Comparing $\mathcal{L}$ and χ^2 approaches when obtaining cosmological constraints with type Ia Supernovae using SALT2

Cássia Nascimento

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1 Overview

Following [1], here we test two different approaches when constraining cosmological parameters for ΛCDM model using emcee python package to perform a MCMC with the SALT2 fitting results from [2]. The basic equations for the theoretical distance modulus are

$$\mu_{th}(z; \Omega_{m0}, \Omega_{\Lambda 0}, h) = 5 \log_{10} \mathcal{D}_L(z; \Omega_{m0}, \Omega_{\Lambda 0}) + \mu_0(h), \tag{1}$$

where $\mu_0(h) = 5 \log_{10} \left(\frac{100c/(km/s)}{h} \right)$. The luminosity distance is given by

$$\mathcal{D}_L(z; \Omega_{m0}, \Omega_{\Lambda 0}) = \begin{cases} (1+z) \frac{1}{\sqrt{\Omega_{k0}}} \sinh \left(\sqrt{\Omega_{k0}} \int_0^z \frac{dz'}{E(z')} \right), & \Omega_{k0} > 0 \\ (1+z) \int_0^z \frac{dz'}{E(z')}, & \Omega_{k0} = 0 \ , \\ (1+z) \frac{1}{\sqrt{-\Omega_{k0}}} \sin \left(\sqrt{-\Omega_{k0}} \int_0^z \frac{dz'}{E(z')} \right), & \Omega_{k0} < 0 \end{cases} \tag{2}$$

with $E(z; \Omega_{m0}, \Omega_{\Lambda 0}) = \sqrt{\Omega_{m0}(1+z)^3 + \Omega_{\Lambda 0} + \Omega_{k0}(1+z)^2}$, and recalling $\Omega_{k0} = 1 - \Omega_{m0} - \Omega_{\Lambda 0}$.

While the distance modulus obtained from observations after standardization with SALT2 model is

$$\mu_{SALT2}(\alpha, \beta, M_B) = m_B^* - M_B + \alpha x_1 - \beta c. \tag{3}$$

To constrain the cosmological parameters with the observational data we write a χ^2 as follows

$$\chi^2 = \sum_i^N \frac{[\mu_{th}(z_i; \Omega_{m0}, \Omega_{\Lambda 0}, h) - \mu_{SALT2,i}(\alpha, \beta, M_B)]^2}{\sigma^2_{SALT2,i}(\alpha, \beta) + \sigma^2_{int}}. \tag{4}$$

And the correspoding log-likelihood as

$$-2\log\mathcal{L} = \log(2\pi)N + \sum_i^N \log(\sigma^2_{SALT2,i}(\alpha,\beta) + \sigma^2_{int}) + \quad (5)$$

$$+\sum_i^N \frac{[\mu_{th}(z_i;\Omega_{m0},\Omega_{\Lambda 0},h) - \mu_{SALT2,i}(\alpha,\beta,M_B)]^2}{\sigma^2_{SALT2,i}(\alpha,\beta)+\sigma^2_{int}}. \quad (6)$$

Where $\sigma^2_{SALT2,i}(\alpha,\beta) = \sigma^2_{m^*_B,i} + \alpha^2\sigma^2_{x_1,i} + \beta^2\sigma_{c,i} + 2\alpha\sigma_{m^*_B,x_1,i} - 2\beta\sigma_{m^*_B,c,i} - 2\alpha\beta\sigma_{x_1,c,i} + \sigma^2_{\mu,z,i}$.

So the two different approaches we follow are finding the cosmological parameters whether using the entire log-likelihood expression or only the chi-squared term. When performing the fit only with the χ^2 expression an iterative method is performed where the nuisance parameters in the denominator are kept fixed, once convergence is reached the value of σ_{int} is estimated by requiring $\chi^2/ndof = 1$ for the best fit parameters.

We compare both approaches and also investigate the impact of keeping a fixed or a varying σ_{int} in the complete likelihood approach.

2 Results

In Fig. 1 we have the triangle plots using both $\mathcal{L}$ (with σ_{int} as a free parameter) and χ^2 approaches. We can see good agreement with the cosmological parameters and only the nuisance parameters shows a little bias.

In Fig. 2 we have the constraints for σ_{int} fixed at 0.16 and σ_{int} as a free parameter, both considering $\mathcal{L}$ approach. There is now a slight difference also in the cosmological parameters. However, the constraint are still in agreement inside the 68% confidence region.

