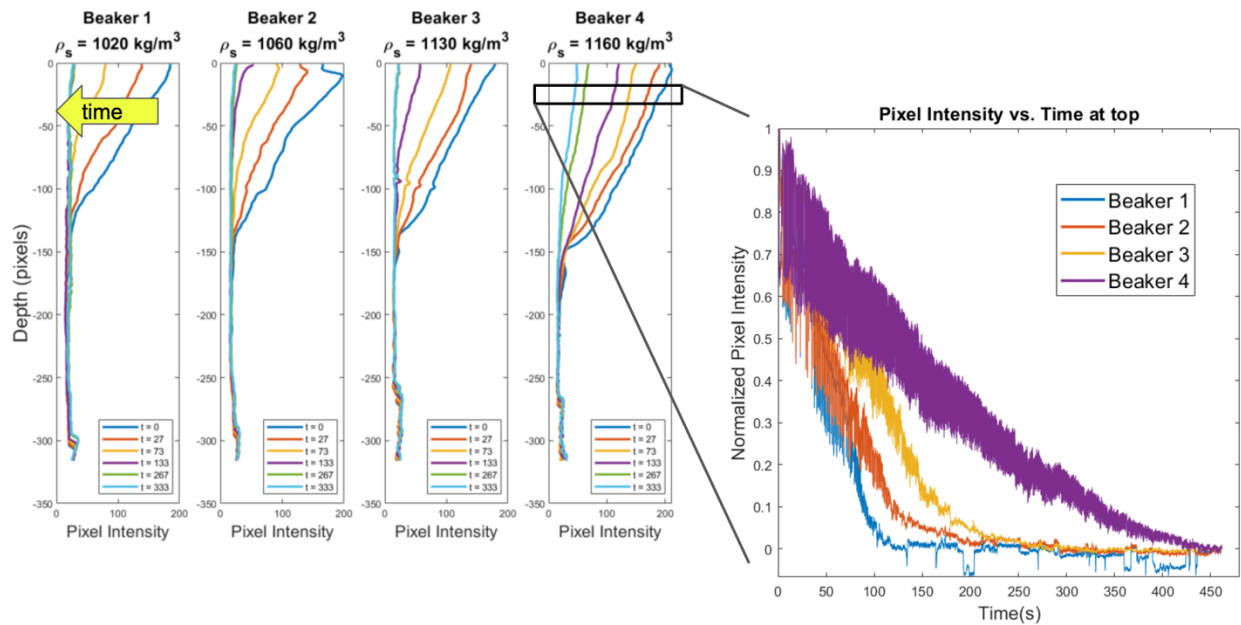


Figure 3. Data from computation set-up. Pixel intensity at the top tends to become darker as the fluid is increasingly mixed. However, pixel intensity at the bottom does not tend to get lighter with

more mixing. This means that too much dye was used to create the dense water. Pixel intensity vs. time is noisy due to the stirring rod.

Quantative set-up

In stratified geophysical flows, we are interested in K_p :

$$K_p = \frac{\text{flux}}{\text{gradient}} = \frac{\Gamma}{\rho_{\text{mean}} H} \cdot \frac{\varepsilon}{N^2} = \frac{\Gamma}{\rho_{\text{mean}} H \cdot N^2} \left(W_o - \frac{\Delta GPE}{\Delta T} \right)$$

$$K_p N^2 = \frac{\Gamma}{\rho_{\text{mean}} H} W_o - \frac{1}{\rho_{\text{mean}} H} \Gamma \frac{\Delta GPE}{\Delta T}$$

H is depth of the fluid. The squared buoyancy frequency term is:

$$N^2 = - \left(\frac{g}{\rho_0} \right) \left(\frac{d\bar{\rho}}{dz} \right) = - \frac{g}{\rho_{\text{mean}}} \left(\frac{\rho_f - \rho_s}{H} \right)$$

The change in gravitation potential energy is:

$$\Delta GPE = GPE_{\text{mixed}} - GPE_{\text{unmixed}} = \frac{1}{4} (\rho_s + \rho_f) g H^2 - \frac{1}{8} \rho_s g H^2 - \frac{3}{8} \rho_f g H^2$$

$$= \frac{1}{8} g H^2 (2\rho_s + 2\rho_f - \rho_s - 3\rho_f) = \frac{1}{8} (\rho_s - \rho_f) g H^2 = \frac{1}{8} \Delta \rho g H^2$$

$$\frac{\Delta GPE}{\Delta T} \approx \left(1 - \frac{1}{e} \right) \Delta GPE \frac{1}{t_{\text{mixed}}}$$

We then fit the exponential decay curve to pixel intensity at the top of the fluid to estimate the mixing timescales.

$$\rho_{\text{top}} \propto e^{-\frac{t}{t_{\text{mixed}}}}$$

Using a diffusion simulation with an initial step function profile and no flux boundary conditions, we find the following relation between the mixing timescale and diffusivity:

$$K_p = c \frac{H^2}{t_{\text{mixed}}}$$

where c is a constant that depends on the initial configuration and boundary conditions. In this experiment, $c \approx 0.11$.

