Chapter 47

Red-Black Trees

Objectives

- To know what a red-black tree is (§47.1).
- To convert a red-black tree to a 2-4 tree and vice versa (§47.2).
- To design the `RBTree` class that extends the `BinaryTree` class (§47.3).
- To insert an element in a red-black tree and resolve the double-red violation if necessary (§47.4).
- To delete an element from a red-black tree and resolve the double-black problem if necessary (§47.5).
- To implement and test the `RBTree` class (§§47.6–47.7).
- To compare the performance of AVL trees, 2-4 trees, and `RBTree` (§47.8).

47.1 Introduction

derived from 2-4
color attribute
external

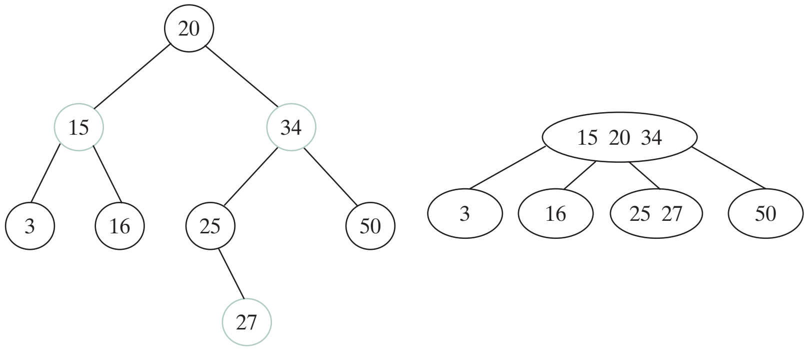
A red-black tree is a binary search tree derived from a *2-4 tree*. A red-black tree corresponds to a 2-4 tree. Each node in a red-black tree has a *color attribute* red or black, as shown in Figure 47.1(a). A node is called *external* if its left or right subtree is empty. Note that a leaf node is external, but an external node is not necessarily a leaf node. For example, node **25** is external, but it is not a leaf. The *black depth* of a node is defined as the number of black nodes in a path from the node to the root. For example, the black depth of node **25** is **2** and that of node **27** is **2**.

black depth

A red-black tree has the following properties:

1. The root is black.
2. Two adjacent nodes cannot be both red.
3. All external nodes have the same black depth.

The red-black tree in Figure 47.1(a) satisfies all three properties. A red-black tree can be converted to a 2-4 tree, and vice versa. Figure 47.1(b) shows an equivalent 2-4 tree for the red-black tree in Figure 47.1(a).



(a) A red-black tree (b) A 2-4 tree

FIGURE 47.1 A red-black tree can be represented using a 2-4 tree, and vice versa.

Note

The red nodes appear in blue in the text.

47.2 Conversion between Red-Black Trees and 2-4 Trees

You can design insertion and deletion algorithms for red-black trees without having knowledge of 2-4 trees. However, the correspondence between red-black trees and 2-4 trees provides useful intuition about the structure of red-black trees and operations. For this reason, this section discusses the correspondence between these two types of trees.

red-black to 2-4

To convert a red-black tree to a 2-4 tree, simply merge every red node with its parent to create a 3-node or a 4-node. For example, the red nodes **15** and **34** are merged to their parent to create a 4-node, and the red node **27** is merged to its parent to create a 3-node, as shown in Figure 47.1(b).

2-4 to red-black

To convert a 2-4 tree to a red-black tree, perform the following transformations for each node **u**:

converting 2-node

1. If **u** is a 2-node, color it black, as shown in Figure 47.2(a).

converting 3-node

2. If **u** is a 3-node with element values **e0** and **e1**, there are two ways to convert it. Either make **e0** the parent of **e1** or make **e1** the parent of **e0**. In any case, color the parent black and the child red, as shown in Figure 47.2(b).

converting 4-node

3. If **u** is a 4-node with element values **e0**, **e1**, and **e2**, make **e1** the parent of **e0** and **e2**. Color **e1** black and **e0** and **e2** red, as shown in Figure 47.2(c).

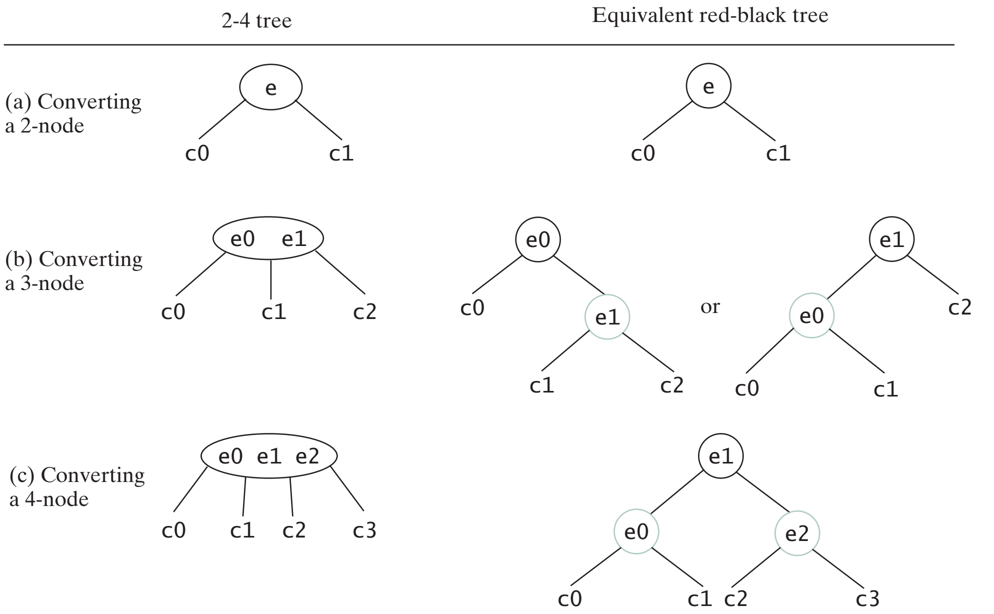


FIGURE 47.2 A node in a 2-4 tree can be transformed to nodes in a red-black tree.

Let us apply the transformation for the 2-4 tree in Figure 47.1(b). After transforming the 4-node, the tree is as shown in Figure 47.3(a). After transforming the 3-node, the tree is as shown in Figure 47.3(b). Note that the transformation for a 3-node is not unique. Therefore, the conversion from a 2-4 tree to a red-black tree is *not unique*. After transforming the 3-node, the tree could also be as shown in Figure 47.3(c).

not unique

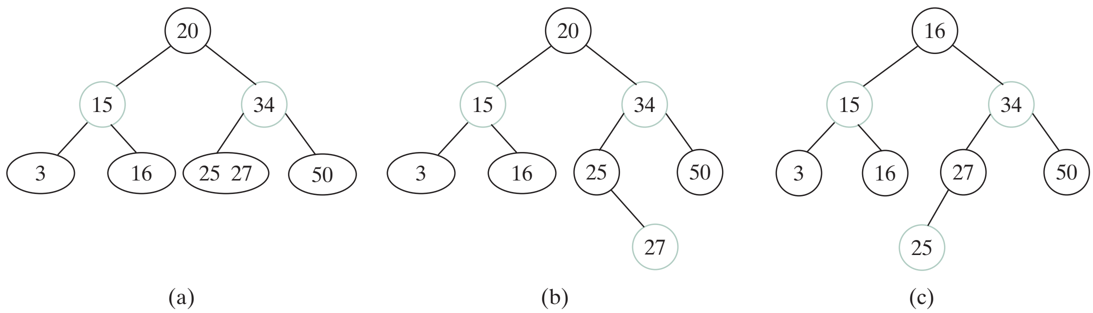


FIGURE 47.3 The conversion from a 2-4 tree to a red-black tree is not unique.

You can prove that the conversion results in a red-black tree that satisfies all three properties.

Property 1. The root is black.
Proof: If the root of a 2-4 tree is a 2-node, the root of the red-black tree is black. If the root of a 2-4 tree is a 3-node or 4-node, the transformation produces a black parent at the root.

Property 1 proof

Property 2. Two adjacent nodes cannot be both red.
Proof: Since the parent of a red node is always black, no two adjacent nodes can be both red.

Property 2 proof

Property 3. All external nodes have the same black depth.
Proof: When you covert a node in a 2-4 tree to red-black tree nodes, you get one black node and zero, one, or two red nodes as its children, depending on whether the original node is a 2-, 3-, or 4-node. Only a leaf 2-4 node may produce external red-black nodes.

Property 3 proof

Since a 2-4 tree is perfectly balanced, the number of black nodes in any path from the root to an external node is the same.

47.3 Designing Classes for Red-Black Trees

A red-black tree is a binary search tree. So, you can define the `RBTree` class to extend the `BinaryTree` class, as shown in Figure 47.4. The `BinaryTree` and `TreeNode` classes are defined in §26.2.5.

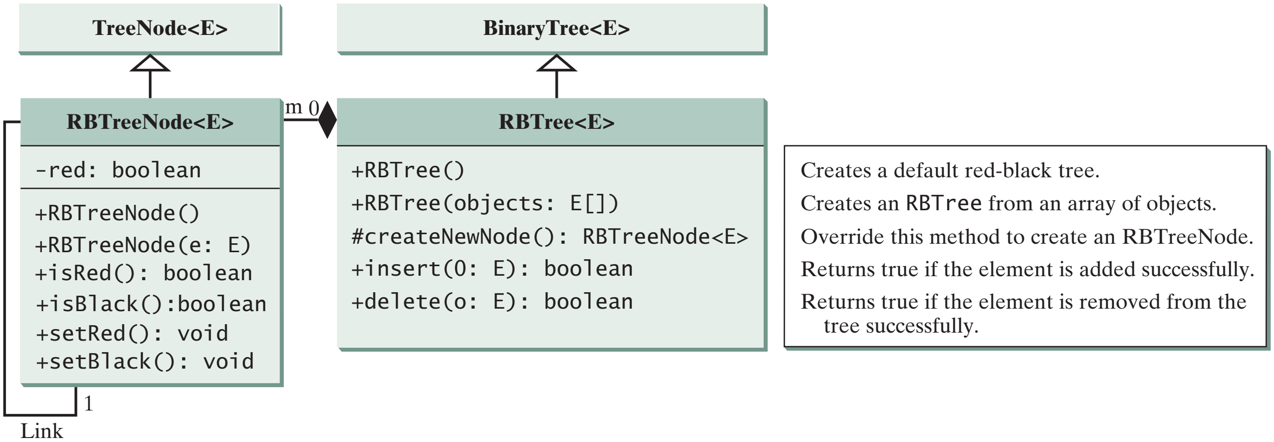


FIGURE 47.4 The `RBTree` class extends `BinaryTree` with new implementations for the `insert` and `delete` methods.

RBTreeNode

Each node in a red-black tree has a color property. Because the color is either red or black, it is efficient to use the `boolean` type to denote it. The `RBTreeNode` class can be defined to extend `BinaryTree.TreeNode` with the color property. For convenience, we also provide the methods for checking the color and setting a new color. Note that `TreeNode` is defined as a static inner class in `BinaryTree`. `RBTreeNode` will be defined as a static inner class in `RBTree`. Note that `BinaryTreeNode` contains the data fields `element`, `left`, and `right`, which are inherited in `RBTreeNode`. So, `RBTreeNode` contains four data fields, as pictured in Figure 47.5.

