Since I have already taken the midterm, I will only be completing the exercises related to the second part of the course (from differential kinematics onwar

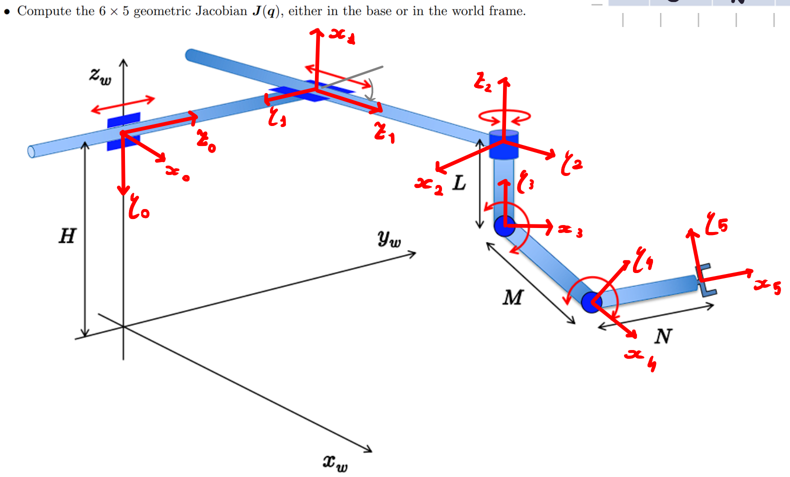
Exercise 3

Consider the 2P3R spatial robot in Fig. 1, shown together with a world frame RF_w. This 5-dof robot is hanging from the ceiling at an height $H > 0$ from the plane $(\boldsymbol{x}_w, \boldsymbol{y}_w)$.

- Assign the link frames and fill in the associated table of parameters according to the Denavit–Hartenberg (DH) convention (use the extra sheet). The origin O_5 of the last DH frame should be placed at the center of the gripper.
- Provide the symbolic expression of the end-effector position, both as ${}^0\boldsymbol{p}_5$ (i.e., in terms of the robot base frame) and as ${}^w\boldsymbol{p}_5$ (expressed in the world frame).
- Compute the 6×5 geometric Jacobian $\boldsymbol{J}(\boldsymbol{q})$, either in the base or in the world frame.

i	α_i	a_i	d_i	θ_i
1	$-\pi/2$	0	q_1	$-\pi/2$
2	$\pi/2$	0	q_2	$\pi/2$
3	$\pi/2$	0	L	q_3
4	0	M	0	q_4
5	0	N	0	q_5

And $${}^wT_0 = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & -1 & 0 & H \\ 0 & 0 & 0 & 1 \end{bmatrix}$$



$${}^0T_1 = \begin{bmatrix} 0 & 0 & 1 & 0 \\ -1 & 0 & 0 & 0 \\ 0 & -1 & 0 & q_1 \\ 0 & 0 & 0 & 1 \end{bmatrix} \qquad {}^1T_2 = \begin{bmatrix} 0 & 0 & 1 & 0 \\ 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & q_2 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$${}^2T_3 = \begin{bmatrix} c_3 & 0 & s_3 & 0 \\ s_3 & 0 & -c_3 & 0 \\ 0 & 1 & 0 & L \\ 0 & 0 & 0 & 1 \end{bmatrix} \qquad {}^3T_4 = \begin{bmatrix} c_4 & -s_4 & 0 & Mc_4 \\ s_4 & c_4 & 0 & Ms_4 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$${}^4T_5 = \begin{bmatrix} c_5 & -s_5 & 0 & Nc_5 \\ s_5 & c_5 & 0 & Ns_5 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

