

```
!pip install ipympl torchviz
!pip install torch==1.13 --extra-index-url https://download.pytorch.org/whl/cpu
# restart your runtime after this step
```

```

Looking in indexes: https://pypi.org/simple, https://us-python.pkg.dev/colab-wheels/public/simple/
Collecting ipympl
  Downloading ipympl-0.9.2-py2.py3-none-any.whl (510 kB) 510.3/510.3 KB 5.0 MB/s eta 0:00:00
Collecting torchviz
  Downloading torchviz-0.0.2.tar.gz (4.9 kB)
    Preparing metadata (setup.py) ... done
Collecting matplotlib<4,>=3.4.0
  Downloading matplotlib-3.6.3-cp38-cp38-manylinux_2_12_x86_64.manylinux2010_x86_64.whl (9.4 MB) 9.4/9.4 MB 51.9 MB/s eta 0:00:00
Requirement already satisfied: pillow in /usr/local/lib/python3.8/dist-packages (from ipympl) (7.1.2)
Requirement already satisfied: ipython-genutils in /usr/local/lib/python3.8/dist-packages (from ipympl) (0.2.0)
Requirement already satisfied: traitlets<6 in /usr/local/lib/python3.8/dist-packages (from ipympl) (5.7.1)
Requirement already satisfied: numpy in /usr/local/lib/python3.8/dist-packages (from ipympl) (1.21.6)
Requirement already satisfied: ipython9 in /usr/local/lib/python3.8/dist-packages (from ipympl) (7.9.0)
Requirement already satisfied: ipywidgets<9,>=7.6.0 in /usr/local/lib/python3.8/dist-packages (from ipympl) (7.7.1)
Requirement already satisfied: torch in /usr/local/lib/python3.8/dist-packages (from torchviz) (1.13.1+cu116)

import math
import matplotlib.pyplot as plt
import numpy as np
import torch
import torch.nn as nn
from torch.autograd import Variable
import tqdm

import IPython
from ipywidgets import interactive, widgets, Layout
from IPython.display import display, HTML

Requirement already satisfied: ipykernel>=4.5.1 in /usr/local/lib/python3.8/dist-packages (from ipywidgets<9,>=7.6.0>ipympl) (5.3.4)
print(torch.__version__, torch.cuda.is_available())
# Homework 0 does not require a GPU

1. 13.1+cu116 False

Collecting fonttools>=4.22.0
# enable matplotlib widgets;

# on Google Colab
from google.colab import output
output.enable_custom_widget_manager()

%matplotlib widget
Requirement already satisfied: parso>0.5.0 in /usr/local/lib/python3.8/dist-packages (from jupyter-core>4.4.0>notebook>4.4.1>ipython>7.1.0>ipython>7.1.0>ipympy>0.2.0)

# Constants
cap_value = 1e-06 # Farads
R_init = 500 # Ohms
cutoff_mag = 1. / math.sqrt(2)
cutoff_dB = 20 * math.log10(cutoff_mag)
dataset_size = 1000
max_training_steps = 100000

Requirement already satisfied: Send2Trash in /usr/local/lib/python3.8/dist-packages (from notebook>=4.4.1>widetoolsnbextension>=3.6.0>ipywidgets>=7.6.0>ipykernel>=5.3.4>ipython>7.1.0>ipython>7.1.0>ipympy>0.2.0)

# Transfer function: evaluates magnitude of given frequencies for a resistor value in the low pass circuit
def evaluate_lp_circuit(freqs, R_low):
    return 1. / torch.sqrt(1 + (R_low * cap_value * freqs) ** 2)

Requirement already satisfied: testpath in /usr/local/lib/python3.8/dist-packages (from notebook>=4.4.1>widetoolsnbextension>=3.6.0>ipywidgets>=7.6.0>ipykernel>=5.3.4>ipython>7.1.0>ipython>7.1.0>ipympy>0.2.0)

# Plot transfer function for a given low pass circuit
fig = plt.figure(figsize=(9, 4))
ws = 2 * math.pi * 10 ** torch.linspace(0, 6, 1000)
mags = 20 * torch.log10(evaluate_lp_circuit(ws, R_init))
R_low_des = 1 / (2 * math.pi * 800 * cap_value)
mags_des = 20 * torch.log10(evaluate_lp_circuit(ws, R_low_des))
tf, = plt.semilogx(ws / (2 * math.pi), mags, linewidth=3)
tf_des, = plt.semilogx(ws / (2 * math.pi), mags_des, linestyle="--", linewidth=3)
plt.xlim([1, 1e06])
plt.ylim([-60, 1])
plt.title("Low Pass Transfer Functions")
plt.xlabel("Frequency (Hz)")
plt.ylabel("dB")
plt.grid(which="both")
leg = plt.legend(["Predicted Transfer Function", "Desired Transfer Function"])
plt.tight_layout()

# Main update function for interactive plot
def update_tfs(R=R_init):
    mags = 20 * torch.log10(evaluate_lp_circuit(ws, R))
    tf.set_data(ws / (2 * math.pi), mags)
    fig.canvas.draw_idle()

```

▼ (a) Designing a Low Pass Filter by Matching Transfer Functions

```

Requirement already satisfied: platformdirs>=2.0 in /usr/local/lib/python3.8/dist-packages (from jupyter-core>4.4.0>notebook>4.4.1>ipywidgets>=7.6.0>ipykernel>=5.3.4>ipython>7.1.0>ipython>7.1.0>ipympy>0.2.0)

# Transfer function: evaluates magnitude of given frequencies for a resistor value in the low pass circuit
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    return 1. / torch.sqrt(1 + (R_low * cap_value * freqs) ** 2)

Requirement already satisfied: testpath in /usr/local/lib/python3.8/dist-packages (from notebook>=4.4.1>widetoolsnbextension>=3.6.0>ipywidgets>=7.6.0>ipykernel>=5.3.4>ipython>7.1.0>ipython>7.1.0>ipympy>0.2.0)

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ws = 2 * math.pi * 10 ** torch.linspace(0, 6, 1000)
mags = 20 * torch.log10(evaluate_lp_circuit(ws, R_init))
R_low_des = 1 / (2 * math.pi * 800 * cap_value)
mags_des = 20 * torch.log10(evaluate_lp_circuit(ws, R_low_des))
tf, = plt.semilogx(ws / (2 * math.pi), mags, linewidth=3)
tf_des, = plt.semilogx(ws / (2 * math.pi), mags_des, linestyle="--", linewidth=3)
plt.xlim([1, 1e06])
plt.ylim([-60, 1])
plt.title("Low Pass Transfer Functions")
plt.xlabel("Frequency (Hz)")
plt.ylabel("dB")
plt.grid(which="both")
leg = plt.legend(["Predicted Transfer Function", "Desired Transfer Function"])
plt.tight_layout()

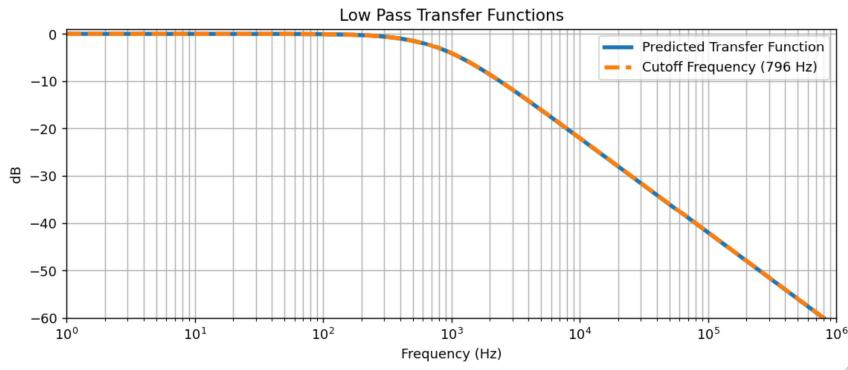
# Main update function for interactive plot
def update_tfs(R=R_init):
    mags = 20 * torch.log10(evaluate_lp_circuit(ws, R))
    tf.set_data(ws / (2 * math.pi), mags)
    fig.canvas.draw_idle()

```

```
# Include sliders for relevant quantities
ip = interactive(update_tfs,
                 R=widgets.IntSlider(value=R_init, min=1, max=1000, step=1, description="R", layout=Layout(width='100%')))
ip
```



