color_organ_learning

January 31, 2023

[1]:
```
!pip install ipympl torchviz
!pip install torch==1.13 --extra-index-url https://download.pytorch.org/whl/cpu
# restart your runtime after this step
```

```
Looking in indexes: https://pypi.org/simple, https://us-python.pkg.dev/colab-
wheels/public/simple/
Collecting ipympl
  Downloading ipympl-0.9.2-py2.py3-none-any.whl (510 kB)
                            510.3/510.3

KB 5.0 MB/s eta 0:00:00
Collecting torchviz
  Downloading torchviz-0.0.2.tar.gz (4.9 kB)
  Preparing metadata (setup.py) … done
Collecting matplotlib<4,>=3.4.0
  Downloading
matplotlib-3.6.3-cp38-cp38-manylinux_2_12_x86_64.manylinux2010_x86_64.whl (9.4
MB)
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Requirement already satisfied: pillow in /usr/local/lib/python3.8/dist-
packages (from ipympl) (7.1.2)
Requirement already satisfied: ipython-genutils in
/usr/local/lib/python3.8/dist-packages (from ipympl) (0.2.0)
Requirement already satisfied: traitlets<6 in /usr/local/lib/python3.8/dist-
packages (from ipympl) (5.7.1)
Requirement already satisfied: numpy in /usr/local/lib/python3.8/dist-packages
(from ipympl) (1.21.6)
Requirement already satisfied: ipython<9 in /usr/local/lib/python3.8/dist-
packages (from ipympl) (7.9.0)
Requirement already satisfied: ipywidgets<9,>=7.6.0 in
/usr/local/lib/python3.8/dist-packages (from ipympl) (7.7.1)
Requirement already satisfied: torch in /usr/local/lib/python3.8/dist-packages
(from torchviz) (1.13.1+cu116)
Requirement already satisfied: graphviz in /usr/local/lib/python3.8/dist-
packages (from torchviz) (0.10.1)
Requirement already satisfied: pygments in /usr/local/lib/python3.8/dist-
packages (from ipython<9->ipympl) (2.6.1)
Requirement already satisfied: backcall in /usr/local/lib/python3.8/dist-
```

```
packages (from ipython<9->ipympl) (0.2.0)
Requirement already satisfied: decorator in /usr/local/lib/python3.8/dist-
packages (from ipython<9->ipympl) (4.4.2)
Collecting jedi>=0.10
  Downloading jedi-0.18.2-py2.py3-none-any.whl (1.6 MB)
                           1.6/1.6 MB
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Requirement already satisfied: pexpect in /usr/local/lib/python3.8/dist-
packages (from ipython<9->ipympl) (4.8.0)
Requirement already satisfied: pickleshare in /usr/local/lib/python3.8/dist-
packages (from ipython<9->ipympl) (0.7.5)
Requirement already satisfied: prompt-toolkit<2.1.0,>=2.0.0 in
/usr/local/lib/python3.8/dist-packages (from ipython<9->ipympl) (2.0.10)
Requirement already satisfied: setuptools>=18.5 in
/usr/local/lib/python3.8/dist-packages (from ipython<9->ipympl) (57.4.0)
Requirement already satisfied: jupyterlab-widgets>=1.0.0 in
/usr/local/lib/python3.8/dist-packages (from ipywidgets<9,>=7.6.0->ipympl)
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Requirement already satisfied: widgetsnbextension~=3.6.0 in
/usr/local/lib/python3.8/dist-packages (from ipywidgets<9,>=7.6.0->ipympl)
(3.6.1)
Requirement already satisfied: ipykernel>=4.5.1 in
/usr/local/lib/python3.8/dist-packages (from ipywidgets<9,>=7.6.0->ipympl)
(5.3.4)
Requirement already satisfied: packaging>=20.0 in /usr/local/lib/python3.8/dist-
packages (from matplotlib<4,>=3.4.0->ipympl) (21.3)
Collecting contourpy>=1.0.1
  Downloading
contourpy-1.0.7-cp38-cp38-manylinux_2_17_x86_64.manylinux2014_x86_64.whl (300
kB)
                           300.0/300.0

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Requirement already satisfied: cycler>=0.10 in
/usr/local/lib/python3.8/dist-packages (from matplotlib<4,>=3.4.0->ipympl)
(0.11.0)
Collecting fonttools>=4.22.0
  Downloading fonttools-4.38.0-py3-none-any.whl (965 kB)
                          965.4/965.4 KB
26.5 MB/s eta 0:00:00
Requirement already satisfied: python-dateutil>=2.7 in
/usr/local/lib/python3.8/dist-packages (from matplotlib<4,>=3.4.0->ipympl)
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Requirement already satisfied: pyparsing>=2.2.1 in
/usr/local/lib/python3.8/dist-packages (from matplotlib<4,>=3.4.0->ipympl)
(3.0.9)
Requirement already satisfied: kiwisolver>=1.0.1 in
/usr/local/lib/python3.8/dist-packages (from matplotlib<4,>=3.4.0->ipympl)
```

```
(1.4.4)
Requirement already satisfied: typing-extensions in
/usr/local/lib/python3.8/dist-packages (from torch->torchviz) (4.4.0)
Requirement already satisfied: jupyter-client in /usr/local/lib/python3.8/dist-
packages (from ipykernel>=4.5.1->ipywidgets<9,>=7.6.0->ipympl) (6.1.12)
Requirement already satisfied: tornado>=4.2 in /usr/local/lib/python3.8/dist-
packages (from ipykernel>=4.5.1->ipywidgets<9,>=7.6.0->ipympl) (6.0.4)
Requirement already satisfied: parso<0.9.0,>=0.8.0 in
/usr/local/lib/python3.8/dist-packages (from jedi>=0.10->ipython<9->ipympl)
(0.8.3)
Requirement already satisfied: six>=1.9.0 in /usr/local/lib/python3.8/dist-
packages (from prompt-toolkit<2.1.0,>=2.0.0->ipython<9->ipympl) (1.15.0)
Requirement already satisfied: wcwidth in /usr/local/lib/python3.8/dist-packages
(from prompt-toolkit<2.1.0,>=2.0.0->ipython<9->ipympl) (0.2.5)
Requirement already satisfied: notebook>=4.4.1 in /usr/local/lib/python3.8/dist-
packages (from widgetsnbextension~=3.6.0->ipywidgets<9,>=7.6.0->ipympl) (5.7.16)
Requirement already satisfied: ptyprocess>=0.5 in /usr/local/lib/python3.8/dist-
packages (from pexpect->ipython<9->ipympl) (0.7.0)
Requirement already satisfied: pyzmq>=17 in /usr/local/lib/python3.8/dist-
packages (from
notebook>=4.4.1->widgetsnbextension~=3.6.0->ipywidgets<9,>=7.6.0->ipympl)
(23.2.1)
Requirement already satisfied: nbformat in /usr/local/lib/python3.8/dist-
packages (from
notebook>=4.4.1->widgetsnbextension~=3.6.0->ipywidgets<9,>=7.6.0->ipympl)
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Requirement already satisfied: nbconvert<6.0 in /usr/local/lib/python3.8/dist-
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notebook>=4.4.1->widgetsnbextension~=3.6.0->ipywidgets<9,>=7.6.0->ipympl)
(5.6.1)
Requirement already satisfied: prometheus-client in
/usr/local/lib/python3.8/dist-packages (from
notebook>=4.4.1->widgetsnbextension~=3.6.0->ipywidgets<9,>=7.6.0->ipympl)
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Requirement already satisfied: terminado>=0.8.1 in
/usr/local/lib/python3.8/dist-packages (from
notebook>=4.4.1->widgetsnbextension~=3.6.0->ipywidgets<9,>=7.6.0->ipympl)
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Requirement already satisfied: Send2Trash in /usr/local/lib/python3.8/dist-
packages (from
notebook>=4.4.1->widgetsnbextension~=3.6.0->ipywidgets<9,>=7.6.0->ipympl)
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Requirement already satisfied: jinja2<=3.0.0 in /usr/local/lib/python3.8/dist-
packages (from
notebook>=4.4.1->widgetsnbextension~=3.6.0->ipywidgets<9,>=7.6.0->ipympl)
(2.11.3)
Requirement already satisfied: jupyter-core>=4.4.0 in
/usr/local/lib/python3.8/dist-packages (from
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notebook>=4.4.1->widgetsnbextension~=3.6.0->ipywidgets<9,>=7.6.0->ipympl)
(5.1.3)
Requirement already satisfied: MarkupSafe>=0.23 in
/usr/local/lib/python3.8/dist-packages (from jinja2<=3.0.0->notebook>=4.4.1->wid
getsnbextension~=3.6.0->ipywidgets<9,>=7.6.0->ipympl) (2.0.1)
Requirement already satisfied: platformdirs>=2.5 in
/usr/local/lib/python3.8/dist-packages (from jupyter-core>=4.4.0->notebook>=4.4.
1->widgetsnbextension~=3.6.0->ipywidgets<9,>=7.6.0->ipympl) (2.6.2)
Requirement already satisfied: bleach in /usr/local/lib/python3.8/dist-packages
(from nbconvert<6.0->notebook>=4.4.1->widgetsnbextension~=3.6.0->ipywidgets<9,>=
7.6.0->ipympl) (5.0.1)
Requirement already satisfied: entrypoints>=0.2.2 in
/usr/local/lib/python3.8/dist-packages (from nbconvert<6.0->notebook>=4.4.1->wid
getsnbextension~=3.6.0->ipywidgets<9,>=7.6.0->ipympl) (0.4)
Requirement already satisfied: defusedxml in /usr/local/lib/python3.8/dist-
packages (from nbconvert<6.0->notebook>=4.4.1->widgetsnbextension~=3.6.0->ipywid
gets<9,>=7.6.0->ipympl) (0.7.1)
Requirement already satisfied: mistune<2,>=0.8.1 in
/usr/local/lib/python3.8/dist-packages (from nbconvert<6.0->notebook>=4.4.1->wid
getsnbextension~=3.6.0->ipywidgets<9,>=7.6.0->ipympl) (0.8.4)
Requirement already satisfied: testpath in /usr/local/lib/python3.8/dist-
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gets<9,>=7.6.0->ipympl) (0.6.0)
Requirement already satisfied: pandocfilters>=1.4.1 in
/usr/local/lib/python3.8/dist-packages (from nbconvert<6.0->notebook>=4.4.1->wid
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Requirement already satisfied: jsonschema>=2.6 in /usr/local/lib/python3.8/dist-
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9,>=7.6.0->ipympl) (4.3.3)
Requirement already satisfied: fastjsonschema in /usr/local/lib/python3.8/dist-
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9,>=7.6.0->ipympl) (2.16.2)
Requirement already satisfied: attrs>=17.4.0 in /usr/local/lib/python3.8/dist-
packages (from jsonschema>=2.6->nbformat->notebook>=4.4.1->widgetsnbextension~=3
.6.0->ipywidgets<9,>=7.6.0->ipympl) (22.2.0)
Requirement already satisfied: importlib-resources>=1.4.0 in
/usr/local/lib/python3.8/dist-packages (from jsonschema>=2.6->nbformat->notebook
>=4.4.1->widgetsnbextension~=3.6.0->ipywidgets<9,>=7.6.0->ipympl) (5.10.2)
Requirement already satisfied: pyrsistent!=0.17.0,!=0.17.1,!=0.17.2,>=0.14.0 in
/usr/local/lib/python3.8/dist-packages (from jsonschema>=2.6->nbformat->notebook
>=4.4.1->widgetsnbextension~=3.6.0->ipywidgets<9,>=7.6.0->ipympl) (0.19.3)
Requirement already satisfied: webencodings in /usr/local/lib/python3.8/dist-
packages (from bleach->nbconvert<6.0->notebook>=4.4.1->widgetsnbextension~=3.6.0
->ipywidgets<9,>=7.6.0->ipympl) (0.5.1)
Requirement already satisfied: zipp>=3.1.0 in /usr/local/lib/python3.8/dist-
packages (from importlib-resources>=1.4.0->jsonschema>=2.6->nbformat->notebook>=
4.4.1->widgetsnbextension~=3.6.0->ipywidgets<9,>=7.6.0->ipympl) (3.11.0)
Building wheels for collected packages: torchviz
```

```
  Building wheel for torchviz (setup.py) … done
  Created wheel for torchviz: filename=torchviz-0.0.2-py3-none-any.whl size=4151
sha256=4a07ca4dda0fb37c587e2f2e3eb3f2ee5f88ec1e372edf02cadb98529c3c31aa
  Stored in directory: /root/.cache/pip/wheels/05/7d/1b/8306781244e42ede119edbb0
53bdcda1c1f424ca226165a417
Successfully built torchviz
Installing collected packages: jedi, fonttools, contourpy, torchviz, matplotlib,
ipympl
  Attempting uninstall: matplotlib
    Found existing installation: matplotlib 3.2.2
    Uninstalling matplotlib-3.2.2:
      Successfully uninstalled matplotlib-3.2.2
Successfully installed contourpy-1.0.7 fonttools-4.38.0 ipympl-0.9.2 jedi-0.18.2
matplotlib-3.6.3 torchviz-0.0.2
```

```
Looking in indexes: https://pypi.org/simple, https://us-python.pkg.dev/colab-
wheels/public/simple/, https://download.pytorch.org/whl/cpu
Collecting torch==1.13
  Downloading https://download.pytorch.org/whl/cpu/torch-1.13.0%2Bcpu-
cp38-cp38-linux_x86_64.whl (198.5 MB)
                           198.5/198.5

MB 5.3 MB/s eta 0:00:00
Requirement already satisfied: typing-extensions in
/usr/local/lib/python3.8/dist-packages (from torch==1.13) (4.4.0)
Installing collected packages: torch
  Attempting uninstall: torch
    Found existing installation: torch 1.13.1+cu116
    Uninstalling torch-1.13.1+cu116:
ERROR: Operation cancelled by user
```

