Since I have already taken the midterm, I will only be completing the exercises related to the second part of the course (from differential kinematics onwards).

Exercise #1

Consider the 3-dof PPR robot in Fig. 1, with a jaw gripper mounted on the end effector.

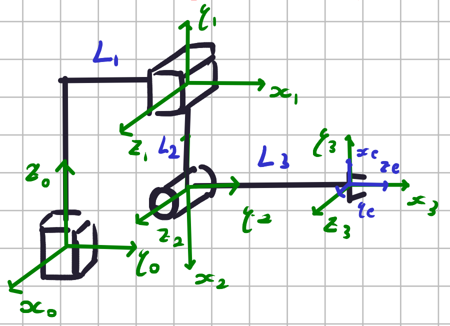
a) Assign and draw the robot frames according to the Denavit-Hartenberg (DH) convention. Place the origin of frame 0 on the floor and the origin of the last frame at the center of the gripper. Compile the associated table of DH parameters.

b) Check whether the last DH frame assigned coincides in orientation with the definition of the standard frame $(\boldsymbol{n}, \boldsymbol{s}, \boldsymbol{a})$ attached to a jaw gripper. If not, determine the rotation matrix ${}^3\boldsymbol{R}_g$ needed to align the two frames.

c) Provide the expression of the direct kinematics $\boldsymbol{p} = \boldsymbol{f}(\boldsymbol{q})$ between $\boldsymbol{q} = (q_1, q_2, q_3)$ and the position $\boldsymbol{p} = (p_x, p_y, p_z)$ of the center of the gripper.

d) Derive the 3×3 Jacobian matrix $\boldsymbol{J}(\boldsymbol{q})$ relating $\dot{\boldsymbol{q}}$ to the linear velocity $\boldsymbol{v} = \dot{\boldsymbol{p}}$ in two different ways, as part of the geometric Jacobian of the robot and using differentiation w.r.t. time.

e) Find all the singular configurations of matrix $\boldsymbol{J}(\boldsymbol{q})$. In one of such configurations $\boldsymbol{q}_s$, characterize which Cartesian directions are instantaneously accessible by the robot gripper and which not.



Joint	α_i	a_i	d_i	θ_i
1	$\pi/2$	L_1	q_1	$\pi/2$
2	0	L_2	q_2	$-\pi/2$
3	0	L_3	0	q_3

$$ {}^0T_1 = \begin{pmatrix} 0 & 0 & 1 & 0 \\ 1 & 0 & 0 & L_1 \\ 0 & 1 & 0 & q_1 \\ 0 & 0 & 0 & 1 \end{pmatrix} \quad {}^1T_2 = \begin{pmatrix} 0 & 1 & 0 & 0 \\ -1 & 0 & 0 & -L_2 \\ 0 & 0 & 1 & q_2 \\ 0 & 0 & 0 & 1 \end{pmatrix} \quad {}^2T_3 = \begin{pmatrix} C_3 & -S_3 & 0 & L_3C_3 \\ S_3 & C_3 & 0 & L_3S_3 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix} $$

$$ {}^0T_2 = \begin{pmatrix} 0 & 0 & 1 & q_2 \\ 0 & 1 & 0 & L_1 \\ -1 & 0 & 0 & q_1-L_2 \\ 0 & 0 & 0 & 1 \end{pmatrix} \quad {}^0T_3 = \begin{pmatrix} 0 & 0 & 1 & q_2 \\ 0 & 1 & 0 & L_1 \\ -1 & 0 & 0 & q_1-L_2 \\ 0 & 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} C_3 & -S_3 & 0 & L_3C_3 \\ S_3 & C_3 & 0 & L_3S_3 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix} = \begin{pmatrix} 0 & 0 & 1 & q_2 \\ S_3 & C_3 & 0 & L_3S_3+L_1 \\ -C_3 & S_3 & 0 & q_1-L_2-L_3C_3 \\ 0 & 0 & 0 & 1 \end{pmatrix} $$

$$ {}^0T_e = \begin{pmatrix} 0 & 1 & 0 & q_2 \\ C_3 & 0 & S_3 & L_3S_3+L_1 \\ S_3 & 0 & -C_3 & q_1-L_2-L_3C_3 \\ 0 & 0 & 0 & 1 \end{pmatrix} \Rightarrow J(q) = \begin{pmatrix} 0 & 1 & 0 \\ 0 & 0 & L_3C_3 \\ 1 & 0 & L_3S_3 \end{pmatrix} \Rightarrow \det J = L_3C_3 = 0 \Leftrightarrow q_3 = \pm\frac{\pi}{2} $$

