Formulario

Cinematica

moto rettilineo uniforme

$$\begin{cases} x(t) = x_0 + v_0 t \\ v_0 \text{ costante} \end{cases}$$

moto uniformemente accelerato

$$\begin{cases} x(t) = x_0 + v_0 t + \frac{1}{2} a_0 t^2 \\ v(t) = v_0 + a_0 t \\ a_0 \text{ costante} \end{cases}$$

moto armonico

$x(t) = A\sin(\phi + \omega t)$

$v(t) = A\omega\cos(\phi + \omega t)$

$a(t) = -A\omega^2 \sin(\phi + \omega t) = -\omega^2 x(t)$

periodo $= T = \frac{2\pi}{\omega}$

Frequenza $: \nu = \frac{1}{T} = \frac{\omega}{2\pi}$

pulsazione $: \omega = 2\pi\nu$

moto circolare (R :: raggio)

$\omega(t)$ = velocità angolare

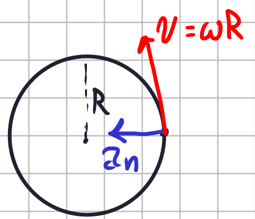
$v = \omega R$ velocità tangenziale

$a_t = \frac{dv}{dt}$ accelerazione tangenziale

$a_n = \omega^2 R$ accelerazione normale

$a = \sqrt{a_t^2 + a_n^2}$ acc. totale

$\bar{v} = \bar{\omega} \times \bar{R}$



moto circ. unifor.

$$\begin{cases} x(t) = R\cos(\theta(t)) \\ y(t) = R\sin(\theta(t)) \\ \theta = \omega t \end{cases}$$

Dinamica

$\bar{F} = m \cdot \bar{a}$

quantità di moto $\bar{p} = \frac{d}{dt}(m\bar{a}) = m\bar{v}$

impulso $I = \int_{t_0}^{t} \bar{F}\,dt$ e $I = \Delta\bar{p}$

Forza media $= \frac{I}{\Delta t} = \frac{\Delta\bar{p}}{\Delta t} = \frac{|\bar{p}(t_1) - \bar{p}(t_2)|}{t_2 - t_1}$

