

Gli esercizi presentati in questo documento, esposti negli appunti di Optimization di Simone Bianco, sono tratti da esercizi del libro, e da esercizi che il professore ha presentato in classe durante il corso delle lezioni.

Consider the following linear program:

$$\begin{aligned} \max \quad & x_1 + 2x_2 - 3x_3 + 7x_5 \\ \text{s.t.} \quad & x_1 + 2x_2 + 2x_3 + x_4 = 3 \\ & x_1 + 2x_2 + 7x_3 + x_5 = 3 \\ & 2x_1 + 4x_2 + 7x_3 + x_6 = 6 \\ & x_1, x_2, x_3, x_4, x_5, x_6 \geq 0 \end{aligned}$$

Determine and justify which of the following vectors is a basic feasible solution (BFS):

1. $a = [1 \ 1 \ 0 \ 0 \ 0 \ 0]$

2. $b = [1 \ 0 \ 0 \ 2 \ 2 \ 4]$

3. $c = [0 \ 0 \ 0 \ 3 \ 3 \ 6]$

La matrice dei vincoli e'

$$A = \begin{bmatrix} 1 & 2 & 2 & 1 & 0 & 0 \\ 1 & 2 & 7 & 0 & 1 & 0 \\ 2 & 4 & 7 & 0 & 0 & 1 \end{bmatrix} \Rightarrow \text{Le colonne 4, 5, 6 sono lin. indep. quindi } C \text{ e' una BFS associata a tale base } B = \{4, 5, 6\}$$

Given a general linear program in equational form

$$\begin{aligned} \max \quad & c^T x \\ \text{s.t.} \quad & Ax = b \\ & x \geq 0 \end{aligned}$$

let \bar{x} and \bar{y} be two optimal solutions to the problem. Prove or disprove the following statements:

1. Any convex combination of \bar{x} and \bar{y} is also an optimal solution to the linear program

2. There exists an optimal solution \bar{z} on the line through \bar{x} and \bar{y} which is also basic

Una combinazione convessa di due punti \bar{x}, \bar{y} e' il segmento di linea.

$$c^T \bar{z} = c^T \bar{y} = \gamma \quad z = \alpha \bar{x} + (1-\alpha) \bar{y}$$
$$c^T z = c^T (\alpha \bar{x} + (1-\alpha) \bar{y}) = \alpha c^T \bar{x} + (1-\alpha) c^T \bar{y} = \alpha \gamma + (1-\alpha) \gamma = \gamma \Rightarrow z \text{ e' ottimale.}$$

La 2 e' falsa perche' \bar{x} e \bar{y} sono BFS, ossia vertici del poliedro, e ogni punto sul segmento di linea di 2 vertici non e' (per definizione) un vertice.

Consider the polyhedron $P = \{x \in \mathbb{R}^n \mid Ax \leq b, x \geq 0\}$ and let $v \in P$. Prove that v is a vertex of P if and only if there exists a vector c such that v is the unique optimal basic feasible solution to the linear program:

$$\begin{aligned} \max \quad & c^T x \\ \text{s.t.} \quad & x \in P \end{aligned}$$

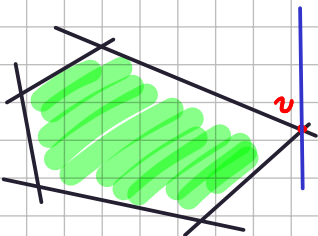
[\Rightarrow]: v e' un vertice di $P \Rightarrow \exists \mathcal{H} = \{x : c^T x = \alpha\}$ t.c.

$$c^T x < \alpha \quad \forall x \in P \setminus \{v\} \quad \wedge \quad c^T v = \alpha$$

ma allora v e' sol. ottimale per $\max_{x \in P} c^T x$.

[\Leftarrow]: v e' l'unica sol. di $\max_{x \in P} c^T x \Rightarrow$ necessariamente v e' una BFS

$\Rightarrow v$ e' un vertice di P .



