ECE2150J Introduction to Circuits

Chapter 3. Methods of Analysis

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3.1 Introduction

- In this chapter, we develop two powerful techniques for circuit analysis:
  1) Nodal analysis: based on KCL
  2) Mesh analysis: based on KVL

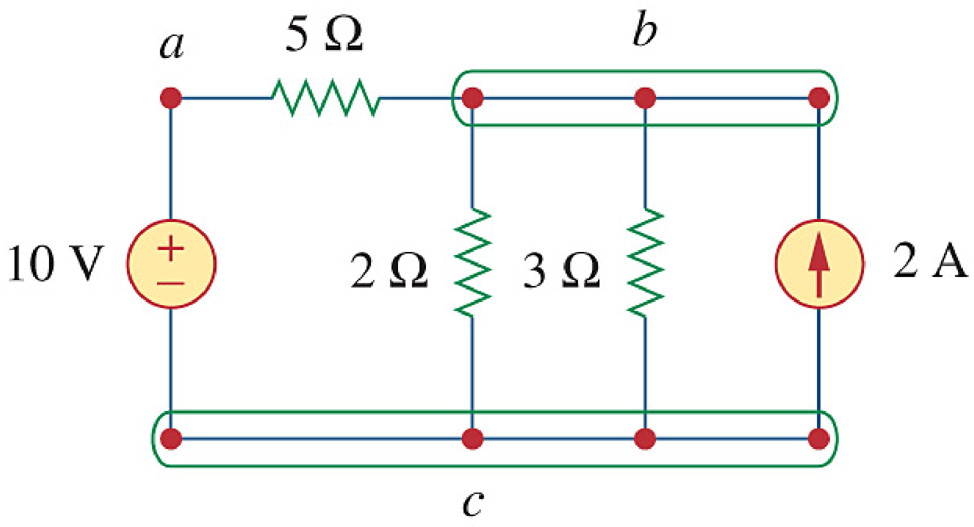
- We will use these two methods throughout this term and in the advanced classes.

3.2 Nodal Analysis

- Nodal analysis provides a general procedure for analyzing circuits using ***node voltages*** as the circuit variables.

- **Choosing node voltages instead of element voltages** as circuit variables

This circuit has three nodes, but we can set only two variables. How?

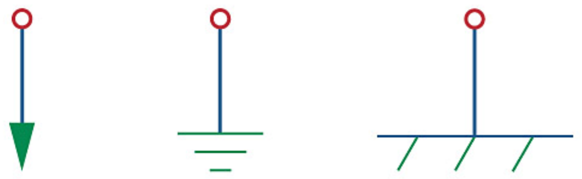


We will make one node **as a reference**, i.e. ground.
→ reducing one variable

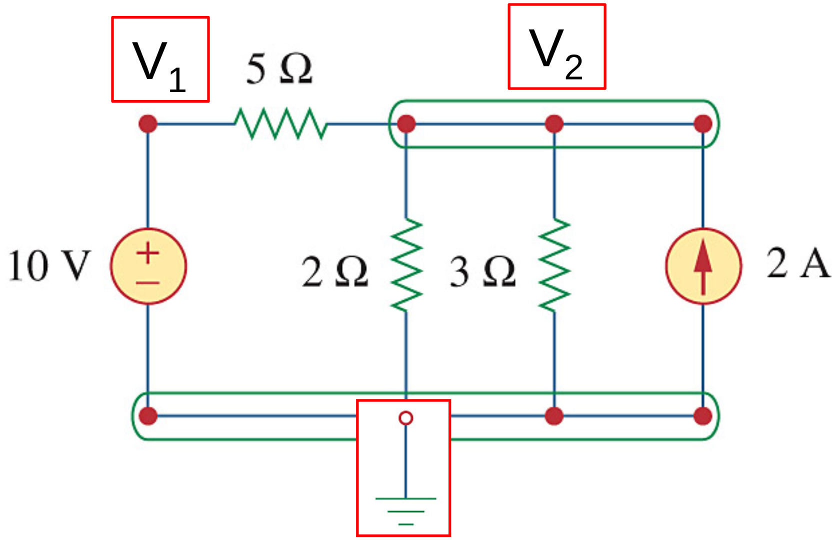
Steps to Determine Node Voltages

Given a circuit with n nodes **without voltage sources**, the nodal analysis involves taking the following three steps.

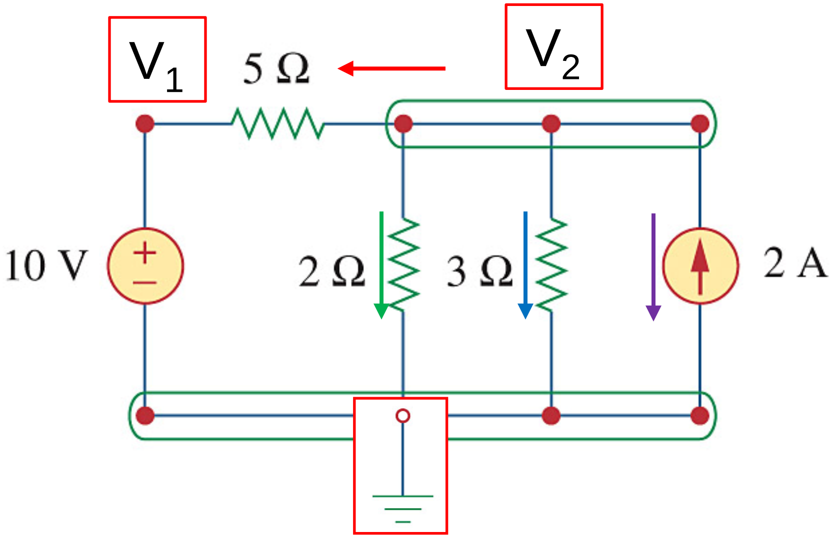
(1) Select a node as the reference node, i.e. ground. Then, assign voltages v_1, v_2, …, v_{n-1} to the remaining n–1 nodes.



Symbol for a reference node



(2) Apply KCL to each of the n−1 nonreference nodes. Use Ohm's law to express the branch current in terms of node voltages.



e.g. KCL from V_2 node gives

$$\frac{V_2 - V_1}{5} + \frac{V_2 - 0}{2} + \frac{V_2 - 0}{3} + (-2) = 0$$

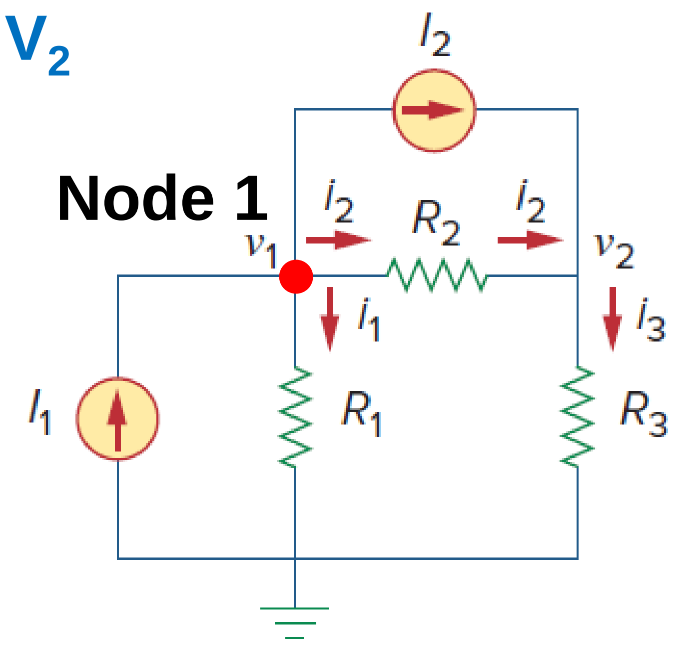
→ A sum of currents leaving a node = 0

(3) Solve the resulting simultaneous equations to obtain the unknown node voltages.

***Because resistance is a passive element, by the passive sign convention, **current must always flow from a higher potential to a lower potential**.

Let's assume that **V_1 is higher than V_2** in the following example.