Attrito

Statico $\bar{F}_s \leq \mu_s R_n$ $\quad R_n$:= reazione normale

Dinamico $\bar{F}_d = -\mu_d \cdot R_n \cdot \hat{\tau}$

versore velocità

$\hat{\tau} = \frac{\bar{v}}{|\bar{v}|}$

attrito dell'aria : $\bar{F} = -b\bar{v}$

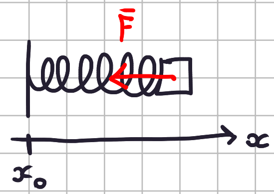
coefficiente

Forza Elastica

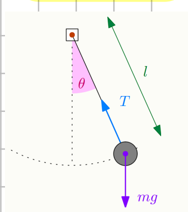
coeff   centro

$\bar{F} = -K(x - x_0)$

pos



Pendolo



$-mg\sin\theta = -m\frac{d^2 S}{dt^2}$

$-mg\sin\left(\frac{S}{l}\right) = -m\frac{d^2 S}{dt^2}$

per $\theta \to 0 \Rightarrow -mg\frac{S}{l} = -m\frac{d^2 S}{dt}$

$\Rightarrow S = A\cos(\omega t + \phi)$

$\omega = \sqrt{\frac{g}{l}}$

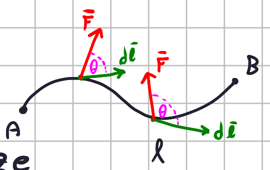
Gravi

$$\begin{cases} h: \text{quota lancio} \\ v_0: \text{vel iniziale} \end{cases}$$

si fermerà in $\Rightarrow t^* = \frac{1}{g} v_0$

quota max raggiunta: $h + \frac{1}{2g} v_0^2$

Lavoro



$\bar{F}: \mathbb{R}^3 \to \mathbb{R}^3$ campo di Forze

$$L = \int_{l} \bar{F}\, d\bar{l} = \int_A^B F\, dl \cos\theta = \Delta T$$

Impulso

$$I = \Delta \bar{p} = \Delta(m\bar{v})$$ variazione della quantita' di moto

Teorema dell'impulso

$$\frac{I}{\Delta t} = \frac{\Delta \bar{p}}{\Delta t} = \frac{1}{\Delta t}\int_{t_1}^{t_2} \bar{F}\, dt$$ Forza media

energia cinetica $T = \frac{1}{2} m v^2$

$$L = \frac{1}{2} m v^2(B) - \frac{1}{2} m v^2(A)$$

$\bar{F}$ e' conservativa se

$L = -\Delta U$ dove U energia potenziale

$$\oint_{l} \bar{F}\, d\bar{l} = 0$$

Potenziale gravita' $U(y) = mgy$ (altezza)

Potenziale gravitazionale $U(r) = -\frac{GMm}{r}$ (distanza)

Potenziale elastica $U(x) = \frac{1}{2} K (x - x_0)^2$

Energia meccanica $E_m = U + T$

$$= U + \frac{1}{2} m v^2$$

Se $\bar{F}$ e' conservativa $\Delta E_m = 0$

$$\Rightarrow \begin{cases} L = \Delta T \\ L = -\Delta U \end{cases} \Rightarrow \Delta T + \Delta U = 0 \Rightarrow \Delta(T+U) = 0$$

$$\Delta E_m = \text{LAVORO FORZE NON CONSERVATIVE}$$

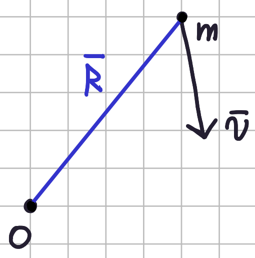
Potenza

$$P = \frac{dL}{dt}$$

Momento Angolare

$$\bar{M} = \bar{R} \times \bar{p} = \bar{R} \times \frac{d}{dt} m\bar{v}$$

($\bar{p}$: quantita' di moto)



momento della quantita' di moto

$$\bar{b} = \bar{R} \times m\bar{v}$$

$$\bar{M} = \frac{d\bar{b}}{dt} - \frac{d\bar{R}}{dt} \times m\bar{v}$$

essendo $\bar{v} = \bar{\omega} \times \bar{R}$

$$\bar{b} = \bar{R} \times m(\bar{\omega} \times \bar{R}) = mR^2 \bar{\omega}$$

$$\bar{b} = I\bar{\omega}$$

momento di inerzia $I = mR^2$

il momento si puo' scrivere $\bar{M} = \frac{d}{dt} I\bar{\omega} = I\dot{\bar{\omega}}$ se I cost.

Sistema di Punti

le Forze interne si annullano. n punti

$$\bar{F}_{(est)} = \sum_{i=1}^{n} m_i a_i$$

centro di massa $\bar{r}_c = \sum_i m_i \bar{r}_i \cdot \frac{1}{\sum_i m_i}$ ($\bar{r}_i$: pos. di ogni punto)

vel. del sistema (centro di massa)

$$\frac{d\bar{r}_c}{dt} = \bar{v}_c$$

somma masse $M = \sum_i m_i$

$\Rightarrow$ quantita' di moto del sistema

$$\bar{p}_c = M\bar{v}_c = \sum_i m_i \bar{v}_i$$

$$\frac{d}{dt}\bar{p}_c = \sum_i \frac{d}{dt} m_i \bar{v}_i = \sum_i m_i \bar{a}_i = \bar{F}_{(est)}$$

Urti

•elastici : la quantità di moto si conserva.

