Esercizio 1

$x_i = t \cdot c_i$

(a) $\begin{cases} 8x \equiv 2 \ (18) \\ 9x \equiv 12 \ (21) \\ 14x \equiv 10 \ (22) \end{cases} \Rightarrow \begin{cases} 4x \equiv 1 \ (9) \\ 3x \equiv 4 \ (7) \\ 7x \equiv 5 \ (11) \end{cases} \Rightarrow \begin{cases} 7\cdot 4x \equiv 7 \ (9) \\ 5\cdot 3x \equiv 5\cdot 4 \ (7) \\ 8\cdot 7x \equiv 8\cdot 5 \ (11) \end{cases} \Rightarrow \begin{cases} x \equiv 7 \ (9) \\ x \equiv 6 \ (7) \\ x \equiv 7 \ (11) \end{cases} \Rightarrow R = 11\cdot 7\cdot 9 \Rightarrow$

$R_1 = 77$ risolvo $77t + 9K = 1 \Rightarrow t = 2 \Rightarrow \tilde{x} = 2\cdot 7 = 14 \equiv 5$

$R_2 = 99$ risolvo $99t + 7K = 1 \Rightarrow t = 1 \Rightarrow \tilde{x} = 6$

$R_3 = 63$ risolvo $63t + 11K = 1 \Rightarrow t = -4 \Rightarrow \tilde{x} = -4\cdot 7 = -28 \equiv 5$

$\tilde{x} = 77\cdot 5 + 99\cdot 6 + 63\cdot 5 = 385 + 594 + 315 = 1294 \ (\mathrm{mod}\ 9\cdot 7\cdot 11)$

(b) $\begin{cases} x \equiv 5 \ (18) \\ x \equiv 3 \ (20) \\ x \equiv 11 \ (24) \end{cases} \Rightarrow \begin{cases} x = 5 + 18l \\ 18l \equiv 18 \ (20) \\ x \equiv 11 \ (24) \end{cases} \Rightarrow \begin{cases} x = 5 + 18l \\ l = 1 + 20K \\ 〃 \end{cases} \Rightarrow \begin{cases} x = 5 + 18(1 + 20K) \\ 〃 \\ 5 + 18 + 18\cdot 20K \equiv 11 \ (24) \end{cases} \Rightarrow 18\cdot 20K \equiv 11 - 23 \ (24) \Rightarrow 360K \equiv -12 \ (24) \Rightarrow$

$360K \equiv -12 \ (24) \Rightarrow 0\cdot K \equiv 12 \ (24) \Rightarrow$ NON C'E' SOLUZIONE.

(c) $\begin{cases} x \equiv 7 \ (18) \\ x \equiv 13 \ (20) \\ x \equiv 19 \ (21) \end{cases} \Rightarrow \begin{cases} x = 7 + 18l \\ 18l \equiv 6 \ (20) \\ x \equiv 19 \ (21) \end{cases} \Rightarrow \begin{cases} x = 7 + 18l \\ 9l \equiv 3 \ (10) \\ x \equiv 19 \ (21) \end{cases} \Rightarrow \begin{cases} x = 7 + 18l \\ l \equiv 7 \ (10) \\ x \equiv 19 \ (21) \end{cases} \Rightarrow \begin{cases} x = 7 + 18\cdot 7 + 18\cdot 10K \\ l = 7 + 10K \\ x \equiv 19 \ (21) \end{cases} \Rightarrow \begin{cases} x = 133 + 18\cdot 10K \\ l = 7 + 10K \\ 18\cdot 10K \equiv 19 - 133 \ (21) \end{cases} \Rightarrow \begin{cases} x = 133 + 18\cdot 10K \\ 180K \equiv -114 \ (21) \end{cases} \Rightarrow \begin{cases} x = 133 + 18\cdot 10K \\ 12K \equiv 12 \ (21) \end{cases}$

$\Rightarrow \begin{cases} x = 133 + 18\cdot 10K \\ K = 1 + 21t \end{cases} \Rightarrow x = 133 + 18\cdot 10\cdot (1 + 21t) = 133 + 180 + 18\cdot 10\cdot 21t = 313 + (18\cdot 10\cdot 21)t$