In Fig. 3 we compare the results for the iterative chi-squared process and the process without fixating the denominator nuisance parameters but with fixed σ_{int}. Ω_{m0} is the less affected parameter. $\Omega_{\Lambda 0}$ and $\mathcal{M}$ are biased in different directions and the biggest effect is seen on the nuisance parameters, with significant shift towards increasing values. This behaviour is understood as being due to an error in the best fit estimation due to the nuisance parameters appearing both in the numerator and denominator of the chi-squared expression.

In Fig. 5 we see again a comparison between chi-squared iterative process and the complete likelihood. While the nuisance parameters show a bias, as they are proportional to their values and in the same direction, the effect over the final value of the distance modulus is little, as seen in the RHS, not affecting the cosmology.

Finally, Fig. 5, Fig. 6 and Fig. 7 show the one-dimensional marginalized distributions. We can conclude that the apparent bias doe not significantly affect the cosmological parameters, except when not considering the iterative method for the chi-squared approach.

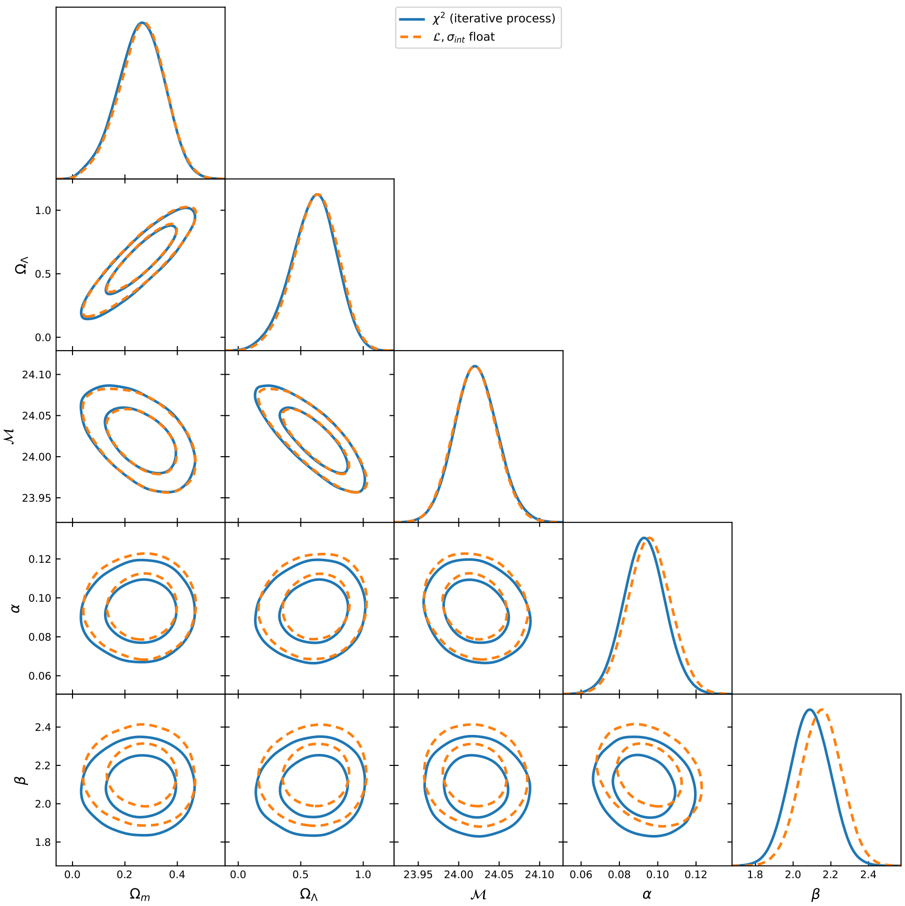

Figure 1: Triangle plot comparing complete likelihood and chi-squared approaches.

