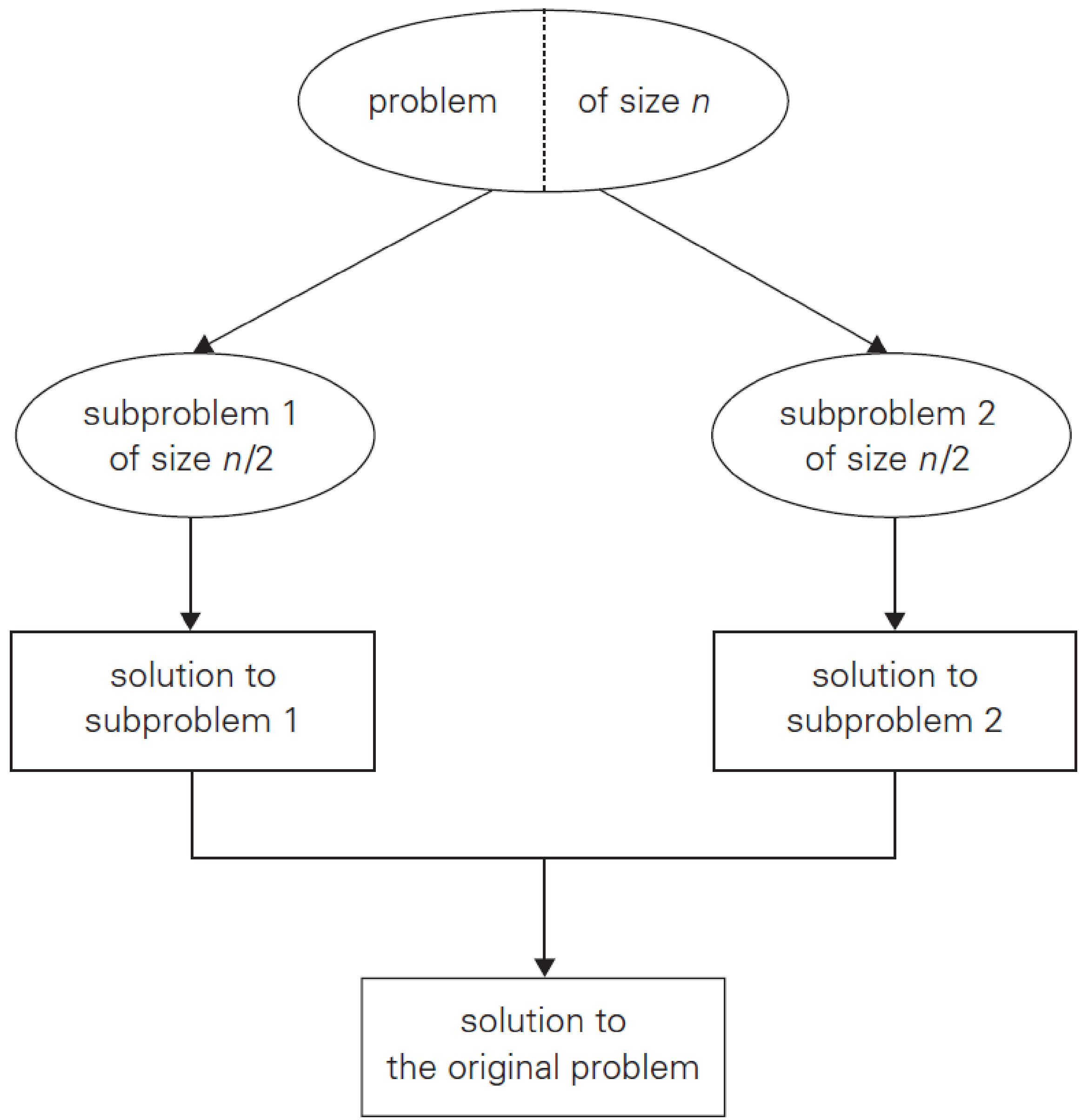
Divide and Conquer

(Chapter 5)

This week:

- Divide and Conquer technique
- Count a specific key in an array
- Master theorem
- Merge sort

Divide and Conquer technique



Divide and Conquer technique

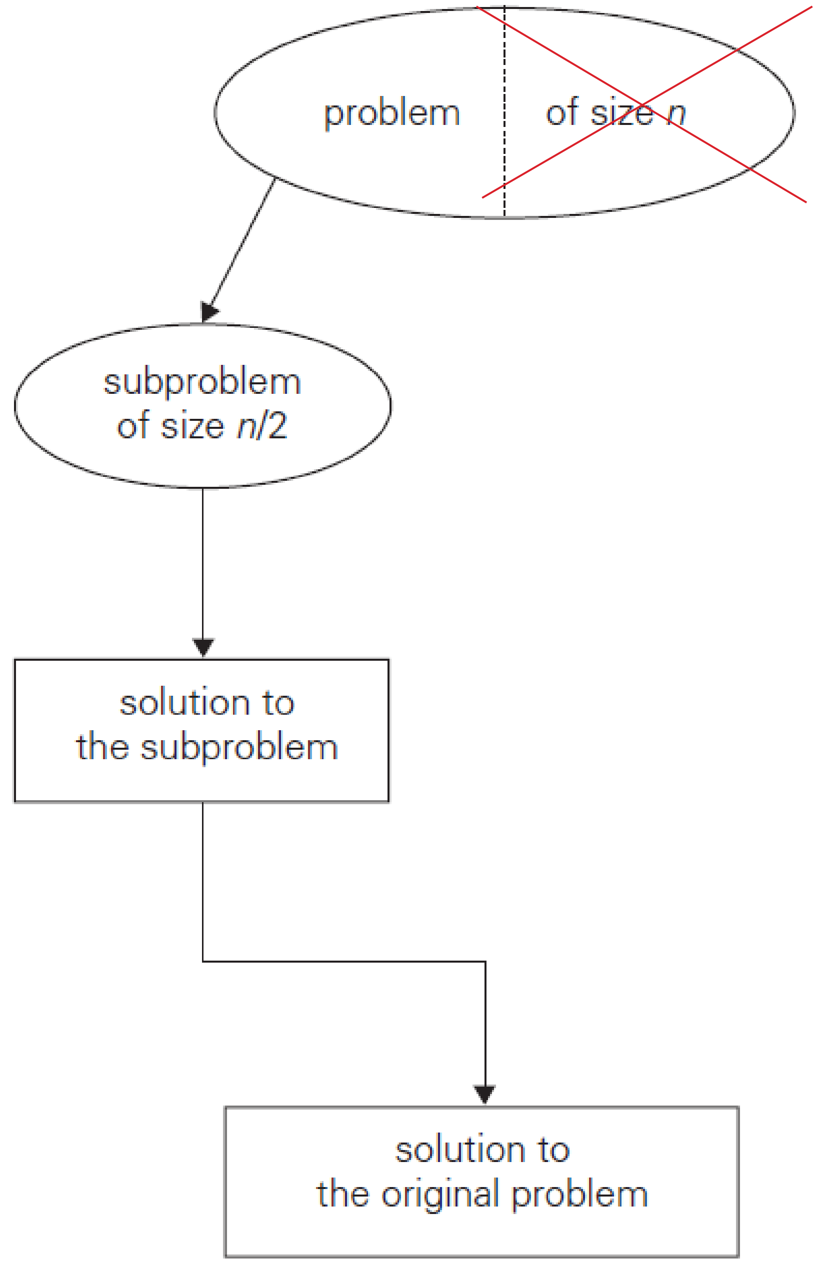
A well known algorithm design technique:

1. Divide instance of problem into two or more smaller instances

2. Solve smaller instances (usually recursively)

3. Obtain solution to original (larger) instance by combining these solutions

Decrease and Conquer (last week)



A Natural Question

- How is this different from decrease and conquer technique
- Think of the fake coin problem:
  - We discarded half the coins at each step
  - So we didn't do any work on those "sub problems"
- For divide and conquer…
  - You need to solve all of the sub problems

