Since I have already taken the midterm, I will only be completing the exercises related to the second part of the course (from differential kinematics onwards).

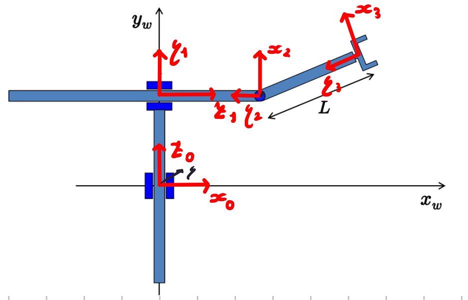
Exercise 3

With reference to the DH assignment in Exercise 1, determine the 6×3 geometric Jacobian ${}^w\boldsymbol{J}(\boldsymbol{q})$ relating the joint velocity $\dot{\boldsymbol{q}} \in \mathbb{R}^3$ to the end-effector velocities ${}^w\boldsymbol{v}_e$ and ${}^w\boldsymbol{\omega}_e$, all expressed in the world frame. Find then all possible configurations for which this matrix loses rank. Next, with the robot in the configuration $\bar{\boldsymbol{q}} = (1, 1, 0)$ [m,m,rad] and with $L = 0.5$ m:

a) compute the joint velocity $\dot{\boldsymbol{q}}_a$ producing the end-effector velocity ${}^w\boldsymbol{v}_e = (-2, 1, 0)$ [m/s];

b) compute the joint velocity $\dot{\boldsymbol{q}}_b$ producing the angular velocity ${}^w\boldsymbol{\omega}_e = (0, 0, -3)$ [rad/s] of the end-effector frame;

c) determine whether the end-effector (twist) velocity $\left({}^w\boldsymbol{v}_e^T \; {}^w\boldsymbol{\omega}_e^T\right)^T = \begin{pmatrix} 1 & 0 & 0 & 0 & 0 & 1 \end{pmatrix}^T$ is admissible for this robot and, if so, compute the joint velocity $\dot{\boldsymbol{q}}_c$ that realizes it.



Joint	α_i	a_i	d_i	θ_i
1	$\pi/2$	0	q_1	$\pi/2$
2	$-\pi/2$	0	q_2	$\pi/2$
3	0	L	0	q_3

$${}^wT_0 = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & -1 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \qquad {}^wT_1 = {}^wT_0 {}^0T_1 = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & -1 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 0 & 0 & 1 & 0 \\ 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & q_1 \\ 0 & 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} 0 & 0 & 1 & 0 \\ 0 & 1 & 0 & q_1 \\ -1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$${}^wT_2 = {}^wT_1 {}^1T_2 = \begin{bmatrix} 0 & 0 & 1 & 0 \\ 0 & 1 & 0 & q_1 \\ -1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 0 & 0 & -1 & 0 \\ 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & q_2 \\ 0 & 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} 0 & 1 & 0 & q_2 \\ 1 & 0 & 0 & q_1 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$${}^wT_3 = {}^wT_2 {}^2T_3 = \begin{bmatrix} 0 & 1 & 0 & q_2 \\ 1 & 0 & 0 & q_1 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} \cos q_3 & -\sin q_3 & 0 & L\cos q_3 \\ \sin q_3 & \cos q_3 & 0 & L\sin q_3 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} \sin q_3 & \cos q_3 & 0 & L\sin q_3 + q_2 \\ \cos q_3 & -\sin q_3 & 0 & L\cos q_3 + q_1 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$\Rightarrow \quad z_0 = \begin{pmatrix} 0 \\ 1 \\ 0 \end{pmatrix} \quad z_1 = \begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix} \quad z_2 = \begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix} \qquad p_e - p_2 = \begin{pmatrix} L\sin q_3 \\ L\cos q_3 \\ 0 \end{pmatrix} \Rightarrow J = \begin{bmatrix} 0 & 1 & -LC_3 \\ 1 & 0 & LS_3 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$p_0 = \begin{pmatrix} 0 \\ 0 \\ 0 \end{pmatrix} \quad p_1 = \begin{pmatrix} 0 \\ q_1 \\ 0 \end{pmatrix} \quad p_2 = \begin{pmatrix} q_2 \\ q_1 \\ 0 \end{pmatrix}$$

J can always realize any ee-velocity (planar)

