Since I have already taken the midterm, I will only be completing the exercises related to the second part of the course (from differential kinematics onwards).

Exercise #2

The 2-dof Cartesian robot in Fig. 2 should execute with its end-effector the following desired eight-shaped periodic trajectory

$$\boldsymbol{p}_d(t) = \begin{pmatrix} c + a\sin 2\omega t \\ c + b\sin\omega t \end{pmatrix}, \qquad \text{with } a, b, c, \omega > 0, \text{ for } t \in \left[0, \frac{2\pi}{\omega}\right]. \tag{1}$$

The robot joint velocities and accelerations are bounded as

$$|\dot{q}_i| \le V_i > 0, \qquad |\ddot{q}_i| \le A_i > 0, \qquad i = 1, 2,$$

while the velocity along the Cartesian path is bounded in norm as $\|\dot{\boldsymbol{p}}_d(t)\| \le V_{c,max} > 0$. The robot is commanded by joint accelerations.

Give the symbolic expressions of the needed robot joint commands, and determine the maximum value ω_{max} of the angular frequency ω in (1) so that the robot motion satisfies all the constraints. Provide then the numerical value of ω_{max} using the following data: $a = 1$ [m], $b = 1.5$ [m], $c = 3$ [m], $V_1 = V_2 = 2$ [m/s], $V_{c,max} = 1.8$ [m/s], $A_1 = 2$ [m/s²], $A_2 = 1.5$ [m/s²].

Since the DK is $f(q) = \begin{cases} p_x = q_2 \\ p_y = q_1 \end{cases}$ we have

$$q_d(t) = \begin{cases} c + b\sin(\omega t) \\ c + b\sin(2\omega t) \end{cases} \qquad \dot{q}_d(t) = \begin{cases} \omega b\cos(\omega t) \\ 2\omega b\cos(2\omega t) \end{cases} \Rightarrow \begin{cases} max|\dot{q}_{d_1}| = \omega b \le V_1 \\ max|\dot{q}_{d_2}| = 2\omega b \le V_2 \end{cases} \Rightarrow \begin{cases} \omega \le V_1/b = \alpha \\ \omega \le V_2/2b = \beta \end{cases}$$

$$\dot{p}_d(t) : \begin{cases} 2\omega b\cos(2\omega t) \\ \omega b\cos(\omega t) \end{cases} \Rightarrow \|\dot{p}_d\| = \left[(2\omega b)^2\cos^2(2\omega t) + (\omega b^2)\cos^2(\omega t)\right]^{1/2} \le \sqrt{5\omega^2 b^2} = \omega b\sqrt{5} \le V_c \Rightarrow \omega \le \underbrace{V_c/b\sqrt{5}}_{\eta}$$

$$\ddot{q}_d(t) = \begin{cases} -\omega^2 b\sin(\omega t) \\ -4\omega^2 b\sin(2\omega t) \end{cases} \Rightarrow \begin{cases} max|\ddot{q}_{d_1}| = \omega^2 b \le A_1 \\ max|\ddot{q}_{d_2}| = 4\omega^2 b \le A_2 \end{cases} \Rightarrow \begin{cases} \omega \le (A_1/b)^{1/2} = \gamma \\ \omega \le (A_2/4b)^{1/2} = \delta \end{cases} \Rightarrow \omega_{max} = \min\{\alpha, \beta, \gamma, \delta, \eta\}$$

$\Rightarrow$ with $c = 3$, $V_1 = V_2 = 2$, $V_c = 1.8$, $A_1 = 2$, $A_2 = 1.5$, $b = 1.5$ we have:

$$\omega = \min\left\{\frac{4}{3}, \frac{2}{3}, \frac{6\sqrt{5}}{25}, \frac{2\sqrt{3}}{3}, \frac{1}{2}\right\} = \frac{1}{2}$$

Exercise #4

With reference to Fig. 3, a 3R planar robot with equal link lengths $\ell = 2$ [m] executes a linear Cartesian path from point $\boldsymbol{A} = (3, 2.5)$ [m] (at $t = 0$) to point $\boldsymbol{B} = (0.75, 1.8)$ [m] with constant speed $v = 0.5$ [m/s], while keeping its end-effector always orthogonal to the path. Provide the value of the joint velocity $\dot{\boldsymbol{q}} \in \mathbb{R}^3$ realizing the task at $t = 1$ [s]. Sketch graphically the situation.