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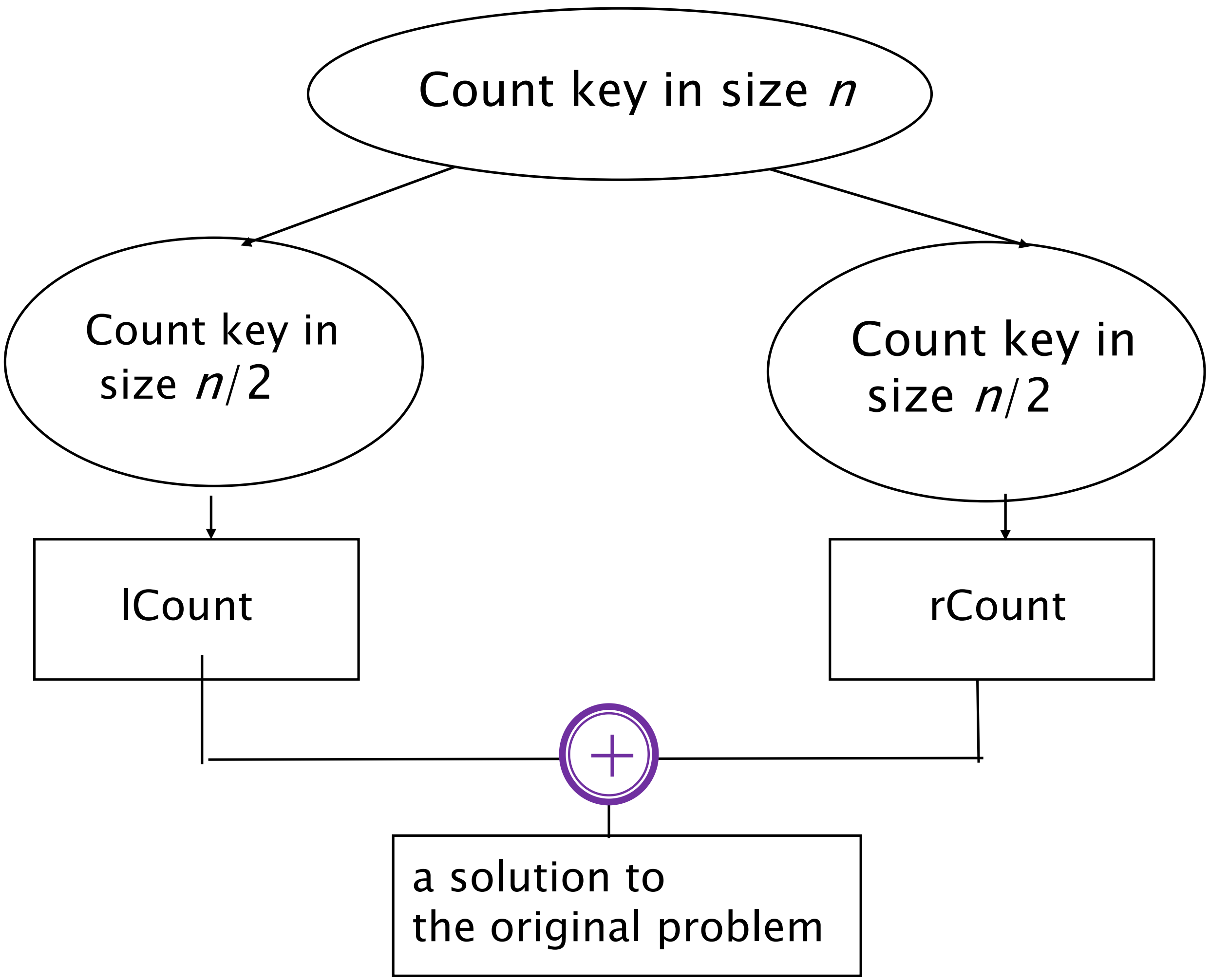
Count a specific key in an array

Problem:
Count the number of times a specific key occurs in an array.

For example:
If input array is A=[2,7,6,6,2,4,6,9,2] and key=6…
... should return the value 3.

Design an algorithm using divide and conquer technique

Count a specific key in an array



Count a specific key in an array

Algorithm CountKey(A[], L, R, Key)

```
//Input: A[] is an array A[0..n-1] from indices l and r (L ≤ R)
//Output: A count of the number of time Key exists in A[L..R]

if L = R
   if (A[L] = Key) return 1
   else return 0
else
   lCount = CountKey(A[],        L,      ⌊(L+R)/2⌋, Key)
   rCount = CountKey(A[], ⌊(L+R)/2⌋+1,     R,     Key)
   return lCount + rCount
```

Count a specific key in an array

- *CountKey* looks familiar…
- What's the difference between *Binary Search and CountKey?*

- We have to search both sides
  - In the counter, both sides must be searched
  - In Binary Search, one half gets ignored

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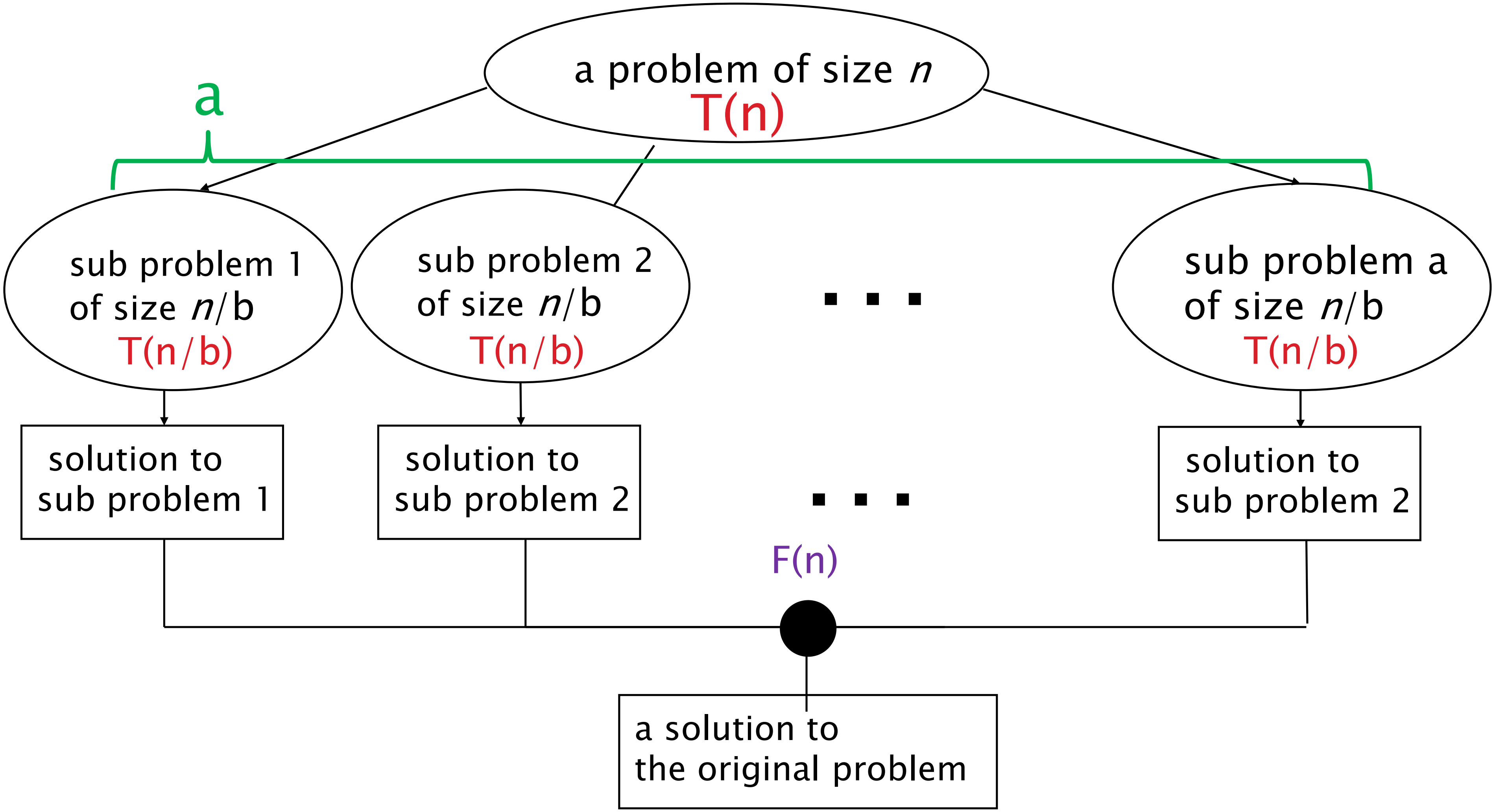
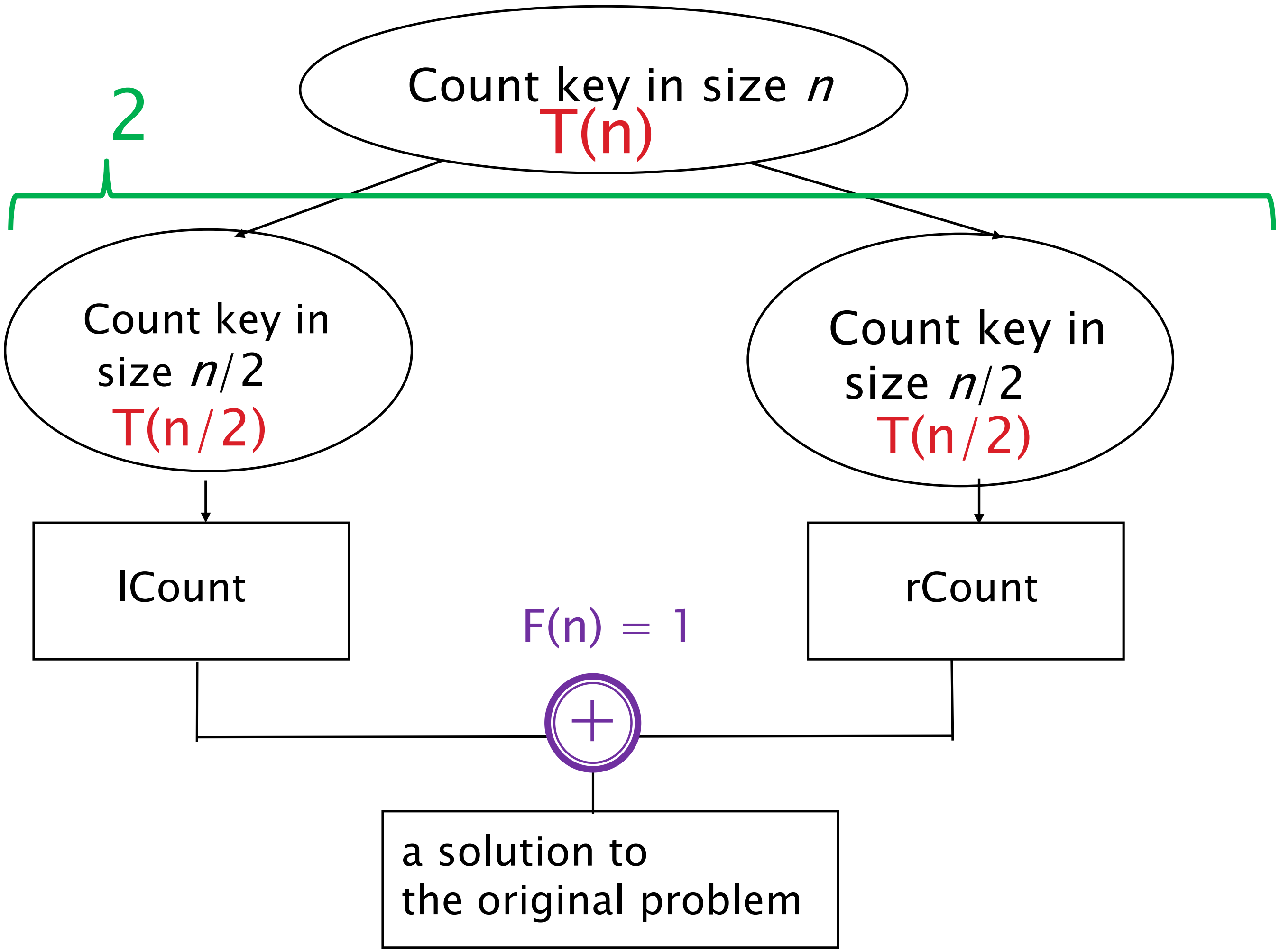
Analysis of a divide and conquer algorithm



