- Risolvere con il metodo iterativo e con il metodo principale:
  $T(n) = 3T(n/2) + \Theta(n)$ e $T(1) = \Theta(1)$

metodo iterativo

$$T(n) = 3^k T\left(\frac{n}{2^k}\right) + \sum_{i=0}^{k-1} 3^i \Theta\left(\frac{n}{2^i}\right)$$

fino a $k = \log_2(n)$

$$T(n) = \Theta\left(3^{\log_2(n)}\right) + \Theta(n) \sum \left(\frac{3}{2}\right)^i$$

$$T(n) = \Theta\left(n^{\log_2(3)}\right) + \Theta(n) \left[2\left(\frac{3}{2}\right)^{\log(n)} - 2\right]$$

$$T(n) = \Theta\left(n^{\log_2(3)}\right) + \Theta(n) \left[2\,\frac{3^{\log_2(n)}}{n} - 2\right]$$

$$T(n) = \Theta\left(n^{\log_2(3)}\right) + \Theta\left(2 \cdot 3^{\log_2(n)} - n\right) = \Theta\left(n^{\log_2 3}\right)$$

metodo principale

$$n^{\log_b(a)} = n^{\log_2(3)}$$

$$f(n) = \Theta(n)$$

$$f(n) = O\left(n^{\log_2 3 - \varepsilon}\right) \Rightarrow T(n) = \Theta\left(n^{\log_2(3)}\right)$$

T(n) = 2T(n/3) + Θ(n) e T(1) = Θ(1)

$$T(n) = 2^k T\left(\frac{n}{3^k}\right) + \sum^{k-1} 2^i \Theta\left(\frac{n}{3^i}\right) \quad \text{fino a} \quad k = \log_3(n)$$

$$T(n) = \Theta(1)\, 2^{\log_{(3)}(n)} + \Theta(n) \underbrace{\sum \left(\frac{2}{3}\right)^i}_{\Theta(1)}$$

$$T(n) = \underbrace{\Theta\left(n^{\log_3(2)}\right)}_{\Theta(n^\alpha) \text{ DOVE } \alpha < 1} + \Theta(n) = \Theta(n)$$

metodo principale

$$n^{\log_b a} = n^{\log_3 2}$$

$$f(n) = n$$

$$f(n) = \Omega\left(n^{\log_3 2 + \varepsilon}\right) \quad e$$

$$f\left(\frac{n}{3}\right) = \Theta\left(\frac{n}{3}\right) \leq c \cdot f(n)? \quad \text{SI}, \; c = \frac{1}{2}$$

QUINDI

$$T(n) = f(n) \quad \text{OSSIA} \quad T(n) = \Theta(n)$$

- Data l'equazione:
  $T(n)=2T(n/2)+\Theta(n \log n)$
  $T(1)=\Theta(1)$
  Dimostrare per induzione che $T(n)=\Omega(n\ log^2 n)$.

VOGLIO DIMOSTRARE CHE

$$T(n) \geq K n \log^2(n)$$

RISCRIVO
$$\begin{cases} T(n) = 2T\left(\frac{n}{2}\right) + cn \\ T(1) = d \end{cases}$$

CASO BASE

$$T(1) \geq K \cdot 1 \cdot \log(1)$$

$$d \geq 0 \checkmark$$

IP. INDUTTIVA

$$\forall m < n \rightarrow T(m) \geq K m \log^2(m)$$

P. INDUTTIVO

$$\underbrace{2T\left(\frac{n}{2}\right)}_{\text{i. IND.}} + cn \geq K n \log^2(n)$$

$$2\left[K \frac{n}{2} \log^2\left(\frac{n}{2}\right)\right] + cn \geq K n \log^2(n)$$

$$K n \left[\log(n)^2 - 1\right] + cn \geq K n \log^2(n)$$

$$K n \log^2(n) - Kn + cn \geq K n \log^2(n)$$

$$cn \geq Kn$$

$$c \geq K \checkmark$$

DIMOSTRATO

- Calcolare l' equazione di ricorrenza associata al seguente algoritmo e risolverla con tutti i metodi possibili:

```
def Test (n)
  k = 0
  for i in range(1, n):
     k = k + 1
  if n ≤ 1: return k
     else: return (Test(n DIV 2)+Test(n DIV 4))
```

$\Theta(1)$

$\Theta(n)$

$T(\frac{n}{2}) + T(\frac{n}{4})$

$$\begin{cases} T(n) = T\left(\frac{n}{2}\right) + T\left(\frac{n}{4}\right) + \Theta(n) \\ T(1) = \Theta(1) \end{cases}$$

