Since I have already taken the midterm, I will only be completing the exercises related to the second part of the course (from differential kinematics onwar

Exercise 3

With reference to the cooperative task in Fig. 3, the RP robot a and the PP robot b (two 2-dof planar robots) should jointly move a payload along a parametrized Cartesian path. The path is an arc of a circle to be traced counterclockwise, centered at the origin of the world frame, starting at $P_i = (0.4, 0.3)$ [m], and of length $L = 0.8$ m. The base frame of robot b is placed at $O_b = (0.7, 0.6)$ [m]. The RP robot has symmetric joint velocity and acceleration bounds given by

$$V_{a,1} = 1 \text{ rad/s} \qquad V_{a,2} = 0.7 \text{ m/s} \qquad A_{a,1} = 3 \text{ rad/s}^2 \qquad A_{a,2} = 5 \text{ m/s}^2,$$

while the joints of the PP robot have symmetric (and equal) velocity bounds

$$V_{b,1} = V_{b,2} = 0.6 \text{ m/s},$$

while their acceleration is in practice unlimited. Determine the rest-to-rest *coordinated* trajectory that traces the given path in *minimum time* (coordination means that both robots start and end their motions at the same time). Plot the resulting time profiles of the position and velocity of the joints for the two robots.

Since it's an arc of circle, $q_{a,2}$ will be fixed at $\|P_i\| = (0.4^2 + 0.3^2)^{1/2} = 1/2$ and only $q_{a,1}$ will move. The arc of 0.8 [m] corresponds to $\frac{8}{5}$ rad.

So $q_{a,1}$ starts at $\operatorname{atan}\left(\frac{0.3}{0.4}\right) \approx 0.64$, so:

$$q_{1,a}(\sigma) = 0.64 + \frac{\sigma}{0.8}\left(\frac{8}{5} - 0.64\right) \text{ [rad]}, \quad \sigma \in [0, 0.8] \qquad \sigma: \text{arc parameter}$$

Now i have to plan the path for robot b. The ee-position of the second robot in the reference frame RF_ω is:

$$p_b = O_b - \begin{pmatrix} q_{b,2} \\ q_{b,1} \end{pmatrix} = \begin{pmatrix} 0.7 - q_{b,2} \\ 0.6 - q_{b,1} \end{pmatrix} \text{ should realize } \begin{pmatrix} \frac{1}{2}\cos(\phi) \\ \frac{1}{2}\sin(\phi) \end{pmatrix} \quad \phi \in \left[0.64, \frac{8}{5}\right]$$

$$\Rightarrow q_b = \begin{pmatrix} 0.7 - \frac{1}{2}\cos(\phi) \\ 0.6 - \frac{1}{2}\sin(\phi) \end{pmatrix} \quad \phi \in \left[0.64, \frac{8}{5}\right] \Rightarrow q_b(\sigma) = \begin{pmatrix} 0.7 - \frac{1}{2}\cos\left(0.64 + \frac{\sigma}{0.8}\left(\frac{8}{5} - 0.64\right)\right) \\ 0.6 - \frac{1}{2}\sin\left(0.64 + \frac{\sigma}{0.8}\left(\frac{8}{5} - 0.64\right)\right) \end{pmatrix} \quad \sigma \in [0, 0.8]$$

Now, i have to decide a timing law for σ (shared between the two robots). I'll use a bang-coast-bang acceleration by taking in account the limitation of both robots using $V = 0.6$ and $A = 3$ as bounds.

