Esercitazione del 20/12/2023

1) (1.1) considero $V_\lambda = Ker \begin{vmatrix} -\lambda & -2 & -1 \\ 0 & 1-\lambda & 0 \\ -1 & -2 & -\lambda \end{vmatrix}$ e trovo: $\det \begin{vmatrix} -\lambda & -2 & -1 \\ 0 & 1-\lambda & 0 \\ -1 & -2 & -\lambda \end{vmatrix} = (-1)^4 \cdot (1-\lambda) \cdot \det \begin{vmatrix} -\lambda & -1 \\ -1 & -\lambda \end{vmatrix} = (1-\lambda)(\lambda^2-1) = (1-\lambda)(\lambda-1)(\lambda+1)$

gli autovalori sono 1 (con molt. 2) e -1 (con molt. 1).

(1.2) consider $V_1 = Ker(A-id) = Ker \begin{vmatrix} -1 & -2 & -1 \\ 0 & 0 & 0 \\ -1 & -2 & -1 \end{vmatrix} = Ker \begin{vmatrix} -1 & -2 & -1 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{vmatrix} \Rightarrow \dim(V_1) = 2$ e $V_1 = \{ \tilde{x} \in \mathbb{R}^3 \mid x_1 = -2x_2 - x_3 \}$

considero $V_1 = Ker(A+id) = Ker \begin{vmatrix} 1 & -2 & -1 \\ 0 & 2 & 0 \\ -1 & -2 & 1 \end{vmatrix} = Ker \begin{vmatrix} 1 & -2 & -1 \\ 0 & 2 & 0 \\ 0 & 0 & 0 \end{vmatrix} \Rightarrow \dim(V_1) = 1$ e $V_1 = \left\{ \tilde{x} \in \mathbb{R}^3 \mid \begin{cases} x_1 = x_3 \\ x_2 = 0 \end{cases} \right\}$

ogni autovalore ha molteplicità algebrica identica a quella geometrica.

(1.3) $V_1 = Ker \begin{vmatrix} -1 & -2 & -1 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{vmatrix} \Rightarrow \begin{cases} x_1 = -2t_2 - t_3 \\ x_2 = t_2 \\ x_3 = t_3 \end{cases} \Rightarrow V_1 = Span\left(\begin{vmatrix} -2 \\ 1 \\ 0 \end{vmatrix}, \begin{vmatrix} -1 \\ 0 \\ 1 \end{vmatrix} \right).$

(1.4) base di $\mathbb{R}^3$: $\mathcal{B} = \left\{ \begin{vmatrix} -2 \\ 1 \\ 0 \end{vmatrix}, \begin{vmatrix} -1 \\ 0 \\ 1 \end{vmatrix}, \begin{vmatrix} 1 \\ 0 \\ 1 \end{vmatrix} \right\}$ non e' unica.

$V_{-1} = Ker \begin{vmatrix} 1 & -2 & -1 \\ 0 & 2 & 0 \\ 0 & 0 & 0 \end{vmatrix} = \begin{cases} x_1 = 2x_2 + x_3 \\ 2x_2 = 0 \\ x_3 = t \end{cases} \Rightarrow \begin{cases} x_1 = t \\ x_2 = 0 \\ x_3 = t \end{cases} \Rightarrow V_{-1} = Span\left(\begin{vmatrix} 1 \\ 0 \\ 1 \end{vmatrix} \right)$

(1.5) $L_A \begin{vmatrix} -2 \\ 1 \\ 0 \end{vmatrix} = \begin{vmatrix} -2 \\ 1 \\ 0 \end{vmatrix} = 1 \cdot \begin{vmatrix} -2 \\ 1 \\ 0 \end{vmatrix} + 0 \cdot \begin{vmatrix} -1 \\ 0 \\ 1 \end{vmatrix} + 0 \cdot \begin{vmatrix} 1 \\ 0 \\ 1 \end{vmatrix}$ $\quad L_A \begin{vmatrix} -1 \\ 0 \\ 1 \end{vmatrix} = \begin{vmatrix} -1 \\ 0 \\ 1 \end{vmatrix} = 0 \cdot \begin{vmatrix} -2 \\ 1 \\ 0 \end{vmatrix} + 1 \cdot \begin{vmatrix} -1 \\ 0 \\ 1 \end{vmatrix} + 0 \cdot \begin{vmatrix} 1 \\ 0 \\ 1 \end{vmatrix}$ $\quad L_A \begin{vmatrix} 1 \\ 0 \\ 1 \end{vmatrix} = \begin{vmatrix} -1 \\ 0 \\ -1 \end{vmatrix} = 0 \cdot \begin{vmatrix} -2 \\ 1 \\ 0 \end{vmatrix} + 0 \cdot \begin{vmatrix} -1 \\ 0 \\ 1 \end{vmatrix} + (-1) \cdot \begin{vmatrix} 1 \\ 0 \\ 1 \end{vmatrix}$

