Formulario

Cinematica

moto rettilineo uniforme

$$\begin{cases} x(t) = x_0 + v_0 t \\ v_0 \text{ costante} \end{cases}$$

moto uniformemente accelerato

$$\begin{cases} x(t) = x_0 + v_0 t + \frac{1}{2} a_0 t^2 \\ v(t) = v_0 + a_0 t \\ a_0 \text{ costante} \end{cases}$$

moto armonico

$x(t) = A \sin(\phi + \omega t)$

$v(t) = A\omega \cos(\phi + \omega t)$

$a(t) = -A\omega^2 \sin(\phi + \omega t) = -\omega^2 x(t)$

periodo $= T = \frac{2\pi}{\omega}$

Frequenza $: \nu = \frac{1}{T} = \frac{\omega}{2\pi}$

pulsazione $: \omega = 2\pi\nu$

moto circolare (R :: raggio)

$\omega(t) =$ velocità angolare

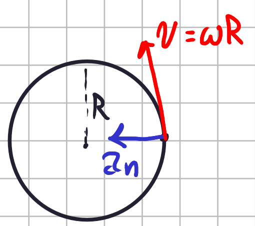
$v = \omega R$ velocità tangenziale

$a_t = \frac{dv}{dt}$ accelerazione tangenziale

$a_n = \omega^2 R$ accelerazione normale

$a = \sqrt{a_t^2 + a_n^2}$ acc. totale

$\bar{v} = \bar{\omega} \times \bar{R}$



moto circ. unifor.

$$\begin{cases} x(t) = R\cos(\theta(t)) \\ y(t) = R\sin(\theta(t)) \\ \theta = \omega t \end{cases}$$

Dinamica

$\bar{F} = m \cdot \bar{a}$

quantità di moto

$\bar{p} = \frac{d}{dt}(m\bar{a}) = m\bar{v}$

impulso

$I = \int_{t_0}^{t} \bar{F}\, dt$ e $I = \Delta\bar{p}$

Forza media

$: \frac{I}{\Delta t} = \frac{\Delta \bar{p}}{\Delta t} = \frac{|\bar{p}(t_1) - \bar{p}(t_2)|}{t_2 - t_1}$

Attrito

Statico $\bar{F}_s \leq \mu_s R_n$ $\quad R_n :=$ reazione normale

Dinamico $\bar{F}_d = -\mu_d \cdot R_n \cdot \hat{\tau}$

versore velocità

$\hat{\tau} = \frac{\bar{v}}{|\bar{v}|}$

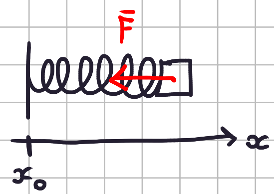
attrito dell'aria : $\bar{F} = -b\bar{v}$

coefficiente

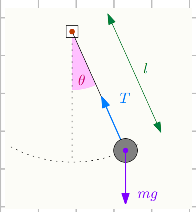
Forza Elastica

$\bar{F} = -K(x - x_0)$

coeff, centro, pos



Pendolo



$-mg\sin\theta = -m\frac{d^2 s}{dt^2}$

$-mg\sin\left(\frac{s}{l}\right) = -m\frac{d^2 s}{dt^2}$

per $\theta \to 0 \Rightarrow -mg\frac{s}{l} = -m\frac{d^2 s}{dt}$

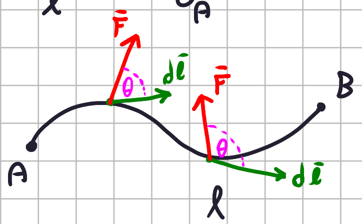
$\Rightarrow s = A\cos(\omega t + \phi)$