[1]:

```
import math
import matplotlib.pyplot as plt
import numpy as np
import torch
import torch.nn as nn
from torch.autograd import Variable
import tqdm

import IPython
from ipywidgets import interactive, widgets, Layout
from IPython.display import display, HTML
```

[2]:

```
print(torch.__version__, torch.cuda.is_available())
# Homework 0 does not require a GPU
```

```
1.13.1+cu116 False
```

[3]:
```python
# enable matplotlib widgets;

# on Google Colab
from google.colab import output
output.enable_custom_widget_manager()

%matplotlib widget
```

[38]:
```python
# Constants
cap_value = 1e-6           # Farads
R_init = 500               # Ohms
cutoff_mag = 1. / math.sqrt(2)
cutoff_dB = 20 * math.log10(cutoff_mag)
dataset_size = 1000
max_training_steps = 100000
```

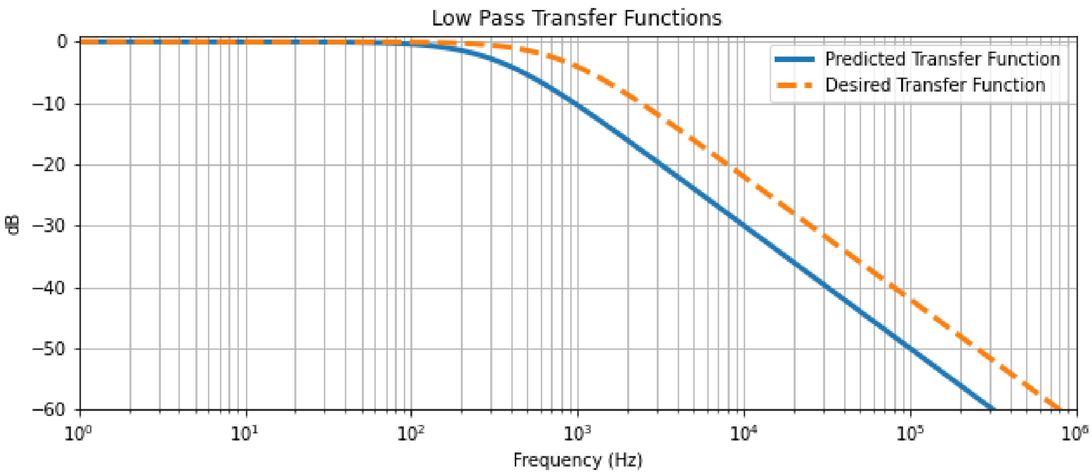
0.1 (a) Designing a Low Pass Filter by Matching Transfer Functions

[39]:
```python
# Transfer function: evaluates magnitude of given frequencies for a resistor␣
↪value in the low pass circuit
def evaluate_lp_circuit(freqs, R_low):
    return 1. / torch.sqrt(1 + (R_low * cap_value * freqs) ** 2)
```

[40]:
```python
# Plot transfer function for a given low pass circuit
fig = plt.figure(figsize=(9, 4))
ws = 2 * math.pi * 10 ** torch.linspace(0, 6, 1000)
mags = 20 * torch.log10(evaluate_lp_circuit(ws, R_init))
R_low_des = 1 / (2 * math.pi * 800 * cap_value)
mags_des = 20 * torch.log10(evaluate_lp_circuit(ws, R_low_des))
tf, = plt.semilogx(ws / (2 * math.pi), mags, linewidth=3)
tf_des, = plt.semilogx(ws / (2 * math.pi), mags_des, linestyle="--",␣
↪linewidth=3)
plt.xlim([1, 1e6])
plt.ylim([-60, 1])
plt.title("Low Pass Transfer Functions")
plt.xlabel("Frequency (Hz)")
plt.ylabel("dB")
plt.grid(which="both")
leg = plt.legend(["Predicted Transfer Function", "Desired Transfer Function"])
plt.tight_layout()

# Main update function for interactive plot
def update_tfs(R=R_init):
    mags = 20 * torch.log10(evaluate_lp_circuit(ws, R))
    tf.set_data(ws / (2 * math.pi), mags)
    fig.canvas.draw_idle()
```

```
# Include sliders for relevant quantities
ip = interactive(update_tfs,
                 R=widgets.IntSlider(value=R_init, min=1, max=1000, step=1,␣
 ↪description="R", layout=Layout(width='100%')))
ip
```

```
interactive(children=(IntSlider(value=500, description='R',␣
↪layout=Layout(width='100%'), max=1000, min=1), Out…
```



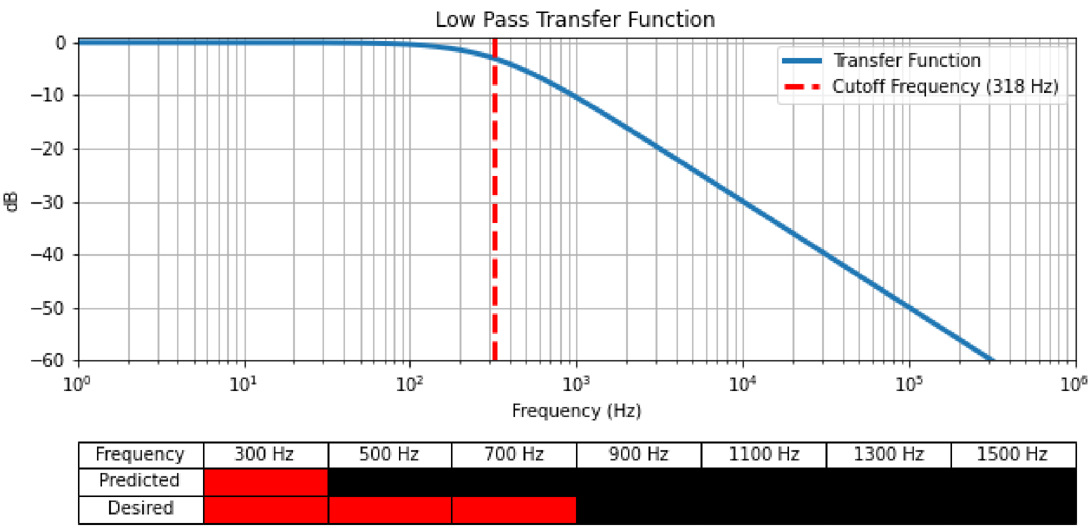
0.2 (b) Designing a Low pass Filter from Binary Data

[31]:

```
# Plot transfer function for a given low pass circuit
fig = plt.figure(figsize=(9, 5))
ws = 2 * math.pi * 10 ** torch.linspace(0, 6, 1000)
mags = 20 * torch.log10(evaluate_lp_circuit(ws, R_init))
cutoff = ws[np.argmax(mags < cutoff_dB)]
tf, = plt.semilogx(ws / (2 * math.pi), mags, linewidth=3)
cut = plt.axvline(cutoff / (2 * math.pi), c="red", linestyle="--", linewidth=3)
plt.xlim([1, 1e6])
plt.ylim([-60, 1])
plt.title("Low Pass Transfer Function")
plt.xlabel("Frequency (Hz)")
plt.ylabel("dB")
plt.grid(which="both")
leg = plt.legend(["Transfer Function", f"Cutoff Frequency ({1 / (2 * math.pi *␣
 ↪R_init * cap_value):.0f} Hz)"])

# Plot table of LED on/off values (predicted and desired)
ws_test = 2 * math.pi * np.linspace(300, 1500, num=7)
table_txt = np.zeros((3, len(ws_test) + 1), dtype="U15")
```

```python
table_txt[0, :] = ["Frequency"] + [f"{w / (2 * math.pi):.0f} Hz" for w in␣
↪ws_test]
table_txt[1:, 0] = ["Predicted", "Desired"]
table_colors = np.zeros_like(table_txt, dtype=(np.int32, (3,)))
table_colors[-1, 1:4] = (1, 0, 0)
table_colors[1, 1] = (1, 0, 0)
table_colors[:, :1] = (1, 1, 1)
table_colors[:1, :] = (1, 1, 1)
tab = plt.table(table_txt, table_colors, bbox=[0.0, -0.5, 1.0, 0.25],␣
↪cellLoc="center")
plt.tight_layout()

# Main update function for interactive plot
def update_lights(R=R_init):
    mags = 20 * torch.log10(evaluate_lp_circuit(ws, R))
    cutoff = ws[np.argmax(mags < cutoff_dB)]
    tf.set_data(ws / (2 * math.pi), mags)
    cut.set_xdata(cutoff / (2 * math.pi))
    for i, w in enumerate(ws_test):
        if w < cutoff:
            tab[(1, i+1)].set_facecolor((1, 0, 0))
        else:
            tab[(1, i+1)].set_facecolor((0, 0, 0))
    leg.get_texts()[1].set_text(f"Cutoff Frequency ({1 / (2 * math.pi * R *␣
↪cap_value):.0f} Hz)")
    fig.canvas.draw_idle()

# Include sliders for relevant quantities
ip = interactive(update_lights,
                 R=widgets.IntSlider(value=R_init, min=1, max=1000, step=1,␣
↪description="R", layout=Layout(width='100%')))
ip
```

```
interactive(children=(IntSlider(value=500, description='R',␣
↪layout=Layout(width='100%'), max=1000, min=1), Out…
```



0.3 (c) Learning a Low Pass Filter from Desired Transfer Function Samples

[32]:
```python
# PyTorch model of the low pass circuit (for training)
class LowPassCircuit(nn.Module):
    def __init__(self, R=None):
        super().__init__()
        self.R = nn.Parameter(torch.tensor(R, dtype=float) if R is not None else torch.rand(1) * 1000)

    # Note: the forward function is called automatically when the __call__ function of this object is called
    def forward(self, freqs):
        return evaluate_lp_circuit(freqs, self.R)

# Generate training data in a uniform log scale of frequences, then evaluate using the true transfer function
def generate_lp_training_data(n):
    rand_ws = 2 * math.pi * torch.pow(10, torch.rand(n) * 6)
    labels = evaluate_lp_circuit(rand_ws, R_low_des)
    return rand_ws, labels

# Train a given low pass filter
def train_lp_circuit_tf(circuit, loss_fn, dataset_size, max_training_steps, lr):

    R_values = [float(circuit.R.data)]
    grad_values = [np.nan]
```

```
    train_data = generate_lp_training_data(dataset_size)
    print(f"Initial Resistor Value: R = {float(circuit.R.data):.0f}")
    iter_bar = tqdm.trange(max_training_steps, desc="Training Iter")
    for i in iter_bar:
        pred = circuit(train_data[0])
        loss = loss_fn(pred, train_data[1]).mean()
        grad = torch.autograd.grad(loss, circuit.R)
        with torch.no_grad():
            circuit.R -= lr * grad[0]

        R_values.append(float(circuit.R.data))
        grad_values.append(float(grad[0].data))
        iter_bar.set_postfix_str(f"Loss: {float(loss.data):.3f},␣
↪R={float(circuit.R.data):.0f}")
        if loss.data < 1e-6 or abs(grad[0].data) < 1e-6:
            break

    print(f"Final Resistor Value: R = {float(circuit.R.data):.0f}")
    return train_data, R_values, grad_values
```

[ ]:
```
# Create a circuit, use mean squared error loss w/ learning rate of 200
circuit = LowPassCircuit(200)
loss_fn = lambda x, y: (x - y) ** 2
lr = 200
train_data_low_tf, R_values_low_tf, grad_values_low_tf =␣
↪train_lp_circuit_tf(circuit, loss_fn, dataset_size, max_training_steps, lr)
```

