

Formulario

Cinematica

moto rettilineo uniforme

$$\begin{cases} x(t) = x_0 + v_0 t \\ v_0 \text{ costante} \end{cases}$$

moto uniformemente accelerato

$$\begin{cases} x(t) = x_0 + v_0 t + \frac{1}{2} a_0 t^2 \\ v(t) = v_0 + a_0 t \\ a_0 \text{ costante} \end{cases}$$

moto armonico

$$x(t) = A \sin(\phi + \omega t)$$

$$v(t) = A\omega \cos(\phi + \omega t)$$

$$a(t) = -A\omega^2 \sin(\phi + \omega t) = -\omega^2 x(t)$$

$$\text{periodo} = T = \frac{2\pi}{\omega}$$

$$\text{frequenza} = \nu = \frac{1}{T} = \frac{\omega}{2\pi}$$

$$\text{pulsazione} = \omega = 2\pi\nu$$

moto circolare (R::raggio)

$$\omega(t) = \text{velocità angolare}$$

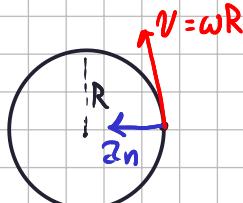
$$v = \omega R \text{ velocità tangenziale}$$

$$a_t = \frac{dv}{dt} \text{ accelerazione tangenziale}$$

$$a_n = \omega^2 R \text{ accelerazione normale}$$

$$a = \sqrt{a_t^2 + a_n^2} \text{ acc. totale}$$

$$\bar{v} = \bar{\omega} \times \bar{R}$$



moto circ. unifor.

$$\begin{cases} x(t) = R \cos(\theta(t)) \\ y(t) = R \sin(\theta(t)) \\ \theta = \omega t \end{cases}$$

Dinamica

$$\bar{F} = m \cdot \bar{a}$$

$$\text{quantità di moto } \bar{p} = \frac{d}{dt}(m\bar{v}) = m\bar{v}$$

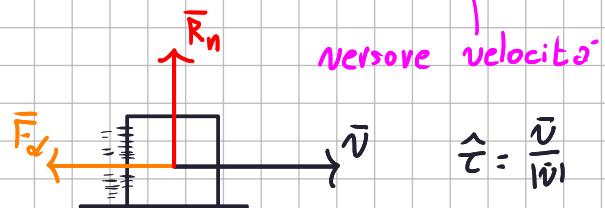
$$\text{impulso } I = \int_{t_0}^t \bar{F} dt \quad \text{e} \quad I = \Delta \bar{p}$$

$$\text{Forza media} = \frac{I}{\Delta t} = \frac{\Delta \bar{p}}{\Delta t} = \frac{|\bar{p}(t_2) - \bar{p}(t_1)|}{t_2 - t_1}$$

Arrivo

$$\text{statico} \quad \bar{F}_s \leq \mu_s R_n \quad R_n := \text{reazione normale}$$

$$\text{dinamico} \quad \bar{F}_d = -\mu_d \cdot R_n \cdot \hat{\vec{v}}$$



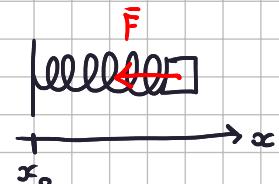
$$\text{altrito dell'aria} : \bar{F} = -b \bar{v}$$

b coefficiente

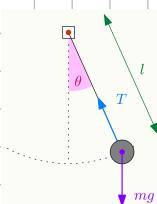
Forza Elastica

$$\bar{F} = -k(x - x_0)$$

x_0



Pendolo



$$-mg \sin \theta = -m \frac{d^2 S}{dt^2}$$

$$-mg \sin\left(\frac{S}{l}\right) = -m \frac{d^2 S}{dt^2}$$

$$\text{per } \theta \rightarrow 0 \Rightarrow -mg \frac{S}{l} = -m \frac{d^2 S}{dt^2}$$

$$\Rightarrow S = A \cos(\omega t + \phi)$$

$$\omega = \sqrt{\frac{g}{l}}$$

Gravi

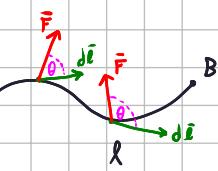
$\begin{cases} h: \text{quota lancio} \\ v_0: \text{vel iniziale} \end{cases} \Rightarrow t^* = \frac{1}{g} v_0$

si fermerà in quota mass raggiunta: $h + \frac{1}{2} \frac{v_0^2}{g}$

Lavoro

$\bar{F} : \mathbb{R}^3 \rightarrow \mathbb{R}^3$ campo di Forze

$$L = \int_{\ell} \bar{F} d\bar{\ell} = \int_A^B \bar{F} d\ell \cos \theta = \Delta T$$



Impulso

$$I = \Delta \bar{p} = \Delta(m\bar{v}) \quad \text{variazione della quantità di moto}$$

Teorema dell'impulso

$$\frac{I}{\Delta t} = \frac{\Delta \bar{p}}{\Delta t} = \frac{1}{\Delta t} \int_{t_1}^{t_2} \bar{F} dt \quad \text{forza media}$$

$$\text{energia cinetica } T = \frac{1}{2} m v^2$$

$$L = \frac{1}{2} m v^2(B) - \frac{1}{2} m v^2(A)$$

\bar{F} e' conservativa se

$L = -\Delta U$ dove U energia potenziale

$$\oint_L \bar{F} d\bar{\ell} = 0$$

$$\text{Potenziale gravitazionale } U(y) = mgy \quad \text{altezza}$$

$$\text{Potenziale gravitazionale } U(r) = -\frac{G M m}{r} \quad \text{distanza}$$

$$\text{Potenziale elastica } U(x) = \frac{1}{2} k (x - x_0)^2$$

$$\text{Energia meccanica } E_m = U + T = U + \frac{1}{2} m v^2$$

Se \bar{F} e' conservativa $\Delta E_m = 0$

$$\Rightarrow \begin{cases} L = \Delta T \\ L = -\Delta U \end{cases} \Rightarrow \Delta T + \Delta U = 0 \Rightarrow \Delta(T+U) = 0$$

