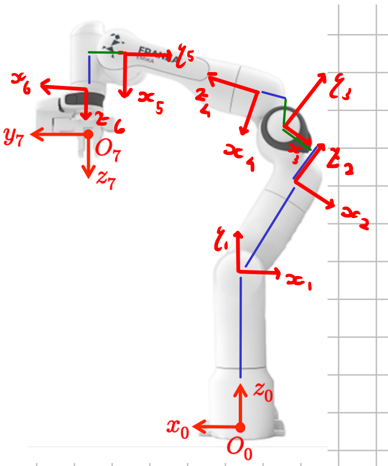
Since I have already taken the midterm, I will only be completing the exercises related to the second part of the course (from differential kinematics onwards).

Exercise 1

The Panda by Franka Emika shown in Fig. 1 is an innovative lightweight robot intended for friendly and safe human-robot interaction. The robot has seven revolute joints and its kinematics is characterized by a spherical shoulder, an elbow with two offsets, and a non-spherical wrist. This combination allows eliminating unaccessible 'holes' close to the robot base, thus increasing the robot workspace.

- Assign the link frames according to the Denavit-Hartenberg (DH) convention and complete the associated symbolic table of parameters, specifying also the signs of the non-zero constant parameters. Draw the frames and fill in the table directly on the extra sheet #1 provided separately. Therein, the two DH frames 0 and 7 are already assigned and should not be modified. [Please, make clean drawings and return the completed sheet with your name written on it.]
- Write explicitly the seven resulting DH homogeneous transformation matrices ${}^0\boldsymbol{A}_1(q_1)$ to ${}^6\boldsymbol{A}_7(q_7)$. [Do NOT attempt to write the direct kinematics in symbolic form!]



i	α_i	a_i	d_i	θ_i
1	$\pi/2$	0	d_1	q_1
2	$-\pi/2$	0	d_2	q_2
3	$\pi/2$	a_3	d_3	q_3
4	$\pi/2$	a_4	d_4	q_4
5	$-\pi/2$	0	d_5	q_5
6	$-\pi/2$	a_6	d_6	q_6
7	0	0	d_7	q_7

$${}^0A_1 = \begin{pmatrix} c_1 & 0 & s_1 & 0 \\ s_1 & 0 & -c_1 & 0 \\ 0 & 1 & 0 & d_1 \\ 0 & 0 & 0 & 1 \end{pmatrix}$$

$${}^1A_2 = \begin{pmatrix} c_2 & 0 & -s_2 & 0 \\ s_2 & 0 & c_2 & 0 \\ 0 & -1 & 0 & d_2 \\ 0 & 0 & 0 & 1 \end{pmatrix}$$

$${}^2A_3 = \begin{pmatrix} c_3 & 0 & s_3 & a_3c_3 \\ s_3 & 0 & -c_3 & a_3s_3 \\ 0 & 1 & 0 & d_3 \\ 0 & 0 & 0 & 1 \end{pmatrix} \quad {}^3A_4 = \begin{pmatrix} c_4 & 0 & s_4 & a_4c_4 \\ s_4 & 0 & -c_4 & a_4s_4 \\ 0 & 1 & 0 & d_4 \\ 0 & 0 & 0 & 1 \end{pmatrix} \quad {}^4A_5 = \begin{pmatrix} c_5 & 0 & -s_5 & 0 \\ s_5 & 0 & c_5 & 0 \\ 0 & -1 & 0 & d_5 \\ 0 & 0 & 0 & 1 \end{pmatrix} \quad {}^5A_6 = \begin{pmatrix} c_6 & 0 & -s_6 & a_6c_6 \\ s_6 & 0 & c_6 & a_6s_6 \\ 0 & -1 & 0 & d_6 \\ 0 & 0 & 0 & 1 \end{pmatrix}$$

$${}^6A_7 = \begin{pmatrix} c_7 & -s_7 & 0 & 0 \\ s_7 & c_7 & 0 & 0 \\ 0 & 0 & 1 & d_7 \\ 0 & 0 & 0 & 1 \end{pmatrix}$$