(i) $V_1 > V_2$



Node 1:

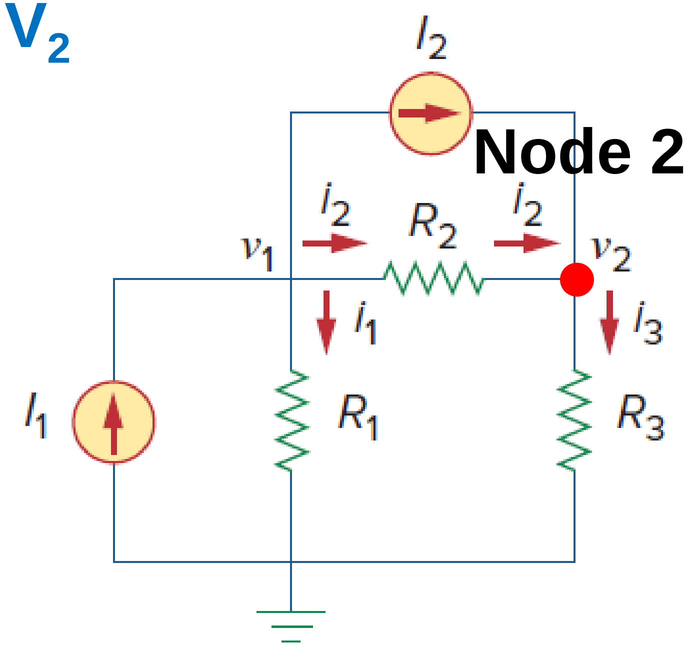
By KCL $\quad I_1 = I_2 + i_1 + i_2$

By Ohm's law

$$i_1 = \frac{v_1 - 0}{R_1} \quad \text{or} \quad i_1 = G_1 v_1$$

$$i_2 = \frac{v_1 - v_2}{R_2} \quad \text{or} \quad i_2 = G_2(v_1 - v_2)$$

(i) $V_1 > V_2$



Node 2:

By KCL $\quad I_2 + i_2 = i_3$

By Ohm's law

$$i_2 = \frac{v_1 - v_2}{R_2} \quad \text{or} \quad i_2 = G_2(v_1 - v_2)$$

$$i_3 = \frac{v_2 - 0}{R_3} \quad \text{or} \quad i_3 = G_3 v_2$$

Therefore,

Node 1: $\quad I_1 = I_2 + i_1 + i_2 \quad \rightarrow \quad I_1 = I_2 + \dfrac{v_1}{R_1} + \dfrac{v_1 - v_2}{R_2}$

Node 2: $\quad I_2 + i_2 = i_3 \quad \rightarrow \quad I_2 + \dfrac{v_1 - v_2}{R_2} = \dfrac{v_2}{R_3}$

(i) $V_1 > V_2$

We rewrite the form

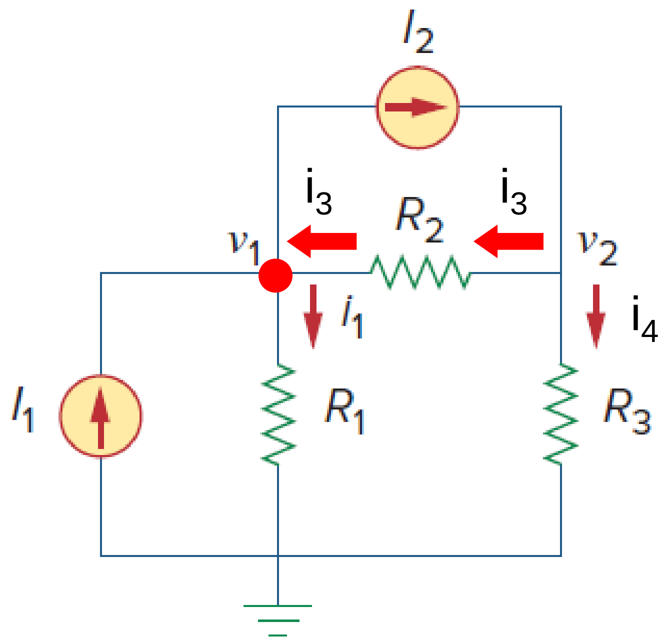
$$I_1 = I_2 + \frac{v_1}{R_1} + \frac{v_1 - v_2}{R_2} \qquad\qquad I_1 = I_2 + G_1 v_1 + G_2(v_1 - v_2)$$
$$\rightarrow$$
$$I_2 + \frac{v_1 - v_2}{R_2} = \frac{v_2}{R_3} \qquad\qquad I_2 + G_2(v_1 - v_2) = G_3 v_2$$

Using a matrix form, we get

$$\begin{bmatrix} G_1 + G_2 & -G_2 \\ -G_2 & G_2 + G_3 \end{bmatrix} \begin{bmatrix} v_1 \\ v_2 \end{bmatrix} = \begin{bmatrix} I_1 - I_2 \\ I_2 \end{bmatrix}$$

But.. we assumed that V_1 is higher than V_2 in the following example.
What if V_2 is higher than V_1?

(ii) $V_2 > V_1$



Node 1:

By KCL $\quad I_1 + i_3 = i_1 + I_2$

By Ohm's law

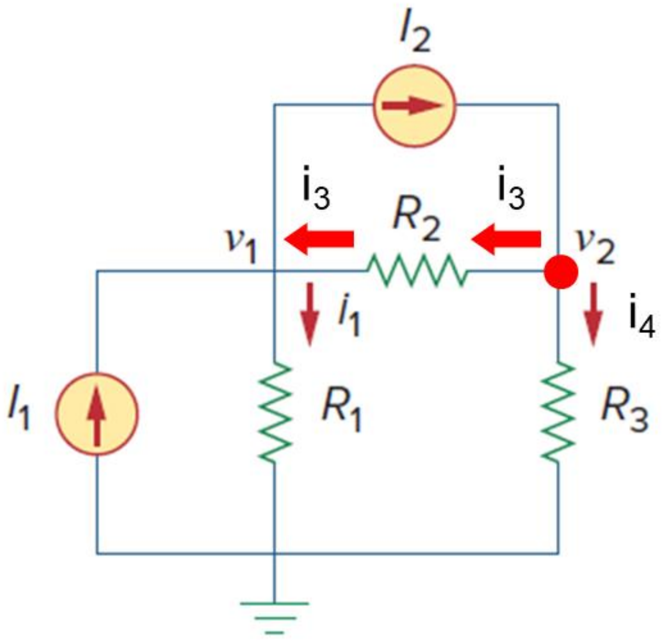
$$i_1 = \frac{v_1 - 0}{R_1} \qquad i_3 = \frac{v_2 - v_1}{R_2}$$

When $V_2 > V_1$ $\quad I_1 + i_3 = i_1 + I_2 \;\longrightarrow\; I_1 + \frac{v_2 - v_1}{R_2} = \frac{v_1 - 0}{R_1} + I_2$

When $V_1 > V_2$ $\quad I_1 = I_2 + i_1 + i_2 \;\longrightarrow\; I_1 = I_2 + \frac{v_1}{R_1} + \frac{v_1 - v_2}{R_2}$

Two equations are the same

(ii) $V_2 > V_1$



Node 2:

By KCL $\quad I_2 = i_3 + i_4$

By Ohm's law

$$i_3 = \frac{v_2 - v_1}{R_2} \qquad i_4 = \frac{v_2 - 0}{R_3}$$

When $V_2 > V_1$ $\quad I_2 = i_3 + i_4 \quad \longrightarrow \quad I_2 = \frac{v_2 - v_1}{R_2} + \frac{v_2 - 0}{R_3}$

When $V_1 > V_2$ $\quad I_2 + i_2 = i_3 \quad \longrightarrow \quad I_2 + \frac{v_1 - v_2}{R_2} = \frac{v_2}{R_3}$

Two equations are the same

Example 3.1 Calculate the node voltages in the circuit shown in Fig. 3.3(a).

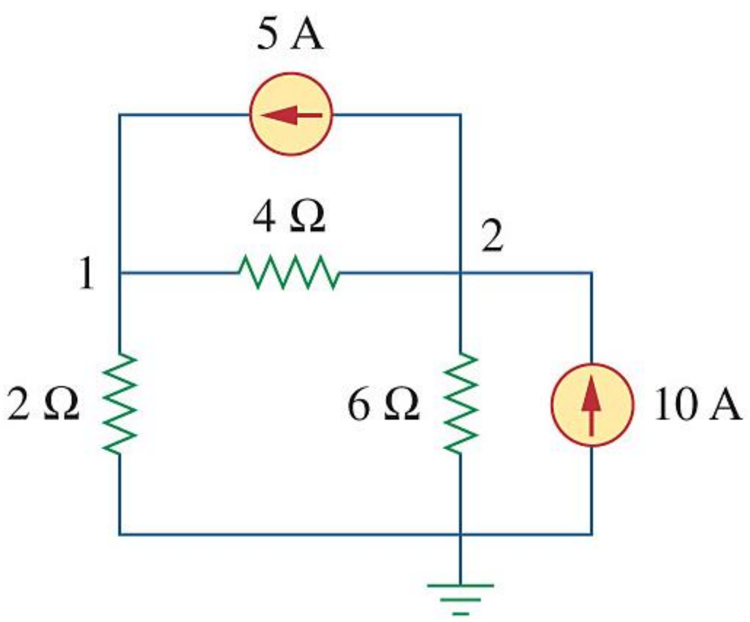


Figure 3.3 (a)