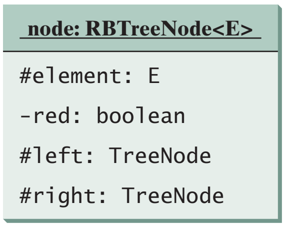


FIGURE 47.5 An `RBTreeNode` contains data fields `element`, `red`, `left`, and `right`.

createNewNode()

In the `BinaryTree` class, the `createNewNode()` method creates a `TreeNode` object. This method is overridden in the `RBTree` class to create an `RBTreeNode`. Note that the return type of the `createNewNode()` method in the `BinaryTree` class is `TreeNode`, but the return type of the `createNewNode()` method in `RBTree` class is `RBTreeNode`. This is fine, since `RBTreeNode` is a subtype of `TreeNode`.

Searching an element in a red-black tree is the same as searching in a regular binary search tree. So, the `search` method defined in the `BinaryTree` class also works for `RBTree`.

The `insert` and `delete` methods are overridden to insert and delete an element and perform operations for coloring and restructuring if necessary to ensure that the three properties of the red-black tree are satisfied.

Pedagogical NOTE

Run from http://www.cs.armstrong.edu/liang/animation/RBTreeAnimation.html to see how a red-black tree works, as shown in Figure 47.6. Red-Black tree animation

47.4 Overriding the `insert` Method

A new element is always inserted as a leaf node. If the new node is the root, color it black. Otherwise, color it red. If the parent of the new node is red, it violates Property 2 of the red-black tree. We call this a *double-red* violation. double red

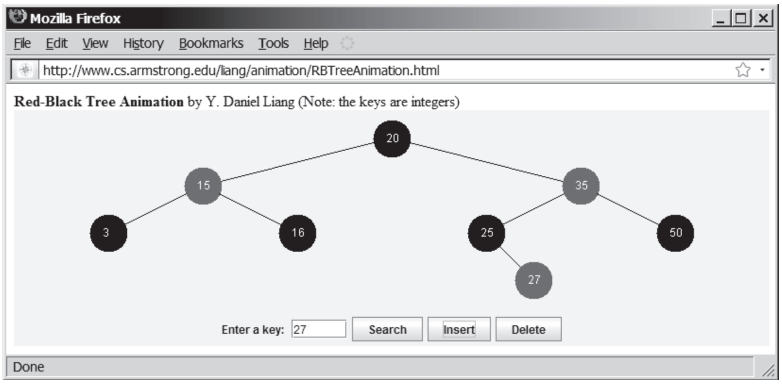


Figure 47.6 The animation tool enables you to insert, delete, and search elements in a red-black tree visually.

Let `u` denote the new node inserted, `v` the parent of `u`, `w` the parent of `v`, and `x` the sibling of `v`. To fix the double-red violation, consider two cases:

Case 1: `x` is black or `x` is null. There are four possible configurations for `u`, `v`, `w`, and `x`, as shown in Figures 47.7(a), 47.8(a), 47.9(a), and 47.10(a). In this case, `u`, `v`, and `w` form a 4-node in the corresponding 2-4 tree, as shown in Figures 47.7(c), 47.8(c), 47.9(c), and 47.10(c), but are represented incorrectly in the red-black tree. To correct this error, restructure and recolor three nodes `u`, `v`, and `w`, as shown in Figures 47.7(b), 47.8(b), 47.9(b), and 47.10(b). Note that `x`, `y1`, `y2`, and `y3` may be null.

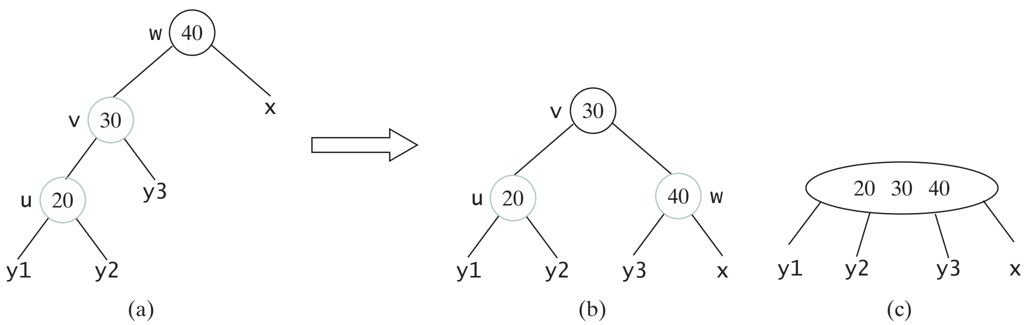


Figure 47.7 Case 1.1: `u` < `v` < `w`.

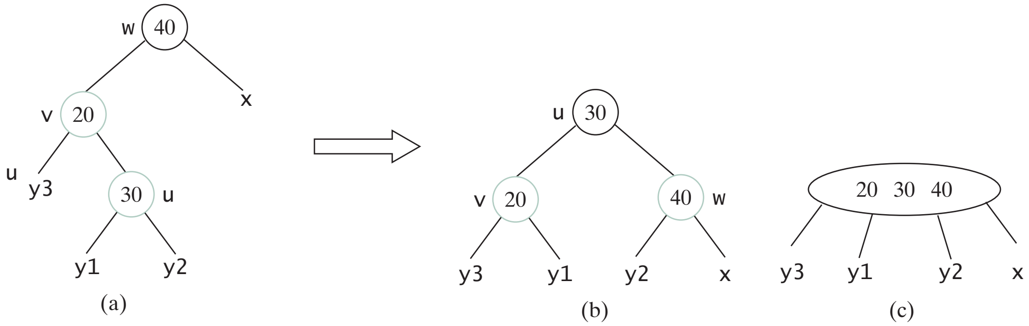


FIGURE 47.8 Case 1.2: v < u < w

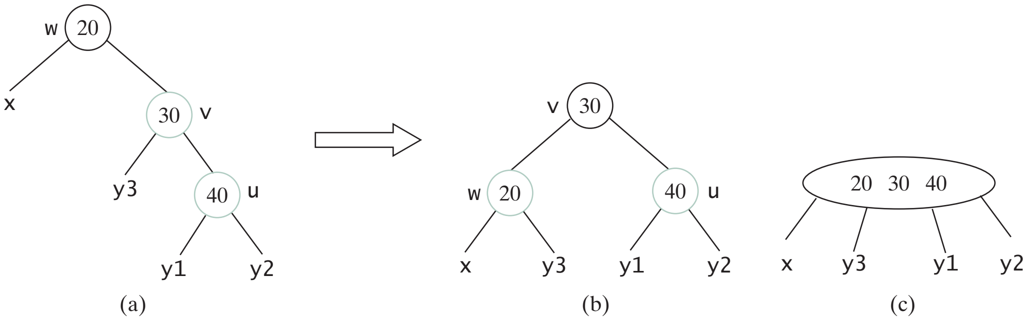


FIGURE 47.9 Case 1.3: w < v < u

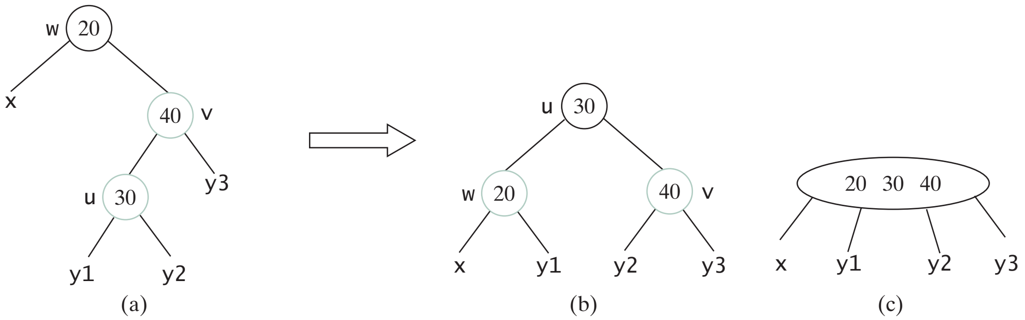


FIGURE 47.10 Case 1.4: w < u < v

Case 2: x is red. There are four possible configurations for u, v, w, and x, as shown in Figures 47.11(a), 47.11(b), 47.11(c), and 47.11(d). All of these configurations correspond to an overflow situation in the corresponding 4-node in a 2-4 tree, as shown in Figure 47.12(a). A splitting operation is performed to fix the overflow problem in a 2-4 tree, as shown in Figure 47.12(b). We perform an equivalent recoloring operation to fix the problem in a red-black tree. Color w and u red and color two children of w black. Assume u is a left child of v, as shown in Figure 47.11(a). After recoloring, the nodes are shown in Figure 47.12(c). Now w is red, if w's parent is black, the double-red w problem is fixed. Otherwise, a new double-red violation occurs at node w. We need to continue the same process to eliminate the double-red violation at w, recursively.

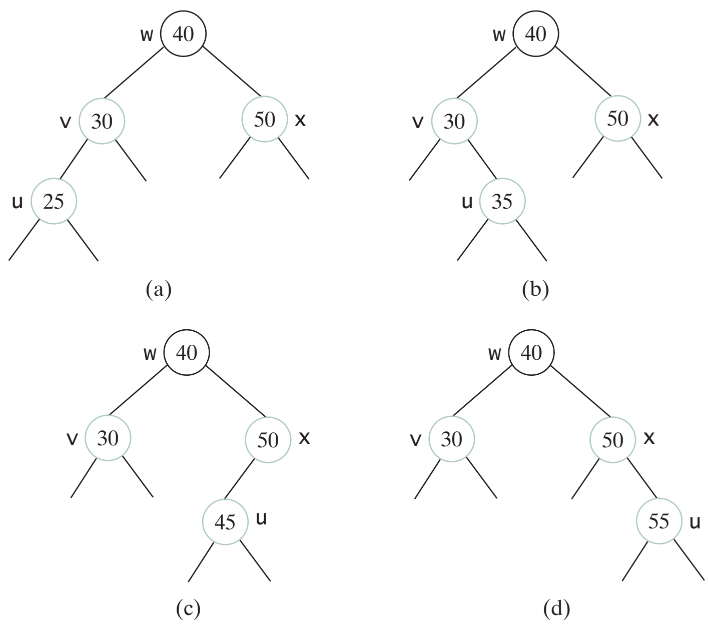


FIGURE 47.11 Case 2 has four possible configurations.

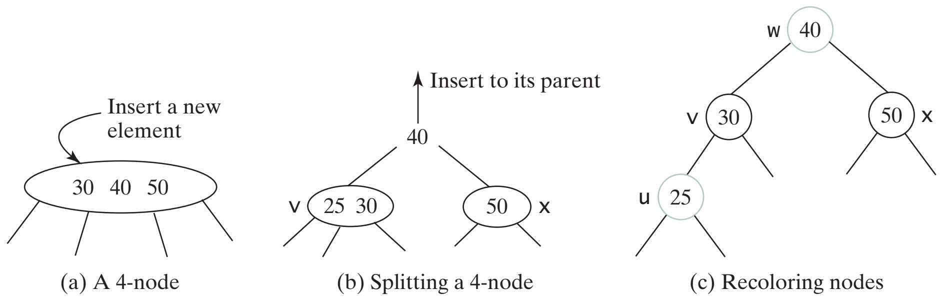


FIGURE 47.12 Splitting a 4-node corresponds to recoloring the nodes in the red-black tree.

A more detailed algorithm for inserting an element is described in Listing 47.1.

