Since I have already taken the midterm, I will only be completing the exercises related to the second part of the course (from differential kinematics onwards).

**Exercise 4**

A 2R planar robot should move from $\boldsymbol{q}_s = (0, \pi/4)$ to $\boldsymbol{q}_f = (\pi/2, -\pi/2)$ [rad] in a coordinated time $T$ and with a smooth profile for $t \in [0, T]$ satisfying zero boundary conditions on velocity and acceleration. If the maximum absolute joint velocity and joint acceleration are $\boldsymbol{V}_{max} = (V_{max,1}, V_{max,2}) = (2, 3.5)$ [rad/s] and, respectively, $\boldsymbol{A}_{max} = (A_{max,1}, A_{max,2}) = (3, 6)$ [rad/s$^2$], determine the minimum motion time $T^*$.

Suppose now that at $t = T^*/4$ an emergency is detected and the robot should come as soon as possible to a complete stop. Which would be the new motion profile and the minimum time instant $T_s$ at which $\dot{\boldsymbol{q}}(T_s) = \boldsymbol{0}$? Which is the final reached configuration $\boldsymbol{q}(T_s)$? In this situation, sketch the overall position, velocity and acceleration profiles for $t \in [0, T_s]$.

to satisfy the 6 boundary conditions, i choose a 5-degree polynomial function.

$$q_1(s) = \sum_{i=0}^{5} a_i s^i, \quad q_1'(s) = \sum_{i=1}^{5} i a_i s^{i-1} \qquad s \in [0,1]$$

$$\begin{cases} q_1(0) = a_0 = 0 \\ q_1(1) = a_3 + a_4 + a_5 = \frac{\pi}{2} \\ q_1'(0) = a_1 = 0 \\ q_1'(1) = 5a_5 + 4a_4 + 3a_3 = 0 \\ q_1''(0) = 2a_2 = 0 \\ q_1''(1) = 20a_5 + 12a_4 + 6a_3 = 0 \end{cases} \qquad \begin{cases} q_2(0) = a_0 = \frac{\pi}{4} \\ q_2(1) = +a_3 + a_4 + a_5 + \frac{\pi}{4} = -\frac{\pi}{2} \\ q_2'(0) = a_1 = 0 \\ q_2'(1) = 5a_5 + 4a_4 + 3a_3 = 0 \\ q_2''(0) = 2a_2 = 0 \\ q_2''(1) = 20a_5 + 12a_4 + 6a_3 = 0 \end{cases}$$

$$\Downarrow \qquad\qquad \Downarrow$$

$$\begin{cases} a_3 = 5\pi \\ a_4 = -\frac{15}{2}\pi \\ a_5 = 3\pi \end{cases} \qquad \begin{cases} a_3 = -\frac{15}{2}\pi \\ a_4 = \frac{45}{4}\pi \\ a_5 = -\frac{9}{2}\pi \end{cases}$$

$$q_1(s) = 5\pi s^3 - \frac{15}{2}\pi s^4 + 3\pi s^5 \qquad q_2(s) = \frac{\pi}{4} - \frac{15}{2}\pi s^3 + \frac{45}{4}\pi s^4 - \frac{9}{2}\pi s^5$$

then, i consider $\max |q_1'|$

$$\left.\begin{aligned} q_1'(s) &= \pi(15s^2 - 30s^3 + 15s^4) \\ q_1''(s) &= \pi(30s - 90s^2 + 60s^3) \\ q_1''(s) &= 0 \Rightarrow s^* = 1/2 \end{aligned}\right\} \Rightarrow \max|q_1'| = \frac{15}{16}\pi, \quad |\dot{q}_1| = |q_1'|\dot{s} \le 2$$

$$\Rightarrow |\dot{s}| \le \frac{2}{|q_1'|} \Rightarrow |\dot{s}| \le 0.679$$

$$q_1'''(s) = \pi(30 - 180s + 180s^2) = 0$$

$$\Rightarrow s^* = \frac{3 \pm \sqrt{3}}{6}$$

$$\Rightarrow \max|q_1''(s^*)| = 9.068$$

$$\max|\ddot{q}| = \max\left|q''\dot{s}^2 + q'\ddot{s}\right| \le \max|q''\dot{s}^2| + \max|q'\ddot{s}| \le$$

$$9.068 \cdot 0.461 + \frac{15}{16}\pi \cdot \max|\ddot{s}| \le 3 \Rightarrow \max|\ddot{s}| \le 0.4$$

$q_2(s) = \frac{\pi}{4} - \frac{15}{2}\pi s^3 + \frac{45}{4}\pi s^4 - \frac{9}{2}\pi s^5 \Rightarrow$

$q_2'(s) = \pi\left(-\frac{45}{2}s^2 + 45 s^3 - \frac{45}{2}s^4\right) \Rightarrow \max q_2' = 4.417 \Rightarrow |\dot{s}| \leq \frac{3.5}{|q_2'|} \Rightarrow |\dot{s}| \leq 0.792$

$q_2''(s) = \pi\left(-45s + 135s^2 - 90s^3\right) = 0 \Rightarrow s^* \Rightarrow \max q_2'' = 13.603$

$q_2'''(s) = \pi\left(-45 + 270s - 270s^2\right) = 0 \Rightarrow s^* = \frac{3 \pm \sqrt{3}}{6}$

$\Rightarrow \quad \max|q''\dot{s}^2| + \max|q'\ddot{s}| \Rightarrow |\ddot{s}| \leq 0.06 \quad \Rightarrow \quad a_m = 0.06, \; v_m = 0.679$

$\Rightarrow T^* = 12.789$