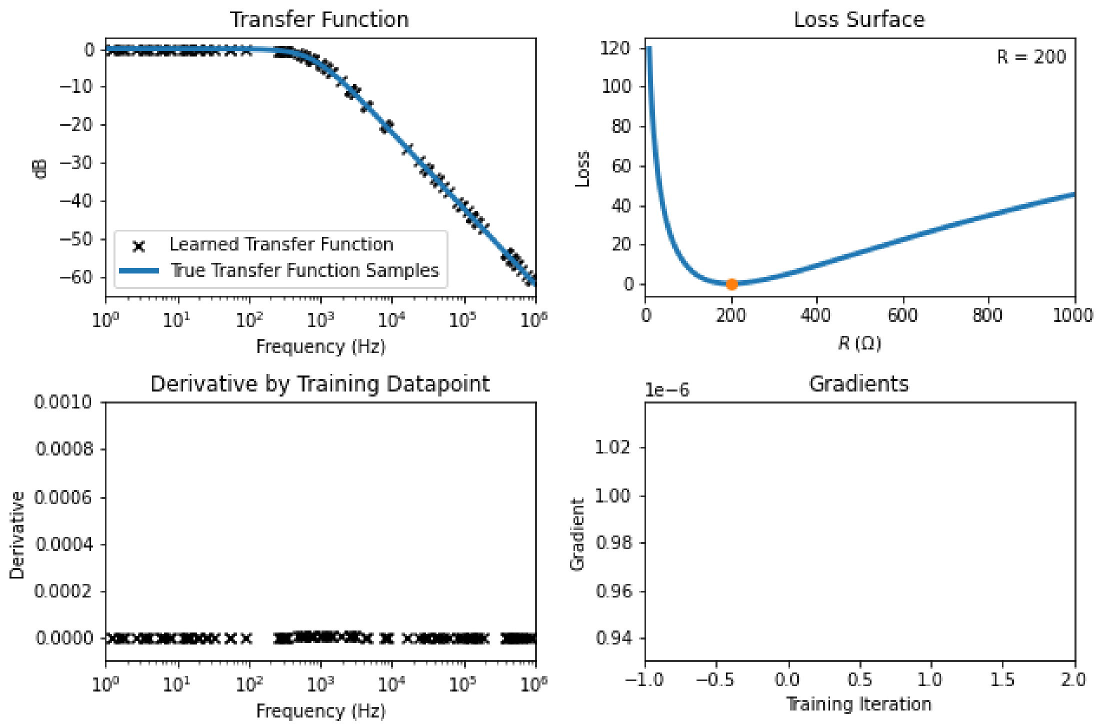
```
Initial Resistor Value: R = 200

Training Iter:   0%|          | 0/100000 [00:00<?, ?it/s, Loss: 0.000, R=200]

Final Resistor Value: R = 200
```

[ ]:
```
# Plot transfer function over training
fig, ((ax1, ax2), (ax3, ax4)) = plt.subplots(2, 2, figsize=(9, 6))
ws = 2 * math.pi * 10 ** torch.linspace(0, 6, 1000)
subsample = int(dataset_size / 100)
ax1.scatter(train_data_low_tf[0][::subsample] / (2 * math.pi), 20 * torch.
↪log10(train_data_low_tf[1][::subsample]), c="k", marker="x")
learned_tf, = ax1.semilogx(ws / (2 * math.pi), 20 * torch.
↪log10(evaluate_lp_circuit(ws, R_values_low_tf[0])), linewidth=3)
ax1.set_xlim([1, 1e6])
ax1.set_title("Transfer Function")
ax1.set_xlabel("Frequency (Hz)")
ax1.set_ylabel("dB")
ax1.legend(["Learned Transfer Function", "True Transfer Function Samples"])
```

```python
# Show loss surface over training
eval_pts = torch.arange(10, 1001, 1)
eval_vals = evaluate_lp_circuit(train_data_low_tf[0][:, None], eval_pts[None, :
↪])
loss_surface_mse = loss_fn(eval_vals, train_data_low_tf[1][:, None].
↪expand(eval_vals.shape))
ax2.plot(eval_pts, loss_surface_mse.sum(0), linewidth=3)
cur_loss, = ax2.plot(R_values_low_tf[0], loss_surface_mse[:,␣
↪int(R_values_low_tf[0] - 10)].sum(0), marker="o")
cur_loss_label = ax2.annotate(f"R = {R_values_low_tf[0]:.0f}", (0, 0),␣
↪xytext=(0.82, 0.9), textcoords='axes fraction')
ax2.set_title("Loss Surface")
ax2.set_xlim([0, 1000])
ax2.set_xlabel("$R \; (\Omega)$")
ax2.set_ylabel("Loss")

# Show loss contributions of each data point
cur_circuit = LowPassCircuit(R_values_low_tf[0])
data_losses = loss_fn(cur_circuit(train_data_low_tf[0][::subsample]),␣
↪(train_data_low_tf[1][::subsample]).float())
data_grads = torch.zeros(len(data_losses))
for i, dl in enumerate(data_losses):
    data_grads[i] = torch.autograd.grad(dl, cur_circuit.R, retain_graph=True)[0]
data_grads_scat = ax3.scatter(train_data_low_tf[0][::subsample] / (2 * math.
↪pi), data_grads, marker="x", c="k")
ax3.set_xscale("log")
ax3.set_ylabel("Derivative")
ax3.set_xlim([1, 1e6])
ax3.set_ylim([-1e-4, 1e-3])
ax3.set_xlabel("Frequency (Hz)")
ax3.set_title("Derivative by Training Datapoint")

# Show total gradient at each training iteration
ax4.plot(np.arange(len(grad_values_low_tf)), grad_values_low_tf, linewidth=3)
cur_iter, = ax4.plot(0, grad_values_low_tf[0], marker="o")
cur_grad_label = ax4.annotate(f"Grad = {grad_values_low_tf[0]:.2e}", (0, 0),␣
↪xytext=(0.65, 0.9), textcoords='axes fraction')
ax4.set_xlabel("Training Iteration")
ax4.set_ylabel("Gradient")
ax4.set_title("Gradients")
ax4.set_xlim([-1, len(grad_values_low_tf)])

plt.tight_layout()

# Main update function for interactive plots
def update_iter_tf(t=0):
```

```
    learned_tf.set_data(ws / (2 * math.pi), 20 * torch.
→log10(evaluate_lp_circuit(ws, R_values_low_tf[t])))
    cur_loss.set_data(R_values_low_tf[t], loss_surface_mse[:,␣
→int(R_values_low_tf[t] - 10)].sum(0))
    cur_loss_label.set_text(f"R = {R_values_low_tf[t]:.0f}")
    cur_iter.set_data(t, grad_values_low_tf[t])
    cur_grad_label.set_text(f"Grad = {grad_values_low_tf[t]:.2e}")
    cur_circuit = LowPassCircuit(R_values_low_tf[t])
    data_losses = loss_fn(cur_circuit(train_data_low_tf[0][::subsample]),␣
→(train_data_low_tf[1][::subsample]).float())
    data_grads = torch.zeros(len(data_losses))
    for i, dl in enumerate(data_losses):
        data_grads[i] = torch.autograd.grad(dl, cur_circuit.R,␣
→retain_graph=True)[0]
    data_grads_scat.set_offsets(torch.stack((train_data_low_tf[0][::subsample] /
→ (2 * math.pi), data_grads)).T)
    fig.canvas.draw_idle()

# Include sliders for relevant quantities
ip = interactive(update_iter_tf,
                 t=widgets.IntSlider(value=0, min=0, max=len(R_values_low_tf) -␣
→1, step=1, description="Training Iteration", style={'description_width':␣
→'initial'}, layout=Layout(width='100%')))
ip
```

```
interactive(children=(IntSlider(value=0, description='Training Iteration',␣
→layout=Layout(width='100%'), max=1,…
```



0.4 (d) Learning a Low Pass Filter from Binary Data with Mean Squared Error Loss

[41]:
```
# Train a given low pass filter from binary data
def train_lp_circuit_binary(circuit, loss_fn, dataset_size, max_training_steps,␣
↪lr):

    R_values = [float(circuit.R.data)]
    grad_values = [np.nan]
    train_data = generate_lp_training_data(dataset_size)
    print(f"Initial Resistor Value: R = {float(circuit.R.data):.0f}")
    iter_bar = tqdm.trange(max_training_steps, desc="Training Iter")
    for i in iter_bar:
        pred = circuit(train_data[0])
        ### YOUR CODE HERE
        loss = loss_fn(pred, (train_data[1] > cutoff_mag).float()).mean()
        # loss = loss_fn(?, ?).mean()
        ### END YOUR CODE
        grad = torch.autograd.grad(loss, circuit.R)
        with torch.no_grad():
            circuit.R -= lr * grad[0]

        R_values.append(float(circuit.R.data))
```

```
        grad_values.append(float(grad[0].data))
        iter_bar.set_postfix_str(f"Loss: {float(loss.data):.3f},␣
↪R={float(circuit.R.data):.0f}")
        if loss.data < 1e-6 or abs(grad[0].data) < 1e-6:
            break

    print(f"Final Resistor Value: R = {float(circuit.R.data):.0f}")
    return train_data, R_values, grad_values
```

[42]:
```
# Create a circuit, use MSE loss with learning rate of 200
circuit = LowPassCircuit(500)
loss_fn = lambda x, y: (x - y) ** 2 # x:pred
lr = 200
train_data_low_bin, R_values_low_bin, grad_values_low_bin =␣
↪train_lp_circuit_binary(circuit, loss_fn, dataset_size, max_training_steps,␣
↪lr)
```

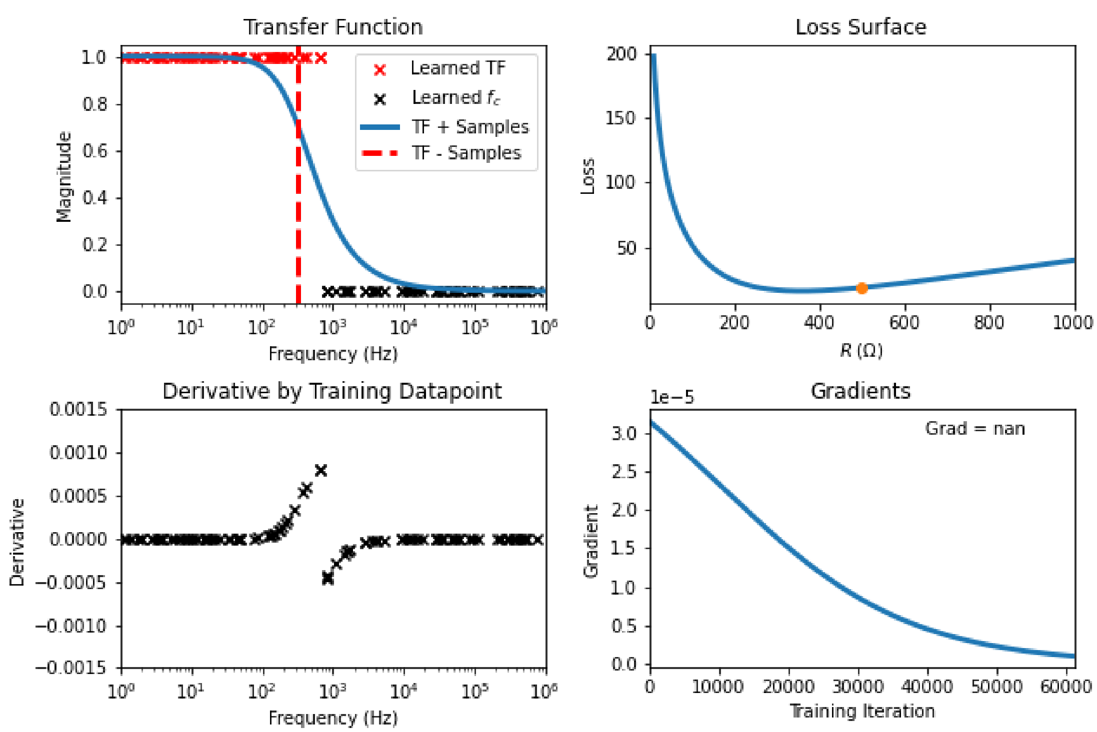
```
Initial Resistor Value: R = 500

Training Iter:  61%|        | 61101/100000 [03:27<02:12, 294.31it/s, Loss:
0.016, R=361]

Final Resistor Value: R = 361
```