2 corpi

Masse : m_1 , m_2
vel. post urto V_1 V_2
vel. pre urto v_1 v_2

Cons. quantità di moto

$$m_1 v_1 + m_2 v_2 = m_1 V_1 + m_2 V_2$$

cons. energia cinetica

$$\frac{1}{2} m_1 v_1^2 + \frac{1}{2} m_2 v_2^2 = \frac{1}{2} m_1 V_1^2 + \frac{1}{2} m_2 V_2^2$$

Momento sistema di punti

$$\begin{cases} \bar{M}_1 = \frac{d\bar{b}_1}{dt} + \bar{v}_0 \times \bar{r}_1 \\ \vdots \\ \bar{M}_n = \frac{d\bar{b}_n}{dt} + \bar{v}_0 \times \bar{r}_n \end{cases}$$

i momenti interni del sistema si annullano

$$\left(\frac{d}{dt} \sum \bar{b}_i \right) + \bar{v}_0 \times \left(\sum \bar{r}_i \right)$$

si può riscrivere

$$\frac{d}{dt} \left(\sum \bar{r}_i \times m_i \bar{v}_i \right) + \bar{v}_0 \times \left(\sum \bar{r}_i \right)$$

Il momento del sistema non è uguale alla somma dei momenti

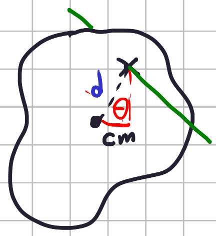
Sistema Continuo

per un oggetto puntiforme $I = Rm^2$

$dI = R^2 dm = R^2 \lambda dR \Rightarrow \lambda$ densità: $\frac{dm}{dR} = \lambda$

$I = \int_S dI$ ← sup. oggetto

Pendolo Composto



d = dist. centro di massa - punto inF.

per $\theta \to 0$

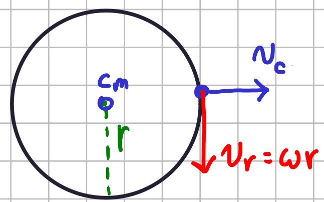
$$-mg \cdot d \cdot \theta = -I_0 \frac{d^2\theta}{dt^2}$$

I_0 = momento di inerzia con polo in cm

$$I_0 = I_{cm} + m \cdot d^2$$

I_{cm} ← momento in. centro massa

Rotolamento



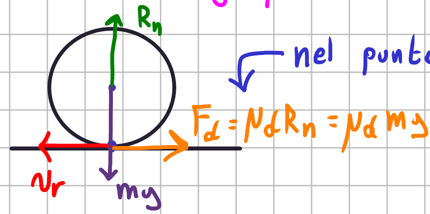
velocità centro di massa

velocità di rotazione

$v_a = v_c + v_r$

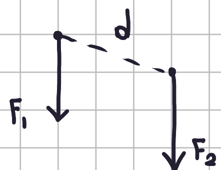
↑ vel. assoluta di ogni punto

$|v_c| = |v_r| \Rightarrow$ rotolamento puro

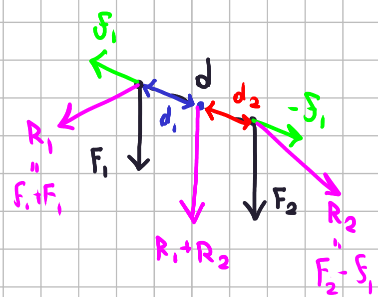


nel punto di contatto

Sistemi eq. di Forze



$$\frac{d_1}{d_2} = \frac{F_2}{F_1}$$

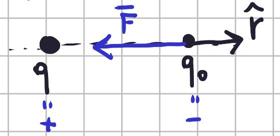


Campo Elettrico

Coloumb

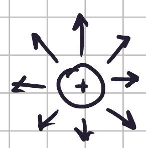
q esercita su q_0 $\quad \bar{F} = \frac{1}{4\pi\varepsilon_0}\frac{q \cdot q_0}{r^2}\hat{r}$