$$T(n) = a\,T(n/b) + F(n)$$

Analysis of Count a specific key in an array



$$T(n) = 2\,T(n/2) + 1$$

Master theorem

$$T(n) = a\,T(n/b) + F(n)$$

1) If $n^{\log_b a} < F(n)$, $T(n) \in O(F(n))$
2) If $n^{\log_b a} > F(n)$, $T(n) \in O(n^{\log_b a})$
3) If $n^{\log_b a} = F(n)$, $T(n) \in O(n^{\log_b a} \log n)$

Example 1: $T(n) = 4T(n/2) + n^3 \Rightarrow T(n) \in ?$

$$\left.\begin{array}{l} a = 4,\ b = 2 \Rightarrow n^{\log_b a} \Rightarrow n^{\log_2 4} \Rightarrow n^2 \\ F(n) = n^3 \end{array}\right\} \Rightarrow T(n) \in O(n^3)$$

Master theorem

Example 2: $T(n) = 4T(n/2) + n \Rightarrow T(n) \in ?$

$a = 4$
$b = 2$ $\Rightarrow n^{\log_b a} \Rightarrow n^{\log_2 4} \Rightarrow n^2$

$F(n) = n$

$\Rightarrow T(n) \in O(n^2)$

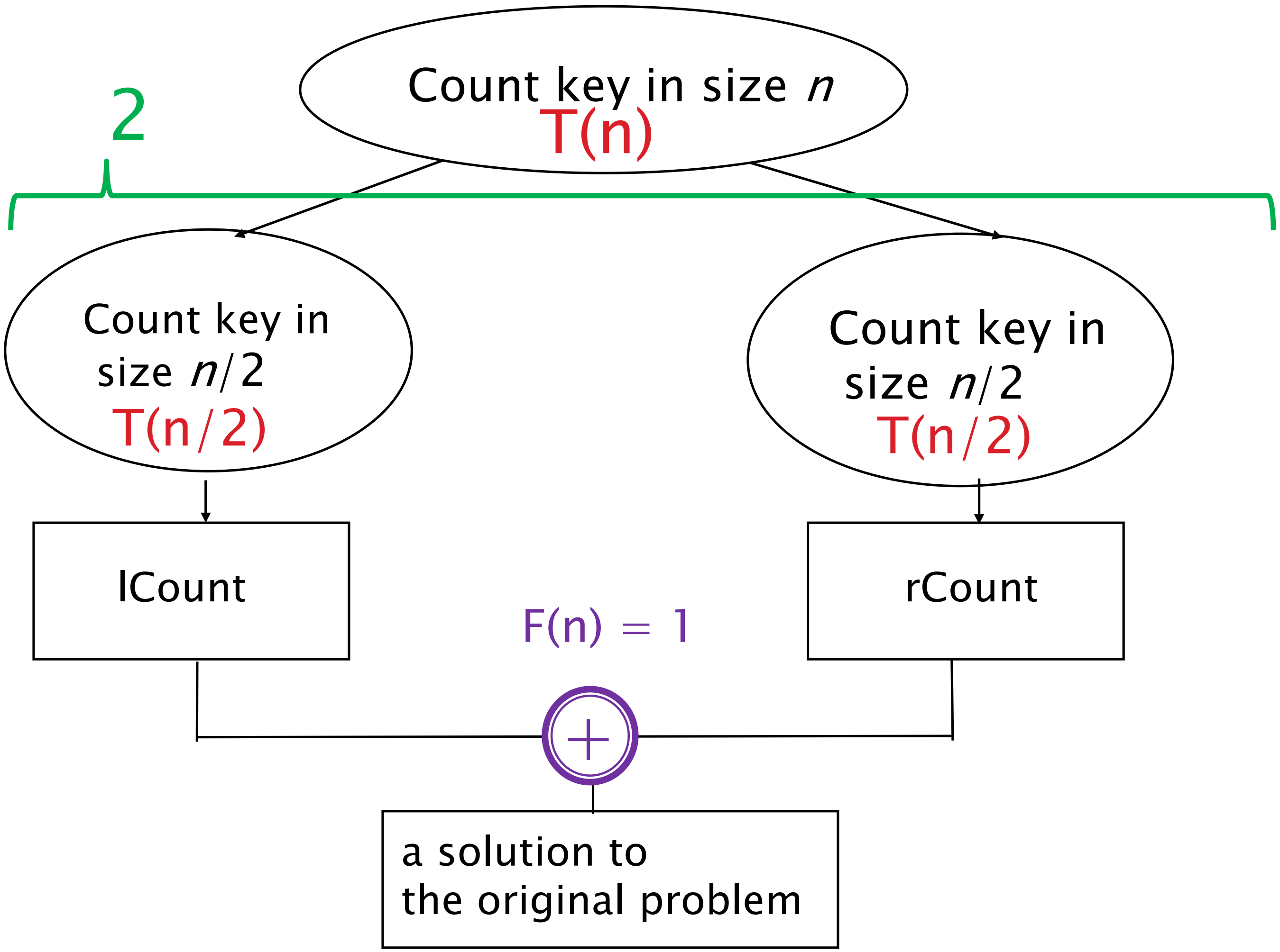
Example 3: $T(n) = 4T(n/2) + n^2 \Rightarrow T(n) \in ?$

$a = 4$
$b = 2$ $\Rightarrow n^{\log_b a} \Rightarrow n^{\log_2 4} \Rightarrow n^2$

$F(n) = n^2$

$\Rightarrow T(n) \in O(n^2 \log n)$

Analysis of count a specific key in an array



$$T(n) = 2\,T(n/2) + 1 \;\Rightarrow\; T(n) \in O(n)$$

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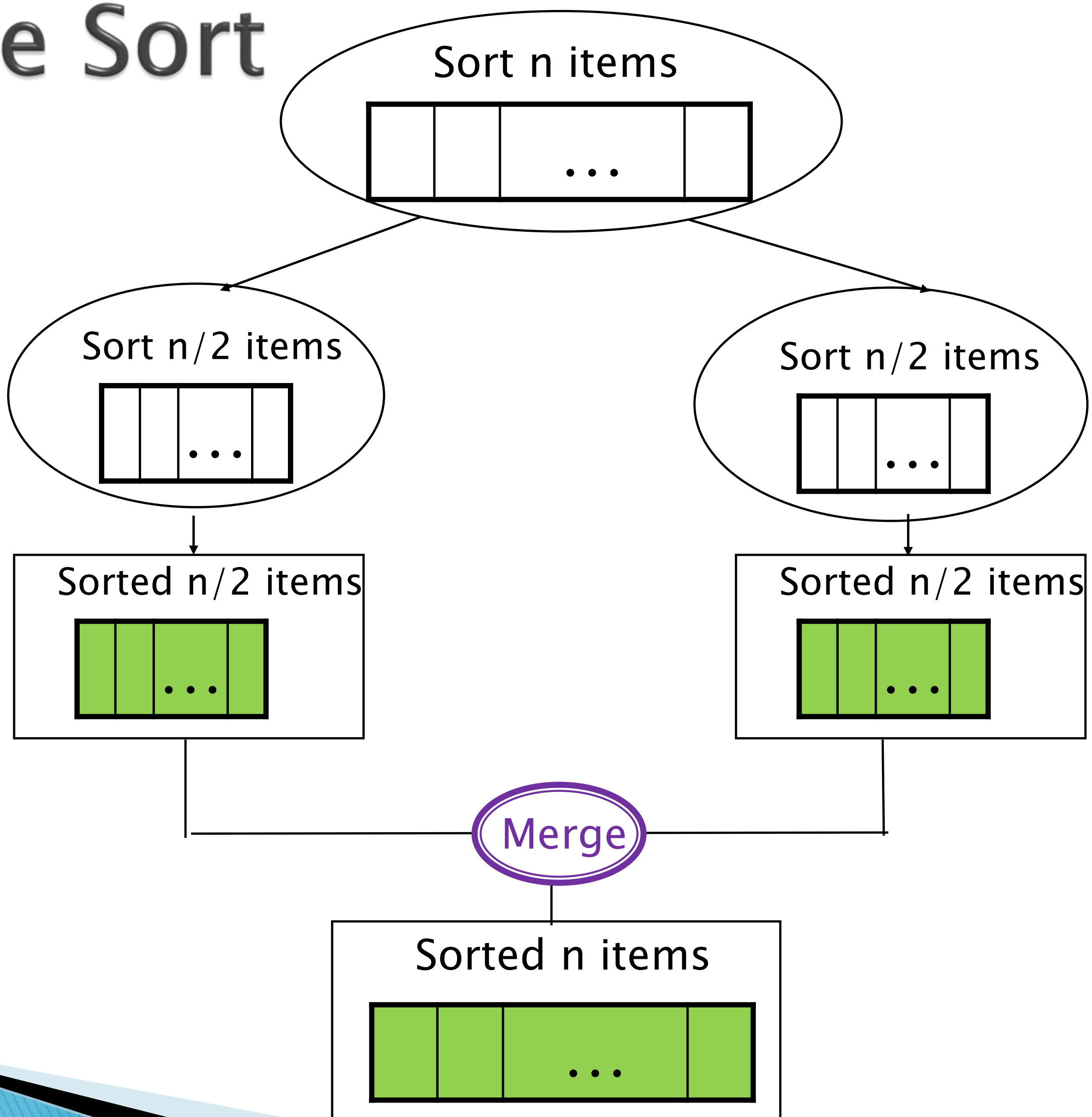
Merge Sort



Sort n items
. . .
Sort n/2 items
. . .
Sort n/2 items
. . .
Sorted n/2 items
. . .
Sorted n/2 items
. . .
Merge
Sorted n items
. . .