Figure 11



▼ (b) Designing a Low pass Filter from Binary Data

```
# Plot transfer function for a given low pass circuit
fig = plt.figure(figsize=(9, 5))
ws = 2 * math.pi * 10 ** torch.linspace(0, 6, 1000)
mags = 20 * torch.log10(evaluate_lp_circuit(ws, R_init))
cutoff = ws[np.argmax(mags < cutoff_dB)]
tf, = plt.semilogx(ws / (2 * math.pi), mags, linewidth=3)
cut = plt.axvline(cutoff / (2 * math.pi), c="red", linestyle="--", linewidth=3)
plt.xlim([1, 1e6])
plt.ylim([-60, 1])
plt.title("Low Pass Transfer Function")
plt.xlabel("Frequency (Hz)")
plt.ylabel("dB")
plt.grid(which="both")
leg = plt.legend(["Transfer Function", f"Cutoff Frequency ({1 / (2 * math.pi * R_init * cap_value):.0f} Hz)"])

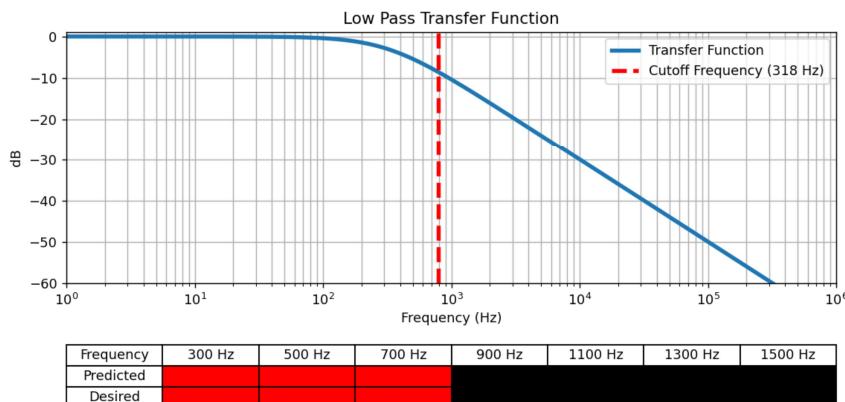
# Plot table of LED on/off values (predicted and desired)
ws_test = 2 * math.pi * np.linspace(300, 1500, num=7)
table_txt = np.zeros((3, len(ws_test) + 1, dtype="U15"))
table_txt[0, :] = ["Frequency"] + [f"{w / (2 * math.pi):.0f} Hz" for w in ws_test]
table_txt[1:, 0] = ["Predicted", "Desired"]
table_colors = np.zeros_like(table_txt, dtype=(np.int32, (3,)))
table_colors[-1, 1:4] = (1, 0, 0)
table_colors[1, 1] = (1, 0, 0)
table_colors[:, :1] = (1, 1, 1)
table_colors[:1, :] = (1, 1, 1)
tab = plt.table(table_txt, table_colors, bbox=[0.0, -0.5, 1.0, 0.25], cellLoc="center")
plt.tight_layout()

# Main update function for interactive plot
def update_lights(R=R_init):
    mags = 20 * torch.log10(evaluate_lp_circuit(ws, R))
    cutoff = ws[np.argmax(mags < cutoff_dB)]
    tf.set_data(ws / (2 * math.pi), mags)
    cut.set_xdata(cutoff / (2 * math.pi))
    for i, w in enumerate(ws_test):
        if w < cutoff:
            tab[(1, i+1)].set_facecolor((1, 0, 0))
        else:
            tab[(1, i+1)].set_facecolor((0, 0, 0))
    leg.get_texts()[1].set_text(f"Cutoff Frequency ({1 / (2 * math.pi * R * cap_value):.0f} Hz)")
    fig.canvas.draw_idle()

# Include sliders for relevant quantities
ip = interactive(update_lights,
                 R=widgets.IntSlider(value=R_init, min=1, max=1000, step=1, description="R", layout=Layout(width='100%')))
ip
```

R

Figure 8



▼ (c) Learning a Low Pass Filter from Desired Transfer Function Samples

```
# PyTorch model of the low pass circuit (for training)
class LowPassCircuit(nn.Module):
    def __init__(self, R=None):
        super().__init__()
        self.R = nn.Parameter(torch.tensor(R, dtype=float) if R is not None else torch.rand(1) * 1000)

    # Note: the forward function is called automatically when the __call__ function of this object is called
    def forward(self, freqs):
        return evaluate_lp_circuit(freqs, self.R)

# Generate training data in a uniform log scale of frequencies, then evaluate using the true transfer function
def generate_lp_training_data(n):
    rand_ws = 2 * math.pi * torch.pow(10, torch.rand(n) * 6)
    labels = evaluate_lp_circuit(rand_ws, R_low_des)
    return rand_ws, labels

# Train a given low pass filter
def train_lp_circuit_tf(circuit, loss_fn, dataset_size, max_training_steps, lr):

    R_values = [float(circuit.R.data)]
    grad_values = [np.nan]
    train_data = generate_lp_training_data(dataset_size)
    print(f"Initial Resistor Value: R = {float(circuit.R.data):.0f}")
    iter_bar = tqdm.trange(max_training_steps, desc="Training Iter")
    for i in iter_bar:
        pred = circuit(train_data[0])
        loss = loss_fn(pred, train_data[1]).mean()
        grad = torch.autograd.grad(loss, circuit.R)
        with torch.no_grad():
            circuit.R -= lr * grad[0]

        R_values.append(float(circuit.R.data))
        grad_values.append(float(grad[0].data))
        iter_bar.set_postfix_str(f"Loss: {float(loss.data):.3f}, R={float(circuit.R.data):.0f}")
        if loss.data < 1e-6 or abs(grad[0].data) < 1e-6:
            break

    print(f"Final Resistor Value: R = {float(circuit.R.data):.0f}")
    return train_data, R_values, grad_values

# Create a circuit, use mean squared error loss w/ learning rate of 200
circuit = LowPassCircuit(200)
loss_fn = lambda x, y: (x - y) ** 2
lr = 200
train_data_low_tf, R_values_low_tf, grad_values_low_tf = train_lp_circuit_tf(circuit, loss_fn, dataset_size, max_training_steps, lr)

Initial Resistor Value: R = 200
Training Iter: 0% | 0/100000 [00:00<?, ?it/s, Loss: 0.000, R=200]Final Resistor Value: R = 200

# Plot transfer function over training
fig, ((ax1, ax2), (ax3, ax4)) = plt.subplots(2, 2, figsize=(9, 6))
ws = 2 * math.pi * 10 ** np.linspace(0, 6, 1000)
```

```

ws = np.pi * 10 * torch.linspace(0, 0, 1000)
subsample = int(dataset_size / 100)
ax1.scatter(train_data_low_tf[0][:subsample] / (2 * math.pi), 20 * torch.log10(train_data_low_tf[1][:subsample]), c="k", marker="x")
learned_tf, = ax1.semilogx(ws / (2 * math.pi), 20 * torch.log10(evaluate_lp_circuit(ws, R_values_low_tf[0])), linewidth=3)
ax1.set_xlim([1, 1e6])
ax1.set_title("Transfer Function")
ax1.set_xlabel("Frequency (Hz)")
ax1.set_ylabel("dB")
ax1.legend(["Learned Transfer Function", "True Transfer Function Samples"])

# Show loss surface over training
eval_pts = torch.arange(10, 1001, 1)
eval_vals = evaluate_lp_circuit(train_data_low_tf[0][:, None], eval_pts[None, :])
loss_surface_mse = loss_fn(eval_vals, train_data_low_tf[1][:, None].expand(eval_vals.shape))
ax2.plot(eval_pts, loss_surface_mse.sum(0), linewidth=3)
cur_loss, = ax2.plot(R_values_low_tf[0], loss_surface_mse[:, int(R_values_low_tf[0] - 10)].sum(0), marker="o")
cur_loss_label = ax2.annotate(f'R = {R_values_low_tf[0][0]}', (0, 0), xytext=(0.82, 0.9), textcoords='axes fraction')
ax2.set_title("Loss Surface")
ax2.set_xlim([0, 1000])
ax2.set_xlabel("$R$; $(\Omega)$")
ax2.set_ylabel("Loss")

# Show loss contributions of each data point
cur_circuit = LowPassCircuit(R_values_low_tf[0])
data_losses = loss_fn(cur_circuit(train_data_low_tf[0][:subsample]), (train_data_low_tf[1][:subsample]).float())
data_grads = torch.zeros(len(data_losses))
for i, dl in enumerate(data_losses):
    data_grads[i] = torch.autograd.grad(dl, cur_circuit.R, retain_graph=True)[0]
data_grads_scat = ax3.scatter(train_data_low_tf[0][:subsample] / (2 * math.pi), data_grads, marker="x", c="k")
ax3.set_xscale("log")
ax3.set_ylabel("Derivative")
ax3.set_xlim([1, 1e6])
ax3.set_ylim([-1e-4, 1e-3])
ax3.set_xlabel("Frequency (Hz)")
ax3.set_title("Derivative by Training Datapoint")

# Show total gradient at each training iteration
ax4.plot(np.arange(len(grad_values_low_tf)), grad_values_low_tf, linewidth=3)
cur_iter, = ax4.plot(0, grad_values_low_tf[0], marker="o")
cur_grad_label = ax4.annotate(f'Grad = {grad_values_low_tf[0]:.2e}', (0, 0), xytext=(0.65, 0.9), textcoords='axes fraction')
ax4.set_xlabel("Training Iteration")
ax4.set_ylabel("Gradient")
ax4.set_title("Gradients")
ax4.set_xlim([-1, len(grad_values_low_tf)])

plt.tight_layout()

# Main update function for interactive plots
def update_iter_tf(t=0):
    learned_tf.set_data(ws / (2 * math.pi), 20 * torch.log10(evaluate_lp_circuit(ws, R_values_low_tf[t])))
    cur_loss.set_data(R_values_low_tf[t], loss_surface_mse[:, int(R_values_low_tf[t] - 10)].sum(0))
    cur_loss_label.set_text(f'R = {R_values_low_tf[t][0]}')
    cur_iter.set_data(t, grad_values_low_tf[t])
    cur_grad_label.set_text(f'Grad = {grad_values_low_tf[t]:.2e}')
    cur_circuit = LowPassCircuit(R_values_low_tf[t])
    data_losses = loss_fn(cur_circuit(train_data_low_tf[0][:subsample]), (train_data_low_tf[1][:subsample]).float())
    data_grads = torch.zeros(len(data_losses))
    for i, dl in enumerate(data_losses):
        data_grads[i] = torch.autograd.grad(dl, cur_circuit.R, retain_graph=True)[0]
    data_grads_scat.set_offsets(torch.stack((train_data_low_tf[0][:subsample] / (2 * math.pi), data_grads)).T)
    fig.canvas.draw_idle()

# Include sliders for relevant quantities
ip = interactive(update_iter_tf,
                  t=widgets.IntSlider(value=0, min=0, max=len(R_values_low_tf) - 1, step=1, description="Training Iteration"))