$r := \text{dist}(q, q_0)$



Campo Elettrico

$$\bar{E} = \lim_{q_0 \to 0}\frac{\bar{F}}{q_0} = \frac{1}{4\pi\varepsilon_0}\frac{q}{r^2}\hat{r}$$

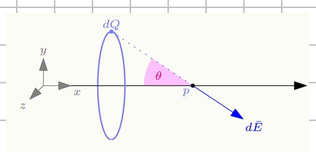


q_0 immersa in $\bar{E}$ sente $\bar{F} = q_0\bar{E}$

Sistema di cariche : $\bar{E} = \frac{1}{4\pi\varepsilon_0}\sum_{i=1}^{n}\frac{q_i}{r_i^2}\hat{r}_i$

Sup. carica : $\bar{E} = \frac{1}{4\pi\varepsilon_0}\int_S \frac{dQ}{r^2}\hat{r}$

Anello Carico



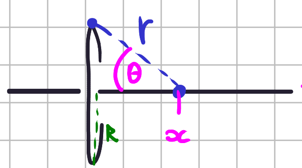
$dE_x = \frac{1}{4\pi\varepsilon_0}\frac{dQ}{r^2}\cos\alpha$

λ densità $\Rightarrow dQ = \lambda d\ell$

$d\ell$ = punto circonferenza

$$E_x = \frac{1}{4\pi\varepsilon_0}\int_A \frac{\lambda d\ell}{r^2}\cos\theta = \frac{1}{4\pi\varepsilon_0}\frac{1}{r^2}\cos\theta\, Q$$

non dipende dal punto sulla circ.



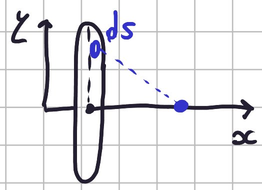
$\Rightarrow r\cos\theta = x \Rightarrow \cos\theta = \frac{x}{r}$

$$E = \frac{1}{4\pi\varepsilon_0}\frac{x}{r^3}Q \qquad r = \sqrt{R^2 + x^2}$$

diperde da x — raggio

Disco Carico

$dQ = \lambda dS \qquad dS$ = sup. infinitesima

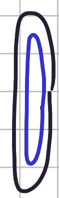


$$E = \frac{1}{4\pi\varepsilon_0}\iint_S \frac{\lambda dS}{r^2}$$

superf.

Cambio :

dS = anello infinitesimo



spessore anello

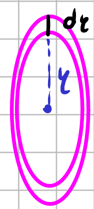
$dS = (2\pi r) \cdot dr$

raggio

$$dE = \frac{1}{4\pi\varepsilon_0}\cdot\frac{x}{\underbrace{(x^2+\gamma^2)^{3/2}}_{\text{dist}}}\cdot dQ$$

γ = raggio r anello

$$\overset{\text{dens.}}{dQ} = \sigma \cdot \underbrace{(2\pi\gamma)\,d\gamma}_{\text{sup. anello inf.}}$$



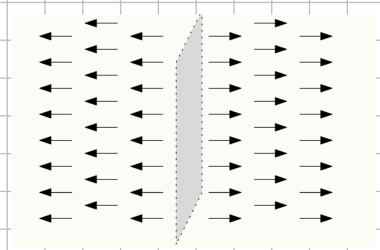
RAGGIO DISCO → R

$$E = \frac{1}{4\pi\varepsilon_0}\cdot\sigma 2\pi \cdot x\int_0^R \frac{\gamma}{(x^2+\gamma^2)^{3/2}}d\gamma$$

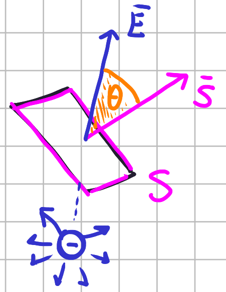
$$E(x) = \frac{\sigma x}{2\varepsilon_0}\cdot\left(\frac{1}{|x|} - \frac{1}{\sqrt{x^2+R^2}}\right)$$

