Linear Constraints

$$\mathbf{r}_{\mathrm{l}} \leq \mathbf{A}\mathbf{x} \leq \mathbf{r}_{\mathrm{u}}$$

$$\begin{aligned} \mathbf{A}\mathbf{x} &\leq \mathbf{b} \\ \mathbf{A}_{\mathrm{eq}}\mathbf{x} &= \mathbf{b}_{\mathrm{eq}} \end{aligned}$$

Nonlinear Constraints

$$\mathbf{c}_{\mathrm{l}} \leq \mathbf{nlcon}\left(\mathbf{x}\right) \leq \mathbf{c}_{\mathrm{u}}$$

$$\mathbf{nlcon}\left(\mathbf{x}\right) \underbrace{[\leq \mathit{or} \geq \mathit{or} =]}_{\mathbf{nle}} \mathbf{nlrhs}$$

System of Linear Equations

$$\mathbf{A}\mathbf{x} = \mathbf{b}$$

$$\begin{aligned} a + c &= 2200 \\ 4a + 1.5c &= 5050 \end{aligned}$$

$$\begin{bmatrix} 1 & 1 \\ 4 & 1.5 \end{bmatrix} \begin{bmatrix} a \\ c \end{bmatrix} = \begin{bmatrix} 2200 \\ 5050 \end{bmatrix}$$

Linear Programs

$$\begin{aligned} &\min_{\mathbf{x}} \ \mathbf{f}^T\mathbf{x} \\ \text{subject to: } &\mathbf{A}\mathbf{x} \leq \mathbf{b} \\ &\mathbf{A}_{\mathrm{eq}}\mathbf{x} = \mathbf{b}_{\mathrm{eq}} \\ &\mathbf{l}_{\mathrm{b}} \leq \mathbf{x} \leq \mathbf{u}_{\mathrm{b}} \end{aligned}$$

$$\text{return} = -0.07x - 0.08y - 0.12z$$

$$
\begin{aligned}
x + y + z &= 12000 \\
z &\leq 2000 \\
-\frac{1}{3}x + y &\leq 0 \\
x, y, z &\geq 0
\end{aligned}
$$

$$
\begin{aligned}
\min_{\mathbf{x}} & -\begin{bmatrix} 0.07 \\ 0.08 \\ 0.12 \end{bmatrix}^T \mathbf{x} \\
\text{subject to: } & \begin{bmatrix} -\frac{1}{3} & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \mathbf{x} \leq \begin{bmatrix} 0 \\ 2000 \end{bmatrix} \\
& \begin{bmatrix} 1 & 1 & 1 \end{bmatrix} \mathbf{x} = \begin{bmatrix} 12000 \end{bmatrix} \\
& \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix} \leq \mathbf{x}
\end{aligned}
$$

Binary Integer Linear Programs

$$
\begin{aligned}
\min_{\mathbf{x}} \ & \mathbf{f}^T \mathbf{x} \\
\text{subject to: } & \mathbf{A}\mathbf{x} \leq \mathbf{b} \\
& \mathbf{A}_{\text{eq}}\mathbf{x} = \mathbf{b}_{\text{eq}} \\
& \mathbf{x} \in \{0, 1\}
\end{aligned}
$$

$$
\text{return} = -0.2x_1 - 0.3x_2 - 0.5x_3 - 0.1x_4
$$

$$
\begin{aligned}
0.5x_1 + 1.0x_2 + 1.5x_3 + 0.1x_4 &\leq 3.1 \\
0.3x_1 + 0.8x_2 + 1.5x_3 + 0.4x_4 &\leq 2.5 \\
0.2x_1 + 0.2x_2 + 0.3x_3 + 0.1x_4 &\leq 0.4
\end{aligned}
$$

$$
\begin{aligned}
\min_{\mathbf{x}} & -\begin{bmatrix}0.2\\0.3\\0.5\\0.1\end{bmatrix}^T \mathbf{x} \\
\text{subject to: } & \begin{bmatrix}0.5 & 1.0 & 1.5 & 0.1\\0.3 & 0.8 & 1.5 & 0.4\\0.2 & 0.2 & 0.3 & 0.1\end{bmatrix} \mathbf{x} \leq \begin{bmatrix}3.1\\2.5\\0.4\end{bmatrix} \\
& \mathbf{x} \in \{0,1\}
\end{aligned}
$$