Potenza

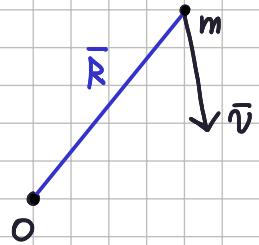
$$P = \frac{dL}{dt}$$

$$\Delta E_m = \begin{matrix} \text{LAVORO FORZE} \\ \text{NON CONSERVATIVE} \end{matrix}$$

Momento Angolare

$$\bar{M} = \bar{R} \times \bar{p} = \bar{R} \times \frac{d}{dt} m \bar{v}$$

quantità di moto



momento dell'a
quantità di moto

$$\bar{b} = \bar{R} \times m \bar{v}$$

$$\bar{M} = \frac{d\bar{b}}{dt} - \frac{d\bar{R}}{dt} \times m \bar{v}$$

essendo $\bar{v} = \bar{\omega} \times \bar{R}$

$$\bar{b} = \bar{R} \times m (\bar{\omega} \times \bar{R}) = m R^2 \bar{\omega}$$

$$\text{momento di inerzia } I = m R^2$$

$$\bar{b} = I \bar{\omega}$$

$$\text{il momento si puo' scrivere } \bar{M} = \frac{d}{dt} I \bar{\omega} = I \dot{\bar{\omega}}$$

se I cost.

Sistema di Punti

le forze interne si annullano. In punti

$$\bar{F}_{(\text{est})} = \sum_{i=1}^n m_i \bar{a}_i$$

per ogni punto

$$\text{centro di massa } \bar{R}_c = \sum_i m_i \bar{r}_i \cdot \frac{1}{\sum m_i}$$

vel. del sistema (centro di massa)

$$\frac{d\bar{R}_c}{dt} = \bar{v}_c$$

$$\text{Somma masse } M = \sum_i m_i$$

\Rightarrow quantità di moto del sistema

$$\bar{p}_c = M \bar{v}_c = \sum_i m_i \bar{v}_i$$

$$\frac{d}{dt} \bar{p}_c = \sum_i \frac{d}{dt} m_i \bar{v}_i = \sum_i m_i \bar{a}_i = \bar{F}_{(\text{est})}$$

Urti

• elastici: le quantità di moto si conservano.

2 corpi

Masse: m_1, m_2

Vel. post urto v_1, v_2

Vel. pre urto v_1, v_2

Cons. quantità di moto

$$m_1 v_1 + m_2 v_2 = m_1 v_1' + m_2 v_2'$$

cons. energia cinetica

$$\frac{1}{2} m_1 v_1^2 + \frac{1}{2} m_2 v_2^2 = \frac{1}{2} m_1 v_1'^2 + \frac{1}{2} m_2 v_2'^2$$

Momento Sistema di punti

$$\left\{ \begin{array}{l} \bar{M}_1 = \frac{d\bar{b}_1}{dt} + \bar{v}_0 \times \bar{r}_1 \\ \vdots \\ \bar{M}_n = \frac{d\bar{b}_n}{dt} + \bar{v}_0 \times \bar{r}_n \end{array} \right. \quad \begin{array}{l} \text{i momenti interni} \\ \text{del sistema si} \\ \text{annullano} \end{array}$$

$$\left(\frac{d}{dt} \sum \bar{b}_i \right) + \bar{v}_0 \times \left(\sum \bar{r}_i \right)$$

si può riscrivere

$$\frac{d}{dt} \left(\sum \bar{r}_i \times m_i \bar{v}_i \right) + \bar{v}_0 \times \left(\sum \bar{r}_i \right)$$

Il momento del sistema non è uguale alla somma dei momenti

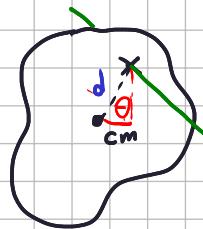
Sistema Continuo

per un oggetto puntiforme $I = R^2 m^2$

$dI = R^2 dm = R^2 \lambda dR \Rightarrow \lambda$ densità: $\frac{dm}{dR} = \lambda$

$$I = \int_S dI \quad \text{sup. oggetto}$$

Pendolo Composto



d = dist. centro di massa - punto inf.

per
 $\theta \rightarrow 0$

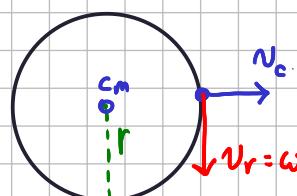
$$-mg \cdot d \cdot \theta = -I_0 \frac{d^2 \theta}{dt^2}$$

I_0 = momento di inerzia con polo in cm

$$I_0 = I_{cm} + M \cdot d^2$$

momento in
centro
massa

Rotolamento

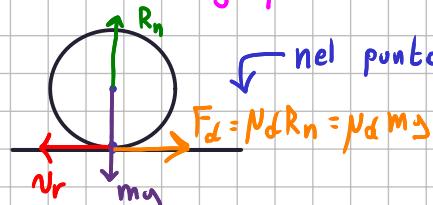


Velocità centro di massa

$$v_a = v_c + v_r$$

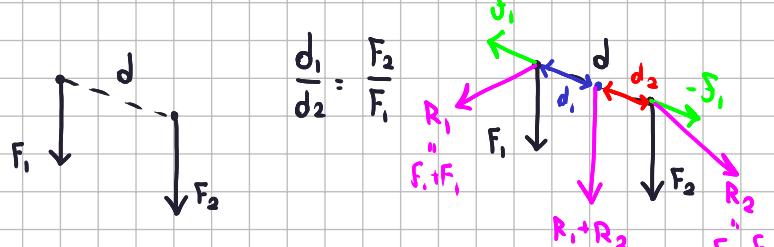
$|v_c| = |v_r| \Rightarrow$ rotolamento puro

Vel. assoluta di ogni punto



nel punto di contatto

Sistemi eq. di Forze



Campo Elettrico

Coloumb

q esercita su q_0 $\vec{F} = \frac{1}{4\pi\epsilon_0} \frac{q \cdot q_0}{r^2} \hat{r}$

