Esercitazione del 13/12/2023

1)

$$C=\begin{bmatrix} 9+4\cdot\sqrt{2}+2\pi & 14 \\ -\sqrt{2}-\pi+1 & 2 \\ \sqrt{6}+2\pi-1 & 2 \\ -7\pi+1 & -12 \\ 3 & 6 \end{bmatrix}$$

2)

$$A\cdot B=\begin{vmatrix} 1 & -1 & 0 & \sqrt{2} & 1 \end{vmatrix}\cdot\begin{vmatrix} 0 \\ 2 \\ -1 \\ 1 \\ 0 \end{vmatrix}=[\sqrt{2}-2] \qquad B\cdot A=\begin{vmatrix} 0 \\ 2 \\ -1 \\ 1 \\ 0 \end{vmatrix}\cdot\begin{vmatrix} 1 & -1 & 0 & \sqrt{2} & 1 \end{vmatrix}=\begin{vmatrix} 0 & 0 & 0 & 0 & 0 \\ 2 & -2 & 0 & 2\sqrt{2} & 2 \\ -1 & 1 & 0 & -\sqrt{2} & -1 \\ 1 & -1 & 0 & \sqrt{2} & 1 \\ 0 & 0 & 0 & 0 & 0 \end{vmatrix}$$

3) Controllo quali siano le colonne lin. indipendenti della matrice:

$$\begin{vmatrix} 1 & -1 & -1 \\ -1 & 2 & 3 \\ 3 & -6 & -9 \\ 2 & -4 & -6 \end{vmatrix} \xrightarrow{A_4+2A_2} \begin{vmatrix} 1 & -1 & -1 \\ -1 & 2 & 3 \\ 3 & -6 & -9 \\ 0 & 0 & 0 \end{vmatrix} \xrightarrow[A_3-3A_1]{A_2+A_1} \begin{vmatrix} 1 & -1 & -1 \\ 0 & 1 & 2 \\ 0 & 3 & 6 \\ 0 & 0 & 0 \end{vmatrix} \xrightarrow{A_3+3A_2} \begin{vmatrix} 1 & -1 & -1 \\ 0 & 1 & 2 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{vmatrix} \Rightarrow \text{base di } W: \left\{ \begin{vmatrix} 1 \\ -1 \\ 3 \\ 2 \end{vmatrix}, \begin{vmatrix} -1 \\ 2 \\ -6 \\ -4 \end{vmatrix} \right\}$$

$$\begin{vmatrix} 1 & -1 & -1 & 1 & 0 & 0 & 0 \\ -1 & 2 & 3 & 0 & 1 & 0 & 0 \\ 3 & -6 & -9 & 0 & 0 & 1 & 0 \\ 2 & -4 & -6 & 0 & 0 & 0 & 1 \end{vmatrix} \xrightarrow{A_4+2A_2} \begin{vmatrix} 1 & -1 & -1 & 1 & 0 & 0 & 0 \\ -1 & 2 & 3 & 0 & 1 & 0 & 0 \\ 3 & -6 & -9 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 2 & 0 & 1 \end{vmatrix} \xrightarrow{A_3+3A_2} \begin{vmatrix} 1 & -1 & -1 & 1 & 0 & 0 & 0 \\ -1 & 2 & 3 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 3 & 1 & 0 \\ 0 & 0 & 0 & 0 & 2 & 0 & 1 \end{vmatrix} \xrightarrow{A_2+A_1} \begin{vmatrix} 1 & -1 & -1 & 1 & 0 & 0 & 0 \\ 0 & 1 & 2 & 1 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 3 & 1 & 0 \\ 0 & 0 & 0 & 0 & 2 & 0 & 1 \end{vmatrix}$$

$$\begin{vmatrix} 1 & -1 & -1 & 1 & 0 & 0 & 0 \\ 0 & 1 & 2 & 1 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 3 & 1 & 0 \\ 0 & 0 & 0 & 0 & 2 & 0 & 1 \end{vmatrix} \xrightarrow{A_4-\frac{2}{3}A_3} \begin{vmatrix} 1 & -1 & -1 & 1 & 0 & 0 & 0 \\ 0 & 1 & 2 & 1 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 3 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & -2/3 & 1 \end{vmatrix} \Rightarrow \text{un completamento e' } \left\{ \begin{vmatrix} 0 \\ 1 \\ 0 \\ 0 \end{vmatrix}, \begin{vmatrix} 0 \\ 0 \\ 1 \\ 0 \end{vmatrix} \right\}$$

