3

$$T(n) = 2T\left(\frac{n}{3}\right) + \Theta(n) \quad \text{con} \quad T(1) = \Theta(1)$$

metodo iterativo

$$T(n) = 2\left[2T\left(\frac{n}{3^2}\right) + \Theta\left(\frac{n}{3}\right)\right] + \Theta(n)$$

$$T(n) = 2\left[2\left[2T\left(\frac{n}{3^3}\right) + \Theta\left(\frac{n}{3^2}\right)\right] + \Theta\left(\frac{n}{3}\right)\right] + \Theta(n)$$

$$T(n) = 2^K T\left(\frac{n}{3^K}\right) + \sum_{i=0}^{K-1} 2^i \cdot \Theta\left(\frac{n}{3^i}\right)$$

$$T(n) = 2^K T\left(\frac{n}{3^K}\right) + \Theta(n) \sum_{i=0} \left(\frac{2}{3}\right)^i$$

$$T(n) = 2^K T\left(\frac{n}{3^K}\right) + \Theta(n)\left[\frac{\left(\frac{2}{3}\right)^K - 1}{\left(\frac{1}{3}\right)}\right]$$

Sia quindi $K = \log_3(n)$

$$T(n) = \Theta\left(2^{\log_3(n)}\right) + \Theta(n)\left[-3 \cdot \left[\left(\frac{2}{3}\right)^{\log_3(n)} - 1\right]\right]$$

QUESTO TERMINE TENDE A 0

$$T(n) = \Theta\left(2^{\log_3(n)}\right) + \Theta(n)$$

$$T(n) = \Theta(n$$

Metodo principale

$f(n) = \Theta(n)$ $\quad \log_3(2) \approx 0,6$ $\quad \Theta(n^1) = \Omega(n^{0,6+\varepsilon})$

$n^{\log_b a} = n^{\log_3(2)}$ $\quad f(n) = \Theta(n^1)$ $\quad$ QUINDI RIENTRA NEL TERZO CASO $\quad T(n) = f(n)$

$T(n) = \Theta(n)$

Metodo di sostituzione $T(n) = 2T(\frac{n}{3}) + c$

IPOTIZZO CHE $T(n) = O(n)$ $T(1) = d$

$T(n) \leq Kn \Rightarrow$ CASO BASE $d \leq K$

IPOTESI INDUTTIVA

$\forall m < n, \; T(m) \leq Km$

PASSO INDUTTIVO

$T(n) \leq K \cdot n$ ma $T(n) = 2T(\frac{n}{3}) + c$

$2T(\frac{n}{3}) + c \leq Kn$ ma essendo vera l'ipotesi mediamo che $\frac{n}{3} < n$ QUINDI

$T(\frac{n}{3}) \leq K\frac{n}{3}$ allora riscrivo

$2\left[K\frac{n}{3}\right] + c \leq Kn \Rightarrow \frac{2}{3}Kn + c \leq Kn$

$c \leq Kn - \frac{2}{3}Kn \Rightarrow c \leq \frac{Kn}{3}$ ✓

analogamente per $T(n) = \Omega(n)$

$d \geq K$ e $c \geq \frac{Kn}{3}$ ✓

quindi $T(n) = \Theta(n)$