```

▼ (d) Learning a Low Pass Filter from Binary Data with Mean Squared Error Loss

```
# Train a given low pass filter from binary data
def train_lp_circuit_binary(circuit, loss_fn, dataset_size, max_training_steps, lr):

    R_values = [float(circuit.R.data)]
    grad_values = [np.nan]
    train_data = generate_lp_training_data(dataset_size)
    print(f"Initial Resistor Value: R = {float(circuit.R.data):.0f}")
    iter_bar = tqdm.trange(max_training_steps, desc="Training Iter")
    for i in iter_bar:
        pred = circuit(train_data[0])
        #### YOUR CODE HERE
        loss = loss_fn(pred, (train_data[1] > cutoff_mag).float()).mean()
        # loss = loss_fn(?, ?).mean()
        #### END YOUR CODE
        grad = torch.autograd.grad(loss, circuit.R)
        with torch.no_grad():
            circuit.R -= lr * grad[0]

        R_values.append(float(circuit.R.data))
        grad_values.append(float(grad[0].data))
        iter_bar.set_postfix_str(f"Loss: {float(loss.data):.3f}, R={float(circuit.R.data):.0f}")
        if loss.data < 1e-6 or abs(grad[0].data) < 1e-6:
            break

    print(f"Final Resistor Value: R = {float(circuit.R.data):.0f}")
    return train_data, R_values, grad_values
```

```
# Create a circuit, use MSE loss with learning rate of 200
circuit = LowPassCircuit(500)
loss_fn = lambda x, y: (x - y) ** 2 # x:pred
lr = 200
train_data_low_bin, R_values_low_bin, grad_values_low_bin = train_lp_circuit_binary(circuit, loss_fn, dataset_size, max_training_steps, lr)

Initial Resistor Value: R = 500
Training Iter: 61% |██████████| 61101/100000 [03:27<02:12, 294.31it/s, Loss: 0.016, R=361]Final Resistor Value: R = 361
```

```
# Plot transfer function over training
fig, ((ax1, ax2), (ax3, ax4)) = plt.subplots(2, 2, figsize=(9, 6))
ws = 2 * math.pi * 10 ** torch.linspace(0, 6, 1000)
subsample = int(dataset_size / 100)
train_data_mask = train_data_low_bin[1][:subsample] > cutoff_mag
ax1.scatter(train_data_low_bin[0][:subsample][train_data_mask] / (2 * math.pi), np.ones(train_data_mask.sum()), c="r", marker="x")
ax1.scatter(train_data_low_bin[0][:subsample][~train_data_mask] / (2 * math.pi), np.zeros(~train_data_mask.sum()), c="k", marker="x")
mags = evaluate_lp_circuit(ws, R_values_low_bin[0])
learned_tf, = ax1.semilogx(ws / (2 * math.pi), mags, linewidth=3)
cutoff = ws[np.argmax(mags < cutoff_mag)]
cut = ax1.axvline(cutoff / (2 * math.pi), c="red", linestyle="--", linewidth=3)
ax1.set_xlim([1, 1e6])
ax1.set_title("Transfer Function")
ax1.set_xlabel("Frequency (Hz)")
ax1.set_ylabel("Magnitude")
ax1.legend(["Learned TF", "Learned $f_c$", "TF + Samples", "TF - Samples"])

# Show loss surface over training
eval_pts = torch.arange(10, 1001, 1)
eval_vals = evaluate_lp_circuit(train_data_low_bin[0][:, None], eval_pts[None, :])
loss_surface_mse = loss_fn(eval_vals, (train_data_low_bin[1][:, None].expand(eval_vals.shape) > cutoff_mag).float())
ax2.plot(eval_pts, loss_surface_mse.sum(0), linewidth=3)
cur_loss, = ax2.plot(R_values_low_bin[0], loss_surface_mse[:, int(R_values_low_bin[0] - 10)].sum(0), marker="o")
cur_loss_label = ax2.annotate(f'R = {R_values_low_bin[0]:.0f}', (0, 0), xytext=(0.82, 0.9), textcoords='axes fraction')
ax2.set_title("Loss Surface")
ax2.set_xlim([0, 1000])
ax2.set_xlabel("$R$ ; ($\omega$)")
ax2.set_ylabel("Loss")

# Show loss contributions of each data point
cur_circuit = LowPassCircuit(R_values_low_bin[0])
data_losses = loss_fn(cur_circuit(train_data_low_bin[0][:subsample]), (train_data_low_bin[1][:subsample] > cutoff_mag).float())
data_grads = torch.zeros(len(data_losses))
for i, dl in enumerate(data_losses):
    data_grads[i] = torch.autograd.grad(dl, cur_circuit.R, retain_graph=True)[0]
data_grads_scat = ax3.scatter(train_data_low_bin[0][:subsample] / (2 * math.pi), data_grads, marker="x", c="k")
ax3.set_xscale("log")
ax3.set_ylabel("Derivative")
ax3.set_xlim([1, 1e6])
```

```

ax3.set_yscale([-1.5e-3, 1.5e-3])
ax3.set_xlabel("Frequency (Hz)")
ax3.set_title("Derivative by Training Datapoint")

# Show gradient at each training iteration
ax4.plot(np.arange(len(grad_values_low_bin)), grad_values_low_bin, linewidth=3)
cur_iter, = ax4.plot(0, grad_values_low_bin[0], marker="o")
cur_grad_label = ax4.annotate(f"Grad = {grad_values_low_bin[0]:.2e}", (0, 0), xytext=(0.65, 0.9), textcoords='axes fraction')
ax4.set_xlabel("Training Iteration")
ax4.set_ylabel("Gradient")
ax4.set_title("Gradients")
ax4.set_xlim([-1, len(grad_values_low_bin)])

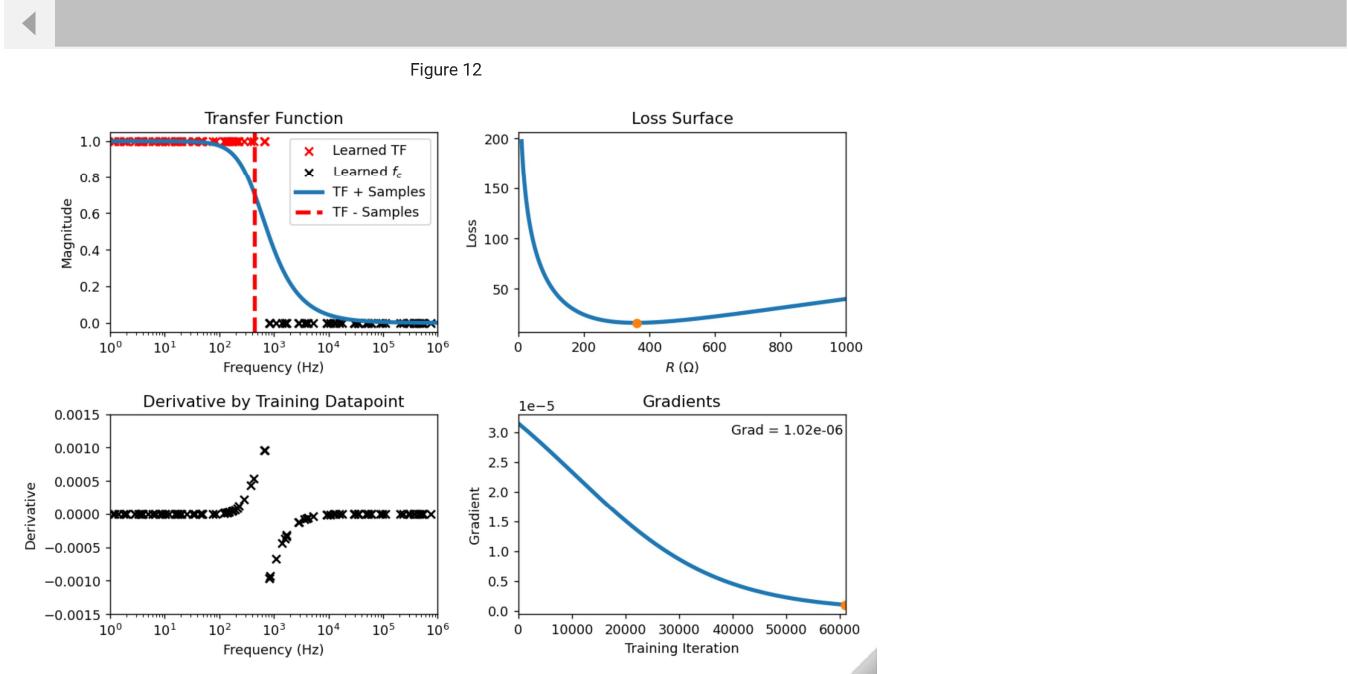
plt.tight_layout()

# Main update function for interactive plots
def update_iter_low_bin(t=0):
    mags = evaluate_lp_circuit(ws, R_values_low_bin[t])
    learned_tf.set_data(ws / (2 * math.pi), mags)
    cutoff = ws[np.argmax(mags < cutoff_mag)]
    cut.set_xdata(cutoff / (2 * math.pi))
    cur_loss.set_data(R_values_low_bin[t], loss_surface_mse[:, int(R_values_low_bin[t] - 10)].sum(0))
    cur_loss_label.set_text(f"R = {R_values_low_bin[t]:.0f}")
    cur_iter.set_data(t, grad_values_low_bin[t])
    cur_grad_label.set_text(f"Grad = {grad_values_low_bin[t]:.2e}")
    cur_circuit = LowPassCircuit(R_values_low_bin[t])
    data_losses = loss_fn(cur_circuit(train_data_low_bin[0][::subsample]), (train_data_low_bin[1][::subsample] > cutoff_mag).float())
    data_grads = torch.zeros(len(data_losses))
    for i, dl in enumerate(data_losses):
        data_grads[i] = torch.autograd.grad(dl, cur_circuit.R, retain_graph=True)[0]
    data_grads_scat.set_offsets(torch.stack((train_data_low_bin[0][::subsample] / (2 * math.pi), data_grads)).T)
    fig.canvas.draw_idle()

# Include sliders for relevant quantities
ip = interactive(update_iter_low_bin,
                  t=widgets.IntSlider(value=0, min=0, max=len(R_values_low_bin) - 1, step=1, description="Training Iteration"))
ip