Exercise 3

With reference to the setup in Fig. 2, two identical planar 3R manipulators, a master robot M and a slave robot S having link lengths $\ell_1 = \ell_2 = 0.5$ and $\ell_3 = 0.25$ [m], should perform a Cartesian motion task in coordination. The base frames of the robots are displaced by $\boldsymbol{p}_{MS} = (\Delta x, \Delta y, 0) = (1.6, 0.9, 0)$ [m] and rotated by $\alpha_{MS} = \pi$ [rad] around the common z_0 axis. The desired Cartesian motion starts at $t = t_0$ from the position $\boldsymbol{p}_M(t_0) \in \mathbb{R}^2$ assumed by the end-effector of the master robot in the configuration $\boldsymbol{q}_M(t_0) = (\pi/2, -\pi/3, 0)$ [rad] and will proceed along a straight line path, which is specified by the initial direction of the end-effector velocity $\boldsymbol{v}_M = \dot{\boldsymbol{p}}_M(t_0) \in \mathbb{R}^2$ resulting from $\dot{\boldsymbol{q}}_M(t_0) = (-\pi/6, 0, -\pi/2)$ [rad/s]. The slave robot should execute the same Cartesian motion in position, while keeping its end-effector always oriented orthogonally to the linear path (more specifically, rotated by a constant angle $\beta = -\pi/2$ [rad] with respect to the vector $\boldsymbol{v}_M$).

- Determine an initial configuration $\boldsymbol{q}_S(t_0)$ of the slave robot such that its end-effector position $\boldsymbol{p}_S(t_0) \in \mathbb{R}^2$ and orientation are initially matched with those required by the motion task.
- Determine the initial joint velocity $\dot{\boldsymbol{q}}_S(t_0) \in \mathbb{R}^3$ of the slave robot, in order to match also the initial desired Cartesian velocity (i.e., $\boldsymbol{v}_S = \dot{\boldsymbol{p}}_S(t_0)$ is equal to $\boldsymbol{v}_M$).
- If the initial configuration of the slave robot is not matched with the desired Cartesian motion, how can this robot still perform the task after an initial transient, with its task error decreasing exponentially to zero?

The transformation between the two robot is:

$${}^MA_S = \begin{pmatrix} -1 & 0 & 0 & 1.6 \\ 0 & -1 & 0 & 0.9 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix} \Rightarrow {}^Mp = {}^MA_S\,{}^Sp \quad \text{and} \quad {}^Sp = {}^MA_S^{-1}\,{}^Mp = {}^SA_M\,{}^Mp$$

$${}^SA_M = \begin{pmatrix} -1 & 0 & 0 & 1.6 \\ 0 & -1 & 0 & 0.9 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}.$$

I solve the IK for:

$$q_M(t_0) = \begin{pmatrix} \pi/2 \\ -\pi/2 \\ 0 \end{pmatrix} \Rightarrow {}^Mp_M(t_0) = \begin{pmatrix} 3/4 \\ 1/2 \end{pmatrix}. \qquad {}^M\dot{p}_M = J(q_M(t_0)) \cdot \dot{q}_M(t_0) = \begin{pmatrix} -\frac{1}{2} & 0 & 0 \\ \frac{3}{4} & \frac{3}{4} & \frac{1}{4} \end{pmatrix} \begin{pmatrix} -\frac{\pi}{6} \\ 0 \\ -\pi/2 \end{pmatrix} = \begin{pmatrix} \pi/12 \\ -\frac{7}{24}\pi \end{pmatrix}$$

the direction of the path is specified by ${}^M\dot{p}_M$, the angle respect to x_{0M} is $\text{atan2}\{-\frac{7}{24}\pi, \pi/12\} = -1.292$. So the angle that the slave should have is ${}^M\phi = -1.292 - \pi/2 = -2.862$. The position of S in t_0 should be ${}^Mp_M(t_0)$, in his frame is

$${}^Sp_M(t_0) = {}^SA_M\,{}^Mp_M(t_0) = \begin{pmatrix} -1 & 0 & 0 & 1.6 \\ 0 & -1 & 0 & 0.9 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} 3/4 \\ 1/2 \\ 0 \\ 1 \end{pmatrix} = \begin{pmatrix} 0.85 \\ 0.4 \end{pmatrix} \quad \text{and} \quad {}^S\phi = {}^M\phi + \pi = 0.28$$