[43]:
```
# Plot transfer function over training
fig, ((ax1, ax2), (ax3, ax4)) = plt.subplots(2, 2, figsize=(9, 6))
ws = 2 * math.pi * 10 ** torch.linspace(0, 6, 1000)
subsample = int(dataset_size / 100)
train_data_mask = train_data_low_bin[1][::subsample] > cutoff_mag
ax1.scatter(train_data_low_bin[0][::subsample][train_data_mask] / (2 * math.
↪pi), np.ones(train_data_mask.sum()), c="r", marker="x")
ax1.scatter(train_data_low_bin[0][::subsample][~train_data_mask] / (2 * math.
↪pi), np.zeros((~train_data_mask).sum()), c="k", marker="x")
mags = evaluate_lp_circuit(ws, R_values_low_bin[0])
learned_tf, = ax1.semilogx(ws / (2 * math.pi), mags, linewidth=3)
cutoff = ws[np.argmax(mags < cutoff_mag)]
cut = ax1.axvline(cutoff / (2 * math.pi), c="red", linestyle="--", linewidth=3)
ax1.set_xlim([1, 1e6])
ax1.set_title("Transfer Function")
ax1.set_xlabel("Frequency (Hz)")
ax1.set_ylabel("Magnitude")
ax1.legend(["Learned TF", "Learned $f_c$", "TF + Samples", "TF - Samples"])

# Show loss surface over training
eval_pts = torch.arange(10, 1001, 1)
eval_vals = evaluate_lp_circuit(train_data_low_bin[0][:, None], eval_pts[None, :
↪])
```

```
loss_surface_mse = loss_fn(eval_vals, (train_data_low_bin[1][:, None].
↪expand(eval_vals.shape) > cutoff_mag).float())
ax2.plot(eval_pts, loss_surface_mse.sum(0), linewidth=3)
cur_loss, = ax2.plot(R_values_low_bin[0], loss_surface_mse[:,␣
↪int(R_values_low_bin[0] - 10)].sum(0), marker="o")
cur_loss_label = ax2.annotate(f"R = {R_values_low_bin[0]:.0f}", (0, 0),␣
↪xytext=(0.82, 0.9), textcoords='axes fraction')
ax2.set_title("Loss Surface")
ax2.set_xlim([0, 1000])
ax2.set_xlabel("$R \; (\Omega)$")
ax2.set_ylabel("Loss")

# Show loss contributions of each data point
cur_circuit = LowPassCircuit(R_values_low_bin[0])
data_losses = loss_fn(cur_circuit(train_data_low_bin[0][::subsample]),␣
↪(train_data_low_bin[1][::subsample] > cutoff_mag).float())
data_grads = torch.zeros(len(data_losses))
for i, dl in enumerate(data_losses):
    data_grads[i] = torch.autograd.grad(dl, cur_circuit.R, retain_graph=True)[0]
data_grads_scat = ax3.scatter(train_data_low_bin[0][::subsample] / (2 * math.
↪pi), data_grads, marker="x", c="k")
ax3.set_xscale("log")
ax3.set_ylabel("Derivative")
ax3.set_xlim([1, 1e6])
ax3.set_ylim([-1.5e-3, 1.5e-3])
ax3.set_xlabel("Frequency (Hz)")
ax3.set_title("Derivative by Training Datapoint")

# Show gradient at each training iteration
ax4.plot(np.arange(len(grad_values_low_bin)), grad_values_low_bin, linewidth=3)
cur_iter, = ax4.plot(0, grad_values_low_bin[0], marker="o")
cur_grad_label = ax4.annotate(f"Grad = {grad_values_low_bin[0]:.2e}", (0, 0),␣
↪xytext=(0.65, 0.9), textcoords='axes fraction')
ax4.set_xlabel("Training Iteration")
ax4.set_ylabel("Gradient")
ax4.set_title("Gradients")
ax4.set_xlim([-1, len(grad_values_low_bin)])

plt.tight_layout()

# Main update function for interactive plots
def update_iter_low_bin(t=0):
    mags = evaluate_lp_circuit(ws, R_values_low_bin[t])
    learned_tf.set_data(ws / (2 * math.pi), mags)
    cutoff = ws[np.argmax(mags < cutoff_mag)]
    cut.set_xdata(cutoff / (2 * math.pi))
```

```
    cur_loss.set_data(R_values_low_bin[t], loss_surface_mse[:,␣
↪int(R_values_low_bin[t] - 10)].sum(0))
    cur_loss_label.set_text(f"R = {R_values_low_bin[t]:.0f}")
    cur_iter.set_data(t, grad_values_low_bin[t])
    cur_grad_label.set_text(f"Grad = {grad_values_low_bin[t]:.2e}")
    cur_circuit = LowPassCircuit(R_values_low_bin[t])
    data_losses = loss_fn(cur_circuit(train_data_low_bin[0][::subsample]),␣
↪(train_data_low_bin[1][::subsample] > cutoff_mag).float())
    data_grads = torch.zeros(len(data_losses))
    for i, dl in enumerate(data_losses):
        data_grads[i] = torch.autograd.grad(dl, cur_circuit.R,␣
↪retain_graph=True)[0]
    data_grads_scat.set_offsets(torch.stack((train_data_low_bin[0][::subsample]␣
↪/ (2 * math.pi), data_grads)).T)
    fig.canvas.draw_idle()

# Include sliders for relevant quantities
ip = interactive(update_iter_low_bin,
                 t=widgets.IntSlider(value=0, min=0, max=len(R_values_low_bin)␣
↪- 1, step=1, description="Training Iteration", style={'description_width':␣
↪'initial'}, layout=Layout(width='100%')))
ip
```

```
interactive(children=(IntSlider(value=0, description='Training Iteration',␣
↪layout=Layout(width='100%'), max=61…
```



0.5 (e) Learning a Low Pass Filter from Binary Data with a Different Loss

[ ]:
```
circuit = LowPassCircuit(500)
### YOUR CODE HERE
lr = 200
loss_fn = lambda x, y: (1-y) * torch.where(x-cutoff_mag>0,x-cutoff_mag, 0) + y␣
↪* torch.where(cutoff_mag-x>0,cutoff_mag-x, 0)# x:pred
# loss_fn = lambda x, y: ?
### END YOUR CODE
train_data_low_bin, R_values_low_bin, grad_values_low_bin =␣
↪train_lp_circuit_binary(circuit, loss_fn, dataset_size, max_training_steps,␣
↪lr)
```

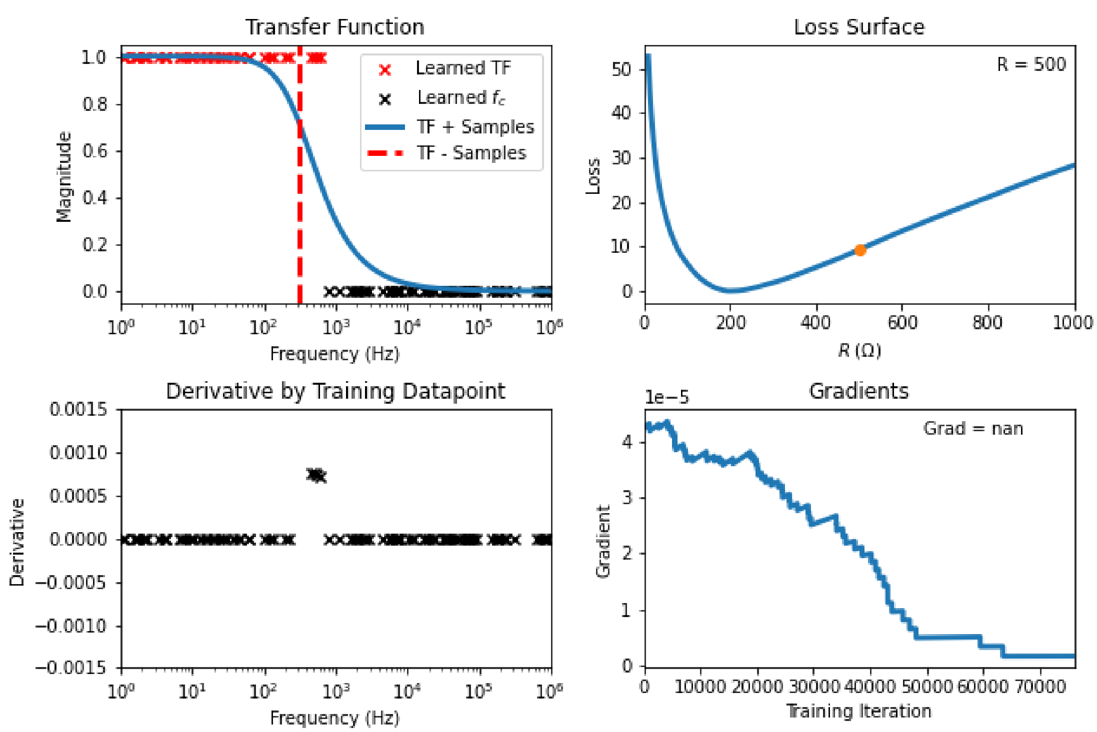
```
Initial Resistor Value: R = 500

Training Iter:  76%|      | 76095/100000 [05:23<01:41, 235.03it/s, Loss:
0.000, R=200]

Final Resistor Value: R = 200
```

[ ]:
```
# Plot transfer function over training
fig, ((ax1, ax2), (ax3, ax4)) = plt.subplots(2, 2, figsize=(9, 6))
ws = 2 * math.pi * 10 ** torch.linspace(0, 6, 1000)
subsample = int(dataset_size / 100)
train_data_mask = train_data_low_bin[1][::subsample] > cutoff_mag
ax1.scatter(train_data_low_bin[0][::subsample][train_data_mask] / (2 * math.
↪pi), np.ones(train_data_mask.sum()), c="r", marker="x")
ax1.scatter(train_data_low_bin[0][::subsample][~train_data_mask] / (2 * math.
↪pi), np.zeros((~train_data_mask).sum()), c="k", marker="x")
mags = evaluate_lp_circuit(ws, R_values_low_bin[0])
learned_tf, = ax1.semilogx(ws / (2 * math.pi), mags, linewidth=3)
cutoff = ws[np.argmax(mags < cutoff_mag)]
cut = ax1.axvline(cutoff / (2 * math.pi), c="red", linestyle="--", linewidth=3)
ax1.set_xlim([1, 1e6])
ax1.set_title("Transfer Function")
ax1.set_xlabel("Frequency (Hz)")
ax1.set_ylabel("Magnitude")
ax1.legend(["Learned TF", "Learned $f_c$", "TF + Samples", "TF - Samples"])

# Show loss surface over training
eval_pts = torch.arange(10, 1001, 1)
eval_vals = evaluate_lp_circuit(train_data_low_bin[0][:, None], eval_pts[None, :
↪])
```

```
loss_surface_mse = loss_fn(eval_vals, (train_data_low_bin[1][:, None].
↪expand(eval_vals.shape) > cutoff_mag).float())
ax2.plot(eval_pts, loss_surface_mse.sum(0), linewidth=3)
cur_loss, = ax2.plot(R_values_low_bin[0], loss_surface_mse[:,␣
↪int(R_values_low_bin[0] - 10)].sum(0), marker="o")
cur_loss_label = ax2.annotate(f"R = {R_values_low_bin[0]:.0f}", (0, 0),␣
↪xytext=(0.82, 0.9), textcoords='axes fraction')
ax2.set_title("Loss Surface")
ax2.set_xlim([0, 1000])
ax2.set_xlabel("$R \; (\Omega)$")
ax2.set_ylabel("Loss")

# Show loss contributions of each data point
cur_circuit = LowPassCircuit(R_values_low_bin[0])
data_losses = loss_fn(cur_circuit(train_data_low_bin[0][::subsample]),␣
↪(train_data_low_bin[1][::subsample] > cutoff_mag).float())
data_grads = torch.zeros(len(data_losses))
for i, dl in enumerate(data_losses):
    data_grads[i] = torch.autograd.grad(dl, cur_circuit.R, retain_graph=True)[0]
data_grads_scat = ax3.scatter(train_data_low_bin[0][::subsample] / (2 * math.
↪pi), data_grads, marker="x", c="k")
ax3.set_xscale("log")
ax3.set_ylabel("Derivative")
ax3.set_xlim([1, 1e6])
ax3.set_ylim([-1.5e-3, 1.5e-3])
ax3.set_xlabel("Frequency (Hz)")
ax3.set_title("Derivative by Training Datapoint")

# Show gradient at each training iteration
ax4.plot(np.arange(len(grad_values_low_bin)), grad_values_low_bin, linewidth=3)
cur_iter, = ax4.plot(0, grad_values_low_bin[0], marker="o")
cur_grad_label = ax4.annotate(f"Grad = {grad_values_low_bin[0]:.2e}", (0, 0),␣
↪xytext=(0.65, 0.9), textcoords='axes fraction')
ax4.set_xlabel("Training Iteration")
ax4.set_ylabel("Gradient")
ax4.set_title("Gradients")
ax4.set_xlim([-1, len(grad_values_low_bin)])

plt.tight_layout()

# Main update function for interactive plots
def update_iter_low_bin(t=0):
    mags = evaluate_lp_circuit(ws, R_values_low_bin[t])
    learned_tf.set_data(ws / (2 * math.pi), mags)
    cutoff = ws[np.argmax(mags < cutoff_mag)]
    cut.set_xdata(cutoff / (2 * math.pi))
```

```
    cur_loss.set_data(R_values_low_bin[t], loss_surface_mse[:,␣
↪int(R_values_low_bin[t] - 10)].sum(0))
    cur_loss_label.set_text(f"R = {R_values_low_bin[t]:.0f}")
    cur_iter.set_data(t, grad_values_low_bin[t])
    cur_grad_label.set_text(f"Grad = {grad_values_low_bin[t]:.2e}")
    cur_circuit = LowPassCircuit(R_values_low_bin[t])
    data_losses = loss_fn(cur_circuit(train_data_low_bin[0][::subsample]),␣
↪(train_data_low_bin[1][::subsample] > cutoff_mag).float())
    data_grads = torch.zeros(len(data_losses))
    for i, dl in enumerate(data_losses):
        data_grads[i] = torch.autograd.grad(dl, cur_circuit.R,␣
↪retain_graph=True)[0]
    data_grads_scat.set_offsets(torch.stack((train_data_low_bin[0][::subsample]␣
↪/ (2 * math.pi), data_grads)).T)
    fig.canvas.draw_idle()

# Include sliders for relevant quantities
ip = interactive(update_iter_low_bin,
                 t=widgets.IntSlider(value=0, min=0, max=len(R_values_low_bin)␣
↪- 1, step=1, description="Training Iteration", style={'description_width':␣
↪'initial'}, layout=Layout(width='100%')))
ip
```

```
interactive(children=(IntSlider(value=0, description='Training Iteration',␣
↪layout=Layout(width='100%'), max=76…
```