I have to compute ${}^0T_5$

$${}^0T_2 = {}^0T_1\,{}^1T_2 = \begin{bmatrix} 0 & 0 & 1 & 0 \\ -1 & 0 & 0 & 0 \\ 0 & -1 & 0 & q_1 \\ 0 & 0 & 0 & 1 \end{bmatrix}\begin{bmatrix} 0 & 0 & 1 & 0 \\ 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & q_2 \\ 0 & 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} 0 & 1 & 0 & q_2 \\ 0 & 0 & -1 & 0 \\ -1 & 0 & 0 & q_1 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$${}^2T_4 = \begin{bmatrix} c_3 & 0 & s_3 & 0 \\ s_3 & 0 & -c_3 & 0 \\ 0 & 1 & 0 & L \\ 0 & 0 & 0 & 1 \end{bmatrix}\begin{bmatrix} c_4 & -s_4 & 0 & Mc_4 \\ s_4 & c_4 & 0 & Ms_4 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} c_3c_4 & -c_3s_4 & s_3 & Mc_3c_4 \\ s_3c_4 & -s_3s_4 & -c_3 & Ms_3c_4 \\ s_4 & c_4 & 0 & Ms_4+L \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$${}^2T_5 = \begin{bmatrix} c_3c_4 & -c_3s_4 & s_3 & Mc_3c_4 \\ s_3c_4 & -s_3s_4 & -c_3 & Ms_3c_4 \\ s_4 & c_4 & 0 & Ms_4+L \\ 0 & 0 & 0 & 1 \end{bmatrix}\begin{bmatrix} c_5 & -s_5 & 0 & Nc_5 \\ s_5 & c_5 & 0 & Ns_5 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} =$$

$$\begin{bmatrix} c_3c_4c_5 - c_3s_4s_5 & -c_3c_4s_5 - c_3s_4c_5 & s_3 & Nc_3c_4c_5 - Nc_3s_4s_5 + Mc_3c_4 \\ s_3c_4c_5 - s_3s_4s_5 & -s_3c_4s_5 - s_3s_4c_5 & -c_3 & Ns_3c_4c_5 - Ns_3s_4s_5 + Ms_3c_4 \\ s_4c_5 + c_4s_5 & -s_4s_5 + c_4c_5 & 0 & Ns_4c_5 + Nc_4c_5 + Ms_4 + L \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$${}^0T_5 = \begin{bmatrix} 0 & 1 & 0 & q_2 \\ 0 & 0 & -1 & 0 \\ -1 & 0 & 0 & q_1 \\ 0 & 0 & 0 & 1 \end{bmatrix}\begin{bmatrix} c_3c_4c_5 - c_3s_4s_5 & -c_3c_4s_5 - c_3s_4c_5 & s_3 & Nc_3c_4c_5 - Nc_3s_4s_5 + Mc_3c_4 \\ s_3c_4c_5 - s_3s_4s_5 & -s_3c_4s_5 - s_3s_4c_5 & -c_3 & Ns_3c_4c_5 - Ns_3s_4s_5 + Ms_3c_4 \\ s_4c_5 + c_4s_5 & -s_4s_5 + c_4c_5 & 0 & Ns_4c_5 + Nc_4c_5 + Ms_4 + L \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

the DK from RF_0 to RF_5 is : $$\begin{bmatrix} Ns_3(c_4c_5 - s_4s_5) + Ms_3c_4 + q_2 \\ -Nc_5(s_4 + c_4) - Ms_4 - L \\ -Nc_3(c_4c_5 - s_4s_5) - Mc_3c_4 + q_1 \end{bmatrix}$$

Exercise 5

A 2R planar robot with link lengths $l_1 = 1.2$, $l_2 = 0.8$ [m] is at rest at $t = 0$ in the configuration $\boldsymbol{q}_0 = \boldsymbol{0}$ (stretched along the $\boldsymbol{x}_0$ axis). A pointwise target moves at constant speed $v = 1.5$ m/s on a straight line with an angle $\delta = 15°$ from the $\boldsymbol{x}_0$ axis, being in $\boldsymbol{p}_0 = \begin{pmatrix} -2 & 1 \end{pmatrix}^T$ [m] at $t = 0$ and entering after in the robot workspace. Solve the following rendez-vous problem:

a) define a trajectory that will bring the robot end-effector on the target when the latter crosses the $\boldsymbol{y}_0$ axis; the end-effector should have then the same velocity $\boldsymbol{v}_t \in \mathbb{R}^2$ of the target;

b) provide the rendez-vous time $t_{rv} > 0$ and the expression of the command $\dot{\boldsymbol{q}}(t) \in \mathbb{R}^2$, $t \in [0, t_{rv}]$.

How would you modify the velocity command $\dot{\boldsymbol{q}}(t)$ as a function of $\boldsymbol{q}(t)$ so as to reach the target at the rendez-vous position if the robot starts from a configuration close but different from $\boldsymbol{q}_0$?