$\omega = \sqrt{\frac{g}{l}}$

Lavoro

$\bar F : \mathbb{R}^3 \to \mathbb{R}^3$ campo di Forze

$$L = \int_\ell \bar F \, d\bar\ell = \int_A^B F \, d\ell \cos\theta = \Delta T$$



energia cinetica

$$T = \frac{1}{2} m v^2$$

$$L = \frac{1}{2} m v^2(B) - \frac{1}{2} m v^2(A)$$

$\bar F$ e' conservativa se

$L = -\Delta U$ dove U energia potenziale

$$\oint_\ell \bar F \, d\bar\ell = 0$$

altezza

Potenziale gravita' $U(y) = mgy$

Potenziale gravitazionale $U(r) = -\dfrac{GMm}{r}$

distanza

Potenziale elastica $U(x) = \frac{1}{2} k (x - x_0)^2$

Energia meccanica

$$E_m = U + T = U + \frac{1}{2} m v^2$$

Se $\bar F$ e' conservativa $\Delta E_m = 0$

$$\Rightarrow \begin{cases} L = \Delta T \\ L = -\Delta U \end{cases} \Rightarrow \Delta T + \Delta U = 0 \Rightarrow \Delta(T+U) = 0$$

ΔE_m = LAVORO FORZE NON CONSERVATIVE

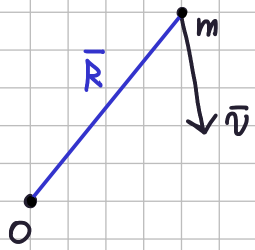
Potenza

$$P = \frac{dL}{dt}$$

Momento Angolare

$$\bar M = \bar R \times \bar p = \bar R \times \frac{d}{dt} m \bar v$$

quantita' di moto



momento della quantita' di moto

$$\bar b = \bar R \times m \bar v$$

$$\bar M = \frac{d\bar b}{dt} - \frac{d\bar R}{dt} \times m \bar v$$

essendo $\bar v = \bar \omega \times \bar R$

$$\bar b = \bar R \times m(\bar\omega \times \bar R) = m R^2 \bar\omega$$

$$\bar b = I \bar \omega$$

momento di inerzia $I = mR^2$

il momento si puo' scrivere $\bar M = \dfrac{d}{dt} I \bar\omega = I \dot{\bar\omega}$ se I cost.

Sistema di Punti

le forze interne si annullano. n punti

$$\bar F_{(est)} = \sum_{i=1}^{n} m_i a_i$$

pos. di ogni punto

centro di massa $\bar r_c = \sum_i m_i \bar r_i \cdot \dfrac{1}{\sum_i m_i}$

vel. del sistema (centro di massa)

$$\frac{d\bar r_c}{dt} = \bar v_c$$

somma masse $M = \sum_i m_i$

$\Rightarrow$ quantita' di moto del sistema

$$\bar p_c = M \bar v_c = \sum_i m_i \bar v_i$$

$$\frac{d}{dt} \bar p_c = \sum_i \frac{d}{dt} m_i \bar v_i = \sum_i m_i \bar a_i = \bar F_{(est)}$$

Urti

• elastici: la quantità di moto si conserva.

2 corpi

Masse: m_1, m_2
vel. post urto V_1 V_2
vel. pre urto v_1 v_2

Cons. quantità di moto

$$m_1 v_1 + m_2 v_2 = m_1 V_1 + m_2 V_2$$

cons. energia cinetica

$$\frac{1}{2} m_1 v_1^2 + \frac{1}{2} m_2 v_2^2 = \frac{1}{2} m_1 V_1^2 + \frac{1}{2} m_2 V_2^2$$

Momento sistema di punti

$$\begin{cases} \bar{M}_1 = \frac{d\bar{b}_1}{dt} + \bar{v}_0 \times \bar{r}_1 \\ \vdots \\ \bar{M}_n = \frac{d\bar{b}_n}{dt} + \bar{v}_0 \times \bar{r}_n \end{cases}$$

i momenti interni del sistema si annullano

$$\left(\frac{d}{dt} \sum \bar{b}_i\right) + \bar{v}_0 \times \left(\sum \bar{r}_i\right)$$

si può riscrivere

$$\frac{d}{dt}\left(\sum \bar{r}_i \times m_i \bar{v}_i\right) + \bar{v}_0 \times \left(\sum \bar{r}_i\right)$$