Pseudocode of Mergesort

ALGORITHM *Mergesort*$(A[0..n-1])$

//Sorts array $A[0..n-1]$ by recursive mergesort
//Input: An array $A[0..n-1]$ of orderable elements
//Output: Array $A[0..n-1]$ sorted in nondecreasing order
if $n > 1$
copy $A[0..\lfloor n/2 \rfloor - 1]$ to $B[0..\lfloor n/2 \rfloor - 1]$
copy $A[\lfloor n/2 \rfloor..n-1]$ to $C[0..\lceil n/2 \rceil - 1]$
$Mergesort(B[0..\lfloor n/2 \rfloor - 1])$
$Mergesort(C[0..\lceil n/2 \rceil - 1])$
$Merge(B, C, A)$

Merge Sort

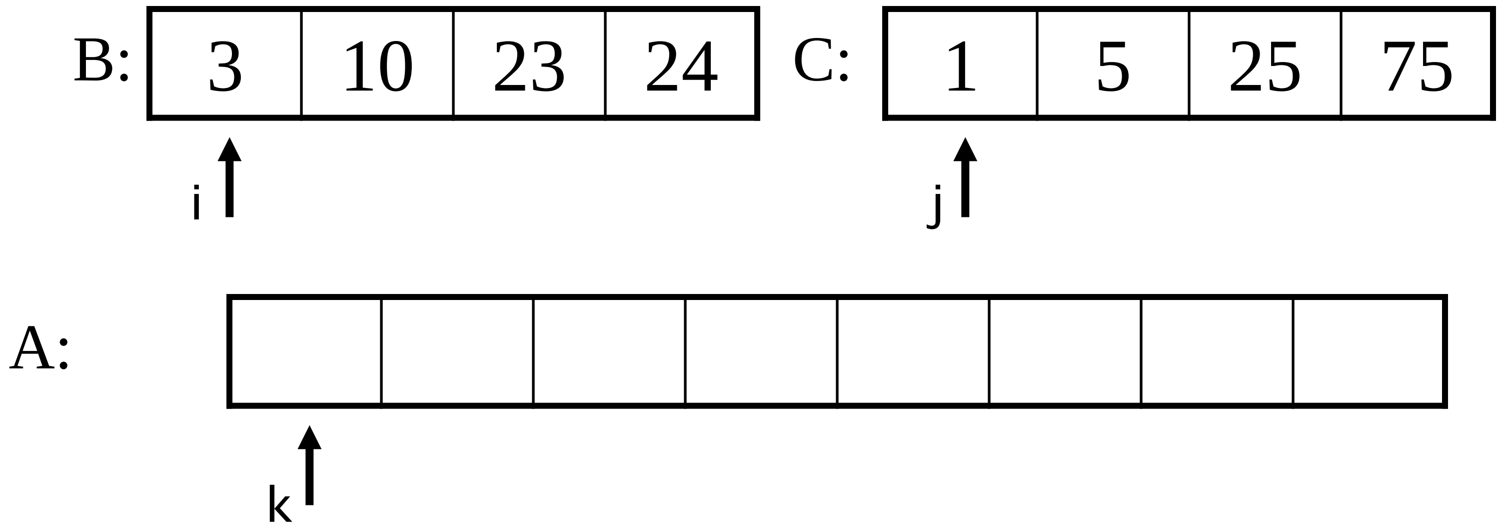
- Important part in merge sort is to merge two sorted array into one

- Example:

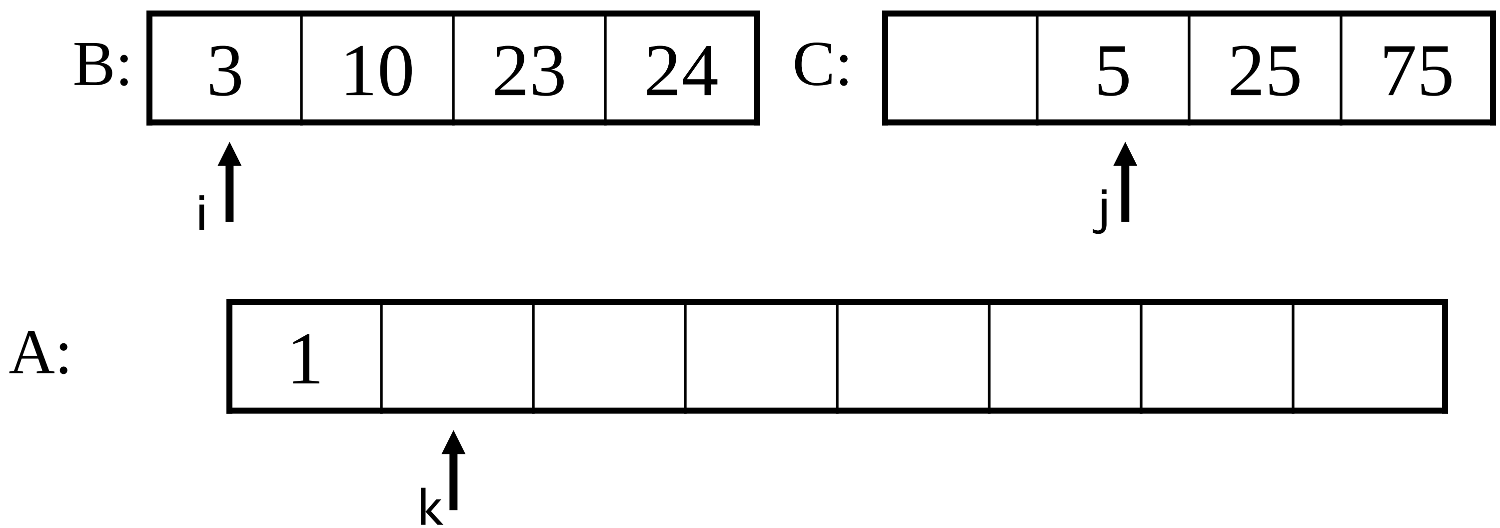
B = { 3 8 9 }   C = { 1 5 7 }

merge(B, C) = { 1 3 5 7 8 9 }

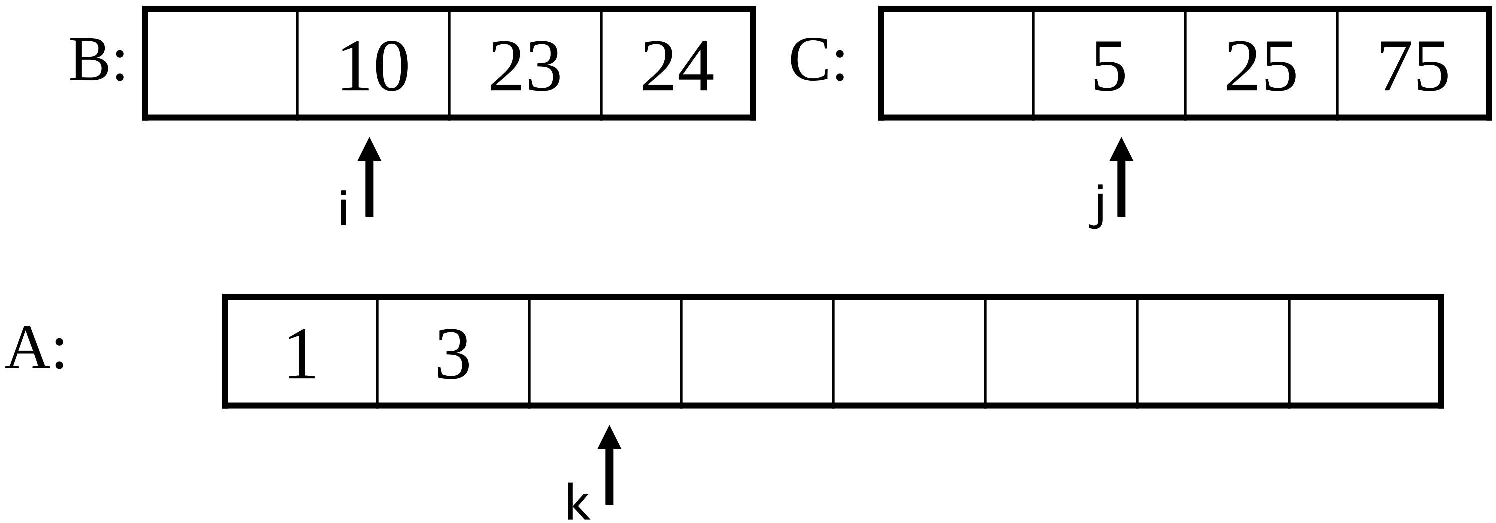
Merging (cont.)