LISTING 47.1 Inserting an Element to a Red-Black Tree

```
public boolean insert(E e) {
  boolean successful = super.insert(e);
  if (!successful)
    return false; // e is already in the tree
  else {
    ensureRBTree(e);
  }

  return true; // e is inserted
}

/** Ensure that the tree is a red-black tree */
private void ensureRBTree(E e) {
  Get the path that leads to element e from the root.
  int i = path.size() – 1; // Index to the current node in the path
  Get u, v from the path. u is the node that contains e and v
```

insert to tree
invoke super.insert
duplicate element
ensure color and depth
ensure color and depth
get path
node index
get **u**, **v**

u is root?
double-red violation
fix double red
get **w**
get **x**
Case 1
Case 1.1
Case 1.2
Case 1.3
Case 1.4
Case 2
recoloring
w is root?
propagate upward
fix new double red

```
17      is the parent of u.
18    Color u red;
19
20    if (u == root) // If e is inserted as the root, set root black
21      u.setBlack();
22    else if (v.isRed())
23      fixDoubleRed(u, v, path, i); // Fix double-red violation at u
24  }
25
26  /** Fix double-red violation at node u */
27  private void fixDoubleRed(RBTreeNode<E> u, RBTreeNode<E> v,
28      ArrayList<TreeNode<E>> path, int i) {
29    Get w from the path. w is the grandparent of u.
30
31    // Get v's sibling named x
32    RBTreeNode<E> x = (w.left == v) ?
33      (RBTreeNode<E>)(w.right) : (RBTreeNode<E>)(w.left);
34
35    if (x == null || x.isBlack()) {
36      // Case 1: v's sibling x is black
37      if (w.left == v && v.left == u) {
38        // Case 1.1: u < v < w, Restructure and recolor nodes
39      }
40      else if (w.left == v && v.right == u) {
41        // Case 1.2: v < u < w, Restructure and recolor nodes
42      }
43      else if (w.right == v && v.right == u) {
44        // Case 1.3: w < v < u, Restructure and recolor nodes
45      }
46      else {
47        // Case 1.4: w < u < v, Restructure and recolor nodes
48      }
49    }
50    else { // Case 2: v's sibling x is red
51      Color w and u red
52      Color two children of w black.
53
54      if (w is root) {
55        Set w black;
56      }
57      else if (the parent of w is red) {
58        // Propagate along the path to fix new double-red violation
59        u = w;
60        v = parent of w;
61        fixDoubleRed(u, v, path, i - 2); // i – 2 propagates upward
62      }
63    }
64  }
```

`insert(E, e)`

The **`insert(E e)`** method (lines 1–10) invokes the **`insert`** method in the **`BinaryTree`** class to create a new leaf node for the element (line 2). If the element is already in the tree, return false (line 4). Otherwise, invoke **`ensureRBTree(e)`** (line 6) to ensure that the tree satisfies the color and black depth property of the red-black tree.

`ensureRBTree(E, e)`

The **`ensureRBTree(E e)`** method (lines 13–24) obtains the path that leads to **`e`** from the root (line 14), as shown in Figure 47.13. This path plays an important role to implement the algorithm. From this path, you get nodes **`u`** and **`v`** (lines 16–17). If **`u`** is the root, color **`u`** black

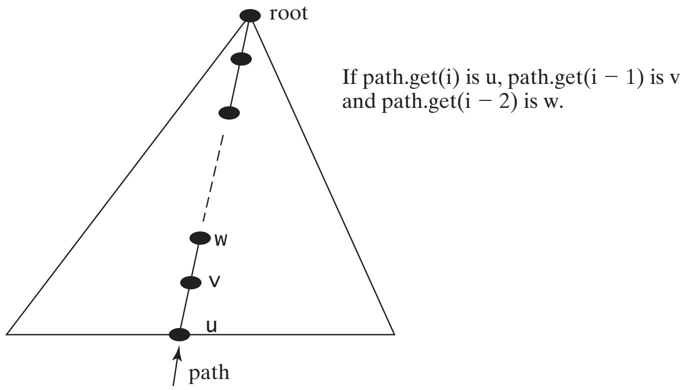


FIGURE 47.13 The path consists of the nodes from `u` to the root.

(lines 20–21). If `v` is red, a double-red violation occurs at node `u`. Invoke `fixDoubleRed` to fix the problem.

`fixDoubleRed`

The `fixDoubleRed` method (lines 27–63) fixes the double-red violation. It first obtains `w` (the parent of `v`) from the path (line 29) and `x` (the sibling of `v`) (lines 32–33). If `x` is empty or a black node, restructure and recolor three nodes `u`, `v`, and `w` to eliminate the problem (lines 35–49). If `x` is a red node, recolor the nodes `u`, `v`, `w` and `x` (lines 51–52). If `w` is the root, color `w` black (lines 54–56). If the parent of `w` is red, the double-red violation reappears at `w`. Invoke `fixDoubleRed` with new `u` and `v` to fix the problem (line 61). Note that now `i – 2` points to the new `u` in the path. This adjustment is necessary to locate the new nodes `w` and parent of `w` along the path.

insertion example

Figure 47.14 shows the steps of inserting `34`, `3`, `50`, `20`, `15`, `16`, `25`, and `27` into an empty red-black tree. When inserting `20` into the tree in (d), Case 2 applies to recolor `3` and `50` to black. When inserting `15` into the tree in (g), Case 1.4 applies to restructure and recolor nodes `15`, `20`, and `3`. When inserting `16` into the tree in (i), Case 2 applies to recolor nodes `3` and `20` to black and nodes `15` and `16` to red. When inserting `27` into the tree in (l), Case 2 applies to recolor nodes `16` and `25` to black and nodes `20` and `27` to red. Now a new double-red problem occurs at node `20`. Apply Case 1.2 to restructure and recolor nodes. The new tree is shown in (n).

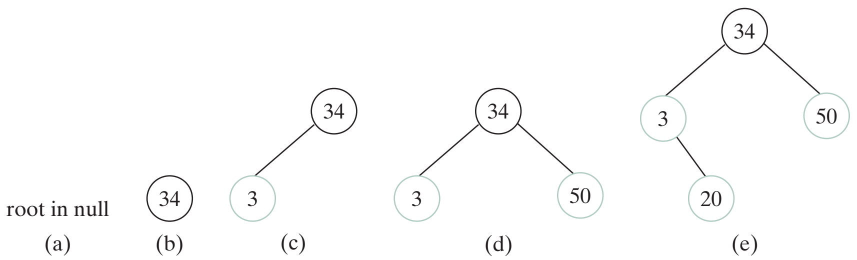


FIGURE 47.14 Inserting into a red-black tree: (a) initial empty tree; (b) inserting `34`; (c) inserting `3`; (d) inserting `50`; (e) inserting `20` causes a double red; (f) after recoloring (Case 2); (g) inserting `15` causes a double red; (h) after restructuring and recoloring (Case 1.4); (i) inserting `16` causes a double red; (j) after recoloring (Case 2); (k) inserting `25`; (l) inserting `27` causes a double red at `27`; (m) a double red at `20` reappears after recoloring (Case 2); (n) after restructuring and recoloring (Case 1.2).

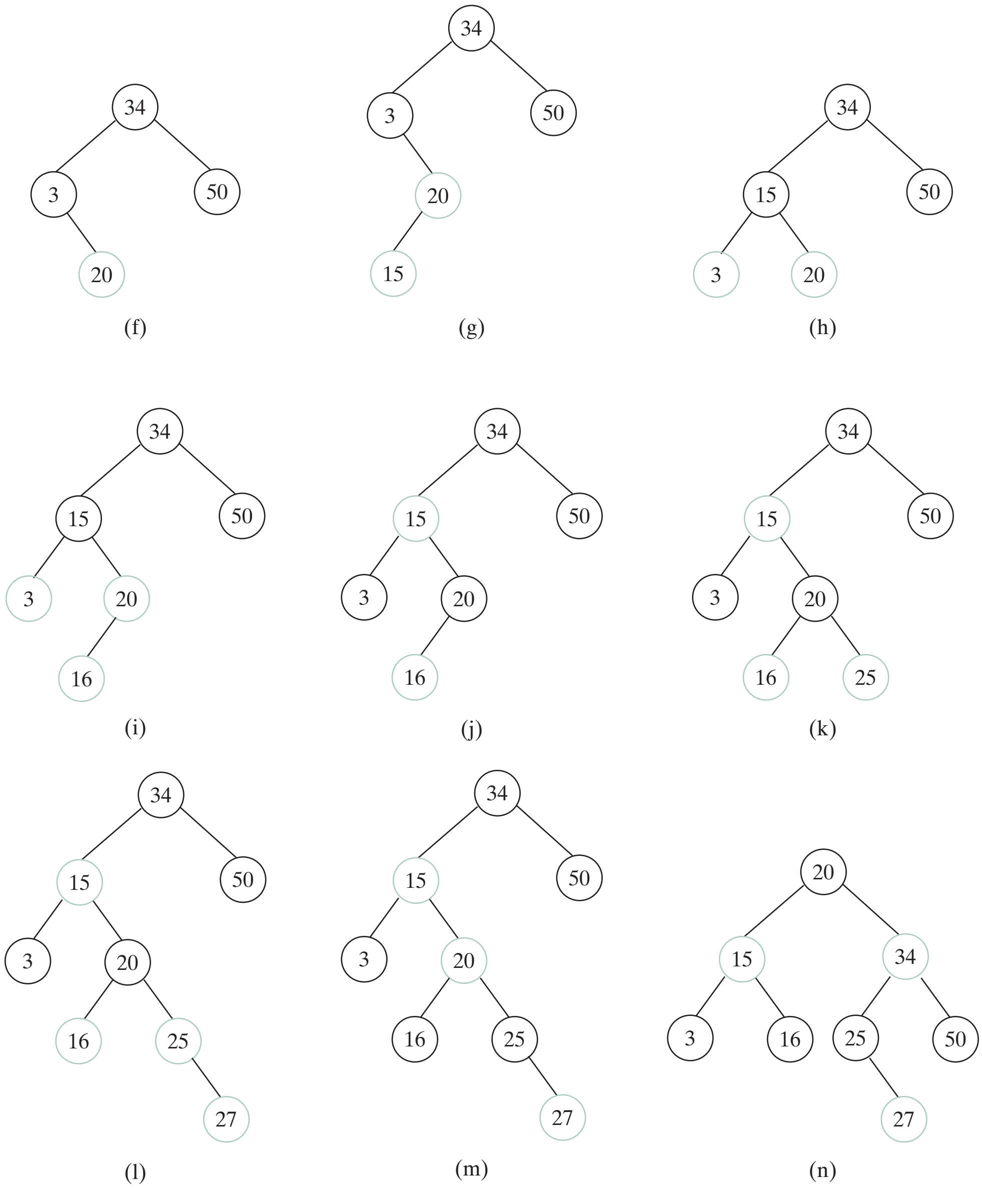
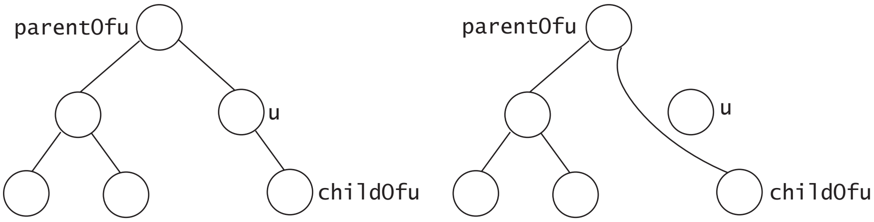


FIGURE 47.14 *continued*

47.5 Overriding the `delete` Method

To delete an element from a red-black tree, first search the element in the tree to locate the node that contains the element. If the element is not in the tree, the method returns false. Let `u` be the node that contains the element. If `u` is an internal node with both left and right children, find the rightmost node in the left subtree of `u`. Replace the element in `u` with the element in the rightmost node. Now we will only consider deleting external nodes.

