Since I have already taken the midterm, I will only be completing the exercises related to the second part of the course (from differential kinematics onwards).

Exercise 2

For a 2-dof robot, consider the differential mapping between the joint velocity $\dot{\boldsymbol{q}} \in \mathbb{R}^2$ and the task velocity $\dot{\boldsymbol{r}} \in \mathbb{R}^2$ at a given configuration $\boldsymbol{q}$. Joint velocities are bounded as $|\dot{q}_i| \le V_i$, for $i = 1, 2$. The task velocity is specified as $\dot{\boldsymbol{r}} = s\dot{\boldsymbol{r}}_d$, with a scaling factor $s \in [0, 1]$ to reduce as little as possible the original task velocity $\dot{\boldsymbol{r}}_d$ in case of an unfeasible joint velocity (i.e., that violates one or both bounds). Draw in the plane $(\dot{q}_1, \dot{q}_2)$ a qualitative picture for each of the following situations.

a) Regular robot configuration and feasible joint velocity. Represent geometrically the two linear equations of the differential map $\boldsymbol{J}(\boldsymbol{q})\dot{\boldsymbol{q}} = \dot{\boldsymbol{r}}_d$.

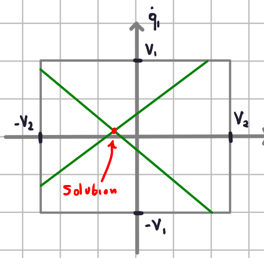
b) Regular configuration and unfeasible joint velocity. Find geometrically the largest task scaling factor that recovers feasibility, together with the corresponding joint velocity.

c) Singular configuration, realizable $\dot{\boldsymbol{r}}_d$, but unfeasible joint velocity. Find geometrically the largest task scaling factor that recovers feasibility and the corresponding joint velocity.

d) Singular configuration and unrealizable $\dot{\boldsymbol{r}}_d$. Find geometrically the smallest joint velocity in norm that minimizes the norm of the task velocity error $\|\dot{\boldsymbol{r}}_d - \boldsymbol{J}(\boldsymbol{q})\dot{\boldsymbol{q}}\|$. When the joint velocity obtained in this way is unfeasible, find geometrically the largest task scaling factor that recovers feasibility and the corresponding joint velocity.

let $J = \begin{pmatrix} a & b \\ c & d \end{pmatrix}$ with $a, b, c, d \in \mathbb{R}$.

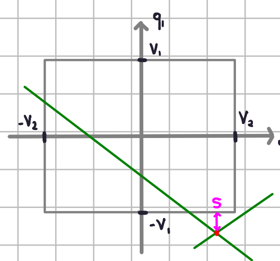
a) In this case, $ad - bc \neq 0$.



$J\dot{q} = \dot{r}_d$

$\begin{cases} a\dot{q}_1 + b\dot{q}_2 = \dot{r}_{d_1} \\ c\dot{q}_1 + d\dot{q}_2 = \dot{r}_{d_2} \end{cases}$

b) In this case the intersection is outside the rectangle

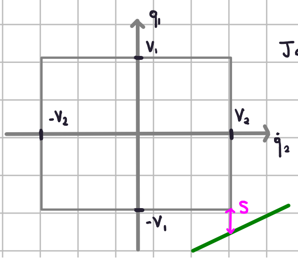


$J\dot{q} = \dot{r}_d$

$\begin{cases} a\dot{q}_1 + b\dot{q}_2 = \dot{r}_{d_1} \\ c\dot{q}_1 + d\dot{q}_2 = \dot{r}_{d_2} \end{cases}$

by changing s we traslate the linear equation on the y-axis.

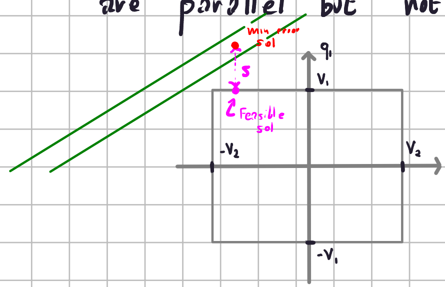
c) In that case the 2 eq. are identical and we have ∞ solutions outside the rectangle.