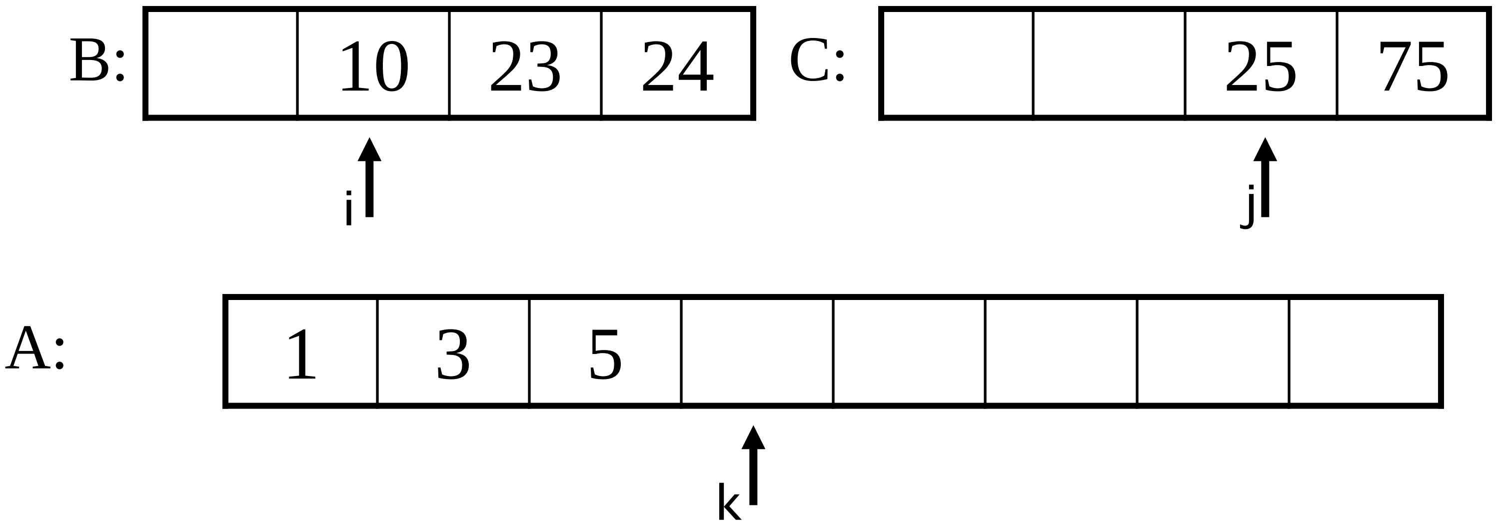
Merging (cont.)



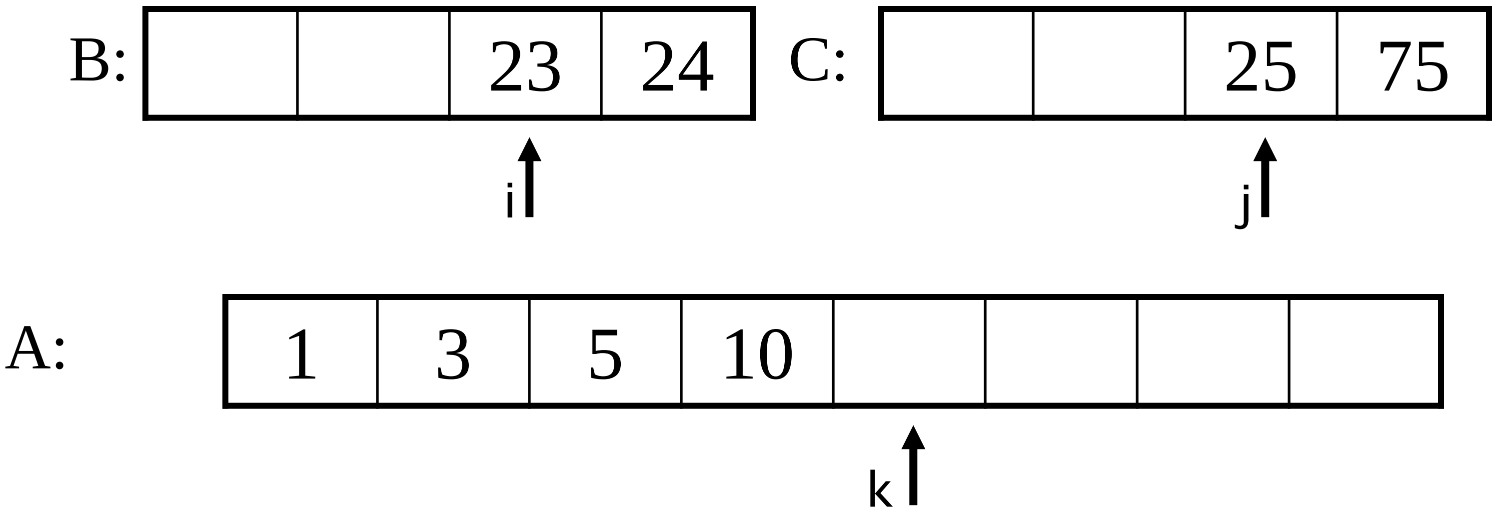
Merging (cont.)



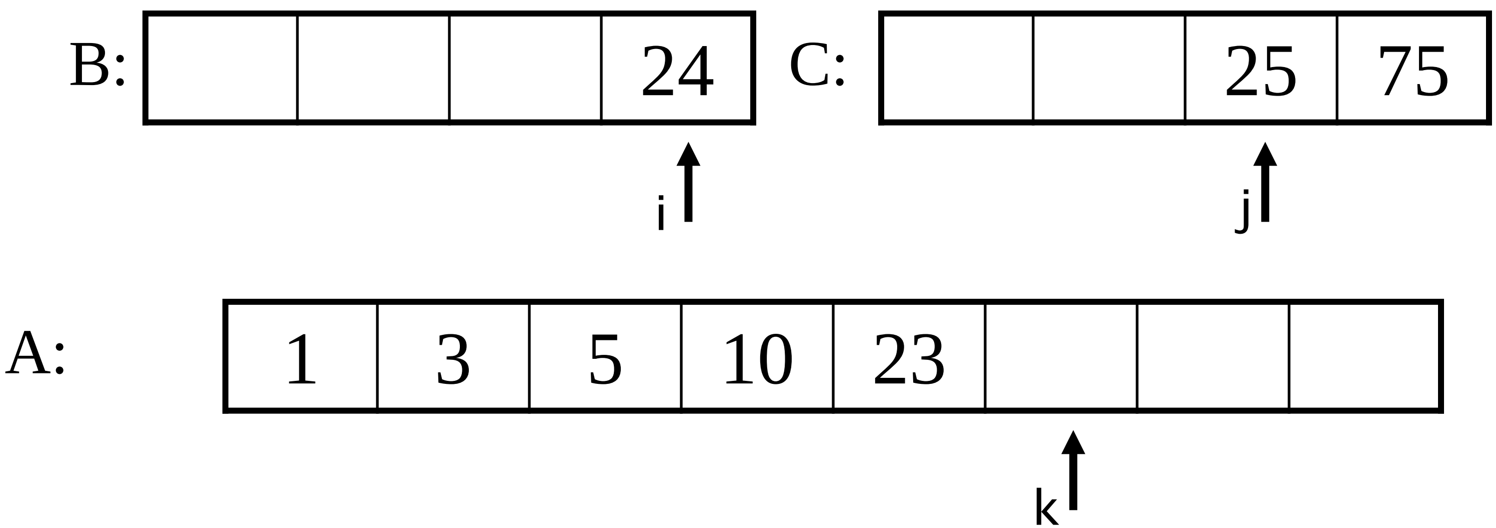
Merging (cont.)



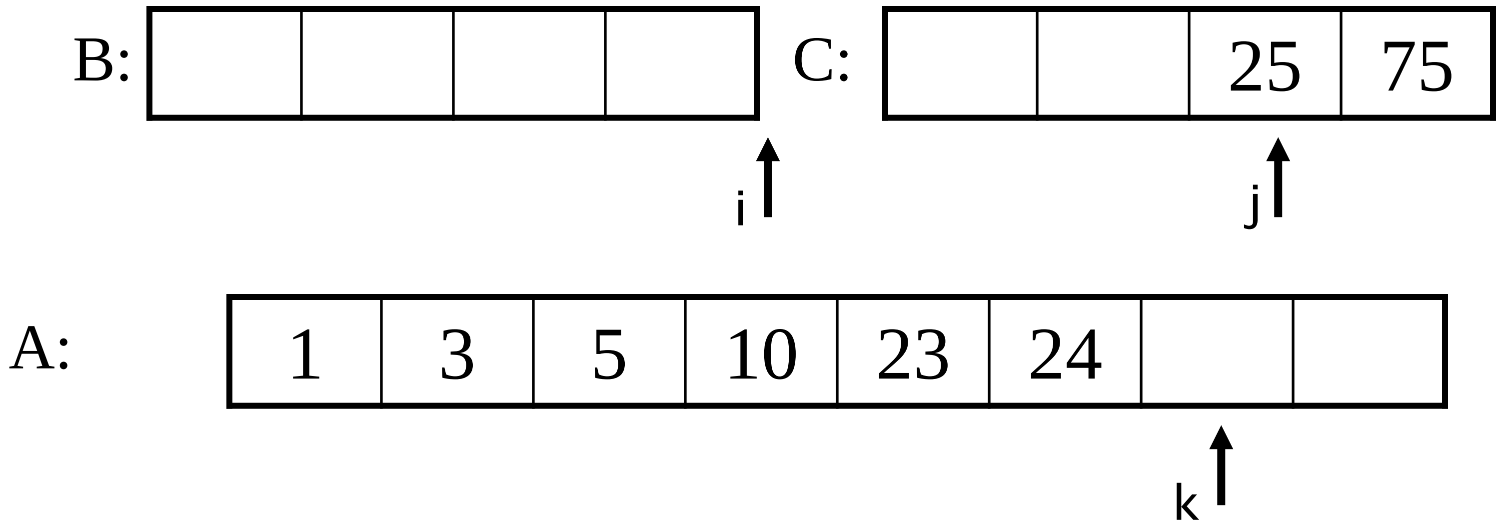
Merging (cont.)



