Since I have already taken the midterm, I will only be completing the exercises related to the second part of the course (from differential kinematics onwards).

Exercise 1

The kinematics of a 3R robot is defined by the following Denavit-Hartenberg table (units in [m] or [rad]):

i	α_i	a_i	d_i	θ_i
1	$\pi/2$	0	$d_1 = 5$	q_1
2	0	$a_2 = 4$	0	q_2
3	0	$a_3 = 3$	0	q_3

Determine the 3×3 linear part of the geometric Jacobian $\boldsymbol{J}(\boldsymbol{q})$ of this robot. When the robot is in the configuration $\boldsymbol{q}_0 = (\pi/2, \pi/4, \pi/2)$ [rad] and has a joint velocity $\dot{\boldsymbol{q}}_0 = (1, 2, -2)$ [rad/s], determine, if possible, a joint acceleration $\ddot{\boldsymbol{q}}$ that realizes a zero end-effector acceleration, i.e., $\ddot{\boldsymbol{p}} = \boldsymbol{0}$. *[Bonus: What if the second link parameter is changed to $a_2 = 3$?]*

$$ {}^0T_1 = \begin{pmatrix} C_1 & 0 & S_1 & 0 \\ S_1 & 0 & -C_1 & 0 \\ 0 & 1 & 0 & 5 \\ 0 & 0 & 0 & 1 \end{pmatrix} \quad {}^1T_2 = \begin{pmatrix} C_2 & -S_2 & 0 & a_2C_2 \\ S_2 & C_2 & 0 & a_2S_2 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix} \quad {}^2T_3 = \begin{pmatrix} C_3 & -S_3 & 0 & a_3C_3 \\ S_3 & C_3 & 0 & a_3S_3 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix} $$

$$ {}^1T_2\,{}^2T_3 = \begin{pmatrix} C_{23} & -S_{23} & 0 & a_2C_2+a_3C_{23} \\ S_{23} & C_{23} & 0 & a_2S_2+a_3S_{23} \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix} \quad {}^0T_1\,{}^1T_3 = \begin{pmatrix} C_1C_{23} & -C_1S_{23} & S_1 & C_1(a_2C_2+a_3C_{23}) \\ S_1C_{23} & -S_1S_{23} & -C_1 & S_1(a_2C_2+a_3C_{23}) \\ S_{23} & C_{23} & 0 & a_2S_2+a_3S_{23}+5 \\ 0 & 0 & 0 & 1 \end{pmatrix} $$

$$ p(q) = \begin{cases} C_1(a_2C_2+a_3C_{23}) \\ S_1(a_2C_2+a_3C_{23}) \\ a_2S_2+a_3S_{23}+5 \end{cases} \Rightarrow J = \frac{dp}{dq} = \begin{pmatrix} -S_1(a_2C_2+a_3C_{23}) & -C_1(a_2S_2+a_3S_{23}) & -a_3C_1S_{23} \\ C_1(a_2C_2+a_3C_{23}) & -S_1(a_2S_2+a_3S_{23}) & -a_3S_1S_{23} \\ 0 & a_2C_2+a_3C_{23} & a_3C_{23} \end{pmatrix} $$

IF $q_0 = \pi \begin{pmatrix} 1/2 \\ 1/4 \\ 1/2 \end{pmatrix}$ we have: $S_1 = 1$, $C_1 = 0$, $S_2 = C_2 = \sqrt2/2$, $C_{23} = -\sqrt2/2$, $S_{23} = \sqrt2/2$ so:

$$ J = \begin{pmatrix} \frac{\sqrt2}{2}(a_3-a_2) & 0 & 0 \\ 0 & -\sqrt2/2(a_2+a_3) & -\frac{\sqrt2}{2}a_3 \\ 0 & a_2\frac{\sqrt2}{2}-a_3\frac{\sqrt2}{2} & -\frac{\sqrt2}{2}a_3 \end{pmatrix} = \begin{pmatrix} -\sqrt2/2 & 0 & 0 \\ 0 & -\frac72\sqrt2 & -\frac32\sqrt2 \\ 0 & \sqrt2/2 & -\frac32\sqrt2 \end{pmatrix} $$

Since $\ddot p = J\ddot q + \dot J \dot q$, i have to compute $\dot J$.

