Gli esercizi presentati in questo documento, esposti negli appunti di Optimization di Simone Bianco, sono tratti da esercizi del libro, e da esercizi che il professore ha presentato in classe durante il corso delle lezioni.

Consider the following linear program:

$$\max\ x_1 + 2x_2 - 3x_3 + 7x_5$$

$$\begin{aligned} x_1 + 2x_2 + 2x_3 + x_4 &= 3 \\ x_1 + 2x_2 + 7x_3 + x_5 &= 3 \\ 2x_1 + 4x_2 + 7x_3 + x_6 &= 6 \\ x_1, x_2, x_3, x_4, x_5, x_6 &\geq 0 \end{aligned}$$

Determine and justify which of the following vectors is a basic feasible solution (BFS):

1. $a = [1\ \ 1\ \ 0\ \ 0\ \ 0\ \ 0]$
2. $b = [1\ \ 0\ \ 0\ \ 2\ \ 2\ \ 4]$
3. $c = [0\ \ 0\ \ 0\ \ 3\ \ 3\ \ 6]$

La matrice dei vincoli e'

$$A = \begin{bmatrix} 1 & 2 & 2 & 1 & 0 & 0 \\ 1 & 2 & 7 & 0 & 1 & 0 \\ 2 & 4 & 7 & 0 & 0 & 1 \end{bmatrix} \Rightarrow$$

Le colonne 4, 5, 6 sono lin. indip. quindi c e' una BFS associata a tale base $\mathcal{B} = \{4, 5, 6\}$

Given a general linear program in equational form

$$\max\ c^T x \qquad Ax = b \qquad x \geq 0$$

let $\overline{x}$ and $\overline{y}$ be two optimal solutions to the problem. Prove or disprove the following statements:

1. Any convex combination of $\overline{x}$ and $\overline{y}$ is also an optimal solution to the linear program
2. There exists an optimal solution $\overline{z}$ on the line through $\overline{x}$ and $\overline{y}$ which is also basic

Una combinazione convessa di due punti $\overline{x}, \overline{y}$ e' il segmento di linea.

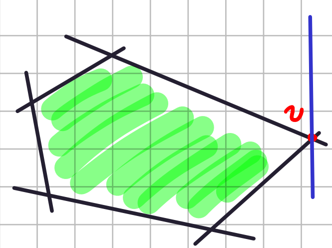
$c^T\overline{x} = c^T\overline{y} = \gamma \qquad z = \alpha\overline{x} + (1-\alpha)\overline{y}$

$c^T z = c^T(\alpha x + (1-\alpha)y) = \alpha c^T \overline{x} + (1-\alpha) c^T \overline{y} = \alpha\gamma + (1-\alpha)\gamma = \gamma \Rightarrow z$ e' ottimale.

La 2 e' falsa perche' $\overline{x}$ e $\overline{y}$ sono BFS, ossia vertici del poliedro, e ogni punto sul segmento di linea di 2 vertici non e' (per definizione) un vertice.

Consider the polyhedron $P = \{x \in \mathbb{R}^n \mid Ax \leq b, x \geq 0\}$ and let $v \in P$. Prove that v is a vertex of P if and only if there exists a vector c such that v is the unique optimal basic feasible solution to the linear program:

$$\max\ c^T x \qquad x \in P$$



$[\Rightarrow]$: v e' un vertice di $P \Rightarrow \exists\ \mathcal{H} = \{x : c^T x = \alpha\}$ t.c.

$c^T x < \alpha \quad \forall x \in P \setminus \{v\} \quad \wedge \quad c^T v = \alpha$

ma allora v e' sol. ottimale per $\max_{x \in P} c^T x$.

$[\Leftarrow]$: v e' l'unica sol. di $\max_{x \in P} c^T x \Rightarrow$ necessariamente v e' una BFS $\Rightarrow v$ e' un vertice di P.