RESULTS

Beaker	$\rho_{\text{salty}} \left(\frac{\text{kg}}{\text{m}^3} \right)$	$K_p \left(\frac{\text{m}^2}{\text{s}} \right)$
1	1020	1.35×10^{-5}
2	1060	1.01×10^{-5}
3	1130	7.09×10^{-6}
4	1160	3.76×10^{-6}

Table 1. (Left) K_p for each beaker.

After fitting exponential curves to pixel intensity at the top of the beaker and estimating t_{mixed} for each beaker, we estimated K_p . As the density difference increases, diffusivity decreases (**Fig. 4a**).

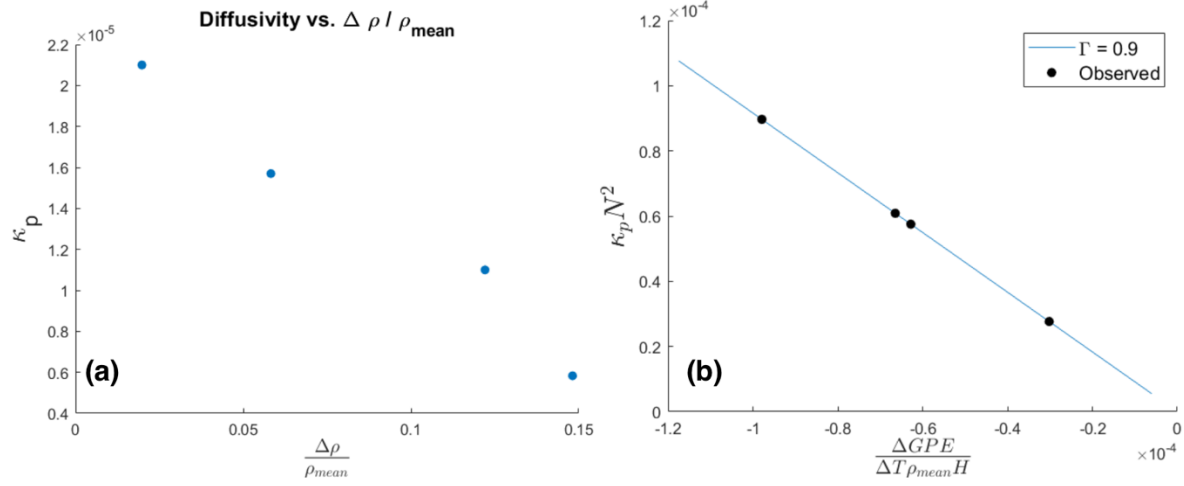


Figure 4. (a) Scatterplot of diffusivity vs. the change in density over the mean density, (b) using a linear fit to solve for Γ

$$\kappa_\rho N^2 = \Gamma \frac{\Delta GPE}{\Delta T \rho_{\text{mean}} H}$$

Using a linear fit to solve for Γ , we determine $\Gamma = 0.9$.

DISCUSSION AND CONCLUSION

$\Gamma = 0.9$ is a physical result as it is < 1 and > 0 . However, one cause for concern is that all the points fall on the linear fit line. The linear fit in Figure 4b is perfect ($r = 1$). As a sanity check, we check

$$\begin{aligned} \kappa_\rho N^2 &\propto \Gamma \frac{\Delta GPE}{\Delta T \rho_{\text{mean}} H} \\ c \frac{H_{\text{cropped}}^2}{t_{\text{mixed}}} \frac{g}{\rho_{\text{mean}}} \frac{2\Delta\rho}{H_{\text{beaker}}} &= \Gamma \frac{1}{8} g \Delta \frac{\rho H_{\text{beaker}}^2}{t_{\text{mixed}}} \frac{1}{\rho_{\text{mean}} H_{\text{beaker}}} \\ \Gamma &= 16c \frac{H_{\text{cropped}}^2}{H_{\text{beaker}}^2} \end{aligned}$$

This means that the way we computed Γ is independent of our data.

