Exercise 1

The end-effector of a robot manipulator has an initial orientation specified by the ZXY Euler angles $(\alpha, \beta, \gamma) = (\pi/2, \pi/4, -\pi/4)$ [rad] and should reach a final orientation specified by an axis-angle pair (r, θ), with $r = (0, -\sqrt{2}/2, \sqrt{2}/2)$ and $\theta = \pi/6$ rad. What is the required rotation matrix R_{if} between these two orientations? Represent R_{if} by the RPY-type angles (ϕ, χ, ψ) around the fixed-axes sequence YXY.

Given the base frame RF_0, we compute ${}^0R_i$ and ${}^0R_5$ and then consider ${}^iR_0 {}^0R_5 = {}^0R_i^T {}^0R_5 = {}^iR_5$.

$${}^0R_i = R_z(\tfrac{\pi}{2}) R_x(\tfrac{\pi}{4}) R_y(-\tfrac{\pi}{4}) = \begin{bmatrix} 1/2 & -\sqrt{2}/2 & 1/2 \\ \sqrt{2}/2 & 0 & -\sqrt{2}/2 \\ 1/2 & \sqrt{2}/2 & 1/2 \end{bmatrix}$$

Then given r and θ we consider $rr^T + (I - rr^T)\cos\theta + S(r)\sin\theta$

— rounded →

$$rr^T + (I - rr^T)\cos\theta + S(r)\sin\theta = {}^0R_5 \approx \begin{bmatrix} 0.86 & -0.35 & -0.35 \\ 0.35 & 0.93 & -0.06 \\ 0.35 & -0.06 & 0.93 \end{bmatrix}$$

We have ${}^0R_i^T {}^0R_5 \approx \begin{bmatrix} 0.85 & 0.44 & 0.24 \\ -0.36 & 0.2 & 0.9 \\ 0.35 & -0.87 & 0.33 \end{bmatrix}$

Since the RPY for angles (ϕ, χ, ψ) in axes YXY have a matrix:

$$R_c(\psi) R_x(\chi) R_c(\phi) = \begin{bmatrix} \cos\psi & 0 & \sin\psi \\ 0 & 1 & 0 \\ -\sin\psi & 0 & \cos\psi \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 \\ 0 & \cos\chi & -\sin\chi \\ 0 & \sin\chi & \cos\chi \end{bmatrix} \begin{bmatrix} \cos\phi & 0 & \sin\phi \\ 0 & 1 & 0 \\ -\sin\phi & 0 & \cos\phi \end{bmatrix}$$

$$\begin{bmatrix} \cos\psi & \sin\psi\sin\chi & \sin\psi\cos\chi \\ 0 & \cos\chi & -\sin\chi \\ -\sin\psi & \cos\psi\sin\chi & \cos\psi\cos\chi \end{bmatrix} \begin{bmatrix} \cos\phi & 0 & \sin\phi \\ 0 & 1 & 0 \\ -\sin\phi & 0 & \cos\phi \end{bmatrix}$$

$\cos\psi\cos\phi - \sin\psi\cos\chi\sin\phi$ 　　　$\sin\psi\sin\chi$ 　　　$\cos\psi\sin\phi + s\psi c\chi c\phi$

$\sin\chi\sin\phi$ 　　　$\cos\chi$ 　　　$-\sin\chi\cos\phi$

$-\sin\psi\cos\phi - \sin\phi\cos\psi\cos\chi$ 　　　$\cos\psi\sin\chi$ 　　　$-\sin\psi\sin\phi + c\psi c\chi c\phi$

$$\begin{bmatrix} c\psi c\phi - s\psi c\chi s\phi & c\psi s\chi & c\psi s\phi + s\psi c\chi c\phi \\ s\chi s\phi & c\chi & -s\chi c\phi \\ -s\psi c\phi - s\phi c\psi c\chi & c\psi s\chi & -s\psi s\phi + c\phi c\chi c\psi \end{bmatrix} = \begin{bmatrix} 0.85 & 0.44 & 0.24 \\ -0.36 & 0.2 & 0.9 \\ 0.35 & -0.87 & 0.33 \end{bmatrix}$$

$\cos \chi = 0.2026 \Rightarrow \cos^2 \chi = 0.04104 \Rightarrow \sin^2 = 1 - 0.04104 = 0.9589$

$\Rightarrow \sin \chi = \pm 0.979262 \Rightarrow \chi^{.\dagger} = \begin{cases} 1,366 \\ -1,366 \end{cases}$

$\Rightarrow \sin \phi = -\dfrac{0.36}{\pm 0.97} \Rightarrow \cos \phi = \dfrac{0.9}{\mp 0.97}$

$\Rightarrow \phi = atan2 \left\{ \dfrac{-0.36}{\pm 0.97} , \dfrac{0.9}{\mp 0.97} \right\} = \begin{cases} -2.76 \\ 0.38 \end{cases}$