Solve the following linear program using the Simplex method:

$$\max\ 2x_1 + x_2 - x_3$$

$$\begin{aligned} x_1 + 2x_2 + x_3 &\leq 8 \\ -x_1 + x_2 - 2x_3 &\leq 4 \\ x_1, x_2, x_3 &\geq 0 \end{aligned}$$

Il problema in forma matriciale e'

$$A = \begin{bmatrix} 1 & 2 & 1 \\ -1 & 1 & -2 \end{bmatrix} \qquad b = [8\ \ 4]^T$$

$\Rightarrow$ in forma di eq: $A' = \begin{bmatrix} 1 & 2 & 1 & 1 & 0 \\ -1 & 1 & -2 & 0 & 1 \end{bmatrix}$

La prima base e' $\mathcal{B} = \{4, 5\}$

$$\begin{aligned} x_1 + 2x_2 + x_3 + x_4 &= 8 \\ -x_1 + x_2 - 2x_3 + x_5 &= 4 \\ z &= 2x_1 + x_2 - x_3 \end{aligned} \Rightarrow$$

$\mathcal{B} = \{4, 5\}$

$$\begin{array}{|l} x_4 = 8 - x_1 - 2x_2 - x_3 \\ x_5 = 4 + x_1 - x_2 + 2x_3 \\ \hline z = 2x_1 + x_2 - x_3 \end{array} \Rightarrow$$

$\mathcal{B} = \{1, 5\}$

$$\begin{array}{l} x_1 = 8 - x_4 - 2x_2 - x_3 \\ x_5 = 4 + x_1 - x_2 + 2x_3 \\ \hline z = 16 - 2x_4 - 3x_2 - 3x_3 \end{array}$$

$\Rightarrow$ la sol ott. e' $\begin{cases} x_1 = 8 \\ x_5 = 12 \end{cases} \Rightarrow$ per il problema originale e' $x = [8\ \ 0\ \ 0]^T$

Consider the following linear program

$$\min\ 3x_1 + 2x_2 - x_3$$
$$x_1 - 2x_2 + x_3 - x_4 = 0$$
$$x_1 + x_2 + 2x_3 - x_4 = 1$$
$$x_1, x_2, x_3, x_4 \geq 0$$

Beginning with the basis $\mathcal{B} = \{1,2\}$, solve the linear program with the Simplex method

Riscrivo il problema in Forma max e parto con il primo tableu

$\mathcal{B} = \{1,2\}$

$$\begin{cases} x_1 - 2x_2 + x_3 - x_4 = 0 \\ x_1 + x_2 + 2x_3 - x_4 = 1 \end{cases} \qquad \begin{cases} x_1 = -\frac{5}{3}x_3 + x_4 + \frac{2}{3} \\ x_2 = \frac{1}{3} - \frac{1}{3}x_3 \end{cases}$$

$\mathcal{B} = \{1,2\}$

$$x_1 = -\frac{5}{3}x_3 + x_4 + \frac{2}{3}$$
$$x_2 = \frac{1}{3} - \frac{1}{3}x_3$$
$$z = -\frac{8}{3} + \frac{20}{3}x_3 + 3x_4$$

$\Rightarrow$

$\mathcal{B} = \{2,4\}$

$$x_4 = -x_1 - \frac{5}{3}x_3 + \frac{2}{3}$$
$$x_2 = 1/3 - 1/3\, x_3$$
$$z = -\frac{2}{3} + \frac{20}{3}x_3 - 3x_1 - 5x_3$$

$\Rightarrow$

$\mathcal{B} = \{3,4\}$

$$x_4 = -x_1 - \frac{5}{3} + 5x_2 + \frac{2}{3}$$
$$x_3 = 1 - 3x_2$$
$$z = 1 - 17x_2 - 3x_1$$

Finito, il massimo e' 1, il minimo del problema originale e' -1,

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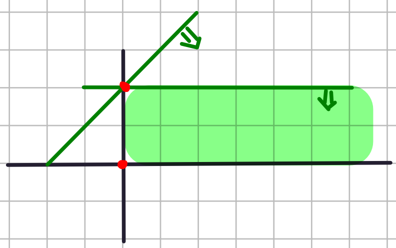
Give an explicit example of a linear program in the form of:

$$\max c^T x$$
$$Ax = b$$
$$x \geq 0$$

such that the problem is unbounded, it has a degenerate BFS and infinitely many optimal solutions

