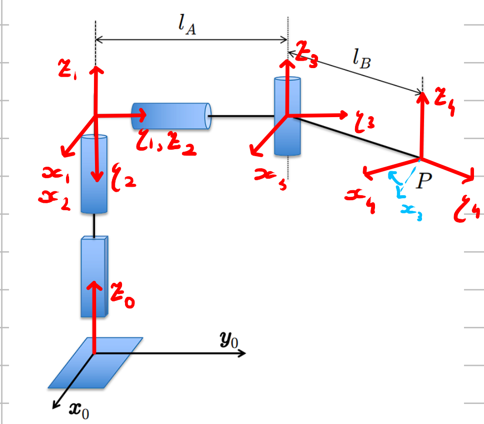
Since I have already taken the midterm, I will only be completing the exercises related to the second part of the course (from differential kinematics onwards).

Exercise 1

Consider the 4-dof manipulator in Fig. 1. The robot has the first joint prismatic and the other three revolute. Determine a frame assignment and the associated table of parameters following the Denavit-Hartenberg (DH) convention. Assign the given geometric data l_A and l_B to the corresponding constant DH parameters. The origin of the first DH frame RF_0 is already specified, while the origin of the last frame RF_4 should be placed in P. Use the provided Extra Sheet #1 to draw the frames, and complete the DH table there. Add your name on the sheet and return it.



i	α_i	a_i	d_i	θ_i
1	0	0	q_1	0
2	$-\pi/2$	0	0	q_2
3	$\pi/2$	0	l_A	q_3
4	0	l_B	0	q_4

$$ {}^0T_1 = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & q_1 \\ 0 & 0 & 0 & 1 \end{pmatrix} \quad {}^1T_2 = \begin{pmatrix} c_2 & 0 & -s_2 & 0 \\ s_2 & 0 & c_2 & 0 \\ 0 & -1 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix} $$

$$ {}^2T_3 = \begin{pmatrix} c_3 & 0 & s_3 & 0 \\ s_3 & 0 & -c_3 & 0 \\ 0 & 1 & 0 & l_A \\ 0 & 0 & 0 & 1 \end{pmatrix} \quad {}^3T_4 = \begin{pmatrix} c_4 & -s_4 & 0 & l_B c_4 \\ s_4 & c_4 & 0 & l_B s_4 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix} $$

$$ {}^0T_2 = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & q_1 \\ 0 & 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} c_2 & 0 & -s_2 & 0 \\ s_2 & 0 & c_2 & 0 \\ 0 & -1 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix} = \begin{pmatrix} c_2 & 0 & -s_2 & 0 \\ s_2 & 0 & c_2 & 0 \\ 0 & -1 & 0 & q_1 \\ 0 & 0 & 0 & 1 \end{pmatrix} $$

$$ {}^0T_3 = \begin{pmatrix} c_2 & 0 & -s_2 & 0 \\ s_2 & 0 & c_2 & 0 \\ 0 & -1 & 0 & q_1 \\ 0 & 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} c_3 & 0 & s_3 & 0 \\ s_3 & 0 & -c_3 & 0 \\ 0 & 1 & 0 & l_A \\ 0 & 0 & 0 & 1 \end{pmatrix} = \begin{pmatrix} c_2c_3 & -s_2 & c_2s_3 & -l_A s_2 \\ s_2c_3 & c_2 & s_2s_3 & l_A c_2 \\ -s_3 & 0 & c_3 & q_1 \\ 0 & 0 & 0 & 0 \end{pmatrix} $$