$r := \text{dist}(q, q_0)$

Campo Elettrico

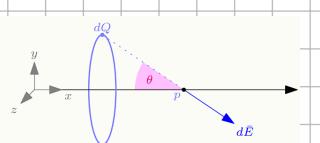
$$\bar{E} = \lim_{q_0 \rightarrow 0} \frac{\bar{F}}{q_0} = \frac{1}{4\pi\epsilon_0} \frac{q}{r^2} \hat{r}$$

q_0 immersa in \bar{E} sente $\bar{F} = q_0 \bar{E}$

Sistema di cariche : $\bar{E} = \frac{1}{4\pi\epsilon_0} \sum_{i=1}^n \frac{q_i}{r_i^2} \hat{r}_i$

Sup. carica : $\bar{E} = \frac{1}{4\pi\epsilon_0} \int_S \frac{dQ}{r^2} \hat{r}$

Anello Carico



$$dE_x = \frac{1}{4\pi\epsilon_0} \frac{dQ}{r^2} \cos\alpha$$

λ densità $\Rightarrow dQ = \lambda dl$

dl = punto circonferenza

$$E_x = \frac{1}{4\pi\epsilon_0} \int_A \frac{\lambda dl}{r^2} \cos\theta = \frac{1}{4\pi\epsilon_0} \frac{1}{r^2} \cos\theta Q$$

$\text{non dipende dal punto sulla circ.}$

$\Rightarrow r \cos\theta = x \Rightarrow \cos\theta = \frac{x}{r}$

$$E = \frac{1}{4\pi\epsilon_0} \frac{x}{r^3} Q$$

$\text{dipende da } x = \text{raggio}$

Disco Carico

$$dQ = \lambda ds \quad ds = \text{sup. infinitesima}$$

$$E = \frac{1}{4\pi\epsilon_0} \iint_S \frac{\lambda ds}{r^2}$$

superf.

Cambio :

$$ds = \text{anello infinitesimo}$$

$\xrightarrow{\text{spessore anello}}$

$$ds = (2\pi r) \cdot dr$$

$\xrightarrow{\text{raggio}}$

$$dE = \frac{1}{4\pi\epsilon_0} \cdot \underbrace{\frac{\infty}{(x^2 + z^2)^{3/2}}}_{\text{dist}} \cdot dQ$$

$\lambda = \text{dens.}$

$dQ = \sigma \cdot (2\pi r) dr$

$\xrightarrow{\text{sup. anello inf.}}$

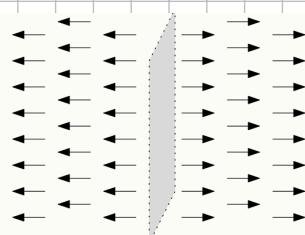
RAGGIO DISCO

$$E = \frac{1}{4\pi\epsilon_0} \cdot \sigma \cdot 2\pi \cdot \infty \int_0^R \frac{z}{(x^2 + z^2)^{3/2}} dz$$

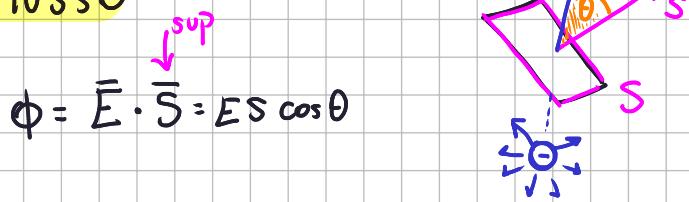
$$E(x) = \frac{\sigma x}{2\epsilon_0} \cdot \left(\frac{1}{|x|} - \frac{1}{\sqrt{x^2 + R^2}} \right)$$

Piano Carico

$$\lim_{R \rightarrow \infty} \frac{\sigma x}{2\epsilon_0} \cdot \left(\frac{1}{|x|} - \frac{1}{\sqrt{x^2 + R^2}} \right) = \pm \frac{\sigma}{2\epsilon_0}$$



Flusso



$S := \text{sup. finita}$

$$\phi = \int_S d\phi = \int_S E \cdot d\bar{s}$$

Legge di Gauss

$S := \text{sup. chiusa}$

$$\phi = \oint_S E \cdot d\bar{s} = \frac{q}{\epsilon_0}$$



$$d\phi = E \cdot d\bar{s} = \frac{1}{4\pi\epsilon_0} \frac{q}{r^2} \hat{r} d\bar{s} = \frac{q}{4\pi\epsilon_0} \cdot \frac{\hat{r} d\bar{s}}{r^2} = \frac{q}{4\pi\epsilon_0} d\Omega$$

$\hat{r} d\bar{s} \Rightarrow \oint d\Omega = 4\pi$

Si scrive anche

$$\oint_S E \cdot d\bar{s} = \frac{1}{\epsilon_0} \int_{V_S} \rho d\tau$$

carica int.
 zllz sup S
 $V_S \Rightarrow$ volume di S

Campo Sfera Carica

$$\text{Diagram of a sphere of radius } r \text{ with charge } +R. \quad r \geq R$$

$$\Phi = \frac{1}{\epsilon_0} \int_{V_S} \rho d\tau = \frac{Q}{\epsilon_0}$$

$$\frac{Q}{\epsilon_0} = \oint_S E \cdot d\bar{s} = E \int_S d\bar{s} = E 4\pi r^2 \Rightarrow E = \frac{1}{4\pi\epsilon_0} \frac{Q}{r^2}$$

$$\text{Diagram of a sphere of radius } r < R \text{ with charge } +R.$$

$$\Phi = \frac{1}{\epsilon_0} \int_{V_S} \rho d\tau = \rho \int_0^r d\tau = \frac{\rho}{\epsilon_0} \frac{4}{3}\pi r^3$$

$$E 4\pi r^2 = \frac{\rho}{\epsilon_0} \frac{4}{3}\pi r^3$$

$$E = \frac{\rho}{\epsilon_0} \frac{1}{3} r = \frac{Q}{4\pi\epsilon_0} \cdot \frac{r}{R^3}$$