So i solve the IK:

$$\begin{cases} \frac{1}{2}C_1+\frac{1}{2}C_{12}+\frac{1}{4}C_{123} = 0.85 \\ \frac{1}{2}S_1+\frac{1}{2}S_{12}+\frac{1}{4}S_{123} = 0.4 \\ q_1+q_2+q_3 = 0.28 \end{cases} \Rightarrow \begin{cases} 1/2\,(C_1+C_{12}) = 0.61 \\ 1/2\,(S_1+S_{12}) = 0.331 \end{cases} \Rightarrow \frac{1}{4}\left(C_1^2+C_{12}^2+2C_1C_{12}+S_1^2+S_{12}^2+2S_1S_{12}\right) = 0.482$$

$$\frac{1}{4}(2+2C_2) = 0.482 \Rightarrow C_2 = -0.036 \Rightarrow q_2 = 1.607$$

$$S_2 \approx 1$$

$$\Rightarrow \begin{cases} C_1+C_{12} = 1.22 \\ S_1+S_{12} = 0.662 \end{cases} \Rightarrow \begin{cases} C_1+C_1C_2-S_1S_2 = 1.22 \\ S_1+S_1C_2+C_1S_2 = 0.662 \end{cases} \Rightarrow \begin{cases} C_1+C_1(-0.036)-S_1 = 1.22 \\ S_1+S_1(-0.036)+C_1 = 0.662 \end{cases} \Rightarrow \begin{cases} C_1 = 0.952 \\ S_1 = -0.301 \end{cases} \Rightarrow \begin{cases} q_1 = -0.306 \\ q_3 = -1.021 \end{cases}$$

$$q_s(t_0) = \begin{pmatrix} -0.306 \\ 1.607 \\ -1.021 \end{pmatrix}.$$ the Joint velocity that S should have is

$$^S\dot{P}_M = {^SA_M}\,{^M\dot{P}_M}(t_0) = \begin{pmatrix} -1 & 0 & 0 & 1.6 \\ 0 & -1 & 0 & 0.9 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}\begin{pmatrix} \pi/12 \\ -\frac{7}{24}\pi \\ 0 \\ 1 \end{pmatrix} = \begin{pmatrix} 1.338 \\ 1.816 \\ 0 \end{pmatrix}$$ and the angular velocity should be zero too.

let $r = \begin{pmatrix} p_x \\ p_y \\ \phi \end{pmatrix} \Rightarrow J_r(q_s(t_0)) = \begin{pmatrix} -0.4 & -0.551 & -0.069 \\ 0.85 & 0.373 & 0.24 \\ 1 & 1 & 1 \end{pmatrix} \Rightarrow \dot{q}_s(t_0) = J_r^{-1}(q_s(t_0))\begin{pmatrix} 1.338 \\ 1.816 \\ 0 \end{pmatrix} = \begin{pmatrix} 4.213 \\ -5.669 \\ 1.456 \end{pmatrix}$

let ${^Se} = {^Sp_M} - {^Sp_S}$ to be the error. $\dot{e} = {^S\dot{p}_M} - {^S\dot{p}_S} = {^S\dot{p}_M} - J\dot{q}_S$, i impose the joint velocity command: $\dot{q}_S = J^{\#}({^S\dot{p}_M}+Ke)$ so: $\dot{e} = {^S\dot{p}_M} - JJ^{\#}({^S\dot{p}_M}+Ke) = -Ke$ since $\dot{e} = -Ke$ and $K = \mathrm{diag}\{2,2\}$: $e(t) = \begin{cases} e_{0x}\cdot\exp(-2t) \\ e_{0y}\cdot\exp(-2t) \end{cases}$

Exercise 4

Consider the planning of a smooth trajectory for a planar RP robot (with unlimited joint ranges) between the configurations $\boldsymbol{q}(0) = (\pi/4, -1)$ [rad,m] at $t = 0$ and $\boldsymbol{q}(T) = (-\pi/2, 1)$ [rad,m] at $t = T$. The initial and final joint velocity and acceleration should be zero, and the acceleration should be continuous in the entire time interval $[0, T]$. The following joint velocity and acceleration bounds are also present:

$$|\dot{q}_1| \le V_1 = 120°/\text{s}, \quad |\dot{q}_2| \le V_2 = 180 \text{ cm/s}, \quad |\ddot{q}_1| \le A_1 = 150°/\text{s}^2, \quad |\ddot{q}_2| \le A_2 = 200 \text{ cm/s}^2. \qquad (1)$$

Define a suitable class of trajectories and choose a final time $T = 3$ s. Will the resulting robot motion be feasible with respect to the bounds in (1)? Using uniform time scaling, find the minimum feasible motion time T^* to perform the desired reconfiguration along the chosen trajectory. Sketch a plot of the resulting joint velocity and acceleration profiles. Will the robot cross a singular configuration during its motion?