Si consideri il problema

$$\max\ -x_1$$
$$\text{s.t. } x_2 - x_1 \leq 2$$
$$x_2 \leq 2$$



e' illimitato, le due BFS $[0\ 0\ 0\ 0]$ e $[0\ 2\ 0\ 0]$ sono ottimali

in Forma di eq: $x_2 - x_1 + x_3 = 2$

$x_2 + x_4 = 2$

Si consideri $\mathcal{B} = \{1,2\} \Rightarrow \begin{cases} x_1 = -x_4 + x_3 - 2 + 2 \\ x_2 = -x_4 + 2 \end{cases} \Rightarrow$ Sol : $[0\ 2\ 0\ 0]^T \Rightarrow$ Una variabile di base e' $0 \Rightarrow [0\ 2\ 0\ 0]^T$ e' degenere.

Solve the following linear program using the Simplex method:

$$\min\ 12x_1 + 3x_2 + 4x_3$$
$$4x_1 + 2x_2 + 3x_3 \geq 2$$
$$8x_1 + x_2 + 2x_3 \geq 3$$
$$x_1, x_2, x_3 \geq 0$$

Trasformo il problema in forma canonica, e poi in forma di equazione.

$$\max\ -12x_1 - 3x_2 - 4x_3$$
$$-4x_1 - 2x_2 - 3x_3 \leq -2$$
$$-8x_1 - x_2 - 2x_3 \leq -3$$

$\Rightarrow$

$$\max\ -12x_1 - 3x_2 - 4x_3$$
$$-4x_1 - 2x_2 - 3x_3 + x_4 = -2$$
$$-8x_1 - x_2 - 2x_3 + x_5 = -3$$

la base $\mathcal{B} = \{4,5\}$ ha BFS associata $[0\ 0\ 0\ {-2}\ {-3}]^T \Rightarrow$ non e' ammissibile.

$\mathcal{B} = \{1,2\}$

$$x_1 = -\frac{7}{20}x_3 + \frac{1}{20}x_4 + \frac{1}{10}x_5 + \frac{2}{5}$$
$$x_2 = -\frac{4}{5}x_3 + \frac{2}{5}x_4 - \frac{1}{5}x_5 + \frac{1}{5}$$
$$z = \frac{13}{5}x_3 - \frac{9}{5}x_4 - \frac{3}{5}x_5 - \frac{27}{5}$$

3

2

$\Rightarrow$

$\mathcal{B} = \{1,3\}$

$$x_1 = \frac{35}{80}x_2 + \frac{3}{16}x_3 - \frac{1}{8}x_4 + \frac{5}{16}$$
$$x_3 = -\frac{5}{4}x_2 + \frac{1}{2}x_4 - \frac{1}{4}x_5 + \frac{1}{4}$$
$$z = -\frac{13}{4}x_2 - \frac{1}{2}x_4 - \frac{25}{20}x_5 - \frac{19}{4}$$

$\Rightarrow$

Sol : $x^* = \left[\frac{5}{16}, 0, \frac{1}{4}\right]$

opt: $\frac{19}{4}$

per l'LP originale

Consider the standard form polyhedron $P = \{x \mid Ax = b, x \geq 0\}$. Suppose that the matrix A of dimensions $m \times n$ has linearly independent rows and that all basic feasible solutions are non-degenerate.

Given $\overline{x} \in P$ such that $\overline{x}$ has exactly m positive components:

1. Show that $\overline{x}$ is a BFS
2. Show that the previous point is false if we remove the non-degeneracy assumption

Se $\bar{x}$ ha m componenti positive, e non ci sono soluzioni degenere, allora $B = \{i : \bar{x}_i > 0\}$ e' la base di $\bar{x}$, ed e' una BFS solo se A_B, che ha m colonne, e' non singolare, essendo che il rango delle righe di A e' m, lo e' anche quello di $A_B \Rightarrow$ le colonne di A_B sono lin. ind. $\Rightarrow A_B$ e' non singolare.