Piano Carico

$$\lim_{R\to\infty}\frac{\sigma x}{2\varepsilon_0}\cdot\left(\frac{1}{|x|} - \frac{1}{\sqrt{x^2+R^2}}\right) = \pm\frac{\sigma}{2\varepsilon_0}$$



Flusso



$$\phi = \bar{E} \cdot \bar{S} = ES\cos\theta$$

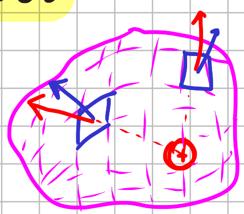
$\bar{S}$ ← sup

S = sup Finita

$$\phi = \int_S d\phi = \int_S \bar{E} \cdot d\bar{s}$$

Legge di Gauss

S := sup. chiusa



$$\phi = \oint_S \bar{E} \cdot d\bar{s} = \frac{q}{\varepsilon_0}$$

$$d\phi = \bar{E} \cdot d\bar{s} = \frac{1}{4\pi\varepsilon_0}\frac{q}{r^2}\hat{r}\, d\bar{s} = \frac{q}{4\pi\varepsilon_0} \cdot \underbrace{\frac{\hat{r}\, d\bar{s}}{r^2}}_{d\Omega} = \frac{q}{4\pi\varepsilon_0} d\Omega$$

$d\Omega \Rightarrow \oint d\Omega = 4\pi$

Si scrive anche

$$\boxed{\oint_S \bar{E}\, d\bar{s} = \frac{1}{\varepsilon_0}\int_{\tau_S} \rho\, d\tau}$$

carica int. alla sup S

$\tau_S \Rightarrow$ volume di S

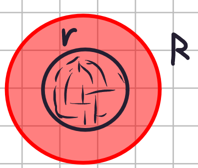
Campo Sfera carica

$r \geq R$

$$\phi = \frac{1}{\varepsilon_0}\int_{\tau_S} \rho\, d\tau = \frac{Q}{\varepsilon_0}$$

$$\frac{Q}{\varepsilon_0} = \oint_S E\, d\bar{s} = E\int_S d\bar{s} = E 4\pi r^2 \Rightarrow E = \frac{1}{4\pi\varepsilon_0}\frac{Q}{r^2}$$

$r < R$

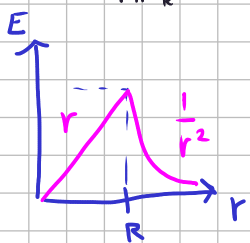


$$\phi = \frac{1}{\varepsilon_0}\int_{\tau_S} \rho\, d\tau = \frac{\rho}{\varepsilon_0}\int_0^r d\tau = \frac{\rho}{\varepsilon_0}\frac{4}{3}\pi r^3$$

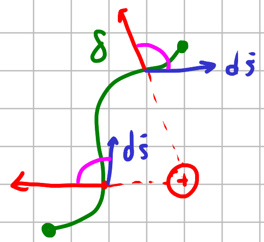
$$\boxed{\rho \frac{4}{3}\pi R^3 = Q \Downarrow \rho = Q\frac{3}{4}\frac{1}{\pi}\frac{1}{R^3}}$$

$$E 4\pi r^2 = \frac{\rho}{\varepsilon_0}\frac{4}{3}\pi r^3$$

$$E = \frac{\rho}{\varepsilon_0}\frac{1}{3} r = \frac{Q}{4\pi\varepsilon_0} \cdot \frac{r}{R^3}$$



Circuitazione



$$L = \int_\delta q_0 \bar{E} = q_0 \int_\delta \bar{E}\, d\bar{s}$$

L lavoro

Sia C una curva chiusa

$$\mathcal{E} = \oint_C \bar{E}\, d\bar{s} \qquad \int_A^B \bar{E}\, d\bar{s} = V(A) - V(B) = \Delta V$$