```

Training Iteration



• (e) Learning a Low Pass Filter from Binary Data with a Different Loss

```

circuit = LowPassCircuit(500)
### YOUR CODE HERE
lr = 200
loss_fn = lambda x, y: (1-y) * torch.where(x-cutoff_mag>0, x-cutoff_mag, 0) + y * torch.where(cutoff_mag-x>0, cutoff_mag-x, 0) # x:pred
# loss_fn = lambda x, y: ?
### END YOUR CODE
train_data_low_bin, R_values_low_bin, grad_values_low_bin = train_lp_circuit_binary(circuit, loss_fn, dataset_size, max_training_steps, lr)

```

```
Initial Resistor Value: R = 500
Training Iter: 76%|██████████| 76095/100000 [05:23<01:41, 235.03it/s, Loss: 0.000, R=200]Final Resistor Value: R = 200
```

```
# Plot transfer function over training
fig, ((ax1, ax2), (ax3, ax4)) = plt.subplots(2, 2, figsize=(9, 6))
ws = 2 * math.pi * 10 ** torch.linspace(0, 6, 1000)
subsample = int(dataset_size / 100)
train_data_mask = train_data_low_bin[1][:subsample] > cutoff_mag
ax1.scatter(train_data_low_bin[0][:subsample][train_data_mask] / (2 * math.pi), np.ones(train_data_mask.sum()), c="r", marker="x")
ax1.scatter(train_data_low_bin[0][:subsample][~train_data_mask] / (2 * math.pi), np.zeros(~train_data_mask.sum()), c="k", marker="x")
mags = evaluate_lp_circuit(ws, R_values_low_bin[0])
learned_tf, = ax1.semilogx(ws / (2 * math.pi), mags, linewidth=3)
cutoff = ws[np.argmax(mags < cutoff_mag)]
cut = ax1.axvline(cutoff / (2 * math.pi), c="red", linestyle="--", linewidth=3)
ax1.set_xlim([1, 1e6])
ax1.set_title("Transfer Function")
ax1.set_xlabel("Frequency (Hz)")
ax1.set_ylabel("Magnitude")
ax1.legend(["Learned TF", "Learned $f_c$", "TF + Samples", "TF - Samples"])

# Show loss surface over training
eval_pts = torch.arange(10, 1001, 1)
eval_vals = evaluate_lp_circuit(train_data_low_bin[0][:, None], eval_pts[None, :])
loss_surface_mse = loss_fn(eval_vals, (train_data_low_bin[1][:, None].expand(eval_vals.shape) > cutoff_mag).float())
ax2.plot(eval_pts, loss_surface_mse.sum(0), linewidth=3)
cur_loss, = ax2.plot(R_values_low_bin[0], loss_surface_mse[:, int(R_values_low_bin[0] - 10)].sum(0), marker="o")
cur_loss_label = ax2.annotate(f'R = {R_values_low_bin[0]:.0f}', (0, 0), xytext=(0.82, 0.9), textcoords='axes fraction')
ax2.set_title("Loss Surface")
ax2.set_xlim([0, 1000])
ax2.set_xlabel("$R$ ; ($\omega$)")
ax2.set_ylabel("Loss")

# Show loss contributions of each data point
cur_circuit = LowPassCircuit(R_values_low_bin[0])
data_losses = loss_fn(cur_circuit(train_data_low_bin[0][:subsample]), (train_data_low_bin[1][:subsample] > cutoff_mag).float())
data_grads = torch.zeros(len(data_losses))
for i, dl in enumerate(data_losses):
    data_grads[i] = torch.autograd.grad(dl, cur_circuit.R, retain_graph=True)[0]
data_grads_scat = ax3.scatter(train_data_low_bin[0][:subsample] / (2 * math.pi), data_grads, marker="x", c="k")
ax3.set_xscale("log")
ax3.set_xlabel("Derivative")
ax3.set_xlim([1, 1e6])
ax3.set_ylim([-1.5e-3, 1.5e-3])
ax3.set_xlabel("Frequency (Hz)")
ax3.set_title("Derivative by Training Datapoint")

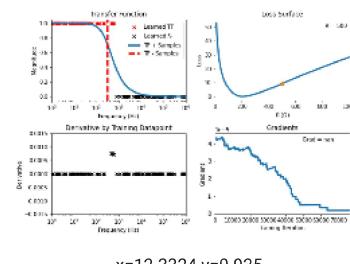
# Show gradient at each training iteration
ax4.plot(np.arange(len(grad_values_low_bin)), grad_values_low_bin, linewidth=3)
cur_iter, = ax4.plot(0, grad_values_low_bin[0], marker="o")
cur_grad_label = ax4.annotate(f'Grad = {grad_values_low_bin[0]:.2e}', (0, 0), xytext=(0.65, 0.9), textcoords='axes fraction')
ax4.set_xlabel("Training Iteration")
ax4.set_ylabel("Gradient")
ax4.set_title("Gradients")
ax4.set_xlim([-1, len(grad_values_low_bin)])
plt.tight_layout()

# Main update function for interactive plots
def update_iter_low_bin(t=0):
    mags = evaluate_lp_circuit(ws, R_values_low_bin[t])
    learned_tf.set_data(ws / (2 * math.pi), mags)
    cutoff = ws[np.argmax(mags < cutoff_mag)]
    cut.set_xdata(cutoff / (2 * math.pi))
    cur_loss.set_data(R_values_low_bin[t], loss_surface_mse[:, int(R_values_low_bin[t] - 10)].sum(0))
    cur_loss_label.set_text(f'R = {R_values_low_bin[t]:.0f}')
    cur_iter.set_data(t, grad_values_low_bin[t])
    cur_grad_label.set_text(f'Grad = {grad_values_low_bin[t]:.2e}')
    cur_circuit = LowPassCircuit(R_values_low_bin[t])
    data_losses = loss_fn(cur_circuit(train_data_low_bin[0][:subsample]), (train_data_low_bin[1][:subsample] > cutoff_mag).float())
    data_grads = torch.zeros(len(data_losses))
    for i, dl in enumerate(data_losses):
        data_grads[i] = torch.autograd.grad(dl, cur_circuit.R, retain_graph=True)[0]
    data_grads_scat.set_offsets(torch.stack((train_data_low_bin[0][:subsample] / (2 * math.pi), data_grads)).T)
    fig.canvas.draw_idle()

# Include sliders for relevant quantities
ip = interactive(update_iter_low_bin,
                 t=widgets.IntSlider(value=0, min=0, max=len(R_values_low_bin) - 1, step=1, description="Training Iteration",
ip
```



