

作业 2.2

一、求下列函数的导数: (40 分)

$$(1) \quad y = 3x^3 - 4^x + 5e^{2x} \quad \text{解: } y' = 9x^2 - 2\ln 2 \cdot 4^x + 10e^{2x}$$

$$(2) \quad y = \sin x \cdot \cos x \quad \text{解: } y' = \cos^2 x - \sin^2 x$$

$$(3) \quad y = \frac{\ln x}{x^2} \quad \text{解: } y' = \frac{1 - 2\ln x}{x^3}$$

$$(4) \quad y = (3x + 4)^5 \quad \text{解: } y' = 15(3x + 4)^4$$

$$(5) \quad y = \sin(3 - 5x) \quad \text{解: } y' = -5\cos(3 - 5x)$$

$$(6) \quad y = \ln(1 + x^3) \quad \text{解: } y' = \frac{3x^2}{1 + x^3}$$

$$(7) \quad y = \arctan \sqrt{x} \quad \text{解: } y' = \frac{1}{2(1+x)\sqrt{x}}$$

$$(8) \quad y = \ln(x + \sqrt{a^2 + x^2}) \quad \text{解: } y' = \frac{1}{\sqrt{a^2 + x^2}}$$

二、设 $f(x) = (x+9)^5$, 求 $f'''(1)$. (10 分)

$$\text{解: 因为 } f'(x) = 5(x+9)^4, \quad f''(x) = 20(x+9)^3, \quad f'''(x) = 60(x+9)^2$$

$$\text{所以 } f'''(1) = 60(1+9)^2 = 6000$$

三、求下列函数所指定的阶的导数: (20 分)

$$(1) \quad y = e^x \cdot \sin x, \text{ 求 } y^{(6)};$$

$$\text{解: } y^{(6)} = -8e^x \cos x$$

$$(2) \quad y = x \cos 5x, \text{ 求 } y^{(50)}.$$

$$y^{(50)} = C_{50}^{49} x' [\cos x]^{(49)} + x^{(0)} [\cos x]^{(50)}$$

$$\begin{aligned} \text{解: } &= 50 \cdot 5^{49} \cos(5x + 49 \cdot \frac{\pi}{2}) + x \cdot 5^{50} \cdot \cos(5x + 50 \cdot \frac{\pi}{2}) \\ &= -5^{50} (10 \sin 5x + x \cos 5x) \end{aligned}$$

四、选择题和填空题(30 分)

1. 设 $y = f[\sin(-x)]$, 则 $y' =$ (D)

(A) $f'[\sin(-x)]$ (B) $f'[\sin(-x)]\cos(-x)$

(C) $f'[\sin(-x)]\cos x$ (D) $-f'[\sin(-x)]\cos x$

2. 设 $f(x)$ 可导, 则 $y = e^{f(x^2)}$ 的导数 $y' =$ (D)

A $e^{f(x^2)}$ B $e^{f(x^2)}f'(x)$ C $e^{f(x^2)}f'(x^2)$ D $2xe^{f(x^2)}f'(x^2)$

3. 设 $f(x) = \frac{1}{\sqrt[3]{x^2}} - \frac{1}{x\sqrt{x}}$, 则 $f'(1) =$ ()

A $-\frac{1}{6}$ B $\frac{7}{6}$ C $-\frac{7}{6}$ D $\frac{5}{6}$

4. 已知 $y = f(u) = f\left(\frac{3x-2}{3x+2}\right)$, $f'(x) = \arctan x^2$, 则 $\left.\frac{dy}{dx}\right|_{x=0} = \underline{\underline{\frac{3\pi}{4}}}$

5. 设 $f'(x_0) = -1$, 则 $\lim_{x \rightarrow 0} \frac{x}{f(x_0 - 2x) - f(x_0 - x)} = \underline{\underline{1}}$

6. 曲线 $y = \ln x$ 上与直线 $x + y = 1$ 垂直的切线方程为 $y = x - 1$