Se l'assunzione sulle sol. non degenere e' Falsa, non si puo' affermare che $B = \{i : \bar{x}_i > 0\}$ sia la base di $\bar{x}$.

Consider the following primal program:

$$\begin{aligned} \min\ & c^T x \\ & Ax \geq b \\ & x \geq 0 \end{aligned}$$

1. Form the dual program and state it as an equivalent minimization problem
2. Derive the conditions on A, b and c such that the dual is identical to the primal, meaning that the problem is *self-dual*
3. Give a concrete example of a primal program identical to its dual

Il duale e' $\begin{cases} \max\ b^T y \\ A^T y \leq c \\ y \leq 0 \end{cases} \equiv \begin{cases} \min\ -b^T y \\ A^T y \leq c \\ y \leq 0 \end{cases}$, e' self-dual se

$$\{x : Ax \geq b\} = \{x : A^T x \leq c\} \Rightarrow$$

$$\{x : Ax \geq b\} = \{x : -A^T x \geq -c\} \Longrightarrow \begin{cases} b = -c \\ A = -A^T \end{cases}$$

Un'esempio e':

$$\begin{aligned} \max\ & [1\ -1]\, x \\ \text{s.t.}\ & \begin{bmatrix} 0 & -1 \\ 1 & 0 \end{bmatrix} x \leq \begin{bmatrix} -1 \\ 1 \end{bmatrix} \\ & x \geq 0 \end{aligned}$$

Consider the following linear program:

$$\begin{aligned} \max\ & 2x_1 + 16x_2 + 12x_3 \\ & 2x_1 + x_2 - x_3 \leq 3 \\ & -3x_1 + 8x_2 + 2x_3 \leq 12 \\ & x_1, x_2, x_3 \geq 0 \end{aligned}$$

Determine if the feasible solution $x^* = \begin{bmatrix} 6 & 0 & 12 \end{bmatrix}$ is also optimal.

Considero il problema duale:

$$\begin{cases} \min\ 3y_1 + 12y_2 \\ 2y_1 - 3y_2 \geq 2 \\ y_1 + 8y_2 \geq 16 \\ -y_1 + 2y_2 \geq 12 \\ y_1, y_2 \geq 0 \end{cases}$$

Essendo $x_1^*, x_3^* > 0 \Rightarrow$

$$\begin{cases} 2y_1 - 3y_2 = 2 \\ y_1 + 8y_2 \geq 16 \\ -y_1 + 2y_2 = 12 \end{cases} \Rightarrow \begin{cases} 2y_1 = 2 + 3y_2 \\ 2y_2 = 12 + y_1 \end{cases} \Rightarrow \begin{cases} y_1 = \frac{3}{2}y_2 + 1 \\ 2y_2 = 12 + \frac{3}{2}y_2 + 1 \end{cases} \Rightarrow \begin{cases} y_1 = 40 \\ y_2 = 26 \end{cases} \Rightarrow y^* = [40\ \ 26]^T$$

y^* dovrebbe essere la sol oltimale per il duale, verifichiamo

$$2x_1^* + 16x_2^* + 12x_3^* = 12 + 144 = 156$$

$3y_1^* + 12y_2^* = 3 \cdot 40 + 12 \cdot 26 = 432$, gli oltimi non coincidono $\Rightarrow x^*$ non e' oltimale.

Consider the following linear program:

$$\max\ 2x_1 + \alpha x_2 + \beta x_3$$

$$\begin{aligned} 2x_1 + x_2 - x_3 &\le 3 \\ -3x_1 + 8x_2 + 2x_3 &\le 12 \\ x_1, x_2, x_3 &\ge 0 \end{aligned}$$

Find all the values α, β such that $x^* = \begin{bmatrix} 0 & 0 & 6 \end{bmatrix}$ is an optimal solution.

