Factorizaciones triangulares

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Pregunta 1

Encuentra las factorizaciones A = LU o PA = LU de las siguientes matrices:

A1)

```
A1 = rbind(c(0,2,-3,4),c(0,0,-5,-1),c(5,-1,-2,0),c(-2,0,4,6))
luA1=LU(A1)
P=luA1$P
P
```

```
     [,1] [,2] [,3] [,4]
[1,]    0    1    0    0
[2,]    1    0    0    0
[3,]    0    0    1    0
[4,]    0    0    0    1
```

```
U=luA1$U
U
```

```
          [,1] [,2] [,3]      [,4]
[1,] 0.000000    0 -5.0 -1.000000
[2,] 0.000000    2 -3.0  4.000000
[3,] 5.000000    0 -3.5  2.000000
[4,] 3.714286    0  0.0  8.285714
```

```
L=luA1$L
L
```

```
     [,1] [,2]       [,3] [,4]
[1,]    1  0.0  0.000000    0
[2,]    0  1.0  0.000000    0
[3,]    0 -0.5  1.000000    0
[4,]    0  0.0 -1.142857    1
```

```
P%*%A1
```

```
     [,1] [,2] [,3] [,4]
[1,]    0    0   -5   -1
[2,]    0    2   -3    4
[3,]    5   -1   -2    0
[4,]   -2    0    4    6
```

```r
L%*%U
```

```
     [,1] [,2] [,3] [,4]
[1,]    0    0   -5   -1
[2,]    0    2   -3    4
[3,]    5   -1   -2    0
[4,]   -2    0    4    6
```

```r
P%*%A1==L%*%U
```

```
       [,1] [,2]  [,3]  [,4]
[1,]   TRUE TRUE  TRUE  TRUE
[2,]   TRUE TRUE  TRUE  TRUE
[3,]   TRUE TRUE  TRUE  TRUE
[4,] FALSE TRUE FALSE FALSE
```

#Curiosamente, a pesar de ser iguales estas os ultimnos cálculos, R me devuelve que la última fila no e

```python
import scipy
import scipy.linalg
A1 = scipy.array([[0,2,-3,4],[0,0,-5,-1],[5,-1,-2,0],[-2,0,4,6]])
P, L, U = scipy.linalg.lu(A1)
L
```

```
array([[ 1.  ,  0.  ,  0.  ,  0.  ],
       [ 0.  ,  1.  ,  0.  ,  0.  ],
       [ 0.  ,  0.  ,  1.  ,  0.  ],
       [-0.4 , -0.2 , -0.52,  1.  ]])
```

```python
U
```

```
array([[ 5.  , -1.  , -2.  ,  0.  ],
       [ 0.  ,  2.  , -3.  ,  4.  ],
       [ 0.  ,  0.  , -5.  , -1.  ],
       [ 0.  ,  0.  ,  0.  ,  6.28]])
```

```python
P
```

```
array([[0., 1., 0., 0.],
       [0., 0., 1., 0.],
       [1., 0., 0., 0.],
       [0., 0., 0., 1.]])
```

```
A1 = [0 2 -3 4; 0 0 -5 -1; 5 -1 -2 0; -2 0 4 6];
[L U P] = lu(A1)
```

```
L =

   1.00000   0.00000   0.00000   0.00000
```

```
     0.00000    1.00000    0.00000    0.00000
     0.00000    0.00000    1.00000    0.00000
    -0.40000   -0.20000   -0.52000    1.00000
```

U =

```
     5.00000   -1.00000   -2.00000    0.00000
     0.00000    2.00000   -3.00000    4.00000
     0.00000    0.00000   -5.00000   -1.00000
     0.00000    0.00000    0.00000    6.28000
```

P =

Permutation Matrix

```
    0   0   1   0
    1   0   0   0
    0   1   0   0
    0   0   0   1
```