Figure 3



v=12.2224 u=0.025

▼ (f) Learning a High Pass Filter from Binary Data

```
# Transfer function: evaluates magnitude of given frequencies for a resistor value in the high pass circuit
def evaluate_hp_circuit(freqs, R_high):
    ### YOUR CODE HERE
    return torch.sqrt((R_high * cap_value * freqs) ** 2) / torch.sqrt(1 + (R_high * cap_value * freqs) ** 2)
    # return ?
    ### END YOUR CODE

# PyTorch model of the high pass circuit (for training)
class HighPassCircuit(nn.Module):
    def __init__(self, R=None):
        super().__init__()
        self.R = nn.Parameter(torch.tensor(R, dtype=float) if R is not None else torch.rand(1) * 1000)

    def forward(self, freqs):
        return evaluate_hp_circuit(freqs, self.R)

# Generate training data in a uniform log scale of frequencies, then evaluate using the true transfer function
R_high_des = 1 / (2 * math.pi * 5000 * cap_value)
def generate_hp_training_data(n):
    rand_ws = 2 * math.pi * torch.pow(10, torch.rand(n) * 6)
    labels = evaluate_hp_circuit(rand_ws, R_high_des)
    return rand_ws, labels

# Train a given low pass filter from binary data
def train_hp_circuit_binary(circuit, loss_fn, dataset_size, max_training_steps, lr):

    R_values = [float(circuit.R.data)]
    grad_values = [np.nan]
    train_data = generate_hp_training_data(dataset_size)
    print(f"Initial Resistor Value: R = {float(circuit.R.data):.0f}")
    iter_bar = tqdm.trange(max_training_steps, desc="Training Iter")
    for i in iter_bar:
        pred = circuit(train_data[0])
        loss = loss_fn(pred, (train_data[1] > cutoff_mag).float()).mean()
        ### YOUR CODE HERE
        grad = torch.autograd.grad(loss, circuit.R)
        # grad = torch.autograd.grad(?, ?)
        ### END YOUR CODE
        with torch.no_grad():
            ### YOUR CODE HERE
            # circuit.R -= ?
            circuit.R -= lr * grad[0]
            ### END YOUR CODE

        R_values.append(float(circuit.R.data))
        grad_values.append(float(grad[0].data))
        iter_bar.set_postfix_str(f"Loss: {float(loss.data):.3f}, R={float(circuit.R.data):.0f}")
        if loss.data < 1e-6 or abs(grad[0].data) < 1e-6:
            break

    print(f"Final Resistor Value: R = {float(circuit.R.data):.0f}")
    return train_data, R_values, grad_values

# Create a circuit, use loss_fn with learning rate of 1000
circuit = HighPassCircuit(500)
### YOUR CODE HERE
loss_fn = lambda x, y: (1-y) * torch.where(x-cutoff_mag>0, x-cutoff_mag, 0) + y * torch.where(cutoff_mag-x>0, cutoff_mag-x, 0) # x:pred
### END YOUR CODE
lr = 1000
train_data_high_bin, R_values_high_bin, grad_values_high_bin = train_hp_circuit_binary(circuit, loss_fn, dataset_size, max_training_steps, lr)
```

```
Initial Resistor Value: R = 500
Training Iter: 7% | 7115/100000 [00:26<05:42, 271.02it/s, Loss: 0.000, R=32]Final Resistor Value: R = 32
```

```
# Plot transfer function over training
fig, ((ax1, ax2), (ax3, ax4)) = plt.subplots(2, 2, figsize=(9, 6))
ws = 2 * math.pi * 10 ** torch.linspace(0, 6, 1000)
subsample = int(dataset_size / 100)
train_data_mask = train_data_high_bin[1][:subsample] > cutoff_mag
ax1.scatter(train_data_high_bin[0][:subsample][train_data_mask] / (2 * math.pi), np.ones(train_data_mask.sum()), c="r", marker="x")
ax1.scatter(train_data_high_bin[0][:subsample][~train_data_mask] / (2 * math.pi), np.zeros(~train_data_mask.sum()), c="k", marker="x")
mags = evaluate_hp_circuit(ws, R_values_high_bin[0])
learned_tf, = ax1.semilogx(ws / (2 * math.pi), mags, linewidth=3)
cutoff = ws[np.argmax(mags > cutoff_mag)]
cut = ax1.axvline(cutoff / (2 * math.pi), c="red", linestyle="--", linewidth=3)
ax1.set_xlim([1, 1e6])
ax1.set_title("Transfer Function")
ax1.set_xlabel("Frequency (Hz)")
ax1.set_ylabel("Magnitude")
ax1.legend(["Learned TF", "Learned $f_c$", "TF + Samples", "TF - Samples"])

# Show loss surface over training
eval_pts = torch.arange(10, 1001, 1)
eval_vals = evaluate_hp_circuit(train_data_high_bin[0][:, None], eval_pts[None, :])
loss_surface_mse = loss_fn(eval_vals, (train_data_high_bin[1][:, None].expand(eval_vals.shape) > cutoff_mag).float())
ax2.plot(eval_pts, loss_surface_mse.sum(0), linewidth=3)
cur_loss, = ax2.plot(R_values_high_bin[0], loss_surface_mse[:, int(R_values_high_bin[0] - 10)].sum(0), marker="o")
cur_loss_label = ax2.annotate(f'R = {R_values_high_bin[0]:.0f}', (0, 0), xytext=(0.82, 0.9), textcoords='axes fraction')
ax2.set_title("Loss Surface")
ax2.set_xlim([0, 1000])
ax2.set_xlabel("$R$ ; ($\omega$)")
ax2.set_ylabel("Loss")

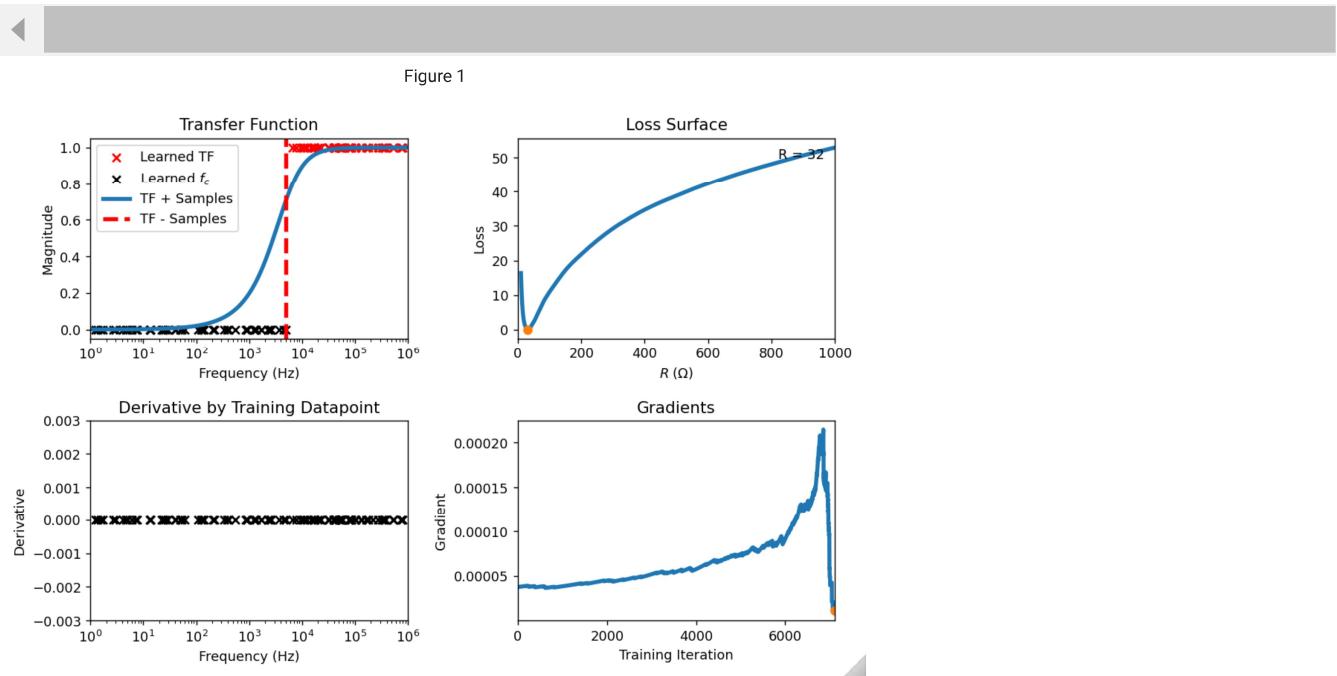
# Show loss contributions of each data point
cur_circuit = HighPassCircuit(R_values_high_bin[0])
data_losses = loss_fn(cur_circuit(train_data_high_bin[0][:subsample]), (train_data_high_bin[1][:subsample] > cutoff_mag).float())
data_grads = torch.zeros(len(data_losses))
for i, dl in enumerate(data_losses):
    data_grads[i] = torch.autograd.grad(dl, cur_circuit.R, retain_graph=True)[0]
data_grads_scat = ax3.scatter(train_data_high_bin[0][:subsample] / (2 * math.pi), data_grads, marker="x", c="k")
ax3.set_xscale("log")
ax3.set_xlabel("Derivative")
ax3.set_xlim([1, 1e6])
ax3.set_ylim([-3e-3, 3e-3])
ax3.set_xlabel("Frequency (Hz)")
ax3.set_title("Derivative by Training Datapoint")

# Show gradient at each training iteration
ax4.plot(np.arange(len(grad_values_high_bin)), grad_values_high_bin, linewidth=3)
cur_iter, = ax4.plot(0, grad_values_high_bin[0], marker="o")
cur_grad_label = ax4.annotate(f'Grad = {grad_values_high_bin[0]:.2e}', (0, 0), xytext=(0.65, 0.9), textcoords='axes fraction')
ax4.set_xlabel("Training Iteration")
ax4.set_ylabel("Gradient")
ax4.set_title("Gradients")
ax4.set_xlim([-1, len(grad_values_high_bin)])
plt.tight_layout()

# Main update function for interactive plots
def update_iter_high_bin(t=0):
    mags = evaluate_hp_circuit(ws, R_values_high_bin[t])
    learned_tf.set_data(ws / (2 * math.pi), mags)
    cutoff = ws[np.argmax(mags > cutoff_mag)]
    cut.set_xdata(cutoff / (2 * math.pi))
    cur_loss.set_data(R_values_high_bin[t], loss_surface_mse[:, int(R_values_high_bin[t] - 10)].sum(0))
    cur_loss_label.set_text(f'R = {R_values_high_bin[t]:.0f}')
    cur_iter.set_data(t, grad_values_high_bin[t])
    cur_grad_label.set_text(f'Grad = {grad_values_high_bin[t]:.2e}')
    cur_circuit = HighPassCircuit(R_values_high_bin[t])
    data_losses = loss_fn(cur_circuit(train_data_high_bin[0][:subsample]), (train_data_high_bin[1][:subsample] > cutoff_mag).float())
    data_grads = torch.zeros(len(data_losses))
    for i, dl in enumerate(data_losses):
        data_grads[i] = torch.autograd.grad(dl, cur_circuit.R, retain_graph=True)[0]
    data_grads_scat.set_offsets(torch.stack((train_data_high_bin[0][:subsample] / (2 * math.pi), data_grads)).T)
    fig.canvas.draw_idle()

# Include sliders for relevant quantities
ip = interactive(update_iter_high_bin,
                 t=widgets.IntSlider(value=0, min=0, max=len(R_values_high_bin) - 1, step=1, description="Training Iteration",
ip
```