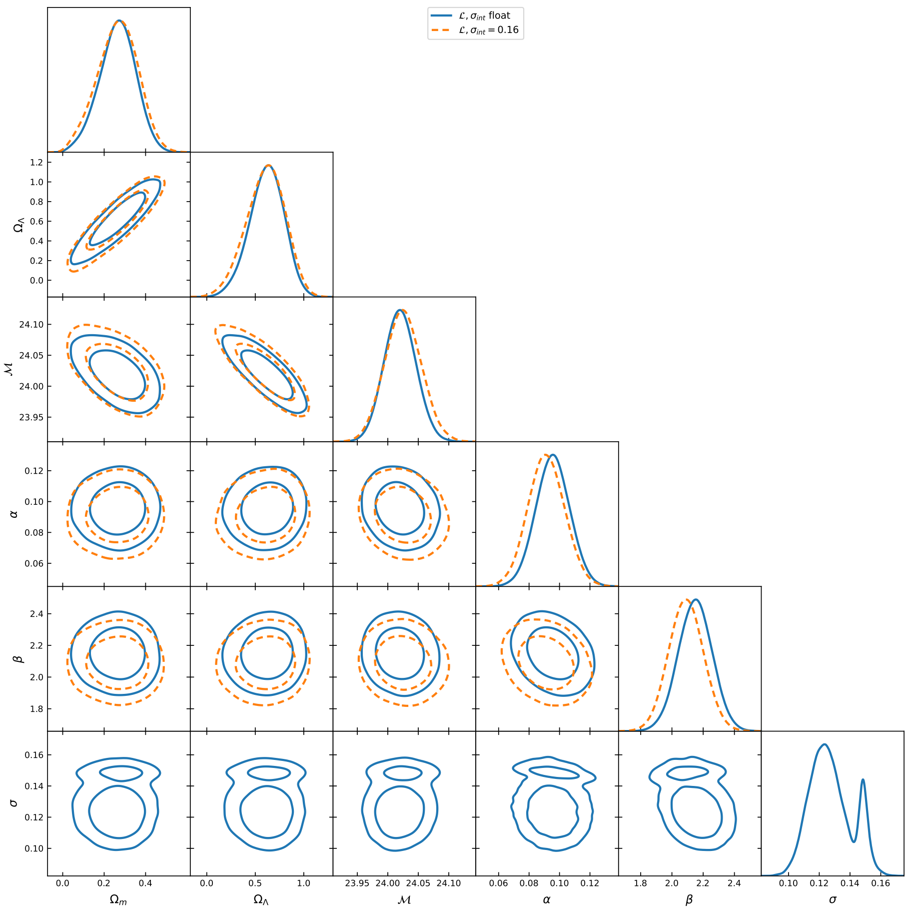

Figure 2: Triangle plot comparing complete likelihood approach with fixed of varying σ_{int}.

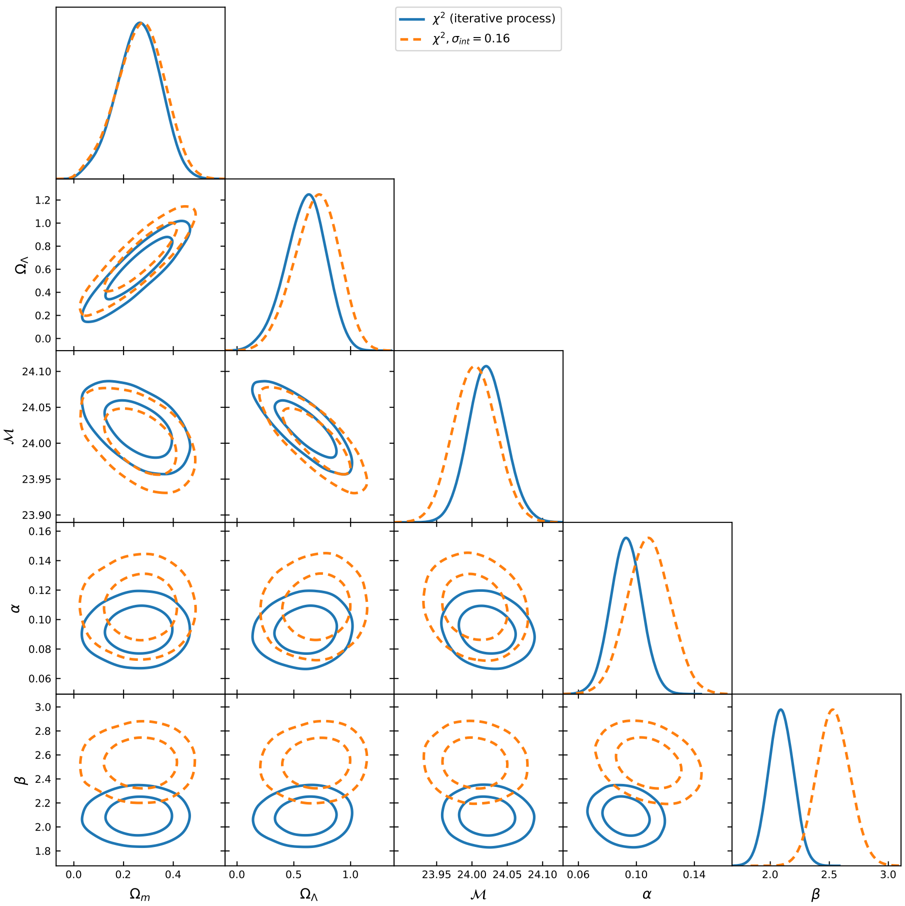

Figure 3: Triangle plot comparing chi-squared approach with and without the iterative process.