$$ {}^0T_4 = \begin{pmatrix} c_2c_3 & -s_2 & c_2s_3 & -l_A s_2 \\ s_2c_3 & c_2 & s_2s_3 & l_A c_2 \\ -s_3 & 0 & c_3 & q_1 \\ 0 & 0 & 0 & 0 \end{pmatrix} \begin{pmatrix} c_4 & -s_4 & 0 & l_B c_4 \\ s_4 & c_4 & 0 & l_B s_4 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix} \Rightarrow {}^0R_4 = \begin{pmatrix} c_2c_3c_4 - s_2s_4 & -c_2c_3s_4 - s_2c_4 & c_2s_3 \\ s_2c_3c_4 + c_2s_4 & -s_2c_3s_4 + c_2c_4 & s_2s_3 \\ -s_3c_4 & s_3s_4 & c_3 \\ 0 & 0 & 0 \end{pmatrix} $$

$$ \Rightarrow f(q) = \begin{pmatrix} c_2c_3 & -s_2 & c_2s_3 & -l_A s_2 \\ s_2c_3 & c_2 & s_2s_3 & l_A c_2 \\ -s_3 & 0 & c_3 & q_1 \\ 0 & 0 & 0 & 0 \end{pmatrix} \begin{pmatrix} l_B c_4 \\ l_B s_4 \\ 0 \\ 1 \end{pmatrix} = \begin{cases} l_B(c_2c_3c_4 - s_2s_4) - l_A s_2 \\ l_B(s_2c_3c_4 + c_2s_4) + l_A c_2 \\ -l_B s_3 c_4 + q_1 \end{cases} $$

Exercise 2

For the 4-dof manipulator of Exercise 1:

a) Determine the direct kinematics $\boldsymbol{p} = \boldsymbol{f}(\boldsymbol{q})$ of point P in symbolic form. Using the numerical values $l_A = 0.5$ [m] and $l_B = 0.75$ [m], evaluate the position $\boldsymbol{p}$ in the two configurations $\boldsymbol{q}^I = \boldsymbol{0}$ and $\boldsymbol{q}^{II} = \begin{pmatrix} 1 & 0 & -\pi/2 & \pi/2 \end{pmatrix}^T$.

b) Find the symbolic expression of the generalized joint force/torque $\boldsymbol{\tau} \in \mathbb{R}^4$ that balances the purely vertical, downward force $\boldsymbol{F} \in \mathbb{R}^3$ applied at the point P as shown in Fig. 1, so that the robot remains in a static equilibrium. Using the same previous numerical values for l_A and l_B, evaluate $\boldsymbol{\tau}$ at the two given configurations $\boldsymbol{q}^I$ and $\boldsymbol{q}^{II}$.

c) Determine the angular part of the geometric Jacobian $\boldsymbol{J}_A(\boldsymbol{q})$ that relates the joint velocity $\dot{\boldsymbol{q}} \in \mathbb{R}^4$ of the robot to the angular velocity $\boldsymbol{\omega} \in \mathbb{R}^3$ of its end-effector frame RF_4. Study the singular configurations of $\boldsymbol{J}_A(\boldsymbol{q})$. Find a basis for all possible $\dot{\boldsymbol{q}} \in \mathbb{R}^4$ that produce $\boldsymbol{\omega} = \boldsymbol{0}$ when the Jacobian $\boldsymbol{J}_A(\boldsymbol{q})$ looses rank.

let $l_A = 1/2$, $l_B = 3/4$.

$$ f(0) = \begin{cases} 0.75 \\ 0.5 \\ 0 \end{cases} \qquad f(q^{II}) = \begin{cases} 0 \\ 1.25 \\ 1 \end{cases} $$

$$ J_L = \begin{pmatrix} 0 & l_B(-s_2c_3c_4 - c_2s_4) - l_A c_2 & -l_B c_2 s_3 c_4 & l_B(-c_2c_3s_4 - s_2c_4) \\ 0 & l_B(c_2c_3c_4 - s_2s_4) - l_A s_2 & -l_B s_2 s_3 c_4 & l_B(-s_2c_3s_4 + c_2c_4) \\ 1 & 0 & -l_B c_3 c_4 & l_B s_3 s_4 \end{pmatrix}. $$