Solve the following linear program using the Simplex method:

$$\begin{aligned} \max \quad & 2x_1 + x_2 - x_3 \\ \text{s.t.} \quad & x_1 + 2x_2 + x_3 \leq 8 \\ & -x_1 + x_2 - 2x_3 \leq 4 \\ & x_1, x_2, x_3 \geq 0 \end{aligned}$$

Il problema in forma matriciale e'

$$A = \begin{bmatrix} 1 & 2 & 1 \\ -1 & 1 & -2 \end{bmatrix} \Rightarrow \text{in forma di eq: } A' = \begin{bmatrix} 1 & 2 & 1 & 1 & 0 \\ -1 & 1 & -2 & 0 & 1 \end{bmatrix}$$
$$b = [8 \ 4]^T$$

La prima base e' $B = \{4, 5\}$

$$B = \{1, 5\}$$

$$\begin{aligned} x_1 + 2x_2 + x_3 &= 8 \\ -x_1 + x_2 - 2x_3 &= 4 \end{aligned} \Rightarrow \begin{aligned} x_4 &= 8 - x_1 - 2x_2 - x_3 \\ x_5 &= 4 + x_1 - x_2 + 2x_3 \end{aligned} \Rightarrow$$
$$z = 2x_1 + x_2 - x_3$$
$$z = 2x_4 + x_2 - x_3$$

$B = \{1, 5\}$

$$\begin{aligned} x_1 &= 8 - x_4 - 2x_2 - x_3 \\ x_5 &= 4 + x_1 - x_2 + 2x_3 \end{aligned} \Rightarrow \text{12 sol ott. e'}$$
$$\begin{aligned} x_1 &= 8 \\ x_5 &= 12 \end{aligned} \Rightarrow \text{per il problema originale e' } x = [8 \ 0 \ 0]^T$$
$$z = 16 - 2x_4 - 3x_2 - 3x_3$$

Consider the following linear program

$$\begin{aligned} \min \quad & 3x_1 + 2x_2 - x_3 \\ & x_1 - 2x_2 + x_3 - x_4 = 0 \\ & x_1 + x_2 + 2x_3 - x_4 = 1 \\ & x_1, x_2, x_3, x_4 \geq 0 \end{aligned}$$

Beginning with the basis $B = \{1, 2\}$, solve the linear program with the Simplex method

$$B = \{1, 2\} \quad \begin{cases} x_1 - 2x_2 + x_3 - x_4 = 0 \\ x_1 + x_2 + 2x_3 - x_4 = 1 \end{cases} \quad \begin{cases} x_1 = -\frac{5}{3}x_3 + x_4 + \frac{2}{3} \\ x_2 = \frac{1}{3} - \frac{1}{3}x_3 \end{cases}$$

$$B = \{1, 2\}$$

$$\begin{aligned} x_1 &= -\frac{5}{3}x_3 + x_4 + \frac{2}{3} \\ x_2 &= \frac{1}{3} - \frac{1}{3}x_3 \\ \hline z &= -\frac{8}{3} + \frac{20}{3}x_3 + 3x_4 \end{aligned}$$

$$B = \{2, 4\}$$

$$\begin{aligned} x_4 &= -x_1 - \frac{5}{3}x_3 + \frac{2}{3} \\ x_2 &= \frac{1}{3} - \frac{1}{3}x_3 \\ \hline z &= -\frac{2}{3} + \frac{20}{3}x_3 - 3x_1 - 5x_3 \end{aligned}$$

$$B = \{3, 4\}$$

$$\begin{aligned} x_4 &= -x_1 - \frac{5}{3} + 5x_2 + \frac{2}{3} \\ x_3 &= 1 - 3x_2 \\ \hline z &= 1 - 17x_2 - 3x_1 \end{aligned}$$

Finito, il massimo e' 1, il minimo del problema originale e' -1,

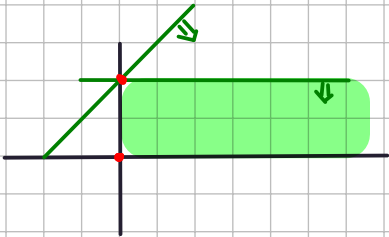
Give an explicit example of a linear program in the form of:

$$\begin{aligned} \max \quad & c^T x \\ & Ax = b \\ & x \geq 0 \end{aligned}$$

such that the problem is unbounded, it has a degenerate BFS and infinitely many optimal solutions