0.6 (f) Learning a High Pass Filter from Binary Data

[7]:
```
# Transfer function: evaluates magnitude of given frequencies for a resistor␣
↪value in the high pass circuit
def evaluate_hp_circuit(freqs, R_high):
    ### YOUR CODE HERE
    return torch.sqrt((R_high * cap_value * freqs) ** 2) / torch.sqrt(1 +␣
↪(R_high * cap_value * freqs) ** 2)
    # return ?
    ### END YOUR CODE

# PyTorch model of the high pass circuit (for training)
class HighPassCircuit(nn.Module):
    def __init__(self, R=None):
        super().__init__()
        self.R = nn.Parameter(torch.tensor(R, dtype=float) if R is not None␣
↪else torch.rand(1) * 1000)

    def forward(self, freqs):
        return evaluate_hp_circuit(freqs, self.R)

# Generate training data in a uniform log scale of frequences, then evaluate␣
↪using the true transfer function
R_high_des = 1 / (2 * math.pi * 5000 * cap_value)
def generate_hp_training_data(n):
    rand_ws = 2 * math.pi * torch.pow(10, torch.rand(n) * 6)
    labels = evaluate_hp_circuit(rand_ws, R_high_des)
    return rand_ws, labels

# Train a given low pass filter from binary data
def train_hp_circuit_binary(circuit, loss_fn, dataset_size, max_training_steps,␣
↪lr):

    R_values = [float(circuit.R.data)]
    grad_values = [np.nan]
    train_data = generate_hp_training_data(dataset_size)
    print(f"Initial Resistor Value: R = {float(circuit.R.data):.0f}")
    iter_bar = tqdm.trange(max_training_steps, desc="Training Iter")
    for i in iter_bar:
        pred = circuit(train_data[0])
        loss = loss_fn(pred, (train_data[1] > cutoff_mag).float()).mean()
        ### YOUR CODE HERE
        grad = torch.autograd.grad(loss, circuit.R)
        # grad = torch.autograd.grad(?, ?)
```

```
        ### END YOUR CODE
        with torch.no_grad():
            ### YOUR CODE HERE
            # circuit.R -= ?
            circuit.R -= lr * grad[0]
            ### END YOUR CODE

        R_values.append(float(circuit.R.data))
        grad_values.append(float(grad[0].data))
        iter_bar.set_postfix_str(f"Loss: {float(loss.data):.3f},␣
↪R={float(circuit.R.data):.0f}")
        if loss.data < 1e-6 or abs(grad[0].data) < 1e-6:
            break

    print(f"Final Resistor Value: R = {float(circuit.R.data):.0f}")
    return train_data, R_values, grad_values
```

[9]:
```
# Create a circuit, use loss_fn with learning rate of 1000
circuit = HighPassCircuit(500)
### YOUR CODE HERE
loss_fn = lambda x, y: (1-y) * torch.where(x-cutoff_mag>0,x-cutoff_mag, 0) + y␣
↪* torch.where(cutoff_mag-x>0,cutoff_mag-x, 0)# x:pred
### END YOUR CODE
lr = 1000
train_data_high_bin, R_values_high_bin, grad_values_high_bin =␣
↪train_hp_circuit_binary(circuit, loss_fn, dataset_size, max_training_steps,␣
↪lr)
```

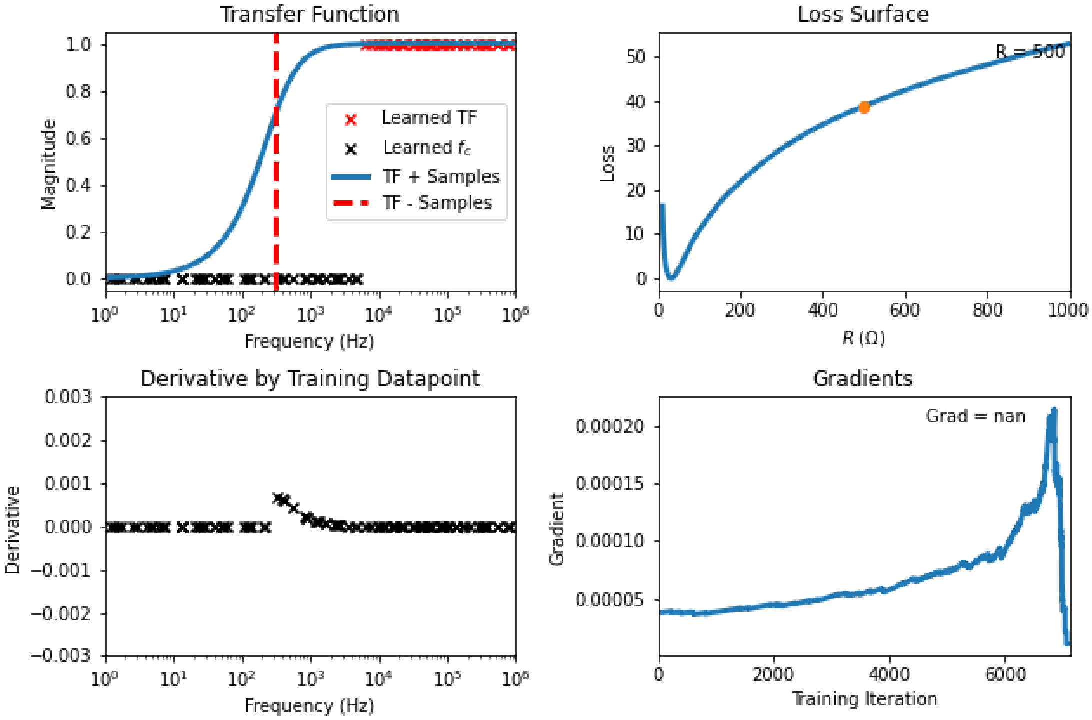
```
Initial Resistor Value: R = 500

Training Iter:   7%|          | 7115/100000 [00:26<05:42, 271.02it/s, Loss:
0.000, R=32]

Final Resistor Value: R = 32
```

[10]:
```
# Plot transfer function over training
fig, ((ax1, ax2), (ax3, ax4)) = plt.subplots(2, 2, figsize=(9, 6))
ws = 2 * math.pi * 10 ** torch.linspace(0, 6, 1000)
subsample = int(dataset_size / 100)
train_data_mask = train_data_high_bin[1][::subsample] > cutoff_mag
ax1.scatter(train_data_high_bin[0][::subsample][train_data_mask] / (2 * math.
↪pi), np.ones(train_data_mask.sum()), c="r", marker="x")
ax1.scatter(train_data_high_bin[0][::subsample][~train_data_mask] / (2 * math.
↪pi), np.zeros((~train_data_mask).sum()), c="k", marker="x")
mags = evaluate_hp_circuit(ws, R_values_high_bin[0])
learned_tf, = ax1.semilogx(ws / (2 * math.pi), mags, linewidth=3)
cutoff = ws[np.argmax(mags > cutoff_mag)]
```

```python
cut = ax1.axvline(cutoff / (2 * math.pi), c="red", linestyle="--", linewidth=3)
ax1.set_xlim([1, 1e6])
ax1.set_title("Transfer Function")
ax1.set_xlabel("Frequency (Hz)")
ax1.set_ylabel("Magnitude")
ax1.legend(["Learned TF", "Learned $f_c$", "TF + Samples", "TF - Samples"])

# Show loss surface over training
eval_pts = torch.arange(10, 1001, 1)
eval_vals = evaluate_hp_circuit(train_data_high_bin[0][:, None], eval_pts[None, :])
loss_surface_mse = loss_fn(eval_vals, (train_data_high_bin[1][:, None].expand(eval_vals.shape) > cutoff_mag).float())
ax2.plot(eval_pts, loss_surface_mse.sum(0), linewidth=3)
cur_loss, = ax2.plot(R_values_high_bin[0], loss_surface_mse[:, int(R_values_high_bin[0] - 10)].sum(0), marker="o")
cur_loss_label = ax2.annotate(f"R = {R_values_high_bin[0]:.0f}", (0, 0), xytext=(0.82, 0.9), textcoords='axes fraction')
ax2.set_title("Loss Surface")
ax2.set_xlim([0, 1000])
ax2.set_xlabel("$R \; (\Omega)$")
ax2.set_ylabel("Loss")

# Show loss contributions of each data point
cur_circuit = HighPassCircuit(R_values_high_bin[0])
data_losses = loss_fn(cur_circuit(train_data_high_bin[0][::subsample]), (train_data_high_bin[1][::subsample] > cutoff_mag).float())
data_grads = torch.zeros(len(data_losses))
for i, dl in enumerate(data_losses):
    data_grads[i] = torch.autograd.grad(dl, cur_circuit.R, retain_graph=True)[0]
data_grads_scat = ax3.scatter(train_data_high_bin[0][::subsample] / (2 * math.pi), data_grads, marker="x", c="k")
ax3.set_xscale("log")
ax3.set_ylabel("Derivative")
ax3.set_xlim([1, 1e6])
ax3.set_ylim([-3e-3, 3e-3])
ax3.set_xlabel("Frequency (Hz)")
ax3.set_title("Derivative by Training Datapoint")

# Show gradient at each training iteration
ax4.plot(np.arange(len(grad_values_high_bin)), grad_values_high_bin, linewidth=3)
cur_iter, = ax4.plot(0, grad_values_high_bin[0], marker="o")
cur_grad_label = ax4.annotate(f"Grad = {grad_values_high_bin[0]:.2e}", (0, 0), xytext=(0.65, 0.9), textcoords='axes fraction')
ax4.set_xlabel("Training Iteration")
```

```
ax4.set_ylabel("Gradient")
ax4.set_title("Gradients")
ax4.set_xlim([-1, len(grad_values_high_bin)])

plt.tight_layout()

# Main update function for interactive plots
def update_iter_high_bin(t=0):
    mags = evaluate_hp_circuit(ws, R_values_high_bin[t])
    learned_tf.set_data(ws / (2 * math.pi), mags)
    cutoff = ws[np.argmax(mags > cutoff_mag)]
    cut.set_xdata(cutoff / (2 * math.pi))
    cur_loss.set_data(R_values_high_bin[t], loss_surface_mse[:,␣
↪int(R_values_high_bin[t] - 10)].sum(0))
    cur_loss_label.set_text(f"R = {R_values_high_bin[t]:.0f}")
    cur_iter.set_data(t, grad_values_high_bin[t])
    cur_grad_label.set_text(f"Grad = {grad_values_high_bin[t]:.2e}")
    cur_circuit = HighPassCircuit(R_values_high_bin[t])
    data_losses = loss_fn(cur_circuit(train_data_high_bin[0][::subsample]),␣
↪(train_data_high_bin[1][::subsample] > cutoff_mag).float())
    data_grads = torch.zeros(len(data_losses))
    for i, dl in enumerate(data_losses):
        data_grads[i] = torch.autograd.grad(dl, cur_circuit.R,␣
↪retain_graph=True)[0]
    data_grads_scat.set_offsets(torch.stack((train_data_high_bin[0][::
↪subsample] / (2 * math.pi), data_grads)).T)
    fig.canvas.draw_idle()

# Include sliders for relevant quantities
ip = interactive(update_iter_high_bin,
                 t=widgets.IntSlider(value=0, min=0, max=len(R_values_high_bin)␣
↪- 1, step=1, description="Training Iteration", style={'description_width':␣
↪'initial'}, layout=Layout(width='100%')))
ip
```

```
interactive(children=(IntSlider(value=0, description='Training Iteration',␣
↪layout=Layout(width='100%'), max=71…
```