CIRCUITAZIONE — potenziale

$$L = -q_0 \Delta V = -\Delta U$$

CAMPO ELETTROSTATICO $\Rightarrow$ CONSERVATIVO $\Rightarrow \mathcal{E} = 0$

Potenziale di una Carica

$$\bar{E} = \frac{1}{4\pi\varepsilon_0}\frac{Q}{r^2}\hat{r} \Rightarrow \bar{E}\, d\bar{s} = \frac{1}{4\pi\varepsilon_0}\frac{Q}{r^2}\hat{r}\, d\bar{s} = \frac{1}{4\pi\varepsilon_0}\frac{Q}{r^2} dr$$

$$\mathcal{E} = \int_C \bar{E}\, d\bar{s} = \int_A^B \bar{E}\, d\bar{s} = \frac{Q}{4\pi\varepsilon_0}\int_{r_A}^{r_B} \frac{1}{r^2}\, dr$$

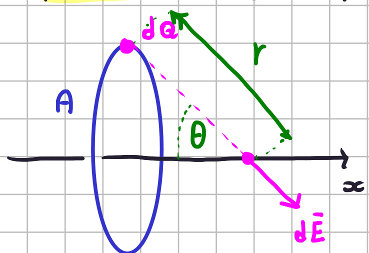
$$= \frac{Q}{4\pi\varepsilon_0}\left(\frac{1}{r_B} - \frac{1}{r_A}\right) \Rightarrow V(r) = \frac{Q}{4\pi\varepsilon_0} \cdot \frac{1}{r}$$

r ← dist da Q

Gradiente

$$\bar{E} = -\nabla V = \left[-\frac{\partial V}{\partial x}, -\frac{\partial V}{\partial y}, -\frac{\partial V}{\partial z}\right]$$

Potenziale Anello Carico



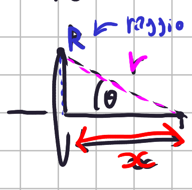
Potenziale di dQ é

$$\frac{1}{4\pi\varepsilon_0}\frac{dQ}{r}$$

$$\boxed{\int_A dQ = Q}$$

r non dipende dalla pos. di dQ

$$\text{Pot. totale } V(r) = \frac{1}{4\pi\varepsilon_0}\int_A \frac{dQ}{r} = \frac{1}{4\pi\varepsilon_0 r} \cdot Q$$



$$\Rightarrow \sqrt{x^2 + R^2} = r \Rightarrow V(r) = \frac{Q}{4\pi\varepsilon_0} \cdot \frac{1}{\sqrt{x^2 + R^2}}$$

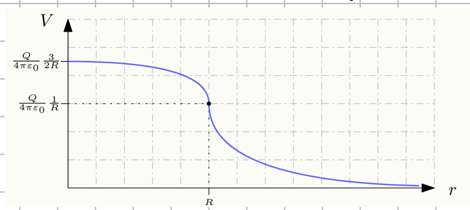
Potenziale Sfera Carica

raggio sfera: R

$$E(r) = \begin{cases} \frac{1}{r^2 4\pi\varepsilon_0} Q & \text{se } r \geq R \\ \frac{Q}{4\pi\varepsilon_0} \cdot \frac{r}{R^3} & \text{se } r \leq R \end{cases}$$

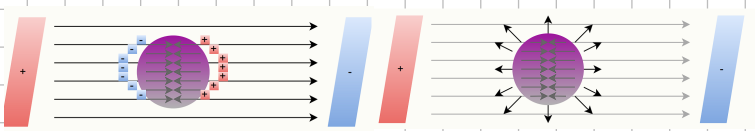
Interno: $V(r) = \int_r^\infty \frac{1}{r^2} \cdot \frac{Q}{4\pi\varepsilon_0} = \frac{1}{4\pi\varepsilon_0} \cdot \frac{Q}{r}$ $\begin{cases} \text{come se} \\ \text{fosse una} \\ \text{carica punt.} \end{cases}$

esterno: $V(r) = \frac{Q}{4\pi\varepsilon_0} \frac{1}{R^3} \int_r^R r\, dr = \frac{1}{4\pi\varepsilon_0} \left(\frac{3}{2}R - \frac{r^2}{2R^3} \right)$