Si consideri il problema

$$\begin{aligned} \max \quad & -x_1 \\ \text{s.t.} \quad & x_2 - x_1 \leq 2 \\ & x_2 \leq 2 \end{aligned}$$



e' illimitato, le due BFS $[0 \ 0 \ 0 \ 0]$ e $[0 \ 2 \ 0 \ 0]$ sono ottimali;

in forma di eq:
$$\begin{aligned} x_2 - x_1 + x_3 &= 2 \\ x_2 + x_4 &= 2 \end{aligned}$$

Si consideri $B = \{1, 2\} \Rightarrow \begin{cases} x_1 = -x_4 + x_3 - 2 + 2 \\ x_2 = -x_4 + 2 \end{cases} \Rightarrow \text{Sol: } [0 \ 2 \ 0 \ 0]^T \Rightarrow \text{Una variabile di base e' } 0 \Rightarrow [0 \ 2 \ 0 \ 0]^T \text{ e' degenera.}$

Solve the following linear program using the Simplex method:

$$\begin{aligned} \min \quad & 12x_1 + 3x_2 + 4x_3 \\ & 4x_1 + 2x_2 + 3x_3 \geq 2 \\ & 8x_1 + x_2 + 2x_3 \geq 3 \\ & x_1, x_2, x_3 \geq 0 \end{aligned}$$

Trasformo il problema in forma canonica, e poi in forma di equazione.

$$\begin{aligned} \max \quad & -12x_1 - 3x_2 - 4x_3 \\ & -4x_1 - 2x_2 - 3x_3 \leq -2 \\ & -8x_1 - x_2 - 2x_3 \leq -3 \end{aligned} \Rightarrow \begin{aligned} & -4x_1 - 2x_2 - 3x_3 + x_4 = -2 \\ & -8x_1 - x_2 - 2x_3 + x_5 = -3 \end{aligned}$$

la base $B = \{4, 5\}$ ha BFS associata $[0 \ 0 \ 0 \ -2 \ -3]^T \Rightarrow$ non e' ammissibile.

$$B = \{1, 2\}$$

$$\begin{aligned} x_1 &= -\frac{3}{20}x_3 + \frac{1}{20}x_4 + \frac{1}{10}x_5 + \frac{2}{5} \\ x_2 &= -\frac{4}{5}x_3 + \frac{2}{5}x_4 - \frac{1}{5}x_5 + \frac{1}{5} \\ \hline z &= \frac{13}{5}x_3 - \frac{9}{5}x_4 - \frac{3}{5}x_5 - \frac{27}{5} \end{aligned}$$

$$B = \{1, 3\}$$

$$\begin{aligned} x_1 &= \frac{35}{80}x_2 + \frac{3}{16}x_3 - \frac{1}{8}x_4 + \frac{5}{16} \\ x_3 &= -\frac{5}{4}x_2 + \frac{1}{2}x_4 - \frac{1}{4}x_5 + \frac{1}{4} \\ \hline z &= -\frac{13}{4}x_2 - \frac{1}{2}x_4 - \frac{25}{20}x_5 - \frac{19}{4} \end{aligned}$$

Sol: $x^* = [\frac{5}{16}, 0, \frac{1}{4}]$
 \Rightarrow opt: $\frac{19}{4}$
 \uparrow
per l'LP originale

Consider the standard form polyhedron $P = \{x \mid Ax = b, x \geq 0\}$. Suppose that the matrix A of dimensions $m \times n$ has linearly independent rows and that all basic feasible solutions are non-degenerate.

Given $\bar{x} \in P$ such that \bar{x} has exactly m positive components:

1. Show that \bar{x} is a BFS
2. Show that the previous point is false if we remove the non-degeneracy assumption

Se \bar{x} ha m componenti positive, e non ci sono soluzioni degenerate, allora $B = \{i : \bar{x}_i > 0\}$ e' la base di \bar{x} , ed e' una BFS solo se A_B , che ha m colonne, e' non singolare, essendo che il rango delle righe di A e' m , lo e' anche quello di $A_B \Rightarrow$ le colonne di A_B sono lin. ind. $\Rightarrow A_B$ e' non singolare.