Il momento del sistema non è uguale alla somma dei momenti

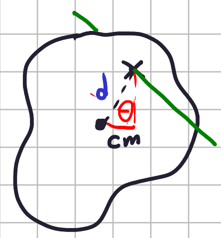
Sistema Continuo

per un oggetto puntiforme $I = Rm^2$

$dI = R^2 dm = R^2 \lambda dR \Rightarrow \lambda$ densità: $\frac{dm}{dR} = \lambda$

$I = \int_S dI$ ← sup. oggetto

Pendolo Composto



d = dist. centro di massa - punto inf.

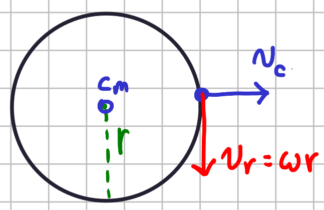
per $\theta \to 0$

$$-mg \cdot d \cdot \theta = -I_0 \frac{d^2\theta}{dt^2}$$

I_0 = momento di inerzia con polo in cm

$$I_0 = \underbrace{I_{cm}}_{\text{momento in. centro massa}} + m \cdot d^2$$

Rotolamento

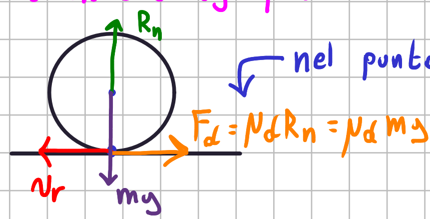


Velocità centro di massa

Velocità di rotazione

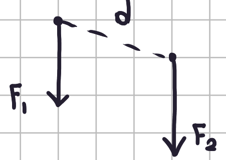
$v_a = v_c + v_r$ $\boxed{|v_c| = |v_r| \Rightarrow \text{rotolamento puro}}$

↑ vel. assoluta di ogni punto

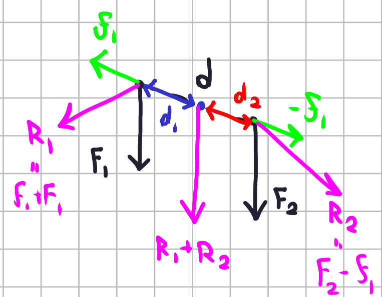


nel punto di contatto

Sistemi eq. di Forze



$$\frac{d_1}{d_2} = \frac{F_2}{F_1}$$

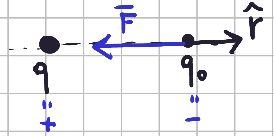


Campo Elettrico

Coloumb

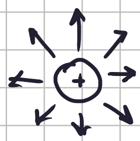
q esercita su q_0 $\quad \bar{F} = \frac{1}{4\pi\varepsilon_0} \frac{q \cdot q_0}{r^2} \hat{r}$

$r := \text{dist}(q, q_0)$



Campo Elettrico

$$\bar{E} = \lim_{q_0 \to 0} \frac{\bar{F}}{q_0} = \frac{1}{4\pi\varepsilon_0} \frac{q}{r^2} \hat{r}$$

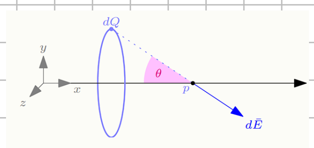


q_0 immersa in $\bar{E}$ sente $\bar{F} = q_0 \bar{E}$

Sistema di cariche : $\bar{E} = \frac{1}{4\pi\varepsilon_0} \sum_{i=1}^{n} \frac{q_i}{r_i^2} \hat{r}_i$

Sup. carica : $\bar{E} = \frac{1}{4\pi\varepsilon_0} \int_S \frac{dQ}{r^2} \hat{r}$

Anello Carico



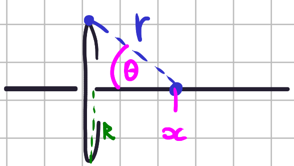
$dE_x = \frac{1}{4\pi\varepsilon_0} \frac{dQ}{r^2} \cos\alpha$

λ densità $\Rightarrow$ $dQ = \lambda dl$

dl = punto circonferenza

$$E_x = \frac{1}{4\pi\varepsilon_0} \int_A \frac{\lambda dl}{r^2} \cos\theta = \frac{1}{4\pi\varepsilon_0} \frac{1}{r^2} \cos\theta \, Q$$

non dipende dal punto sulla circ.