Let `u` be an external node to be deleted. Since `u` is an external node, it has at most one child, denoted by `childOfu`. `childOfu` may be `null`. Let `parentOfu` denote the parent of `u` as shown in Figure 47.15(a). Delete `u` by connecting `childOfu` with `parentOfu`, as shown in Figure 47.15(b).



(a) Before deleting `u` (b) After deleting `u`

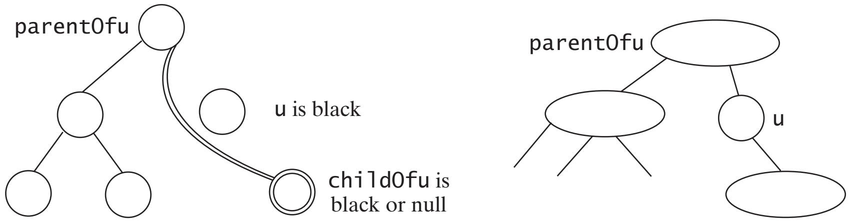
FIGURE 47.15 `u` is an external node and `childOfu` may be null.

Consider the following case:

- If `u` is red, we are done.
- If `u` is black and `childOfu` is red, color `childOfu` black to maintain the black height for `childOfu`.
- Otherwise, assign childOfu a fictitious *double black*, as shown in Figure 47.16(a). We call this a double-black problem, which indicates that the black-depth is short by 1, caused by deleting a black node `u`.

double-black problem

A double black in a red-black tree corresponds to an empty node for `u` (i.e., underflow situation) in the corresponding 2-4 tree, as shown in Figure 47.16(b). To fix the double-black problem, we will perform equivalent transfer and fusion operations. Consider three cases:

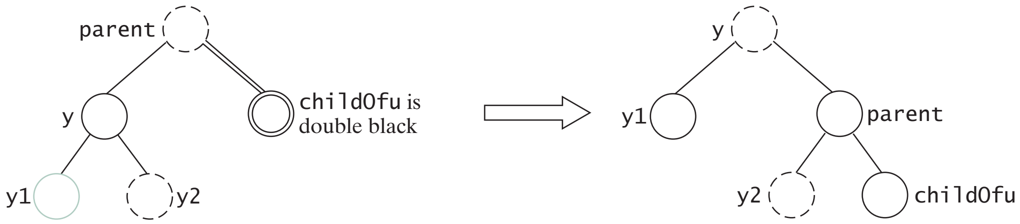


(a) (b)

FIGURE 47.16 (a) `childOfu` is denoted double black. (b) `u` corresponds to an empty node in a 2-4 tree.

Case 1

Case 1: The sibling `y` of `childOfu` is black and has a red child. This case has four possible configurations, as shown in Figures 47.17(a), 47.18(a), 47.19(a), and 47.20(a). The dashed circle denotes that the node is either red or black. To eliminate the double-black problem, restructure and recolor the nodes, as shown in Figures 47.17(b), 47.18(b), 47.19(b), and 47.20(b).



(a) (b)

FIGURE 47.17 Case 1.1: The sibling `y` of `childOfu` is black and `y1` is red.

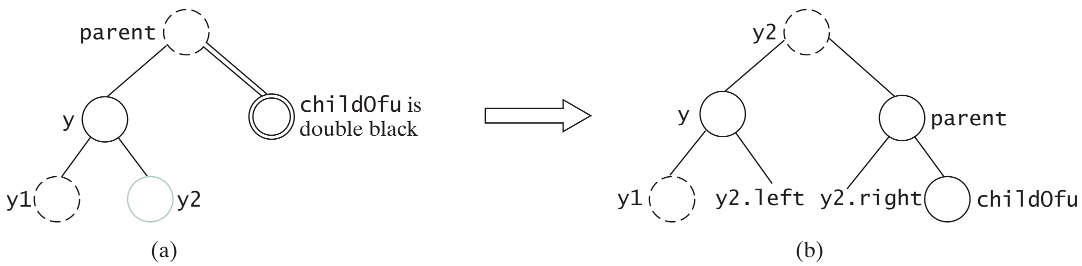


Figure 47.18 Case 1.2: The sibling y of childOfu is black and y2 is red.

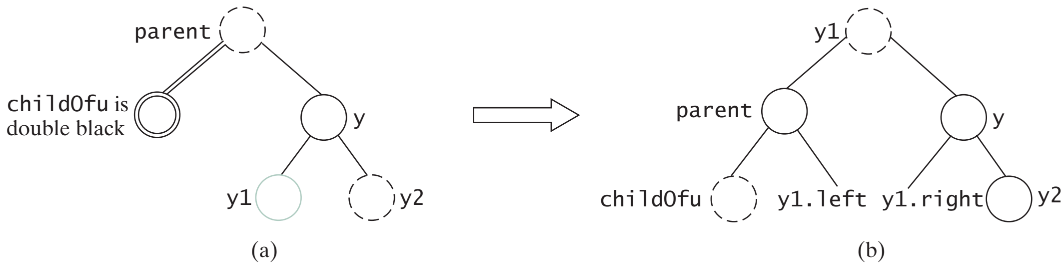


Figure 47.19 Case 1.3: The sibling y of childOfu is black and y1 is red.

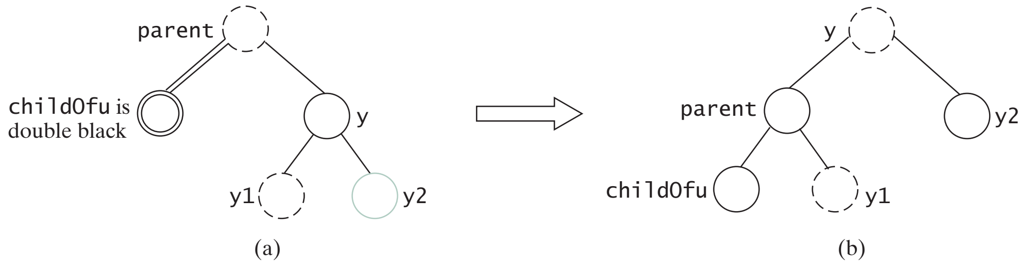


Figure 47.20 Case 1.4: the sibling y of childOfu is black and y2 is red.

transfer operation

Note

Case 1 corresponds to a *transfer* operation in the 2-4 tree. For example, the corresponding 2-4 tree for Figure 47.17(a) is shown in Figure 47.21(a), and it is transformed into 47.21(b) through a transfer operation.

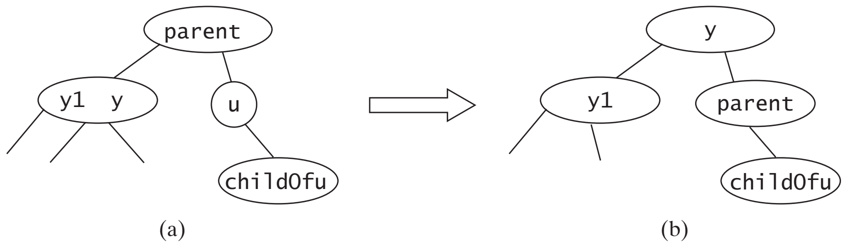


Figure 47.21 Case 1 corresponds to a transfer operation in the corresponding 2-4 tree.

Case 2: The sibling `y` of childOfu is black and its children are black or `null`. In this case, change `y`'s color to red. If `parent` is red, change it to black, and we are done, as shown in Figure 47.22. If `parent` is black, we denote `parent` double black, as shown in Figure 47.23. The double-black problem *propagates* to the parent node. Case 2 propagate

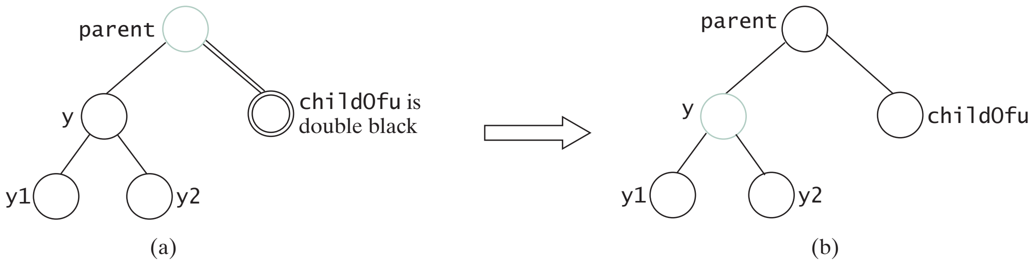


FIGURE 47.22 Case 2: Recoloring eliminates the double-black problem if `parent` is red.

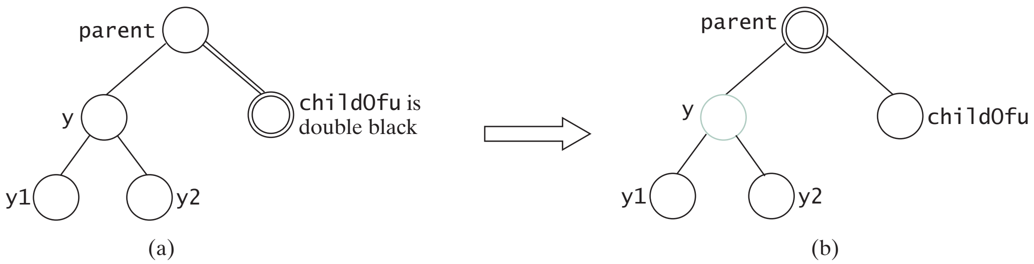


FIGURE 47.23 Case 2: Recoloring propagates the double-black problem if `parent` is black.

Note

Figures 47.22 and 47.22 show that `childOfu` is a right child of `parent`. If `childOfu` is a left child of `parent`, recoloring is performed identically. left childOfu

Note

Case 2 corresponds to a *fusion* operation in the 2-4 tree. For example, the corresponding 2-4 tree for Figure 47.22(a) is shown in Figure 47.24(a), and it is transformed into 47.24(b) through a fusion operation. fusion operation

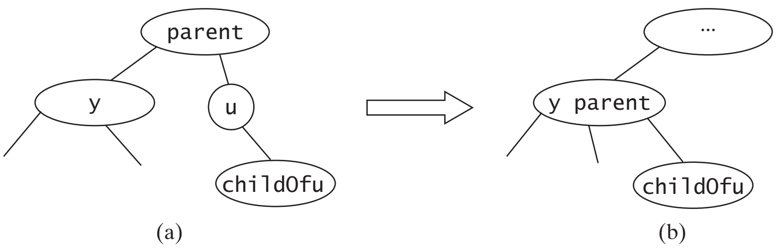


FIGURE 47.24 Case 2 corresponds to a fusion operation in the corresponding 2-4 tree.

