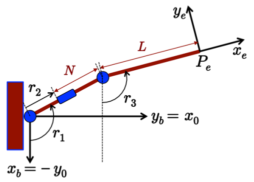
Exercise 1 [8 points]

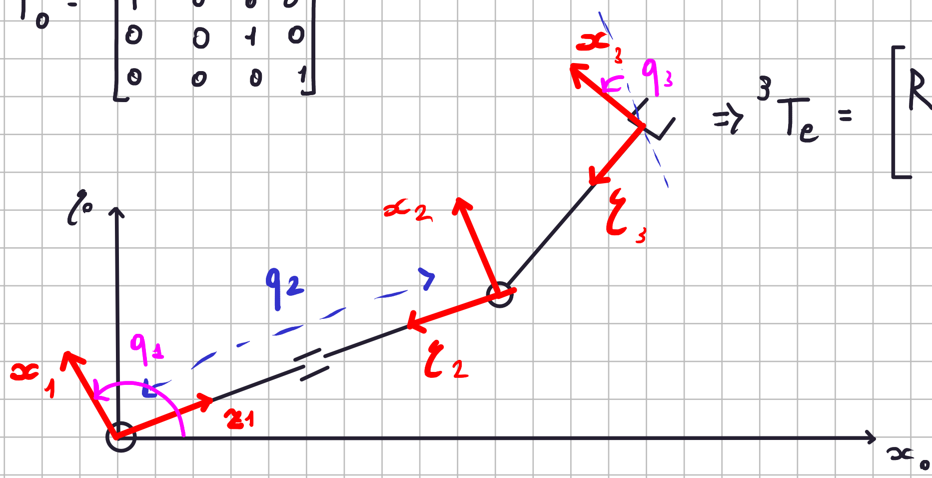
Consider the 3-dof (RPR) planar robot in Fig. 1, where the joint coordinates $\boldsymbol{r} = \begin{pmatrix} r_1 & r_2 & r_3 \end{pmatrix}^T$ have been defined in a free, arbitrary way, with reference to a base frame RF_b.



$$\begin{cases} p_x = (N + r_2)\cos r_1 + L\cos(r_1 + r_3) \\ p_y = (N + r_2)\sin r_1 + L\sin(r_1 + r_3) \end{cases} \quad \text{in reference frame } RF_B$$

$${}^{B}T_0 = \begin{bmatrix} 0 & -1 & 0 & 0 \\ 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

I now assign the DH Frames



$$\Rightarrow {}^{3}T_e = \begin{bmatrix} R_x(\frac{\pi}{2}) & 0 \\ 0 & 1 \end{bmatrix}$$

	α_i	a_i	d_i	θ_i
1	$\pi/2$	0	0	q_1
2	$-\pi/2$	0	q_2	0
3	0	L	0	q_3

$$\begin{bmatrix} C_1 & 0 & S_1 & 0 \\ S_1 & 0 & -C_1 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \cdot \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & -1 & 0 & q_2 \\ 0 & 0 & 0 & 1 \end{bmatrix} \cdot \begin{bmatrix} C_3 & -S_3 & 0 & LC_3 \\ S_3 & C_3 & 0 & LS_3 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} =$$

$$\begin{bmatrix} C_1 & -S_1 & 0 & q_2 S_1 \\ S_1 & C_1 & 0 & -q_2 C_1 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} C_3 & -S_3 & 0 & LC_3 \\ S_3 & C_3 & 0 & LS_3 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} =$$

$$\begin{bmatrix} C_1C_3 - S_1S_3 & -C_1S_3 - S_1C_3 & 0 & LC_1C_3 - LS_1S_3 + q_2S_1 \\ S_1C_3 + C_1S_3 & -S_1S_3 + C_1C_3 & 0 & LS_1C_3 + LC_1S_3 - q_2C_1 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

We have to solve:

$$\begin{cases} Lc_1c_3 - Ls_1s_3 + q_2 s_1 = (N+r_2)\cos r_1 + L\cos(r_1+r_3) \\ Ls_1c_3 + Lc_1s_3 - q_2 c_1 = (N+r_2)\sin r_1 + L\sin(r_1+r_3) \end{cases}$$

For q

$$\begin{cases} Lc_{13} + q_2 s_1 = (N+r_2)\cos r_1 + L\cos(r_1+r_3) \\ Ls_{13} - q_2 c_1 = (N+r_2)\sin r_1 + L\sin(r_1+r_3) \end{cases}$$