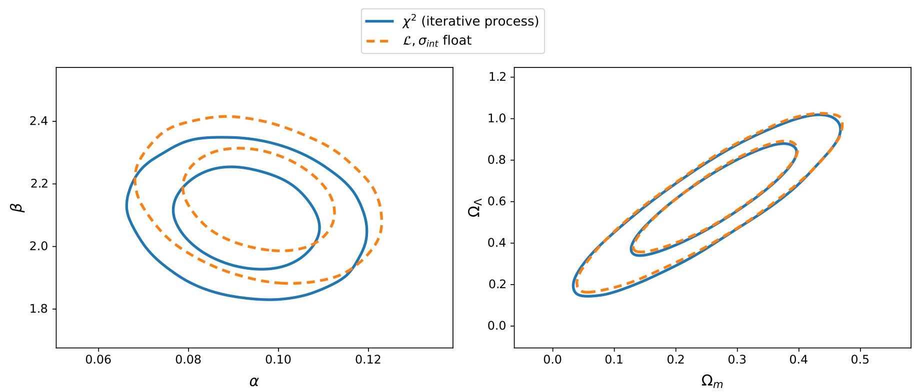

Figure 4: Contours comparing complete likelihood and chi-squared approaches effect onto nuisance and cosmological parameters.

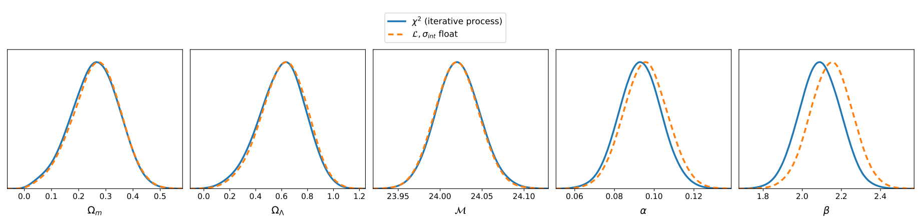

Figure 5: 1-d marginalized distributions for complete likelihood and chi-squared approaches.

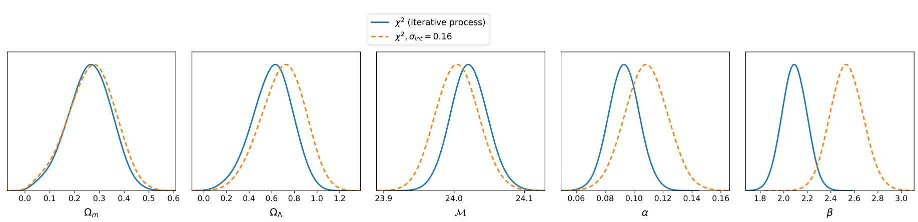

Figure 6: 1-d marginalized distributions for chi-squared with and without iterative process.

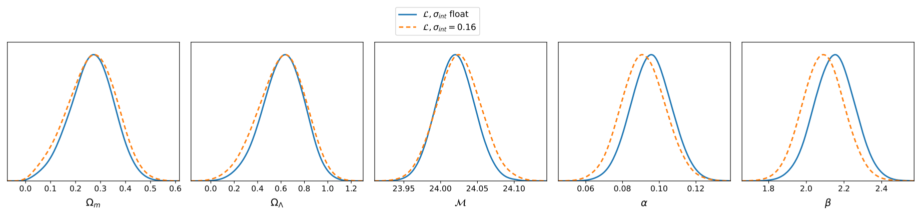

Figure 7: 1-d marginalized distributions for complete likelihood with varying and fixed σ_{int}.

References

[1] Bruno L Lago, Mauricio O Calvao, Sérgio E Jorás, Ribamar RR Reis, Ioav Waga, and Ramón Giostri. Type ia supernova parameter estimation: a comparison of two approaches using current datasets. *Astronomy & Astrophysics*, 541:A110, 2012.

[2] Richard Kessler, Andrew C Becker, David Cinabro, Jake Vanderplas, Joshua A Frieman, John Marriner, Tamara M Davis, Benjamin Dilday, Jon Holtzman, Saurabh W Jha, et al. First-year sloan digital sky survey-ii supernova results: Hubble diagram and cosmological parameters. *The Astrophysical Journal Supplement Series*, 185(1):32, 2009.