ESERCIZI VARI DA ESONERI/PRE-ESONERI

$$\begin{cases} y'(t) = (9 + y^2(t))\, e^{3t} \\ y(0) = 0 \end{cases}$$

$$\frac{y'(t)}{9 + y^2(t)} = e^{3t} \Longrightarrow \int_0^t \frac{y'(s)}{9 + y^2(s)}\, ds = \int_0^t e^{3s}\, ds$$

$$\Longrightarrow \int_0^{y(t)} \frac{1}{9 + z^2}\, dz = \frac{e^{3t}}{3} - \frac{1}{3} \Rightarrow \frac{1}{3} \operatorname{arctan}\left(\frac{y(t)}{3}\right) = \frac{e^{3t}}{3} - \frac{1}{3}$$

$$\operatorname{arctan}\left(\frac{y(t)}{3}\right) = e^{3t} - 1$$

$$y(t) = 3 \tan\left(e^{3t} - 1\right)$$

$$\begin{cases} y''(t) - 8y'(t) + 16y(t) = e^{5t} \\ y(0) = 1 \qquad y'(0) = 6 \end{cases}$$

$P(\lambda) \Rightarrow \lambda^2 - 8\lambda + 16 = 0 \rightarrow \frac{8 \pm 0}{2} = 4 \qquad y_0(t) = (C + Dt)e^{4t}$

$\bar{y}(t) = ae^{5t} \qquad \bar{y}'(t) = 5ae^{5t} \qquad \bar{y}''(t) = 25ae^{5t}$

$25ae^{5t} - 40ae^{5t} + 16ae^{5t} = e^{5t}$

$ae^{5t} = e^{5t} \Rightarrow a = 1 \qquad \bar{y}(t) = e^{5t}$

$y(t) = (C + Dt)e^{4t} + e^{5t}$

$y'(t) = De^{4t} + (C + Dt)4e^{4t} + 5e^{5t}$

$y(0) = 1 \rightarrow 1 = (C + 0) \cdot 1 + 1 \rightarrow 1 = C + 1 \rightarrow C = 0$

$y'(0) = 6 \rightarrow 6 = D + 4C + 5 \rightarrow 6 = D + 5 \rightarrow D = 1$

$y(t) = te^{4t} + e^{5t}$

$y'(t) = t\left(y^3(t) - 9y(t)\right)$

4) Si consideri l'equazione differenziale $y'(t) = t\left(y(t)^3 - 9y(t)\right)$.

4A) Se $y(0) = 3$, la soluzione non è costante. FALSO

4B) Se $y(0) = 1$, si ha $y'(0) \neq 0$. FALSO

4C) Se $y(0) = -1$, si ha $T_2(y(t); 0) = -1 + 4\,t^2$ VERO

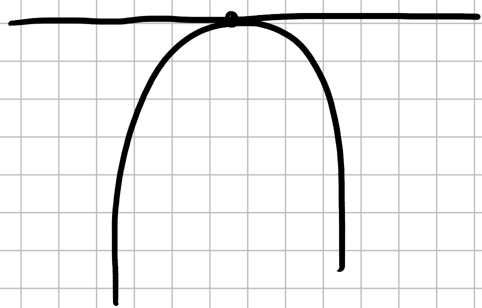
4D) Se $y(0) = 1$, la soluzione ha un minimo relativo per $t = 0$. FALSO

$$y''(t) = \left(y^3(t) - 9y(t)\right) + t\left[3y^2(t)y'(t) - 9y'(t)\right]$$

$y''(0) = 8$

$y'(0) = 0$

-8



6) Si risolvano le seguenti equazioni differenziali a variabili separabili.

a) $y'(t) = 8\,y(t) + 13$, se $y(0) = 0$.

b) $y'(t) = 4\,t\,(36 + y^2(t))$, se $y(0) = 0$.

c) $y'(t) = \mathrm{e}^{-y(t)}\,\mathrm{e}^{5\,t}$, se $y(0) = 0$.

d) $y'(t) = \frac{7\,(1+y^2(t))}{y(t)}$, se $y(0) = 1$.