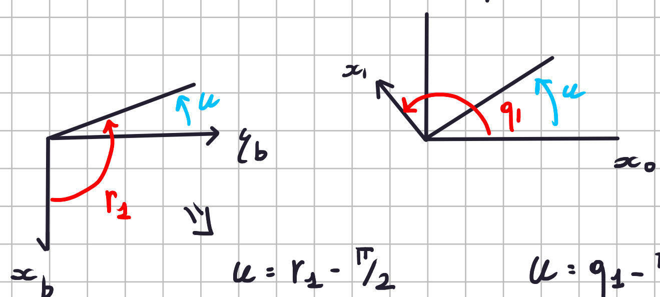
$$L^2 + q_2^2 + 2Lc_{13}q_2s_1 - 2Lq_2c_1s_{13} = (N+r_2)^2 + L^2 + 2(N+r_2)\cos r_1 \cdot L \cdot \cos(r_1+r_3) + 2(N+r_2)\sin r_1 \, L\sin(r_1+r_3)$$

$$\Rightarrow L^2 + q_2^2 - 2Lq_2 S_3 = (N+r_2)^2 + L^2 - 2(N+r_2)L\cos(r_3)$$ { I notice that $q_2 = N+r_2$

$$\Rightarrow \cancel{L^2} + \cancel{q_2^2} - 2Lq_2 S_3 = \cancel{q_2^2} + \cancel{L^2} - 2\, q_2 \, L\cos(r_3) \Rightarrow$$

$$\sin(q_3) = \cos(r_3) \Rightarrow \cos(q_3) = \pm\sqrt{1-\cos^2(r_3)} \Rightarrow q_3 = \operatorname{atan2}\left\{\cos(r_3), \pm\sqrt{1-\cos^2(r_3)}\right\}$$

I notice that $q_1 = r_1$



$$\Downarrow$$

$$u = r_1 - \pi/2 \qquad u = q_1 - \pi/2 \Rightarrow q_1 = r_1$$

Given q_1 and q_3 the following

$$\begin{cases} Lc_{13} + q_2 s_1 = (N+r_2)\cos r_1 + L\cos(r_1+r_3) \\ Ls_{13} - q_2 c_1 = (N+r_2)\sin r_1 + L\sin(r_1+r_3) \end{cases}$$

become a linear system in q_2 and can be solved.

Exercise 2 [5 points]

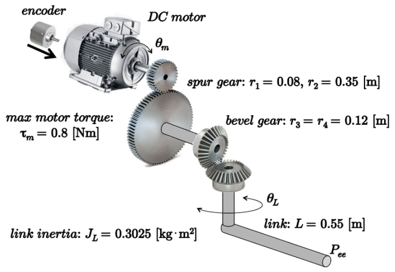


Figure 2: A DC servomotor that drives a robot link through transmissions.

With reference to the servo-drive sketched in Fig. 2 and the data therein, we need to measure the position of the tip point P_{ee} of the link with a resolution of 0.01 [mm]. A suitable incremental

A difference of 0.01 [mm], is in angle $\Delta\theta_L$ such that $\frac{360°}{2\pi L} = \frac{x}{0.01}$

we have to consider $L = 0.55$ m $= 550$ mm $\Rightarrow$

$$\frac{360°}{2\pi \cdot 550} = \frac{x}{0.01} \Rightarrow \Delta\theta_L = 1.04 \times 10^{-3} \text{ degrees.}$$

I have to calculate the reduction ratio n_r.

From θ_m to spur gear $\Rightarrow n = \frac{0.35}{0.08} = \frac{35}{8}$

the bevel gear ratio are 1:

$\theta_m = \frac{35}{8}\theta_L \Rightarrow$ the resolution to measure on the motor

should be $\Delta\theta_m = \frac{35}{8} \cdot 1.04 \times 10^{-3} = 4.55 \times 10^{-3}$ degrees

$\Rightarrow$ With quadrature, we have $\overset{\text{Angular res}}{AR} = \frac{360°}{\underset{\text{pulse per revolut}}{PPR} \cdot 4} \Rightarrow 19780.21978$ PPR

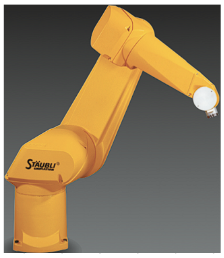
the number of bits should be $\lceil \log_2 PPR \rceil = 15$ bits

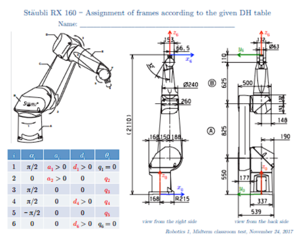
the optimal ratio is $n_r = \sqrt{J_L / J_m} \Rightarrow$ best $J_m = 0.0158$ kg·m²

$\Rightarrow$ the achivable acc. is $\ddot{\theta}_L = 5.78 \frac{\text{degrees}}{\text{second}}$.