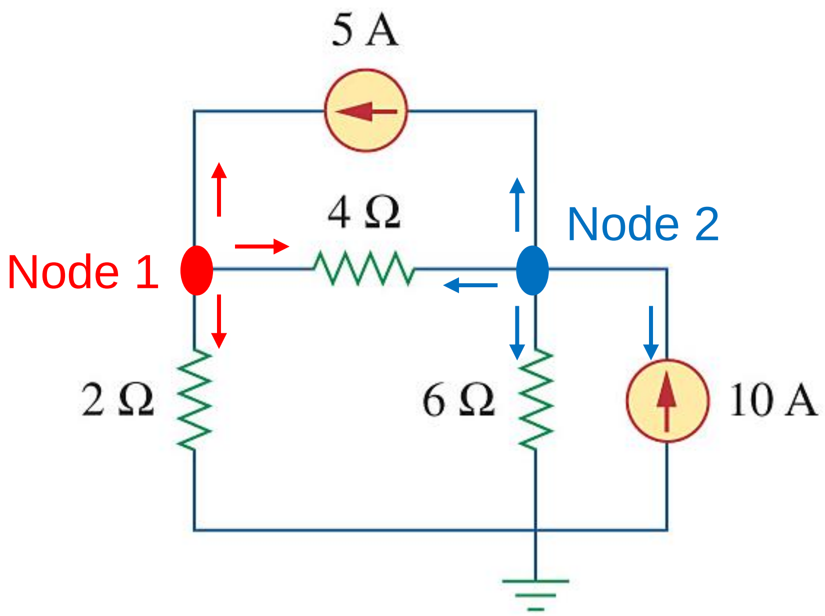


Figure 3.3 (a)

At node 1,

$$\frac{v_1}{2}+\frac{v_1-v_2}{4}-5=0$$

$$\left(\frac{1}{4}+\frac{1}{2}\right)v_1-\frac{1}{4}v_2=5$$

At node 2,

$$\frac{v_2-v_1}{4}+\frac{v_2}{6}-10+5=0$$

$$-\frac{1}{4}v_1+\left(\frac{1}{4}+\frac{1}{6}\right)v_2=10-5$$

$$\left(\frac{1}{4}+\frac{1}{2}\right)v_1 - \frac{1}{4}v_2 = 5$$

$$-\frac{1}{4}v_1 + \left(\frac{1}{4}+\frac{1}{6}\right)v_2 = 10-5$$

$$\Rightarrow \begin{bmatrix} \frac{1}{4}+\frac{1}{2} & -\frac{1}{4} \\ -\frac{1}{4} & \frac{1}{4}+\frac{1}{6} \end{bmatrix} \begin{bmatrix} v_1 \\ v_2 \end{bmatrix} = \begin{bmatrix} 5 \\ 10-5 \end{bmatrix} \begin{array}{l} \text{X 4} \\ \text{X 12} \end{array}$$

We can use an elimination method, but let's try Cramer's rule.

Cramer’s rule

Two equations in the matrix form: $\begin{bmatrix} a & b \\ c & d \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} e \\ f \end{bmatrix}$

Solution x and y can be calculated by Cramer’s rule

$$x = \frac{\begin{vmatrix} e & b \\ f & d \end{vmatrix}}{\begin{vmatrix} a & b \\ c & d \end{vmatrix}} = \frac{ed - bf}{ad - bc} \qquad y = \frac{\begin{vmatrix} a & e \\ c & f \end{vmatrix}}{\begin{vmatrix} a & b \\ c & d \end{vmatrix}} = \frac{af - ec}{ad - bc}$$

Similarly, three equations in the matrix form:

$$\begin{bmatrix} a & b & c \\ d & e & f \\ g & h & i \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} j \\ k \\ l \end{bmatrix}$$

$$x = \frac{\begin{vmatrix} j & b & c \\ k & e & f \\ l & h & i \end{vmatrix}}{\begin{vmatrix} a & b & c \\ d & e & f \\ g & h & i \end{vmatrix}} \quad y = \frac{\begin{vmatrix} a & j & c \\ d & k & f \\ g & l & i \end{vmatrix}}{\begin{vmatrix} a & b & c \\ d & e & f \\ g & h & i \end{vmatrix}} \quad z = \frac{\begin{vmatrix} a & b & j \\ d & e & k \\ g & h & l \end{vmatrix}}{\begin{vmatrix} a & b & c \\ d & e & f \\ g & h & i \end{vmatrix}}$$

https://courses.lumenlearning.com/

$$\begin{bmatrix} 3 & -1 \\ -3 & 5 \end{bmatrix} \begin{bmatrix} v_1 \\ v_2 \end{bmatrix} = \begin{bmatrix} 20 \\ 60 \end{bmatrix}$$

Use Cramer's rule,

$$\Delta = \begin{vmatrix} 3 & -1 \\ -3 & 5 \end{vmatrix} = 3 \times 5 - (-1) \times (-3) = 12$$

$$\Delta_1 = \begin{vmatrix} 20 & -1 \\ 60 & 5 \end{vmatrix} = 20 \times 5 - (-1) \times 60 = 160$$

$$\Delta_2 = \begin{vmatrix} 3 & 20 \\ -3 & 60 \end{vmatrix} = 3 \times 60 - 20 \times (-3) = 240$$

$$v_1 = \frac{\Delta_1}{\Delta} = \frac{160}{12} = \frac{40}{3} \approx 13.33 \text{ (V)}$$

$$v_2 = \frac{\Delta_2}{\Delta} = \frac{240}{12} = 20 \text{ (V)}$$

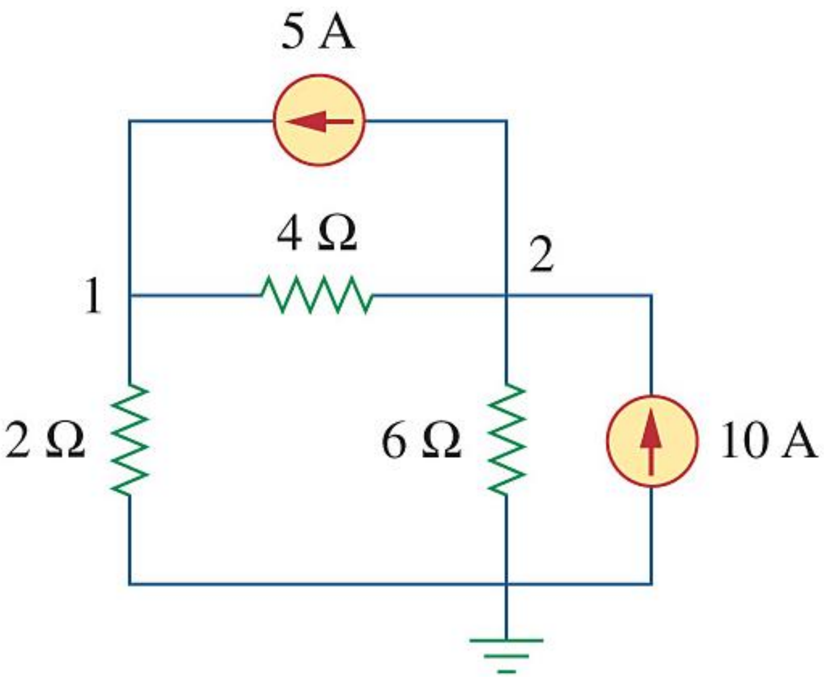


Figure 3.3 (a)

$$\begin{cases} \left(\frac{1}{4}+\frac{1}{2}\right)v_1 - \frac{1}{4}v_2 = 5 \\ -\frac{1}{4}v_1 + \left(\frac{1}{4}+\frac{1}{6}\right)v_2 = 10-5 \end{cases}$$

$$\begin{bmatrix} \frac{1}{4}+\frac{1}{2} & -\frac{1}{4} \\ -\frac{1}{4} & \frac{1}{4}+\frac{1}{6} \end{bmatrix} \begin{bmatrix} v_1 \\ v_2 \end{bmatrix} = \begin{bmatrix} 5 \\ 10-5 \end{bmatrix}$$

Nodal Analysis by Inspection (Section 3.6)

If a circuit with only independent current sources has N nonreference nodes, the node-voltage equations can be written as

$$\begin{bmatrix} G_{11} & G_{12} & \cdots & G_{1N} \\ G_{21} & G_{22} & \cdots & G_{2N} \\ \vdots & \vdots & \vdots & \vdots \\ G_{N1} & G_{N2} & \cdots & G_{NN} \end{bmatrix} \begin{bmatrix} v_1 \\ v_2 \\ \vdots \\ v_N \end{bmatrix} = \begin{bmatrix} i_1 \\ i_2 \\ \vdots \\ i_N \end{bmatrix}$$

***Only valid for circuits with <u>current sources</u> and linear resistors.**

$$\begin{bmatrix} G_{11} & G_{12} & \cdots & G_{1N} \\ G_{21} & G_{22} & \cdots & G_{2N} \\ \vdots & \vdots & \vdots & \vdots \\ G_{N1} & G_{N2} & \cdots & G_{NN} \end{bmatrix} \begin{bmatrix} v_1 \\ v_2 \\ \vdots \\ v_N \end{bmatrix} = \begin{bmatrix} i_1 \\ i_2 \\ \vdots \\ i_N \end{bmatrix}$$