\downarrow
 $P = Q \frac{3}{4} \frac{1}{\pi} \frac{1}{R^3}$
 E

Circuazione

$$L = \int_C q_0 E = q_0 \int_C E d\bar{s}$$

Sia C una curva chiusa

$$E = \oint_C E d\bar{s} \quad \int_A^B E d\bar{s} = V(A) - V(B) = \Delta V$$

CIRCUITATIONE

potenziale

$$L = -q_0 \Delta V = -\Delta U$$

CAMPO ELETROSTATICO \Rightarrow CONSERVATIVO $\Rightarrow \vec{E} = 0$

Potenziale di una Carica

$$\vec{E} = \frac{1}{4\pi\epsilon_0} \frac{Q}{r^2} \hat{r} \Rightarrow E d\bar{s} = \frac{1}{4\pi\epsilon_0} \frac{Q}{r^2} \hat{r} d\bar{s} = \frac{1}{4\pi\epsilon_0} \frac{Q}{r^2} dr$$

$$\begin{aligned} \mathcal{E} = \oint_C E d\bar{s} &= \int_A^B E d\bar{s} = \frac{Q}{4\pi\epsilon_0} \int_{r_A}^{r_B} \frac{1}{r^2} dr \\ &= \frac{Q}{4\pi\epsilon_0} \left(\frac{1}{r_B} - \frac{1}{r_A} \right) \Rightarrow V(r) = \frac{Q}{4\pi\epsilon_0} \cdot \frac{1}{r} \end{aligned}$$

\uparrow dist da Q

Gradiente

$$\vec{E} = -\nabla V = \left[-\frac{\partial V}{\partial x}, -\frac{\partial V}{\partial y}, -\frac{\partial V}{\partial z} \right]$$

Potenziale Anello Carico

$$\text{Potenziale di } dQ \text{ e}'$$

A
 r
 $d\bar{s}$
 θ
 ∞
 $d\bar{E}$

$$\frac{1}{4\pi\epsilon_0} \frac{dQ}{r}$$

r non dipende dalla pos. di dQ

$$\text{Pot. totale } V(r) = \frac{1}{4\pi\epsilon_0} \int_A \frac{dQ}{r} = \frac{1}{4\pi\epsilon_0 r} \cdot Q$$

$R \leftarrow$ maggio anello
 r
 θ
 x

$$\Rightarrow \sqrt{x^2 + R^2} = r \Rightarrow V(r) = \frac{Q}{4\pi\epsilon_0} \cdot \frac{1}{\sqrt{x^2 + R^2}}$$

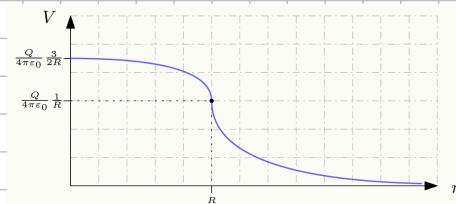
Potenziale Sfera Carica

raggio sfera: R

$$E(r) = \begin{cases} \frac{1}{r^2 4\pi \epsilon_0} Q & \text{se } r \geq R \\ \frac{Q}{4\pi \epsilon_0 \cdot R^3} \cdot r & \text{se } r \leq R \end{cases}$$

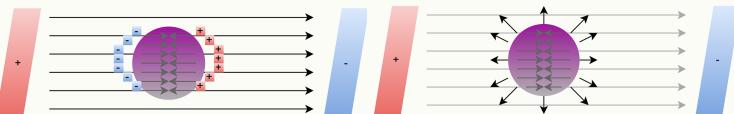
Interno: $V(r) = \int_r^\infty \frac{1}{r^2} \cdot \frac{Q}{4\pi \epsilon_0} = \frac{1}{4\pi \epsilon_0} \cdot \frac{Q}{r}$ { come se fosse una carica punt.

esterno: $V(r) = \frac{Q}{4\pi \epsilon_0 R^3} \int_r^R r dr = \frac{Q}{4\pi \epsilon_0} \left(\frac{3}{2} R - \frac{r^2}{2 R^3} \right)$

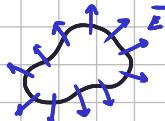


Conduttori

le cariche nei conduttori sono libere e perturbano il campo elettrico



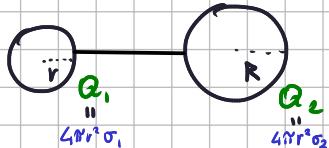
Un conduttore ha \vec{E} nullo all'interno. Sulla sup. e' normale ad essa:



Equipotenziale

in un conduttore \vec{E} il campo \vec{E} e' nullo $\Rightarrow \int \vec{E} d\vec{l} = 0 = \Delta V \Rightarrow$ in ogni punto il potenziale e' identico.

Effetto delle punte: il campo e' più intenso dove la curvatura e' maggiore.



$$\frac{\sigma_1}{\sigma_2} = \frac{R}{r} \Rightarrow \sigma_1 = \sigma_2 \frac{R}{r}$$

Questo perche' la superficie e' equipotenz.

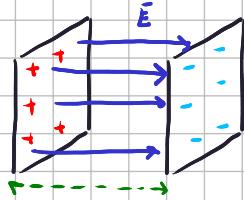
Capacità

$$\text{conduttore di carica } Q \text{ e pot. } V \Rightarrow C = \frac{Q}{V}$$

nel caso della sfera $V = \frac{1}{4\pi \epsilon_0} \frac{Q}{R} \Rightarrow C = 4\pi \epsilon_0 R$

Condensatore

solamente fra le lastre

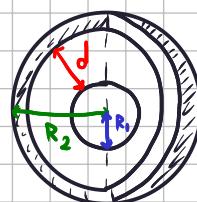


$$E = \frac{\sigma}{\epsilon_0} \Rightarrow V = \int_0^\infty E dz = \int_0^d \frac{\sigma}{\epsilon_0} dz = \frac{\sigma}{\epsilon_0} d$$

A: sup. lastre $\Rightarrow Q = \sigma A$

$$\Rightarrow C = \frac{Q}{V} = \epsilon_0 \frac{A}{d}$$

Condensatore Sferico



$$V_1 = \frac{1}{4\pi \epsilon_0} \frac{Q}{R_1} \quad V_2 = \frac{1}{4\pi \epsilon_0} \frac{Q}{R_2} \Rightarrow \Delta V = \frac{1}{4\pi \epsilon_0} Q \frac{d}{R_1 R_2}$$

$$V_n \quad C \text{ eq.} \Rightarrow C = \frac{Q}{V_1} - \frac{Q}{V_2} = \frac{Q}{\Delta V} = 4\pi \epsilon_0 \frac{R_1 R_2}{d}$$