Since $J^T F = \tau$ to balance a force $F = (0 \; 0 \; -1)^T$; consider:

$$ -J_L^T \begin{pmatrix} 0 \\ 0 \\ -1 \end{pmatrix} = \begin{pmatrix} 1 & 0 & -l_B c_3 c_4 & l_B s_3 s_4 \end{pmatrix}^T $$ so the τ are: $$ \left\{ a \cdot \begin{pmatrix} 1 \\ 0 \\ -l_B c_3 c_4 \\ l_B s_3 s_4 \end{pmatrix} : a \in \mathbb{R}^+ \right\} $$

With $q^I = 0$: $\tau = (1 \;\; 0 \;\; -\frac{3}{4} \;\; 0)$ with q^{II}: $\tau = (1 \;\; 0 \;\; 0 \;\; -\frac{3}{4})$

For the angular part:

$$z_0 = \begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix} \quad z_1 = \begin{pmatrix} -S_2 \\ C_2 \\ 0 \end{pmatrix} \quad z_2 = \begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix} \quad z_3 = \begin{pmatrix} C_2 S_3 \\ S_2 S_3 \\ C_3 \end{pmatrix} \Rightarrow$$

$$J_A = \begin{pmatrix} 0 & -S_2 & 0 & C_2 S_3 \\ 0 & C_2 & 0 & S_2 S_3 \\ 1 & 0 & 1 & C_3 \end{pmatrix}.$$ the Jacobian is singular if $q_3 = 0$:

$$J_A(q_3 = 0) = \begin{pmatrix} 0 & -S_2 & 0 & 0 \\ 0 & C_2 & 0 & 0 \\ 1 & 0 & 1 & 1 \end{pmatrix}$$ rank $= 2 < 3$. Since $\omega = J\dot{q}$ i study $\operatorname{Ker} J$ when J is singular.

$$\begin{pmatrix} 0 & -S_2 & 0 & 0 \\ 0 & C_2 & 0 & 0 \\ 1 & 0 & 1 & 1 \end{pmatrix} \begin{pmatrix} \dot{q}_1 \\ \dot{q}_2 \\ \dot{q}_3 \\ \dot{q}_4 \end{pmatrix} = \begin{cases} -\dot{q}_2 S_2 = 0 \\ -\dot{q}_2 C_2 = 0 \\ \dot{q}_1 + \dot{q}_3 + \dot{q}_4 = 0 \end{cases} \Rightarrow \begin{cases} \dot{q}_2 = 0 \\ \sum_{i=1}^{3} \dot{q}_i = 0 \end{cases} \Rightarrow \mathcal{N}(J) = \operatorname{Span}\left\{ \begin{pmatrix} 2 \\ 0 \\ -1 \\ -1 \end{pmatrix} \right\}$$

Exercise 3

Consider the 3-dof, planar PPR robot in Fig. 2, with a third link of length $L > 0$. The robot end-effector should move at a constant speed $v > 0$, tracing counterclockwise a full circle of radius $R > 0$ centered in $\boldsymbol{P}_c$, and keep its end-effector always aligned with the normal to the surface, pointing toward the circle center. Neglect for simplicity any possible collision between the robot body and the circle (e.g., they may live on two parallel, but different horizontal planes). At time $t = 0$, the robot end-effector is correctly at the initial point $\boldsymbol{A}$ with the right orientation. Suppose that the two prismatic joints have a (common) maximum velocity limit $|\dot{q}_i| \leq V$, $i = 1, 2$, while the revolute joint velocity is limited by $|\dot{q}_3| \leq \Omega$, with $V > 0$ and $\Omega > 0$. Similarly, for the joint accelerations, there are the bounds $|\ddot{q}_i| \leq A$, $i = 1, 2$, and $|\ddot{q}_3| \leq \Psi$, with $A > 0$ and $\Psi > 0$.

