$$\begin{cases} y''(t)+y(t)=\sin(t) \\ y(0)=y'(0)=0 \end{cases}$$

$P(\lambda) \to \lambda^2+1=0$ $\quad \lambda_{1,2}=\frac{\pm\sqrt{-4}}{2}=\frac{\pm i\sqrt{4}}{2}=\pm i$ $\quad \alpha=0$, $\beta=1$

$$y_0(t)=C\cdot\cos(t)+D\cdot\sin(t)$$

SOL. PARTICOLARE

CERCHIAMO $\bar{y}(t)=Q\cos(t)+P\sin(t)$, MA ESSA PUO' ESSERE SOLUZIONE DELL'OMOGENEA ASSOCIATA! QUINDI MOLTIPLICHIAMO PER t.

$\bar{y}(t)=Qt\cos(t)+Pt\sin(t)$, TROVIAMO P e Q

$$\bar{y}'(t)=Q\cos(t)-Qt\sin(t)+P\sin(t)+Pt\cos(t)$$

$$\bar{y}''(t)=-2Q\sin(t)-Qt\cos(t)+2P\cos(t)-Pt\sin(t)$$

SOSTITUIAMO ALL'EDO

$$-2Q\sin(t)-Qt\cos(t)+2P\cos(t)-Pt\sin(t)+Qt\cos(t)+Pt\sin(t)=\sin(t)$$

$$2P\cos(t)-2Q\sin(t)=\sin(t)$$

$$\begin{cases} 2P=0 \\ -2Q=1 \end{cases} \begin{cases} P=0 \\ Q=-\frac{1}{2} \end{cases} \qquad \bar{y}(t)=-\frac{t}{2}\cos(t)$$

$$y(t)=C\cos(t)+D\sin(t)-\frac{t}{2}\cos(t)$$

$$y'(t)=-C\sin(t)+D\cos(t)-\frac{1}{2}\cos(t)+\frac{t}{2}\sin(t)$$

$y(0)=0 \to 0=C \to C=0$

$y'(0)=0 \to D-\frac{1}{2}=0 \to D=\frac{1}{2}$ $\qquad y(t)=\frac{1}{2}\sin(t)-\frac{t}{2}\cos(t)$