Mixed Integer Linear Programs

$$
\begin{aligned}
\min_{\mathbf{x}} \ & \mathbf{f}^T\mathbf{x} \\
\text{subject to: } & \mathbf{A}\mathbf{x} \leq \mathbf{b} \\
& \mathbf{A}_{\text{eq}}\mathbf{x} = \mathbf{b}_{\text{eq}} \\
& \mathbf{l}_{\text{b}} \leq \mathbf{x} \leq \mathbf{u}_{\text{b}} \\
& x_i \in \mathbb{Z} \\
& x_j \in \{0,1\}
\end{aligned}
$$

$$\text{profit} = -12x_{11} - 16x_{12} - 12x_{21} - 16x_{22} + 45e3f_{11} + 76e3f_{12} + 45e3f_{21} + 76e3f_{22}$$

$$
\begin{aligned}
\frac{1}{52}x_{11} + \frac{1}{38}x_{12} &\leq 480 \\
\frac{1}{42}x_{21} + \frac{1}{23}x_{22} &\leq 720
\end{aligned}
$$

$$
\begin{aligned}
x_{11} - 52\,(480)\,f_{11} &\leq 0 \\
x_{12} - 38\,(480)\,f_{12} &\leq 0 \\
x_{21} - 42\,(720)\,f_{21} &\leq 0 \\
x_{22} - 23\,(720)\,f_{22} &\leq 0
\end{aligned}
$$

$$\min_{\mathbf{x}} -\begin{bmatrix}12 & 16 & 12 & 16 & -45e3 & -76e3 & -45e3 & -76e3\end{bmatrix}\mathbf{x}$$

$$\text{subject to: } \begin{bmatrix} \frac{1}{52} & \frac{1}{38} & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & \frac{1}{42} & \frac{1}{23} & 0 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 & -52\,(480) & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & -38\,(480) & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & -42\,(720) & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 & -23\,(720) \end{bmatrix}\mathbf{x} \leq \begin{bmatrix}480\\720\\0\\0\\0\\0\end{bmatrix}$$

$$\mathbf{0} \leq \mathbf{x}$$

$$x_{1,2,3,4} \in \mathbb{Z}$$

$$x_{5,6,7,8} \in \{0,1\}$$

Quadratic Programs

$$\begin{aligned} \min_{\mathbf{x}} \ & \frac{1}{2}\mathbf{x}^T\mathbf{H}\mathbf{x} + \mathbf{f}^T\mathbf{x} \\ \text{subject to: } & \mathbf{A}\mathbf{x} \leq \mathbf{b} \\ & \mathbf{A}_{\text{eq}}\mathbf{x} = \mathbf{b}_{\text{eq}} \\ & \mathbf{l}_{\text{b}} \leq \mathbf{x} \leq \mathbf{u}_{\text{b}} \end{aligned}$$

$$\begin{aligned} \min_{\mathbf{x}} \ & 0.5x_1^2 + x_2^2 - x_1x_2 - 2x_1 - 6x_2 \\ \text{subject to: } & x_1 + x_2 \leq 2 \\ & -x_1 + 2x_2 \leq 2 \\ & 2x_1 + x_2 \leq 3 \\ & \mathbf{0} \leq \mathbf{x} \end{aligned}$$

$$\min_{\mathbf{x}} \ \frac{1}{2}\mathbf{x}^T\begin{bmatrix}1 & -1\\ -1 & 2\end{bmatrix}\mathbf{x} + \begin{bmatrix}-2\\-6\end{bmatrix}^T\mathbf{x}$$

$$\text{subject to: } \begin{bmatrix}1 & 1\\ -1 & 2\\ 2 & 1\end{bmatrix}\mathbf{x} \leq \begin{bmatrix}2\\2\\3\end{bmatrix}$$

$$\mathbf{0} \leq \mathbf{x}$$

Quadratically Constrained Quadratic Programs

$$\begin{aligned} \min_{\mathbf{x}} \ & \frac{1}{2}\mathbf{x}^T\mathbf{H}\mathbf{x} + \mathbf{f}^T\mathbf{x} \\ \text{subject to: } & \mathbf{A}\mathbf{x} \leq \mathbf{b} \\ & \mathbf{A}_{\text{eq}}\mathbf{x} = \mathbf{b}_{\text{eq}} \\ & \mathbf{l}_{\text{b}} \leq \mathbf{x} \leq \mathbf{u}_{\text{b}} \\ & \forall i = 0...q : \mathbf{x}^T\mathbf{Q}_i\mathbf{x} + \mathbf{l}_i^T\mathbf{x} \leq \mathbf{r}_i \end{aligned}$$

$$\begin{aligned} \min_{\mathbf{x}} \ & 0.5x_1^2 + 0.5x_2^2 - 2x_1 - 2x_2 \\ \text{subject to: } & -x_1 + x_2 \leq 2 \\ & x_1 + 3x_2 \leq 5 \\ & x_1^2 + x_2^2 - 2x_2 \leq 1 \\ & \mathbf{0} \leq \mathbf{x} \end{aligned}$$