a) Provide the expressions of the time evolution of the end-effector position $\boldsymbol{p} \in \mathbb{R}^2$, velocity $\dot{\boldsymbol{p}}$ and acceleration $\ddot{\boldsymbol{p}}$, possibly using separation in space and time, when the given task is perfectly executed. Similarly, provide the expressions of the time evolution of the end-effector absolute orientation angle $\phi \in \mathbb{R}$ and of its derivatives $\dot{\phi}$ and $\ddot{\phi}$. Sketch qualitative plots of all these quantities (keep symbolic values).

b) Provide the associated expressions of the time evolution of the joint position $\boldsymbol{q} \in \mathbb{R}^3$, velocity $\dot{\boldsymbol{q}}$ and acceleration $\ddot{\boldsymbol{q}}$. Sketch qualitative plots also of these quantities.

c) Determine, as a function of the parametric data, the expression of the maximum constant speed v at which the task can be completed without violating any of the physical limits of the robot. Accordingly, give the minimum time T for completing one full round of the circle while remaining feasible.

the parametrization of that curve is:

$$r(s) = \begin{pmatrix} P_d(s) \\ \phi_d(s) \end{pmatrix} = \begin{cases} P_{Cx} + R\cos(2\pi s) \\ P_{Cy} + R\sin(2\pi s) \\ -2\pi s - \pi \end{cases}$$ with $s \in [0, 1]$ or $s = \frac{\sigma}{2\pi R}$ and $\sigma \in [0, 2\pi R]$

with $s(t) = vt$ $\quad t \in [0, \frac{1}{v}]$, $\dot{s} = v$ and $\ddot{s} = 0$.

$$\dot{r}(t) = \frac{dr}{ds} \cdot \dot{s} = \begin{cases} -2\pi v R\sin(2\pi vt) \\ 2\pi v R\cos(2\pi vt) \\ -2\pi v \end{cases} \qquad \ddot{r}(t) = \frac{dr^2}{ds^2}\dot{s}^2 + \frac{dr}{ds}\ddot{s} = \frac{dr^2}{ds^2}\dot{s}^2 = \begin{cases} -4\pi^2 v^2 R\cos(2\pi vt) \\ -4\pi^2 v^2 R\sin(2\pi vt) \\ 0 \end{cases}$$

the DK is $$\delta_r(q) = \begin{cases} q_1 + LC_3 \\ q_2 + LS_3 \\ q_3 \end{cases} \Rightarrow$$ Now: solve the IK

$$\begin{cases} P_x = q_1 + LC_3 \\ P_y = q_2 + LS_3 \\ \phi = q_3 \end{cases} \Rightarrow \begin{cases} P_x = q_1 + LC\phi \\ P_y = q_2 + LS\phi \end{cases} \Rightarrow \begin{cases} q_1 = P_x - LC\phi \\ q_2 = P_y - LS\phi \end{cases} \Rightarrow q(s) = \begin{pmatrix} P_x(s) \\ P_y(s) \\ \phi_d(s) \end{pmatrix} - \begin{pmatrix} L\cos(\phi_d(s)) \\ L\sin(\phi_d(s)) \\ 0 \end{pmatrix} = \begin{cases} P_{Cx} + R\cos(2\pi s) - L\cos(-2\pi s - \pi) \\ P_{Cy} + R\sin(2\pi s) - L\sin(-2\pi s - \pi) \\ -2\pi s - \pi \end{cases}$$

$$q(t) = \begin{cases} P_{Cx} + R\cos(2\pi vt) - L\cos(-2\pi vt - \pi) \\ P_{Cy} + R\sin(2\pi vt) - L\sin(-2\pi vt - \pi) \\ -2\pi vt - \pi \end{cases} \Rightarrow \dot{q}(t) = \begin{cases} -2\pi vR\sin(2\pi vt) - 2\pi vL\sin(-2\pi vt - \pi) \\ 2\pi vR\cos(2\pi vt) + 2\pi vL\cos(-2\pi vt - \pi) \\ -2\pi v \end{cases}$$

$$\ddot{q}(t) = \begin{cases} -(2\pi v)^2 R\cos(2\pi vt) + (2\pi v)^2 L\cos(-2\pi vt - \pi) \\ -(2\pi v)^2 R\sin(2\pi vt) + (2\pi v)^2 L\sin(-2\pi vt - \pi) \\ 0 \end{cases}$$