G_{kk} = Sum of the conductances connected to node k

$G_{kj} = G_{jk}$ = Negative of the sum of the conductances directly connecting nodes k and j, $k \neq j$.

v_k = Unknown voltage at node k

i_k = Sum of all independent current sources directly connected to node k, with currents entering the node treated as positive

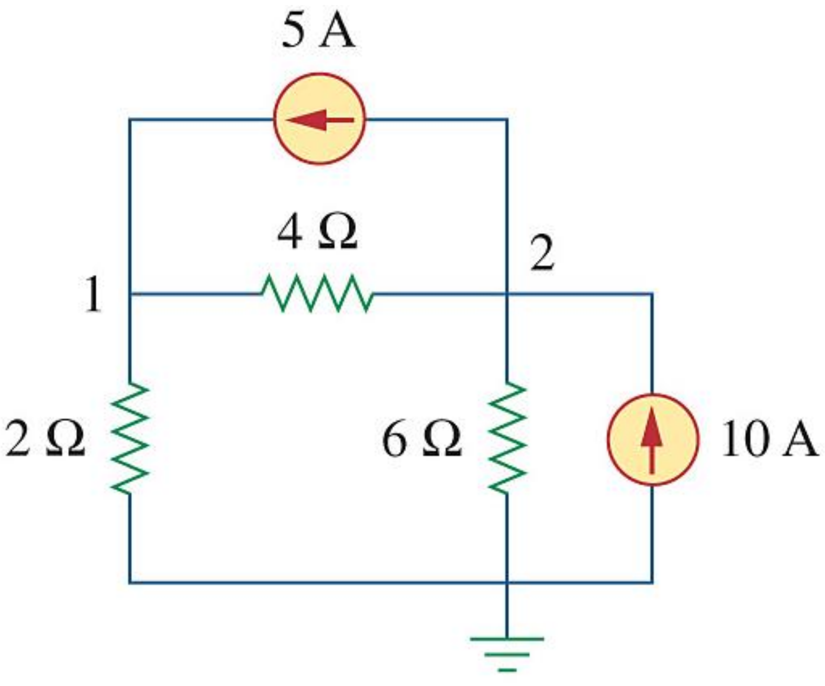


Figure 3.3 (a)

$$\begin{cases} \left(\dfrac{1}{4}+\dfrac{1}{2}\right)v_1 - \dfrac{1}{4}v_2 = 5 \\ -\dfrac{1}{4}v_1 + \left(\dfrac{1}{4}+\dfrac{1}{6}\right)v_2 = 10-5 \end{cases}$$

$$\begin{bmatrix} \dfrac{1}{4}+\dfrac{1}{2} & -\dfrac{1}{4} \\ -\dfrac{1}{4} & \dfrac{1}{4}+\dfrac{1}{6} \end{bmatrix} \begin{bmatrix} v_1 \\ v_2 \end{bmatrix} = \begin{bmatrix} 5 \\ 10-5 \end{bmatrix}$$

Example 3.8 Write the node-voltage matrix equations for the circuit in Fig. 3.27 by inspection.

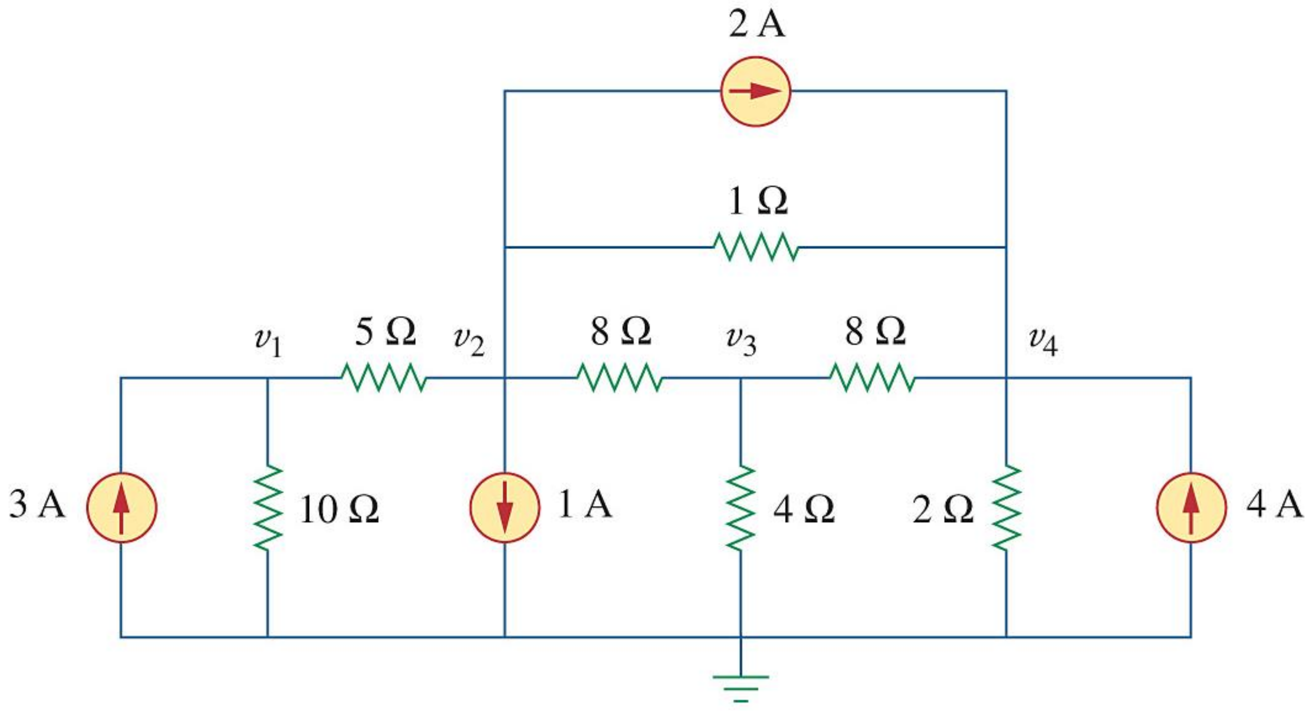
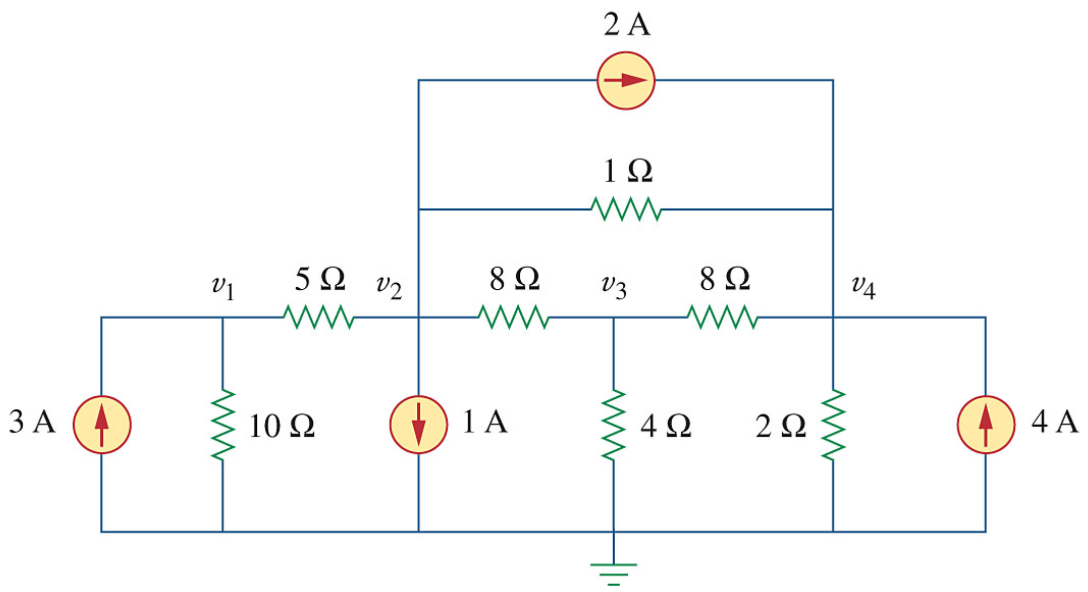
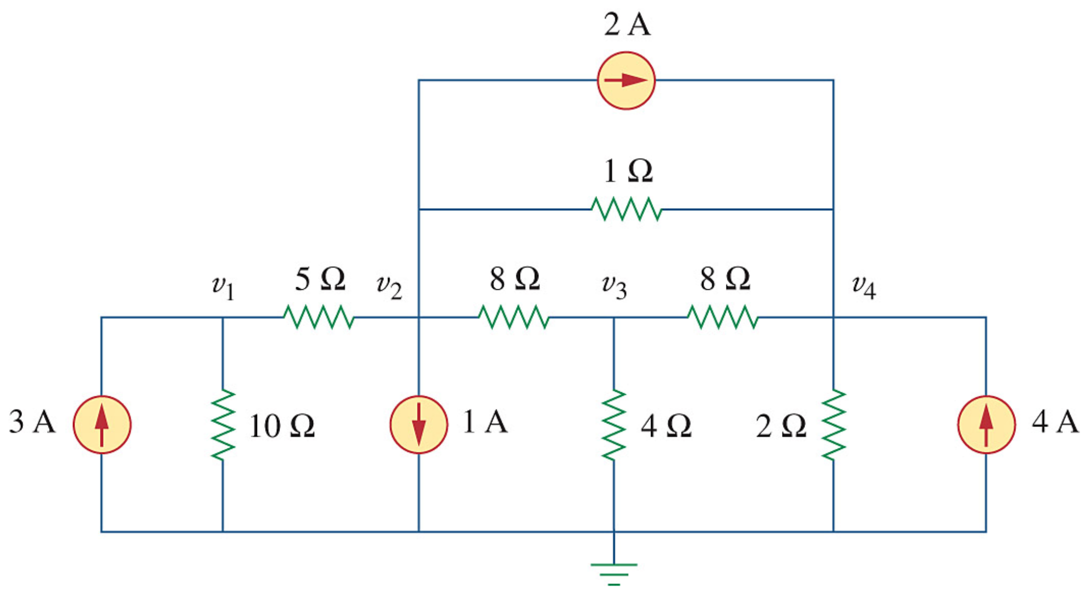