SO CHE $T(n) \ge 2T\left(\frac{n}{4}\right) + \Theta(n)$ E $T(n) \le 2T\left(\frac{n}{2}\right) + \Theta(n)$

metodo iterativo

$$T(n) \ge 2^k T\left(\frac{n}{4^k}\right) + \sum 2^i \Theta\left(\frac{n}{4^i}\right)$$

$$k = \log_4(n)$$

$$T(n) \ge 2^{\log_4 n}\Theta(1) + \Theta(n)\sum\left(\frac{1}{2}\right)^i$$

$$T(n) \ge n^{\log_4 2}\Theta(1) + \Theta(n) = \boxed{\Theta(n)}$$

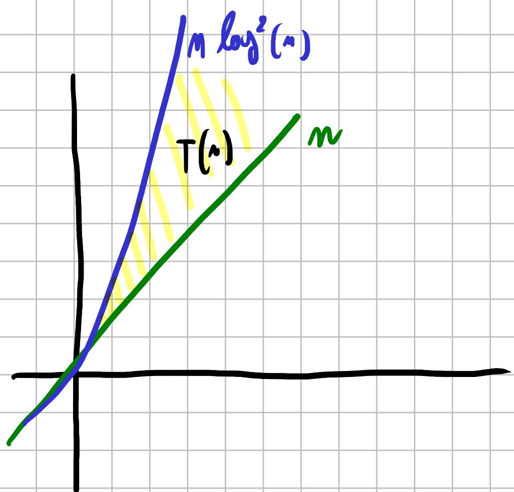
$$T(n) \le 2T\left(\frac{n}{2}\right) + \Theta(n)$$

$$T(n) \le 2^k T\left(\frac{n}{2^k}\right) + \sum 2^i \Theta\left(\frac{n}{2^i}\right) \quad k = \log n$$

$$T(n) \le \Theta(n) + \Theta(n)\sum i$$

$$T(n) \le \Theta(n) + \Theta(n)\left[\frac{\log^2 n - \log(n)}{2}\right]$$

$$T(n) \le \boxed{\Theta(n \log^2(n))}$$



$$\Theta(n) \le T(n) \le \Theta(n \log(n))$$

metodo di sostituzione

VOGLIO DIMOSTRARE CHE $T(n) = \Omega(n \log(n))$

$$T(n) \geq K n \log(n) \qquad \begin{cases} T(n) = T(\frac{n}{2}) + T(\frac{n}{4}) + Cn \\ T(1) = d \end{cases}$$

C. BASE $T(1) \geq k \cdot 1 \cdot \log^2(1) \Rightarrow d \geq 0$ ✓

I. IND. $\forall m < n \rightarrow T(m) \geq K m \log^2(m)$

P. IND.

$$\underbrace{T(\tfrac{n}{2}) + T(\tfrac{n}{4})}_{\text{i. IND.}} + Cn \geq K n \log^2(n)$$

$$\left[K \frac{\cancel{n}}{2} \log^2(\frac{n}{2})\right] + \left[K \frac{\cancel{n}}{4} \log^2(\frac{n}{4})\right] + C\cancel{n} \geq K \cancel{n} \log^2(n)$$

$$\frac{K}{2} \log(n) - \frac{K}{2} + \frac{K}{4} \log(n) - 2 + Cn \geq K \log^2(n)$$

$$\frac{K}{4} \log^2(n) - \frac{K}{2} - 4 + Cn \geq \frac{K}{2} \log^2(n)$$

$$Cn - 4 - \frac{K}{2} \geq \frac{K}{4} \log^2(n)$$

$$Cn \geq \frac{K}{4} \log(n) + 4 + \frac{K}{2}$$

$$Cn \geq \frac{K}{2}\left[1 + \frac{K}{2} \log(n)\right] + 4$$

$T(n) = \Omega(n \log(n))$?

$$\begin{cases} T(n) = T\left(\frac{n}{2}\right) + T\left(\frac{n}{4}\right) + cn \\ T(1) = d \end{cases}$$

CASO B.

$$d \geq K \cdot 1 \cdot \log(1) \rightarrow d \geq K \cdot 0 \rightarrow d \geq 0 \checkmark$$

I. IND.

$$\forall m < n \rightarrow T(m) \geq K m \log(m)$$

P. IND.

$$T\left(\frac{n}{2}\right) + T\left(\frac{n}{4}\right) + cn \geq K n \log(n)$$

I. IND.

$$\left[K \frac{\cancel{n}}{2} \log\left(\frac{n}{2}\right)\right] + \left[K \frac{\cancel{n}}{4} \log\left(\frac{n}{4}\right)\right] + c\cancel{n} \geq K \cancel{n} \log(n)$$

$$\frac{K}{2}\left[\log(n) - 1\right] + \frac{K}{4}\left[\log(n) - 2\right] + c \geq K \log(n)$$

$$\frac{K}{2}\log(n) - \frac{K}{2} + \frac{K}{4}\log(n) - \frac{K}{4} + c \geq K \log(n)$$

SO CHE $\quad Kx - \frac{K}{2}x - \frac{K}{4}x = \frac{K}{4}x$

$$c - \frac{K}{2} - \frac{K}{4} \geq \frac{K}{4}\log(n)$$

DIVIDO PER $K/4$

$$\frac{4c}{K} - 3 \geq \log(n) \Rightarrow \frac{4c}{K} \geq \log(n) + 3 \checkmark$$