0.7 (g) Learning a Band Pass Filter from Binary Data

[16]:
```python
# Transfer function: evaluates magnitude of given frequencies for resistor values in the band pass circuit
def evaluate_bp_circuit(freqs, R_low, R_high):
    ### YOUR CODE HERE
    tmp = torch.sqrt((R_high * cap_value * freqs) ** 2) / torch.sqrt(1 + (R_high * cap_value * freqs) ** 2)
    return 1 / torch.sqrt(1 + (R_low * cap_value * freqs) ** 2) * tmp
    ### END YOUR CODE

# PyTorch model of the band pass circuit (for training)
class BandPassCircuit(nn.Module):
    def __init__(self, R_low=None, R_high=None):
        super().__init__()
        self.R_low = nn.Parameter(torch.tensor(R_low, dtype=float) if R_low is not None else torch.rand(1) * 1000)
        self.R_high = nn.Parameter(torch.tensor(R_high, dtype=float) if R_high is not None else torch.rand(1) * 1000)

    def forward(self, freqs):
        return evaluate_bp_circuit(freqs, self.R_low, self.R_high)
```

```python
# Generate training data in a uniform log scale of frequences, then evaluate␣
↪using true transfer function
R_low_des = 1 / (2 * math.pi * 4000 * cap_value)
R_high_des = 1 / (2 * math.pi * 1000 * cap_value)
def generate_bp_training_data(n):
    rand_ws = 2 * math.pi * torch.pow(10, torch.rand(n) * 6)
    labels = evaluate_bp_circuit(rand_ws, R_low_des, R_high_des)
    return rand_ws, labels

# Train a given low pass filter from binary data
def train_bp_circuit_binary(circuit, loss_fn, dataset_size, max_training_steps,␣
↪lr):

    R_values = [[float(circuit.R_low.data), float(circuit.R_high.data)]]
    grad_values = [[np.nan, np.nan]]
    train_data = generate_bp_training_data(dataset_size)
    print(f"Initial Resistor Values: R_low = {float(circuit.R_low.data):.0f},␣
↪R_high = {float(circuit.R_high.data):.0f}")
    iter_bar = tqdm.trange(max_training_steps, desc="Training Iter")
    for i in iter_bar:
        pred = circuit(train_data[0])
        loss = loss_fn(pred, (train_data[1] > cutoff_mag).float()).mean()
        ### YOUR CODE HERE
        grad = torch.autograd.grad(loss, [circuit.R_low, circuit.R_high])
        ### END YOUR CODE
        with torch.no_grad():
            ### YOUR CODE HERE
            # circuit.R_low -= ?
            # circuit.R_high -= ?
            circuit.R_low -= lr * grad[0]
            circuit.R_high -= lr * grad[1]
            ### END YOUR CODE

        R_values.append([float(circuit.R_low.data), float(circuit.R_high.data)])
        grad_values.append([float(grad[0].data), float(grad[1].data)])
        iter_bar.set_postfix_str(f"Loss: {float(loss.data):.3f},␣
↪R_low={float(circuit.R_low.data):.0f}, R_high={float(circuit.R_high.data):.
↪0f}")
        if loss.data < 1e-6 or (abs(grad[0].data) < 1e-6 and abs(grad[1].data)␣
↪< 1e-6):
            break

    print(f"Final Resistor Values: R_low = {float(circuit.R_low.data):.0f},␣
↪R_high = {float(circuit.R_high.data):.0f}")
    return train_data, R_values, grad_values
```

[26]:
```
# Create a circuit, use loss_fn with learning rate of 1000
circuit = BandPassCircuit(900, 900)
# circuit = BandPassCircuit(500, 500)
lr = 1000
loss_fn = lambda x, y: (1-y) * torch.where(x-cutoff_mag>0,x-cutoff_mag, 0) + y␣
↪* torch.where(cutoff_mag-x>0,cutoff_mag-x, 0)# x:pred
train_data_band_bin, R_values_band_bin, grad_values_band_bin =␣
↪train_bp_circuit_binary(circuit, loss_fn, dataset_size, max_training_steps,␣
↪lr)
```

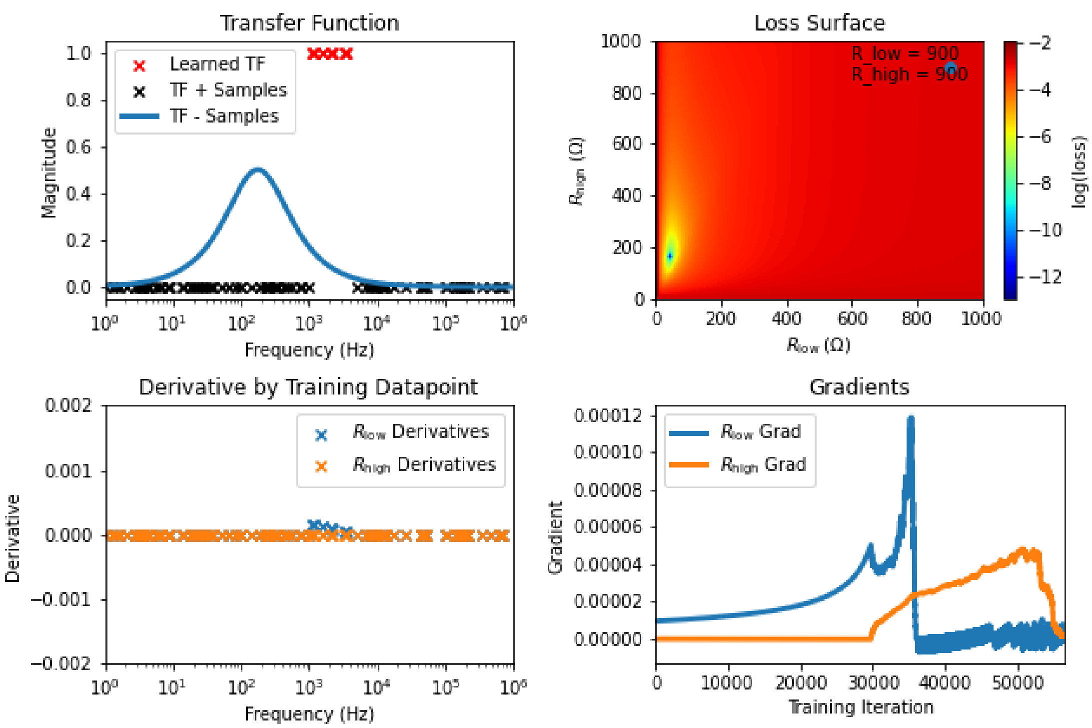
```
Initial Resistor Values: R_low = 900, R_high = 900

Training Iter:  56%|        | 56469/100000 [04:04<03:08, 231.28it/s, Loss:
0.000, R_low=40, R_high=161]

Final Resistor Values: R_low = 40, R_high = 161
```

[28]:
```
# Plot transfer function over training
fig, ((ax1, ax2), (ax3, ax4)) = plt.subplots(2, 2, figsize=(9, 6))
ws = 2 * math.pi * 10 ** torch.linspace(0, 6, 1000)
subsample = int(dataset_size / 100)
train_data_mask = train_data_band_bin[1][::subsample] > cutoff_mag
ax1.scatter(train_data_band_bin[0][::subsample][train_data_mask] / (2 * math.
↪pi), np.ones(train_data_mask.sum()), c="r", marker="x")
ax1.scatter(train_data_band_bin[0][::subsample][~train_data_mask] / (2 * math.
↪pi), np.zeros((~train_data_mask).sum()), c="k", marker="x")
learned_tf, = ax1.semilogx(ws / (2 * math.pi), evaluate_bp_circuit(ws,␣
↪*R_values_band_bin[0]), linewidth=3)
ax1.set_xlim([1, 1e6])
ax1.set_title("Transfer Function")
ax1.set_xlabel("Frequency (Hz)")
ax1.set_ylabel("Magnitude")
ax1.legend(["Learned TF", "TF + Samples", "TF - Samples"])

# Show loss surfaces for BCE and MSE Loss
eval_pts = torch.stack(torch.meshgrid((torch.arange(0, 1000, 10), torch.
↪arange(0, 1000, 10)), indexing="ij"))
eval_vals = evaluate_bp_circuit(train_data_band_bin[0][:, None, None],␣
↪eval_pts[0][None, ...], eval_pts[1][None, ...])
loss_surface = loss_fn(eval_vals, (train_data_band_bin[1][..., None, None].
↪expand(eval_vals.shape) > cutoff_mag).float())
loss_surf = ax2.imshow(torch.log(loss_surface.mean(0)).T, cmap=plt.cm.jet,␣
↪extent=(0, 1000, 0, 1000), aspect="auto", origin="lower")
cur_loss, = ax2.plot(*R_values_band_bin[0], marker="o")
```

```
cur_loss_label = ax2.annotate(f"R_low = {R_values_band_bin[0][0]:.0f}\nR_high =␣
↪{R_values_band_bin[0][1]:.0f}", (0, 0), xytext=(0.6, 0.85), textcoords='axes␣
↪fraction')
ax2.set_title("Loss Surface")
ax2.set_xlabel("$R_\mathrm{low} \; (\Omega)$")
ax2.set_ylabel("$R_\mathrm{high} \; (\Omega)$")
fig.colorbar(loss_surf, ax=ax2, label="log(loss)")

# Show loss contributions of each data point
cur_circuit = BandPassCircuit(*R_values_band_bin[0])
data_losses = loss_fn(cur_circuit(train_data_band_bin[0][::subsample]),␣
↪(train_data_band_bin[1][::subsample] > cutoff_mag).float())
data_grads = torch.zeros((len(data_losses), 2))
for i, dl in enumerate(data_losses):
    data_grads[i] = torch.tensor(torch.autograd.grad(dl, (cur_circuit.R_low,␣
↪cur_circuit.R_high), retain_graph=True))
data_grads_scat1 = ax3.scatter(train_data_band_bin[0][::subsample] / (2 * math.
↪pi), data_grads[:, 0], marker="x")
data_grads_scat2 = ax3.scatter(train_data_band_bin[0][::subsample] / (2 * math.
↪pi), data_grads[:, 1], marker="x")
ax3.set_xscale("log")
ax3.set_ylabel("Derivative")
ax3.set_xlim([1, 1e6])
ax3.set_ylim([-2e-3, 2e-3])
ax3.set_xlabel("Frequency (Hz)")
ax3.set_title("Derivative by Training Datapoint")
ax3.legend(["$R_\mathrm{low}$ Derivatives", "$R_\mathrm{high}$ Derivatives"])

# Show gradient at each training iteration
ax4.plot(np.arange(len(grad_values_band_bin)), grad_values_band_bin,␣
↪linewidth=3)
cur_grad0, = ax4.plot(0, grad_values_band_bin[0][0], marker="o")
cur_grad1, = ax4.plot(0, grad_values_band_bin[0][1], marker="o")
ax4.set_xlabel("Training Iteration")
ax4.set_ylabel("Gradient")
ax4.set_title("Gradients")
ax4.set_xlim([-1, len(grad_values_band_bin)])
ax4.legend(["$R_\mathrm{low}$ Grad", "$R_\mathrm{high}$ Grad"])

plt.tight_layout()

# Main update function for interactive plots
def update_iter_band_bin(t=0):
    mags = evaluate_bp_circuit(ws, *R_values_band_bin[t])
    learned_tf.set_data(ws / (2 * math.pi), mags)
    cur_loss.set_data(*R_values_band_bin[t])
```

```python
    cur_loss_label.set_text(f"R_low = {R_values_band_bin[t][0]:.0f}\nR_high = {R_values_band_bin[t][1]:.0f}")
    cur_grad0.set_data(t, grad_values_band_bin[t][0])
    cur_grad1.set_data(t, grad_values_band_bin[t][1])
    cur_circuit = BandPassCircuit(*R_values_band_bin[t])
    data_losses = loss_fn(cur_circuit(train_data_band_bin[0][::subsample]), (train_data_band_bin[1][::subsample] > cutoff_mag).float())
    data_grads = torch.zeros((len(data_losses), 2))
    for i, dl in enumerate(data_losses):
        data_grads[i] = torch.tensor(torch.autograd.grad(dl, (cur_circuit.R_low, cur_circuit.R_high), retain_graph=True))
    data_grads_scat1.set_offsets(torch.stack((train_data_band_bin[0][::subsample] / (2 * math.pi), data_grads[:, 0])).T)
    data_grads_scat2.set_offsets(torch.stack((train_data_band_bin[0][::subsample] / (2 * math.pi), data_grads[:, 1])).T)
    fig.canvas.draw_idle()

# Include sliders for relevant quantities
ip = interactive(update_iter_band_bin,
                 t=widgets.IntSlider(value=0, min=0, max=len(R_values_band_bin) - 1, step=1, description="Training Iteration", style={'description_width': 'initial'}, layout=Layout(width='100%')))
ip
```

```
interactive(children=(IntSlider(value=0, description='Training Iteration', layout=Layout(width='100%'), max=56…
```