Collegamento Condensatori

$$\frac{C_1}{Q_1}, \frac{C_2}{Q_2} \quad C_1 = C_2 \Rightarrow C_{eq} = \frac{C_1 C_2}{C_1 + C_2}$$

$$\frac{C_1}{Q_1}, \frac{C_2}{Q_2} \quad V_1 = V_2 = V \Rightarrow C_1 = \frac{Q_1}{V} \quad C_2 = \frac{Q_2}{V} \Rightarrow C_{eq} = \frac{(Q_1 + Q_2)}{V} \Rightarrow C_{eq} = C_1 + C_2$$

Energia Condensatore

$$\text{Lavoro per caricarlo: } \int_0^Q \frac{1}{C} dQ = \frac{Q^2}{2C}$$

$$\text{e' l'energia ACCUMULATA: } U_e = \frac{Q^2}{2C} = \frac{1}{2} \frac{Q^2}{C}$$

Condensatore piano:

$$C = \epsilon_0 \frac{A}{d} \Rightarrow U_e = \sigma^2 A \frac{d}{2 \epsilon_0} = \frac{1}{2} \epsilon_0 E^2 A d$$

In generale $U_e = \int \frac{1}{2} \epsilon_0 E^2 d\tau$ τ : VOLUME DOVE CONTENUTO E

VOLUME
CAMPO
 E

DENSITÀ
DI ENERGIA

Corrente Elettrica

\vec{j} : vettore dens. di corrente



$$|\vec{j}| = j = \frac{dq}{dsdt} \quad I = \int_S j d\vec{s}$$

Intensità di corrente

in un conduttore

$$\Delta V = \int_A^B \bar{E} d\bar{l} = V(A) - V(B) = R \cdot I$$

lunghezza

$$R: \text{resistenza} : R = \rho \cdot l \cdot \frac{1}{A}$$

dipende dal materiale

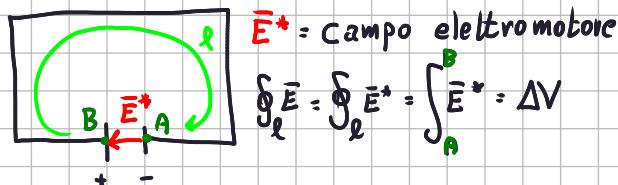
Collegamento Resistenze

$$\text{serie: } R_1 \parallel R_2 \Rightarrow R = R_1 + R_2$$

$$\text{parallelo} \quad \frac{1}{R} = \frac{1}{R_1} + \frac{1}{R_2} \Rightarrow R = \frac{R_1 \cdot R_2}{R_1 + R_2}$$

Campo Elettromotore

$$\Delta V = RI \quad \Delta V = \oint \bar{E} d\bar{l} \neq 0 \Leftrightarrow \bar{E} \text{ non elettrostatico}$$



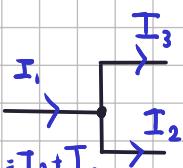
\bar{E}^* = campo elettromotore

$$\oint \bar{E} = \oint \bar{E}^* = \int_A^B \bar{E}^* = \Delta V$$

Generatore di forza elettromotrice: un dispositivo capace di mantenere una d.d.p. costante fra due punti di un conduttore.

Leggi di Kirchhoff

1°) correnti entranti = uscenti

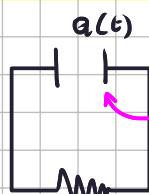


su una maglia $\Delta V_1 + \Delta V_2 = 0$

2°) $\sum \Delta V_i = 0$

$$\text{Potenza} \quad P = \frac{dL}{dt} = \frac{\Delta V dq}{dt} = \Delta V \cdot I = RI^2 \quad \frac{\text{J}}{\text{s}}$$

Andamento Circuito

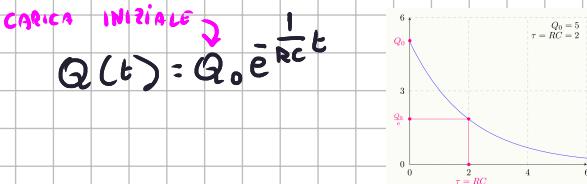


$$\left\{ \begin{array}{l} \text{si scarica} \quad I = -\frac{dQ}{dt} \\ \text{si carica} \quad I = \frac{dQ}{dt} \end{array} \right.$$

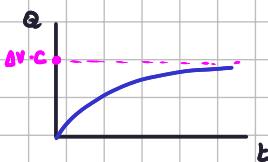
$\Rightarrow \Delta V$ ai capi del cond. e' $\frac{Q}{C}$

$$\Rightarrow RI - \frac{Q}{C} = 0 \Rightarrow \frac{1}{C} dQ = -\frac{1}{R} dt \Rightarrow$$

$$\text{CARICA INIZIALE} \quad Q(t) = Q_0 e^{-\frac{t}{RC}}$$



$$\left\{ \begin{array}{l} \Delta V - IR - \frac{Q}{C} = 0 \Rightarrow \Delta V - \frac{dQ}{dt} - \frac{1}{C} Q = 0 \\ \Rightarrow Q(t) = \Delta V \cdot C \cdot (1 - e^{-\frac{t}{RC}}) \end{array} \right.$$



Campo Magnetico

Una carica q a velocità \vec{v} in un campo \vec{B} subisce una forza $\vec{F} = q(\vec{v} \times \vec{B})$

Un filo \bar{l} in cui scorre corrente I subisce $\vec{F} = I(\bar{l} \times \vec{B})$