A2)

```r
A2 = rbind(c(1,2,-1,4),c(0,-1,5,8),c(2,3,1,4),c(1,-1,6,4))
luA2=LU(A2,fractions = TRUE)
P=luA2$P
P
```

```
     [,1] [,2] [,3] [,4]
[1,] 1    0    0    0
[2,] 0    1    0    0
[3,] 0    0    1    0
[4,] 0    0    0    1
```

```r
L=luA2$L
L
```

```
     [,1] [,2] [,3] [,4]
[1,] 1    0    0    0
[2,] 0    1    0    0
[3,] 2    1    1    0
[4,] 1    3    4    1
```

```r
U=luA2$U
U
```

```
     [,1] [,2] [,3] [,4]
[1,]   1    2   -1    4
[2,]   0   -1    5    8
[3,]   0    0   -2  -12
[4,]   0    0    0   24
```

```
A2==L%*%U
```

```
     [,1] [,2] [,3] [,4]
[1,] TRUE TRUE TRUE TRUE
[2,] TRUE TRUE TRUE TRUE
[3,] TRUE TRUE TRUE TRUE
[4,] TRUE TRUE TRUE TRUE
```

```python
import scipy
import scipy.linalg
A2 = scipy.array([[1,2,-1,4],[0,-1,5,8],[2,3,1,4],[1,-1,6,4]])
P, L, U = scipy.linalg.lu(A2)
P
```

```
array([[0., 0., 0., 1.],
       [0., 0., 1., 0.],
       [1., 0., 0., 0.],
       [0., 1., 0., 0.]])
```

```
L
```

```
array([[ 1.        ,  0.        ,  0.        ,  0.        ],
       [ 0.5       ,  1.        ,  0.        ,  0.        ],
       [ 0.        ,  0.4       ,  1.        ,  0.        ],
       [ 0.5       , -0.2       , -0.14285714,  1.        ]])
```

```
U
```

```
array([[ 2.        ,  3.        ,  1.        ,  4.        ],
       [ 0.        , -2.5       ,  5.5       ,  2.        ],
       [ 0.        ,  0.        ,  2.8       ,  7.2       ],
       [ 0.        ,  0.        ,  0.        ,  3.42857143]])
```

```
A2 = [1 2 -1 4; 0 -1 5 8; 2 3 1 4; 1 -1 6 4];
[P L U] = lu(A2)
```

```
P =

   1.00000   0.00000   0.00000   0.00000
   0.50000   1.00000   0.00000   0.00000
   0.00000   0.40000   1.00000   0.00000
   0.50000  -0.20000  -0.14286   1.00000

L =

   2.00000   3.00000   1.00000   4.00000
   0.00000  -2.50000   5.50000   2.00000
   0.00000   0.00000   2.80000   7.20000
   0.00000   0.00000   0.00000   3.42857

U =
```

Permutation Matrix

```
   0   0   1   0
   0   0   0   1
   0   1   0   0
   1   0   0   0
```

A3)

```r
A3=rbind(c(2,4,-2,0),c(3,7,5,-4),c(-1,2,-2,5),c(6,1,0,2))
luA3=LU(A3,fractions = TRUE)
P=luA3$P
P
```

```
     [,1] [,2] [,3] [,4]
[1,] 1    0    0    0
[2,] 0    1    0    0
[3,] 0    0    1    0
[4,] 0    0    0    1
```

```r
L=luA3$L
L
```

```
      [,1]   [,2]   [,3]   [,4]
[1,]     1      0      0      0
[2,]   3/2      1      0      0
[3,]  -1/2      4      1      0
[4,]     3    -11 -94/35      1
```

```r
U=luA3$U
U
```

```
     [,1] [,2] [,3] [,4]
[1,]    2    4   -2    0
[2,]    0    1    8   -4
[3,]    0    0  -35   21
[4,]    0    0    0 72/5
```

```r
A3==L%*%U
```

```
     [,1] [,2] [,3] [,4]
[1,] TRUE TRUE TRUE TRUE
[2,] TRUE TRUE TRUE TRUE
[3,] TRUE TRUE TRUE TRUE
[4,] TRUE TRUE TRUE TRUE
```

```python
import scipy
import scipy.linalg
A3 = scipy.array([[2,4,-2,0],[3,7,5,-4],[-1,2,-2,5],[6,1,0,2]])
P, L, U = scipy.linalg.lu(A3)
P
```

```
array([[0., 0., 1., 0.],
       [0., 1., 0., 0.],
       [0., 0., 0., 1.],
       [1., 0., 0., 0.]])
```