Esercizio 2

Il sistema ha matrice associata:

$\left|\begin{array}{ccc|c} 1 & -2 & 3 & 4 \\ 2 & -3 & a & 5 \\ 3 & -4 & 5 & b \end{array}\right| \Rightarrow \begin{array}{c} A_2 := A_2 - 2A_1 \\ A_3 := A_2 - 2A_1 \end{array} \sim \left|\begin{array}{ccc|c} 1 & -2 & 3 & 4 \\ 0 & 1 & a-6 & -3 \\ 0 & 2 & -4 & b-12 \end{array}\right| \Rightarrow A_3 := A_3 - 2A_2 \ \left|\begin{array}{ccc|c} 1 & -2 & 3 & 4 \\ 0 & 1 & a-6 & -3 \\ 0 & 0 & 2-2a & b-6 \end{array}\right| \Rightarrow \begin{cases} x - 2y + 3z = 4 \\ y + (a-6)z = -3 \\ (2-2a)z = b-6 \end{cases} \Rightarrow$

il sistema e' compatibile se e solo se $a \neq 1$, se $a = 1 \wedge b \neq 6$ il sistema e' incompatibile, se $a = 1 \wedge b = 6$ il sistema e' indeterminato.

Esercizio 3

(a) $\sigma_1^{-1} = \begin{pmatrix} 1 & 2 & 3 & 4 & 5 & 6 & 7 & 8 & 9 \\ 7 & 6 & 5 & 8 & 1 & 9 & 3 & 4 & 2 \end{pmatrix} \quad \sigma_2^{-1} = \begin{pmatrix} 1 & 2 & 3 & 4 & 5 & 6 & 7 & 8 & 9 \\ 6 & 9 & 1 & 7 & 5 & 3 & 8 & 2 & 4 \end{pmatrix} \quad \sigma_3^{-1} = \begin{pmatrix} 1 & 2 & 3 & 4 & 5 & 6 & 7 & 8 & 9 \\ 7 & 1 & 5 & 8 & 3 & 2 & 6 & 9 & 4 \end{pmatrix}$

$\sigma_1^{-1} \circ \sigma_2^{-1} \circ \sigma_3^{-1} = \begin{pmatrix} 1 & 2 & 3 & 4 & 5 & 6 & 7 & 8 & 9 \\ 9 & 2 & 7 & 3 & 1 & 5 & 4 & 6 & 8 \end{pmatrix} \begin{pmatrix} 1 & 2 & 3 & 4 & 5 & 6 & 7 & 8 & 9 \\ 7 & 1 & 5 & 8 & 3 & 2 & 6 & 9 & 4 \end{pmatrix} = \begin{pmatrix} 1 & 2 & 3 & 4 & 5 & 6 & 7 & 8 & 9 \\ 4 & 9 & 1 & 6 & 7 & 2 & 5 & 8 & 3 \end{pmatrix}$

(b) $\sigma_1^{-1} = (1\ 7\ 3\ 5)(2\ 6\ 9)(4\ 8) = (1\ 5)(1\ 3)(1\ 7)(2\ 9)(2\ 6)(4\ 8)$ e' pari

$\sigma_2^{-1} = (1\ 6\ 3)(2\ 9\ 4\ 7\ 8) = (1\ 3)(1\ 6)(2\ 8)(2\ 7)(2\ 4)(2\ 9)$ e' pari

$\sigma_3^{-1} = (1\ 7\ 6\ 2)(3\ 5)(4\ 8\ 9) = (1\ 2)(1\ 6)(1\ 7)(3\ 5)(4\ 9)(4\ 8)$ e' pari

Al posto di $\sigma_1, \sigma_2, \sigma_3$ ho preso il loro inverso per distrazione.

(c) σ_3^{-1} e σ_1^{-1} sono coniugate e condividono la stessa struttura ciclica.

$\alpha = \begin{pmatrix} 1 & 2 & 3 & 4 & 5 & 6 & 7 & 8 & 9 \\ 1 & 4 & 6 & 3 & 2 & 8 & 7 & 5 & 9 \end{pmatrix}$