We need a 5 degree polynomial due to the 6 boundary conditions.

$$q_1(t) = at^5+bt^4+ct^3+dt^2+et+F$$
$$\dot{q}_1(t) = 5at^4+4bt^3+3ct^2+2dt+e$$
$$\ddot{q}_1(t) = 20at^3+12bt^2+6ct+2d$$

$$\begin{cases} q_1(0) = F = \pi/4 \\ \dot{q}_1(0) = e = 0 \\ \ddot{q}_1(0) = 2d = 0 \\ q_1(3) = 243a+81b+27c = -1.785 \\ \dot{q}_1(3) = 405a+108b+27c = 0 \\ \ddot{q}_1(3) = 540a+108b+6c = 0 \end{cases} \Rightarrow \begin{cases} a = 0.109 \\ b = -0.551 \\ c = 0.661 \end{cases}$$

$$\Rightarrow \begin{cases} \dot{q}_1(t) = 5at^4 + 4bt^3 + 3ct^2 \\ \ddot{q}_1(t) = 20at^3 + 12bt^2 + 6ct \end{cases}$$

$rad \cdot \frac{180}{\pi} = grad$

$grad \cdot \frac{\pi}{180} = rad$

$$\Rightarrow \ddot{q}_1(t) = 0 \Rightarrow t^* = \{2.2097, 0.823, 0\} \Rightarrow \max|\dot{q}_1| = \max_{t \in t^*}\{|\dot{q}_1(t)|\} = 1.103 \frac{rad}{sec} = 63.197°/s \leq V_1$$

$$\dddot{q}_1(t) = 0 \Rightarrow t^* = \{1.665, 0.3662\} \quad \max|\ddot{q}_1| = \max_{t \in t^*}\{|\ddot{q}_1(t)|\} = 1.664 \frac{rad}{sec} = 95.34°/s \leq A_1$$

q_1 is feasible and respect the constrains.

$q_2(t) = at^5 + bt^4 + ct^3 + dt^2 + et + F$

$\dot{q}_2(t) = 5at^4 + 4bt^3 + 3ct^2 + 2dt + e$

$\ddot{q}_2(t) = 20at^3 + 12bt^2 + 6ct + 2d$

$$\begin{cases} q_2(0) = F = -1 \\ \dot{q}_2(0) = e = 0 \\ \ddot{q}_2(0) = 2d = 0 \\ q_2(3) = 243a + 81b + 27c - 2 \\ \dot{q}_2(3) = 405a + 108b + 27c = 0 \\ \ddot{q}_2(3) = 540a + 108b + 6c = 0 \end{cases} \Rightarrow \begin{cases} a = -0.115 \\ b = 0.617 \\ c = -0.741 \end{cases}$$

$$\begin{cases} \dot{q}_2(t) = -0.575t^4 + 2.468t^3 - 2.223t^2 \\ \ddot{q}_2(t) = -2.3t^3 + 7.404t^2 - 4.446t \\ \dddot{q}_2(t) = -6.9t^2 + 14.808t^2 - 4.446 \end{cases}$$

$$\ddot{q}_2(t) = 0 \Rightarrow t^* = \{2.42, 0.798, 0\} \Rightarrow \max|\dot{q}_2| = \max_{t \in t^*}\{|\dot{q}_2(t)|\} = 2.2379 \frac{m}{s} > V_2 = 1.8 \frac{m}{s} \;!$$

$$\dddot{q}_2(t) = 0 \Rightarrow t^* = \{1.785, 0.360\} \Rightarrow \max|\ddot{q}_2| = \max_{t \in t^*}\{|\ddot{q}_2(t)|\} = 2.5736 \frac{m}{s} > A_2 = 2 \frac{m}{s} \;!$$

$$\Rightarrow k = \max\left\{\frac{2.2379}{1.8}, \sqrt{\frac{2.5736}{2}}\right\} = 1.244 \Rightarrow \text{minimum time } T^* = 3.732$$