4) A e' invertibile se $\begin{vmatrix} 3 \\ 0 \\ 1 \end{vmatrix}$ $\begin{vmatrix} 1 \\ -1 \\ 1 \end{vmatrix}$ e $\begin{vmatrix} 0 \\ 2 \\ 1 \end{vmatrix}$ sono lin. indipedenti, verifico:

$$A=\begin{vmatrix} 3 & 1 & 0 \\ 0 & -1 & 2 \\ 1 & 1 & 1 \end{vmatrix} \xrightarrow{A_3-\frac{1}{3}A_1} \begin{vmatrix} 3 & 1 & 0 \\ 0 & -1 & 2 \\ 0 & 2/3 & 1 \end{vmatrix} \xrightarrow{A_3+\frac{2}{3}A_2} \begin{vmatrix} 3 & 1 & 0 \\ 0 & -1 & 2 \\ 0 & 0 & 7/3 \end{vmatrix} \Rightarrow A \text{ e' invertibile.}$$

considero:

$$\begin{vmatrix} 3 & 1 & 0 & 1 & 0 & 0 \\ 0 & -1 & 2 & 0 & 1 & 0 \\ 1 & 1 & 1 & 0 & 0 & 1 \end{vmatrix} \equiv \begin{vmatrix} 1 & 1 & 1 & 0 & 0 & 1 \\ 3 & 1 & 0 & 1 & 0 & 0 \\ 0 & -1 & 2 & 0 & 1 & 0 \end{vmatrix} \xrightarrow{A_2-3A_1} \begin{vmatrix} 1 & 1 & 1 & 0 & 0 & 1 \\ 0 & -2 & -3 & 1 & 0 & -3 \\ 0 & -1 & 2 & 0 & 1 & 0 \end{vmatrix} \xrightarrow{A_3-\frac{1}{2}A_2} \begin{vmatrix} 1 & 1 & 1 & 0 & 0 & 1 \\ 0 & -2 & -3 & 1 & 0 & -3 \\ 0 & 0 & 7/2 & -1/2 & 1 & 3/2 \end{vmatrix} \xrightarrow{A_3=2A_3} \begin{vmatrix} 1 & 1 & 1 & 0 & 0 & 1 \\ 0 & -2 & -3 & 1 & 0 & -3 \\ 0 & 0 & 7 & -1 & 2 & 3 \end{vmatrix}$$

$$\begin{vmatrix} 1 & 1 & 1 & 0 & 0 & 1 \\ 0 & -2 & -3 & 1 & 0 & -3 \\ 0 & 0 & 7 & -1 & 2 & 3 \end{vmatrix} \xrightarrow{A_2+\frac{3}{7}A_3} \begin{vmatrix} 1 & 1 & 1 & 0 & 0 & 1 \\ 0 & -2 & 0 & 4/7 & 6/7 & -12/7 \\ 0 & 0 & 7 & -1 & 2 & 3 \end{vmatrix} \xrightarrow{A_1-\frac{1}{7}A_3} \begin{vmatrix} 1 & 1 & 0 & 1/7 & -2/7 & 4/7 \\ 0 & -2 & 0 & 4/7 & 6/7 & -12/7 \\ 0 & 0 & 7 & -1 & 2 & 3 \end{vmatrix} \xrightarrow{A_1+\frac{1}{2}A_2} \begin{vmatrix} 1 & 0 & 0 & 3/7 & 1/7 & -2/7 \\ 0 & -2 & 0 & 4/7 & 6/7 & -12/7 \\ 0 & 0 & 7 & -1 & 2 & 3 \end{vmatrix} \xrightarrow[A_3\cdot\frac{1}{7}]{A_2\cdot-\frac{1}{2}} \begin{vmatrix} 1 & 0 & 0 & 3/7 & 1/7 & -2/7 \\ 0 & 1 & 0 & -\frac{4}{14} & -\frac{6}{14} & 6/7 \\ 0 & 0 & 1 & -1/7 & 2/7 & 3/7 \end{vmatrix}$$

La matrice inversa e':

$$\begin{vmatrix} 3/7 & 1/7 & -2/7 \\ -\frac{4}{14} & -\frac{6}{14} & 6/7 \\ -1/7 & 2/7 & 3/7 \end{vmatrix} = \frac{1}{7}\cdot\begin{vmatrix} 3 & 1 & -2 \\ -2 & -3 & 6 \\ -1 & 2 & 3 \end{vmatrix}$$