Case 3: The sibling `y` of `childOfu` is red. In this case, perform an *adjustment* operation. If `y` is a left child of `parent`, let `y1` and `y2` be the left and right child of `y`, as shown in Case 3 adjustment

Figure 47.25. If y is a right child of parent, let y1 and y2 be the left and right childof y, as shown in Figure 47.26. In both cases, color y black and parent red. childOfu is still a fictitious double-black node. After the adjustment, the sibling of childOfu is now black, and either Case 1 or Case 2 applies. If Case 1 applies, a one-time restructuring and recoloring operation eliminates the double-black problem. If Case 2 applies, the double-black problem cannot reappear, since parent is now red. Therefore, one-time application of Case 1 or Case 2 will complete Case 3.

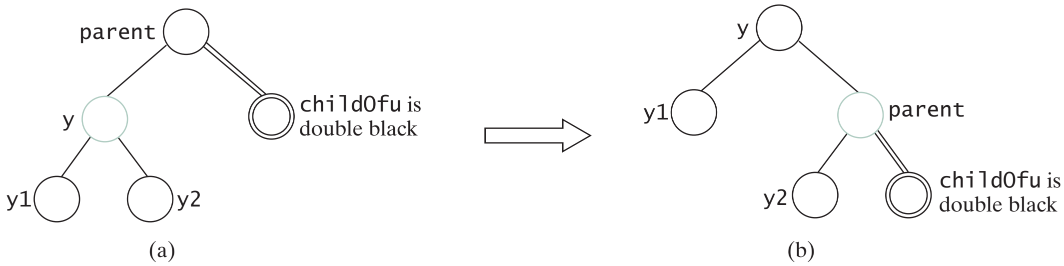


FIGURE 47.25 Case 3.1: y is a left red child of parent.

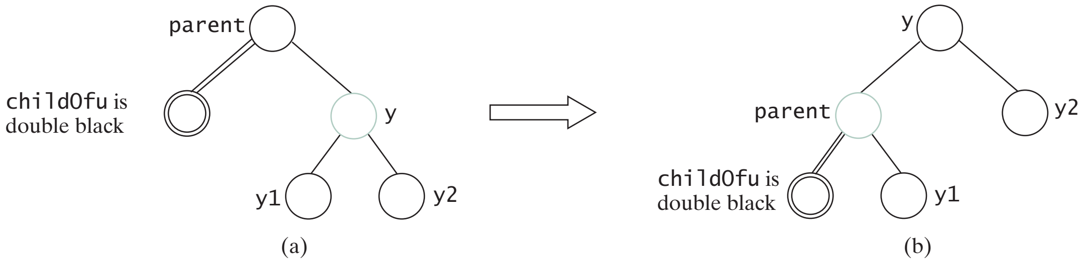


FIGURE 47.26 Case 3.2: y is a right red child of parent.

nonunique transform of 3-node

Note

Case 3 results from the fact that a 3-node may be transformed in two ways to a red-black tree, as shown in Figure 47.27.

Based on the foregoing discussion, Listing 47.2 presents a more detailed algorithm for deleting an element.

LISTING 47.2 Deleting an Element from a Red-Black Tree

delete e from tree
locate the node
element not found
internal element?
rightmost node
path to external node

```
public boolean delete(E e) {
  Locate the node to be deleted
  if (the node is not found)
    return false;

  if (the node is an internal node) {
    Find the rightmost node in the subtree of the node;
    Replace the element in the node with the one in rightmost;
    The rightmost node is the node to be deleted now;
  }

  Obtain the path from the root to the node to be deleted;

  // Delete the last node in the path and propagate if needed
```

```
  deleteLastNodeInPath(path);

  size--; // After one element deleted
  return true; // Element deleted
}

/** Delete the last node from the path. */
public void deleteLastNodeInPath(ArrayList<TreeNode<E>> path) {
  Get the last node u in the path;
  Get parentOfu and grandparentOfu in the path;
  Get childOfu from u;
  Delete node u. Connect childOfu with parentOfu

  // Recolor the nodes and fix double black if needed
  if (childOfu == root || u.isRed())
    return; // Done if childOfu is root or if u is red
  else if (childOfu != null && childOfu.isRed())
    childOfu.setBlack(); // Set it black, done
  else // u is black, childOfu is null or black
    // Fix double black on parentOfu
    fixDoubleBlack(grandparentOfu, parentOfu, childOfu, path, i);
}

/** Fix the double black problem at node parent */
private void fixDoubleBlack(
    RBTreeNode<E> grandparent, RBTreeNode<E> parent,
    RBTreeNode<E> db, ArrayList<TreeNode<E>> path, int i) {
  Obtain y, y1, and y2

  if (y.isBlack() && y1 != null && y1.isRed()) {
    if (parent.right == db) {
      // Case 1.1: y is a left black sibling and y1 is red
      Restructure and recolor parent, y, and y1 to fix the problem;
    }
    else {
      // Case 1.3: y is a right black sibling and y1 is red
      Restructure and recolor parent, y1, and y to fix the problem;
    }
  }
  else if (y.isBlack() && y2 != null && y2.isRed()) {
    if (parent.right == db) {
      // Case 1.2: y is a left black sibling and y2 is red
      Restructure and recolor parent, y2, and y to fix the problem;
    }
    else {
      // Case 1.4: y is a right black sibling and y2 is red
      Restructure and recolor parent, y, and y2 to fix the problem;
    }
  }
  else if (y.isBlack()) {
    // Case 2: y is black and y's children are black or null
    Recolor y to red;

    if (parent.isRed())
      parent.setBlack(); // Done
    else if (parent != root) {
      // Propagate double black to the parent node
      // Fix new appearance of double black recursively
      db = parent;
      parent = grandparent;
```

delete the node
one element deleted
deletion successful
delete a node
u
childOfu
parentOfu, **grandparentOfu**
delete u
done
set **childOfu** black
fix double black
fix double black
y, y1, y2
process Case 1.1
process Case 1.3
process Case 1.2
process Case 1.4
process Case 2

propagate double black

process Case 3.1

process Case 3.2

fix double black

```
75        grandparent =
76          (i >= 3) ? (RBTreeNode<E>)(path.get(i - 3)) : null;
77        fixDoubleBlack(grandparent, parent, db, path, i - 1);
78      }
79    }
80    else if (y.isRed()) {
81      if (parent.right == db) {
82        // Case 3.1: y is a left red child of parent
83        parent.left = y2;
84        y.right = parent;
85      }
86      else {
87        // Case 3.2: y is a right red child of parent
88        parent.right = y.left;
89        y.left = parent;
90      }
91
92      parent.setRed(); // Color parent red
93      y.setBlack(); // Color y black
94      connectNewParent(grandparent, parent, y); // y is new parent
95      fixDoubleBlack(y, parent, db, path, i - 1);
96    }
97  }
```

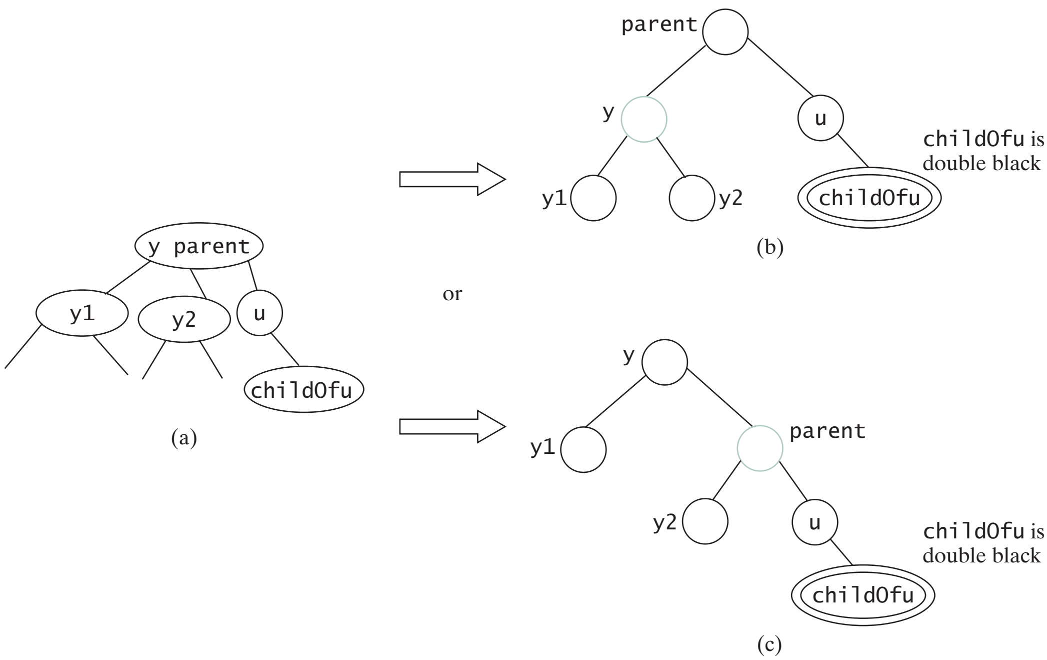


FIGURE 47.27 A 3-node may be transformed in two ways to red-black tree nodes.

delete(E, e)

The `delete(E e)` method (lines 1–19) locates the node that contains `e` (line 2). If the node does not exist, return `false` (lines 3–4). If the node is an internal node, find the right most node in its left subtree and replace the element in the node with the element in the right most node (lines 6–9). Now the node to be deleted is an external node. Obtain the path from the root to the node (line 12). Invoke `deleteLastNodeInPath(path)` to delete the last node in the path and ensure that the tree is still a red-black tree (line 15).

The `deleteLastNodeInPath` method (lines 22–36) obtains the last node `u`, `parentOfu`, `grandparendOfu`, and `childOfu` (lines 23–26). If `childOfu` is the root or `u` is red, the tree is fine (lines 29–30). If `childOfu` is red, color it black (lines 31–32). We are done. Otherwise, `u` is black and `childOfu` is `null` or black. Invoke `fixDoubleBlack` to eliminate the double-black problem (line 35).

deleteLastNodeInPath (path)

The `fixDoubleBlack` method (lines 39–97) eliminates the double-black problem. Obtain `y`, `y1`, and `y2` (line 42). `y` is the sibling of the double-black node. `y1` and `y2` are the left and right children of `y`. Consider three cases:

fixDoubleBlack

1. If `y` is black and one of its children is red, the double-black problem can be fixed by one-time restructuring and recoloring in Case 1 (lines 44–63).
2. If `y` is black and its children are `null` or black, change `y` to red. If `parent` of `y` is black, denote `parent` to be the new double-black node and invoke `fixDoubleBlack` recursively (line 77).
3. If `y` is red, adjust the nodes to make `parent` a child of `y` (lines 84, 89) and color `parent` red and `y` black (lines 92–93). Make `y` the new parent (line 94). Recursively invoke `fixDoubleBlack` on the same double-black node with a different color for `parent` (line 95).

Figure 47.28 shows the steps of deleting elements. To delete `50` from the tree in Figure 47.28(a), apply Case 1.2, as shown in Figure 47.28(b). After restructuring and recoloring, the new tree is as shown in Figure 47.28(c).

deletion example

When deleting `20` in Figure 47.28(c), `20` is an internal node, and it is replaced by `16`, as shown in Figure 47.28(d). Now Case 2 applies to deleting the rightmost node, as shown in Figure 47.28(e). Recolor the nodes results in a new tree, as shown in Figure 47.28(f).

When deleting `15`, connect node 3 with node 20 and color node 3 black, as shown in Figure 47.28(g). We are done.

