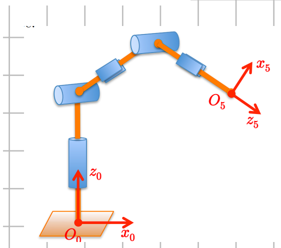
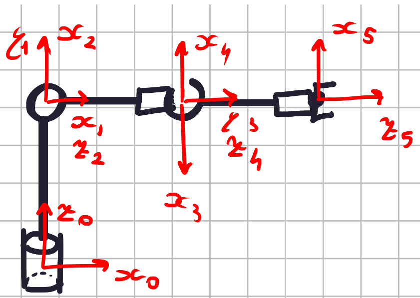
**Exercise 1**

Consider the 5-dof spatial robot in Fig. 1, having the third and fifth joints of the prismatic type while the others are revolute.

- Assign the link frames according to the Denavit-Hartenberg (DH) convention and complete the associated table of parameters so that all constant parameters are *non-negative*. Draw the frames and fill in the table directly on the extra sheet #1 provided separately. The two DH frames 0 and 5 are already assigned and should not be modified. [Please, make clean drawings and return the completed sheet with your name written on it.]
- Sketch the robot in the configuration $\boldsymbol{q}_a = \begin{pmatrix} 0 & \frac{\pi}{2} & 1 & \frac{\pi}{2} & 1 \end{pmatrix}^T$ [rad, rad, m, rad, m].
- For which value $\boldsymbol{q}_b \in \mathbb{R}^5$ does the robot assume a stretched upward configuration?




| | $\alpha_i$ | $a_i$ | $d_i$ | $\theta_i$ |
|---|---|---|---|---|
| 1 | $\pi/2$ | 0 | $d_1$ | $q_1$ |
| 2 | $\pi/2$ | 0 | 0 | $q_2$ |
| 3 | $\pi/2$ | 0 | $q_3$ | $\pi$ |
| 4 | $\pi/2$ | 0 | 0 | $q_4$ |
| 5 | 0 | 0 | $q_5$ | 0 |

$$^0T_1 = \begin{pmatrix} c_1 & 0 & s_1 & 0 \\ s_1 & 0 & -c_1 & 0 \\ 0 & 1 & 0 & d_1 \\ 0 & 0 & 0 & 1 \end{pmatrix}$$

$$^1T_2 = \begin{pmatrix} c_2 & 0 & s_2 & 0 \\ s_2 & 0 & -c_2 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix} \quad ^2T_3 = \begin{pmatrix} -1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 1 & 0 & q_3 \\ 0 & 0 & 0 & 0 \end{pmatrix} \quad ^3T_4 = \begin{pmatrix} c_4 & 0 & s_4 & 0 \\ s_4 & 0 & -c_4 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix} \quad ^4T_5 = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & q_5 \\ 0 & 0 & 0 & 1 \end{pmatrix}$$

$$^0T_2 = \begin{pmatrix} c_1 & 0 & s_1 & 0 \\ s_1 & 0 & -c_1 & 0 \\ 0 & 1 & 0 & d_1 \\ 0 & 0 & 0 & 1 \end{pmatrix}\begin{pmatrix} c_2 & 0 & s_2 & 0 \\ s_2 & 0 & -c_2 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix} = \begin{pmatrix} c_1c_2 & s_1 & c_1s_2 & 0 \\ s_1c_2 & -c_1 & s_1s_2 & 0 \\ s_2 & 0 & -c_2 & d_1 \\ 0 & 0 & 0 & 1 \end{pmatrix}$$

$$^0T_3 = \begin{pmatrix} c_1c_2 & s_1 & c_1s_2 & 0 \\ s_1c_2 & -c_1 & s_1s_2 & 0 \\ s_2 & 0 & -c_2 & d_1 \\ 0 & 0 & 0 & 1 \end{pmatrix}\begin{pmatrix} -1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 1 & 0 & q_3 \\ 0 & 0 & 0 & 0 \end{pmatrix} = \begin{pmatrix} -c_1c_2 & c_1s_2 & s_1 & q_3c_1s_2 \\ -s_1c_2 & s_1s_2 & -c_1 & q_3s_1s_2 \\ -s_2 & -c_2 & 0 & d_1-q_3c_2 \\ 0 & 0 & 0 & 1 \end{pmatrix}$$

