Since I have already taken the midterm, I will only be completing the exercises related to the second part of the course (from differential kinematics onwards).

Exercise 2

A unitary mass moves along a circular path centered at the origin of the (x, y) plane and having radius $R > 0$. At the initial time $t = 0$, the mass is in $A = (R, 0)$ while at the final time $t = T$ it should be in $B = (-R, 0)$. The timing law is chosen as a cubic rest-to-rest profile. If the norm of the Cartesian acceleration $\|\ddot{p}\|$ is bounded by $A > 0$, what is the minimum feasible time T to execute the desired trajectory? At which time instant(s) is the bound attained? Provide a closed-form solution to the problem in symbolic form, and then evaluate it with the data $R = 1.5$ [m], $A = 3$ [m/s^2]. Sketch the time profile of the norm $\|\ddot{p}(t)\|$ and of the components $\ddot{p}_x(t)$ and $\ddot{p}_y(t)$ of the obtained Cartesian acceleration $\ddot{p}(t)$.

the parametric equation is $p(s) = \begin{pmatrix} R\cos(\pi s) \\ R\sin(\pi s) \end{pmatrix}$ $s \in [0, 1]$. the path length is πR, so, with the arc parameter $p(s) = \begin{pmatrix} R\cos(s/R) \\ R\sin(s/R) \end{pmatrix}$ $s \in [0, \pi R]$.

the timing law is cubic: $s(t) = as^3 + bs^2 + cs + d$ with $s(0) = 0$, $s(T) = \pi R \Rightarrow$

$s(t) = \pi R(-2\tau^3 + 3\tau^2)$ $\tau = \frac{t}{T} \Rightarrow s(t) = \frac{\pi R}{T^2}\left(-\frac{2}{T}t^3 + 3t^2\right)$

now i consider $\ddot{p} = \frac{d^2p}{ds^2}\dot{s}^2 + \frac{dp}{ds}\ddot{s} = p''\dot{s}^2 + p'\ddot{s}$

$p' = \begin{pmatrix} -\pi R\sin(\pi s) \\ \pi R\cos(\pi s) \end{pmatrix}$ $p'' = \begin{pmatrix} -\pi^2 R\cos(\pi s) \\ -\pi^2 R\sin(\pi s) \end{pmatrix} \Rightarrow \|p'\| = \pi R$, $\|p''\| = \pi^2 R$

$$\|\ddot{p}\| \leq A \Rightarrow \left\|\frac{d^2p}{ds^2}\left(\frac{ds}{dt}\right)^2 + \frac{dp}{ds}\frac{d^2s}{dt^2}\right\| = \|p''\dot{s}^2\| + \|p'\ddot{s}\| \leq A \Rightarrow$$

$$\|p''\| \cdot |\dot{s}^2| + \|p'\| \cdot |\ddot{s}| \leq A \Rightarrow$$

$$\pi^2 R \cdot \dot{s}^2 + \pi R \cdot |\ddot{s}| \leq A \Rightarrow$$

$$|\ddot{s}| \leq \frac{A}{\pi R} - \pi\dot{s}^2 \quad \text{with } A = 3,\ R = 3/2 \Rightarrow |\ddot{s}| \leq \left|\frac{2}{\pi} - \pi\dot{s}^2\right|$$

i consider $V_{max} = 1 \Rightarrow A_{max} = 2.5049$

since the acceleration of s is a linear function, the maximum (or minimum) is reached at t=0 or t=T

$\Rightarrow \ddot{s}(t) = \frac{\pi R}{T^2}\left(-\frac{12}{T}t + 6\right)$

at $t = 0 \Rightarrow \ddot{s}(0) = \frac{\pi R 6}{T^2} = 2.5049 \Rightarrow T = 3.3597$

Exercise 3

- For the 4R spatial robot in Fig. 2, compute the 6×4 geometric Jacobian $\boldsymbol{J}(\boldsymbol{q})$ and find all its singular configurations $\boldsymbol{q}_s$, i.e., where rank $\boldsymbol{J}(\boldsymbol{q}_s) < 4$.
- Verify that $\boldsymbol{q}_0 = \boldsymbol{0}$ is NOT a singular configuration. With the robot at $\boldsymbol{q}_0$, show that one of the two following six-dimensional end-effector velocities

$$\boldsymbol{V}_a = \begin{pmatrix} \boldsymbol{v}_a \\ \boldsymbol{\omega}_a \end{pmatrix} = \begin{pmatrix} 0 \\ 3 \\ -3 \\ 0 \\ 0 \\ 1 \end{pmatrix}, \qquad \boldsymbol{V}_b = \begin{pmatrix} \boldsymbol{v}_b \\ \boldsymbol{\omega}_b \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \\ 1 \\ 1 \\ 0 \\ 1 \end{pmatrix}$$

is admissible while the other is not, being $\boldsymbol{v} \in \mathbb{R}^3$ the velocity of point $P = O_4$ and $\boldsymbol{\omega} \in \mathbb{R}^3$ the angular velocity of the DH reference frame RF_4.