Prima Legge di Laplace

q : carica a vel. \vec{v}



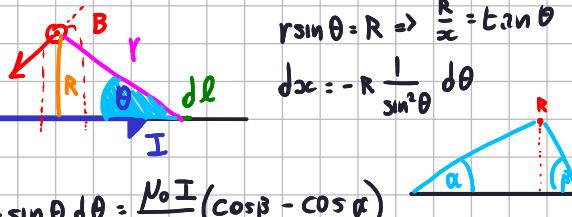
r : dist. da q a P

$$\vec{B} = \frac{1}{r^2} \cdot \frac{\mu_0}{4\pi} q(\hat{r} \times \vec{v})$$

\hat{r} : congiunge P a q

Campo Generato da un Filo

$$dB = \frac{\mu_0}{4\pi} \frac{1}{r^2} I(d\bar{l} \times \hat{r}) = \frac{\mu_0}{4\pi} \frac{1}{r^2} I d\ell \sin \theta$$



$$B = \frac{\mu_0 I}{4\pi} \int_{\alpha}^{\beta} -\frac{1}{r} \sin \theta d\theta = \frac{\mu_0 I}{4\pi r} (\cos \beta - \cos \alpha)$$

$$\Rightarrow \text{Se il filo e' infinito} \quad \begin{cases} \alpha = \pi \\ \beta = 0 \end{cases} \Rightarrow B = \frac{\mu_0 I}{2\pi r}$$

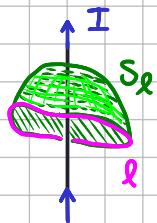
Campo di Una Spira

Forma linee di campo a "ventaglio" che si annullano sulle componenti x e $z \Rightarrow dB = dB \sin \theta$

$$\text{sull'asse } x \Rightarrow B_x = \int_A dB \sin \theta = \int_A \frac{\mu_0 I}{4\pi r^2} \sin \theta d\ell$$

$$= \frac{\mu_0 I}{4\pi r^2} \sin \theta 2\pi r^2 \Rightarrow B_{x0} = \frac{\mu_0 I}{2r^3} r^2$$

Legge di Ampere



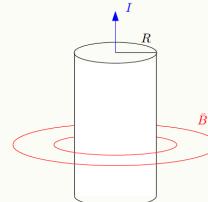
$$\oint \vec{B} d\vec{l} = \mu_0 I$$

$$\Rightarrow \oint \vec{B} d\vec{l} = \mu_0 \iint_S \vec{j} d\vec{s}$$

corrente che scorre in un filo concavo ad ℓ

Cilindro in cui Scorre Corrente

ℓ : circ. di raggio r

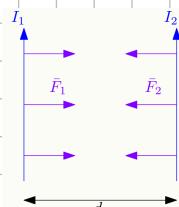


$$r > R \Rightarrow \oint \vec{B} d\vec{l} = B 2\pi r = \mu_0 I \Rightarrow B = \frac{\mu_0 I}{2\pi r}$$

$$r \leq R \Rightarrow \oint \vec{B} d\vec{l} = B 2\pi r = \int_S \vec{j} d\vec{s} = j \int_S d\vec{s} = j \pi r^2$$

$$\Rightarrow B = \mu_0 j \frac{1}{2r}$$

Definizione di Ampere



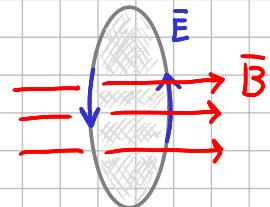
$$I_1 + I_2 = I$$

$$\frac{F_2}{l} = I_2 B_1 = I^2 \frac{\mu_0}{2\pi d}$$

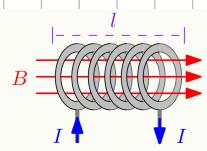
$$i^* = 1 \text{ A} \quad \text{se} \quad \frac{F_2}{l} = (i^*)^2 \frac{\mu_0}{2\pi d} = 2 \cdot 10^{-7} \text{ N}$$

Legge di Faraday - Neumann - Lenz

$$\oint \vec{E} d\vec{l} = - \frac{d}{dt} \iint_S \vec{B} d\vec{s}$$



Campo del Solenoide



l : lunghezza

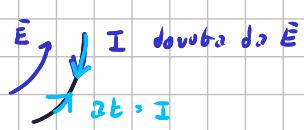
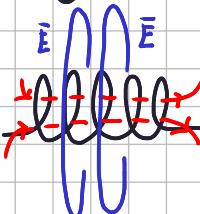
n : spire

S : sez. spira

$$B = \frac{1}{l} \mu_0 n I$$

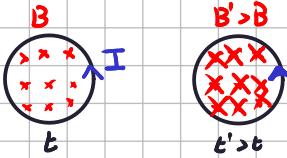
$$\text{Se } I(t) = at \Rightarrow \Phi = \frac{n}{l} \mu_0 n I \cdot S = \frac{n}{l} \mu_0 n (at) S$$

$$\Rightarrow E = \oint \vec{E} = - \frac{d\Phi}{dt} = - \frac{h^2}{l} \mu_0 n^2 S = E z \pi r^2$$



Induttanza

Quando in un circuito I varia si crea un flusso che si oppone alla variazione.



Φ aumenta \Rightarrow
si crea I_L che
si oppone alla
variazione di I

Φ e' in funzione di B che e' in funzione di I

$$\Rightarrow \Phi = L I \Rightarrow L = \frac{\Phi}{I} \Rightarrow -\frac{d\Phi}{dt} = -L \frac{dI}{dt}$$

Induttanza Solenoide

$$B = \mu_0 \frac{n}{l} I \quad \text{---} \quad \Phi = \frac{n^2}{l} \mu_0 I S = n S B$$

$$L = \frac{\Phi}{I} = \frac{1}{l} \frac{n^2}{l} \mu_0 S = \frac{n^2}{l} \mu_0 S = L$$

Induttore nel Circuito



Circuito RL

$$R \parallel \begin{cases} \text{---} \\ \text{---} \\ \text{---} \\ \text{---} \end{cases} \parallel L \quad \Delta V = L \frac{dI}{dt}$$