$$^0T_4 = \begin{pmatrix} -c_1c_2 & c_1s_2 & s_1 & q_3c_1s_2 \\ -s_1c_2 & s_1s_2 & -c_1 & q_3s_1s_2 \\ -s_2 & -c_2 & 0 & d_1-q_3c_2 \\ 0 & 0 & 0 & 1 \end{pmatrix}\begin{pmatrix} c_4 & 0 & s_4 & 0 \\ s_4 & 0 & -c_4 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix} = \begin{pmatrix} -c_1c_{24} & s_1 & -c_1s_{24} & q_3c_1s_2 \\ -c_1c_{24} & -c_1 & -s_1s_{24} & q_3s_1s_2 \\ -s_{24} & 0 & -c_{24} & d_1-q_3c_2 \\ 0 & 0 & 0 & 1 \end{pmatrix}$$

$$f(q) = \begin{pmatrix} -c_1c_{24} & s_1 & -c_1s_{24} & q_3c_1s_2 \\ -c_1c_{24} & -c_1 & -s_1s_{24} & q_3s_1s_2 \\ -s_{24} & 0 & -c_{24} & d_1-q_3c_2 \\ 0 & 0 & 0 & 1 \end{pmatrix}\begin{pmatrix} 0 \\ 0 \\ q_5 \\ 1 \end{pmatrix} = \begin{cases} -q_5c_1s_{24}+q_3c_1s_2 \\ -q_5s_1s_{24}+q_3s_1s_2 \\ -c_{24}q_5+d_1-q_3c_2 \end{cases}$$

$$\frac{df}{dq} = J_L = \begin{bmatrix} q_5s_1s_{24}-q_3s_1s_2 & -q_5c_1c_{24}+q_3c_1c_2 & c_1s_2 & -q_5c_1c_{24} & -c_1s_{24} \\ -q_5c_1s_{24}+q_3c_1s_{24} & -q_5s_1c_{24}+q_3s_1s_{24} & s_1s_2 & -q_5s_1c_{24} & -s_1s_{24} \\ 0 & q_5s_{24}+q_3s_2 & -c_2 & q_5s_{24} & -c_{24} \end{bmatrix}$$

$$J_A = \begin{pmatrix} z_0 & z_1 & z_2 & z_3 & z_4 \end{pmatrix} = \begin{bmatrix} s_1 & s_1 & c_1s_2 & s_1 \\ -c_1 & -c_1 & s_1s_2 & -c_1 \\ 0 & 0 & -c_2 & 0 \end{bmatrix}$$

$$J(q_2)=\begin{pmatrix} 0 & 1 & 1 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 1 \\ 0 & 0 & 0 & 1 & 0 \\ 0 & -1 & -1 & 0 & -1 \\ 1 & 0 & 0 & 0 & 0 \end{pmatrix}, \quad J^T = \begin{pmatrix} 0 & 0 & 0 & 0 & 0 & 1 \\ 1 & 0 & 1 & 0 & -1 & 0 \\ 1 & 0 & 0 & 0 & -1 & 0 \\ 1 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 & -1 & 0 \end{pmatrix} \Rightarrow J^T w = 0 \Rightarrow \begin{cases} \omega_6 = 0 \\ \omega_1 + \omega_3 - \omega_5 = 0 \\ \omega_1 - \omega_5 = 0 \\ \omega_1 + \omega_4 = 0 \\ \omega_3 - \omega_5 = 0 \end{cases}$$

$$J^T w = 0 \Rightarrow \begin{cases} \omega_6 = 0 \\ \omega_1 + \omega_3 - \omega_5 = 0 \\ \omega_1 - \omega_5 = 0 \\ \omega_1 + \omega_4 = 0 \\ \omega_3 - \omega_5 = 0 \end{cases} \begin{cases} \omega_6 = 0 \\ \omega_1 = \omega_5 = 0 \\ \omega_3 = 0 \\ \omega_4 = 0 \end{cases} \Rightarrow N(J^T) = \mathrm{Span}\left\{ \begin{pmatrix} 0 \\ 1 \\ 0 \\ 0 \\ 0 \\ 0 \end{pmatrix} \right\}$$