Training Iteration



▼ (g) Learning a Band Pass Filter from Binary Data

```
# Transfer function: evaluates magnitude of given frequencies for resistor values in the band pass circuit
def evaluate_bp_circuit(freqs, R_low, R_high):
    ### YOUR CODE HERE
    tmp = torch.sqrt((R_high * cap_value * freqs) ** 2) / torch.sqrt(1 + (R_high * cap_value * freqs) ** 2)
    return 1 / torch.sqrt(1 + (R_low * cap_value * freqs) ** 2) * tmp
    ### END YOUR CODE

# PyTorch model of the band pass circuit (for training)
class BandPassCircuit(nn.Module):
    def __init__(self, R_low=None, R_high=None):
        super().__init__()
        self.R_low = nn.Parameter(torch.tensor(R_low, dtype=float) if R_low is not None else torch.rand(1) * 1000)
        self.R_high = nn.Parameter(torch.tensor(R_high, dtype=float) if R_high is not None else torch.rand(1) * 1000)

    def forward(self, freqs):
        return evaluate_bp_circuit(freqs, self.R_low, self.R_high)

# Generate training data in a uniform log scale of frequencies, then evaluate using true transfer function
R_low_des = 1 / (2 * math.pi * 4000 * cap_value)
R_high_des = 1 / (2 * math.pi * 1000 * cap_value)
def generate_bp_training_data(n):
    rand_ws = 2 * math.pi * torch.pow(10, torch.rand(n) * 6)
    labels = evaluate_bp_circuit(rand_ws, R_low_des, R_high_des)
    return rand_ws, labels

# Train a given low pass filter from binary data
def train_bp_circuit_binary(circuit, loss_fn, dataset_size, max_training_steps, lr):

    R_values = [[float(circuit.R_low.data), float(circuit.R_high.data)]]
    grad_values = [[np.nan, np.nan]]
    train_data = generate_bp_training_data(dataset_size)
    print(f"Initial Resistor Values: R_low = {float(circuit.R_low.data):.0f}, R_high = {float(circuit.R_high.data):.0f}")
    iter_bar = tqdm.trange(max_training_steps, desc="Training Iter")
    for i in iter_bar:
        pred = circuit(train_data[0])
        loss = loss_fn(pred, (train_data[1] > cutoff_mag).float()).mean()
        ### YOUR CODE HERE
        grad = torch.autograd.grad(loss, [circuit.R_low, circuit.R_high])
        ### END YOUR CODE
        with torch.no_grad():
            ### YOUR CODE HERE
            # circuit.R_low -= ?
            # circuit.R_high -= ?
            circuit.R_low -= lr * grad[0]
            circuit.R_high -= lr * grad[1]
            ### END YOUR CODE
        R_values.append([float(circuit.R_low.data), float(circuit.R_high.data)])

```

```

grad_values.append([float(grad[0].data), float(grad[1].data)])
iter_bar.set_postfix_str(f"Loss: {float(loss.data):.3f}, R_low={float(circuit.R_low.data):.0f}, R_high={float(circuit.R_high.data):.0f}")
if loss.data < 1e-6 or (abs(grad[0].data) < 1e-6 and abs(grad[1].data) < 1e-6):
    break

print(f"Final Resistor Values: R_low = {float(circuit.R_low.data):.0f}, R_high = {float(circuit.R_high.data):.0f}")
return train_data, R_values, grad_values

# Create a circuit, use loss_fn with learning rate of 1000
circuit = BandPassCircuit(900, 900)
# circuit = BandPassCircuit(500, 500)
lr = 1000
loss_fn = lambda x, y: (1-y) * torch.where(x-cutoff_mag>0, x-cutoff_mag, 0) + y * torch.where(cutoff_mag-x>0, cutoff_mag-x, 0) # x:pred
train_data_band_bin, R_values_band_bin, grad_values_band_bin = train_bp_circuit_binary(circuit, loss_fn, dataset_size, max_training_steps, lr)

Initial Resistor Values: R_low = 900, R_high = 900
Training Iter: 56%|██████████| 56469/100000 [04:04<03:08, 231.28it/s, Loss: 0.000, R_low=40, R_high=161]Final Resistor Values: R_low = 40

```



```

# Plot transfer function over training
fig, ((ax1, ax2), (ax3, ax4)) = plt.subplots(2, 2, figsize=(9, 6))
ws = 2 * math.pi * 10 ** torch.linspace(0, 6, 1000)
subsample = int(dataset_size / 100)
train_data_mask = train_data_band_bin[1][::subsample] > cutoff_mag
ax1.scatter(train_data_band_bin[0][::subsample][train_data_mask] / (2 * math.pi), np.ones(train_data_mask.sum()), c="r", marker="x")
ax1.scatter(train_data_band_bin[0][::subsample][~train_data_mask] / (2 * math.pi), np.zeros(~train_data_mask).sum(), c="k", marker="x")
learned_tf, = ax1.semilogx(ws / (2 * math.pi), evaluate_bp_circuit(ws, *R_values_band_bin[0]), linewidth=3)
ax1.set_xlim([1, 1e6])
ax1.set_title("Transfer Function")
ax1.set_xlabel("Frequency (Hz)")
ax1.set_ylabel("Magnitude")
ax1.legend(["Learned TF", "TF + Samples", "TF - Samples"])

# Show loss surfaces for BCE and MSE Loss
eval_pts = torch.stack(torch.meshgrid((torch.arange(0, 1000, 10), torch.arange(0, 1000, 10)), indexing="ij"))
eval_vals = evaluate_bp_circuit(train_data_band_bin[0][:, None, None], eval_pts[0][None, ...], eval_pts[1][None, ...])
loss_surface = loss_fn(eval_vals, (train_data_band_bin[1][..., None, None].expand(eval_vals.shape) > cutoff_mag).float())
loss_surf = ax2.imshow(torch.log(loss_surface.mean(0)).T, cmap=plt.cm.jet, extent=(0, 1000, 0, 1000), aspect="auto", origin="lower")
cur_loss, = ax2.plot(*R_values_band_bin[0], marker="o")
cur_loss_label = ax2.annotate(f"R_low = {R_values_band_bin[0][0]:.0f}\nR_high = {R_values_band_bin[0][1]:.0f}", (0, 0), xytext=(0.6, 0.85),
ax2.set_title("Loss Surface")
ax2.set_xlabel("$R_{\mathsf{low}}$ \; ($\Omega$)")
ax2.set_ylabel("$R_{\mathsf{high}}$ \; ($\Omega$)")
fig.colorbar(loss_surf, ax=ax2, label="log(loss)")

# Show loss contributions of each data point
cur_circuit = BandPassCircuit(*R_values_band_bin[0])
data_losses = loss_fn(cur_circuit(train_data_band_bin[0][::subsample]), (train_data_band_bin[1][::subsample] > cutoff_mag).float())
data_grads = torch.zeros((len(data_losses), 2))
for i, dl in enumerate(data_losses):
    data_grads[i] = torch.tensor(torch.autograd.grad(dl, (cur_circuit.R_low, cur_circuit.R_high), retain_graph=True))
data_grads_scat1 = ax3.scatter(train_data_band_bin[0][::subsample] / (2 * math.pi), data_grads[:, 0], marker="x")
data_grads_scat2 = ax3.scatter(train_data_band_bin[0][::subsample] / (2 * math.pi), data_grads[:, 1], marker="x")
ax3.set_xscale("log")
ax3.set_ylabel("Derivative")
ax3.set_xlim([1, 1e6])
ax3.set_ylim([-2e-3, 2e-3])
ax3.set_xlabel("Frequency (Hz)")
ax3.set_title("Derivative by Training Datapoint")
ax3.legend(["$R_{\mathsf{low}}$ Derivatives", "$R_{\mathsf{high}}$ Derivatives"])

# Show gradient at each training iteration
ax4.plot(np.arange(len(grad_values_band_bin)), grad_values_band_bin, linewidth=3)
cur_grad0, = ax4.plot(0, grad_values_band_bin[0][0], marker="o")
cur_grad1, = ax4.plot(0, grad_values_band_bin[0][1], marker="o")
ax4.set_xlabel("Training Iteration")
ax4.set_ylabel("Gradient")
ax4.set_title("Gradients")
ax4.set_xlim([-1, len(grad_values_band_bin)])
ax4.legend(["$R_{\mathsf{low}}$ Grad", "$R_{\mathsf{high}}$ Grad"])

plt.tight_layout()

# Main update function for interactive plots
def update_iter_band_bin(t=0):
    mags = evaluate_bp_circuit(ws, *R_values_band_bin[t])
    learned_tf.set_data(ws / (2 * math.pi), mags)
    cur_loss.set_data(*R_values_band_bin[t])
    cur_loss_label.set_text(f"R_low = {R_values_band_bin[t][0]:.0f}\nR_high = {R_values_band_bin[t][1]:.0f}")


```

```

cur_grad0.set_data(t, grad_values_band_bin[t][0])
cur_grad1.set_data(t, grad_values_band_bin[t][1])
cur_circuit = BandPassCircuit(*R_values_band_bin[t])
data_losses = loss_fn(cur_circuit(train_data_band_bin[0][:subsample]), (train_data_band_bin[1][:subsample] > cutoff_mag).float())
data_grads = torch.zeros((len(data_losses), 2))
for i, dl in enumerate(data_losses):
    data_grads[i] = torch.tensor(torch.autograd.grad(dl, (cur_circuit.R_low, cur_circuit.R_high), retain_graph=True))
data_grads_scatt1.set_offsets(torch.stack((train_data_band_bin[0][:subsample] / (2 * math.pi), data_grads[:, 0])).T)
data_grads_scatt2.set_offsets(torch.stack((train_data_band_bin[0][:subsample] / (2 * math.pi), data_grads[:, 1])).T)
fig.canvas.draw_idle()

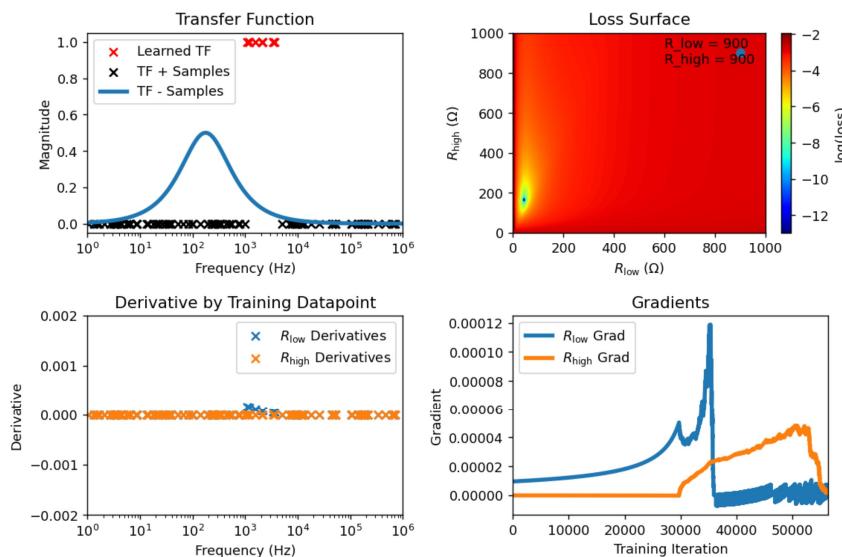
# Include sliders for relevant quantities
ip = interactive(update_iter_band_bin,
                  t=widgets.IntSlider(value=0, min=0, max=len(R_values_band_bin) - 1, step=1, description="Training Iteration",
ip