Se l'assunzione sulle sol. non degenerate e' falsa, non si puo' affermare che $B = \{i : \bar{x}_i > 0\}$ sia la base di \bar{x} .

Consider the following primal program:

$$\begin{aligned} \min \quad & c^T x \\ \text{s.t.} \quad & Ax \geq b \\ & x \geq 0 \end{aligned}$$

1. Form the dual program and state it as an equivalent minimization problem
2. Derive the conditions on A, b and c such that the dual is identical to the primal, meaning that the problem is *self-dual*
3. Give a concrete example of a primal program identical to its dual

$$\text{Il duale e' } \begin{cases} \max & b^T y \\ \text{s.t.} & A^T y \leq c \\ & y \leq 0 \end{cases} \equiv \begin{cases} \min & -b^T y \\ \text{s.t.} & A^T y \leq c \\ & y \leq 0 \end{cases}, \text{ e' self-dual se}$$

$$\begin{aligned} \{x : Ax \geq b\} &= \{x : A^T x \leq c\} \Rightarrow \begin{cases} b = -c \\ A = \bar{A}^T \end{cases} \\ \{x : Ax \geq b\} &= \{x : -A^T x \geq -c\} \Rightarrow \end{aligned}$$

$$\text{Un'esempio e': st } \begin{aligned} \max \quad & [1 \ -1] x \\ \text{s.t.} \quad & \begin{bmatrix} 0 & -1 \\ 1 & 0 \end{bmatrix} x \leq \begin{bmatrix} -1 \\ 1 \end{bmatrix} \\ & x \geq 0 \end{aligned}$$

Consider the following linear program:

$$\begin{aligned} \max \quad & 2x_1 + 16x_2 + 12x_3 \\ \text{s.t.} \quad & 2x_1 + x_2 - x_3 \leq 3 \\ & -3x_1 + 8x_2 + 2x_3 \leq 12 \\ & x_1, x_2, x_3 \geq 0 \end{aligned}$$

Considero il problema duale:

$$\begin{cases} \min & 3y_1 + 12y_2 \\ \text{s.t.} & 2y_1 - 3y_2 \geq 2 \\ & y_1 + 8y_2 \geq 16 \\ & -y_1 + 2y_2 \geq 12 \\ & y_1, y_2 \geq 0 \end{cases}$$

Determine if the feasible solution $x^* = [6 \ 0 \ 12]$ is also optimal.

Essendo $x_1^*, x_3^* > 0 \Rightarrow$

$$\begin{cases} 2y_1 - 3y_2 = 2 \\ y_1 + 8y_2 \geq 16 \\ -y_1 + 2y_2 = 12 \end{cases} \Rightarrow \begin{cases} 2y_1 = 2 + 3y_2 \\ 2y_2 = 12 + y_1 \end{cases} \Rightarrow \begin{cases} y_1 = \frac{3}{2}y_2 + 1 \\ 2y_2 = 12 + \frac{3}{2}y_2 + 1 \end{cases} \Rightarrow \begin{cases} y_1 = 40 \\ y_2 = 26 \end{cases} \Rightarrow y^* = [40 \ 26]^T$$

y^* dovrebbe essere la sol ottimale per il duale, verifichiamo

$$2x_1^* + 16x_2^* + 12x_3^* = 12 + 144 = 156$$

$$3y_1^* + 12y_2^* = 3 \cdot 40 + 12 \cdot 26 = 432, \text{ gli ottimi non coincidono } \Rightarrow x^* \text{ non e' ottimale.}$$

Consider the following linear program:

$$\begin{aligned} \max \quad & 2x_1 + \alpha x_2 + \beta x_3 \\ \text{s.t.} \quad & 2x_1 + x_2 - x_3 \leq 3 \\ & -3x_1 + 8x_2 + 2x_3 \leq 12 \\ & x_1, x_2, x_3 \geq 0 \end{aligned}$$

Find all the values α, β such that $x^* = \begin{bmatrix} 0 & 0 & 6 \end{bmatrix}$ is an optimal solution.