L'ottimo del duale deve coincidere con l'ottimo del primario. Il duale e':

$$\begin{cases} \min\ 3y_1 + 12y_2 \\ 2y_1 - 3y_2 \ge 2 \\ y_1 + 8y_2 \ge \alpha \\ -y_1 + 2y_2 \ge \beta \\ y_1, y_2 \ge 0 \end{cases}$$

essendo $x_3^* > 0$
$\Rightarrow -y_1 + 2y_2 = \beta.$

Inoltre $2x_1^* + x_2^* - x_3^* < 3$
$\Rightarrow y_1^* = 0$

$$\Rightarrow y^* = \begin{bmatrix} 0 & \frac{\beta}{2} \end{bmatrix}$$

Voglio che $\begin{cases} y_1^* + 8y_2^* \ge \alpha \\ 2y_1^* - 3y_2^* \ge 2 \end{cases}$ $\begin{cases} 4\beta \ge \alpha \\ -\frac{3}{2}\beta \ge 2 \end{cases}$ $\begin{cases} \beta \ge \frac{\alpha}{4} \\ \beta \ge -\frac{4}{3} \Leftarrow \text{banale perche' } \beta \ge 0 \end{cases}$

Voglio che gli ottimi siano identici:

$$2x_1^* + \alpha x_2^* + \beta x_3^* = 3y_1^* + 12y_2^* \Rightarrow 6\beta = 12\frac{\beta}{2} \Rightarrow \beta = 1,\ x^* \text{ e' sol per } \beta = 1,\ \alpha \le 4$$

Consider the following linear program:

$$\max\ 5x_1 + \alpha x_2 + 8x_3$$

$$\begin{aligned} x_1 + x_2 + 9x_3 &\le 5 \\ 3x_1 + 4x_2 + 13x_3 &\le 12 \\ 5x_1 + x_2 + 17x_3 &\le \beta \\ x_1, x_2, x_3 &\ge 0 \end{aligned}$$

Find all the values of α, β such that $x^* = \begin{bmatrix} 1 & 2 & 0 \end{bmatrix}$ is an optimal solution.

Il problema duale e':

$$\begin{cases} \min\ 5y_1 + 12y_2 + \beta y_3 \\ y_1 + y_2 + 5y_3 \ge 5 \\ y_1 + 4y_2 + y_3 \ge \alpha \\ 9y_1 + 13y_2 + 17y_3 \ge 8 \\ y_1, y_2, y_3 \ge 0 \end{cases}$$

Sia y^* la sol. ottimale per il duale, se x^* fosse ottimale, dato che $x_1^*, x_2^* > 0$, si avrebbe:

$$\begin{cases} y_1^* + y_2^* + 5y_3^* = 5 \\ y_1^* + 4y_2^* + y_3^* = \alpha \end{cases}$$

inoltre, essendo che $x_1^* + x_2^* + 9x_3^* < 5$ e $3x_1^* + 4x_2^* + 13x_3^* < 12$ $\Rightarrow y_1^*, y_2^* = 0$

$$\Rightarrow \begin{cases} y_1^* + y_2^* + 5y_3^* = 5 \\ y_1^* + 4y_2^* + y_3^* = \alpha \\ 9y_1^* + 13y_2^* + 17y_3^* \ge 8 \end{cases} \Rightarrow \begin{cases} 5y_3^* = 5 \\ y_3^* = \alpha \\ 17y_3^* \ge 8 \end{cases} \Rightarrow \quad y^* = \begin{bmatrix} 0 & 0 & \alpha \end{bmatrix} \quad \alpha = 1$$

allora si vuole che

$$5y_1^* + 12y_2^* + \beta y_3^* = 5x_1^* + \alpha x_2^* + 8x_3^* \Rightarrow$$

$\beta\alpha = 5 + 2\alpha$ ma $\alpha = 1 \Rightarrow$

$\beta = 5 + 2 \Rightarrow (\alpha, \beta) = (1, 7)$

Consider the following linear program:

$$\begin{aligned} \min\ 2x_1 + 4x_2 + 2x_3 \\ x_1 + x_2 + 3x_3 &\le 1 \\ -x_1 + 2x_2 + x_3 &\ge 1 \\ 3x_2 - 6x_3 &= 0 \\ x_1, x_3 &\ge 0 \\ x_2 &\in \mathbb{R} \end{aligned}$$

