Since I have already taken the midterm, I will only be completing the exercises related to the second part of the course (from differential kinematics onwards).

Exercise 4

Consider a trajectory planning problem for the 3R robot in Fig. 1. The robot should move from the start configuration $\boldsymbol{q}_s = (-\pi/4, \pi/4, \pi/4)$ [rad] to the goal configuration $\boldsymbol{q}_g = (0, 0, \pi/4)$ [rad] in a time $T = 2$ s, with continuity up to the acceleration over the whole interval $t \in [0, T]$. The initial joint velocity is chosen so that the end-effector velocity starts with $\dot{\boldsymbol{p}}(0) = (1, -1, 0)$ [m/s], while the final velocity should be zero. Provide the values of the coefficients of the *doubly normalized* joint trajectories satisfying all the given conditions. Sketch the plots of joint position, velocity and acceleration.

For $\dot{p}(0)$ and $\dot{p}(T)$ i need to Find $\dot{q}(0)$ and $\dot{q}(T)$ by using the Jacobian matrix. From the DK i consider:

$$J = \begin{pmatrix} -LS_1 - NS_{12}C_3 & -NS_{12}C_3 & -NC_{12}S_3 \\ LC_1 + NC_{12}C_3 & NC_{12}C_3 & -NS_{12}S_3 \\ 0 & 0 & NC_3 \end{pmatrix} \quad \text{with } N = M = L = \tfrac{1}{2}$$

$$\text{I consider } J(q_s) = \begin{pmatrix} \sqrt{2}/4 & 0 & -\sqrt{2}/4 \\ \sqrt{2}/2 & \sqrt{2}/4 & 0 \\ 0 & 0 & \sqrt{2}/4 \end{pmatrix} \quad \text{to realize } \dot{p} = \begin{pmatrix} 1 \\ -1 \\ 0 \end{pmatrix} \text{ i consider:}$$

$$\Rightarrow \dot{q}(0) = J^{-1}(q_s)\begin{pmatrix} 1 \\ -1 \\ 0 \end{pmatrix} = (2.82, -8.48, 0)^T. \qquad \text{to } \dot{p} = 0 \rightarrow \dot{q} = 0 \ (\text{at } t = T).$$

$$\text{So: } q(0) = q_s \qquad q(T) = q_g \qquad \dot{q}(0) = (2.82, -8.48, 0)^T \qquad \dot{q}(T) = 0$$

i need a 3-degree polynomial.

$$q_i(\tau) = a\tau^3 + b\tau^2 + c\tau + d, \quad \tau = \frac{t}{T} \qquad t \in [0, T] \Rightarrow \tau \in [0, 1]$$

$$\begin{cases} q_1(0) = d = -\pi/4 \\ \dot{q}_1(0) = c = 2.82 \\ q_1(1) = a + b + c + d = 0 \\ \dot{q}(1) = 3a + 2b + c = 0 \end{cases} \qquad \begin{cases} a = 1.249 \\ b = -3.283 \\ c = 2.82 \\ d = -\pi/4 \end{cases}$$

$$J(q_g) = \begin{pmatrix} 0 & 0 & -\frac{\sqrt{2}}{4} \\ \frac{2+\sqrt{2}}{4} & \frac{\sqrt{2}}{4} & 0 \\ 0 & 0 & \frac{\sqrt{2}}{4} \end{pmatrix}$$