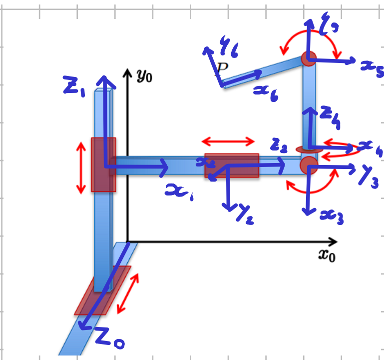
Since I have already taken the midterm, I will only be completing the exercises related to the second part of the course (from differential kinematics onwards).

Exercise 1

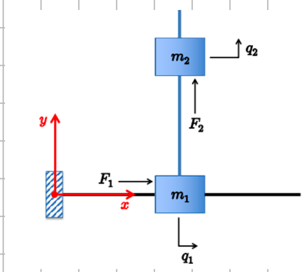
Consider the spatial 6-dof robot in Fig. 1.

- Assign the frames according to the standard Denavit-Hartenberg (DH) convention and provide the corresponding table of parameters. The origin of the last DH frame should coincide with point P. Specify the signs of the linear DH parameters that are constant and non-zero, as well as the signs of the joint variables q_i, $i = 1, \dots, 6$, in the shown configuration.
- Determine the symbolic expression of all elements in the 6×6 geometric Jacobian $\boldsymbol{J}(\boldsymbol{q})$ of this robot and check that $\boldsymbol{q}_0 = (1, 1, 1, -\pi/2, -\pi/2, -\pi/2)$ is a nonsingular configuration.
- At $\boldsymbol{q}_0$, find the position of point P. Moreover, compute a joint velocity $\dot{\boldsymbol{q}} \in \mathbb{R}^6$ that produces the velocity ${}^0\boldsymbol{v} = (0.5, 2, -2)$ [m/s] of P, while the end-effector has an angular velocity ${}^0\boldsymbol{\omega} = (0, 3, 0)$ [rad/s].



Exercise 2

Consider the planar 2P (Cartesian) robot in Fig. 2, where m_1 and m_2 are the masses of the two links in the serial chain. Each input force F_i is bounded in absolute value by $F_{i,max} > 0$, for $i = 1, 2$. Find the expression of the minimum feasible time T for a rest-to-rest robot motion from a start configuration $\boldsymbol{q}_s$ to a goal configuration $\boldsymbol{q}_g$. Compute the numerical value of T with the following data: $m_1 = 5$, $m_2 = 2$ [kg]; $F_{1,max} = 10$, $F_{2,max} = 5$ [N]; $\boldsymbol{q}_s = (0.3, -0.3)$, $\boldsymbol{q}_g = (-0.3, 0.3)$ [m]. Plot the evolutions of $F_i(t)$, $\dot{q}_i(t)$, and $q_i(t)$, for $i = 1, 2$. In your solution, does the mass m_2 trace a linear path during the time-optimal motion?



Since $F_1 = m_1 \ddot{q}_1$, $F_2 = m_2 \ddot{q}_2 \Rightarrow$

$\ddot{q}_1 = \frac{1}{m_1} F_1$ $\quad \ddot{q}_2 = \frac{1}{m_2} F_2$. A rest to rest motion is given by

$q(t) = q_s + \frac{\Delta q}{T^2}\left(-\frac{2}{T}t^3 + 3t^2\right)$ $\quad t \in [0, T]$ and $\Delta q = q_g - q_s$

$\dot{q}(t) = \frac{\Delta q}{T^2}\left(-\frac{6}{T}t^2 + 6t\right)$ $\quad$ if $F_i \le F_{i,max} \Rightarrow$

$m_i \ddot{q}_i \le F_{i,max} \Rightarrow$

$\ddot{q}(t) = \frac{\Delta q}{T^2}\left(-\frac{12}{T}t + 6\right)$ $\quad |\ddot{q}_i| \le \frac{1}{m_i} F_{i,max}$

$\max \ddot{q}_i = |\ddot{q}_i(t = 0 \text{ or } t = T)| \Rightarrow \frac{6}{T^2}\Delta q_i = \frac{1}{m_i} F_{i,max} \Rightarrow \frac{1}{T^2} = F_{i,max} \cdot \frac{1}{6 m_i \Delta q_i} \Rightarrow T = \sqrt{\frac{6 m_i \Delta q_i}{F_{i,max}}}$

With the data: $\sqrt{\frac{6 m_i \Delta q_i}{F_{i,max}}} = \left\{ \frac{3\sqrt{5}}{5}, \frac{6}{5} \right\} \Rightarrow T = \frac{3}{5}\sqrt{5}$

$\Rightarrow q(t) = q_s + \frac{\Delta q}{T^2}\left(-\frac{2}{T}t^3 + 3t^2\right) = \begin{pmatrix} 0.3 \\ -0.3 \end{pmatrix} + \begin{pmatrix} -0.4 \\ 0.4 \end{pmatrix} \frac{5}{9} \cdot \left(-\frac{10}{3\sqrt{5}}t^3 + 3t^2\right)$

the position of m_2 is $\begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} q_1 \\ q_2 \end{pmatrix} = \begin{pmatrix} 0.3 + \frac{\sqrt{5}}{3}t^3 - \frac{3}{2}t^2 \\ -0.3 - \frac{\sqrt{5}}{3}t^3 + \frac{3}{2}t^2 \end{pmatrix} = \begin{pmatrix} s(t) \\ -s(t) \end{pmatrix}$ linear path

$q(t)$