I have the following data: $T_s = 1/5$ [s] $T = 1.5\bar{3}$ [s] $\Rightarrow$

$$\sigma(t) = \begin{cases} 3/2 \cdot t^2 & \text{if } t \in [0, \frac{1}{5}] \\ 0.6 \cdot t - \frac{3}{50} & \text{if } t \in [\frac{1}{5}, 1.\bar{3}] \\ -\frac{3}{2}(t - 1.5\bar{3})^2 + 0.8 & \text{if } t \in [1.\bar{3}, 1.5\bar{3}] \end{cases}$$

Exercise 4

A robot joint has maximum (absolute) speed $v_{max} > 0$ and acceleration $a_{max} > 0$. It should move in minimum time from an initial value q_i to a final value $q_f > q_i$, starting with an initial speed $v_i \geq 0$ and ending with a final speed $v_f \geq 0$ (which may be larger, equal or smaller than v_i). Both v_i and v_f are assumed to be feasible. Find the symbolic expression of the minimum time T and draw the associated position, velocity and acceleration profiles. The result is a generalization of the known rest-to-rest case ($v_i = v_f = 0$), in which a bang-coast-bang (b-c-b) profile is found as a solution. Determine also the condition under which a b-c-b solution is found in the present case. Discuss briefly what happens in the limit case and when such condition is violated.

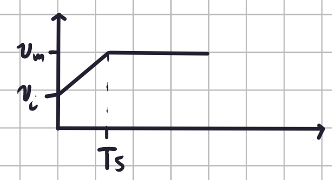
Provide then the numerical value of T and of the other characterizing parameters of this trajectory for the following data

$$q_i = -2 \qquad q_f = 1 \text{ [rad]} \qquad v_i = 0.5 \qquad v_f = 1 \text{ [rad/s]},$$

with the bounds

$$v_{max} = 2 \text{ rad/s} \qquad a_{max} = 3 \text{ rad/s}^2.$$

we have to reach v_m starting from v_i, so we should increase of $v_m - v_i$ in the raising time T_s

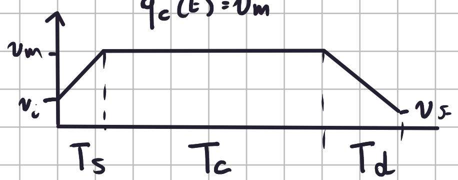


$$\Rightarrow \dot{q}(t) = v_i + t(v_m - v_i) \quad t \in [0, T_s]$$

$$\Rightarrow q(t) = q_i + v_i t + t^2(v_s - v_i) \quad t \in [0, T_s]$$

Let's say there is a coast phase when $q(t)$ reaches $q^* < q_s$, we have to pass from v_m to v_s : $\dot{q}(t) = v_m + t(v_s - v_m)$ $t \in [T - T_d, T]$ where T_d is the time to pass from v_m to v_s.

the speed on T_c is constant.



I denote q_s, q_c, q_d the profiles in the 3 intervals: $T_c = T - T_d$

$$\int_0^T q\,dt = \int_0^{T_s} q_s\,dt + \int_{T_s}^{T-T_d} q_c\,dt + \int_{T-T_d}^{T} q_d\,dt =$$

$$t^2(v_m - v_i) + v_i t \Big|_0^{T_s} + v_m t \Big|_{T_s}^{T-T_d} + t v_m + t^2(v_s - v_m) \Big|_{T-T_d}^{T} =$$

$$T_s^2(v_m - v_i) + v_i T_s + v_m(T - T_d) - v_m T_s + v_m T + T^2(v_s - v_m) - (T - T_d)v_m - (T - T_d)^2(v_s - v_m)$$

For the Fundamental theorem of calculus this is equals to $q_s - q_i$.

T_s is $\dfrac{v_m - v_i}{a_m}$ and $T_d = \dfrac{v_s - v_m}{a_m}$ so

$$\Rightarrow T_s^2 v_m - v_m T_s + v_m T + 2T T_d v_s - 2T T_d v_m - T_d^2 v_s + T_d^2 v_m + (T_s - T_s^2)v_i = q_s - q_i \Rightarrow$$

$$T = \frac{(q_f - q_i) + v_m(T_s - T_s^2 - T_d^2) + T_d^2 v_s - v_i(T_s - T_s^2)}{v_m + 2T_d(v_s - v_m)}$$