$M_{\mathcal{B}\mathcal{B}}(L_A) = \begin{vmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & -1 \end{vmatrix}$ (1.6) So che $M_{\mathcal{B}\mathcal{B}}(L_A)$ e' SIMILE ad $A = M_{\mathcal{E}\mathcal{E}}(L_A)$ dove $\mathcal{E} = \left\{ \begin{vmatrix} 1 \\ 0 \\ 0 \end{vmatrix}, \begin{vmatrix} 0 \\ 1 \\ 0 \end{vmatrix}, \begin{vmatrix} 0 \\ 0 \\ 1 \end{vmatrix} \right\}$.

e so che $M_{\mathcal{B}\mathcal{B}}(L_A) = (M_{\mathcal{E}\mathcal{B}}(L_A))^{-1} \cdot A \cdot M_{\mathcal{E}\mathcal{B}}(L_A)$, trovo $M_{\mathcal{E}\mathcal{B}}$:

$L_A \begin{vmatrix} -2 \\ 1 \\ 0 \end{vmatrix} = \begin{vmatrix} -2 \\ 1 \\ 0 \end{vmatrix} = -2 \cdot \begin{vmatrix} 1 \\ 0 \\ 0 \end{vmatrix} + 1 \cdot \begin{vmatrix} 0 \\ 1 \\ 0 \end{vmatrix} + 0 \cdot \begin{vmatrix} 0 \\ 0 \\ 1 \end{vmatrix}$ $\quad L_A \begin{vmatrix} -1 \\ 0 \\ 1 \end{vmatrix} = \begin{vmatrix} -1 \\ 0 \\ 1 \end{vmatrix} = (-1) \cdot \begin{vmatrix} 1 \\ 0 \\ 0 \end{vmatrix} + 0 \cdot \begin{vmatrix} 0 \\ 1 \\ 0 \end{vmatrix} + 1 \cdot \begin{vmatrix} 0 \\ 0 \\ 1 \end{vmatrix}$ $\quad L_A \begin{vmatrix} 1 \\ 0 \\ 1 \end{vmatrix} = \begin{vmatrix} -1 \\ 0 \\ -1 \end{vmatrix} = (-1) \cdot \begin{vmatrix} 1 \\ 0 \\ 0 \end{vmatrix} + 0 \cdot \begin{vmatrix} 0 \\ 1 \\ 0 \end{vmatrix} + (-1) \cdot \begin{vmatrix} 0 \\ 0 \\ 1 \end{vmatrix}$

$M_{\mathcal{E}\mathcal{B}}(L_A) = \begin{vmatrix} -2 & -1 & -1 \\ 1 & 0 & 0 \\ 0 & 1 & -1 \end{vmatrix}$ (1.7) tale matrice non e' unica.

Verifica: $\begin{vmatrix} -2 & -1 & -1 \\ 1 & 0 & 0 \\ 0 & 1 & -1 \end{vmatrix}^{-1} \cdot A \cdot \begin{vmatrix} -2 & -1 & -1 \\ 1 & 0 & 0 \\ 0 & 1 & -1 \end{vmatrix} = \begin{vmatrix} 0 & 1 & 0 \\ -1/2 & -1 & 1/2 \\ -1/2 & -1 & -1/2 \end{vmatrix} \cdot \begin{vmatrix} 0 & -2 & -1 \\ 0 & 1 & 0 \\ -1 & -2 & 0 \end{vmatrix} \cdot \begin{vmatrix} -2 & -1 & -1 \\ 1 & 0 & 0 \\ 0 & 1 & -1 \end{vmatrix} = \begin{vmatrix} 0 & 1 & 0 \\ -1/2 & -1 & 1/2 \\ 1/2 & 1 & 1/2 \end{vmatrix} \cdot \begin{vmatrix} -2 & -1 & -1 \\ 1 & 0 & 0 \\ 0 & 1 & -1 \end{vmatrix} = \begin{vmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & -1 \end{vmatrix}$

2) (2.1) considero $\det \begin{vmatrix} i-\lambda & 2 & 1 \\ 0 & i-\lambda & 1 \\ 0 & 0 & i-\lambda \end{vmatrix} = (i-\lambda) \cdot \det \begin{vmatrix} i-\lambda & 2 \\ 0 & i-\lambda \end{vmatrix} = (i-\lambda)(i-\lambda)^2 = (i-\lambda)^3 \Rightarrow$ i e' l'unico autovalore, ha molteplicità 3.

(2.2) $V_i = Ker(A - i \cdot id) = Ker \begin{vmatrix} 0 & 2 & 1 \\ 0 & 0 & 1 \\ 0 & 0 & 0 \end{vmatrix} = \begin{cases} x_1 = t \\ 2x_2 + x_3 = 0 \\ x_3 = 0 \end{cases} \Rightarrow \begin{cases} x_1 = t \\ x_2 = 0 \\ x_3 = 0 \end{cases} \Rightarrow V_i = Span\left(\begin{vmatrix} 1 \\ 0 \\ 0 \end{vmatrix} \right)$

(2.3) La molteplicità algebrica dell'autovalore non e' identica alla sua molteplicità geometrica, quindi L_A NON e' diagonalizzabile.