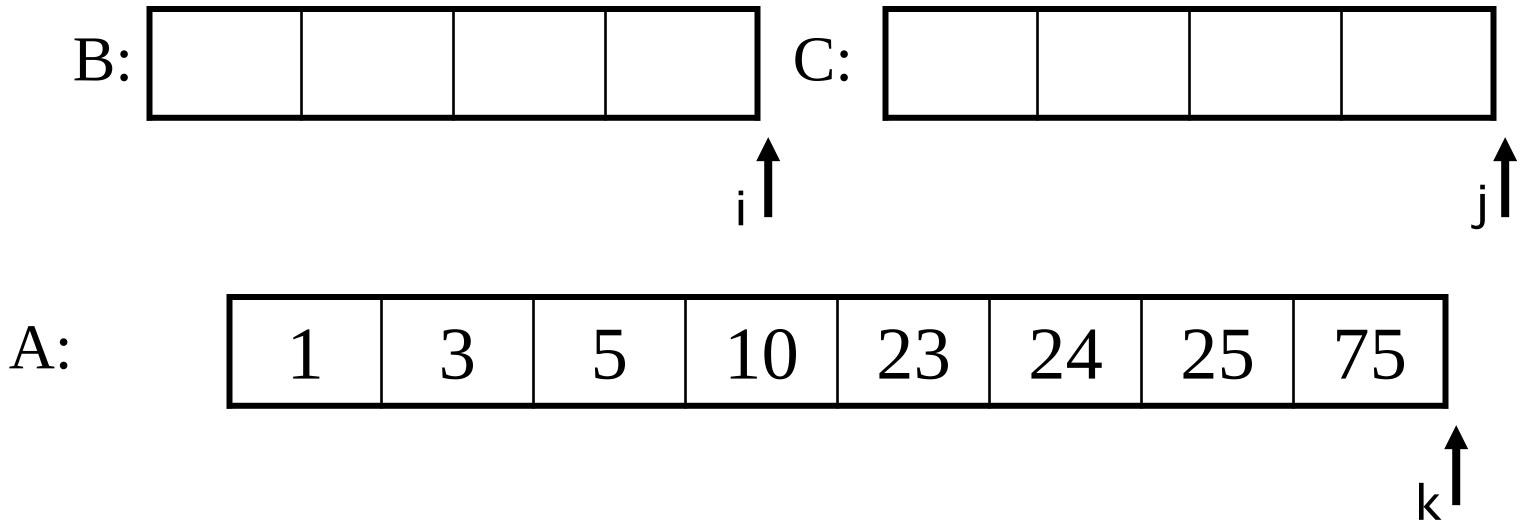
Merging (cont.)



Merging (cont.)



Merging (cont.)



Merging (cont.)

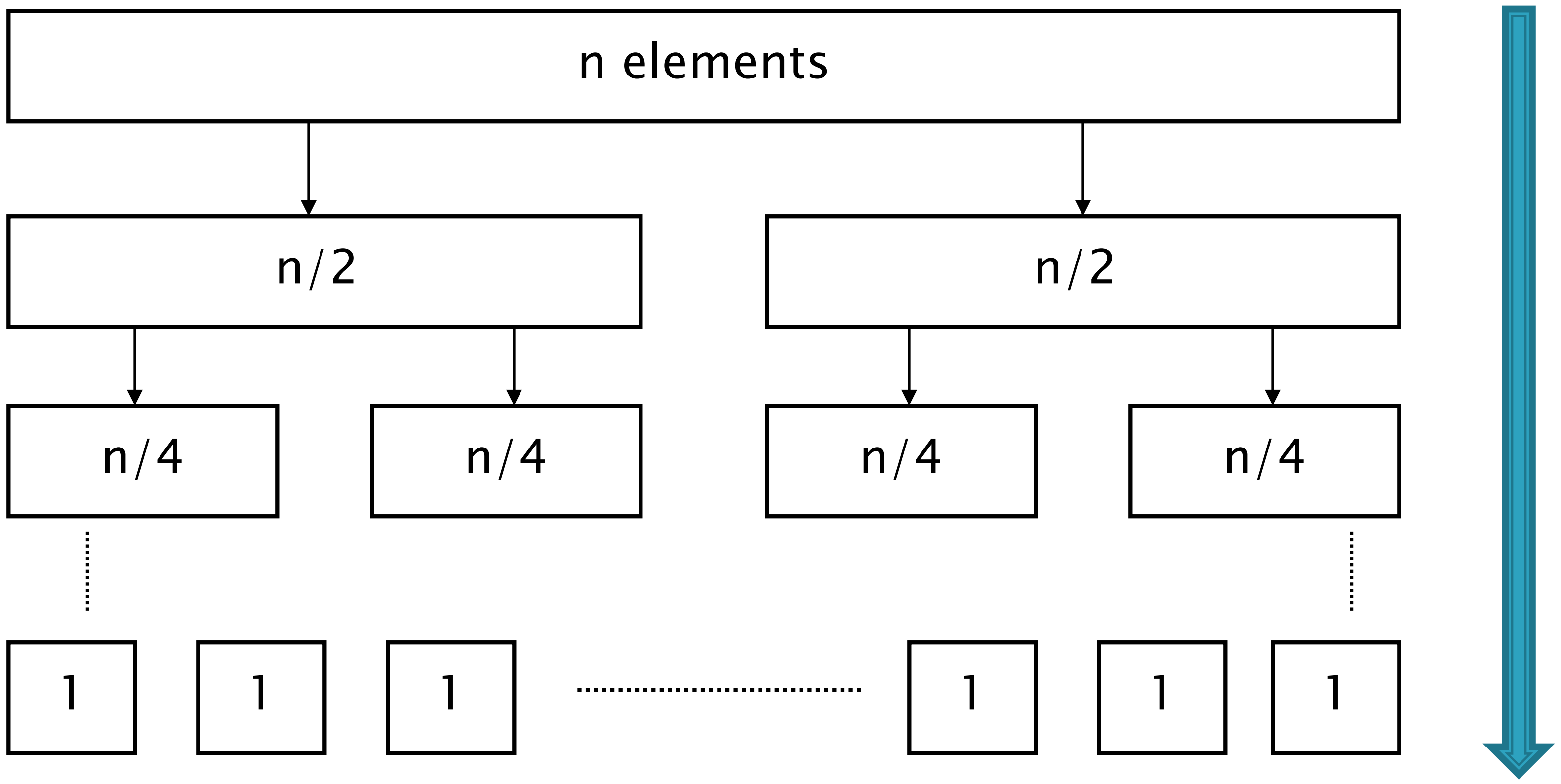
- Must put the next-smallest element into the merged list at each point
- Each next-smallest could come from either list

Pseudocode of Merge

ALGORITHM $Merge(B[0..p-1], C[0..q-1], A[0..p+q-1])$

//Merges two sorted arrays into one sorted array
//Input: Arrays $B[0..p-1]$ and $C[0..q-1]$ both sorted
//Output: Sorted array $A[0..p+q-1]$ of the elements of B and C
$i \leftarrow 0$; $j \leftarrow 0$; $k \leftarrow 0$
while $i < p$ **and** $j < q$ **do**
    if $B[i] \leq C[j]$
        $A[k] \leftarrow B[i]$; $i \leftarrow i + 1$
    else $A[k] \leftarrow C[j]$; $j \leftarrow j + 1$
    $k \leftarrow k + 1$
if $i = p$
    copy $C[j..q-1]$ to $A[k..p+q-1]$
else copy $B[i..p-1]$ to $A[k..p+q-1]$