$\Rightarrow$ to realize ${}^wv_e$ i consider and invert the linear part of J

$$J(\bar{q}) = \begin{pmatrix} 0 & 1 & -L \\ 1 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix} \dot{q} = \begin{pmatrix} -2 \\ 1 \\ 0 \end{pmatrix} \Rightarrow \begin{cases} \dot{q}_2 - L\dot{q}_3 = -2 \\ \dot{q}_1 = 1 \end{cases} \Rightarrow \begin{cases} \dot{q}_2 = L\dot{q}_3 - 2 \\ \dot{q}_1 = 1 \end{cases} \overset{\text{one sol is}}{\Rightarrow} \begin{cases} \dot{q}_1 = 1 \\ \dot{q}_2 = L - 2 \\ \dot{q}_3 = 1 \end{cases}$$

to realize ${}^w\omega_e$ i consider and invert the angular part of J

$$\begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 1 \end{pmatrix} \dot{q} = \begin{pmatrix} 0 \\ 0 \\ -3 \end{pmatrix} \Rightarrow \dot{q}_3 = -3 \quad \text{and} \quad \dot{q}_1, \dot{q}_2 \in \mathbb{R}$$

Now i study when the twist $(1\ 0\ 0\ 0\ 0\ 1)^T$ is possible.

$$\begin{bmatrix} 0 & 1 & -LC_3 \\ 1 & 0 & LS_3 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 1 \end{bmatrix} \dot q = \begin{bmatrix} 1 \\ 0 \\ 0 \\ 0 \\ 0 \\ 1 \end{bmatrix} \Rightarrow \begin{cases} \dot q_2 - \dot q_3 L C_3 = 1 \\ \dot q_1 + \dot q_3 L S_3 = 0 \\ \dot q_3 = 1 \end{cases} \Rightarrow \begin{cases} \dot q_2 - L\cos q_3 = 1 \\ \dot q_1 + L \sin q_3 = 0 \end{cases} \Rightarrow \begin{cases} \cos q_3 = -\frac{1}{L}(1-\dot q_2) \\ \sin q_3 = -\frac{1}{L}\dot q_1 \end{cases}$$

$\Rightarrow$ the twist is realizible if $q_3 = \text{atan2}\left\{-\frac{1}{L}\dot q_1, -\frac{1}{L}(1-\dot q_2)\right\}$

Exercise 4

For the given PPR robot. with a generic length L of the third link, consider the joint variables as in Exercise 1 and the task variables as in Exercise 2. Plan a path $\boldsymbol{r}_d(s)$, parametrized by $s \in [0,1]$, made by a circle of radius $R = 3L/4$ traced counterclockwise, with the gripper always oriented along the path normal and pointing outside the circle. At the start ($s=0$), the path should be matched with the initial robot configuration $\boldsymbol{q}_0 = (1, 2, -\pi/4)$ [m,m,rad].

the DK of the robot gives the Following initial pose:

$$\begin{cases} 2 + L\cos(-\pi/4) \\ 1 + L\sin(-\pi/4) \\ -\pi/4 \end{cases} \Rightarrow \text{the dist. from the origin is} \quad \left((2+L\cos(-\pi/4))^2 + (1+L\sin(-\pi/4))^2\right)^{1/2}$$

i denote this N

I have to find the center of the circle.

Is at $\begin{pmatrix} 2+L\cos(-\pi/4) \\ 1+L\sin(-\pi/4) \end{pmatrix} - \begin{pmatrix} \cos(-\frac{\pi}{4}) \\ \sin(-\frac{\pi}{4}) \end{pmatrix} \frac{3}{4}L \Rightarrow \begin{matrix} 2+LC(-\pi/4) - \frac{3}{4}LC(-\pi/4) \\ 1+LS(-\pi/4) - \frac{3}{4}LS(-\pi/4) \end{matrix} = \begin{cases} LC(-\pi/4)\frac{1}{4} + 2 \\ LS(-\pi/4)\frac{1}{4} + 1 \end{cases}$

$$\Rightarrow \begin{matrix} \frac{L\sqrt{2}}{8} + 2 \\ -\frac{L\sqrt{2}}{8} + 1 \end{matrix}$$

Center of the circle

$$\Rightarrow \quad r_d(s) = \begin{cases} L\frac{3}{4}\cos\left(2\pi s - \frac{\pi}{4}\right) + \frac{\sqrt{2}}{8}L + 2 \\ L\frac{3}{4}\sin\left(2\pi s - \frac{\pi}{4}\right) - \frac{\sqrt{2}}{8}L + 1 \\ -\frac{\pi}{4} + 2\pi s \end{cases}$$