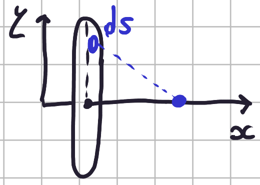
$\Rightarrow r\cos\theta = x \Rightarrow \cos\theta = \frac{x}{r}$

$E = \frac{1}{4\pi\varepsilon_0} \frac{x}{r^3} Q \qquad r = \sqrt{R^2 + x^2}$

dipende da x raggio

Disco Carico

$dQ = \lambda dS \qquad dS$ = sup. infinitesima



$$E = \frac{1}{4\pi\varepsilon_0} \iint_S \frac{\lambda dS}{r^2}$$

superf.

Cambio :

dS = anello infinitesimo

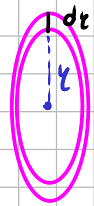
spessore anello

$dS = (2\pi r) \cdot dr$

raggio

$$dE = \frac{1}{4\pi\varepsilon_0} \cdot \frac{x}{\underbrace{(x^2 + z^2)}_{\text{dist}}{}^{3/2}} \cdot dQ$$

z = raggio r anello



dens.

$dQ = \sigma \cdot \underbrace{(2\pi z) \, dz}_{\text{sup. anello inf.}}$

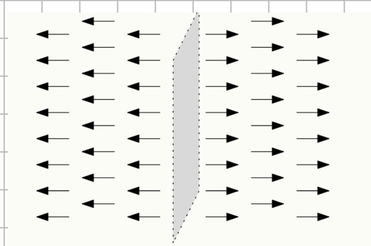
RAGGIO DISCO $\rightarrow R$

$$E = \frac{1}{4\pi\varepsilon_0} \cdot \sigma 2\pi \cdot x \int_0^R \frac{z}{(x^2 + z^2)^{3/2}} dz$$

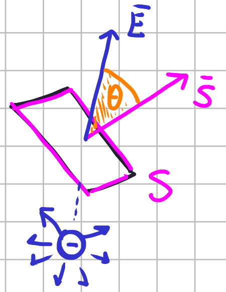
$$E(x) = \frac{\sigma x}{2\varepsilon_0} \cdot \left(\frac{1}{|x|} - \frac{1}{\sqrt{x^2 + R^2}} \right)$$

Piano Carico

$$\lim_{R \to \infty} \frac{\sigma x}{2\varepsilon_0} \cdot \left(\frac{1}{|x|} - \frac{1}{\sqrt{x^2 + R^2}} \right) = \pm \frac{\sigma}{2\varepsilon_0}$$



Flusso



sup

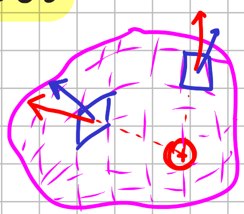
$$\phi = \bar{E}\cdot\bar{S} = ES\cos\theta$$

S = sup finita

$$\phi = \int_S d\phi = \int_S \bar{E}\cdot d\bar{s}$$

Legge di Gauss

$S :=$ sup. chiusa



$$\phi = \oint_S \bar{E}\cdot d\bar{s} = \frac{q}{\varepsilon_0}$$

$$d\phi = \bar{E}\cdot d\bar{s} = \frac{1}{4\pi\varepsilon_0}\frac{q}{r^2}\hat{r}\, d\bar{s} = \frac{q}{4\pi\varepsilon_0}\cdot\frac{\hat{r}\, d\bar{s}}{r^2} = \frac{q}{4\pi\varepsilon_0}\, d\Omega$$

