Since I have already taken the midterm, I will only be completing the exercises related to the second part of the course (from differential kinematics onwards).

the DK for p_x, p_y is:

$$\begin{pmatrix} p_x \\ p_y \end{pmatrix} = f_r(q) = \begin{pmatrix} q_1+q_3 \\ q_2+q_4 \end{pmatrix} \Rightarrow J(q) = \begin{pmatrix} 1 & 0 & 1 & 0 \\ 0 & 1 & 0 & 1 \end{pmatrix}$$

$$\Rightarrow \text{since } J\dot{q}=0 \Rightarrow \begin{cases} \dot{q}_1+\dot{q}_3 = 0 \\ \dot{q}_2+\dot{q}_4 = 0 \end{cases} \Rightarrow N(J) = \left\{ \begin{pmatrix} a \\ b \\ -a \\ -b \end{pmatrix} : a, b \in \mathbb{R} \right\}$$

$$\Rightarrow J^T = \begin{pmatrix} 1 & 0 \\ 0 & 1 \\ 1 & 0 \\ 0 & 1 \end{pmatrix} \Rightarrow J^T F = \begin{cases} F_x = 0 \\ F_y = 0 \end{cases} \Rightarrow N(J^T) = \left\{ \begin{pmatrix} 0 \\ 0 \end{pmatrix} \right\}$$

I need the pseudo-inverse of $J \Rightarrow J^{\#} = J^T (J J^T)^{-1} = \begin{pmatrix} 0.5 & 0 \\ 0 & 0.5 \\ 0.5 & 0 \\ 0 & 0.5 \end{pmatrix}$

$$\Rightarrow \dot{q} = J^{\#} \dot{r} = \begin{pmatrix} 0.5 & 0 \\ 0 & 0.5 \\ 0.5 & 0 \\ 0 & 0.5 \end{pmatrix} \begin{pmatrix} 3 \\ -2 \end{pmatrix} = \begin{pmatrix} 3/2 \\ -1/2 \\ 3/2 \\ -1/2 \end{pmatrix}.$$

And $J^T F = \tau$

$$\begin{pmatrix} 1 & 0 \\ 0 & 1 \\ 1 & 0 \\ 0 & 1 \end{pmatrix} \begin{pmatrix} 2 \\ 1 \end{pmatrix} = \begin{pmatrix} 2 \\ 1 \\ 2 \\ 1 \end{pmatrix} = \tau$$ that balance $F = (2, 1)^T$

I consider: $r(s) = r_{in} + \frac{s}{L}(r_{fin} - r_{in})$, $s \in [0, L]$ and $L = \|r_{fin} - r_{in}\| = \sqrt{52}$

then i choose a b-c-b profile for $s(t)$. I need to know the bounds for $\dot{r}$ and $\ddot{r}$.

$$\begin{pmatrix} 1 & 0 & 1 & 0 \\ 0 & 1 & 0 & 1 \end{pmatrix} \begin{pmatrix} 2 \\ 2 \\ 2 \\ 2 \end{pmatrix} = \begin{pmatrix} 4 \\ 4 \end{pmatrix} \Rightarrow |\dot{p}| \le 5.656 \Rightarrow V = 4\sqrt{2}$$

the max acceleration is $\ddot{p}_m = \begin{pmatrix} 10 \\ 10 \end{pmatrix} \Rightarrow A = 10\sqrt{2}$

So: $T_s = \frac{4\sqrt{2}}{10\sqrt{2}} = 2/5$ and $T = \frac{8+5\sqrt{26}}{20}$ and

$$s(t) = \begin{cases} \frac{10\sqrt{2}}{2} t^2 & t \in [0, 2/5] \\ 4\sqrt{2}\, t - \frac{32}{20\sqrt{2}} & t \in [2/5, T - 2/5] \\ -\frac{10\sqrt{2}}{2}(t-T)^2 & t \in [T - 2/5, T] \end{cases}$$