COMP 3760

Math Review

Math Review

- Logarithm
- Floor and Celling
- Counting
  - Permutations
  - Subsets
- Summation

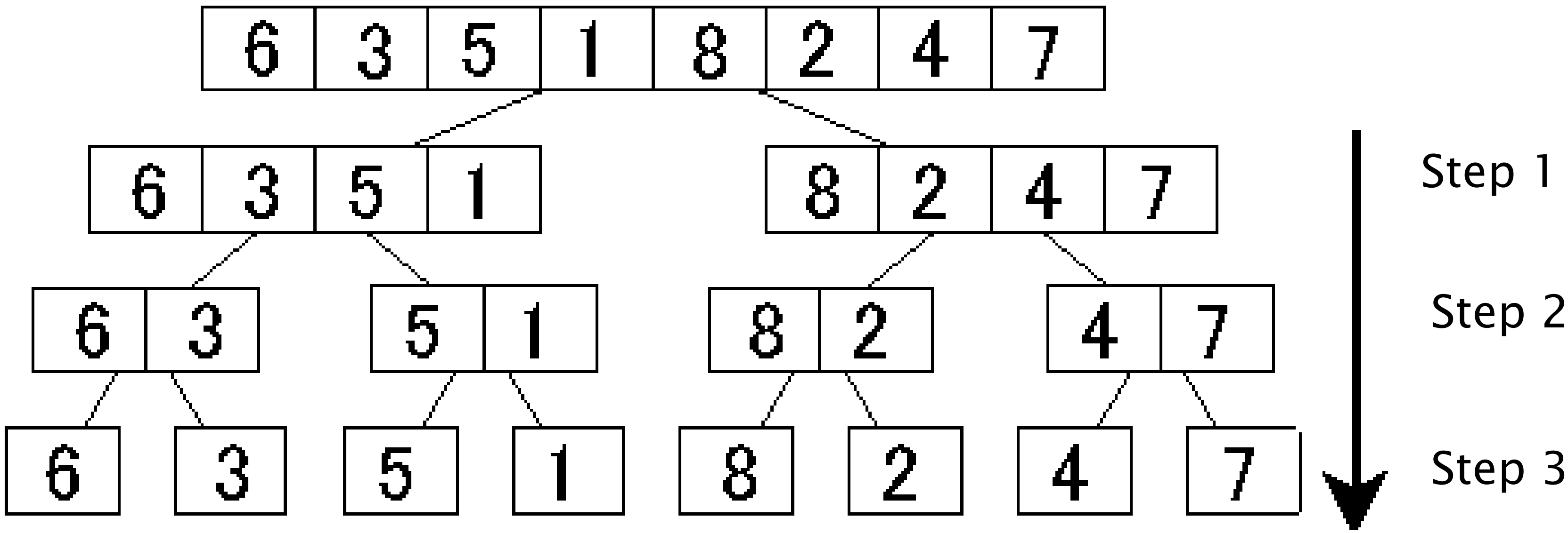
Logarithms

- Mostly what you need to know:
  - $log_b n = e$
  - just means: $b^e = n$
- So these are the same question:
  - $\log_2 16 = ?$
  - $16 = 2^?$
- In words:
  - "What is log base 2 of 16?"
  - "What power of 2 gives 16?"

When We Use Them in This Course

- The most common time to use:
  - Start with n items
  - Divide the group in half at each step
  - How many steps does it take to get down to one?

Example



$$\log_2 8 = 3$$

Floor and Ceiling

- If *x* is not a whole number, these are useful:
  - $\lceil x \rceil$ = The closest whole number *above* x
    (the *ceiling* of x)
  - $\lfloor x \rfloor$ = The closest whole number *below* x
    (the *floor* of x)

So: $\lceil \log 38 \rceil = 6$

$\lfloor \log 38 \rfloor = 5$

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Counting

- Sometimes, we need to count things
- Example

In how many different ways could student sit on the chairs in a class?