$$\frac{\hat{r}\, d\bar{s}}{r^2} = d\Omega \Rightarrow \oint d\Omega = 4\pi$$

Si scrive anche

$$\oint_S \bar{E}\, d\bar{s} = \frac{1}{\varepsilon_0}\int_{\tau_S} \rho\, d\tau$$

carica int. alla sup S

$\tau_S \Rightarrow$ volume di S

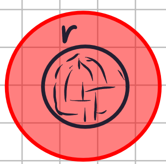
Campo sfera carica

$r \geq R$

$$\phi = \frac{1}{\varepsilon_0}\int_{\tau_S}\rho\, d\tau = \frac{Q}{\varepsilon_0}$$

$$\frac{Q}{\varepsilon_0} = \oint_S E\, d\bar{s} = E\int_S d\bar{s} = E\, 4\pi r^2 \Rightarrow E = \frac{1}{4\pi\varepsilon_0}\frac{Q}{r^2}$$

$r \leq R$

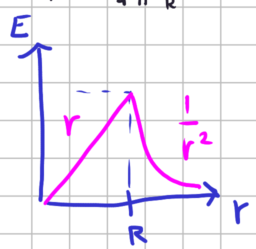


$$\phi = \frac{1}{\varepsilon_0}\int_{\tau_S}\rho\, d\tau = \frac{\rho}{\varepsilon_0}\int_0^r d\tau = \frac{\rho}{\varepsilon_0}\frac{4}{3}\pi r^3$$

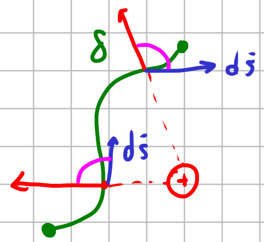
$$\rho\,\frac{4}{3}\pi R^3 = Q \Rightarrow \rho = Q\,\frac{3}{4}\,\frac{1}{\pi}\,\frac{1}{R^3}$$

$$E\, 4\pi r^2 = \frac{\rho}{\varepsilon_0}\,\frac{4}{3}\pi r^3$$

$$E = \frac{\rho}{\varepsilon_0}\,\frac{1}{3}\, r = \frac{Q}{4\pi\varepsilon_0}\cdot\frac{r}{R^3}$$



Circuitazione



$$L = \int_\delta q_0 \bar{E} = q_0\int_\delta \bar{E}\, d\bar{s}$$

L lavoro

Sia C una curva chiusa

$$\mathcal{E} = \oint_C \bar{E}\, d\bar{s} \qquad \int_A^B \bar{E}\, d\bar{s} = V(A) - V(B) = \Delta V$$

CIRCUITAZIONE — potenziale

$$L = -q_0\Delta V = -\Delta U$$

CAMPO ELETTROSTATICO $\Rightarrow$ CONSERVATIVO $\Rightarrow \mathcal{E} = 0$

Potenziale di una Carica

$$\bar{E} = \frac{1}{4\pi\varepsilon_0}\frac{Q}{r^2}\hat{r} \Rightarrow \bar{E}\, d\bar{s} = \frac{1}{4\pi\varepsilon_0}\frac{Q}{r^2}\hat{r}\, d\bar{s} = \frac{1}{4\pi\varepsilon_0}\frac{Q}{r^2}dr$$

$$\mathcal{E} = \int_C \bar{E}\, d\bar{s} = \int_A^B \bar{E}\, d\bar{s} = \frac{Q}{4\pi\varepsilon_0}\int_{r_A}^{r_B}\frac{1}{r^2}\, dr$$

$$= \frac{Q}{4\pi\varepsilon_0}\left(\frac{1}{r_B} - \frac{1}{r_A}\right) \Rightarrow V(r) = \frac{Q}{4\pi\varepsilon_0}\cdot\frac{1}{r}$$

dist da Q

Gradiente

$$\bar{E} = -\nabla V = \left[-\frac{\partial V}{\partial x}, -\frac{\partial V}{\partial y}, -\frac{\partial V}{\partial z}\right]$$