essendo $x_3^* > 0 \Rightarrow -\gamma_1 + 2\gamma_2 = \beta$

$$\begin{cases} \min & 3\gamma_1 + 12\gamma_2 \\ & 2\gamma_1 - 3\gamma_2 \geq 2 \\ & \gamma_1 + 8\gamma_2 \geq \alpha \\ & -\gamma_1 + 2\gamma_2 \geq \beta \\ & \gamma_1, \gamma_2 \geq 0 \end{cases} \Rightarrow \gamma^* = \begin{bmatrix} 0 & \frac{\beta}{2} \end{bmatrix}$$

Inoltre $2x_1^* + x_2^* - x_3^* < 3 \Rightarrow \gamma_1^* = 0$

Voglio che $\begin{cases} \gamma_1^* + 8\gamma_2^* \geq \alpha \\ 2\gamma_1^* - 3\gamma_2^* \geq 2 \end{cases} \Rightarrow \begin{cases} 4\beta \geq \alpha \\ -\frac{3}{2}\beta \geq 2 \end{cases} \Rightarrow \begin{cases} \beta \geq \frac{\alpha}{4} \\ \beta \geq -\frac{4}{3} \end{cases} \Leftarrow \text{banale perche } \beta \geq 0$

Voglio che gli ottimi siano identici:

$$2x_1^* + \alpha x_2^* + \beta x_3^* = 3\gamma_1^* + 12\gamma_2^* \Rightarrow 6\beta = 12 \frac{\beta}{2} \Rightarrow \beta = 1, x^* \text{ e' sol per } \beta = 1, \alpha \leq 4$$

Consider the following linear program:

$$\begin{aligned} \max \quad & 5x_1 + \alpha x_2 + 8x_3 \\ \text{s.t.} \quad & x_1 + x_2 + 9x_3 \leq 5 \\ & 3x_1 + 4x_2 + 13x_3 \leq 12 \\ & 5x_1 + x_2 + 17x_3 \leq \beta \\ & x_1, x_2, x_3 \geq 0 \end{aligned}$$

Find all the values of α, β such that $x^* = \begin{bmatrix} 1 & 2 & 0 \end{bmatrix}$ is an optimal solution.

Il problema duale e' :

$$\begin{cases} \min & 5\gamma_1 + 12\gamma_2 + \beta\gamma_3 \\ & \gamma_1 + \gamma_2 + 5\gamma_3 \geq 5 \\ & \gamma_1 + 4\gamma_2 + \gamma_3 \geq \alpha \\ & 5\gamma_1 + 13\gamma_2 + 17\gamma_3 \geq 8 \\ & \gamma_1, \gamma_2, \gamma_3 \geq 0 \end{cases}$$

Sia γ^* la sol. ottimale per il duale, se x^* fosse ottimale, dato che $x_1^*, x_2^* > 0$, si avrebbe:

$$\begin{cases} \gamma_1^* + \gamma_2^* + 5\gamma_3^* = 5 \\ \gamma_1^* + 4\gamma_2^* + \gamma_3^* = \alpha \end{cases} \text{ inoltre, essendo che } \begin{cases} x_1^* + x_2^* + 9x_3^* < 5 \\ 3x_1^* + 4x_2^* + 13x_3^* < 12 \end{cases} \Rightarrow \gamma_1^*, \gamma_2^* = 0$$

$$\Rightarrow \begin{cases} \gamma_1^* + \gamma_2^* + 5\gamma_3^* = 5 \\ \gamma_1^* + 4\gamma_2^* + \gamma_3^* = \alpha \\ 5\gamma_1^* + 13\gamma_2^* + 17\gamma_3^* = 8 \end{cases} \Rightarrow \begin{cases} 5\gamma_3^* = 5 \\ \gamma_3^* = \alpha \\ 17\gamma_3^* \geq 8 \end{cases} \Rightarrow \gamma^* = \begin{bmatrix} 0 & 0 & \alpha \end{bmatrix} \text{ allora si vuole che } \alpha = 1$$
$$5\gamma_1^* + 12\gamma_2^* + \beta\gamma_3^* = 5x_1^* + \alpha x_2^* + 8x_3^* \Rightarrow \beta\alpha = 5 + 2\alpha \text{ ma } \alpha = 1 \Rightarrow \beta = 5 + 2 \Rightarrow (\alpha, \beta) = (1, 7)$$

