Part1.Comparison in the performance of all 6 portfolios

As calculated in the .ipynb file, given the initial investment of $1000, on the classical efficient frontier, the expected returns for bellowing three portfolios on the out-of-sample data are:

1.Minimum variance portfolio: $28.2242 2. Maximum return portfolio: -$582.1048

3. Optimal Sharpe ratio portfolio: $279.5667

Given the initial investment of $1000, on the resampled efficient frontier, the expected returns for bellowing three portfolios on the out-of-sample data are:

1.Minimum variance portfolio: $59.0626 2. Maximum return portfolio: $37.3341

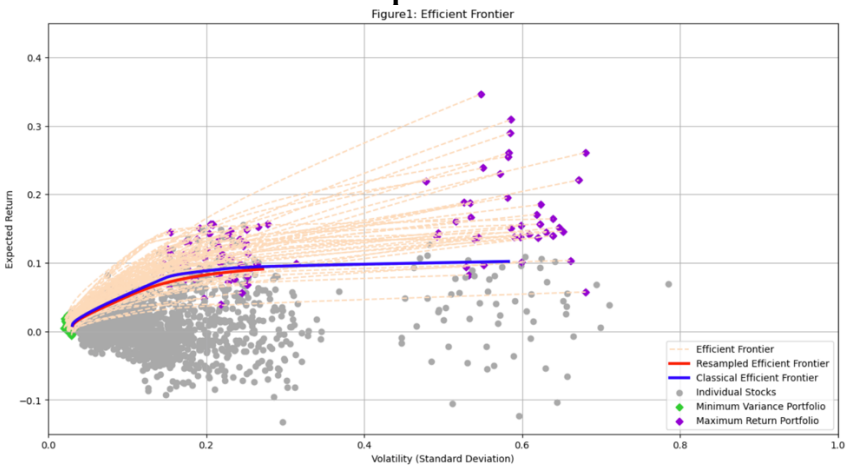
3. Optimal Sharpe ratio portfolio: $327.2898

Findings in part1: By comparing the performance of above six portfolios, the maximum return portfolio on the resampled frontier has the best performance with the expected return = 327.2898. The maximum return portfolio on the classical frontier has the worst performance with the negative return = - 582.1048.

Further, there is a large variation in the performance (expected return) of three portfolios on the classical frontier. On the contrary, there is a relatively slighter variation among the performance (expected return) of three portfolios on the resampled frontier.

Also, by comparing the portfolio of the same nature on the classical frontier and the resampled frontier, we can see overall, the resampled frontier has the better out-of-sample performance.

Part2. The classical and resampled efficient frontiers



Analysis toward Figure1:

Characteristics of the classical frontier:

As seen in Figure1, the classical frontier (blue line) covers the wider range of standard deviations and the wider range of expected returns.

Characteristics of the resampled frontier:

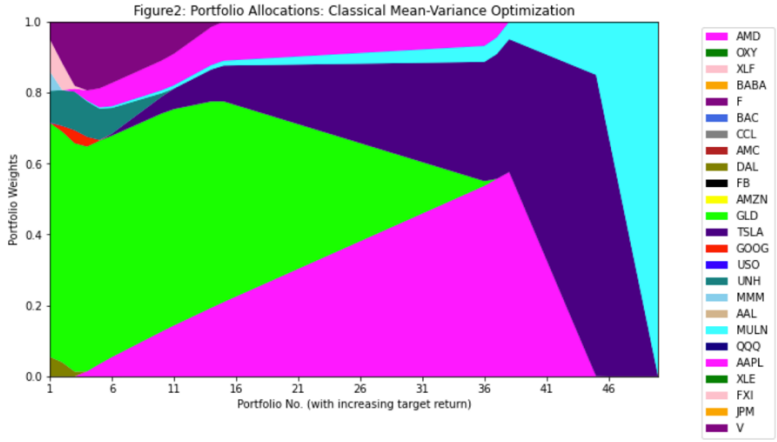
By contrary, the resampled frontier (red line) covers the relatively narrow range of standard deviations and relatively narrow range of returns.

Findings in part2: The resampled frontier lies a little bit lower than the classical frontier and the resampled frontier lies in around the middle of the simulated efficient frontiers.

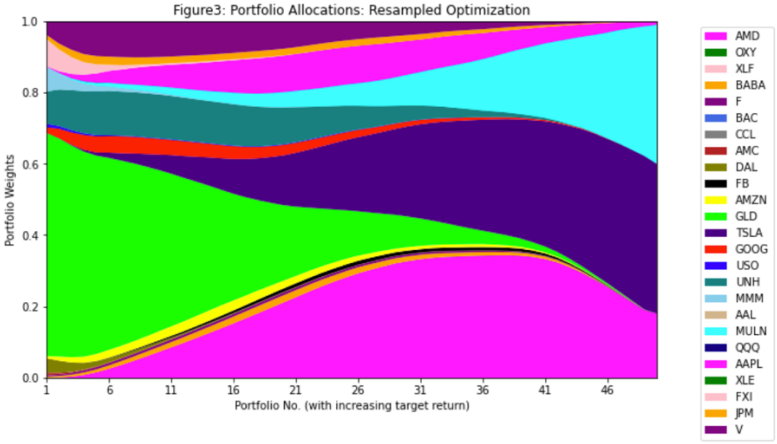
Compared with the classical frontier, the resampled frontier has the lower expected return given the same volatility. Furthermore, the resampled frontier restricts the volatility to a narrower range. As indicated in the recommended paper, lowering and narrowing the frontier contributes to improving the out-of-sample performance. This explains why the resampled frontier has the better out-of-sample performance in part1.

Based on above analysis, we further can conclude that the resampled frontier deal with the investment information uncertainty better since it expects less return as well as less risk, which is consistent with the rational behavior of investors under the uncertainty.

Part3. The portfolio compositions for classical and resampled frontiers



Analysis and findings toward Figure2:
As seen in Figure2, curves are sharp. Further, only around 11 stocks out of 25 stocks show in this composition map by visualizing.
Also, given the certain portfolio No., the number of stocks shown on the vertical slice is relatively rare. This indicates that the compositions of portfolios on the classical frontier are relatively more simple.



Analysis and findings toward Figure3:
As seen in Figure3, curves are smooth and there is a smooth transition of the weight of each stosk from one portfolio to another potfolio along increasing resampled target returns. Further, compared with the Figure2, relatively more stocks are showing in this composition map.

Also, given the certain portfolio No., the number of stocks shown on the vertical slice is relatively more. This indicates that the composition of portfolios on the resampled frontier is relatively more rich.

Findings in part3: by comparing figure2 and figure3, we know that majority of portfolios on the resampled frontier have richer compositions than the portfolios on the classical frontier given the same portfolio No.. This indicates that given the same portfolio No., the portfolio on the resampled frontier is more diversified and less risky than the portfolio on the classical frontier. Meanwhile, the portfolio on the resampled frontier brings relatively less expected return given the same Portfolio No.. Above analysis corresponds with the visualization of resampled frontier and classical frontier in Part2.

Overall, based on above analysis, we find the resampled frontier deals better with the investment information uncertainty and reflect more investment sense since the portfolios on the resampled frontier are more diversified and less risky.

Conclusion for 3 parts: Through analysis in the above three parts, we conclude that 1. The resampled frontier has better out-of-sample performance; 2. Compared with the classical frontier, the resampled frontier expects less return and restricts the volatility (risk) to a narrower range; 3. The portfolios on the resampled frontier have richer compositions, which are more diversified and less risky. Overall, the resampled frontier deals with the investment uncertainty better and reflects more investment sense.