Merge Sort



Merge Sort Example

99	6	86	15	58	35	86	4	0

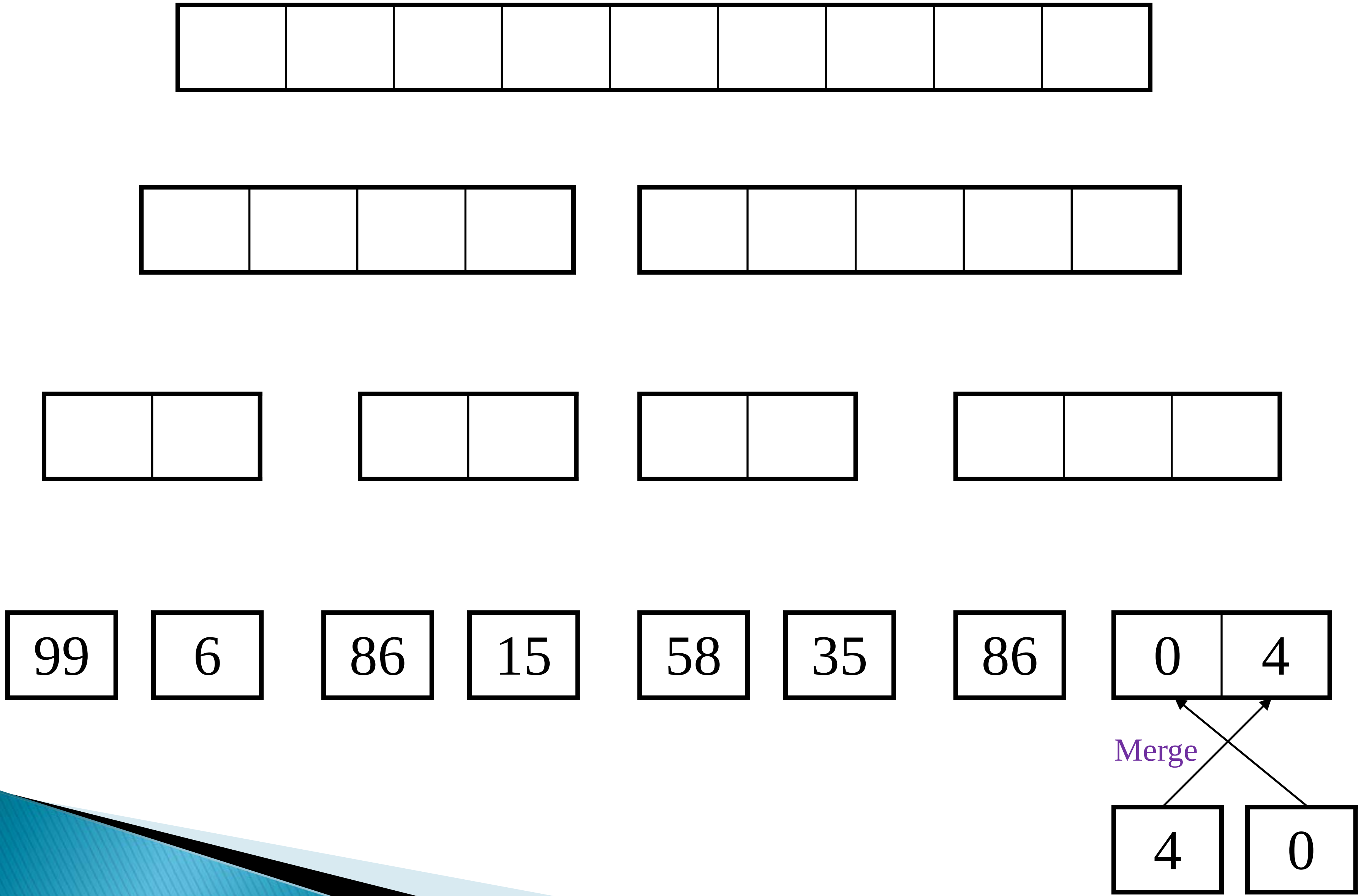
[99, 6, 86, 15] [58, 35, 86, 4, 0]

[99, 6] [86, 15] [58, 35] [86, 4, 0]

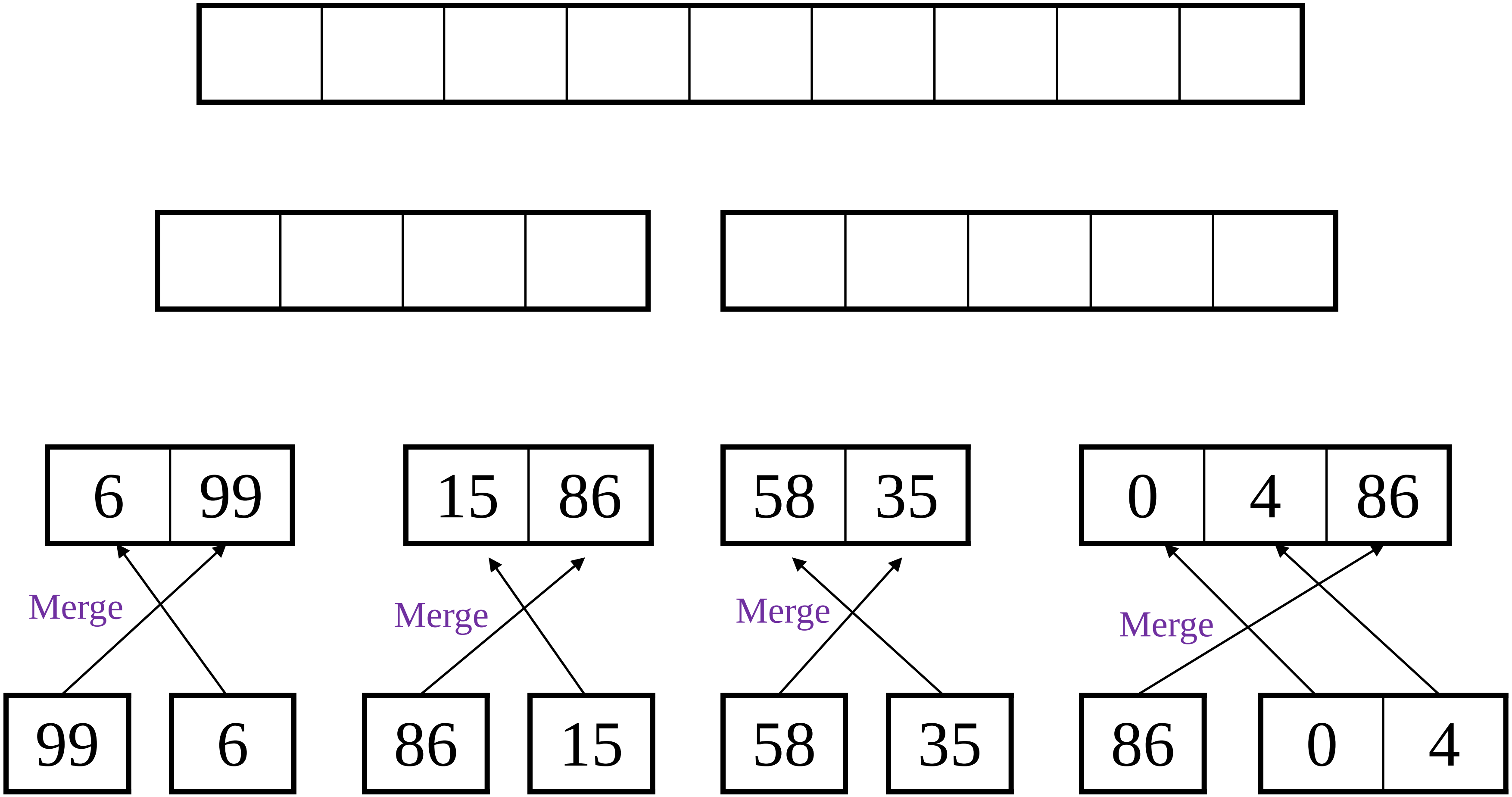
[99] [6] [86] [15] [58] [35] [86] [4, 0]

[4] [0]