Consider the following linear program:

$$\begin{aligned} \min \quad & 2x_1 + 4x_2 + 2x_3 \\ \text{s.t.} \quad & x_1 + x_2 + 3x_3 \leq 1 \\ & -x_1 + 2x_2 + x_3 \geq 1 \\ & 3x_2 - 6x_3 = 0 \\ & x_1, x_3 \geq 0 \\ & x_2 \in \mathbb{R} \end{aligned}$$

Trasformo il terzo vincolo in due vincoli e scrivo il duale:

Determine if $x^* = \begin{bmatrix} 0 & \frac{2}{5} & \frac{1}{5} \end{bmatrix}$ is an optimal solution.

$$(P) = \begin{cases} \min & 2x_1 + 4x_2 + 2x_3 \\ & x_1 + x_2 + 3x_3 \leq 1 \\ & -x_1 + 2x_2 + x_3 \geq 1 \\ & 3x_2 - 6x_3 \geq 0 \\ & 3x_2 - 6x_3 \leq 0 \\ & x_1, x_3 \geq 0, x_2 \in \mathbb{R} \end{cases} \quad (D) = \begin{cases} \max & \gamma_1 + \gamma_2 \\ & \gamma_1 - \gamma_2 \geq 2 \\ & \gamma_1 + 2\gamma_2 + 3\gamma_3 + 3\gamma_4 = 4 \\ & 3\gamma_1 + \gamma_2 - 6\gamma_3 - 6\gamma_4 \geq 2 \\ & \gamma_1, \gamma_4 \geq 0 \\ & \gamma_2, \gamma_3 \leq 0 \end{cases}$$

Sia γ^* la sol. ott. di (D), essendo $x_3^* > 0 \Rightarrow$

$$3\gamma_1^* + \gamma_2^* - 6\gamma_3^* - 6\gamma_4^* = 2$$

Essendo i vincoli di (P) binding per x^* , non si può costruire un sistema di K eq. in K incognite. Risolvero (D) col simplesso.

$$\begin{cases} \max x & x_1 - x_2 \\ x_1 + x_2 + x_5 = 2 \\ x_1 - 2x_2 - 3x_3 + 3x_4 = 4 \\ 3x_1 - x_2 + 6x_3 - 6x_4 = 2 \\ x \geq 0 \end{cases}$$

Serve una base ammissibile,
provo con $B = \{1, 2, 3\}$

A_B e' non singolare.

$$B = \{1, 2, 3\}$$

$$B = \{1, 3, 5\}$$

$$\begin{array}{l} x_1 = -\frac{6}{5}x_4 + \frac{13}{30}x_5 + 2 \\ x_2 = \frac{6}{5}x_4 - \frac{1}{10}x_5 \\ x_3 = -\frac{1}{5}x_4 - \frac{7}{30}x_5 - \frac{2}{3} \\ \hline z = -\frac{12}{5}x_4 + \frac{8}{15}x_5 + 2 \end{array}$$

\Rightarrow

$$\begin{array}{l} x_1 = -\frac{6}{5}x_4 + \frac{13}{30}x_5 + 2 \\ x_5 = -10x_2 + 12x_4 \\ x_3 = -\frac{1}{5}x_4 - \frac{7}{30}x_5 - \frac{2}{3} \\ \hline z = 4x_4 - \frac{16}{3}x_2 + 2 \end{array}$$

\Rightarrow

per far aumentare z serve
 $B = \{3, 4, 5\}$ ma le colonne
3, 4 di A sono lin. dip.

$$x^* = [2 \ 0 \ -\frac{2}{3} \ 0 \ 0] \Rightarrow$$

l'ottimo di (D) e' uguale
all'ottimo di (P)

$$\Rightarrow x_1^* - x_2^* = 2 = 2x_1^* + 4x_2^* + 2x_3^* = 4\frac{2}{5} + 2\frac{1}{5} = 2 \Rightarrow x^* \text{ e' ottimale.}$$

The following tableau corresponds to an optimal basis for a linear program where x_1, x_2 and x_3 are the original primal variables and s_1, s_2 are the slack variables corresponding to the two constraints.