Exercise 3 [12 points]

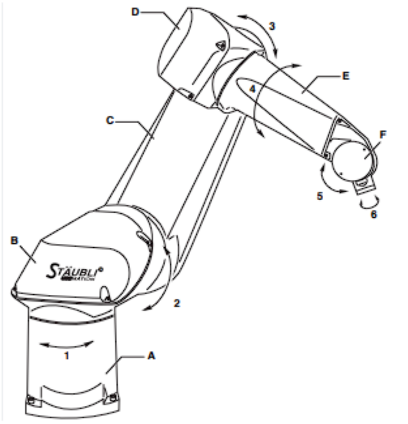
Consider the 6-dof robot Stäubli RX 160 in Fig. 3. In the extra sheet provided separately, the Denavit-Hartenberg (DH) table of parameters is specified, in part numerically and in part symbolically. The two DH frames 0 and 6 are already drawn on the manipulator (in two views). In the shown 'straight upward' robot configuration, the first and last joint variables take the values $q_1 = q_6 = 0$. Draw directly on the extra sheet the remaining DH frames, according to the DH table. Provide all parameters labeled in red in the table, i.e., the missing numerical values of the constant parameters and of the joint variables q_2 to q_5 when the robot is in the 'straight upward' configuration. [Please, make clean drawings and return the sheet with your name written on it.]



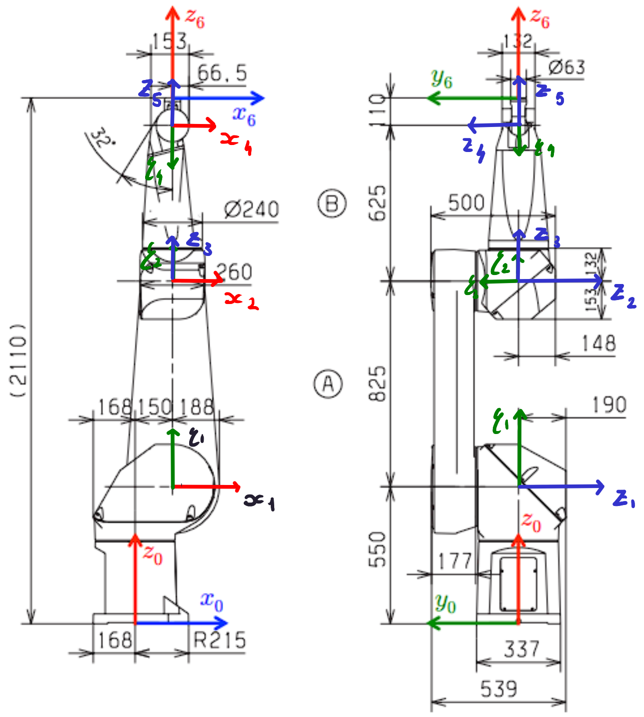


Stäubli RX 160 – Assignment of frames according to the given DH table

Name: ______________________________



i	α_i	a_i	d_i	θ_i
1	$\pi/2$	$a_1 > 0$	$d_1 > 0$	$q_1 = 0$
2	0	$a_2 > 0$	0	q_2
3	$\pi/2$	0	0	q_3
4	$\pi/2$	0	$d_4 > 0$	q_4
5	$-\pi/2$	0	0	q_5
6	0	0	$d_6 > 0$	$q_6 = 0$



view from the right side

view from the back side

Robotics 1, Midterm classroom test, November 24, 2017

i	α_i	a_i	d_i	θ_i
1	$\pi/2$	150	550	$q_1 = 0$
2	0	825	0	q_2
3	$\pi/2$	0	0	q_3
4	$\pi/2$	0	625	q_4
5	$-\pi/2$	0	0	q_5
6	0	0	110	$q_6 = 0$

0

Exercise 4 [5 points]