Densita' di Energia

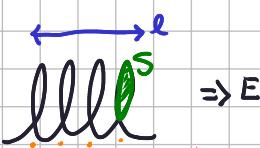
$$\text{Energia} = \int_0^t R I^2 dt = \int_0^t P dt \quad \text{ACCUMULO DI ENERGIA}$$

in un circuito RL : $I = \frac{V}{R} (1 - e^{-\frac{t}{RC}})$

$$\Rightarrow E_n = \int_0^t R \frac{V^2}{R^2} (1 - e^{-\frac{t}{RC}})^2 dt = \frac{V^2}{R} \int_0^t e^{-\frac{2t}{RC}} dt$$

$$= \frac{V^2}{R} \left[-\frac{1}{2} e^{-\frac{2t}{RC}} \right]_0^\infty = \frac{V^2}{R} \frac{L}{2R} = \frac{L}{2} \frac{V^2}{R^2} = \frac{L}{2} I^2$$

nel caso del solenoide $L = \mu_0 \frac{n^2}{l} S$



$$\Rightarrow E_n = \frac{1}{2} \mu_0 \frac{n^2}{l} S I^2 = \frac{1}{2} \frac{1}{\mu_0} B^2 S l$$

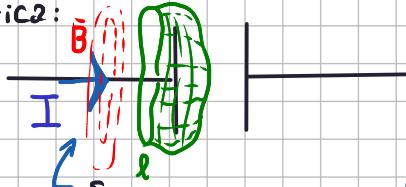
densita' di energia $\frac{1}{2} \frac{B^2}{\mu_0} S l$

τ = Volume dove e' contenuto B

$$\Rightarrow E_n = \int_{\tau} \mu_0 B d\tau = \int_{\tau} \frac{1}{2} \frac{1}{\mu_0} B^2 dt = \frac{1}{2\mu_0} \int_{\tau} B^2 dt$$

Legge di Ampere - Maxwell

Si consideri un condensatore in carica:



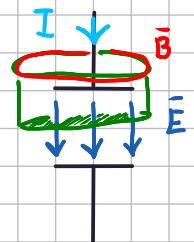
Scorre una corrente NON concatenata ad ℓ

$$\Rightarrow \oint_B d\ell = \mu_0 \int_S ds \quad \text{Ampere non funziona}$$

Si estende la legge:

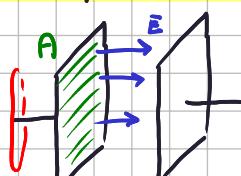
$$\oint \bar{B} d\ell = \mu_0 \int_S \bar{j} ds + \mu_0 \epsilon_0 \frac{d}{dt} \int_S \bar{E} ds$$

corrente di spostamento



Esempio Condensatore

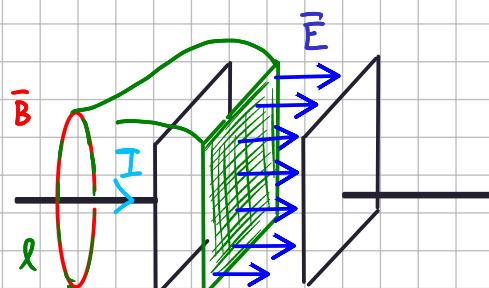
VARIA LA DENSITA' DI CARICA



$$\bar{E} = \frac{\sigma}{\epsilon_0} \Rightarrow \frac{d\phi}{dt} = \frac{d}{dt} \frac{A \sigma(t)}{\epsilon_0}$$

! $A \sigma = Q$ $\Rightarrow \frac{1}{\epsilon_0} \frac{d}{dt} A \sigma(t) = \frac{1}{\epsilon_0} \frac{dQ}{dt}$
caric.² sui cond.

$$\Rightarrow \oint_B = \int_{\text{---}} \bar{j} + \mu_0 \epsilon_0 \frac{1}{\epsilon_0} \frac{dQ}{dt}$$



$$\oint_B = 2\pi r B = \mu_0 \epsilon_0 \frac{dQ}{dt} \cdot \frac{1}{\epsilon_0}$$

$$\Rightarrow 2\pi r B = \mu_0 I \Rightarrow B = \frac{\mu_0 I}{2\pi r}$$

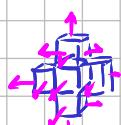
Onde Elettromagnetiche

Divergenza : $\nabla \cdot \vec{A} = \frac{\partial A_x}{\partial x} + \frac{\partial A_y}{\partial y} + \frac{\partial A_z}{\partial z}$

Rotore : $\nabla \times \vec{A} = \hat{i} \left(\frac{\partial A_z}{\partial y} - \frac{\partial A_y}{\partial z} \right) + \hat{j} \left(\frac{\partial A_x}{\partial z} - \frac{\partial A_z}{\partial x} \right) + \hat{k} \left(\frac{\partial A_y}{\partial x} - \frac{\partial A_x}{\partial y} \right)$

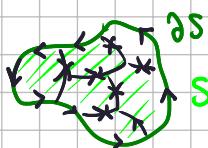
Teo Div

$$\int_S \vec{A} \cdot d\vec{s} = \int_S (\nabla \cdot \vec{A}) ds$$



Teo Rot

$$\oint_S \vec{A} \cdot d\vec{s} = \int_S (\nabla \times \vec{A}) ds$$



Equazioni di Maxwell

$$\begin{cases} \nabla \cdot \vec{E} = \frac{P}{\epsilon_0} \\ \nabla \cdot \vec{B} = 0 \\ \nabla \times \vec{E} = -\frac{\partial \vec{B}}{\partial t} \\ \nabla \times \vec{B} = \mu_0 \vec{j} + \mu_0 \epsilon_0 \frac{\partial \vec{E}}{\partial t} \end{cases}$$

Gauss
Gauss
Farday - Neumann - Lenz
Ampere - Maxwell

Nel caso non si muovono più cariche:

$$\begin{cases} \nabla \cdot \vec{E} = 0 \\ \nabla \cdot \vec{B} = 0 \\ \nabla \times \vec{E} = -\frac{\partial \vec{B}}{\partial t} \\ \nabla \times \vec{B} = \mu_0 \epsilon_0 \frac{\partial \vec{E}}{\partial t} \end{cases}$$

$$\nabla \times \vec{B} = \mu_0 \epsilon_0 \frac{\partial \vec{E}}{\partial t} \Rightarrow \nabla \times \nabla \times \vec{B} = \nabla \times \mu_0 \epsilon_0 \frac{\partial \vec{E}}{\partial t}$$

$$\Rightarrow \nabla(\nabla \cdot \vec{B}) - \nabla^2 \vec{B} = \mu_0 \epsilon_0 \frac{\partial(\nabla \times \vec{E})}{\partial t} = -\frac{\partial \vec{B}}{\partial t}$$

$$\Rightarrow \nabla^2 \vec{B} = \mu_0 \epsilon_0 \frac{\partial^2 \vec{B}}{\partial t^2} \Rightarrow \vec{E} \text{ e } \vec{B} \text{ si propagano}$$

$$\Rightarrow \nabla^2 \vec{B} = \frac{1}{\mu_0 \epsilon_0} \frac{\partial^2 \vec{B}}{\partial t^2}$$

v è la velocità:

$$\frac{1}{v^2} = \epsilon_0 \mu_0 \Rightarrow v = \frac{1}{\sqrt{\epsilon_0 \mu_0}} = c \approx 3 \cdot 10^8 \frac{m}{s} = 300000 \frac{Km}{s}$$