$$ \dot J_1 = \begin{bmatrix} -C_1(a_2C_2+a_3C_{23})\dot q_1 + S_1\left[(a_2S_2+a_3S_{23})\dot q_2 + a_3S_{23}\dot q_3\right] \\ -S_1(a_2C_2+a_3C_{23})\dot q_1 - C_1\left[(a_2S_2+a_3S_{23})\dot q_2 + a_3S_{23}\dot q_3\right] \\ 0 \end{bmatrix} \quad \dot J_2 = \begin{bmatrix} S_1(a_2S_2+a_3S_{23})\dot q_1 - C_1\left[(a_2C_2+a_3C_{23})\dot q_2 + a_3S_{23}\dot q_3\right] \\ -C_1(a_2S_2+a_3S_{23})\dot q_1 - S_1\left[(a_2C_2+a_3C_{23})\dot q_2 + a_3S_{23}\dot q_3\right] \\ -(a_2S_2+a_3S_{23})\dot q_2 - a_3S_{23}\dot q_3 \end{bmatrix} $$

$$ \dot J_3 = \begin{bmatrix} a_3S_1S_{23}\dot q_1 - a_3C_1C_{23}(\dot q_2+\dot q_3) \\ -a_3C_1S_{23}\dot q_1 - a_3S_1C_{23}(\dot q_2+\dot q_3) \\ -a_3S_{23}(\dot q_2+\dot q_3) \end{bmatrix} $$

by using the numerical data and $\dot q_0 = (1 \;\; 2 \;\; -2)^T$ $\Rightarrow$ $\dot J = \begin{pmatrix} 4\sqrt2 & \frac72\sqrt2 & \frac32\sqrt2 \\ -\sqrt2/2 & 2\sqrt2 & 0 \\ 0 & -4\sqrt2 & 0 \end{pmatrix}$

$$ \Rightarrow \ddot p = \begin{pmatrix} -\sqrt2/2 & 0 & 0 \\ 0 & -\frac72\sqrt2 & -\frac32\sqrt2 \\ 0 & \sqrt2/2 & -\frac32\sqrt2 \end{pmatrix}\ddot q + \begin{pmatrix} 4\sqrt2 & \frac72\sqrt2 & \frac32\sqrt2 \\ -\sqrt2/2 & 2\sqrt2 & 0 \\ 0 & -4\sqrt2 & 0 \end{pmatrix}\begin{pmatrix} 1 \\ 2 \\ -2 \end{pmatrix} = \begin{cases} -\frac{\sqrt2}{2}\ddot q_1 + 11.314 \\ -\frac72\sqrt2\ddot q_2 - \frac32\sqrt2\ddot q_3 + 4.95 \\ \sqrt2/2\,\ddot q_2 - \frac32\sqrt2\ddot q_3 - 11.314 \end{cases} $$

i solve for $\ddot q$ with $\ddot p = 0$

$$ \begin{cases} -\frac{\sqrt2}{2}\ddot q_1 + 11.314 = 0 \\ -\frac72\sqrt2\ddot q_2 - \frac32\sqrt2\ddot q_3 + 4.95 = 0 \\ \sqrt2/2\,\ddot q_2 - \frac32\sqrt2\ddot q_3 - 11.314 = 0 \end{cases} $$

3 eq. in 3 unknown $\Rightarrow$

$$ \begin{cases} \ddot q_1 = 16 \\ \ddot q_2 = 2.875 \\ \ddot q_3 = -4.375 \end{cases} $$

if $a_2 = a_3 = 3$:

$$ J = \begin{pmatrix} 0 & 0 & 0 \\ 0 & -3\sqrt2 & -\frac{\sqrt2}{2}3 \\ 0 & 0 & -\frac{\sqrt2}{2}3 \end{pmatrix} \qquad \dot J = \begin{pmatrix} \frac92\sqrt2 & 3\sqrt2 & 0 \\ 0 & 3\sqrt2 & 0 \\ 0 & -3\sqrt2 & \frac32\sqrt2 \end{pmatrix} $$

$$ \Rightarrow \ddot p = \begin{cases} 14.849 = 0 \\ -3\sqrt2\ddot q_2 - \frac32\sqrt2\ddot q_3 + 8.485 = 0 \\ -\frac32\sqrt2\ddot q_3 - 12.728 = 0 \end{cases} $$

$\Rightarrow$ Impossible to realize $\ddot p = 0$ in the given conditions.

Exercise 2

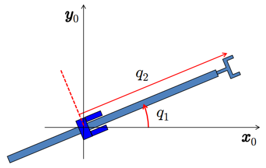


Figure 1: A RP planar robot, with the definition of the joint variables.

The RP robot shown in Fig. 1 starts from rest at time $t = 0$ in the configuration $\boldsymbol{q}(0) = (0, 1)$ [rad; m] and moves under the action of the following discontinuous joint acceleration commands for a time $T = 2$ [s]:

$$\ddot{q}_1(t) = \begin{cases} A_1 = 2 \text{ [rad/s}^2\text{]}, & t \in [0, T/4], \\ 0, & t \in [T/4, 3T/4], \\ -A_1 = -2 \text{ [rad/s}^2\text{]}, & t \in [3T/4, T]; \end{cases} \qquad \ddot{q}_2(t) = \begin{cases} -A_2 = -0.5 \text{ [m/s}^2\text{]}, & t \in [0, T/2], \\ A_2 = 0.5 \text{ [m/s}^2\text{]}, & t \in [T/2, T]. \end{cases}$$

a. Plot the time profiles of $q_i(t)$, $\dot{q}_i(t)$ and $\ddot{q}_i(t)$, for $i = 1, 2$.

