Since I have already taken the midterm, I will only be completing the exercises related to the second part of the course (from differential kinematics onwards).

**Exercise 4**

Consider a rest-to-rest trajectory planning problem for a RP planar robot. The robot should move its end effector along a linear path between the two Cartesian positions $\boldsymbol{p}_i = (0.6, -0.3)$ and $\boldsymbol{p}_f = (-0.3, 0.6)$ [m], using a trapezoidal speed profile. The velocities of the two joints are bounded by $|\dot{q}_1| \le 2$ rad/s and $|\dot{q}_2| \le 1$ m/s, while the acceleration along the path is bounded in norm as $\|\ddot{\boldsymbol{p}}\| \le A = 0.5$ m/s². What is the minimum feasible motion time $T$ for this task? Provide also the corresponding value of the joint velocity $\dot{\boldsymbol{q}}$ at the midpoint of the path.

the length of the path is $\|p_f - p_i\| = L = \frac{9}{10}\sqrt{2}$, since $L \le V^2/A \Rightarrow$ there is no coast phase.

$p(s) = p_i + \frac{s(t)}{L}(p_f - p_i)$ $\quad s \in [0, L]$.

s should go from 0 to L in the minimum possible time.

$$\begin{cases} \ddot{s}(t) = 1/2 & t \in [0, T_s] \\ \ddot{s}(t) = -1/2 & t \in [T_s, T] \end{cases} \quad T_s = T/2 \qquad \begin{cases} \dot{s}(t) = \frac{1}{2}t & t \in [0, T/2] \\ \dot{s}(t) = \frac{T}{2} - \frac{1}{2}t & t \in [T/2, T] \end{cases} \Rightarrow$$

$$\begin{cases} s(t) = \frac{1}{4}t^2 & \text{if } t \in [0, \frac{T}{2}] \\ s(t) = \frac{T^2}{16} + \frac{T}{2}t - \frac{1}{4}t^2 & t \in [\frac{T}{2}, T] \end{cases} \Rightarrow s(T) = \frac{5}{16}T^2 = L \Rightarrow T = 2.0181513$$

$$\dot{p} = \frac{dp}{ds}\frac{ds}{dt} = (p_f - p_i)\frac{1}{2}t\frac{1}{L} \qquad t \in [0, T/2]$$

$$\dot{p}(T/2) = \frac{1}{4}0.5045\begin{pmatrix} -0.9 \\ 0.9 \end{pmatrix} = \frac{1}{L}\begin{pmatrix} -0.454 \\ 0.454 \end{pmatrix} = \begin{pmatrix} -0.356 \\ 0.356 \end{pmatrix}$$

the DK of an RP is : $\begin{cases} q_2\cos q_1 \\ q_2\sin q_1 \end{cases} \Rightarrow J(q) = \begin{pmatrix} -q_2\sin q_1 & \cos q_1 \\ q_2\cos q_1 & \sin q_1 \end{pmatrix}$

$$p(\tfrac{T}{2}) = p_i + T/_{16L}(p_f - p_i) = p_i + 0.099(p_f - p_i) = \begin{pmatrix} 0.51 \\ -0.21 \end{pmatrix}$$ i compute the IK

$$\begin{cases} q_2\cos q_1 = 0.51 \\ q_2\sin q_1 = -0.21 \end{cases} \Rightarrow q_2^2 = 0.3042 \Rightarrow q_2 = 0.551 \Rightarrow \begin{cases} \cos q_1 = 0.925 \\ \sin q_1 = -0.38 \end{cases} \Rightarrow q_1 = -0.39$$

$$\Rightarrow J(q) = \begin{pmatrix} 0.209 & 0.924 \\ 0.509 & -0.38 \end{pmatrix} \Rightarrow J(q)\dot{q} = \dot{p} \Rightarrow \dot{q} = J^{-1}\dot{p} = \begin{pmatrix} 0.6912 & 1.68 \\ 0.925 & -0.38 \end{pmatrix}\begin{pmatrix} -0.356 \\ 0.356 \end{pmatrix} \Rightarrow$$

$$\dot{q}(T/2) = \begin{pmatrix} 0.2545 \\ -0.316 \end{pmatrix}$$