Figure 3.27



Solution :

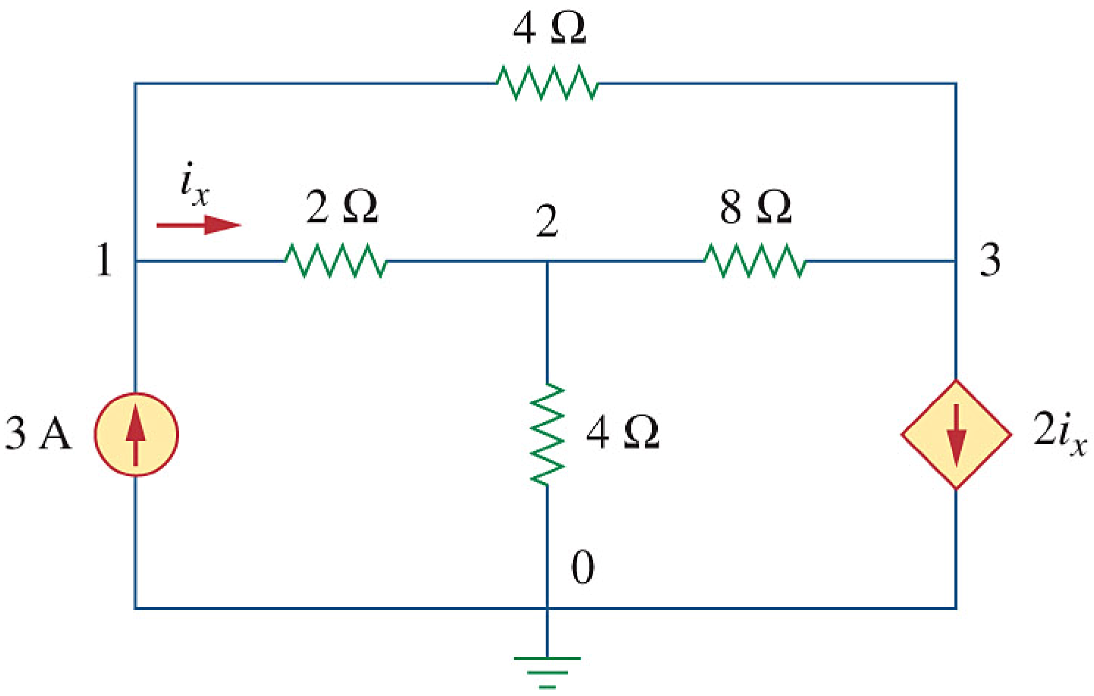
$$\begin{bmatrix} \quad \end{bmatrix}\begin{bmatrix} v_1 \\ v_2 \\ v_3 \\ v_4 \end{bmatrix} = \begin{bmatrix} \quad \end{bmatrix}$$

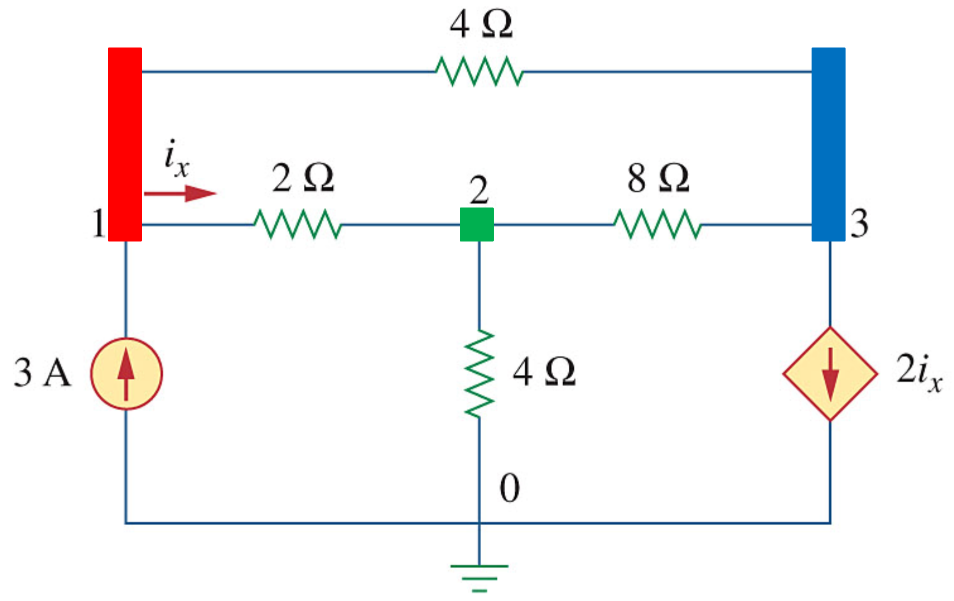


Solution :

$$\begin{bmatrix} \frac{1}{5}+\frac{1}{10} & -\frac{1}{5} & 0 & 0 \\ -\frac{1}{5} & \frac{1}{5}+\frac{1}{8}+1 & -\frac{1}{8} & -1 \\ 0 & -\frac{1}{8} & \frac{1}{4}+\frac{1}{8}+\frac{1}{8} & -\frac{1}{8} \\ 0 & -1 & -\frac{1}{8} & 1+\frac{1}{8}+\frac{1}{2} \end{bmatrix} \begin{bmatrix} v_1 \\ v_2 \\ v_3 \\ v_4 \end{bmatrix} = \begin{bmatrix} 3 \\ -1-2 \\ 0 \\ 2+4 \end{bmatrix}$$

Example 3.2 Determine the voltages at the nodes in Fig. 3.5(a).





(1)KCL at node 1:

$$-3+\frac{V_1-V_2}{2}+\frac{V_1-V_3}{4}=0, \qquad i_x=\frac{V_1-V_2}{2}$$

$$\rightarrow 3V_1-2V_2-V_3=12$$

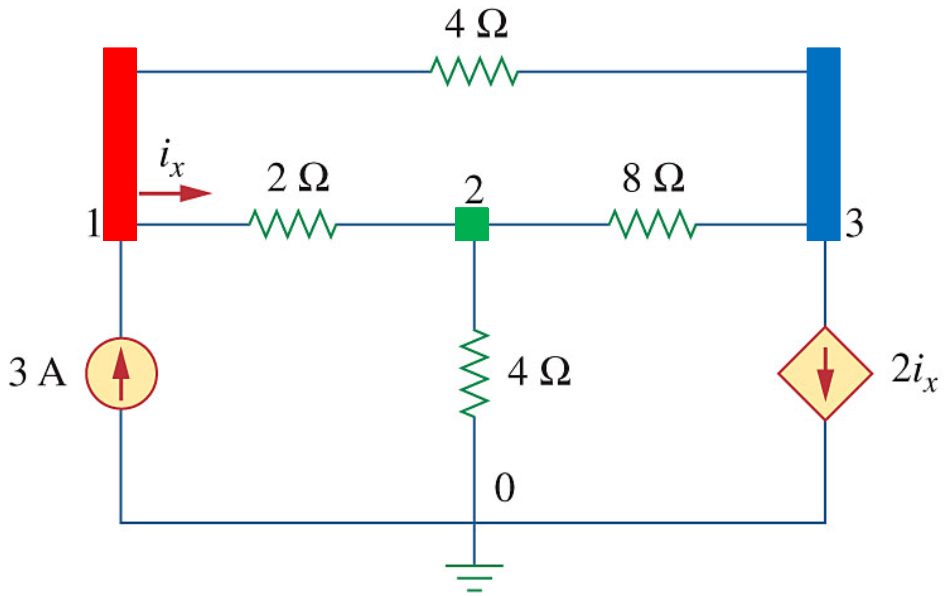
(2) KCL at node 2:

$$\frac{V_2-V_1}{2}+\frac{V_2}{4}+\frac{V_2-V_3}{8}=0$$

$$\rightarrow -4V_1+7V_2-V_3=0$$

(3) KCL at node 3:

$$\frac{V_3-V_1}{4}+\frac{V_3-V_2}{8}+2i_x=0$$



(3) KCL at node 3:

$$\frac{V_3 - V_1}{4} + \frac{V_3 - V_2}{8} + 2i_x = 0, where\ i_x = \frac{V_1 - V_2}{2}$$

$$\rightarrow 2V_1 - 3V_2 + V_3 = 0$$

We can write three individual equations in a matrix form

$$\begin{bmatrix} 3 & -2 & -1 \\ -4 & 7 & -1 \\ 2 & -3 & 1 \end{bmatrix} \begin{bmatrix} V_1 \\ V_2 \\ V_3 \end{bmatrix} = \begin{bmatrix} 12 \\ 0 \\ 0 \end{bmatrix}$$

Similarly, three equations in the matrix form:

$$\begin{bmatrix} a & b & c \\ d & e & f \\ g & h & i \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} j \\ k \\ l \end{bmatrix}$$

$$x = \frac{\begin{vmatrix} j & b & c \\ k & e & f \\ l & h & i \end{vmatrix}}{\begin{vmatrix} a & b & c \\ d & e & f \\ g & h & i \end{vmatrix}} \quad y = \frac{\begin{vmatrix} a & j & c \\ d & k & f \\ g & l & i \end{vmatrix}}{\begin{vmatrix} a & b & c \\ d & e & f \\ g & h & i \end{vmatrix}} \quad z = \frac{\begin{vmatrix} a & b & j \\ d & e & k \\ g & h & l \end{vmatrix}}{\begin{vmatrix} a & b & c \\ d & e & f \\ g & h & i \end{vmatrix}}$$

$$\begin{bmatrix} 3 & -2 & -1 \\ -4 & 7 & -1 \\ 2 & -3 & 1 \end{bmatrix} \begin{bmatrix} V_1 \\ V_2 \\ V_3 \end{bmatrix} = \begin{bmatrix} 12 \\ 0 \\ 0 \end{bmatrix}$$

Determinant of the Matrix (denominator)

$$\begin{vmatrix} 3 & -2 & -1 & 3 & -2 \\ -4 & 7 & -1 & -4 & 7 \\ 2 & -3 & 1 & 2 & -3 \end{vmatrix} = 10$$

$$V_1 = \frac{\begin{vmatrix} 12 & -2 & -1 \\ 0 & 7 & -1 \\ 0 & -3 & 1 \end{vmatrix}}{\begin{vmatrix} 3 & -2 & -1 \\ -4 & 7 & -1 \\ 2 & -3 & 1 \end{vmatrix}} = 4.8 \qquad V_2 = \frac{\begin{vmatrix} 3 & 12 & -1 \\ -4 & 0 & -1 \\ 2 & 0 & 1 \end{vmatrix}}{\begin{vmatrix} 3 & -2 & -1 \\ -4 & 7 & -1 \\ 2 & -3 & 1 \end{vmatrix}} = 2.4$$

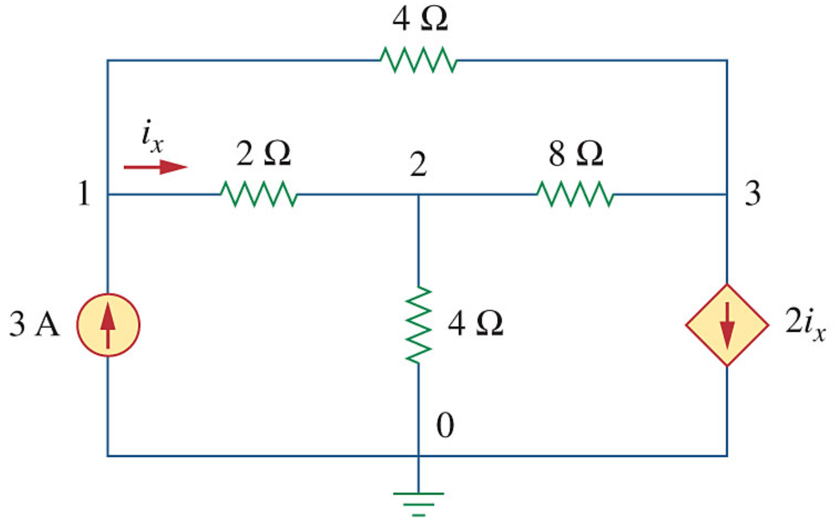
$$\begin{vmatrix} 12 & -2 & -1 & 12 & -2 \\ 0 & 7 & -1 & 0 & 7 \\ 0 & -3 & 1 & 0 & -3 \end{vmatrix} = 48 \text{ (numerator)} \qquad \begin{vmatrix} 3 & 12 & -1 & 12 & -2 \\ -4 & 0 & -1 & 0 & 7 \\ 2 & 0 & 1 & 0 & -3 \end{vmatrix} = 24$$

$$V_3 = \frac{\begin{vmatrix} 3 & -2 & 12 \\ -4 & 7 & 0 \\ 2 & -3 & 0 \end{vmatrix}}{\begin{vmatrix} 3 & -2 & -1 \\ -4 & 7 & -1 \\ 2 & -3 & 1 \end{vmatrix}} = -2.4$$

$$\begin{vmatrix} 3 & -2 & 12 & 3 & -2 \\ -4 & 7 & 0 & -4 & 7 \\ 2 & -3 & 0 & 2 & -3 \end{vmatrix} = -24$$

Therefore, V_1 = 4.8 [V]; V_2 = 2.4 [V]; V_3 = -2.4 [V]

Alternatively, we can set equations by the inspection method



G_{kk} = Sum of the conductances connected to node k

$G_{kj} = G_{jk}$ = Negative of the sum of the conductances directly connecting nodes k and j, $k \neq j$.

i_k = Sum of all independent current sources directly connected to node k, with currents entering the node treated as positive

Solution :

$$\begin{bmatrix} & & \\ & & \\ & & \end{bmatrix} \begin{bmatrix} v_1 \\ v_2 \\ v_3 \end{bmatrix} = \begin{bmatrix} \\ \\ \\ \end{bmatrix}$$

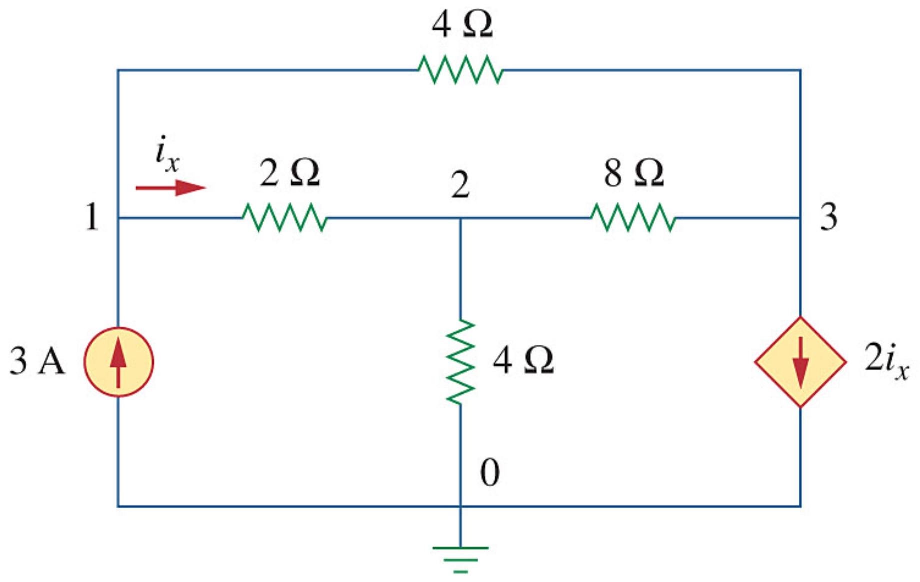
$$i_x = \frac{v_1 - v_2}{2}$$

Alternatively, we can set equations by the inspection method

Solution :