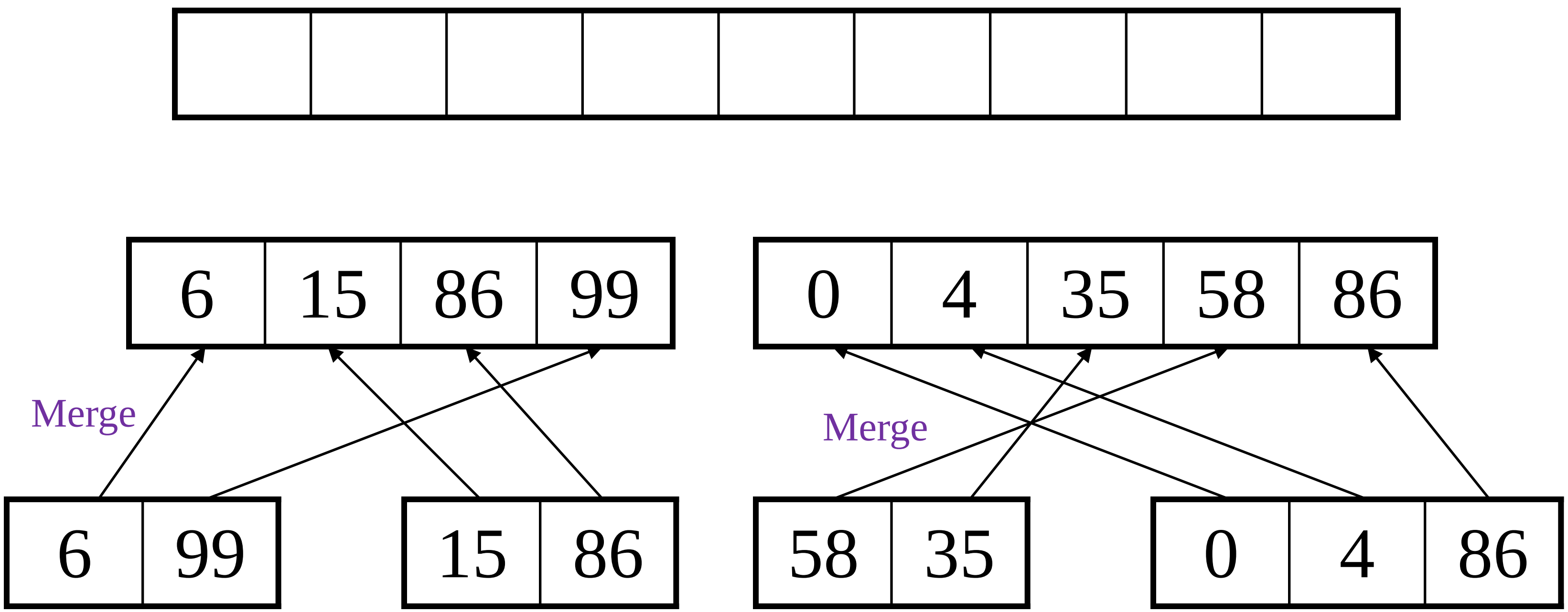
Merge Sort Example



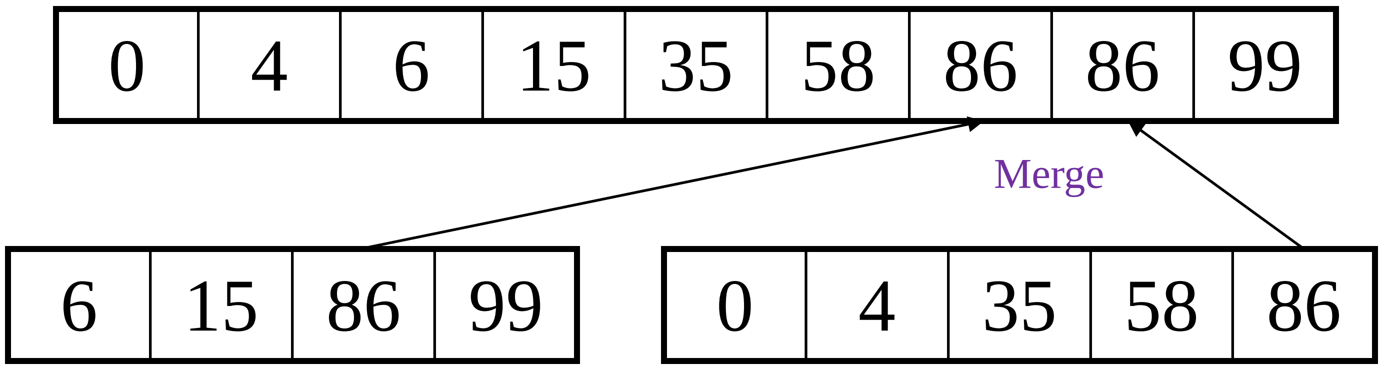
Merge Sort Example



Merge Sort Example



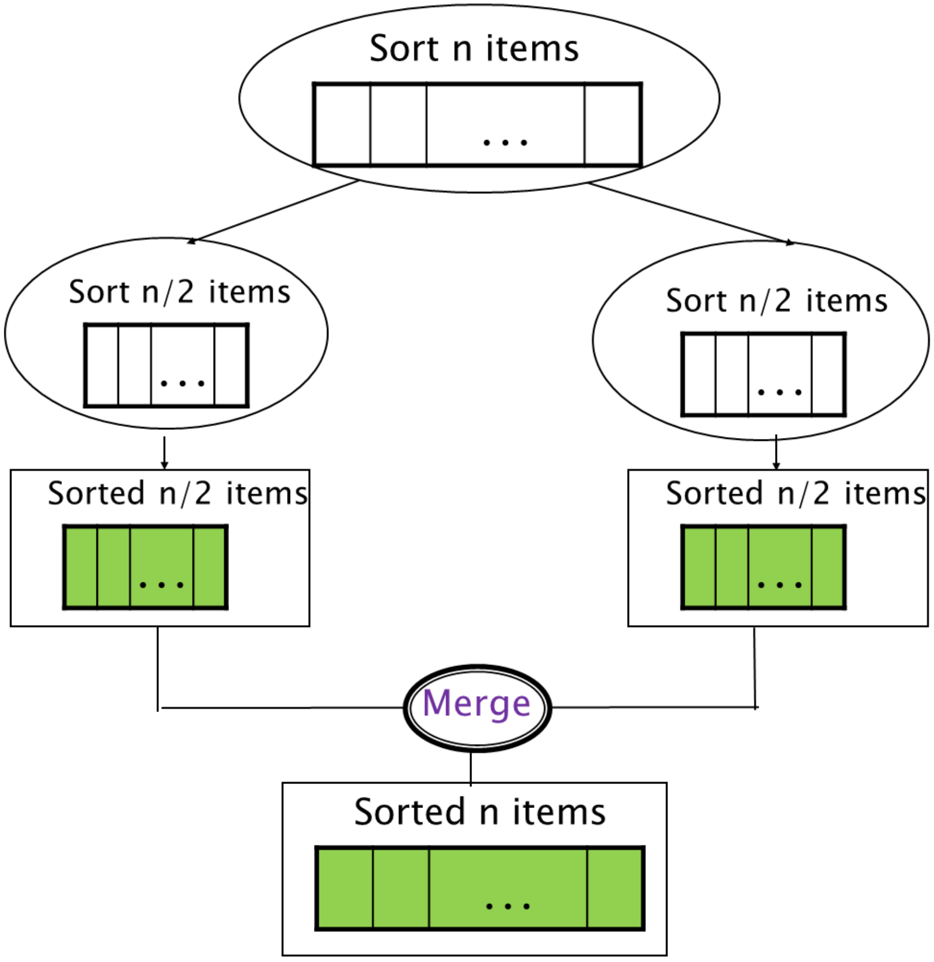
Merge Sort Example



Merge Sort Example

0	4	6	15	35	58	86	86	99

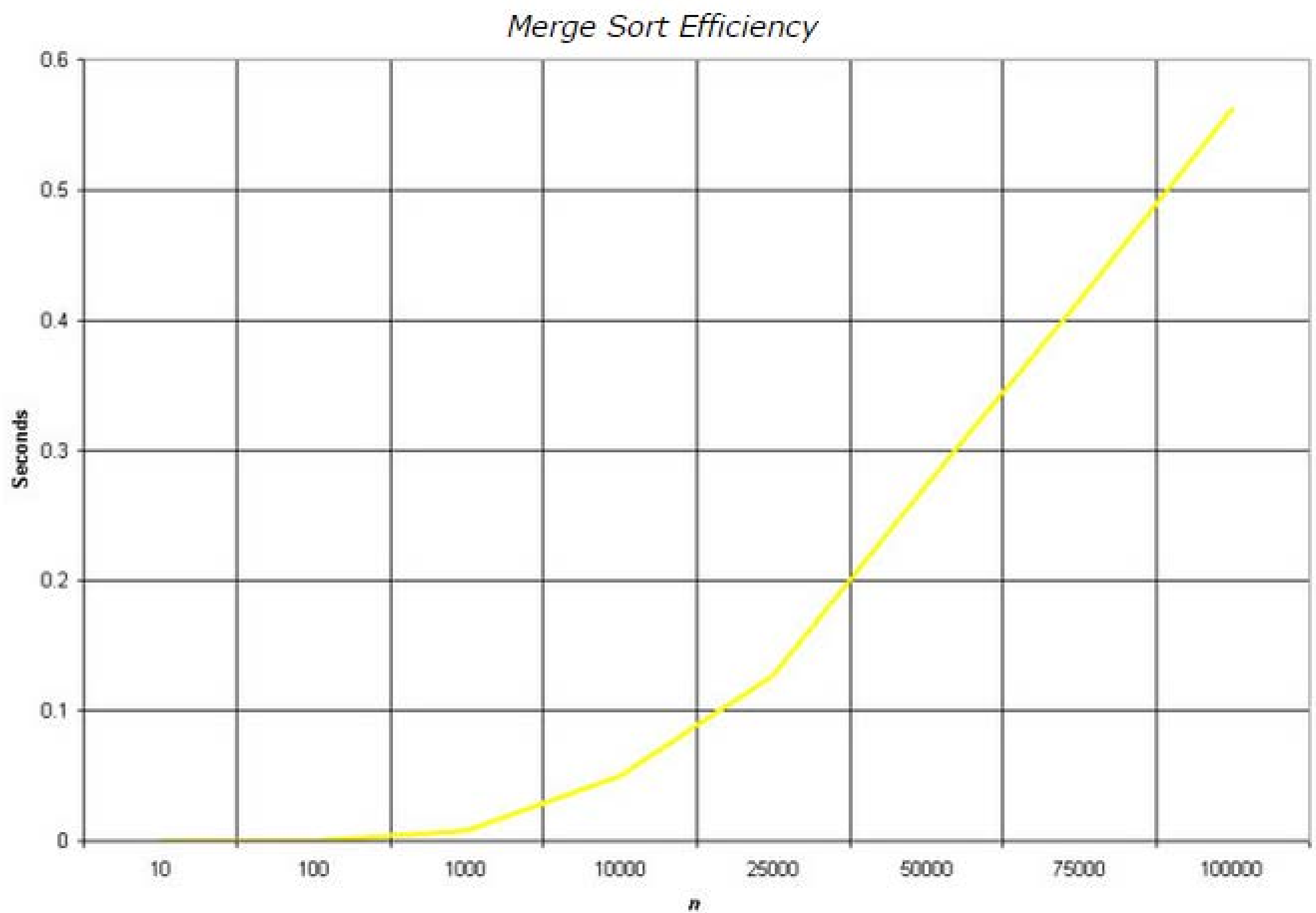
Running Time



$$T(n) = 2\,T(n/2) + n$$

$$T(n) \in O(n \log n)$$

Merge sort



Merge Sort Efficiency
0.6
0.5
0.4
0.3
0.2
0.1
0
Seconds
10
100
1000
10000
25000
50000
75000
100000
n

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Try it/ homework

1. Chapter 5.1, page 174, questions 1, 6
2. Chapter 5.3, page 185, question 2