$B - A = \begin{pmatrix} -2.25 \\ -0.7 \end{pmatrix} \Rightarrow$ the angle of the path respect to x is $\tan\left(\frac{0.7}{2.25}\right) = 0.3016$

So the angle of the EE should be 1.8724.

$$\dot{p}_d(t) = \begin{cases} -v\cos(0.3016) \\ -v\sin(0.3016) \end{cases} \qquad \phi_d(t) = 1.8724$$

$$p_d(t) = \begin{cases} 3 - v\cos(0.3016)t \\ 2.5 - v\sin(0.3016)t \end{cases} = \begin{cases} 3 - 0.4774t \\ 2.5 - 0.1485t \end{cases} \Rightarrow r_d(t) = \begin{cases} 3 - 0.4774t \\ 2.5 - 0.1485t \\ 1.8724 \end{cases}$$

the DK is $r = f(q) = \begin{cases} 2(c_1 + c_{12} + c_{123}) \\ 2(s_1 + s_{12} + s_{123}) \\ q_1 + q_2 + q_3 \end{cases}$; solve the IK:

$$\begin{cases} 2(c_1 + c_{12} + c_{123}) = 3 - 0.4774t \\ 2(s_1 + s_{12} + s_{123}) = 2.5 - 0.1485t \\ q_1 + q_2 + q_3 = 1.8724 \end{cases} \Rightarrow \begin{cases} 2(c_1 + c_{12}) = 3.594 - 0.4774t \\ 2(s_1 + s_{12}) = 0.5902 - 0.1485t \end{cases} \Rightarrow c_2 = \frac{1}{4}(5.265 - 3.606t + 0.25t^2)$$

$$\Rightarrow s_2 = \sqrt{1 - c_2^2} \Rightarrow q_2 = \text{atan2}\{s_2, c_2\}$$

Since $\dot{p} = \begin{pmatrix} -0.4774 \\ -0.1485 \\ 0 \end{pmatrix}$ and $J = 2\begin{pmatrix} -s_1 - s_{12} - s_{123} & -s_{12} - s_{123} & -s_{123} \\ c_1 + c_{12} + c_{123} & c_{12} + c_{123} & c_{123} \\ 1 & 1 & 1 \end{pmatrix}$

$$\dot{q} = J\dot{p} = 2\begin{pmatrix} -s_1 - s_{12} - s_{123} & -s_{12} - s_{123} & -s_{123} \\ c_1 + c_{12} + c_{123} & c_{12} + c_{123} & c_{123} \\ 1 & 1 & 1 \end{pmatrix}\begin{pmatrix} -0.4774 \\ -0.1485 \\ 0 \end{pmatrix} = 2\begin{pmatrix} (s_1+s_{12}+s_{123})0.4774 + 0.1485(s_{12}+s_{123}) \\ -(c_1+c_{12}+c_{123})0.4774 - 0.1485(c_{12}+c_{123}) \\ -0.6259 \end{pmatrix}$$

$$\Rightarrow q(t) = \begin{pmatrix} 2[(s_1+s_{12}+s_{123})0.4774 + 0.1485(s_{12}+s_{123})]t + q_1^0 \\ 2[-(c_1+c_{12}+c_{123})0.4774 - 0.1485(c_{12}+c_{123})]t + q_2^0 \\ -0.6259t + q_3^0 \end{pmatrix} \quad \text{where } q^0 = \begin{pmatrix} q_1^0 \\ q_2^0 \\ q_3^0 \end{pmatrix} = f^{-1}(A)$$

I solve the IK for A

$$\begin{cases} 2(c_1+c_{12}+c_{123}) = 3 \\ 2(s_1+s_{12}+s_{123}) = 2.5 \\ q_1+q_2+q_3 = 1.8724 \end{cases} \Rightarrow \begin{cases} 2(c_1+c_{12}) = 3.594 \\ 2(s_1+s_{12}) = 0.59 \end{cases} \Rightarrow 4(c_1^2+c_{12}^2+2c_1c_{12}) + 4(s_1^2+s_{12}^2+2s_1s_{12}) = 13.265 \Rightarrow$$

$$4(2+2c_2) = 13.265 \Rightarrow c_2 = 0.6581 \Rightarrow q_2^0 = 0.8525 \qquad s_2 = 0.753$$

$$\begin{cases} 2(c_1+c_{12}) = 3.594 \\ 2(s_1+s_{12}) = 0.59 \end{cases} \Rightarrow \begin{cases} 2c_1 + 2(c_1c_2 - s_1s_2) = 3.594 \\ 2s_1 + 2(s_1c_2 + c_1s_2) = 0.59 \end{cases} \Rightarrow \begin{cases} 2c_1 + 2(c_1 \cdot 0.6581 - s_1 \cdot 0.753) = 3.594 \\ 2s_1 + 2(s_1 \cdot 0.6581 + c_1 \cdot 0.753) = 0.59 \end{cases}$$

$$\begin{cases} 3.3162c_1 - 1.506s_1 = 3.594 \\ 1.506c_1 + 3.3162s_1 = 0.59 \end{cases} \Rightarrow \begin{cases} c_1 = 0.965 \\ s_1 = -0.26 \end{cases} \Rightarrow q_1^0 = -0.263 \Rightarrow q_3^0 = 1.2829$$