Exercise #2

For the robot in Fig. 1, using the associated symbolic DH parameters, determine a smooth and coordinated rest-to-rest joint trajectory that will move in T seconds the robot gripper from the initial position $\boldsymbol{p}_i = (a_1 + a_3, 0, 0)$ to the final position $\boldsymbol{p}_f = (a_1, -\delta, 0)$, with $\delta > 0$. Sketch a plot of the obtained joint trajectory $\boldsymbol{q}_d(t) = (q_{1d}(t), q_{2d}(t), q_{3d}(t))$. What will be the maximum value of the norm of the joint velocity $\|\dot{\boldsymbol{q}}_d(t)\|$ during the interval $[0, T]$?

I consider $p(s) = p_i + (p_f - p_i)\frac{s}{L}$ $\quad s \in [0, L]$ and $L = \|p_f - p_i\| = \sqrt{a_3^2 + \delta^2}$

and $s(t) = L(a\tau^3 + b\tau^2 + c\tau - d)$ with $\tau = \frac{t}{T}$ and $t \in [0, T]$

$\Rightarrow$ Since it is rest to rest: $s(t) = \frac{L}{T^2}\left(-2\frac{t^3}{T} + 3t^2\right)$

Inverse Kin:

$$ \begin{cases} p_x = q_2 \\ p_y = L_3S_3 + L_1 \\ p_z = q_1 - L_2 - L_3C_3 \end{cases} \Rightarrow \begin{cases} q_2 = p_x \\ \sin q_3 = \frac{p_y - L_1}{L^3} \Rightarrow \cos q_3 = \left(1 - \left(\frac{p_y - L_1}{L^3}\right)^2\right)^{1/2} \Rightarrow q_3 \text{ is computed through atan2.} \end{cases} $$

then $q_1 = p_z + L_2 + L_3\cos q_3$. and $\dot{q} = J^{-1}(q)\dot{p}$

Since $p(s) = \begin{pmatrix} a_1 + a_3 + \frac{s}{L}(-a_3) \\ -\frac{s}{L}\delta \\ 0 \end{pmatrix} \Rightarrow \dot{p} = p'\dot{s} = \begin{pmatrix} -a_3/L \\ -\delta/L \\ 0 \end{pmatrix} \cdot \frac{L}{T^2}\left(-\frac{6}{T}t^2 + 6t\right)$ $\quad J^{-1} = \frac{1}{L_3C_3}\begin{pmatrix} 0 & -L_3S_3 & L_3C_3 \\ L_3C_3 & 0 & 0 \\ 0 & 1 & 0 \end{pmatrix} \Rightarrow$

$$ \dot{q} = \frac{1}{L_3C_3}\begin{pmatrix} 0 & -L_3S_3 & L_3C_3 \\ L_3C_3 & 0 & 0 \\ 0 & 1 & 0 \end{pmatrix}\begin{pmatrix} -a_3/L \\ -\delta/L \\ 0 \end{pmatrix} \cdot \frac{L}{T^2}\left(-\frac{6}{T}t^2 + 6t\right) = \begin{cases} -\tan(q_3)\frac{\delta}{T^2}\left(\frac{6}{T}t^2 - 6t\right) \\ \frac{a_3}{T^2}\left(\frac{6}{T}t^2 - 6t\right) \\ \frac{\delta}{L_3C_3T^2}\left(\frac{6}{T}t^2 - 6t\right) \end{cases} $$