$J\dot{q} = \dot{r}_d$

$\begin{cases} a\dot{q}_1 + b\dot{q}_2 = \dot{r}_{d_1} \\ c\dot{q}_1 + d\dot{q}_2 = \dot{r}_{d_2} \end{cases}$

d) In this case the two linear eq. are parallel but not coincidents.



$J\dot{q} = \dot{r}_d$

$\begin{cases} a\dot{q}_1 + b\dot{q}_2 = \dot{r}_{d_1} \\ c\dot{q}_1 + d\dot{q}_2 = \dot{r}_{d_2} \end{cases}$

For a planar 2R robot having links of unitary length and velocity bounds $V_1 = 1$ and $V_2 = 1.5$ [rad/s], let $\boldsymbol{r} = \boldsymbol{p} \in \mathbb{R}^2$ be the position of its end-effector. Based on the previous qualitative analysis, provide the solutions for the following two numerical cases, indicating also which of the above situations applies:

- $\boldsymbol{q} = (\pi/3, \pi)$ [rad], $\dot{\boldsymbol{r}}_d = (-\sqrt{3}/2, 0.5)$ [m/s];
- $\boldsymbol{q} = (0, \pi/2)$ [rad], $\dot{\boldsymbol{r}}_d = (1, -1.2)$ [m/s].

the Jacobian matrix is $J = \begin{pmatrix} -S_1 - S_{12} & -S_{12} \\ C_1 + C_{12} & C_{12} \end{pmatrix}$

With $q = \left(\frac{\pi}{3} \;\; \pi\right)^T$, $J = \begin{pmatrix} 0 & \sqrt{3}/2 \\ 0 & -1/2 \end{pmatrix} \Rightarrow \det J = 0$, singular configuration.

$J\dot{q} = \begin{pmatrix} \sqrt{3}/2\,\dot{q}_2 \\ -\frac{1}{2}\dot{q}_2 \end{pmatrix}$, if $\dot{q}_2 = -1 \Rightarrow J\dot{q} = \begin{pmatrix} -\sqrt{3}/2 \\ 1/2 \end{pmatrix} = \dot{r}_d$ $\begin{cases} \text{singular configuration, realizable} \\ \text{velocity, feasible joint velocities.} \end{cases}$

With $q = (0, \pi)$, $J = \begin{pmatrix} 0 & 0 \\ 0 & -1 \end{pmatrix}$. Singular conf.

$J\dot{q} = \begin{pmatrix} 0 \\ -\dot{q}_2 \end{pmatrix} \Rightarrow \dot{r}_d$ is unrealizable.

Exercise 3

A planar RP robot with base at the origin should move its end-effector along a circular path centered at $C_0 = (4,4)$, from point $P_1 = (4,3)$ to point $P_2 = (3,4)$ [m]. The desired motion is rest-to-rest and is executed in a total time $T = 2$ s, with a bang-bang acceleration profile as timing law. Determine:

- the parametric expression of the path $\boldsymbol{p}(s)$, for $s \in [0, L]$, where L is the Cartesian path length, together with its first and second derivatives $\boldsymbol{p}'(s) = d\boldsymbol{p}/ds$ and $\boldsymbol{p}''(s) = d^2\boldsymbol{p}/ds^2$;
- the expression of the corresponding parametrized joint path $\boldsymbol{q}(s) = (q_1(s), q_2(s))$, for $s \in [0, L]$;