The main limitation is that the color intensity of the dye observed by the camera has a non-linear relationship to mass density. We expected the bottom layer to become slightly lighter in color after mixing. However, the entire fluid was saturated (black). To improve our experiment, we can compute $\rho(z)$ if we have $P(z)$, where P = pixel intensity. By estimating $P(z)$ by assuming an initial density profile:

$$\rho(z) = \rho_0 + \frac{\Delta\rho}{2} \left(\tanh\left(\frac{z - \frac{H}{2}}{\delta}\right) + 1 \right)$$

Following through with this improvement of mapping between pixel density and mass density, we computed the calibration curve to determine the calibrated density (**Fig. 5a and 5b**). Using the

calibrating density, we calculate GPE as a function of time (**Fig. 5c**). As the fluid is mixed, GPE increases. From this, we find $\frac{\Delta GPE}{\Delta T}$ for each beaker.

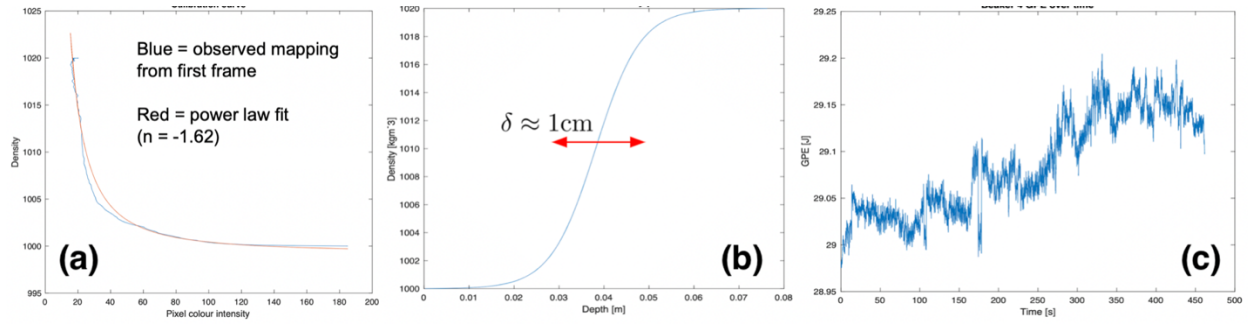
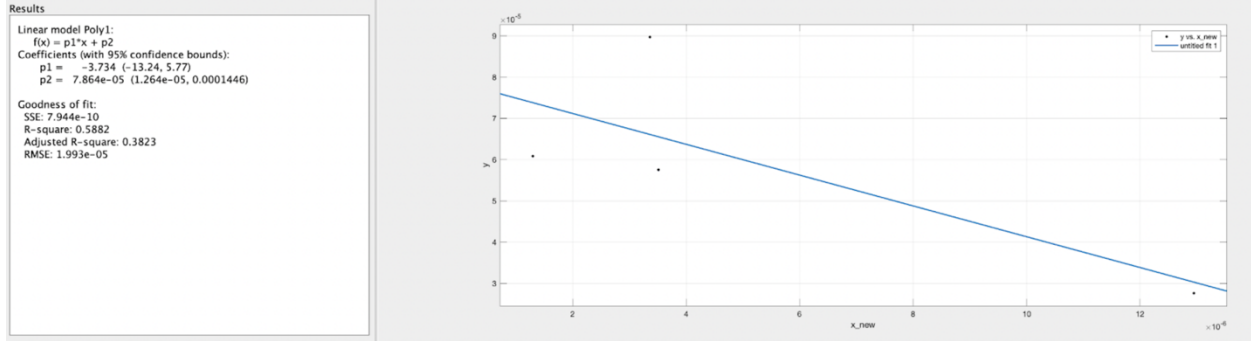


Figure 5 (a) Calibration curve using pixel intensity and intensity (b) assumed initial density profile (c) Recalculated GPE for Beaker 4 (this was repeated for each of the beakers)

With this new $\frac{\Delta GPE}{\Delta T}$, we find a new $\Gamma = 3.734$ with a wide range of upper and lower bounds. Moreover, this new Γ is physically impossible. $\Gamma = 0.2$ is within the upper and lower bound ranges.



However, this means that we would be using our experiment data.

Another limitation is that we assumed the work done by the rod is the same for each beaker. However, the stirring rod contraption was not producing the same amount of turbulence as stirring was manual as well the contraption is rather fragile to disturbance. To improve this set-up, we could have used a mechanical stirring.

The last identified limitation is that we were doing bulk calculations. However, in the beaker, there are effectively 3 layers: (1) fresh top-most layer, (2) mixing middle layer, and (3) densest bottom layer. We are interested in the mixing layer, so we could have limited our calculations to the mixing layer by first determining where and what the length scales of the mixing layer are.

REFERENCES

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