Joint	α_i	a_i	d_i	θ_i
1	0	a_1	d_1	q_1
2	$\pi/2$	0	d_2	q_2
3	0	a_3	0	q_3
4	0	a_4	0	q_4

$${}^0T_1 = \begin{pmatrix} C_1 & -S_1 & 0 & a_1C_1 \\ S_1 & C_1 & 0 & a_1S_1 \\ 0 & 0 & 1 & d_1 \\ 0 & 0 & 0 & 1 \end{pmatrix} \qquad {}^2T_3 = \begin{pmatrix} C_3 & -S_3 & 0 & a_3C_3 \\ S_3 & C_3 & 0 & a_3S_3 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}$$

$${}^1T_2 = \begin{pmatrix} C_2 & 0 & S_2 & 0 \\ S_2 & 0 & -C_2 & 0 \\ 0 & 1 & 0 & d_2 \\ 0 & 0 & 0 & 1 \end{pmatrix} \qquad {}^3T_4 = \begin{pmatrix} C_4 & -S_4 & 0 & a_4C_4 \\ S_4 & C_4 & 0 & a_4S_4 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}$$

$${}^0T_2 = \begin{pmatrix} C_1 & -S_1 & 0 & a_1C_1 \\ S_1 & C_1 & 0 & a_1S_1 \\ 0 & 0 & 1 & d_1 \\ 0 & 0 & 0 & 1 \end{pmatrix}\begin{pmatrix} C_2 & 0 & S_2 & 0 \\ S_2 & 0 & -C_2 & 0 \\ 0 & 1 & 0 & d_2 \\ 0 & 0 & 0 & 1 \end{pmatrix} = \begin{pmatrix} C_{12} & 0 & S_{12} & a_1C_1 \\ S_{12} & 0 & -C_{12} & a_1S_1 \\ 0 & 1 & 0 & d_2+d_1 \\ 0 & 0 & 0 & 1 \end{pmatrix}$$

$${}^0T_3 = \begin{pmatrix} C_{12} & 0 & S_{12} & a_1C_1 \\ S_{12} & 0 & -C_{12} & a_1S_1 \\ 0 & 1 & 0 & d_2+d_1 \\ 0 & 0 & 0 & 1 \end{pmatrix}\begin{pmatrix} C_3 & -S_3 & 0 & a_3C_3 \\ S_3 & C_3 & 0 & a_3S_3 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix} = \begin{pmatrix} C_{12}C_3 & -C_{12}S_3 & S_{12} & a_3C_{12}C_3 + a_1C_1 \\ S_{12}C_3 & -S_{12}S_3 & -C_{12} & a_3S_{12}C_3 + a_1S_1 \\ S_3 & C_3 & 0 & a_3S_3 + d_1 + d_2 \\ 0 & 0 & 0 & 1 \end{pmatrix}$$

$$p_{0,E} = {}^0T_3 \begin{pmatrix} a_4C_4 \\ a_4S_4 \\ 0 \\ 1 \end{pmatrix} =$$

$$\begin{pmatrix} C_{12}C_3 & -C_{12}S_3 & S_{12} & a_3C_{12}C_3 + a_1C_1 \\ S_{12}C_3 & -S_{12}S_3 & -C_{12} & a_3S_{12}C_3 + a_1S_1 \\ S_3 & C_3 & 0 & a_3S_3 + d_1 + d_2 \\ 0 & 0 & 0 & 1 \end{pmatrix}\begin{pmatrix} a_4C_4 \\ a_4S_4 \\ 0 \\ 1 \end{pmatrix} = \begin{pmatrix} a_1C_1 + C_{12}(a_3C_3 + a_4C_{34}) \\ a_1S_1 + S_{12}(a_3C_3 + a_4C_{34}) \\ d_1 + d_2 + a_3S_3 + a_4S_{34} \end{pmatrix} = p$$

$$\Rightarrow J_L(q) = \frac{dp}{dq} = \begin{bmatrix} -a_1S_1 - S_{12}(a_3C_3 + a_4C_{34}) & -S_{12}(a_3C_3 + a_4C_{34}) & -C_{12}(a_3S_3 + a_4S_{34}) & -C_{12}a_4S_{34} \\ a_1C_1 + C_{12}(a_3C_3 + a_4C_{34}) & C_{12}(a_3C_3 + a_4C_{34}) & -S_{12}(a_3S_3 + a_4S_{34}) & -S_{12}a_4S_{34} \\ 0 & 0 & a_3C_3 + a_4C_{34} & a_4C_{34} \end{bmatrix}$$

$$J_A(q) = \begin{pmatrix} 0 & 0 & S_{12} & S_{12} \\ 0 & 0 & -C_{12} & -C_{12} \\ 1 & 1 & 0 & 0 \end{pmatrix}$$