Esercitazione del 6/12/2023

1) $A=\begin{bmatrix}1&1&2&0&1\\1&-1&0&1&1\\1&0&-1&0&-1\\1&2&5&0&3\end{bmatrix}\xrightarrow{\substack{A_2-A_1\\A_3-A_1\\A_4-A_1}}\begin{bmatrix}1&1&2&0&1\\0&-2&-2&1&0\\0&-1&-3&0&-2\\0&1&3&0&2\end{bmatrix}\xrightarrow{A_2\leftrightarrow A_4}\begin{bmatrix}1&1&2&0&1\\0&1&3&0&2\\0&-1&-3&0&-2\\0&-2&-2&1&0\end{bmatrix}\xrightarrow{\substack{A_3+A_2\\A_4+2A_2}}\begin{bmatrix}1&1&2&0&1\\0&1&3&0&2\\0&0&0&0&0\\0&0&4&1&4\end{bmatrix}\xrightarrow{A_3\leftrightarrow A_4}\begin{bmatrix}1&1&2&0&1\\0&1&3&0&2\\0&0&4&1&4\\0&0&0&0&0\end{bmatrix}=S$

Una base per $Im(S)$ e' $\begin{bmatrix}1\\0\\0\\0\end{bmatrix},\begin{bmatrix}1\\1\\0\\0\end{bmatrix},\begin{bmatrix}2\\3\\4\\0\end{bmatrix}$. considero $\begin{cases}x_1+x_2+2x_3+x_5=0\\x_2+3x_3+2x_5=0\\4x_3+x_4+4x_5=0\\x_4=t_4\\x_5=t_5\end{cases}\Rightarrow\begin{cases}x_1=-x_2-2(-\frac{1}{4}t_4-t_5)-t_5\\x_2=-2t_5-3(-\frac{1}{4}t_4-t_5)\\x_3=-\frac{1}{4}t_4-t_5\\x_4=t_4\\x_5=t_5\end{cases}$ errore! :($\Rightarrow\begin{cases}x_1=t_5-\frac14t_4+\frac12t_4+t_5\\x_2=\frac14t_4-t_5\\x_3=-\frac14t_4-t_5\\x_4=t_4\\x_5=t_5\end{cases}$

$\begin{cases}x_1=2t_5+\frac12t_4\\x_2=\frac14t_4-t_5\\x_3=-\frac14t_4-t_5\\x_4=t_4\\x_5=t_5\end{cases}\Rightarrow\Sigma_0=Span\left(\begin{bmatrix}1/2\\1/4\\-1/4\\1\\0\end{bmatrix},\begin{bmatrix}2\\-1\\-1\\0\\1\end{bmatrix}\right)$. Una base per $Im(A)$ e' $\begin{bmatrix}1\\1\\1\\1\end{bmatrix}\begin{bmatrix}1\\-1\\0\\2\end{bmatrix}\begin{bmatrix}2\\0\\-1\\5\end{bmatrix}$.

2) ho $\left[\begin{array}{ccccc|c}1&1&2&0&1&1\\1&-1&0&1&1&2\\1&0&-1&0&-1&1\\1&2&5&0&3&1\end{array}\right]\xrightarrow{\substack{A_2-A_1\\A_3-A_1\\A_4-A_1}}\left[\begin{array}{ccccc|c}1&1&2&0&1&1\\0&-2&-2&1&0&1\\0&-1&-3&0&-2&0\\0&1&3&0&2&0\end{array}\right]\xrightarrow{\substack{A_3+A_4\\A_2+2A_4}}\left[\begin{array}{ccccc|c}1&1&2&0&1&1\\0&0&4&1&4&1\\0&0&0&0&0&0\\0&1&3&0&2&0\end{array}\right]=\left[\begin{array}{ccccc|c}1&1&2&0&1&1\\0&1&3&0&2&0\\0&0&4&1&4&1\\0&0&0&0&0&0\end{array}\right]$

Il sistema e' compatibile: $rg\begin{bmatrix}1&1&2&0&1\\0&1&3&0&2\\0&0&4&1&4\\0&0&0&0&0\end{bmatrix}=rg\begin{bmatrix}1&1&2&0&1&1\\0&1&3&0&2&0\\0&0&4&1&4&1\\0&0&0&0&0&0\end{bmatrix}$. considero $\begin{cases}x_1+x_2+2x_3+x_5=1\\x_2+3x_3+2x_5=0\\4x_3+x_4+4x_5=1\\x_4=t_4\\x_5=t_5\end{cases}\Rightarrow\begin{cases}x_1=1-t_5-x_2+\frac12t_4+2t_5-2\\x_2=-2t_5+\frac34t_4+3t_5-3\\x_3=-\frac14t_4-t_5+1\\x_4=t_4\\x_5=t_5\end{cases}$