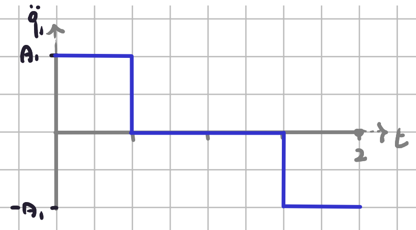
b. Does the robot cross a singularity during this motion?

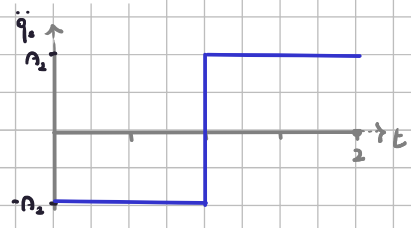
c. Compute the mid time configuration $\boldsymbol{q}(T/2)$ and the final configuration $\boldsymbol{q}(T)$ reached in this motion. Sketch the robot in these two configurations, as well as in the initial one.

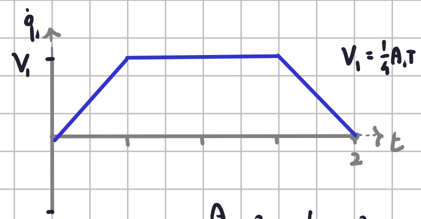
d. Provide the analytic expressions of the end-effector velocity and acceleration norms, i.e., $\|\dot{\boldsymbol{p}}\|$ and $\|\ddot{\boldsymbol{p}}\|$.

e. Draw the end-effector velocity and acceleration vectors $\dot{\boldsymbol{p}}(T/2)$, $\ddot{\boldsymbol{p}}((T/2)^-)$ and $\ddot{\boldsymbol{p}}((T/2)^+)$ on the mid time configuration of the robot sketched at item c. Compute the numerical values of $\|\dot{\boldsymbol{p}}(T/2)\|$, $\|\ddot{\boldsymbol{p}}((T/2)^-)\|$ and $\|\ddot{\boldsymbol{p}}((T/2)^+)\|$.

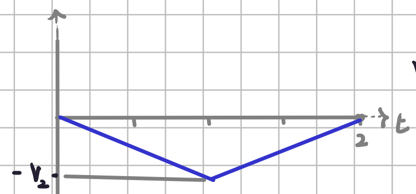
$q_{2C},$
$q_{2S},$





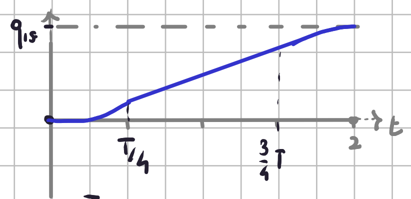


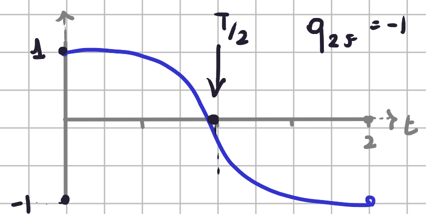
$V_1 = \frac{1}{4} A_1 T$



$V_2 = \frac{1}{2} A_2 T$

$\frac{A_1}{16} T^2 + \frac{1}{8} A_1 T^2$





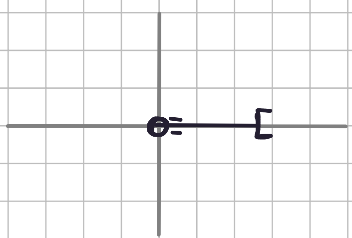
$q_{2s} = -1$

$q_{1s} = \int_0^T \dot{q}_1 \, dt = \frac{3}{16} A_1 T^2 = \frac{3}{2}$

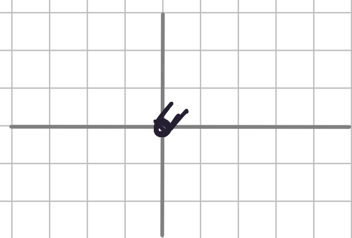
In $t = \frac{T}{2}$ we have $q_2 = 0$, is a singular conf. since $\det J = -q_2$.

here $q(\frac{T}{2}) = \begin{pmatrix} 3/4 \\ 0 \end{pmatrix}$ and $q(T) = \begin{pmatrix} 3/2 \\ -1 \end{pmatrix}$

$t = 0$



$t = 1$



$t = 2$