Determine if $x^* = \begin{bmatrix} 0 & \frac{2}{5} & \frac{1}{5} \end{bmatrix}$ is an optimal solution.

trasformo il terzo vincolo in due vincoli e scrivo il duale:

$$(P): \begin{cases} \min\ 2x_1 + 4x_2 + 2x_3 \\ x_1 + x_2 + 3x_3 \le 1 \\ -x_1 + 2x_2 + x_3 \ge 1 \\ 3x_2 - 6x_3 \ge 0 \\ 3x_2 - 6x_3 \le 0 \\ x_1, x_3 \ge 0, x_2 \in \mathbb{R} \end{cases} \qquad (D): \begin{cases} \max\ y_1 + y_2 \\ y_1 - y_2 \ge 2 \\ y_1 + 2y_2 + 3y_3 + 3y_4 = 4 \\ 3y_1 + y_2 - 6y_3 - 6y_4 \ge 2 \\ y_1, y_4 \ge 0 \\ y_2, y_3 \le 0 \end{cases}$$

Sia y^* la sol. ott. di (D), essendo $x_3^* > 0 \Rightarrow$

$$3y_1^* + y_2^* - 6y_3^* - 6y_4^* = 2$$

Essendo i vincoli di (P) binding per x^*, non si puo' costruire un sistema di K eq. in K incognite. Risolvo (D) col simplesso.

$$\begin{cases} \max\ y_1 - y_2 \\ y_1 + y_2 + y_5 = 2 \\ y_1 - 2y_2 - 3y_3 + 3y_4 = 4 \\ 3y_1 - y_2 + 6y_3 - 6y_4 = 2 \\ y \geq 0 \end{cases}$$

Serve una base ammissibile, provo con $B = \{1, 2, 3\}$

A_B e´ non singolare.

$B = \{1,2,3\}$

$$\begin{array}{|l} y_1 = -\frac{6}{5}y_4 + \frac{13}{30}y_5 + 2 \\ y_2 = \frac{6}{5}y_4 - \frac{1}{10}y_5 \\ y_3 = -\frac{1}{5}y_4 - \frac{7}{30}y_5 - \frac{2}{3} \\ \hline z = -\frac{12}{5}y_4 + \frac{8}{15}y_5 + 2 \end{array}$$

$\Rightarrow$

$B = \{1,3,5\}$

$$\begin{array}{|l} y_1 = -\frac{6}{5}y_4 + \frac{13}{30}y_5 + 2 \\ y_5 = -10y_2 + 12y_4 \\ y_3 = -\frac{1}{5}y_4 - \frac{7}{30}y_5 - \frac{2}{3} \\ \hline z = 4y_4 - \frac{16}{3}y_2 + 2 \end{array}$$

$\Rightarrow$

per Far aumentare z serve $B = \{3, 4, 5\}$ ma le colonne 3, 4 di A sono lin. dip.

$y^* = [2 \quad 0 \quad -\frac{2}{3} \quad 0 \quad 0] \Rightarrow$

l'ottimo di (D) e´ uguale all'ottimo di (P)

$\Rightarrow y_1^* - y_2^* = 2 = 2x_1^* + 4x_2^* + 2x_3^* = 4\frac{2}{5} + 2\frac{1}{5} = 2 \Rightarrow x^*$ e´ ottimale.

The following tableau corresponds to an optimal basis for a linear program where x_1, x_2 and x_3 are the original primal variables and s_1, s_2 are the slack variables corresponding to the two constraints.

$$\begin{array}{ccccccc} x_1 & = & \frac{5}{2} & +\frac{1}{2}x_2 & +\frac{1}{6}s_1 & -\frac{1}{3}s_2 \\ x_3 & = & \frac{5}{2} & -\frac{1}{4}x_2 & -\frac{1}{2}s_1 & \\ \hline z & = & \frac{65}{2} & -2x_2 & -3s_1 & -2s_2 \end{array}$$

1. Write the original linear program
2. Write the dual of the original program
3. Solve the dual problem via Simplex method

Per risalire al LP originale, risolvo il tableu nei termini delle slack variable.

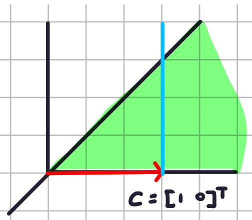
$$\begin{cases} s_2 = -3x_1 + \frac{5}{4}x_2 + 10 - x_3 \\ s_1 = +5 - \frac{1}{2}x_2 - 2x_3 \\ z = 6x_1 + x_2 + 8x_3 + \frac{5}{2} \end{cases} \Rightarrow LP = \begin{cases} \max\ 6x_1 + x_2 + 8x_3 \\ 3x_1 - \frac{5}{4}x_2 + x_3 + s_2 = 10 \\ \frac{1}{2}x_2 + 2x_3 + s_1 = 5 \\ x \geq 0 \end{cases}$$

Il duale e´:

$$\begin{cases} \min\ 10y_1 + 5y_2 \\ 3y_2 \geq 6 \\ \frac{1}{2}y_1 - \frac{5}{4}y_2 \geq 1 \\ 2y_1 - y_2 \geq 8 \\ y_1, y_2 \geq 0 \end{cases} \equiv \begin{cases} \max\ -10y_1 - 5y_2 \\ 3y_2 + s_1 = 6 \\ \frac{1}{2}y_1 - \frac{5}{4}y_2 + s_2 = 1 \\ 2y_1 - y_2 + s_3 = 8 \\ y_1, y_2, s_1, s_2, s_3 \geq 0 \end{cases}$$

Give an example of a linear programs P, P' such that:

1. P is feasible and his dual program D is infeasible
2. Both P' and his dual program D' are infeasible



$$P: \begin{cases} \max x_1 \\ -x_1 + x_2 \leq 0 \\ x_1, x_2 \geq 0 \end{cases}$$ il duale e' $$\begin{matrix} \min 0 \\ -y_1 \geq 1 \\ y_2 \geq 0 \\ y_1, y_2 \geq 0 \end{matrix} \Rightarrow \begin{matrix} y_1 \leq -1 \\ y_1 \geq 0 \end{matrix}$$ impossibile

$$P': \begin{cases} \max x_2 \\ x_1 \leq -1 \\ -x_2 \leq 0 \\ x_1, x_2 \geq 0 \end{cases}$$ e' inammissibile e lo e' anche il duale $$\Rightarrow \begin{cases} \min -y_1 \\ y_1 \geq 0 \\ -y_2 \geq 1 \\ y_1, y_2 \geq 0 \end{cases} \equiv \begin{cases} \min -y_1 \\ y_1 \geq 0 \\ y_2 \leq -1 \\ y_1, y_2 \geq 0 \end{cases} \Rightarrow \begin{matrix} y_2 \leq -1 \\ y_2 \geq 0 \end{matrix}$$ impossibile.

Prove or disprove the following statement: if a linear program has an unique optimal solution then its dual program has an unique optimal solution

E' Falso, si consideri il seguente LP

$$P: \begin{cases} \max x_1 \\ x_1 + x_2 \leq 0 \\ x_1, x_2 \geq 0 \end{cases}$$ Il duale e' $$D: \begin{cases} \min 0 \\ y_1 \geq 1 \\ y_2 \geq 0 \\ y_1, y_2 \geq 0 \end{cases}$$

↑ ha un'unica soluzione $x^* = [0\ 0]^T$

↑ ha infinite sol.