Since $|\dot{q}_i| \le V \quad i=1,2 \Rightarrow |\dot{s}| \le \min_{i\in\{1,2\}} \left\{ \frac{V}{\max|q'_i|} \right\} = \frac{V}{2\pi\nu R} = V_M$

$|\dot{q}_3| \le \Omega \Rightarrow |\dot{s}| \le \frac{\Omega}{2\pi\nu} = \Omega_M$

$$|\ddot{q}_1| \le A \Rightarrow |\ddot{s}| \le \frac{A - V_M^2 \max|q''_i|}{\max|q'_i|} = \left[A - \left(\frac{V}{2\pi\nu R}\right)^2 \cdot 4\pi^2\nu^2\right]\frac{1}{2\pi\nu R} = A_M$$

$$|\ddot{q}_3| \le \Psi \Rightarrow |\ddot{s}| \le \frac{\Psi - \Omega_M^2 \max|q''|}{\max|q'_3|} = \frac{\Psi}{2\pi\nu} \quad \text{since } q''_3 = 0.$$

So: $\nu \le \frac{\Omega}{2\pi\nu} \Rightarrow \nu \le \sqrt{\frac{\Omega}{2\pi}}$ and $\nu \le \frac{V}{2\pi\nu R} \Rightarrow \nu \le \sqrt{\frac{V}{2\pi R}} \Rightarrow$

$$\nu = \min\left\{ \sqrt{\frac{\Omega}{2\pi}}, \sqrt{\frac{V}{2\pi R}} \right\} \Rightarrow T = \frac{1}{\nu}$$

Exercise 4

A number of questions and statements are reported on the Extra Sheet #2. Fill in your answers on the same sheet, providing also a short motivation/explanation for each item. Add your name on the sheet and return it.

1. Order the three classes of infrared, laser, and ultrasound proximity sensors in terms of their typical range of measurement.

2. Order infrared, laser, and ultrasound sensors in terms of their typical angular resolution.

3. Compare the motor-side position resolution of an incremental encoder with 512 pulses per revolution (PPR) and quadrature electronics mounted on the motor with that of an absolute encoder with 16 bits mounted on the link, when the transmission has reduction ratio $n_r = 20$, Which one is better?

4. Given a desired end-effector position for a planar PPR robot, the gradient method will always provide a solution to the inverse kinematics problem without need of restarting procedures. True or false? Why?
   This is true since a PPR doesn't have any singularities so the gradient method will always converge to the correct solution

5. What is the so-called overfly in trajectory planning and which are its pros and cons? Can this concept be applied equally well at the joint level and at the Cartesian level or not? Why?
   When we approximate an edge with a smooth curve, this makes the path differentiable, so there is not discontinuity in the cartesian velocity.

6. We have four positional knots to be interpolated in the 3D Cartesian space, plus a number of boundary conditions and continuity requirements. Should we use 4-3-4 polynomials or cubic splines? If both can be used, which choice is better and why?

7. For a single robot joint, we have computed a spline trajectory interpolating $n = 10$ given knots at some assigned instants of time $t_1 < t_2 < \cdots < t_{10}$. If we modify only one of such time instants, but still satisfying the sequential order —e.g., the kth instant t_k becomes a new $t'_k \in (t_{k-1}, t_{k+1})$, and then redo the computations, will the trajectory change or not? Why?

8. A robot commanded at the joint velocity level has initially zero position and orientation errors with respect to a desired end-effector trajectory, except along the z-component in position. If we apply a Cartesian kinematic control law, the robot will move so that ...