The orientations of two right-handed frames RF_A and RF_B with respect to a third right-handed frame RF_0 (all having the same origin) are specified, respectively, by the rotation matrices

$$ {}^0\boldsymbol{R}_A = \begin{pmatrix} \frac{3}{4} & \sqrt{\frac{3}{8}} & -\frac{1}{4} \\ -\sqrt{\frac{3}{8}} & \frac{1}{2} & -\sqrt{\frac{3}{8}} \\ -\frac{1}{4} & \sqrt{\frac{3}{8}} & \frac{3}{4} \end{pmatrix} \quad \text{and} \quad {}^0\boldsymbol{R}_B = \begin{pmatrix} \frac{1}{\sqrt{2}} & 0 & -\frac{1}{\sqrt{2}} \\ 0 & 1 & 0 \\ \frac{1}{\sqrt{2}} & 0 & \frac{1}{\sqrt{2}} \end{pmatrix}. $$

Determine, if possible, a unit vector $\boldsymbol{r}$ and an angle $\theta < 0$ such that the axis-angle rotation matrix $\boldsymbol{R}(\boldsymbol{r}, \theta)$ provides the orientation of the frame RF_B with respect to the frame RF_A.

I have to calculate $^A R_B = {}^0R_A^T \, {}^0R_B = \begin{bmatrix} 0.35 & -0.61 & -0.7 \\ 0.86 & 0.5 & 0 \\ 0.35 & -0.61 & 0.7 \end{bmatrix}$

only positive

Since $\text{trace}\, R(r,\theta) = 1+2\cos\theta \Rightarrow 1.55 = 1+2\cos\theta \Rightarrow \cos\theta = 0.275 \Rightarrow \sin\theta = \pm 0.96 = -0.96$

$\Rightarrow \theta = -1.29$

Since $r = \frac{1}{2\sin\theta} \begin{bmatrix} R_{32}-R_{23} \\ R_{13}-R_{31} \\ R_{21}-R_{12} \end{bmatrix} \Rightarrow r = -0.52 \cdot \begin{bmatrix} -0.61 \\ -1.05 \\ 1.47 \end{bmatrix} = \begin{bmatrix} 0.3172 \\ 0.546 \\ -0.76 \end{bmatrix}$

Exercise 4

The relative orientation of frame RF_B with respect to frame RF_A is expressed by the rotation matrix

R_{12}

$$
{}^A\boldsymbol{R}_B = {}^0\boldsymbol{R}_A^T \cdot {}^0\boldsymbol{R}_B = \begin{pmatrix} \frac{1}{2\sqrt{2}} & -\sqrt{\frac{3}{8}} & -\frac{1}{\sqrt{2}} \\ -\frac{\sqrt{3}}{2} & \frac{1}{2} & 0 \\ \frac{1}{2\sqrt{2}} & -\sqrt{\frac{3}{8}} & \frac{1}{\sqrt{2}} \end{pmatrix} = \begin{pmatrix} 0.3536 & -0.6124 & -0.7071 \\ 0.8660 & 0.5 & 0 \\ 0.3536 & -0.6124 & 0.7071 \end{pmatrix},
$$

$R_{21} \Rightarrow R_{21} - R_{12} = 1.1124 = \frac{2781}{2500}$

whose elements will be denoted by R_{ij}. Therefore, the equation $\boldsymbol{R}(\boldsymbol{r}, \theta) = {}^A\boldsymbol{R}_B$ should be solved for $\boldsymbol{r}$ and θ, using the inverse mapping of the axis-angle representation. Since

$$
\sin\theta = \pm\frac{1}{2}\sqrt{(R_{12}-R_{21})^2 + (R_{13}-R_{31})^2 + (R_{23}-R_{32})^2} = \pm 0.9599 \neq 0, \tag{1}
$$

the problem at hand is regular, and two distinct solutions can be found depending on the choice of the $+$ or $-$ sign in the expression of $\sin\theta$. From

$$
\cos\theta = \frac{1}{2}(R_{11} + R_{22} + R_{33} - 1) = 0.2803,
$$

taking the $-$ sign in (1) will yield a solution angle $\theta < 0$, as requested. Thus

$$
\theta = \text{ATAN2}\{-0.9599, 0.2803\} = -1.2867 \text{ [rad]} = -73.72°
$$

and

$$
\boldsymbol{r} = \frac{1}{2\sin\theta}\begin{pmatrix} R_{32} - R_{23} \\ R_{13} - R_{31} \\ R_{21} - R_{12} \end{pmatrix} = \begin{pmatrix} 0.3190 \\ 0.5525 \\ -0.7701 \end{pmatrix}.
$$

$\frac{1}{2\sin\theta} = -0.5208$

$\frac{1}{2\sin\theta}(R_{21} - R_{12}) = (-0.5208) \cdot \frac{2781}{2500} = -0.5793$

* * * * *