$$\begin{bmatrix} \frac{1}{2}+\frac{1}{4} & -\frac{1}{2} & -\frac{1}{4} \\ -\frac{1}{2} & \frac{1}{2}+\frac{1}{4}+\frac{1}{8} & -\frac{1}{8} \\ -\frac{1}{4} & -\frac{1}{8} & \frac{1}{4}+\frac{1}{8} \end{bmatrix} \begin{bmatrix} v_1 \\ v_2 \\ v_3 \end{bmatrix} = \begin{bmatrix} 3 \\ 0 \\ -2i_x \end{bmatrix}$$

$$i_x = \frac{v_1 - v_2}{2}$$



$$\begin{bmatrix} \frac{1}{2}+\frac{1}{4} & -\frac{1}{2} & -\frac{1}{4} \\ -\frac{1}{2} & \frac{1}{2}+\frac{1}{4}+\frac{1}{8} & -\frac{1}{8} \\ -\frac{1}{4} & -\frac{1}{8} & \frac{1}{4}+\frac{1}{8} \end{bmatrix} \begin{bmatrix} v_1 \\ v_2 \\ v_3 \end{bmatrix} = \begin{bmatrix} 3 \\ 0 \\ -v_1+v_2 \end{bmatrix}$$

$$\begin{bmatrix} \frac{1}{2}+\frac{1}{4} & -\frac{1}{2} & -\frac{1}{4} \\ -\frac{1}{2} & \frac{1}{2}+\frac{1}{4}+\frac{1}{8} & -\frac{1}{8} \\ -\frac{1}{4}+1 & -\frac{1}{8}-1 & \frac{1}{4}+\frac{1}{8} \end{bmatrix} \begin{bmatrix} v_1 \\ v_2 \\ v_3 \end{bmatrix} = \begin{bmatrix} 3 \\ 0 \\ 0 \end{bmatrix}$$

$$\begin{bmatrix} 3 & -2 & -1 \\ -4 & 7 & -1 \\ 2 & -3 & 1 \end{bmatrix} \begin{bmatrix} v_1 \\ v_2 \\ v_3 \end{bmatrix} = \begin{bmatrix} 12 \\ 0 \\ 0 \end{bmatrix}$$

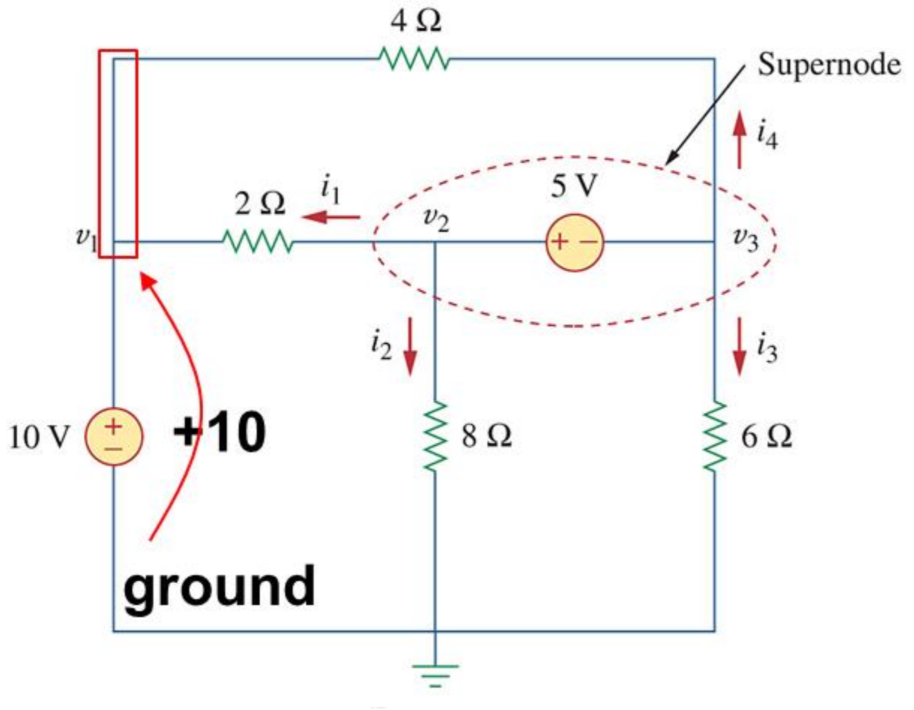
We get the same result.

3.3 Nodal Analysis with Voltage Sources

Now we consider applying nodal analysis to circuits containing **voltage sources** (dependent and independent)

Case 1: If a voltage source is connected between the reference node and a nonreference node, we simply set the voltage at the nonreference node equal to the voltage of the voltage source.

V_1 is simply 10 V



Case 2: If a voltage source is connected between two nonreference nodes, the two nonreference nodes form **a generalized node** or **supernode**. The supernode provides **a constraint** on the two node voltages.

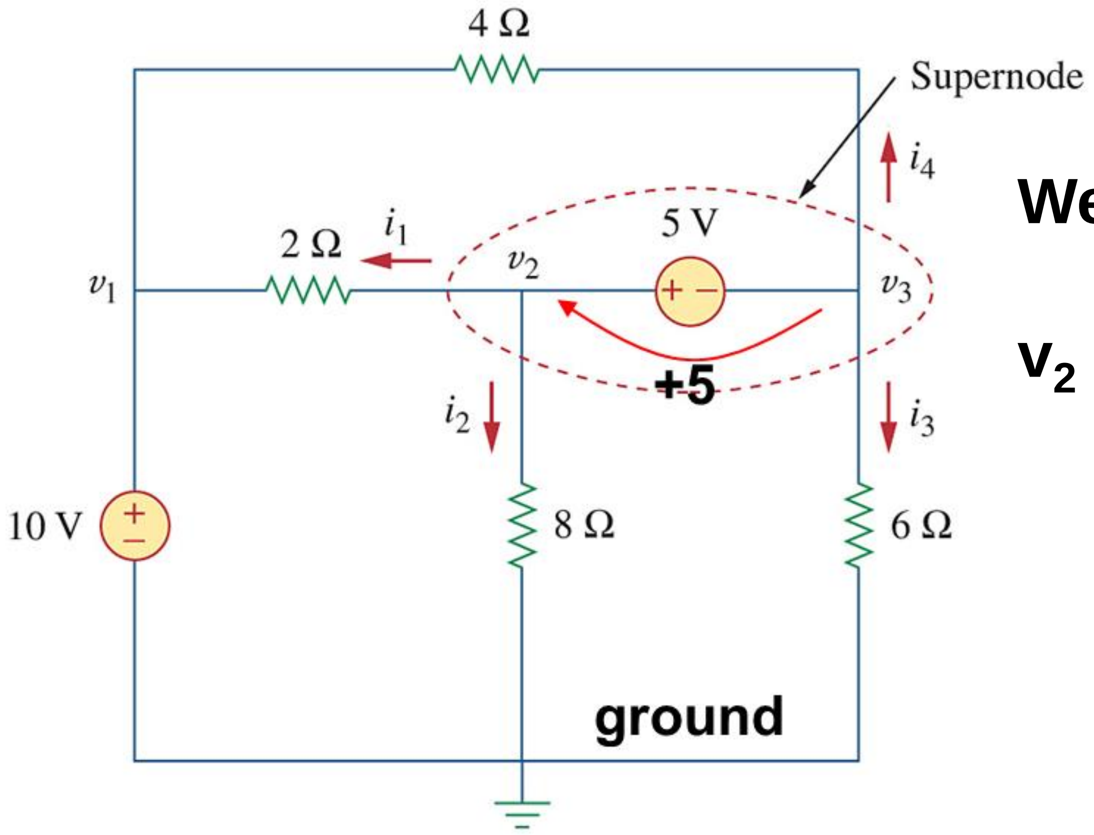


Figure 3.7

We have an additional equation:

$$\mathbf{v_2 = v_3 + 5}$$

A supernode is formed by enclosing a voltage source between two nonreference nodes and **any elements connected in parallel with it**.

The supernodes are treated differently because there is no way of knowing the current through a voltage source in advance.