Counting

- The trick when counting is this:
  - Divide the problem into a sequence of independent choices
  - See how many options there are for each choice
  - Multiply those number together

Counting Permutations

- A permutation is an arrangement in which order matters. ABC differs from BCA
- How many permutations are there on a collection of *3* items, A, B,C?
- ABC, ACB, BAC, BCA, CAB, CBA

Permutations

- Suppose you have n items: $A_1,\ldots,A_n$
- Then you have n independent choices:

....

- Count the # of options for each choice

n 1

- Multiply together:
  - $n*(n-1)*\ldots*1 = n!$ permutations

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Subsets

- Given a set of 3 items {a, b, c}, how many different subsets can we make?

- Subsets are:

  {a, b ,c}, {a, b}, {b, c}, {a, c}, {a}, {b}, {c}, {}

Subsets

- Suppose you have n things: $A_1, \ldots, A_n$
- Then you have n items(choices) to consider:

A_1 .... A_n

- You have 2 options for each item (in/out)

i/o .... i/o

- Multiply together:
  - $2*2*\ldots*2$ (n times) = 2^n subsets

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Summations

- We use compact notation for summations

End by putting this value for i.

Start by putting this value for i.

$$\sum_{i=1}^{n} f(i)$$

f(i) is just a function… Think of it as a formula that has an i in it.

Put all values in between… and add them up.

- So this is really just a shorthand for:

$$\sum_{i=1}^{n} f(i) = f(1) + f(2) + f(3) + \ldots + f(n)$$

Example

- Evaluate this expression:

$$\sum_{i=1}^{4}(2+i^2)$$

- Start with i=1, end with i=4…

$$(2+1^2)+(2+2^2)+(2+3^2)+(2+4^2)$$

- Now you just have numbers… so you can add

$$=3+6+11+18$$

$$=38\ .$$

Sum of a Constant

$$\sum_{i=1}^{n} C$$

- What it means:
  - $\underbrace{C + C + \ldots + C}_{\text{(n times)}}$
  - So: $$\sum_{i=1}^{n} C = nC$$

Sum of a Constant

$$\sum_{i=1}^{n} n$$

- What it means:
  - $\underbrace{n + n + \ldots + n}_{\text{(n times)}}$
  - So:

$$\sum_{i=1}^{n} n = n^2$$

Sums of Sums

- Sometimes you have a sum with two parts added together:

$$\sum_{n=s}^{t} [f(n) + g(n)]$$

- You can just break it into two parts:

$$\sum_{n=s}^{t} f(n) + \sum_{n=s}^{t} g(n)$$

Summation Rules

- There are many more summation rules in the appendix of your text.
- Important examples:

$$\sum_{i=1}^{n} i = 1 + 2 + 3 + \ldots + n = \frac{n(n+1)}{2} \ .$$

$$\sum_{i=1}^{n} i^2 = 1^2 + 2^2 + 3^2 + \ldots + n^2 = \frac{n(n+1)(2n+1)}{6} \ .$$

$$\sum_{i=1}^{n} i^3 = 1^3 + 2^3 + 3^3 + \ldots + n^3 = \frac{n^2(n+1)^2}{4} \ .$$

Sums of Sums

- We will often see things like this:

$$\sum_{j=1}^{i} \sum_{k=j}^{n} 1$$

- What does this mean?
  - It means you have a sum of sums
    - (NOT two sums multiplied)
  - In order to solve it… you work from the inside out.

Sum of Sums

- In this example:

$$\sum_{j=1}^{i}\sum_{k=j}^{n} 1 = \sum_{j=1}^{i}(n-j+1)$$

- Now you can divide into three sums and solve:

$$\sum_{j=1}^{i} n - \sum_{j=1}^{i} j + \sum_{j=1}^{i} 1 = \mathrm{n} * \mathrm{i} - \frac{\mathrm{i} * (\mathrm{i}+1)}{2} + \mathrm{i}$$

We will solve this kind of sum often in the first part of the course… so make sure you understand how to do it.