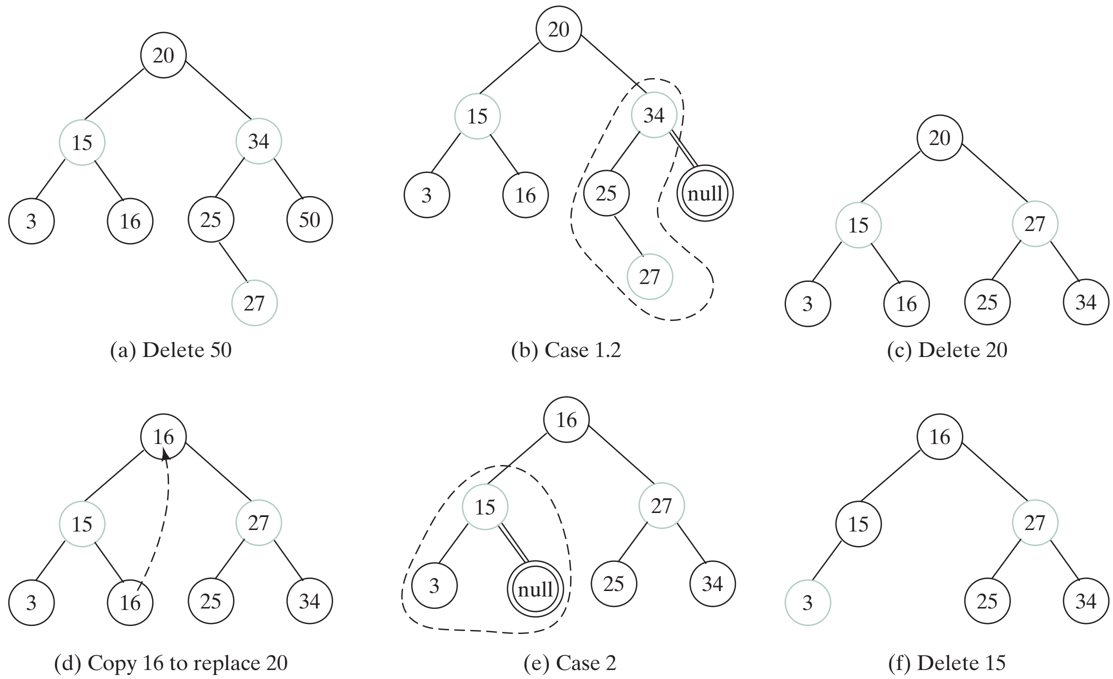


FIGURE 47.28 Delete elements from a red-black tree.

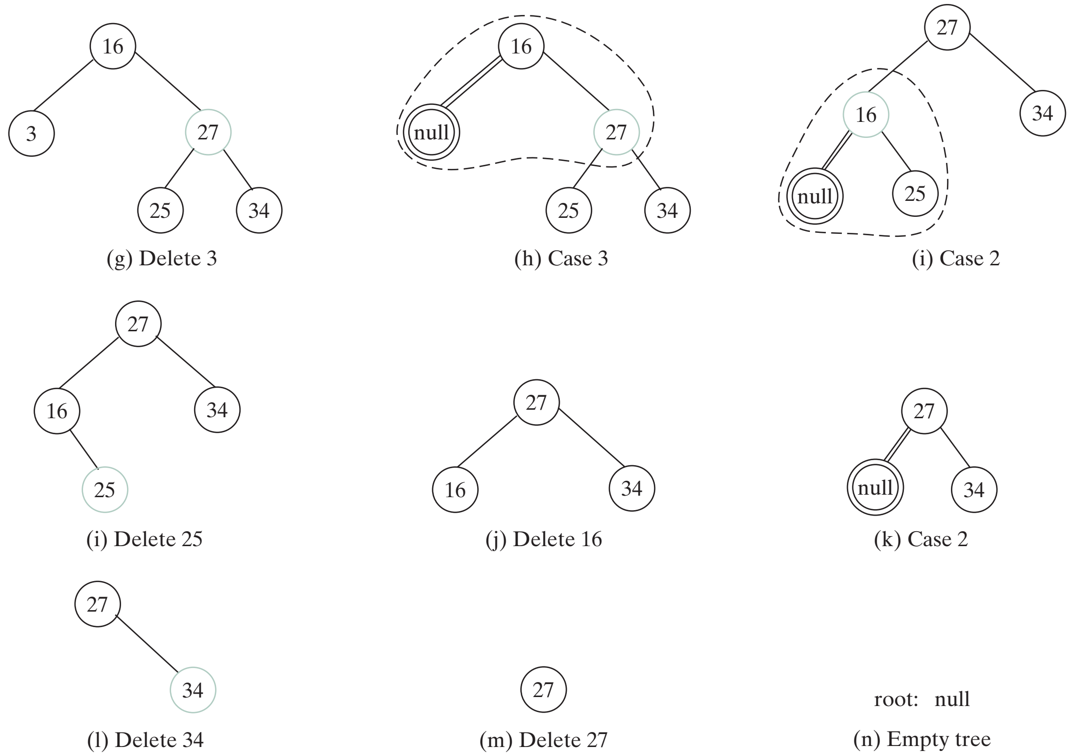


FIGURE 47.28 *continued*

After deleting **25**, the new tree is as shown in Figure 47.28(j). Now delete **16**. Apply Case 2, as shown in Figure 47.28(k). The new tree is shown in Figure 47.29(l).

After deleting **34**, the new tree is as shown in Figure 47.28(m).

After deleting **27**, the new tree is as shown in Figure 47.28(n).

47.6 Implementing **RBTree** Class

Listing 47.3 gives a complete implementation for the **RBTree** class.

LISTING 47.3 RBTree.java

no-arg constructor

constructor

create a new node

```
import java.util.ArrayList;

public class RBTree<E extends Comparable<E>> extends BinaryTree<E> {
  /** Create a default RB tree */
  public RBTree() {
  }

  /** Create an RB tree from an array of elements */
  public RBTree(E[] elements) {
    super(elements);
  }

  /** Override createNewNode to create an RBTreeNode */
  protected RBTreeNode<E> createNewNode(E e) {
    return new RBTreeNode<E>(e);
  }
```

```java

  /** Override the insert method to balance the tree if necessary */
  public boolean insert(E e) {
    boolean successful = super.insert(e);
    if (!successful)
      return false; // e is already in the tree
    else {
      ensureRBTree(e);
    }

    return true; // e is inserted
  }

  /** Ensure that the tree is a red-black tree */
  private void ensureRBTree(E e) {
    // Get the path that leads to element e from the root
    ArrayList<TreeNode<E>> path = path(e);

    int i = path.size() - 1; // Index to the current node in the path

    // u is the last node in the path. u contains element e
    RBTreeNode<E> u = (RBTreeNode<E>)(path.get(i));

    // v is the parent of of u, if exists
    RBTreeNode<E> v = (u == root) ? null :
      (RBTreeNode<E>)(path.get(i - 1));

    u.setRed(); // It is OK to set u red

    if (u == root) // If e is inserted as the root, set root black
      u.setBlack();
    else if (v.isRed())
      fixDoubleRed(u, v, path, i); // Fix double-red violation at u
  }

  /** Fix double-red violation at node u */
  private void fixDoubleRed(RBTreeNode<E> u, RBTreeNode<E> v,
      ArrayList<TreeNode<E>> path, int i) {
    // w is the grandparent of u
    RBTreeNode<E> w = (RBTreeNode<E>)(path.get(i - 2));
    RBTreeNode<E> parentOfw = (w == root) ? null :
      (RBTreeNode<E>)path.get(i - 3);

    // Get v's sibling named x
    RBTreeNode<E> x = (w.left == v) ?
      (RBTreeNode<E>)(w.right) : (RBTreeNode<E>)(w.left);

    if (x == null || x.isBlack()) {
      // Case 1: v's sibling x is black
      if (w.left == v && v.left == u) {
        // Case 1.1: u < v < w, Restructure and recolor nodes
        restructureRecolor(u, v, w, w, parentOfw);

        w.left = v.right; // v.right is y3 in Figure 47.6
        v.right = w;
      }
      else if (w.left == v && v.right == u) {
        // Case 1.2: v < u < w, Restructure and recolor nodes
        restructureRecolor(v, u, w, w, parentOfw);
        v.right = u.left;
```

insert to tree
invoke super.insert
duplicate element
ensure color and depth
ensure color and depth
get path
node index
get **u**
get **v**
u is root?
double-red violation
fix double red
get **w**
get **x**
Case 1
Case 1.1
Case 1.2

```
       w.left = u.right;
       u.left = v;
       u.right = w;
     }
```

Case 1.3

```
     else if (w.right == v && v.right == u) {
       // Case 1.3: w < v < u, Restructure and recolor nodes
       restructureRecolor(w, v, u, w, parentOfw);
       w.right = v.left;
       v.left = w;
     }
```

Case 1.4

```
     else {
       // Case 1.4: w < u < v, Restructure and recolor nodes
       restructureRecolor(w, u, v, w, parentOfw);
       w.right = u.left;
       v.left = u.right;
       u.left = w;
       u.right = v;
     }
   }
```

Case 2

```
   else { // Case 2: v's sibling x is red
     // Recolor nodes
```

recoloring

```
     w.setRed();
     u.setRed();
      ((RBTreeNode<E>)(w.left)).setBlack();
      ((RBTreeNode<E>)(w.right)).setBlack();

```

w is root?

```
      if (w == root) {
        w.setBlack();
      }
      else if (((RBTreeNode<E>)parentOfw).isRed()) {
        // Propagate along the path to fix new double-red violation
```

propagate upward

```
        u = w;
        v = (RBTreeNode<E>)parentOfw;
```

fix new double red

```
        fixDoubleRed(u, v, path, i - 2); // i – 2 propagates upward
      }
    }
  }

  /** Connect b with parentOfw and recolor a, b, c for a < b < c */
```

restructure/recolor

```
  private void restructureRecolor(RBTreeNode<E> a, RBTreeNode<E> b,
      RBTreeNode<E> c, RBTreeNode<E> w, RBTreeNode<E> parentOfw) {
    if (parentOfw == null)
      root = b;
    else if (parentOfw.left == w)
      parentOfw.left = b;
    else
      parentOfw.right = b;

    b.setBlack(); // b becomes the root in the subtree
    a.setRed(); // a becomes the left child of b
    c.setRed(); // c becomes the right child of b
  }

  /** Delete an element from the RBTree.
   * Return true if the element is deleted successfully
   * Return false if the element is not in the tree */
```

delete e from tree

```
  public boolean delete(E e) {
    // Locate the node to be deleted
```