I compute the vector velocity v of the point: $v = \begin{pmatrix} 1.5\cos(\frac{1}{12}\pi) \\ 1.5\sin(\frac{1}{12}\pi) \end{pmatrix} \approx \begin{pmatrix} 1.448 \\ 0.388 \end{pmatrix}$

point pos

$\Rightarrow c(t) = \int_0^t v\, d\tau = \begin{pmatrix} -2 \\ 1 \end{pmatrix} + t \begin{pmatrix} 1.448 \\ 0.388 \end{pmatrix}$ this crosses the y-axis when:

$-2 + t\, 1.448 = 0 \Rightarrow t^* = 1.34408 \Rightarrow c(t^*) = (0\,,\ 1.521)^T$

So i want that the robot goes from $(2,0)^T$ to $(0\,,\ 1.521)^T$ with $v_i = \begin{pmatrix} 0 \\ 0 \end{pmatrix}$ and $v_f = \begin{pmatrix} 1.448 \\ 0.388 \end{pmatrix}$. I can plane these directly in the joint space with $q_i = (0,0)^T$, $q_f = f^{-1}((0,\ 1.521)^T)$, $\dot{q}_i = 0$, $\dot{q}_f = J^{-1} v_f$:

the DK is $f_r(q) = \begin{pmatrix} 1.2\,c_1 + 0.8\,c_{12} \\ 1.2\,s_1 + 0.8\,s_{12} \end{pmatrix}$

IK

$\begin{cases} 1.2\,c_1 + 0.8\,c_{12} = 0 \\ 1.2\,s_1 + 0.8\,s_{12} = 1.521 \end{cases} \Rightarrow \begin{matrix} \cos q_2 = (2.31 - 2.08)\frac{1}{1.92} = 0.119 \\ \sin q_2 = \pm 0.992 \end{matrix} \Rightarrow q_2^+ = 1.451$

$q_1^+ = \text{atan2}\{1.521, 0\} \cdots = \frac{\pi}{2} - \text{atan}\{0.8 \cdot 0.992,\ 1.2 + 0.8 \cdot 0.119\} = \frac{\pi}{2} - 0.549 = 1.021$

$\Rightarrow q_f = (1.021,\ 1.451)^T$ Now i compute J:

$J = \begin{pmatrix} -1.2\,s_1 - 0.8\,s_{12} & -0.8\,s_{12} \\ 1.2\,c_1 + 0.8\,c_{12} & 0.8\,c_{12} \end{pmatrix} \Rightarrow J(q_f) = \begin{pmatrix} -1.51 & -0.49 \\ 0 & -0.62 \end{pmatrix} \Rightarrow J^{-1}(q_f) = \begin{pmatrix} -0.662 & 0.523 \\ 0 & -1.612 \end{pmatrix}$

$\dot{q}_f = \begin{pmatrix} -0.662 & 0.523 \\ 0 & -1.612 \end{pmatrix} \begin{pmatrix} 1.448 \\ 0.388 \end{pmatrix} = -\begin{pmatrix} 0.755 \\ 0.625 \end{pmatrix}$

$\Rightarrow q_1(t) = at^3 + bt^2 + ct + d$

$q_1(0) = d = 0 \qquad \dot{q}_1(0) = c = 0$

$\begin{matrix} q_1(t^*) = q_1(1.344) = \\ \dot{q}_1(t^*) = \dot{q}_1(1.344) = \end{matrix} \begin{cases} a\,2.427 + b\,1.806 = 1.021 \\ a\,1.806 + b\,1.344 = -0.755 \end{cases} \Rightarrow \begin{cases} a = 0.0013 \\ b = -0.5635 \end{cases}$

Same procedure for q_2.

$q_2(t) = at^3 + bt^2 + ct + d \Rightarrow q_2(0) = 0 \Rightarrow d = 0, \quad \dot{q}_2(0) = 0 \Rightarrow c = 0$

$$\Rightarrow \quad \begin{aligned} q_2(t^*) &= q_2(1.344) \\ \dot{q}_2(t^*) &= \dot{q}_2(1.344) \end{aligned} = \begin{cases} a\,2.427 + b\,1.806 = 1.451 \\ a\,1.806 + b\,1.344 = -0.625 \end{cases} \Rightarrow \begin{cases} a = 0.1259 \\ b = -0.6345 \end{cases}$$

$$\Rightarrow \quad q(t) = \begin{cases} 0.0013t^3 - 0.5635t^2 \\ 0.1259t^3 - 0.6345t^2 \end{cases}$$