$$\begin{aligned} \min_{\mathbf{x}} \ & \frac{1}{2}\mathbf{x}^T \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} \mathbf{x} + \begin{bmatrix} -2 \\ -2 \end{bmatrix}^T \mathbf{x} \\ \text{subject to: } & \begin{bmatrix} -1 & 1 \\ 1 & 3 \end{bmatrix} \mathbf{x} \leq \begin{bmatrix} 2 \\ 5 \end{bmatrix} \\ & \mathbf{x}^T \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} \mathbf{x} + \begin{bmatrix} 0 \\ -2 \end{bmatrix}^T \mathbf{x} \leq 1 \\ & \mathbf{0} \leq \mathbf{x} \end{aligned}$$

Mixed Integer Quadratic Programs

$$\begin{aligned} \min_{\mathbf{x}} \ & \frac{1}{2}\mathbf{x}^T\mathbf{H}\mathbf{x} + \mathbf{f}^T\mathbf{x} \\ \text{subject to: } & \mathbf{A}\mathbf{x} \leq \mathbf{b} \\ & \mathbf{A}_{\text{eq}}\mathbf{x} = \mathbf{b}_{\text{eq}} \\ & \mathbf{l}_{\text{b}} \leq \mathbf{x} \leq \mathbf{u}_{\text{b}} \\ & \forall i = 0...n : x_i \in \mathbb{Z} \\ & \forall j = 0...n : x_j \in \{0, 1\} \end{aligned}$$

$$
\begin{aligned}
\min_{\mathbf{x}} \; & 0.5x_1^2 + x_2^2 - x_1x_2 - 2x_1 - 6x_2 \\
\text{subject to:} \quad & x_1 + x_2 \leq 2 \\
& -x_1 + 2x_2 \leq 2 \\
& 2x_1 + x_2 \leq 3 \\
& \mathbf{0} \leq \mathbf{x} \\
& x_1 \in \mathbb{Z}
\end{aligned}
$$

$$
\begin{aligned}
\min_{\mathbf{x}} \; & \frac{1}{2}\mathbf{x}^T \begin{bmatrix} 1 & -1 \\ -1 & 2 \end{bmatrix} \mathbf{x} + \begin{bmatrix} -2 \\ -6 \end{bmatrix}^T \mathbf{x} \\
\text{subject to:} \quad & \begin{bmatrix} 1 & 1 \\ -1 & 2 \\ 2 & 1 \end{bmatrix} \mathbf{x} \leq \begin{bmatrix} 2 \\ 2 \\ 3 \end{bmatrix} \\
& \mathbf{0} \leq \mathbf{x} \\
& x_1 \in \mathbb{Z}
\end{aligned}
$$

Mixed Integer Quadratically Constrained Quadratic Programs

$$
\begin{aligned}
\min_{\mathbf{x}} \; & \frac{1}{2}\mathbf{x}^T\mathbf{H}\mathbf{x} + \mathbf{f}^T\mathbf{x} \\
\text{subject to:} \quad & \mathbf{A}\mathbf{x} \leq \mathbf{b} \\
& \mathbf{A}_{\text{eq}}\mathbf{x} = \mathbf{b}_{\text{eq}} \\
& \mathbf{l}_{\text{b}} \leq \mathbf{x} \leq \mathbf{u}_{\text{b}} \\
& \forall i = 0...q : \mathbf{x}^T\mathbf{Q}_i\mathbf{x} + \mathbf{l}_i^T\mathbf{x} \leq \mathbf{r}_i \\
& \forall j = 0...m : x_j \in \mathbb{Z} \\
& \forall k = 0...n : x_k \in \{0, 1\}
\end{aligned}
$$

$$
\begin{aligned}
\min_{\mathbf{x}} \; & 0.5x_1^2 + 0.5x_2^2 - 2x_1 - 2x_2 \\
\text{subject to:} \quad & -x_1 + x_2 \leq 2 \\
& x_1 + 3x_2 \leq 5 \\
& x_1^2 + x_2^2 - 2x_2 \leq 1 \\
& \mathbf{0} \leq \mathbf{x} \\
& x_1 \in \mathbb{Z}
\end{aligned}
$$

$$\begin{aligned} \min_{\mathbf{x}} \ & \frac{1}{2}\mathbf{x}^T \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} \mathbf{x} + \begin{bmatrix} -2 \\ -2 \end{bmatrix}^T \mathbf{x} \\ \text{subject to:} \quad & \begin{bmatrix} -1 & 1 \\ 1 & 3 \end{bmatrix} \mathbf{x} \leq \begin{bmatrix} 2 \\ 5 \end{bmatrix} \\ & \mathbf{x}^T \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} \mathbf{x} + \begin{bmatrix} 0 \\ -2 \end{bmatrix}^T \mathbf{x} \leq 1 \\ & \mathbf{0} \leq \mathbf{x} \\ & x_1 \in \mathbb{Z} \end{aligned}$$