locate the node

```
    TreeNode<E> current = root;
    while (current != null) {
```

```
      if (e.compareTo(current.element) < 0) {
        current = current.left;
      }
      else if (e.compareTo(current.element) > 0) {
        current = current.right;
      }
      else
        break; // Element is in the tree pointed by current
    }

    if (current == null)
      return false; // Element is not in the tree

    java.util.ArrayList<TreeNode<E>> path;

    // current node is an internal node
    if (current.left != null && current.right != null) {
      // Locate the rightmost node in the left subtree of current
      TreeNode<E> rightMost = current.left;
      while (rightMost.right != null) {
        rightMost = rightMost.right; // Keep going to the right
      }

      path = path(rightMost.element); // Get path before replacement

      // Replace the element in current by the element in rightMost
      current.element = rightMost.element;
    }
    else
      path = path(e); // Get path to current node

    // Delete the last node in the path and propagate if needed
    deleteLastNodeInPath(path);

    size--; // After one element deleted
    return true; // Element deleted
  }

  /** Delete the last node from the path. */
  public void deleteLastNodeInPath(ArrayList<TreeNode<E>> path) {
    int i = path.size() - 1; // Index to the node in the path
    // u is the last node in the path
    RBTreeNode<E> u = (RBTreeNode<E>)(path.get(i));
    RBTreeNode<E> parentOfu = (u == root) ? null :
      (RBTreeNode<E>)(path.get(i - 1));
    RBTreeNode<E> grandparentOfu = (parentOfu == null ||
      parentOfu == root) ? null :
      (RBTreeNode<E>)(path.get(i - 2));
    RBTreeNode<E> childOfu = (u.left == null) ?
      (RBTreeNode<E>)(u.right) : (RBTreeNode<E>)(u.left);

    // Delete node u. Connect childOfu with parentOfu
    connectNewParent(parentOfu, u, childOfu);

    // Recolor the nodes and fix double black if needed
    if (childOfu == root || u.isRed())
      return; // Done if childOfu is root or if u is red
    else if (childOfu != null && childOfu.isRed())
      childOfu.setBlack(); // Set it black, done
    else // u is black, childOfu is null or black
```

element not found
internal element?
rightmost node
path to external node
delete the node
one element deleted
deletion successful
delete a node
u
parentOfu
grandparentOfu
childOfu
delete u
done
set **childOfu** black

```java
      // Fix double black on parentOfu
```

fix double black

```java
      fixDoubleBlack(grandparentOfu, parentOfu, childOfu, path, i);
  }

  /** Fix the double-black problem at node parent */
```

fix double black

```java
  private void fixDoubleBlack(
      RBTreeNode<E> grandparent, RBTreeNode<E> parent,
      RBTreeNode<E> db, ArrayList<TreeNode<E>> path, int i) {
    // Obtain y, y1, and y2
```

y, y1, y2

```java
    RBTreeNode<E> y = (parent.right == db) ?
      (RBTreeNode<E>)(parent.left) : (RBTreeNode<E>)(parent.right);
    RBTreeNode<E> y1 = (RBTreeNode<E>)(y.left);
    RBTreeNode<E> y2 = (RBTreeNode<E>)(y.right);

    if (y.isBlack() && y1 != null && y1.isRed()) {
```

process Case 1.1

```java
      if (parent.right == db) {
        // Case 1.1: y is a left black sibling and y1 is red
        connectNewParent(grandparent, parent, y);
        recolor(parent, y, y1); // Adjust colors

        // Adjust child links
        parent.left = y.right;
        y.right = parent;
      }
```

process Case 1.3

```java
      else {
        // Case 1.3: y is a right black sibling and y1 is red
        connectNewParent(grandparent, parent, y1);
        recolor(parent, y1, y); // Adjust colors

        // Adjust child links
        parent.right = y1.left;
        y.left = y1.right;
        y1.left = parent;
        y1.right = y;
      }
    }
    else if (y.isBlack() && y2 != null && y2.isRed()) {
```

process Case 1.2

```java
      if (parent.right == db) {
        // Case 1.2: y is a left black sibling and y2 is red
        connectNewParent(grandparent, parent, y2);
        recolor(parent, y2, y); // Adjust colors

        // Adjust child links
        y.right = y2.left;
        parent.left = y2.right;
        y2.left = y;
        y2.right = parent;
      }
```

process Case 1.4

```java
      else {
        // Case 1.4: y is a right black sibling and y2 is red
        connectNewParent(grandparent, parent, y);
        recolor(parent, y, y2); // Adjust colors

        // Adjust child links
        y.left = parent;
        parent.right = y1;
      }
    }
```

process Case 2

```java
    else if (y.isBlack()) {
```

```
    // Case 2: y is black and y's children are black or null
    y.setRed(); // Change y to red
    if (parent.isRed())
      parent.setBlack(); // Done
    else if (parent != root) {
      // Propagate double black to the parent node
      // Fix new appearance of double black recursively
      db = parent;
      parent = grandparent;
      grandparent =
        (i >= 3) ? (RBTreeNode<E>)(path.get(i - 3)) : null;
      fixDoubleBlack(grandparent, parent, db, path, i - 1);
    }
  }
  else { // y.isRed()
    if (parent.right == db) {
      // Case 3.1: y is a left red child of parent
      parent.left = y2;
      y.right = parent;
    }
    else {
      // Case 3.2: y is a right red child of parent
      parent.right = y.left;
      y.left = parent;
    }

    parent.setRed(); // Color parent red
    y.setBlack(); // Color y black
    connectNewParent(grandparent, parent, y); // y is new parent
    fixDoubleBlack(y, parent, db, path, i - 1);
  }
}

/** Recolor parent, newParent, and c. Case 1 removal */
private void recolor(RBTreeNode<E> parent,
    RBTreeNode<E> newParent, RBTreeNode<E> c) {
  // Retain the parent's color for newParent
  if (parent.isRed())
    newParent.setRed();
  else
    newParent.setBlack();

  // c and parent become the children of newParent; set them black
  parent.setBlack();
  c.setBlack();
}

/** Connect newParent with grandParent */
private void connectNewParent(RBTreeNode<E> grandparent,
    RBTreeNode<E> parent, RBTreeNode<E> newParent) {
  if (parent == root) {
    root = newParent;
    if (root != null)
      newParent.setBlack();
  }
  else if (grandparent.left == parent)
    grandparent.left = newParent;
  else
    grandparent.right = newParent;
}
```

propagate double black

process Case 3.1

process Case 3.2

fix double black

connect to **grandParent**

override **preorder**

RBTreeNode

```
316
317   /** Preorder traversal from a subtree */
318   protected void preorder(TreeNode<E> root) {
319     if (root == null) return;
320     System.out.print(root.element +
321       (((RBTreeNode<E>)root).isRed() ? " (red) " : " (black) "));
322     preorder(root.left);
323     preorder(root.right);
324   }
325
326   /** RBTreeNode is TreeNode plus color indicator */
327   protected static class RBTreeNode<E extends Comparable<E>> extends
328       BinaryTree.TreeNode<E> {
329     private boolean red = true; // Indicate node color
330
331     public RBTreeNode(E e) {
332       super(e);
333     }
334
335     public boolean isRed() {
336       return red;
337     }
338
339     public boolean isBlack() {
340       return!red;
341     }
342
343     public void setBlack() {
344       red = false;
345     }
346
347     public void setRed() {
348       red = true;
349     }
350
351     int blackHeight;
352   }
353 }
```

constructors

The `RBTree` class extends `BinaryTree`. Like the `BinaryTree` class, the `RBTree` class has a no-arg constructor that constructs an empty `RBTree` (lines 5–6) and a constructor that creates an initial `RBTree` from an array of elements (lines 9–11).

createNewNode()

The `createNewNode()` method defined in the `BinaryTree` class creates a `TreeNode`. This method is overridden to return an `RBTreeNode` (lines 14–16). This method is invoked in the insert method in `BinaryTree` to create a node.

insert

The `insert` method in `RBTree` is overridden in lines 19–28. The method first invokes the `insert` method in `BinaryTree`, then invokes `ensureRBTree(e)` (line 24) to ensure that tree is still a red-black tree after inserting a new element.

ensureRBTree

The `ensureRBTree(E e)` method first obtains the path of nodes that lead to element `e` from the root (line 33). It obtains `u` and `v` (the parent of `u`) from the path. If `u` is the root, color `u` black (lines 46–47). If `v` is red, invoke `fixDoubleRed` to fix the double red on both `u` and `v` (lines 48–49).

fixDoubleRed

The `fixDoubleRed(u, v, path, i)` method fixes the double-red violation at node `u`. The method first obtains `w` (the grandparent of `u` from the path) (line 56), `parentOfw` if exists (lines 57–58), and `x` (the sibling of `v`) (lines 61–62). If `x` is `null` or black, consider four subcases to fix the double-red violation (lines 66–94). If x is red, color `w` and `u` red and color `w`'s two children black (lines 98–101). If `w` is the root, color `w` black (lines 103–105). Otherwise, propagate along the path to fix the new double-red violation (lines 108–110).

The `delete(E e)` method in `RBTree` is overridden in lines 133–173. The method locates the node that contains `e` (lines 135–145). If the node is null, no element is found (lines 147–148). The method considers two cases:

delete

- If the node is internal, find the rightmost node in its left subtree (lines 155–158). Obtain a path from the root to the rightmost node (line 160), and replace the element in the node with the element in the rightmost node (line 163).
- If the node is external, obtain the path from the root to the node (line 166).

The last node in the path is the node to be deleted. Invoke `deleteLastNodeInPath(path)` to delete it and ensure the tree is a red-black after the node is deleted (line 169).

The `deleteLastNodeInPath(path)` method first obtains `u`, `parentOfu`, `grandparendOfu`, and `childOfu` (lines 179–186). `u` is the last node in the path. Connect `childOfu` as a child of `parentOfu` (line 189). This in effect deletes `u` from the tree. Consider three cases:

deleteLastNodeInPath

- If `childOfu` is the root or `childOfu` is red, we are done (lines 192–193).
- Otherwise, if `childOfu` is red, color it black (lines 194–195).
- Otherwise, invoke `fixDoubleBlack` to fix the double-black problem on `childOfu` (line 198).

The `fixDoubleBlack` method first obtains `y`, `y1`, and `y2` (lines 206–209). `y` is the sibling of the first double-black node, and `y1` and `y2` are the left and right children of `y`. Consider three cases:

fixDoubleBlack

- If `y` is black and `y1` or `y2` is red, fix the double-black problem for Case 1 (lines 212–254).
- Otherwise, if `y` is black, fix the double-black problem for Case 2 by recoloring the nodes. If parent is black and not a root, propagate double black to parent and recursively invoke `fixDoubleBlack` (lines 263–267).
- Otherwise, `y` is red. In this case, adjust the nodes to make parent the child of y (lines 271–280). Invoke `fixDoubleBlack` with the adjusted nodes (line 285) to fix the double-black problem.

The `preorder(TreeNode<E> root)` method is overridden to display the node colors (lines 318–324).

preorder

47.7 Testing the **RBTree** Class

Listing 47.4 gives a test program. The program creates an `RBTree` initialized with an array of integers `34`, `3`, and `50` (lines 4–5), inserts elements in lines 8–20, and deletes elements in lines 23–44.