$$\begin{array}{rcl} x_1 & = & \frac{5}{6}s_1 + \frac{1}{2}s_2 + \frac{1}{6}s_1 - \frac{1}{3}s_2 \\ x_3 & = & \frac{3}{2} - \frac{1}{2}s_2 - \frac{1}{2}s_1 \\ z & = & \frac{65}{2} - 2x_2 - 3s_1 - 2s_2 \end{array}$$

1. Write the original linear program
2. Write the dual of the original program
3. Solve the dual problem via Simplex method

$$\Rightarrow \begin{cases} s_2 = -3x_1 + \frac{5}{4}x_2 - x_3 + 10 \\ s_1 = -\frac{1}{2}x_2 - 2x_3 + 5 \\ z = -\frac{5}{2} - 3x_2 + 8x_3 + 6x_1 \end{cases} \Rightarrow \begin{cases} 3x_1 - \frac{5}{4}x_2 + x_3 + s_2 = 10 \\ \frac{1}{2}x_2 + 2x_3 + s_1 = 5 \end{cases}$$

La matrice e': $A = \begin{bmatrix} 0 & \frac{1}{2} & 2 \\ 3 & -\frac{5}{4} & 1 \end{bmatrix}$ il poliedro soddisfa $Ax = \begin{bmatrix} 10 \\ 5 \end{bmatrix}$. la Funzione obiettivo e' $\max c^T x$ con $c^T = [6 \ -3 \ 8]$

Il duale e'

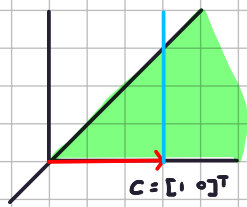
$$\begin{cases} \min 10x_1 + 5x_2 \\ 3x_2 \geq 6 \\ 2x_1 - 5x_2 \geq -12 \\ 2x_1 + x_2 \geq 8 \end{cases} \quad : \quad \begin{cases} \max -10x_1 - 5x_2 \\ 3x_2 + x_3 = 6 \\ 2x_1 - 5x_2 + x_4 = -12 \\ 2x_1 + x_2 + x_5 = 8 \end{cases}$$

Esegui il simplesso

partendo da $B = \{1, 2, 4\}$

Give an example of a linear programs P, P' such that:

1. P is feasible and his dual program D is infeasible
2. Both P' and his dual program D' are infeasible



$$P: \begin{cases} \max x_1 \\ -x_1 + x_2 \leq 0 \\ x_1, x_2 \geq 0 \end{cases} \quad \text{il duale } e'$$

$$\begin{aligned} \min & 0 \\ -z_1 & \geq 1 \\ z_2 & \geq 0 \\ z_1, z_2 & \geq 0 \end{aligned} \Rightarrow \begin{aligned} z_1 & \leq -1 \\ z_1 & \geq 0 \end{aligned} \quad \text{impossibile}$$

$$P': \begin{cases} \max x_2 \\ x_1 \leq -1 \\ -x_2 \leq 0 \\ x_1, x_2 \geq 0 \end{cases} \quad \begin{aligned} & e' \text{ inammissibile} \\ & e' \text{ lo } e' \text{ anche} \\ & \text{il duale} \end{aligned} \Rightarrow \begin{aligned} \min & -z_1 \\ z_1 & \geq 0 \\ -z_2 & \geq 1 \\ z_1, z_2 & \geq 0 \end{aligned} \equiv \begin{aligned} \min & -z_1 \\ z_1 & \geq 0 \\ z_2 & \leq -1 \\ z_1, z_2 & \geq 0 \end{aligned} \Rightarrow \begin{aligned} z_2 & \leq -1 \\ z_2 & \geq 0 \end{aligned} \quad \text{impossibile.}$$

Prove or disprove the following statement: if a linear program has a unique optimal solution then its dual program has a unique optimal solution

E' Falso, si consideri il seguente LP

$$P: \begin{cases} \max x_1 \\ x_1 + x_2 \leq 0 \\ x_1, x_2 \geq 0 \end{cases} \quad \text{Il duale } e' \quad D: \begin{cases} \min 0 \\ z_1 \geq 1 \\ z_2 \geq 0 \\ z_1, z_2 \geq 0 \end{cases}$$

\uparrow ha un'unica soluzione $x^* = [0 \ 0]^T$
 \uparrow ha infinite sol.