System of Nonlinear Equations

$$\mathbf{F}(\mathbf{x}) = \mathbf{0}$$

$$\begin{aligned} 2x_1 - x_2 - e^{-x_1} &= 0 \\ -x_1 + 2x_2 - e^{-x_2} &= 0 \end{aligned}$$

System of Constrained Nonlinear Equations

$$\begin{aligned} & \mathbf{F}(\mathbf{x}) = \mathbf{0} \\ \text{subject to:} \ & \mathbf{A}\mathbf{x} \leq \mathbf{b} \\ & \mathbf{l}_{\mathrm{b}} \leq \mathbf{x} \leq \mathbf{u}_{\mathrm{b}} \\ & \mathbf{c}(\mathbf{x}) \leq \mathbf{d} \end{aligned}$$

Nonlinear Least Squares

$$\begin{aligned} \min_{\mathbf{x}} \ & \|\mathbf{F}(\mathbf{x})\|_2^2 \\ \text{subject to:} \ & \mathbf{A}\mathbf{x} \leq \mathbf{b} \\ & \mathbf{A}_{\mathrm{eq}}\mathbf{x} = \mathbf{b}_{\mathrm{eq}} \\ & \mathbf{l}_{\mathrm{b}} \leq \mathbf{x} \leq \mathbf{u}_{\mathrm{b}} \end{aligned}$$

$$\min_{\mathbf{x}} \sum_i \left(\mathbf{F}(\mathbf{x}, xdata_i) - ydata_i\right)^2$$

$$\mathbf{F}(\mathbf{x}, \mathbf{xdata}) = x_1 e^{x_2 \mathbf{xdata}}$$

Unconstrained Nonlinear Optimization

$$\min_{\mathbf{x}} f(\mathbf{x})$$

$$\min_{\mathbf{x}} (1 - x_1)^2 + 100\left(x_2 - x_1^2\right)^2$$

Nonlinear Programs

$$\begin{aligned} \min_{\mathbf{x}} \ & f(\mathbf{x}) \\ \text{subject to: } & \mathbf{A}\mathbf{x} \leq \mathbf{b} \\ & \mathbf{A}_{\text{eq}}\mathbf{x} = \mathbf{b}_{\text{eq}} \\ & \mathbf{l}_{\text{b}} \leq \mathbf{x} \leq \mathbf{u}_{\text{b}} \\ & \mathbf{c}(\mathbf{x}) \leq \mathbf{d} \\ & \mathbf{c}_{\text{eq}}(\mathbf{x}) = \mathbf{d}_{\text{eq}} \end{aligned}$$

$$\begin{aligned} \min_{\mathbf{x}} \ & \ln\left(1 + x_1^2\right) - x_2 \\ \text{subject to: } & \left(1 + x_1^2\right)^2 + x_2^2 = 4 \end{aligned}$$

Mixed Integer Nonlinear Programs

$$\begin{aligned} \min_{\mathbf{x}} \ & f(\mathbf{x}) \\ \text{subject to: } & \mathbf{A}\mathbf{x} \leq \mathbf{b} \\ & \mathbf{A}_{\text{eq}}\mathbf{x} = \mathbf{b}_{\text{eq}} \\ & \mathbf{l}_{\text{b}} \leq \mathbf{x} \leq \mathbf{u}_{\text{b}} \\ & \mathbf{c}(\mathbf{x}) \leq \mathbf{d} \\ & \mathbf{c}_{\text{eq}}(\mathbf{x}) = \mathbf{d}_{\text{eq}} \\ & \forall i = 0...m : x_i \in \mathbb{Z} \\ & \forall j = 0...n : x_j \in \{0, 1\} \end{aligned}$$

$$
\begin{aligned}
\min_{\mathbf{x}} \ & \sin\left(\pi\frac{x_1}{12}\right)\cos\left(\pi\frac{x_2}{16}\right) \\
\text{subject to:} \ & -x_1 + 2.5x_2 \leq 1 \\
& x_1 + 2.5x_2 \leq -15 \\
& x_1, x_2 \in \mathbb{Z}
\end{aligned}
$$