```

Training Iteration



Figure 7



▼ (h) Learning a Band Pass Filter Bode Plot from Transfer Function Samples

```

def evaluate_bp_bode(freqs, low_cutoff, high_cutoff):
    return -20 * nn.ReLU()(torch.log10(freqs / low_cutoff)) + -20 * nn.ReLU()(torch.log10(high_cutoff / freqs))

# PyTorch model of the band pass bode plot
class BandPassBodePlot(nn.Module):
    def __init__(self, low_cutoff=None, high_cutoff=None):
        super().__init__()
        self.low_cutoff = nn.Parameter(torch.rand(1) * 5000 if low_cutoff is None else torch.tensor(float(low_cutoff)))
        self.high_cutoff = nn.Parameter(torch.rand(1) * 5000 if high_cutoff is None else torch.tensor(float(high_cutoff)))

    def forward(self, freqs):
        return evaluate_bp_bode(freqs, self.low_cutoff, self.high_cutoff)

# Train a given band pass bode plot
def train_bp_bode(bode, loss_fn, dataset_size, max_training_steps, lr):

    cutoff_values = [[float(bode.low_cutoff.data), float(bode.high_cutoff.data)]]
    grad_values = [[np.nan, np.nan]]
    train_data = generate_bp_training_data(dataset_size)
    print(f"Initial Cutoff Values: f_c,l = {float(bode.low_cutoff.data) / (2 * math.pi)} Hz, f_c,h = {float(bode.high_cutoff.data) / (2 * math.pi)} Hz")
    iter_bar = tqdm.trange(max_training_steps, desc="Training Iter")
    for i in iter_bar:

        pred = bode(train_data[0])
        loss = loss_fn(pred, 20 * torch.log10(train_data[1]).mean())
        grad = torch.autograd.grad(loss, (bode.low_cutoff, bode.high_cutoff))
        with torch.no_grad():
            bode.low_cutoff -= lr * grad[0]
            bode.high_cutoff -= lr * grad[1]

    cutoff_values.append([float(bode.low_cutoff.data), float(bode.high_cutoff.data)])
    grad_values.append([float(grad[0].data), float(grad[1].data)])

```

```

        iter_bar.set_postfix_str(f"Loss: {float(loss.data):.3f}, f_c,l = {float(bode.low_cutoff.data / (2 * math.pi)):0f} Hz, f_c,h = {float(bode.high_cutoff.data / (2 * math.pi)):0f} Hz, f_c,h = {float(bode.high_cutoff.data / (2 * math.pi)):0f} Hz")
        if loss.data < 1e-6 or (abs(grad[0].data) < 1e-6 and abs(grad[1].data) < 1e-6):
            break

    print(f"Final Cutoff Values: f_c,l = {float(bode.low_cutoff.data / (2 * math.pi)):0f} Hz, f_c,h = {float(bode.high_cutoff.data / (2 * math.pi)):0f} Hz")
    return train_data, cutoff_values, grad_values

bode = BandPassBodePlot()
loss_fn = lambda x, y: (x - y) ** 2 # MSE loss
lr = 1000
train_data_band_bode, cutoffs_band_bode, grad_values_band_bode = train_bp_bode(bode, loss_fn, dataset_size, max_training_steps, lr)

Initial Cutoff Values: f_c,l = 681 Hz, f_c,h = 693 Hz
Training Iter: 61% [██████] | 61235/100000 [03:42<02:20, 275.43it/s, Loss: 0.905, f_c,l = 3864 Hz, f_c,h = 1029 Hz]Final Cutoff Values:

# Plot transfer function over training
fig, ((ax1, ax2), (ax3, ax4)) = plt.subplots(2, 2, figsize=(9, 6))
ws = 2 * math.pi * 10 ** torch.linspace(0, 6, 1000)
subsample = int(dataset_size / 100)
train_data_mask = train_data_band_bode[1][:subsample] > cutoff_mag
ax1.scatter(train_data_band_bode[0][:subsample]/(2 * math.pi), 20 * torch.log10(train_data_band_bode[1][:subsample]), c="k", marker="x")
learned_tf, = ax1.semilogx(ws / (2 * math.pi), evaluate_bp_bode(ws, *cutoffs_band_bode[0]), linewidth=3)
ax1.set_xlim([1, 1e6])
ax1.set_title("Transfer Function")
ax1.set_xlabel("Frequency (Hz)")
ax1.set_ylabel("dB")
ax1.legend(["Learned Bode Plot", "TF Samples"])

# Show loss surfaces for BCE and MSE Loss
eval_pts = torch.stack(torch.meshgrid((torch.arange(1, 5001, 50), torch.arange(1, 5001, 50)), indexing="ij"))
eval_vals = evaluate_bp_bode(train_data_band_bode[0][:, None, None], 2 * math.pi * eval_pts[0][None, ...], 2 * math.pi * eval_pts[1][None, ...])
loss_surface = loss_fn(eval_vals, 20 * torch.log10(train_data_band_bode[1])[..., None, None].expand(eval_vals.shape))
loss_surf = ax2.imshow(torch.log(loss_surface.mean(0)).T, cmap=plt.cm.jet, extent=(1, 5000, 1, 5000), aspect="auto", origin="lower")
cur_loss, = ax2.plot(cutoffs_band_bode[0][0] / (2 * math.pi), cutoffs_band_bode[0][1] / (2 * math.pi), marker="o")
cur_loss_label = ax2.annotate(f"$f_{\{c,1\}}$ = ${cutoffs_band_bode[0][0]:.0f}$\n$f_{\{c,h\}}$ = ${cutoffs_band_bode[0][1]:.0f}$", (0, 0), xytext=(0, 0))
ax2.set_title("Loss Surface")
ax2.set_xlabel("$f_{\mathcal{C}, \text{low}}$ (Hz)")
ax2.set_ylabel("$f_{\mathcal{C}, \text{high}}$ (Hz)")
fig.colorbar(loss_surf, ax=ax2, label="log(loss)")

# Show loss contributions of each data point
cur_bode = BandPassBodePlot(*cutoffs_band_bode[0])
data_losses = loss_fn(cur_bode(train_data_band_bode[0][:subsample]), 20 * torch.log10(train_data_band_bode[1][:subsample]))
data_grads = torch.zeros((len(data_losses), 2))
for i, dl in enumerate(data_losses):
    data_grads[i] = torch.tensor(torch.autograd.grad(dl, (cur_bode.low_cutoff, cur_bode.high_cutoff), retain_graph=True))
data_grads_scat1 = ax3.scatter(train_data_band_bode[0][:subsample] / (2 * math.pi), data_grads[:, 0], marker="x")
data_grads_scat2 = ax3.scatter(train_data_band_bode[0][:subsample] / (2 * math.pi), data_grads[:, 1], marker="x")
ax3.set_xscale("log")
ax3.set_ylabel("Derivative")
ax3.set_xlim([1, 1e6])
ax3.set_ylim([-5e-3, 5e-3])
ax3.set_xlabel("Frequency (Hz)")
ax3.set_title("Derivative by Training Datapoint")
ax3.legend(["$f_{\mathcal{C},1}$ Derivatives", "$f_{\mathcal{C},h}$ Derivatives"])

# Show gradient at each training iteration
ax4.plot(np.arange(len(grad_values_band_bode)), grad_values_band_bode, linewidth=3)
cur_grad0, = ax4.plot(0, grad_values_band_bode[0][0], marker="o")
cur_grad1, = ax4.plot(0, grad_values_band_bode[0][1], marker="o")
ax4.set_xlabel("Training Iteration")
ax4.set_ylabel("Gradient")
ax4.set_title("Gradients")
ax4.set_xlim([-1, len(grad_values_band_bode)])
ax4.legend(["$f_{\mathcal{C},1}$ Grad", "$f_{\mathcal{C},h}$ Grad"])

plt.tight_layout()

# Main update function for interactive plots
def update_iter_band_bode(t=0):
    learned_tf.set_data(ws / (2 * math.pi), evaluate_bp_bode(ws, *cutoffs_band_bode[t]))
    cur_loss.set_data(cutoffs_band_bode[t][0] / (2 * math.pi), cutoffs_band_bode[t][1] / (2 * math.pi))
    cur_loss_label.set_text(f"$f_{\{c,1\}}$ = ${cutoffs_band_bode[t][0]:.0f}$\n$f_{\{c,h\}}$ = ${cutoffs_band_bode[t][1]:.0f}$")
    cur_grad0.set_data(t, grad_values_band_bode[t][0])
    cur_grad1.set_data(t, grad_values_band_bode[t][1])
    cur_bode = BandPassBodePlot(*cutoffs_band_bode[t])
    data_losses = loss_fn(cur_bode(train_data_band_bode[0][:subsample]), 20 * torch.log10(train_data_band_bode[1][:subsample]))
    data_grads = torch.zeros((len(data_losses), 2))