$$p(s): \begin{cases} 4 + \cos\left(\frac{3}{2L}\pi s - \frac{1}{2}\pi\right) \\ 4 + \sin\left(\frac{3}{2L}\pi s - \frac{1}{2}\pi\right) \end{cases} \quad p'(s): \begin{cases} -\frac{3}{2L}\pi \sin\left(\frac{3}{2L}\pi s - \frac{1}{2}\pi\right) \\ \frac{3}{2L}\pi \cos\left(\frac{3}{2L}\pi s - \frac{1}{2}\pi\right) \end{cases} \quad p''(s): \begin{cases} -\frac{9}{4L^2}\pi^2 \cos\left(\frac{3}{2L}\pi s - \frac{1}{2}\pi\right) \\ -\frac{9}{4L^2}\pi^2 \sin\left(\frac{3}{2L}\pi s - \frac{1}{2}\pi\right) \end{cases}$$

$s \in [0, L]$

$L = \frac{3}{2}\pi$

by replacing L with $\frac{3}{2}\pi$:

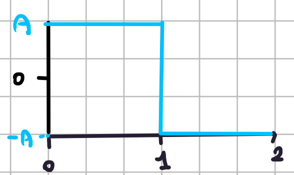
$$p(s): \begin{cases} 4 + \cos\left(s - \frac{1}{2}\pi\right) \\ 4 + \sin\left(s - \frac{1}{2}\pi\right) \end{cases} \quad p'(s): \begin{cases} -\sin\left(s - \frac{1}{2}\pi\right) \\ \cos\left(s - \frac{1}{2}\pi\right) \end{cases} \quad p''(s): \begin{cases} -\cos\left(s - \frac{1}{2}\pi\right) \\ -\sin\left(s - \frac{1}{2}\pi\right) \end{cases}$$

the DK of a RP is $f(q) = \begin{cases} q_2 \cdot \cos q_1 \\ q_2 \cdot \sin q_1 \end{cases}$ so we impose $\forall s \in [0, \frac{3}{2}\pi]$:

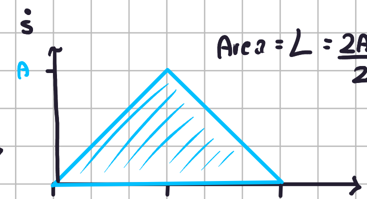
$q(s) = f^{-1}(p(s))$ where the IK is:

$$\begin{cases} p_x = q_2 \cdot \cos q_1 \\ p_y = q_2 \cdot \sin q_1 \end{cases} \Rightarrow \begin{cases} q_2 = \sqrt{p_x^2 + p_y^2} \\ q_1 = \operatorname{atan2}\{p_y, p_x\} \end{cases} \Rightarrow q(s) = \begin{cases} q_1(s) = \operatorname{atan2}\{p_2(s), p_1(s)\} \\ q_2(s) = \|p(s)\| \end{cases}$$

Now, for a bang-bang acceleration profile i need that:



$$\Rightarrow \begin{cases} \ddot{s}(t) = A & \text{if } t \in [0,1] \\ \ddot{s}(t) = -A & \text{if } t \in [1,2] \end{cases} \Rightarrow \begin{cases} \dot{s}(t) = At & \text{if } t \in [0,1] \\ \dot{s}(t) = A - At & \text{if } t \in [1,2] \end{cases} \Rightarrow$$



Area $= L = \frac{2A}{2} = A$

$\Rightarrow A = \frac{3}{2}\pi$. the maximum velocity is reached in $t = 1$.

$$\dot{p} = p'\dot{s} = \begin{pmatrix} -\sin\left(s - \frac{1}{2}\pi\right) \\ \cos\left(s - \frac{1}{2}\pi\right) \end{pmatrix} \dot{s} \quad \text{in } t = 1 \Rightarrow \begin{cases} s(1) = \frac{3}{4}\pi \\ \dot{s}(1) = \frac{3}{2}\pi \end{cases} \Rightarrow$$

$$\dot{p}(1) = \frac{3}{2}\pi \begin{pmatrix} -\sqrt{2}/2 \\ \sqrt{2}/2 \end{pmatrix} \Rightarrow \max \|\dot{p}\| = 4.7123$$