0.8 (h) Learning a Band Pass Filter Bode Plot from Transfer Function Samples

[18]:
```
def evaluate_bp_bode(freqs, low_cutoff, high_cutoff):
    return -20 * nn.ReLU()(torch.log10(freqs / low_cutoff)) + -20 * nn.
 ↪ReLU()(torch.log10(high_cutoff / freqs))

# PyTorch model of the band pass bode plot
class BandPassBodePlot(nn.Module):
    def __init__(self, low_cutoff=None, high_cutoff=None):
        super().__init__()
        self.low_cutoff = nn.Parameter(torch.rand(1) * 5000 if low_cutoff is␣
 ↪None else torch.tensor(float(low_cutoff)))
        self.high_cutoff = nn.Parameter(torch.rand(1) * 5000 if high_cutoff is␣
 ↪None else torch.tensor(float(high_cutoff)))

    def forward(self, freqs):
        return evaluate_bp_bode(freqs, self.low_cutoff, self.high_cutoff)

# Train a given band pass bode plot
def train_bp_bode(bode, loss_fn, dataset_size, max_training_steps, lr):

    cutoff_values = [[float(bode.low_cutoff.data), float(bode.high_cutoff.
 ↪data)]]
    grad_values = [[np.nan, np.nan]]
    train_data = generate_bp_training_data(dataset_size)
    print(f"Initial Cutoff Values: f_c,l = {float(bode.low_cutoff.data / (2 *␣
 ↪math.pi)):.0f} Hz, f_c,h = {float(bode.high_cutoff.data / (2 * math.pi)):.
 ↪0f} Hz")
    iter_bar = tqdm.trange(max_training_steps, desc="Training Iter")
    for i in iter_bar:

        pred = bode(train_data[0])
        loss = loss_fn(pred, 20 * torch.log10(train_data[1])).mean()
        grad = torch.autograd.grad(loss, (bode.low_cutoff, bode.high_cutoff))
        with torch.no_grad():
            bode.low_cutoff -= lr * grad[0]
            bode.high_cutoff -= lr * grad[1]

        cutoff_values.append([float(bode.low_cutoff.data), float(bode.
 ↪high_cutoff.data)])
        grad_values.append([float(grad[0].data), float(grad[1].data)])
        iter_bar.set_postfix_str(f"Loss: {float(loss.data):.3f}, f_c,l =␣
 ↪{float(bode.low_cutoff.data / (2 * math.pi)):.0f} Hz, f_c,h = {float(bode.
 ↪high_cutoff.data / (2 * math.pi)):.0f} Hz")
```

```
        if loss.data < 1e-6 or (abs(grad[0].data) < 1e-6 and abs(grad[1].data)␣
↪< 1e-6):
            break

    print(f"Final Cutoff Values: f_c,l = {float(bode.low_cutoff.data / (2 *␣
↪math.pi)):.0f} Hz, f_c,h = {float(bode.high_cutoff.data / (2 * math.pi)):.
↪0f} Hz")
    return train_data, cutoff_values, grad_values
```

[19]:
```
bode = BandPassBodePlot()
loss_fn = lambda x, y: (x - y) ** 2    # MSE loss
lr = 1000
train_data_band_bode, cutoffs_band_bode, grad_values_band_bode =␣
↪train_bp_bode(bode, loss_fn, dataset_size, max_training_steps, lr)
```

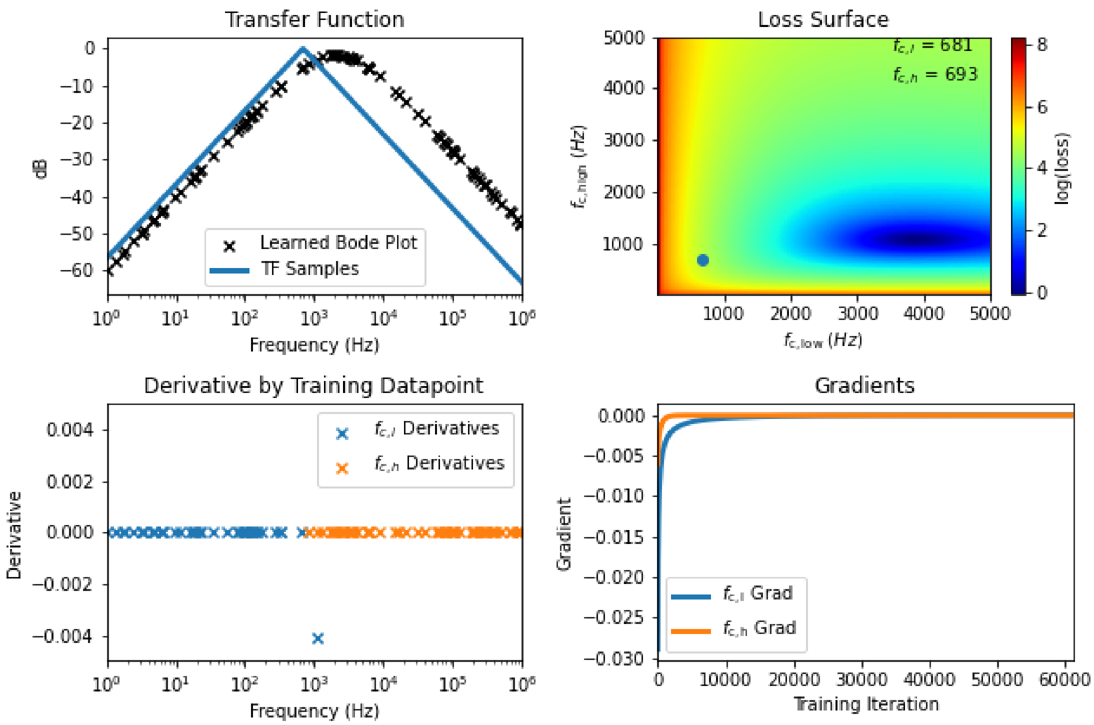
```
Initial Cutoff Values: f_c,l = 681 Hz, f_c,h = 693 Hz

Training Iter:  61%|       | 61235/100000 [03:42<02:20, 275.43it/s, Loss:
0.905, f_c,l = 3864 Hz, f_c,h = 1029 Hz]

Final Cutoff Values: f_c,l = 3864 Hz, f_c,h = 1029 Hz
```

[21]:
```
# Plot transfer function over training
fig, ((ax1, ax2), (ax3, ax4)) = plt.subplots(2, 2, figsize=(9, 6))
ws = 2 * math.pi * 10 ** torch.linspace(0, 6, 1000)
subsample = int(dataset_size / 100)
train_data_mask = train_data_band_bode[1][::subsample] > cutoff_mag
ax1.scatter(train_data_band_bode[0][::subsample]/ (2 * math.pi), 20 * torch.
↪log10(train_data_band_bode[1][::subsample]), c="k", marker="x")
learned_tf, = ax1.semilogx(ws / (2 * math.pi), evaluate_bp_bode(ws,␣
↪*cutoffs_band_bode[0]), linewidth=3)
ax1.set_xlim([1, 1e6])
ax1.set_title("Transfer Function")
ax1.set_xlabel("Frequency (Hz)")
ax1.set_ylabel("dB")
ax1.legend(["Learned Bode Plot", "TF Samples"])

# Show loss surfaces for BCE and MSE Loss
eval_pts = torch.stack(torch.meshgrid((torch.arange(1, 5001, 50), torch.
↪arange(1, 5001, 50)), indexing="ij"))
eval_vals = evaluate_bp_bode(train_data_band_bode[0][:, None, None], 2 * math.
↪pi * eval_pts[0][None, ...], 2 * math.pi * eval_pts[1][None, ...])
loss_surface = loss_fn(eval_vals, 20 * torch.log10(train_data_band_bode[1])[...
↪, None, None].expand(eval_vals.shape))
loss_surf = ax2.imshow(torch.log(loss_surface.mean(0)).T, cmap=plt.cm.jet,␣
↪extent=(1, 5000, 1, 5000), aspect="auto", origin="lower")
```

```
cur_loss, = ax2.plot(cutoffs_band_bode[0][0] / (2 * math.pi),␣
↪cutoffs_band_bode[0][1] / (2 * math.pi), marker="o")
cur_loss_label = ax2.annotate(f"$f_{{c,l}}$ = {cutoffs_band_bode[0][0]:.
↪0f}\n$f_{{c,h}}$ = {cutoffs_band_bode[0][1]:.0f}", (0, 0), xytext=(0.7, 0.
↪82), textcoords='axes fraction')
ax2.set_title("Loss Surface")
ax2.set_xlabel("$f_\mathrm{c,low} \; (Hz)$")
ax2.set_ylabel("$f_\mathrm{c,high} \; (Hz)$")
fig.colorbar(loss_surf, ax=ax2, label="log(loss)")

# Show loss contributions of each data point
cur_bode = BandPassBodePlot(*cutoffs_band_bode[0])
data_losses = loss_fn(cur_bode(train_data_band_bode[0][::subsample]), 20 *␣
↪torch.log10(train_data_band_bode[1][::subsample]))
data_grads = torch.zeros((len(data_losses), 2))
for i, dl in enumerate(data_losses):
    data_grads[i] = torch.tensor(torch.autograd.grad(dl, (cur_bode.low_cutoff,␣
↪cur_bode.high_cutoff), retain_graph=True))
data_grads_scat1 = ax3.scatter(train_data_band_bode[0][::subsample] / (2 * math.
↪pi), data_grads[:, 0], marker="x")
data_grads_scat2 = ax3.scatter(train_data_band_bode[0][::subsample] / (2 * math.
↪pi), data_grads[:, 1], marker="x")
ax3.set_xscale("log")
ax3.set_ylabel("Derivative")
ax3.set_xlim([1, 1e6])
ax3.set_ylim([-5e-3, 5e-3])
ax3.set_xlabel("Frequency (Hz)")
ax3.set_title("Derivative by Training Datapoint")
ax3.legend(["$f_{c,l}$ Derivatives", "$f_{c,h}$ Derivatives"])

# Show gradient at each training iteration
ax4.plot(np.arange(len(grad_values_band_bode)), grad_values_band_bode,␣
↪linewidth=3)
cur_grad0, = ax4.plot(0, grad_values_band_bode[0][0], marker="o")
cur_grad1, = ax4.plot(0, grad_values_band_bode[0][1], marker="o")
ax4.set_xlabel("Training Iteration")
ax4.set_ylabel("Gradient")
ax4.set_title("Gradients")
ax4.set_xlim([-1, len(grad_values_band_bode)])
ax4.legend(["$f_\mathrm{c,l}$ Grad", "$f_\mathrm{c,h}$ Grad"])

plt.tight_layout()

# Main update function for interactive plots
def update_iter_band_bode(t=0):
```

```
    learned_tf.set_data(ws / (2 * math.pi), evaluate_bp_bode(ws,␣
↪*cutoffs_band_bode[t]))
    cur_loss.set_data(cutoffs_band_bode[t][0] / (2 * math.pi),␣
↪cutoffs_band_bode[t][1] / (2 * math.pi))
    cur_loss_label.set_text(f"$f_{{c,l}}$ = {cutoffs_band_bode[t][0] / (2 *␣
↪math.pi):.0f}\n$f_{{c,h}}$ = {cutoffs_band_bode[t][1] / (2 * math.pi):.0f}")
    cur_grad0.set_data(t, grad_values_band_bode[t][0])
    cur_grad1.set_data(t, grad_values_band_bode[t][1])
    cur_bode = BandPassBodePlot(*cutoffs_band_bode[t])
    data_losses = loss_fn(cur_bode(train_data_band_bode[0][::subsample]), 20 *␣
↪torch.log10(train_data_band_bode[1][::subsample]))
    data_grads = torch.zeros((len(data_losses), 2))
    for i, dl in enumerate(data_losses):
        data_grads[i] = torch.tensor(torch.autograd.grad(dl, (cur_bode.
↪low_cutoff, cur_bode.high_cutoff), retain_graph=True))
    data_grads_scat1.set_offsets(torch.stack((train_data_band_bode[0][::
↪subsample] / (2 * math.pi), data_grads[:, 0])).T)
    data_grads_scat2.set_offsets(torch.stack((train_data_band_bode[0][::
↪subsample] / (2 * math.pi), data_grads[:, 1])).T)
    fig.canvas.draw_idle()

# Include sliders for relevant quantities
ip = interactive(update_iter_band_bode,
                 t=widgets.IntSlider(value=0, min=0, max=len(cutoffs_band_bode)␣
↪- 1, step=1, description="Training Iteration", style={'description_width':␣
↪'initial'}, layout=Layout(width='100%')))
ip
```

```
interactive(children=(IntSlider(value=0, description='Training Iteration',␣
↪layout=Layout(width='100%'), max=61…
```