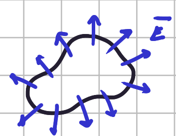


Conduttori

le cariche nei conduttori sono libere e perturbano il campo elettrico



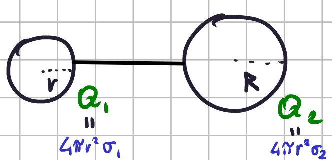
Un conduttore ha $\vec{E}$ nullo all'interno. Sulla sup. e' normale ad essa:



Equipotenziale

in un conduttore $\mathcal{T}$ il campo $\vec{E}$ e' nullo $\Rightarrow \int_\ell \vec{E}\, d\vec{\ell} = 0 = \Delta V \Rightarrow$ in ogni punto il potenziale e' identico.

Effetto delle punte: il campo e' piu' intenso dove la curvatura e' maggiore.



$\frac{\sigma_1}{\sigma_2} = \frac{R}{r} \Rightarrow \sigma_1 = \sigma_2 \frac{R}{r}$ $\vee$ 0

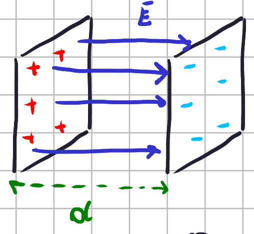
Questo perche' la superfice e' equipotenz.

Capacita'

conduttore di carica Q e pot. V $\Rightarrow C = \frac{Q}{V}$

nel caso della sfera $V = \frac{1}{4\pi\varepsilon_0} \frac{Q}{R} \Rightarrow C = 4\pi\varepsilon_0 R$

Condensatore

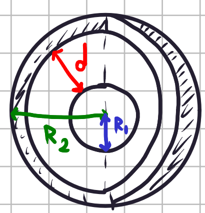


$E = \frac{\sigma}{\varepsilon_0}$ (solo fra le lastre) $\Rightarrow V = \int_0^\infty E = \int_0^d \frac{\sigma}{\varepsilon_0} = d\frac{\sigma}{\varepsilon_0}$ (sulla sup)

A: sup. lastre $\Rightarrow Q = \sigma A$

$\Rightarrow C = \frac{Q}{V} = \boxed{\varepsilon_0 \frac{A}{d}}$

Condensatore Sferico



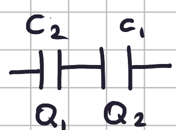
$V_1 = \frac{1}{4\pi\varepsilon_0} \frac{Q}{R_1}$

$V_2 = \frac{1}{4\pi\varepsilon_0} \frac{Q}{R_2}$

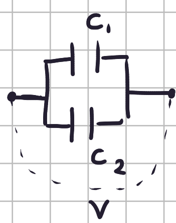
$\Rightarrow \Delta V = \frac{1}{4\pi\varepsilon_0} Q \frac{d}{R_1 R_2}$

Una C eq. $\Rightarrow C = \frac{Q}{V_1} - \frac{Q}{V_2} = \frac{Q}{\Delta V} = 4\pi\varepsilon_0 \frac{R_1 R_2}{d}$

Collegamento Condensatori



$Q_1 = Q_2 \Rightarrow C_{eq} = \frac{C_1 C_2}{C_1 + C_2}$



$V_1 = V_2 = V$

$\Rightarrow C_1 = \frac{Q_1}{V}$

$C_2 = \frac{Q_2}{V}$

$\Rightarrow C_{eq} = \frac{(Q_1 + Q_2)}{V} \Rightarrow C_{eq} = C_1 + C_2$