Listing 47.4 TestRBTree.java

```
1 public class TestRBTree {
2   public static void main(String[] args) {
3     // Create an RB tree
4     RBTree<Integer> tree =
5       new RBTree<Integer>(new Integer[]{34, 3, 50});
6     printTree(tree);
7
8     tree.insert(20);
9     printTree(tree);
```

create an **RBTree**

insert 20

insert 15
insert 16
insert 25
insert 27
delete 50
delete 15
delete 3
delete 25
delete 16
delete 34
delete 27

```java

    tree.insert(15);
    printTree(tree);

    tree.insert(16);
    printTree(tree);

    tree.insert(25);
    printTree(tree);

    tree.insert(27);
    printTree(tree);

    tree.delete(50);
    printTree(tree);

    tree.delete(20);
    printTree(tree);

    tree.delete(15);
    printTree(tree);

    tree.delete(3);
    printTree(tree);

    tree.delete(25);
    printTree(tree);

    tree.delete(16);
    printTree(tree);

    tree.delete(34);
    printTree(tree);

    tree.delete(27);
    printTree(tree);
  }

  public static void printTree(BinaryTree tree) {
    // Traverse tree
    System.out.print("\nInorder (sorted): ");
    tree.inorder();
    System.out.print("\nPostorder: ");
    tree.postorder();
    System.out.print("\nPreorder: ");
    tree.preorder();
    System.out.print("\nThe number of nodes is " + tree.getSize());
    System.out.println();
  }
}
```

```
Inorder (sorted): 3 34 50
Postorder: 3 50 34
Preorder: 34 (black) 3 (red) 50 (red)
The number of nodes is 3

Inorder (sorted): 3 20 34 50
Postorder: 20 3 50 34
```

```
Preorder: 34 (black) 3 (black) 20 (red) 50 (black)
The number of nodes is 4

Inorder (sorted): 3 15 20 34 50
Postorder: 3 20 15 50 34
Preorder: 34 (black) 15 (black) 3 (red) 20 (red) 50 (black)
The number of nodes is 5

Inorder (sorted): 3 15 16 20 34 50
Postorder: 3 16 20 15 50 34
Preorder: 34 (black) 15 (red) 3 (black) 20 (black) 16 (red) 50 (black)
The number of nodes is 6

Inorder (sorted): 3 15 16 20 25 34 50
Postorder: 3 16 25 20 15 50 34
Preorder: 34 (black) 15 (red) 3 (black) 20 (black) 16 (red) 25 (red)
  50 (black)
The number of nodes is 7

Inorder (sorted): 3 15 16 20 25 27 34 50
Postorder: 3 16 15 27 25 50 34 20
Preorder: 20 (black) 15 (red) 3 (black) 16 (black) 34 (red) 25 (black)
  27 (red) 50 (black)
The number of nodes is 8

Inorder (sorted): 3 15 16 20 25 27 34
Postorder: 3 16 15 25 34 27 20
Preorder: 20 (black) 15 (red) 3 (black) 16 (black) 27 (red)
  25 (black) 34 (black)
The number of nodes is 7

Inorder (sorted): 3 15 16 25 27 34
Postorder: 3 15 25 34 27 16
Preorder: 16 (black) 15 (black) 3 (red) 27 (red) 25 (black) 34 (black)
The number of nodes is 6

Inorder (sorted): 3 16 25 27 34
Postorder: 3 25 34 27 16
Preorder: 16 (black) 3 (black) 27 (red) 25 (black) 34 (black)
The number of nodes is 5

Inorder (sorted): 16 25 27 34
Postorder: 25 16 34 27
Preorder: 27 (black) 16 (black) 25 (red) 34 (black)
The number of nodes is 4

Inorder (sorted): 16 27 34
Postorder: 16 34 27
Preorder: 27 (black) 16 (black) 34 (black)
The number of nodes is 3

Inorder (sorted): 27 34
Postorder: 34 27
Preorder: 27 (black) 34 (red)
The number of nodes is 2

Inorder (sorted): 27
Postorder: 27
Preorder: 27 (black)
The number of nodes is 1

Inorder (sorted):
Postorder:
Preorder:
The number of nodes is 0
```

Figure 47.14 shows how the tree evolves as elements are added to it, and Figure 47.28 shows how the tree evolves as elements are deleted from it.

47.8 Performance of the RBTree Class

The search, insertion, and deletion times in a red-black tree depend on the height of the tree. A red-black tree corresponds to a 2-4 tree. When you convert a node in a 2-4 tree to red-black tree nodes, you get one black node and zero, one, or two red nodes as its children, depending on whether the original node is a 2-node, 3-node, or 4-node. So, the height of a red-black tree is at most as twice that of its corresponding 2-4 tree. Since the height of a 2-4 tree is log*n*, the height of a red-black tree is 2log*n*.

2log*n* height

red-black vs. AVL

A red-black tree has the same time complexity as an AVL tree, as shown in Table 47.1. In general, a red-black is more efficient than an AVL tree, because a red-black tree requires only one time restructuring of the nodes for insert and delete operations.

red-black vs. 2-4

A *red-black tree* has the same time complexity as a 2-4 tree, as shown in Table 47.1. In general, a red-black is more efficient than a 2-4 tree for two reasons:

1. A red-black tree requires only one-time restructuring of the nodes for insert and delete operations. However, a 2-4 tree may require many splits for an insert operation and fusion for a delete operation.
2. A red-black tree is a binary search tree. A binary tree can be implemented more space efficiently than a 2-4 tree, because a node in a 2-4 tree has at most three elements and four children. Space is wasted for 2-nodes and 3-nodes in a 2-4 tree.

TABLE 47.1 Time Complexities for Methods in RBTree, AVLTree, and Tree234

Methods	*Red-Black Tree*	*AVL Tree*	*2-4 Tree*
search(e: E)	$O(\log n)$	$O(\log n)$	$O(\log n)$
insert(e: E)	$O(\log n)$	$O(\log n)$	$O(\log n)$
delete(e: E)	$O(\log n)$	$O(\log n)$	$O(\log n)$
getSize()	$O(1)$	$O(1)$	$O(1)$
isEmpty()	$O(1)$	$O(1)$	$O(1)$

Listing 47.5 gives an empirical test of the performance of AVL trees, 2-4 trees, and red-black trees.

LISTING 47.5 TreePerformanceTest.java

an AVL tree

a 2-4 tree

a red-black tree

```
public class TreePerformanceTest {
  public static void main(String[] args) {
    final int TEST_SIZE = 500000; // Tree size used in the test

    // Create an AVL tree
    Tree<Integer> tree1 = new AVLTree<Integer>();
    System.out.println("AVL tree time: " +
      getTime(tree1, TEST_SIZE) + " milliseconds");

    // Create a 2-4 tree
    Tree<Integer> tree2 = new Tree24<Integer>();
    System.out.println("2-4 tree time: "
      + getTime(tree2, TEST_SIZE) + " milliseconds");

    // Create a red-black tree
    Tree<Integer> tree3 = new RBTree<Integer>();
    System.out.println("RB tree time: "
```

```
18        + getTime(tree3, TEST_SIZE) + " milliseconds");
19    }
20
21    public static long getTime(Tree<Integer> tree, int testSize) {
22      long startTime = System.currentTimeMillis(); // Start time
23
24      // Create a list to store distinct integers
25      java.util.List<Integer> list = new java.util.ArrayList<Integer>();
26      for (int i = 0; i < testSize; i++)
27        list.add(i);
28
29      java.util.Collections.shuffle(list); // Shuffle the list
30
31      // Insert elements in the list to the tree
32      for (int i = 0; i < testSize; i++)
33        tree.insert(list.get(i));
34
35      java.util.Collections.shuffle(list); // Shuffle the list
36
37      // Delete elements in the list from the tree
38      for (int i = 0; i < testSize; i++)
39        tree.delete(list.get(i));
40
41      // Return elapse time
42      return System.currentTimeMillis() - startTime;
43    }
44  }
```

start time

shuffle

add to tree

shuffle

remove from container

end time

return elapsed time

```
AVL tree time: 7609 milliseconds
2-4 tree time: 8594 milliseconds
RB tree time: 5515 milliseconds
```

The **getTestTime** method creates a list of distinct integers from **0** to **testSize - 1** (lines 25–27), shuffles the list (line 29), adds the elements from the list to a tree (lines 32–33), shuffles the list again (line 35), removes the elements from the tree (lines 38–39), and finally returns the execution time (line 42).

The program creates an AVL (line 6), a 2-4 tree (line 11), and a red-black tree (line 16). The program obtains the execution time for adding and removing **500000** elements in the three trees.

As you see, the red-black tree performs the best, followed by the AVL tree.

red-black tree best

Note

java.util.TreeSet

The **java.util.TreeSet** class in the Java API is implemented using a red-black tree. Each entry in the set is stored in the tree. Since the **search**, **insert**, and **delete** methods in a red-black tree take $O(\log n)$ time, the **get**, **add**, **remove**, and **contains** methods in **java.util.TreeSet** take $O(\log n)$ time.

Note

java.util.TreeMap

The **java.util.TreeMap** class in the Java API is implemented using a red-black tree. Each entry in the map is stored in the tree. The order of the entries is determined by their keys. Since the **search**, **insert**, and **delete** methods in a red-black tree take $O(\log n)$ time, the **get**, **put**, **remove**, and **containsKey** methods in **java.util.TreeMap** take $O(\log n)$ time.

Key Terms

black depth 47–2
double-black problem 47–11
double-red violation 47–5
external node 47–2
red-black tree 47–2

Chapter Summary

1. A red-black tree is a binary search tree, derived from a *2-4 tree*. A red-black tree corresponds to a 2-4 tree. You can convert a red-black tree to a 2-4 tree or vice versa.
2. In a red-black tree, each node is colored red or black. The root is always black. Two adjacent nodes cannot be both red. All external nodes have the same black depth.
3. Since a red-black tree is a binary search tree, the `RBTree` class extends the `BinaryTree` class.
4. Searching an element in a red-black tree is the same as in binary search tree, since a red-black tree is a binary search tree.
5. A new element is always inserted as a leaf node. If the new node is the root, color it black. Otherwise, color it red. If the parent of the new node is red, we have to fix the *double-red* violation by reassigning the color and/or restructuring the tree.
6. If a node to be deleted is internal, find the rightmost node in its left subtree. Replace the element in the node with the element in the rightmost node. Delete the rightmost node.
7. If the external node to be deleted is red, simply reconnect the parent node of the external node with the child node of the external node.
8. If the external node to be deleted is black, you need to consider several cases to ensure that black height for external nodes in the tree is maintained correctly.
9. The height of a red-black tree is O(logn). So, the time complexities for the `search`, `insert`, and `delete` methods are O(logn).

Review Questions

Sections 47.1–47.2

47.1 What is a red-black tree? What is an external node? What is black-depth?

47.2 Describe the properties of a red-black tree.

47.3 How do you convert a red-black tree to a 2-4 tree? Is the conversion unique?

47.4 How do you convert a 2-4 tree to a red-black tree? Is the conversion unique?

Sections 47.3–47.5

47.5 What are the data fields in `RBTreeNode`?

47.6 How do you insert an element into a red-black tree and how do you fix the double-red violation?

47.7 How do you delete an element from a red-black tree and how do you fix the double-black problem?

47.8 Show the change of the tree when inserting `1`, `2`, `3`, `4`, `10`, `9`, `7`, `5`, `8`, `6` into it, in this order.

47.9 For the tree built in the preceding question, show the change of the tree after deleting `1`, `2`, `3`, `4`, `10`, `9`, `7`, `5`, `8`, `6` from it in this order.

Programming Exercises

47.1* (*red-black tree to 2-4 tree*) Write a program that converts a red-black tree to a 2-4 tree.

47.2* (*2-4 tree to red-black tree*) Write a program that converts a red-black tree to a 2-4 tree.

47.3* ** (*red-black tree animation*) Write a Java applet that animates the red-black tree `insert`, `delete`, and `search` methods, as shown in Figure 47.6.

47.4** (*Parent reference for `RBTree`*) Suppose that the `TreeNode` class defined in `BinaryTree` contains a reference to the node's parent, as shown in Exercise 26.17. Implement the `RBTree` class to support this change. Write a test program that adds numbers `1`, `2`, . . . , `100` to the tree and displays the paths for all leaf nodes.