0.9 (i) Learn a Color Organ Circuit

[33]:
```
# PyTorch model of the color organ circuit
class ColorOrganCircuit(nn.Module):
    def __init__(self, R_low=None, R_high=None, R_band_low=None,␣
↪R_band_high=None):
        super().__init__()
        self.low = LowPassCircuit(R_low)
        self.high = HighPassCircuit(R_high)
        self.band = BandPassCircuit(R_band_low, R_band_high)

    def forward(self, freqs):
        return torch.stack((self.low(freqs), self.band(freqs), self.
↪high(freqs)))


# Generate training data in a uniform log scale of frequences, then evaluate␣
↪using the true transfer function
R_low_des = 1 / (2 * math.pi * 800 * cap_value)
R_band_low_des = 1 / (2 * math.pi * 4000 * cap_value)
R_band_high_des = 1 / (2 * math.pi * 1000 * cap_value)
R_high_des = 1 / (2 * math.pi * 5000 * cap_value)
def generate_co_training_data(n):
```

```python
    rand_ws = 2 * math.pi * torch.pow(10, torch.rand(n) * 6)
    labels = torch.stack((evaluate_lp_circuit(rand_ws, R_low_des), evaluate_bp_circuit(rand_ws, R_band_low_des, R_band_high_des), evaluate_hp_circuit(rand_ws, R_high_des)))
    return rand_ws, labels

# Train a given color organ circuit
def train_co_circuit(circuit, loss_fn, dataset_size, max_training_steps, lr):

    R_values = [[float(circuit.low.R.data), float(circuit.band.R_low.data), float(circuit.band.R_high.data), float(circuit.high.R.data)]]
    grad_values = [[np.nan, np.nan, np.nan, np.nan]]
    train_data = generate_co_training_data(dataset_size)
    print(f"Initial Resistor Values: LP: {float(circuit.low.R.data):.0f} Ohms, BP (Low): {float(circuit.band.R_low.data):.0f} Ohms, BP (High): {float(circuit.band.R_high.data):.0f} Ohms, HP: {float(circuit.high.R.data):.0f} Ohms")

    iter_bar = tqdm.trange(max_training_steps, desc="Training Iter")
    for i in iter_bar:
        pred = circuit(train_data[0])
        loss = loss_fn(pred, (train_data[1] > cutoff_mag).float()).mean()
        grad = torch.autograd.grad(loss, (circuit.low.R, circuit.band.R_low, circuit.band.R_high, circuit.high.R))
        with torch.no_grad():
            circuit.low.R -= lr * grad[0]
            circuit.band.R_low -= lr * grad[1]
            circuit.band.R_high -= lr * grad[2]
            circuit.high.R -= lr * grad[3]

        R_values.append([float(circuit.low.R.data), float(circuit.band.R_low.data), float(circuit.band.R_high.data), float(circuit.high.R.data)])
        grad_values.append([float(grad[0].data), float(grad[1].data), float(grad[2].data), float(grad[3].data)])
        iter_bar.set_postfix_str(f"Loss: {float(loss.data):.3f}, Rs = {float(circuit.low.R.data):.0f}, {float(circuit.band.R_low.data):.0f}, {float(circuit.band.R_high.data):.0f}, {float(circuit.high.R.data):.0f}")
        if loss.data < 1e-6 or (abs(grad[0].data) < 1e-6 and abs(grad[1].data) < 1e-6):
            break

    print(f"Final Resistor Values: LP: {float(circuit.low.R.data):.0f} Ohms, BP (Low): {float(circuit.band.R_low.data):.0f} Ohms, BP (High): {float(circuit.band.R_high.data):.0f} Ohms, HP: {float(circuit.high.R.data):.0f} Ohms")
```

```
    print(f"Final Cutoff Frequencies: LP: {1 / (2 * math.pi * cap_value *␣
↪float(circuit.low.R.data)):.0f} Hz, BP (Low): {1 / (2 * math.pi * cap_value␣
↪* float(circuit.band.R_low.data)):.0f} Hz, BP (High): {1 / (2 * math.pi *␣
↪cap_value * float(circuit.band.R_high.data)):.0f} Hz, HP: {1 / (2 * math.pi␣
↪* cap_value * float(circuit.high.R.data)):.0f} Hz")
    return train_data, R_values, grad_values
```

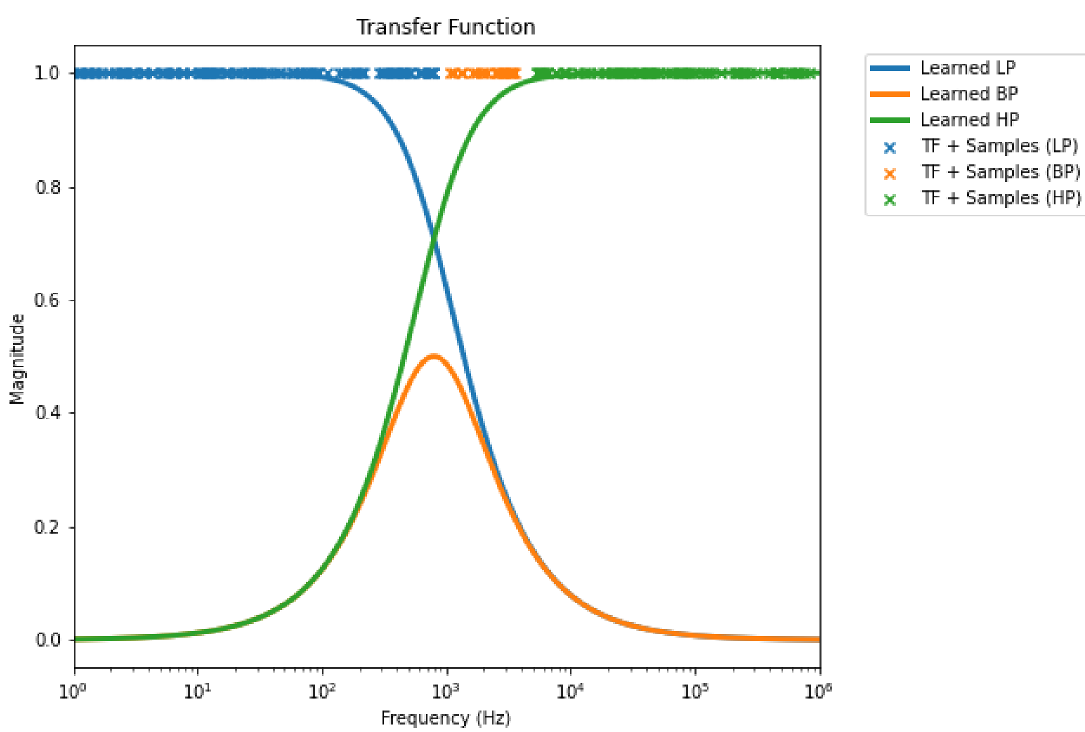
[34]:
```
co = ColorOrganCircuit(200, 200, 200, 200)
loss_fn = lambda x, y: (x - (0.3 + 0.7 * y)) ** 2   # weighted MSE loss
lr = 500
train_data_co, R_values_co, grad_values_co = train_co_circuit(co, loss_fn,␣
↪dataset_size, max_training_steps, lr)
```

```
Initial Resistor Values: LP: 200 Ohms, BP (Low): 200 Ohms, BP (High): 200 Ohms,
HP: 200 Ohms

Training Iter:   9%|          | 9207/100000 [00:43<07:13, 209.52it/s, Loss:
0.047, Rs = 189, 39, 178, 31]

Final Resistor Values: LP: 189 Ohms, BP (Low): 39 Ohms, BP (High): 178 Ohms, HP:
31 Ohms
Final Cutoff Frequencies: LP: 841 Hz, BP (Low): 4130 Hz, BP (High): 894 Hz, HP:
5213 Hz
```

[35]:
```
# Plot transfer function over training
fig, ax1 = plt.subplots(1, 1, figsize=(9, 6))
ws = 2 * math.pi * 10 ** torch.linspace(0, 6, 1000)
subsample = int(dataset_size / 250)
train_data_mask = train_data_co[1][:, ::subsample] > cutoff_mag
learned_tf1, = ax1.semilogx(ws / (2 * math.pi), evaluate_lp_circuit(ws,␣
↪R_values_co[0][0]), linewidth=3)
learned_tf2, = ax1.semilogx(ws / (2 * math.pi), evaluate_bp_circuit(ws,␣
↪*R_values_co[0][1:3]), linewidth=3)
learned_tf3, = ax1.semilogx(ws / (2 * math.pi), evaluate_hp_circuit(ws,␣
↪R_values_co[0][-1]), linewidth=3)
ax1.scatter(train_data_co[0][::subsample][train_data_mask[0]] / (2 * math.pi),␣
↪np.ones(train_data_mask[0].sum()), c=learned_tf1.get_color(), marker="x")
ax1.scatter(train_data_co[0][::subsample][train_data_mask[1]] / (2 * math.pi),␣
↪np.ones(train_data_mask[1].sum()), c=learned_tf2.get_color(), marker="x")
ax1.scatter(train_data_co[0][::subsample][train_data_mask[2]] / (2 * math.pi),␣
↪np.ones(train_data_mask[2].sum()), c=learned_tf3.get_color(), marker="x")
# ax1.scatter(train_data_co[0][::subsample][(~train_data_mask).all(0)] / (2 *␣
↪math.pi), np.zeros((~(train_data_mask.any(0))).sum()), c="k", marker="x")
ax1.set_xlim([1, 1e6])
ax1.set_title("Transfer Function")
ax1.set_xlabel("Frequency (Hz)")
```

```python
ax1.set_ylabel("Magnitude")
ax1.legend(["Learned LP", "Learned BP", "Learned HP",
            "TF + Samples (LP)", "TF + Samples (BP)", "TF + Samples (HP)",
            "TF - Samples"], bbox_to_anchor=(1.05, 1), loc='upper left', ncol=1)

plt.tight_layout()

# Main update function for interactive plots
def update_iter_co(t=0):
    learned_tf1.set_data(ws / (2 * math.pi), evaluate_lp_circuit(ws,␣
 ↪R_values_co[t][0]))
    learned_tf2.set_data(ws / (2 * math.pi), evaluate_bp_circuit(ws,␣
 ↪*R_values_co[t][1:3]))
    learned_tf3.set_data(ws / (2 * math.pi), evaluate_hp_circuit(ws,␣
 ↪R_values_co[t][-1]))
    fig.canvas.draw_idle()

# Include sliders for relevant quantities
ip = interactive(update_iter_co,
                 t=widgets.IntSlider(value=0, min=0, max=len(R_values_co) - 1,␣
 ↪step=1, description="Training Iteration", style={'description_width':␣
 ↪'initial'}, layout=Layout(width='100%')))
ip
```

```
interactive(children=(IntSlider(value=0, description='Training Iteration',␣
 ↪layout=Layout(width='100%'), max=92…
```



0.10 Visualizing the computation graph for the Color Organ

[36]:
```python
from torchviz import make_dot
make_dot(co(generate_co_training_data(dataset_size)[0]), params=dict(co.
 ↪named_parameters()))
```

[36]: