

Formulario

Cinematica

moto rettilineo uniforme

$$\begin{cases} x(t) = x_0 + v_0 t \\ v_0 \text{ costante} \end{cases}$$

moto uniformemente accelerato

$$\begin{cases} x(t) = x_0 + v_0 t + \frac{1}{2} a_0 t^2 \\ v(t) = v_0 + a_0 t \\ a_0 \text{ costante} \end{cases}$$

moto armonico

$$x(t) = A \sin(\phi + \omega t)$$

$$v(t) = A\omega \cos(\phi + \omega t)$$

$$a(t) = -A\omega^2 \sin(\phi + \omega t) = -\omega^2 x(t)$$

$$\text{periodo} = T = \frac{2\pi}{\omega}$$

$$\text{frequenza} = \nu = \frac{1}{T} = \frac{\omega}{2\pi}$$

$$\text{pulsazione} = \omega = 2\pi\nu$$

moto circolare (R::raggio)

$$\omega(t) = \text{velocità angolare}$$

$$v = \omega R \text{ velocità tangenziale}$$

$$a_t = \frac{dv}{dt} \text{ accelerazione tangenziale}$$

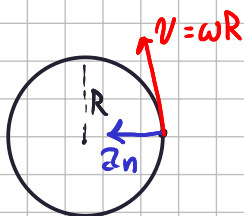
$$a_n = \omega^2 R \text{ accelerazione normale}$$

$$a = \sqrt{a_t^2 + a_n^2} \text{ acc. totale}$$

$$\vec{v} = \vec{\omega} \times \vec{R}$$

moto circ. unifor.

$$\begin{cases} x(t) = R \cos(\theta(t)) \\ y(t) = R \sin(\theta(t)) \\ \theta = \omega t \end{cases}$$



Dinamica

$$\vec{F} = m \cdot \vec{a}$$

quantità di moto $\vec{p} = \frac{d}{dt}(m\vec{a}) = m\vec{v}$

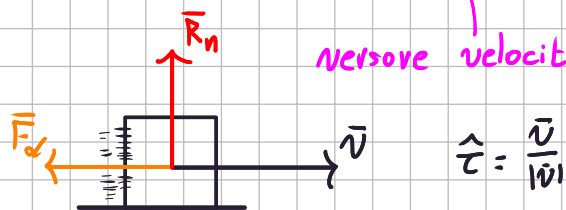
impulso $I = \int_{t_0}^t \vec{F} dt$ e $I = \Delta \vec{p}$

Forza media $= \frac{I}{\Delta t} = \frac{\Delta \vec{p}}{\Delta t} = \frac{|\vec{p}(t_2) - \vec{p}(t_1)|}{t_2 - t_1}$

Attrito

Statico $\vec{F}_s \leq \mu_s R_n$ $R_n :=$ reazione normale

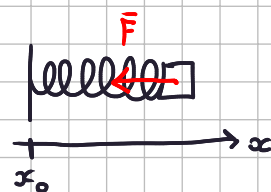
Dinamico $\vec{F}_d = -\mu_d \cdot R_n \cdot \hat{z}$



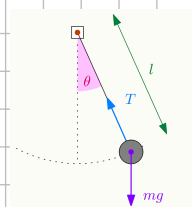
attrito dell'aria: $\vec{F} = -b\vec{v}$ b coefficiente

Forza Elastica

$\vec{F} = -K(x - x_0)$



Pendolo



$$-mg \sin \theta = -m \frac{d^2 s}{dt^2}$$

$$-mg \sin\left(\frac{s}{l}\right) = -m \frac{d^2 s}{dt^2}$$

per $\theta \rightarrow 0 \Rightarrow -mg \frac{s}{l} = -m \frac{d^2 s}{dt^2}$

$$\Rightarrow s = A \cos(\omega t + \phi)$$

$$\omega = \sqrt{\frac{g}{l}}$$

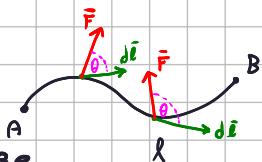
Gravi

$\begin{cases} h: \text{quota lancio} \\ v_0: \text{vel iniziale} \end{cases}$ si fermerà in $\Rightarrow t^* = \frac{1}{g} v_0$
quota max raggiunta: $h + \frac{1}{2g} v_0^2$

Lavoro

$\vec{F}: \mathbb{R}^3 \rightarrow \mathbb{R}^3$ campo di Forze

$$L = \int_A^B \vec{F} d\vec{\ell} = \int_A^B F d\ell \cos \theta = \Delta T$$



Impulso

$I = \Delta \vec{p} = \Delta(m\vec{v})$ variazione della quantità di moto

Teorema dell'impulso

$\frac{I}{\Delta t} = \frac{\Delta \vec{p}}{\Delta t} = \frac{1}{\Delta t} \int_{t_1}^{t_2} \vec{F} dt$ forza media

energia cinetica $T = \frac{1}{2} m v^2$

$$L = \frac{1}{2} m v^2(B) - \frac{1}{2} m v^2(A)$$

\vec{F} e' conservativa se

$L = -\Delta U$ dove U energia potenziale

$$\oint \vec{F} d\vec{\ell} = 0$$

Potenziale gravita' $U(y) = mgy$ ↑ altezza

Potenziale gravitazionale $U(r) = -\frac{GMm}{r}$ ↑ distanza

Potenziale elastica $U(x) = \frac{1}{2} k(x - x_0)^2$

Energia meccanica $E_m = U + T$
 $= U + \frac{1}{2} m v^2$

Se \vec{F} e' conservativa $\Delta E_m = 0$

$\Rightarrow \begin{cases} L = \Delta T \\ L = -\Delta U \end{cases} \Rightarrow \Delta T + \Delta U = 0 \Rightarrow \Delta(T + U) = 0$

$\Delta E_m = \text{LAVORO FORZE NON CONSERVATIVE}$

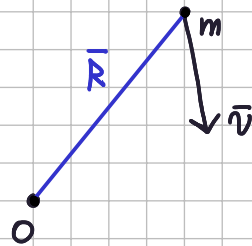
Potenza

$$P = \frac{dL}{dt}$$

Momento Angolare

$$\vec{M} = \vec{R} \times \vec{p} = \vec{R} \times \frac{d}{dt} m\vec{v}$$

↑
quantità di moto



momento della quantità di moto

$$\vec{b} = \vec{R} \times m\vec{v}$$

$$\vec{M} = \frac{d\vec{b}}{dt} = \frac{d\vec{R}}{dt} \times m\vec{v}$$

essendo $\vec{v} = \vec{\omega} \times \vec{R}$

$$\vec{b} = \vec{R} \times m(\vec{\omega} \times \vec{R}) = mR^2 \vec{\omega}$$

momento di inerzia $I = mR^2$

$$\vec{b} = I\vec{\omega}$$

il momento si può scrivere $\vec{M} = \frac{d}{dt} I\vec{\omega} = I \dot{\vec{\omega}}$ se I cost.

Sistema di Punti

le forze interne si annullano. n punti

$$\vec{F}_{\text{est}} = \sum_{i=1}^n m_i \vec{a}_i$$

pos. di ogni punto

centro di massa $\vec{r}_c = \frac{\sum m_i \vec{r}_i}{\sum m_i}$

vel. del sistema (centro di massa)

$\frac{d\vec{r}_c}{dt} = \vec{v}_c$ Somma masse $M = \sum m_i$

\Rightarrow quantità di moto del sistema

$$\vec{p}_c = M\vec{v}_c = \sum m_i \vec{v}_i$$

$$\frac{d}{dt} \vec{p}_c = \sum m_i \frac{d\vec{v}_i}{dt} = \sum m_i \vec{a}_i = \vec{F}_{\text{est}}$$

Urti

• elastici: la quantità di moto si conserva.

2 corpi

Masses: m_1, m_2
vel. post urto v_1, v_2
vel. pre urto u_1, u_2

Cons. quantità di moto

$$m_1 u_1 + m_2 u_2 = m_1 v_1 + m_2 v_2$$

cons. energia cinetica

$$\frac{1}{2} m_1 u_1^2 + \frac{1}{2} m_2 u_2^2 = \frac{1}{2} m_1 v_1^2 + \frac{1}{2} m_2 v_2^2$$

Momento sistema di punti

$$\begin{cases} \vec{M}_i = \frac{d\vec{b}_i}{dt} + \vec{v}_0 \times \vec{r}_i \\ \vdots \\ \vec{M}_n = \frac{d\vec{b}_n}{dt} + \vec{v}_0 \times \vec{r}_n \end{cases}$$

i momenti interni del sistema si annullano

$$\left(\frac{d}{dt} \sum \vec{b}_i \right) + \vec{v}_0 \times \left(\sum \vec{r}_i \right)$$

Si può riscrivere

$$\frac{d}{dt} \left(\sum \vec{r}_i \times m_i \vec{v}_i \right) + \vec{v}_0 \times \left(\sum \vec{r}_i \right)$$

Il momento del sistema non è uguale alla somma dei momenti

Sistema Continuo

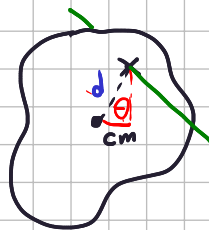
per un oggetto puntiforme $I = R m^2$

$$dI = R^2 dm = R^2 \lambda dR \Rightarrow \lambda \text{ densità: } \frac{dm}{dR} = \lambda$$

$$I = \int_S dI$$

← sup. oggetto

Pendolo Composto



d = dist. centro di massa - punto inf.

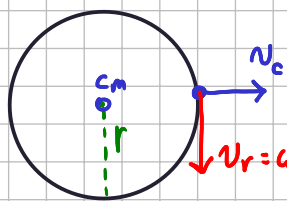
$$-m g \cdot d \cdot \theta = -I_0 \frac{d^2 \theta}{dt^2} \quad \text{per } \theta \rightarrow 0$$

I_0 = momento di inerzia con polo in cm

$$I_0 = I_{cm} + m \cdot d^2$$

↑ momento in. centro massa

Rotolamento



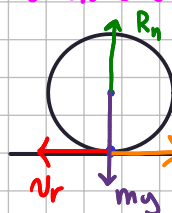
Velocità centro di massa

Velocità di rotazione

$$v_a = v_c + v_r$$

↑ vel. assoluta di ogni punto

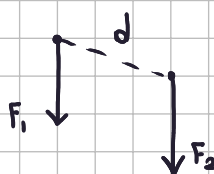
$|v_c| = |v_r| \Rightarrow$ rotolamento puro



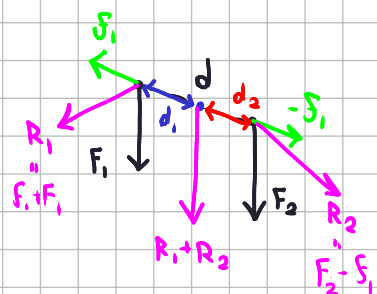
nel punto di contatto

$$F_d = \mu N = \mu m g$$

Sistemi eq. di Forze



$$\frac{d_1}{d_2} = \frac{F_2}{F_1}$$



Campo Elettrico

Coloumb

q esercita su q_0 $\vec{F} = \frac{1}{4\pi\epsilon_0} \frac{q \cdot q_0}{r^2} \hat{r}$

$r := \text{dist}(q, q_0)$

Campo Elettrico

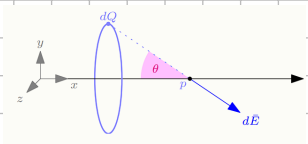
$$\vec{E} = \lim_{q_0 \rightarrow 0} \frac{\vec{F}}{q_0} = \frac{1}{4\pi\epsilon_0} \frac{q}{r^2} \hat{r}$$

q_0 immersa in \vec{E} sente $\vec{F} = q_0 \vec{E}$

Sistema di cariche: $\vec{E} = \frac{1}{4\pi\epsilon_0} \sum_{i=1}^n \frac{q_i}{r_i^2} \hat{r}_i$

Sup. carica: $\vec{E} = \frac{1}{4\pi\epsilon_0} \int_S \frac{dQ}{r^2} \hat{r}$

Anello Carico



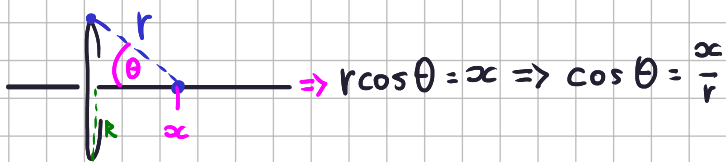
$$dE_x = \frac{1}{4\pi\epsilon_0} \frac{dQ}{r^2} \cos\alpha$$

λ densità $\Rightarrow dQ = \lambda dl$

$dl = \text{punto circonferenza}$

$$E_x = \frac{1}{4\pi\epsilon_0} \int_A \frac{\lambda dl}{r^2} \cos\theta = \frac{1}{4\pi\epsilon_0} \frac{1}{r^2} \cos\theta Q$$

non dipende dal punto sulla circ.



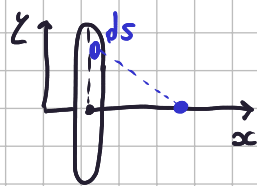
$$E = \frac{1}{4\pi\epsilon_0} \frac{x}{r^3} Q$$

dipende da x raggio

$r = \sqrt{R^2 + x^2}$

Disco Carico

$dQ = \lambda dS$ $dS = \text{sup. infinitesima}$



$$E = \frac{1}{4\pi\epsilon_0} \iint_S \frac{\lambda dS}{r^2}$$

superf.

Cambio:

$dS = \text{anello infinitesimo}$
spessore anello



$$dS = (2\pi r) \cdot dr$$

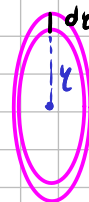
raggio

$$dE = \frac{1}{4\pi\epsilon_0} \cdot \frac{x}{(x^2 + r^2)^{3/2}} \cdot dQ$$

dens. \downarrow dist $z = \text{raggio } r$ anello

$$dQ = \sigma \cdot (2\pi r) dr$$

sup. anello inf.



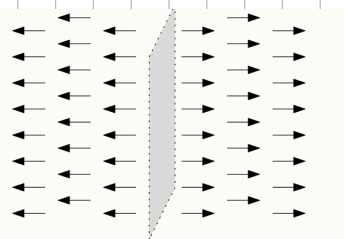
RAGGIO DISCO

$$E = \frac{1}{4\pi\epsilon_0} \cdot \sigma 2\pi \cdot x \int_0^R \frac{z}{(x^2 + r^2)^{3/2}} dr$$

$$E(x) = \frac{\sigma x}{2\epsilon_0} \cdot \left(\frac{1}{|x|} - \frac{1}{\sqrt{x^2 + R^2}} \right)$$

Piano Carico

$$\lim_{R \rightarrow \infty} \frac{\sigma x}{2\epsilon_0} \cdot \left(\frac{1}{|x|} - \frac{1}{\sqrt{x^2 + R^2}} \right) = \pm \frac{\sigma}{2\epsilon_0}$$

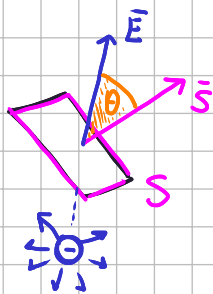


Flusso

$$\Phi = \vec{E} \cdot \vec{S} = ES \cos \theta$$

$S = \text{sup finita}$

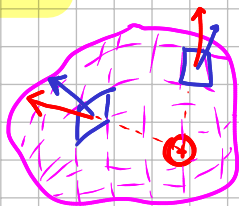
$$\Phi = \int_S d\Phi = \int_S \vec{E} \cdot d\vec{s}$$



Legge di Gauss

$S := \text{sup. chiusa}$

$$\Phi = \oint_S \vec{E} \cdot d\vec{s} = \frac{q}{\epsilon_0}$$



$$d\Phi = \vec{E} \cdot d\vec{s} = \frac{1}{4\pi\epsilon_0} \frac{q}{r^2} \hat{r} \cdot d\vec{s} = \frac{q}{4\pi\epsilon_0} \cdot \frac{\hat{r} \cdot d\vec{s}}{r^2} = \frac{q}{4\pi\epsilon_0} d\Omega$$

$d\Omega \Rightarrow \oint d\Omega = 4\pi$

Si scrive anche

$$\oint_S \vec{E} \cdot d\vec{s} = \frac{1}{\epsilon_0} \int_{\tau_S} \rho d\tau$$

carica int. alla sup S
 $\tau_S \Rightarrow \text{volume di } S$

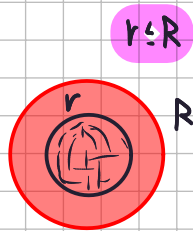
Campo sfera carica



$r \geq R$

$$\Phi = \frac{1}{\epsilon_0} \int_{\tau_S} \rho d\tau = \frac{q}{\epsilon_0}$$

$$\frac{q}{\epsilon_0} = \oint_S \vec{E} \cdot d\vec{s} = E \int_S d\vec{s} = E 4\pi r^2 \Rightarrow E = \frac{1}{4\pi\epsilon_0} \frac{q}{r^2}$$



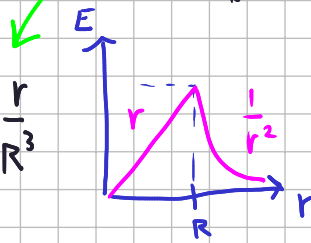
$r \leq R$

$$\Phi = \frac{1}{\epsilon_0} \int_{\tau_S} \rho d\tau = \frac{\rho}{\epsilon_0} \int_0^r d\tau = \frac{\rho}{\epsilon_0} \frac{4}{3} \pi r^3$$

$$E 4\pi r^2 = \frac{\rho}{\epsilon_0} \frac{4}{3} \pi r^3$$

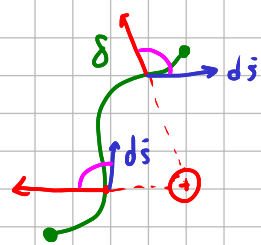
$$E = \frac{\rho}{\epsilon_0} \frac{1}{3} r = \frac{Q}{4\pi\epsilon_0} \cdot \frac{r}{R^3}$$

$\frac{4}{3} \pi R^3 \rho = Q$
 \Downarrow
 $\rho = Q \frac{3}{4\pi R^3}$



Circuitazione

$$L = \int_{\text{cammino}} q_0 \vec{E} = q_0 \int_S \vec{E} \cdot d\vec{s}$$



Sia C una curva chiusa

$$\oint_C \vec{E} \cdot d\vec{s} = \int_A^B \vec{E} \cdot d\vec{s} = V(A) - V(B) = \Delta V$$

CIRCUITAZIONE potenziale

$$L = -q_0 \Delta V = -\Delta U$$

CAMPO ELETTROSTATICO \Rightarrow CONSERVATIVO $\Rightarrow \oint_C \vec{E} \cdot d\vec{s} = 0$

Potenziale di una Carica

$$\vec{E} = \frac{1}{4\pi\epsilon_0} \frac{Q}{r^2} \hat{r} \Rightarrow \vec{E} \cdot d\vec{s} = \frac{1}{4\pi\epsilon_0} \frac{Q}{r^2} \hat{r} \cdot d\vec{s} = \frac{1}{4\pi\epsilon_0} \frac{Q}{r^2} dr$$

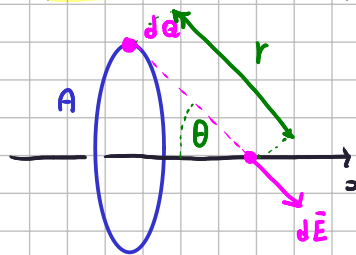
$$\begin{aligned} \oint_C \vec{E} \cdot d\vec{s} &= \int_A^B \vec{E} \cdot d\vec{s} = \frac{Q}{4\pi\epsilon_0} \int_{r_A}^{r_B} \frac{1}{r^2} dr \\ &= \frac{Q}{4\pi\epsilon_0} \left(\frac{1}{r_B} - \frac{1}{r_A} \right) \Rightarrow V(r) = \frac{Q}{4\pi\epsilon_0} \cdot \frac{1}{r} \end{aligned}$$

\uparrow dist da q

Gradiente

$$\vec{E} = -\nabla V = \left[-\frac{\partial V}{\partial x}, -\frac{\partial V}{\partial y}, -\frac{\partial V}{\partial z} \right]$$

Potenziale Anello Carico

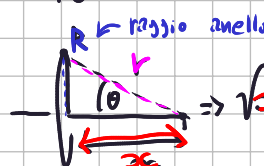


Potenziale di dq e'

$$\frac{1}{4\pi\epsilon_0} \frac{dq}{r}$$

$$\oint_A dq = Q$$

Pot. totale $V(r) = \frac{1}{4\pi\epsilon_0} \int_A \frac{dq}{r} = \frac{1}{4\pi\epsilon_0} \cdot \frac{Q}{r}$



$\Rightarrow \sqrt{x^2 + R^2} = r \Rightarrow V(r) = \frac{Q}{4\pi\epsilon_0} \cdot \frac{1}{\sqrt{x^2 + R^2}}$

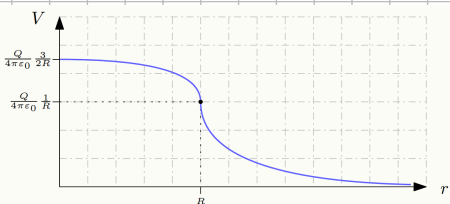
Potenziale Sfera Carica

raggio sfera: R

$$E(r) = \begin{cases} \frac{1}{r^2} \frac{Q}{4\pi\epsilon_0} & \text{se } r \geq R \\ \frac{Q}{4\pi\epsilon_0} \cdot \frac{r}{R^3} & \text{se } r \leq R \end{cases}$$

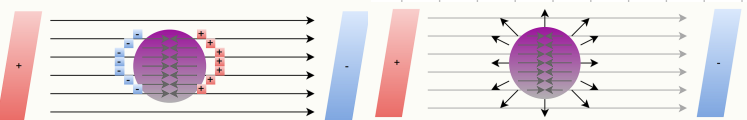
Interno: $V(r) = \int_r^\infty \frac{1}{r^2} \cdot \frac{Q}{4\pi\epsilon_0} = \frac{1}{4\pi\epsilon_0} \cdot \frac{Q}{r}$ { come se fosse una carica punt. }

esterno: $V(r) = \frac{Q}{4\pi\epsilon_0} \frac{1}{R^3} \int_r^R r dr = \frac{1}{4\pi\epsilon_0} \left(\frac{3}{2}R - \frac{r^2}{2R^3} \right)$

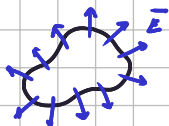


Conduttori

le cariche nei conduttori sono libere e perturbano il campo elettrico



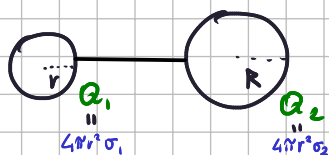
Un conduttore ha \vec{E} nullo all'interno. Sulla sup. e' normale ad essa:



Equipotenziale

in un conduttore τ il campo \vec{E} e' nullo $\Rightarrow \int \vec{E} d\vec{x} = 0 = \Delta V \Rightarrow$ in ogni punto il potenziale e' identico.

Effetto delle punte: il campo e' piu' intenso dove la curvatura e' maggiore.



$$\frac{\sigma_1}{\sigma_2} = \frac{R}{r} \Rightarrow \sigma_1 = \sigma_2 \frac{R}{r}$$

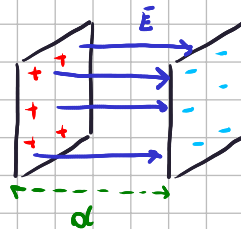
Questo perche' la superficie e' equipotenz.

Capacita'

conduttore di carica Q e pot. $V \Rightarrow C = \frac{Q}{V}$

nel caso della sfera $V = \frac{1}{4\pi\epsilon_0} \frac{Q}{R} \Rightarrow C = 4\pi\epsilon_0 R$

Condensatore

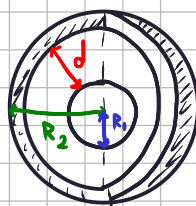


$E = \frac{\sigma}{\epsilon_0} \Rightarrow V = \int_0^d E = \int_0^d \frac{\sigma}{\epsilon_0} = \frac{\sigma d}{\epsilon_0}$ solo fra le lastre
sulla sup

A : sup. lastre $\Rightarrow Q = \sigma A$

$$\Rightarrow C = \frac{Q}{V} = \epsilon_0 \frac{A}{d}$$

Condensatore Sferico



$$V_1 = \frac{1}{4\pi\epsilon_0} \frac{Q}{R_1}, V_2 = \frac{1}{4\pi\epsilon_0} \frac{Q}{R_2} \Rightarrow \Delta V = \frac{1}{4\pi\epsilon_0} Q \frac{d}{R_1 R_2}$$

Un C eq. $\Rightarrow C = \frac{Q}{V_1} - \frac{Q}{V_2} = \frac{Q}{\Delta V} = 4\pi\epsilon_0 \frac{R_1 R_2}{d}$

Collegamento Condensatori

C_1, C_2 in serie $Q_1 = Q_2 \Rightarrow C_{eq} = \frac{C_1 C_2}{C_1 + C_2}$

C_1, C_2 in parallelo $V_1 = V_2 = V \Rightarrow C_1 = \frac{Q_1}{V}, C_2 = \frac{Q_2}{V} \Rightarrow C_{eq} = \frac{(Q_1 + Q_2)}{V} \Rightarrow C_{eq} = C_1 + C_2$

Energia Condensatore

Lavoro per caricarlo: $\int_0^q \frac{1}{C} dq = \frac{q^2}{2C}$
e' l'energia accumulata: $U_e = q^2 \frac{1}{2C} = \frac{1}{2} qV$

Condensatore piano:

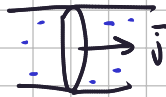
$$C = \epsilon_0 \frac{A}{d} \Rightarrow U_e = \sigma^2 A \frac{d}{2\epsilon_0} = \frac{1}{2} \epsilon_0 E^2 A d$$

In generale $U_e = \int \frac{1}{2} \epsilon_0 E^2 d\tau$ { VOLUME DOVE E' CONTENUTO E }

VOLUME CAMPO E
DENSITA' DI ENERGIA

Corrente Elettrica

\vec{j} : vettore dens. di corrente



$$|\vec{j}| = j = \frac{dQ}{ds dt}$$

$$\vec{I} = \int_S \vec{j} ds$$

Intensita' di corrente

in un conduttore

$$\Delta V = \int_A^B \vec{E} d\vec{l} = V(A) - V(B) = R \cdot I$$

R : resistenza : $R = \rho \cdot l \cdot \frac{1}{A}$

dipende dal materiale

lunghezza

Area sezione

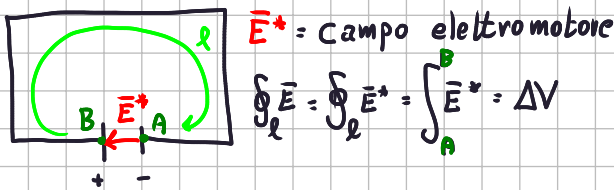
Collegamento Resistenze

serie: $R_1, R_2 \Rightarrow R = R_1 + R_2$

parallelo: $R_1, R_2 \Rightarrow R = \frac{R_1 \cdot R_2}{R_1 + R_2}$

Campo Elettromotore

$\Delta V = RI$ $\Delta V = \oint_L \vec{E} d\vec{l} \neq 0 \Leftrightarrow \vec{E}$ non elettrostatico

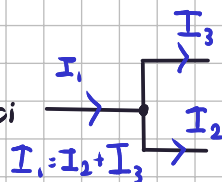


Generatore di forza elettromotrice: un qualsiasi

dispositivo capace di mantenere una d.d.p. costante fra due punti di un conduttore.

Leggi di Kirchhoff

1°) correnti entranti = uscenti



2°) $\sum \Delta V_i = 0$



Potenza

$$P = \frac{dL}{dt} = \frac{\Delta V dq}{dt} = \Delta V \cdot I = RI^2 \quad \frac{J}{s}$$

Andamento Circuito

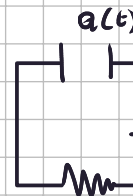
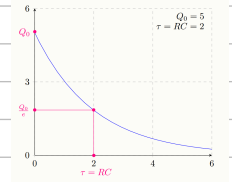


condensatore $\begin{cases} \text{si scarica } I = -\frac{dQ}{dt} \\ \text{si carica } I = \frac{dQ}{dt} \end{cases}$

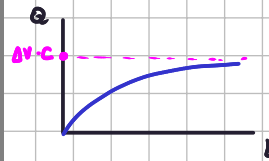
$\Rightarrow \Delta V$ ai capi del cond. e' $\frac{Q}{C}$

$$\Rightarrow RI - \frac{Q}{C} = 0 \Rightarrow \frac{1}{Q} dQ = -\frac{1}{RC} \Rightarrow$$

CARICA INIZIALE $\Rightarrow Q(t) = Q_0 e^{-\frac{1}{RC}t}$



$$\begin{cases} \Delta V - IR - \frac{Q}{C} = 0 \Rightarrow \Delta V - \frac{dQ}{dt} - \frac{1}{C}Q = 0 \\ \Rightarrow Q(t) = \Delta V \cdot C \cdot (1 - e^{-\frac{1}{RC}t}) \end{cases}$$



Campo Magnetico

Una carica q a velocità \vec{v} in un campo \vec{B} subisce una forza $\vec{F} = q(\vec{v} \times \vec{B})$

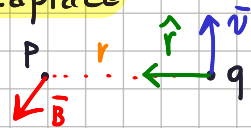
Un filo \vec{l} in cui scorre corrente I subisce $\vec{F} = I(\vec{l} \times \vec{B})$

Prima Legge di Laplace

q = carica a vel. \vec{v}

r = dist. da q a P

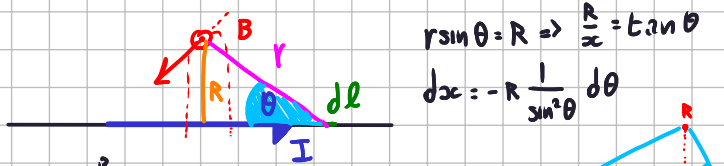
\hat{r} = congiunge P a q



$$\vec{B} = \frac{1}{r^2} \cdot \frac{\mu_0}{4\pi} q(\hat{r} \times \vec{v})$$

Campo Generato da un Filo

$$dB = \frac{\mu_0}{4\pi} \frac{1}{r^2} I(d\vec{l} \times \hat{r}) = \frac{\mu_0}{4\pi} \frac{1}{r^2} I dl \sin \theta$$



$$r \sin \theta = R \Rightarrow \frac{R}{x} = \tan \theta$$

$$dx = -R \frac{1}{\sin^2 \theta} d\theta$$

$$B = \frac{\mu_0 I}{4\pi} \int_{\alpha}^{\beta} -\frac{1}{R} \sin \theta d\theta = \frac{\mu_0 I}{4\pi R} (\cos \beta - \cos \alpha)$$

$$\Rightarrow \text{Se il filo e' infinito} \begin{cases} \alpha = \pi \\ \beta = 0 \end{cases} \Rightarrow B = \frac{\mu_0 I}{2\pi R}$$



$$\cos \alpha = \frac{S}{\sqrt{S^2 + R^2}}$$

$$\text{se } S \rightarrow -\infty \quad \cos \alpha \rightarrow \pi \Rightarrow \alpha = -1$$