ESAME GENNAIO 2020

Esercizio 1

1) E' un estrazione non ordinata senza rimpiazzo: $\Omega=\{(\omega_1,\omega_2,\omega_3)\mid 1\le\omega_1<\omega_2<\omega_3\le 40\}$, $|\Omega|=\binom{40}{3}$

2) $A_i:=\{\text{Napoletana del seme } i\}$, $|A_i|=1 \Rightarrow \mathbb{P}(\text{Napoletana})=\frac{1}{\binom{40}{30}}\cdot 1\cdot 4 \leftarrow \{4:\text{possibili napoletane}\}$

3) $B_i:=\{\text{Carta 1: Seme a, Carta 2: Seme b, Carta 3: Seme c}\}=\binom{10}{1}\cdot\binom{10}{1}\cdot\binom{10}{1}=10^3$

$\mathbb{P}(\{3 \text{ semi diversi}\})=\frac{1}{\binom{40}{3}}\cdot 10^3\cdot\binom{4}{3} \leftarrow \{\binom{4}{3}: \text{combinazioni di 3 semi su 4}\}$

4) Considero $\Omega'=\{(\omega_1,\omega_2,\omega_3)\mid \omega_i\in\{1..40\}\wedge i\neq j\Rightarrow \omega_i\neq\omega_j\}$ :: conta l'ordine! $|\Omega'|=40\cdot 39\cdot 38$

$C=\{\text{estraggo un asso di denari}\}$, $\mathbb{P}(C)=\frac{1}{40}+\frac{1}{39}+\frac{1}{38}=\frac{39\cdot 38+40\cdot 38+40\cdot 39}{40\cdot 39\cdot 38}$ $\quad D=\{1^\circ \text{ e' l'asso}\}$

$\mathbb{P}(D\cap C)=\mathbb{P}(D)$ perche' $D\subset C\Rightarrow \mathbb{P}(D)=\frac{1}{40}\Rightarrow \mathbb{P}(D\mid C)=\frac{\mathbb{P}(C\cap D)}{\mathbb{P}(C)}=\frac{40\cdot 39\cdot 38}{39\cdot 38+40\cdot 38+40\cdot 39}\cdot\frac{1}{40}=\frac{39\cdot 38}{39\cdot 38+40\cdot 38+40\cdot 39}$

Esercizio 2

1) $A=\{\text{Un cavallo vince tutto}\}=\sum_{i:\{a,b,c\}}\{\text{cavallo } i \text{ vince tutto}\}=\sum_{i:\{a,b,c\}}A_i$

$\mathbb{P}(A_a)=0.3\cdot 0.3\cdot 0.3=(0.3)^3 \quad \mathbb{P}(A_b)=(1/2)^3=1/8 \quad \mathbb{P}(A_c)=(0.2)^3 \quad \mathbb{P}(A)=(0.3)^3+(0.2)^3+\frac{1}{8}=\frac{4}{25}$

2) $B'=\{\text{a vince } 1^\circ \text{ gara, b vince } 2^\circ \text{ gara, c vince } 3^\circ \text{ gara}\}$, $\mathbb{P}(B')=\frac{3}{10}\cdot\frac{2}{10}\cdot\frac{5}{10}=\frac{3}{100}$

$B=\{\text{ognuno vince}\}=B'\cdot\text{permutazioni di 3 gare}\Rightarrow \mathbb{P}(B)=\frac{3}{100}\cdot 3!=\frac{3}{100}\cdot 6=\frac{18}{100}=\frac{9}{50}$

Esercizio 3

So che $\mathbb{P}(Z_1=k)=\frac{\lambda_1^k}{k!}e^{-\lambda_1}$ e $\mathbb{P}(Z_2=k)=\frac{\lambda_2^k}{k!}e^{-\lambda_2}$ $\quad$ NOTAZIONE: $x\in \mathrm{Im}(x) :: x\in X$

1) $\mathbb{P}(Z=k)=\mathbb{P}(Z_1+Z_2=k)=\mathbb{P}(Z_1=x\cap Z_2=k-x)\ \forall x\in \mathrm{Im}(Z_1)=\sum_{x\in \mathrm{Im}(Z_1)}\mathbb{P}(Z_1=x)\cdot\mathbb{P}(Z_2=k-x)=\sum_{x\in X}\frac{\lambda_1^x}{x!}e^{-\lambda_1}\cdot\frac{\lambda_2^{k-x}}{(k-x)!}e^{-\lambda_2}$

$=\sum_{x\in X}\frac{\lambda_1^x}{x!}e^{-\lambda_1}\cdot\frac{\lambda_2^{k-x}}{(k-x)!}e^{-\lambda_2}=e^{-(\lambda_1+\lambda_2)}\cdot\sum_{x\in X}\frac{\lambda_1^x\cdot\lambda_2^{(k-x)}}{x!(k-x)!}=\frac{(\lambda_1+\lambda_2)^k}{k!}\cdot e^{-(\lambda_1+\lambda_2)}\sim \text{Poisson}(\lambda_1+\lambda_2)$

2) $\mathbb{P}(Z_1=k\mid Z=x)=\mathbb{P}(Z_1=k\mid Z_1+Z_2=x)=\frac{\mathbb{P}(Z_1=k\cap Z_1+Z_2=x)}{\mathbb{P}(Z=x)}=\frac{\mathbb{P}(Z_1=k\cap Z_2=x-k)}{\mathbb{P}(Z=x)}$

$=\left(\frac{\lambda_1^k}{k!}e^{-\lambda_1}\cdot\frac{\lambda_2^{x-k}}{(x-k!)}\cdot e^{-\lambda_2}\right)\cdot\left(\frac{(\lambda_1+\lambda_2)^x}{x!}e^{-(\lambda_1+\lambda_2)}\right)^{-1}=\frac{x!\cdot\lambda_1^k\cdot\lambda_2^{(x-k)}}{k!(x-k)!\cdot(\lambda_1+\lambda_2)^x}$

3) $\mathbb{E}(Z_1\mid Z)=\sum_{k\in Z_1}k\cdot\mathbb{P}(Z_1=k\mid Z=x)=\sum_{k\in Z_1}k\cdot\frac{x!\cdot\lambda_1^k\cdot\lambda_2^{(x-k)}}{k!(x-k)!\cdot(\lambda_1+\lambda_2)^x}=\sum_{k\in Z_1}\frac{x!\cdot\lambda_1^k\cdot\lambda_2^{(x-k)}}{(k-1)!(x-k)!\cdot(\lambda_1+\lambda_2)^x}$

4) Calcolo prima $\mathbb{P}(Z=k\mid Z_1=x)=\frac{\mathbb{P}(Z=k\cap Z_1=x)}{\mathbb{P}(Z_1=x)}=\frac{\mathbb{P}(x+Z_2=k\cap Z_1=x)}{\mathbb{P}(Z_1=x)}=\frac{\mathbb{P}(Z_2=k-x)\cdot\mathbb{P}(Z_1=x)}{\mathbb{P}(Z_1=x)}=\frac{\lambda_2^{(k-x)}}{(k-x)!}\cdot e^{-\lambda_2}$

$\mathbb{E}(Z\mid Z_1)\ \sum_{k\in Z}k\cdot\mathbb{P}(Z=k\mid Z_1=x)=\sum_{k\in Z}k\cdot\frac{\lambda_2^{(k-x)}}{(k-x)!}\cdot e^{-\lambda_2}$