$\Rightarrow\begin{cases}x_1=1-t_5-t_5-\frac34t_4+3+\frac12t_4+2t_5-2\\x_2=t_5+\frac34t_4-3\\x_3=-\frac14t_4-t_5+1\\x_4=t_4\\x_5=t_5\end{cases}\Rightarrow\begin{cases}x_1=2-\frac14t_4\\x_2=t_5+\frac34t_4-3\\x_3=-\frac14t_4-t_5+1\\x_4=t_4\\x_5=t_5\end{cases}\Rightarrow\Sigma=t_4\begin{vmatrix}-1/4\\3/4\\-1/4\\1\\0\end{vmatrix}+t_5\begin{vmatrix}0\\1\\-1\\0\\1\end{vmatrix}+\begin{vmatrix}2\\-3\\1\\0\\0\end{vmatrix}$

3) $L_A\left(\begin{bmatrix}x_1\\x_2\\x_3\\x_4\end{bmatrix}\right)=\begin{bmatrix}2x_1+x_2+3x_3-x_4\\x_1+x_2+x_3-2x_4\\-x_1+x_2-3x_3-4x_4\end{bmatrix}$ Gauss $\rightarrow\begin{vmatrix}2&1&3&-1\\1&1&1&-2\\-1&1&-3&-4\end{vmatrix}\xrightarrow{A_1\leftrightarrow A_3}\begin{vmatrix}-1&1&-3&-4\\1&1&1&-2\\2&1&3&-1\end{vmatrix}\xrightarrow{\substack{A_2+A_1\\A_3+2A_1}}\begin{vmatrix}-1&1&-3&-4\\0&2&-2&-6\\0&3&-3&-9\end{vmatrix}\xrightarrow{\substack{A_3-\frac32A_2\\A_2=A_2/2}}\begin{vmatrix}-1&1&-3&-4\\0&1&-1&-3\\0&0&0&0\end{vmatrix}$ PIVOT

base di $Im(L_A)=\begin{vmatrix}2\\1\\-1\end{vmatrix},\begin{vmatrix}1\\1\\1\end{vmatrix}\Rightarrow rg(L_A)=2$, considero $\begin{cases}-x_1+x_2-3x_3-4x_4=0\\x_2-x_3-3x_4=0\\x_3=t_3\\x_4=t_4\end{cases}\Rightarrow\begin{cases}x_1=t_3+3t_4-3t_3-4t_4\\x_2=t_3+3t_4\\x_3=t_3\\x_4=t_4\end{cases}\Rightarrow\begin{cases}x_1=-2t_3-t_4\\x_2=t_3+3t_4\\x_3=t_3\\x_4=t_4\end{cases}$

base di $Ker L_A=\begin{vmatrix}-2\\1\\1\\0\end{vmatrix},\begin{vmatrix}-1\\3\\0\\1\end{vmatrix}$. L_A non e' iniettiva $dim(Ker L_A)\neq0$ e non e' suriettiva $rg(L_A)=2\neq dim(\mathbb{R}^4)=4$

4)
- NO, controesempio: $\begin{vmatrix}1\\0\\1\end{vmatrix}+\begin{vmatrix}0\\1\\1\end{vmatrix}=\begin{vmatrix}1\\1\\2\end{vmatrix}\notin W$.
- NO non c'e' il vettore nullo.
- $(x_1,x_2,x_3,x_4)+(y_1,y_2,y_3,y_4)=((x_1+y_1),(x_2+y_2),(x_3+y_3),(x_4+y_4))\Rightarrow x_1+y_1+x_2+y_2-x_3-y_3+x_4+y_4=0\Leftrightarrow\underbrace{(x_1+x_2-x_3+x_4)}_{0}+\underbrace{(y_1+y_2-y_3+y_4)}_{0}=0$ ✓ SI (IPOTESI)
- $(x_1,x_2,x_3,x_4)+(y_1,y_2,y_3,y_4)=((x_1+y_1),(x_2+y_2),(x_3+y_3),(x_4+y_4))\Rightarrow x_1+y_1+x_2+y_2-x_3-y_3+x_4+y_4=1\Leftrightarrow\underbrace{(x_1+x_2-x_3+x_4)}_{1}+\underbrace{(y_1+y_2-y_3+y_4)}_{1}=1\Leftrightarrow 2=1$ NO
- $(x_1,x_2,x_3,x_4)+(y_1,y_2,y_3,y_4)=((x_1+y_1),(x_2+y_2),(x_3+y_3),(x_4+y_4))\Rightarrow(x_1+y_1)^2+(x_2+y_2)-(x_3+y_3)+(x_4+y_4)^2=x_1^2+2x_1y_1+y_1^2+(x_2+y_2)-(x_3+y_3)+x_4^2+2x_4y_4+y_4^2$

$=x_1^2+2x_1y_1+y_1^2+(x_2+y_2)-(x_3+y_3)+x_4^2+2x_4y_4+y_4^2=2(x_1y_1+x_4y_4)+x_1^2+y_1^2+x_4^2+y_4^2+(x_2+y_2)-(x_3+y_3)=0\Leftrightarrow2(x_1y_1+x_4y_4)+x_1^2+y_1^2+x_4^2+y_4^2=(x_1+y_1)+(x_4+y_4)$

$\Rightarrow$ NO
- $(x_1,0,x_3,x_4)+(y_1,0,y_3,y_4)\in W\wedge\lambda\cdot(x_1,0,x_3,x_4)\in W\Rightarrow$ SI
- NO non c'e' il vettore nullo.