```

```

for i, dl in enumerate(data_losses):
    data_grads[i] = torch.tensor(torch.autograd.grad(dl, (cur_bode.low_cutoff, cur_bode.high_cutoff), retain_graph=True))
data_grads_scat1.set_offsets(torch.stack((train_data_band_bode[0][::subsample] / (2 * math.pi), data_grads[:, 0])).T)
data_grads_scat2.set_offsets(torch.stack((train_data_band_bode[0][::subsample] / (2 * math.pi), data_grads[:, 1])).T)
fig.canvas.draw_idle()

# Include sliders for relevant quantities
ip = interactive(update_iter_band_bode,
                  t=widgets.IntSlider(value=0, min=0, max=len(cutoffs_band_bode) - 1, step=1, description="Training Iteration",
ip

```

Training Iteration

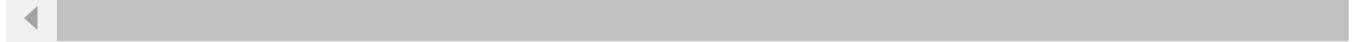
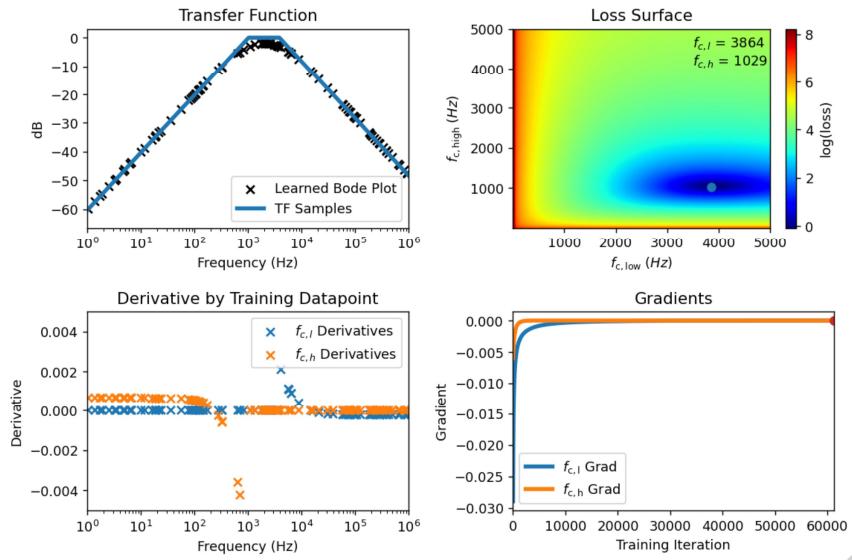


Figure 3



▼ (i) Learn a Color Organ Circuit

```

# PyTorch model of the color organ circuit
class ColorOrganCircuit(nn.Module):
    def __init__(self, R_low=None, R_high=None, R_band_low=None, R_band_high=None):
        super().__init__()
        self.low = LowPassCircuit(R_low)
        self.high = HighPassCircuit(R_high)
        self.band = BandPassCircuit(R_band_low, R_band_high)

    def forward(self, freqs):
        return torch.stack((self.low(freqs), self.band(freqs), self.high(freqs)))

# Generate training data in a uniform log scale of frequencies, then evaluate using the true transfer function
R_low_des = 1 / (2 * math.pi * 800 * cap_value)
R_band_low_des = 1 / (2 * math.pi * 4000 * cap_value)
R_band_high_des = 1 / (2 * math.pi * 1000 * cap_value)
R_high_des = 1 / (2 * math.pi * 5000 * cap_value)
def generate_co_training_data(n):
    rand_ws = 2 * math.pi * torch.pow(10, torch.rand(n) * 6)
    labels = torch.stack((evaluate_lp_circuit(rand_ws, R_low_des), evaluate_bp_circuit(rand_ws, R_band_low_des, R_band_high_des), evaluate_
    return rand_ws, labels

# Train a given color organ circuit
def train_co_circuit(circuit, loss_fn, dataset_size, max_training_steps, lr):

    R_values = [[float(circuit.low.R.data), float(circuit.band.R_low.data), float(circuit.band.R_high.data), float(circuit.high.R.data)]]
    grad_values = [[np.nan, np.nan, np.nan, np.nan]]
    train_data = generate_co_training_data(dataset_size)
    print(f"Initial Resistor Values: LP: {float(circuit.low.R.data):.0f} Ohms, BP (Low): {float(circuit.band.R_low.data):.0f} Ohms, BP

    iter_bar = tqdm.tqdm(max_training_steps, desc="Training Iter")
    for i in iter_bar:
        pred = circuit(train_data[0])
        loss = loss_fn(pred, (train_data[1] > cutoff_mag).float()).mean()
        grad = torch.autograd.grad(loss, (circuit.low.R, circuit.band.R_low, circuit.band.R_high, circuit.high.R))
        with torch.no_grad():
            circuit.low.R -= lr * grad[0]

```

```

circuit.band.R_low -= lr * grad[1]
circuit.band.R_high -= lr * grad[2]
circuit.high.R -= lr * grad[3]

R_values.append([float(circuit.low.R.data), float(circuit.band.R_low.data), float(circuit.band.R_high.data), float(circuit.high.R.data)])
grad_values.append([float(grad[0].data), float(grad[1].data), float(grad[2].data), float(grad[3].data)])
iter_bar.set_postfix_str(f"Loss: {float(loss.data):.3f}, Rs = {float(circuit.low.R.data):.0f}, {float(circuit.band.R_low.data):.0f}, {float(circuit.band.R_high.data):.0f}, {float(circuit.high.R.data):.0f}")
if loss.data < 1e-6 or (abs(grad[0].data) < 1e-6 and abs(grad[1].data) < 1e-6):
    break

print(f"Final Resistor Values: LP: {float(circuit.low.R.data):.0f} Ohms, BP (Low): {float(circuit.band.R_low.data):.0f} Ohms, BP (High): {float(circuit.band.R_high.data):.0f} Ohms")
print(f"Final Cutoff Frequencies: LP: {1 / (2 * math.pi * cap_value * float(circuit.low.R.data)):.0f} Hz, BP (Low): {1 / (2 * math.pi * cap_value * float(circuit.band.R_low.data)):.0f} Hz, BP (High): {1 / (2 * math.pi * cap_value * float(circuit.band.R_high.data)):.0f} Hz")
return train_data, R_values, grad_values

```

```

co = ColorOrganCircuit(200, 200, 200, 200)
loss_fn = lambda x, y: (x - (0.3 + 0.7 * y)) ** 2      # weighted MSE loss
lr = 500
train_data_co, R_values_co, grad_values_co = train_co_circuit(co, loss_fn, dataset_size, max_training_steps, lr)

```

Initial Resistor Values: LP: 200 Ohms, BP (Low): 200 Ohms, BP (High): 200 Ohms, HP: 200 Ohms
Training Iter: 9% |██████████| 9207/100000 [00:43<07:13, 209.52it/s, Loss: 0.047, Rs = 189, 39, 178, 31]Final Resistor Values: LP: 189 Ohms, BP (Low): 189 Ohms, BP (High): 189 Ohms, HP: 5213 Hz



```

# Plot transfer function over training
fig, ax1 = plt.subplots(1, 1, figsize=(9, 6))
ws = 2 * math.pi * 10 ** torch.linspace(0, 6, 1000)
subsample = int(dataset_size / 250)
train_data_mask = train_data_co[1][:, ::subsample] > cutoff_mag
learned_tf1, = ax1.semilogx(ws / (2 * math.pi), evaluate_lp_circuit(ws, R_values_co[0][0]), linewidth=3)
learned_tf2, = ax1.semilogx(ws / (2 * math.pi), evaluate_bp_circuit(ws, *R_values_co[0][1:3]), linewidth=3)
learned_tf3, = ax1.semilogx(ws / (2 * math.pi), evaluate_hp_circuit(ws, R_values_co[0][-1]), linewidth=3)
ax1.scatter(train_data_co[0][:subsample][train_data_mask[0]] / (2 * math.pi), np.ones(train_data_mask[0].sum()), c=learned_tf1.get_color(), marker='o')
ax1.scatter(train_data_co[0][:subsample][train_data_mask[1]] / (2 * math.pi), np.ones(train_data_mask[1].sum()), c=learned_tf2.get_color(), marker='o')
ax1.scatter(train_data_co[0][:subsample][train_data_mask[2]] / (2 * math.pi), np.ones(train_data_mask[2].sum()), c=learned_tf3.get_color(), marker='o')
# ax1.scatter(train_data_co[0][:subsample][~train_data_mask].all(0) / (2 * math.pi), np.zeros(~(train_data_mask.any(0)).sum()), c="k", marker='x')
ax1.set_xlim([1, 1e6])
ax1.set_title("Transfer Function")
ax1.set_xlabel("Frequency (Hz)")
ax1.set_ylabel("Magnitude")
ax1.legend(["Learned LP", "Learned BP", "Learned HP",
           "TF + Samples (LP)", "TF + Samples (BP)", "TF + Samples (HP)",
           "TF - Samples"], bbox_to_anchor=(1.05, 1), loc='upper left', ncol=1)

plt.tight_layout()

```

```

# Main update function for interactive plots
def update_iter_co(t=0):
    learned_tf1.set_data(ws / (2 * math.pi), evaluate_lp_circuit(ws, R_values_co[t][0]))
    learned_tf2.set_data(ws / (2 * math.pi), evaluate_bp_circuit(ws, *R_values_co[t][1:3]))
    learned_tf3.set_data(ws / (2 * math.pi), evaluate_hp_circuit(ws, R_values_co[t][-1]))
    fig.canvas.draw_idle()

# Include sliders for relevant quantities
ip = interactive(update_iter_co,
                 t=widgets.IntSlider(value=0, min=0, max=len(R_values_co) - 1, step=1, description="Training Iteration", style=style)
ip

```

Figure 9



Visualizing the computation graph for the Color Organ