a) $\begin{cases} y'(t) = 8y(t)+13 \\ y(0)=0 \end{cases}$

$$y(t) = e^{8t}\left[\int_0^t 13 e^{-8s}\,ds\right] = e^{8t}\left[13\left[-\frac{e^{-8t}}{8}+\frac{1}{8}\right]\right] =$$

$$= e^{8t}\left[-\frac{13}{8}e^{-8t}+\frac{13}{8}\right] = \frac{13}{8}\left[e^{8t}-1\right] = \frac{13e^{8t}-13}{8}$$

b) $\begin{cases} y'(t) = 4t(36+y^2(t)) \\ y(0)=0 \end{cases}$

$$\frac{y'(t)}{36+y^2(t)} = 4t \Rightarrow \int_0^{y(t)} \frac{1}{36+z^2} = 2t^2 \Rightarrow$$

$$\arctan\left(\frac{y(t)}{6}\right) = 12t^2 \Rightarrow y(t) = 6\tan(12t^2)$$

$$\begin{cases} y'(t) = e^{-y(t)}\, e^{5t} \\ y(0) = 0 \end{cases}$$

$$\frac{y'(t)}{e^{-y(t)}} = e^{5t} \Rightarrow \int_0^{y(t)} \frac{1}{e^{-z}}\, dz = \frac{e^{5t}}{5} - \frac{1}{5}$$

$$e^{y(t)} = \frac{e^{5t}}{5} - \frac{1}{5} + 1 = \frac{e^{5t}+4}{5} \Rightarrow y(t) = \ln\left(\frac{e^{5t}+4}{5} \right)$$

(d)

$$y'(t) = 7\, \frac{1+y^2(t)}{y(t)} \qquad y(0) = 1$$

$$\frac{y'(t)}{\frac{1+y^2(t)}{y(t)}} = 7 \Rightarrow \int_1^{y(t)} \frac{z}{1+z^2} = 7t$$

$$\frac{1}{2} \int_1^{y(t)} \frac{2z}{1+z^2} = 7t \Rightarrow \left[\ln(1+y(t)^2) - \ln(2) \right] = 14t$$

$$\Rightarrow \ln(1+y(t)^2) = 14t + \ln(2)$$

$$1 + y^2(t) = 2e^{14t} \Rightarrow y(t) = \sqrt{2e^{14t} - 1} \quad ✓$$

$$\begin{cases} y'(t) = \dfrac{y(t)(y(t)-14)}{t+13} \\ y(0) = 7 \end{cases}$$

$$\frac{y'(t)}{y(t)(y(t)-14)} = \frac{1}{t+13} \to \int_7^{y(t)} \frac{1}{z^2-14z}\,dz = \ln\left(\frac{t+13}{13}\right)$$

$$\frac{1}{14}\ln\left(\frac{z-14}{z}\right)\Bigg|_7^{y(t)} = \ln\left(\frac{t+13}{13}\right)$$

$$\left[\ln\left(-\frac{y(t)-14}{y(t)}\right)\right] = 14\ln\left(\frac{t+13}{13}\right)$$

$$\frac{1}{y(t)} = \frac{-e^{14\ln\left(\frac{t+13}{13}\right)} - 1}{14}$$

$$y(t) = \frac{14}{-e^{14\ln\left(\frac{t+13}{13}\right)} - 1} \qquad -e^{\ln\left(\frac{t+13}{13}\right)^{14}}$$

$$y(t) = \frac{14}{-\left(\frac{t-13}{13}\right)^{14} - 1}$$

$$\begin{cases} y''(t) - 13y'(t) + 36y(t) = 144 \\ y(0) = 4 \qquad y'(0) = 5 \end{cases}$$

$$P(\lambda) \to \lambda^2 - 13\lambda^2 + 36 = 0 \qquad \frac{13 \pm 5}{2} \to \begin{matrix} 9 \\ 4 \end{matrix}$$

$$y_0(t) = Ce^{9t} + De^{4t}$$

$$\bar{y}(t) = Q \qquad Q = \frac{144}{36} = 4$$

$$y(t) = Ce^{9t} + De^{4t} + 4$$

$$y'(t) = 9Ce^{9t} + 4De^{4t}$$

$$y(0) = 4 \to 4 = C + D + 4 \to C = -D \to D = -1$$

$$y'(0) = 5 \to 5 = 9C + 4D \to 5 = 5C \to C = 1$$

$$e^{9t} - e^{4t} + 4$$