Proprietà Onde EM

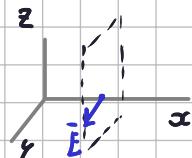
Assunzione: sono onde piane, E e B dipendono solo da x e t

$$\begin{cases} E_x(x, t) \\ E_y(x, t) \\ E_z(x, t) \end{cases} \quad \begin{cases} B_x(x, t) \\ B_y(x, t) \\ B_z(x, t) \end{cases}$$

$$\nabla \cdot \vec{E} = 0 \Rightarrow \frac{\partial E_x}{\partial x} = 0 \Rightarrow \text{Sono trasversali}$$

Se x è la dir. di propagazione.
 $E_x = B_x = 0$

$$\begin{array}{c} \vec{E} \\ \vec{B} \end{array} \rightarrow x \quad \text{componente nulla sulla dir. di propagazione.}$$



Sia y parallelo ad \vec{E}

$$\Rightarrow E_y = 0$$

$$\begin{cases} E_y(x, t) \\ E_z(x, t) \end{cases} \quad \begin{cases} B_y(x, t) \\ B_z(x, t) \end{cases}$$

per la terza eq:

$$\nabla \times \vec{E} = -\frac{\partial \vec{B}}{\partial t} \Rightarrow (\nabla \times \vec{E})_z = \frac{\partial E_x}{\partial z} - \frac{\partial E_y}{\partial y} = -\frac{\partial B_x}{\partial t}$$

$\Rightarrow B_x$ non dipende da t

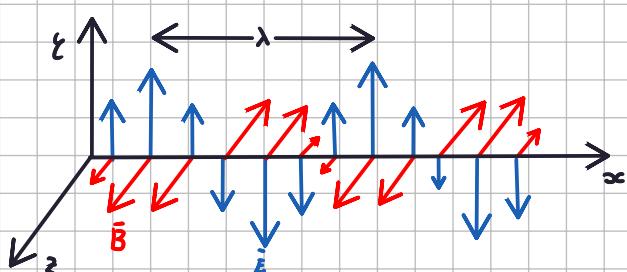
$$\begin{cases} E_y(x, t) \\ B_y(x, t) \\ B_z(x, t) \end{cases}$$

$$4^{\circ} \text{ eq: } \nabla \times \vec{B} = \mu_0 \epsilon_0 \frac{\partial \vec{E}}{\partial t} \Rightarrow \frac{\partial B_z}{\partial x} - \frac{\partial B_x}{\partial z} = \mu_0 \epsilon_0 \frac{\partial E_y}{\partial t}$$

$\Rightarrow B_y$ non dipende da $x \Rightarrow B_y$ è

nullo $\Rightarrow B$ ha componente SOLO su z

$\Rightarrow \vec{B}$ ed \vec{E} sono ortogonali



Una possibile sol. è

$$\vec{E} = \hat{j} \cdot E_0 \cos(Kx + \omega t)$$

$$\vec{B} = \hat{k} \cdot B_0 \cos(Kx + \omega t)$$

$$K = \frac{2\pi}{\lambda} \quad \text{velocità}$$

$$\omega = \frac{2\pi}{\lambda} c \quad \text{lunga d'onda}$$

$$\text{3}^{\circ} \text{ eq } \nabla \times \vec{E} = -\frac{\partial \vec{B}}{\partial t}$$

$$(\nabla \times \vec{E})_z = \frac{\partial E_z}{\partial x} - \frac{\partial E_x}{\partial z} = -\frac{\partial B_z}{\partial t}$$

" 0 "

$$\Rightarrow -KE_0 \sin(kx - \omega t) = -B_0 \omega \sin(kx - \omega t)$$

$$\Rightarrow E_0 = B_0 \frac{\omega}{k} = \frac{2\pi}{\lambda} C \frac{\lambda}{2\pi} = C =$$

$$\Rightarrow E_0 = CB_0.$$

Energia

$$\text{en. campo elettrico : } U_E = \frac{1}{2} \epsilon_0 E^2$$

$$\text{en. campo magnetico : } U_B = \frac{1}{2} \frac{1}{\mu_0} B^2$$

$$U = \frac{1}{2} \epsilon_0 E^2 + \frac{1}{2} \frac{1}{\mu_0} B^2 = \frac{1}{2} \epsilon_0 E^2 + \frac{1}{2\mu_0} \frac{E^2}{C^2}$$

$$= \frac{1}{2} \epsilon_0 E^2 + \frac{1}{2\epsilon_0} E^2 \mu_0 \epsilon_0 = \epsilon_0 E^2 = \frac{1}{\mu_0} B^2$$

$$\text{Intensità onde : } J = CU = \frac{1}{\mu_0} EB$$

Vettore di Poynting

$$\bar{N} = \frac{1}{\mu_0} (\bar{E} \times \bar{B}) \Rightarrow |\bar{N}| = J$$



nel caso

$$\begin{cases} \bar{E} = \hat{j} \cdot E_0 \cos(kx + \omega t) \\ \bar{B} = \hat{k} \cdot B_0 \cos(kx + \omega t) \end{cases} \quad J \text{ varia in } t$$

energia MEDIA periodo T

$$J = \frac{1}{\mu_0} E_0 B_0 \int_0^T \cos^2(kx + \omega t) dt = \frac{1}{2\mu_0} E_0 B_0$$

||
 $\frac{1}{2}$