L

```
array([[ 1.        ,  0.        ,  0.        ,  0.        ],
       [ 0.5       ,  1.        ,  0.        ,  0.        ],
       [ 0.33333333,  0.56410256,  1.        ,  0.        ],
       [-0.16666667,  0.33333333,  0.7606383 ,  1.        ]])
```

U

```
array([[ 6.        ,  1.        ,  0.        ,  2.        ],
       [ 0.        ,  6.5       ,  5.        , -5.        ],
       [ 0.        ,  0.        , -4.82051282,  2.15384615],
       [ 0.        ,  0.        ,  0.        ,  5.36170213]])
```

```
A3 = [2 4 -2 0; 3 7 5 -4; -1 2 -2 5; 6 1 0 2];
[L U P] = lu(A3)
```

```
L =

   1.00000   0.00000   0.00000   0.00000
   0.50000   1.00000   0.00000   0.00000
   0.33333   0.56410   1.00000   0.00000
  -0.16667   0.33333   0.76064   1.00000

U =

   6.00000   1.00000   0.00000   2.00000
   0.00000   6.50000   5.00000  -5.00000
   0.00000   0.00000  -4.82051   2.15385
   0.00000   0.00000   0.00000   5.36170

P =

Permutation Matrix

   0   0   0   1
   0   1   0   0
   1   0   0   0
   0   0   1   0
```

A4)

```
A4=rbind(c(0,2,3,1),c(0,4,-1,5),c(2,0,3,1),c(1,-4,5,6))
luA4=LU(A4,fractions = TRUE)
P=luA4$P
P
```

```
      [,1] [,2] [,3] [,4]
[1,] 0    1    0    0
[2,] 1    0    0    0
[3,] 0    0    1    0
[4,] 0    0    0    1
```

```
L=luA4$L
L
```

```
      [,1] [,2] [,3] [,4]
[1,]    1    0    0    0
[2,]    0    1    0    0
[3,]    0    0    1    0
[4,]    0   -2 11/3    1
```

```
U=luA4$U
U
```

```
       [,1]  [,2]  [,3]  [,4]
[1,]      0     4    -1     5
[2,]      0     2     3     1
[3,]      2     0     3     1
[4,]  -19/3     0     0  13/3
```

```
P%*%A4==L%*%U
```

```
      [,1] [,2] [,3] [,4]
[1,] TRUE TRUE TRUE TRUE
[2,] TRUE TRUE TRUE TRUE
[3,] TRUE TRUE TRUE TRUE
[4,] TRUE TRUE TRUE TRUE
```

```
import scipy
import scipy.linalg
A4 = scipy.array([[0,2,3,1],[0,4,-1,5],[2,0,3,1],[1,-4,5,6]])
P, L, U = scipy.linalg.lu(A4)
P
```

```
array([[0., 0., 1., 0.],
       [0., 1., 0., 0.],
       [1., 0., 0., 0.],
       [0., 0., 0., 1.]])
```

```
L
```

```
array([[ 1.        , 0.        , 0.        , 0.        ],
       [ 0.        , 1.        , 0.        , 0.        ],
       [ 0.        , 0.5       , 1.        , 0.        ],
       [ 0.5       , -1.       , 0.71428571, 1.        ]])
```

```
U
```

```
array([[ 2.        ,  0.        ,  3.        ,  1.        ],
       [ 0.        ,  4.        , -1.        ,  5.        ],
       [ 0.        ,  0.        ,  3.5       , -1.5       ],
       [ 0.        ,  0.        ,  0.        , 11.57142857]])
```

```
A4 = [0 2 3 1; 0 4 -1 5; 2 0 3 1; 1 -4 5 6];
[L U P] = lu(A4)
```

```
L =

   1.00000   0.00000   0.00000   0.00000
   0.00000   1.00000   0.00000   0.00000
   0.00000   0.50000   1.00000   0.00000
   0.50000  -1.00000   0.71429   1.00000

U =

   2.00000   0.00000   3.00000   1.00000
   0.00000   4.00000  -1.00000   5.00000
   0.00000   0.00000   3.50000  -1.50000
   0.00000   0.00000   0.00000  11.57143

P =

Permutation Matrix

   0   0   1   0
   0   1   0   0
   1   0   0   0
   0   0   0   1
```