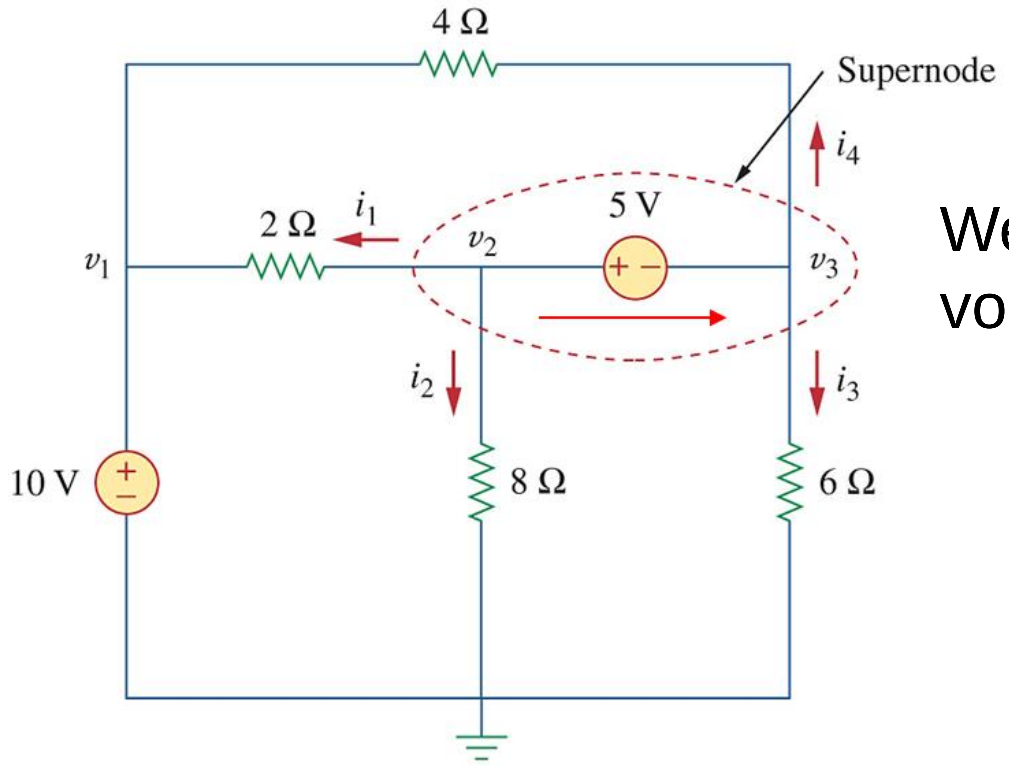


Figure 3.7

We don’t know current through the voltage source 5V.

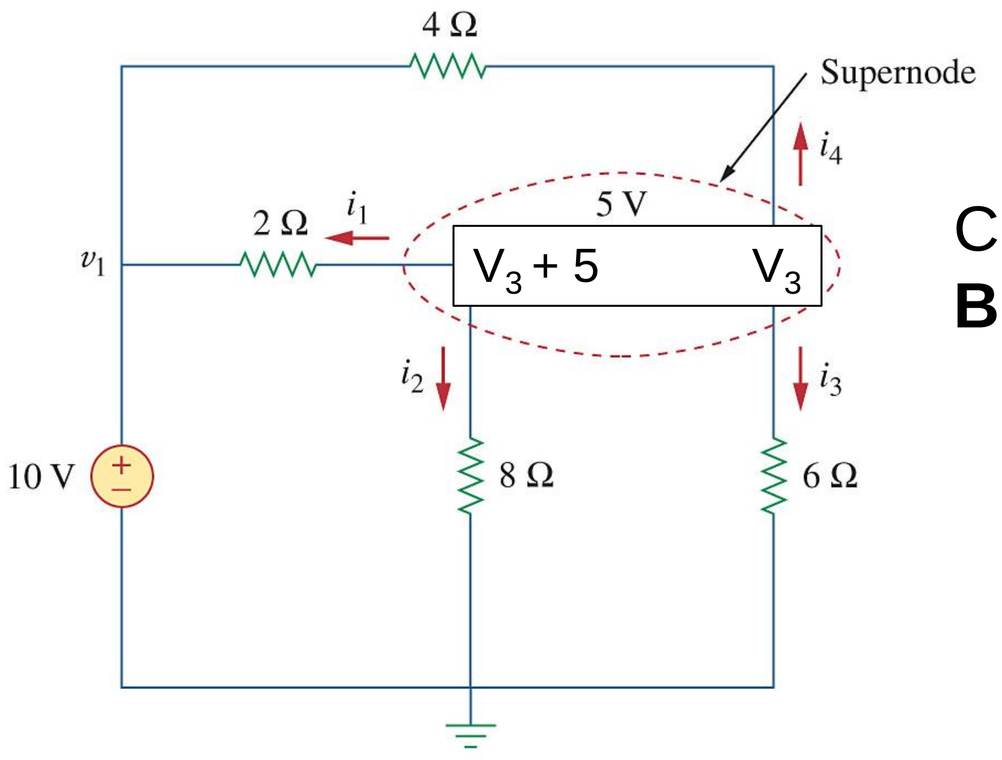


Figure 3.7

Consider it as one node
BUT use two different values

KCL at the supernode:

$$\frac{V_3+5-V_1}{2}+\frac{V_3+5-0}{8}+\frac{V_3-0}{6}+\frac{V_3-V_1}{4}=0$$

i_1 i_2 i_3 i_4

Example 3.3 For the circuit shown in Fig. 3.9, find the node voltages.

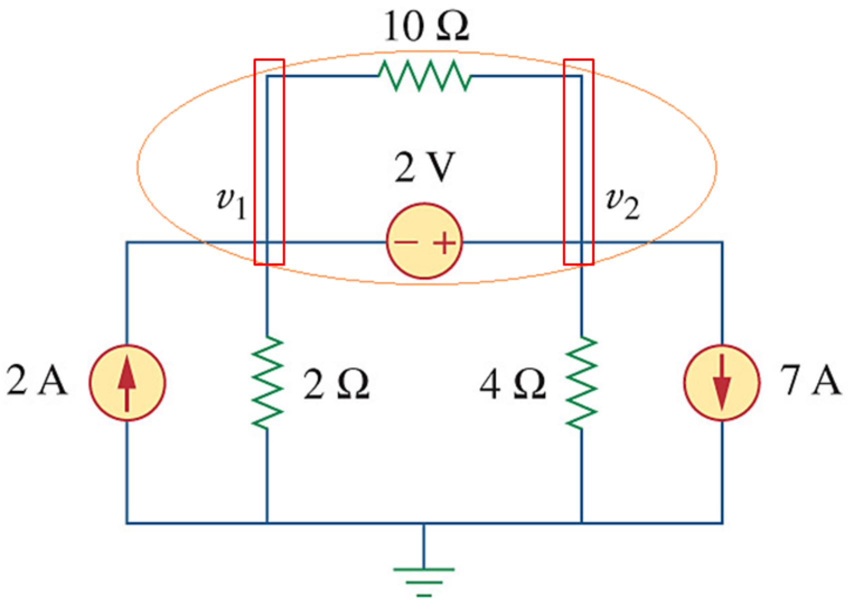
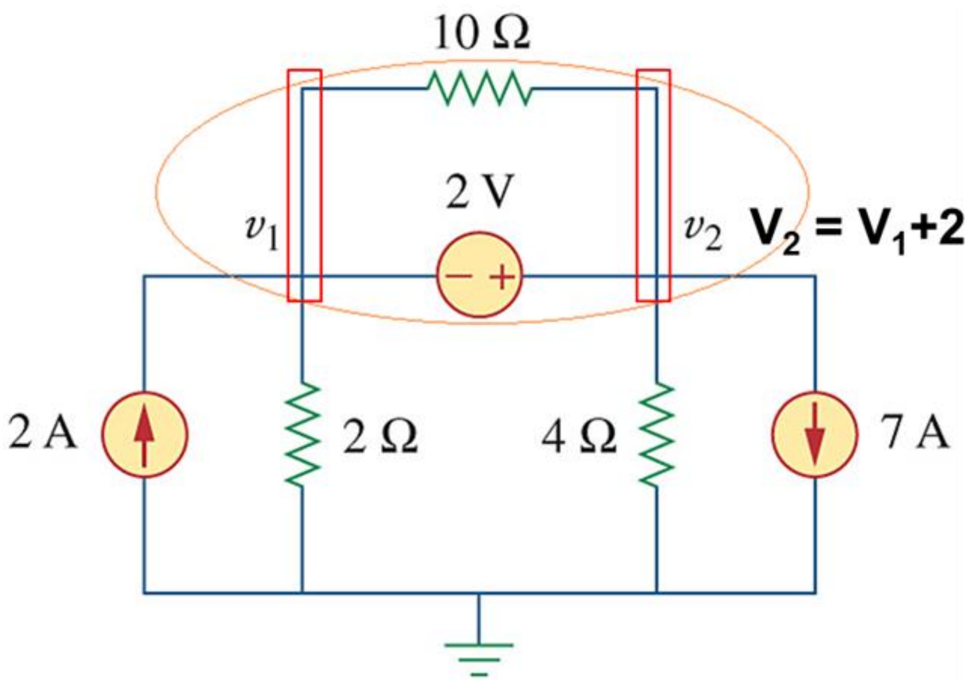


Figure 3.9

From a supernode (including a parallel resistor), we get $V_2 = V_1+2$



$$-2 + \frac{v_1}{2} + \frac{v_1 + 2}{4} + 7 = 0$$

$3v_1 = -22$

thus, $v_1 = -22/3 = -7.33$ V

$v_2 = -16/3 = -5.33$ V

Or, $-2 + \frac{v_1}{2} + \frac{v_2}{4} + 7 = 0$

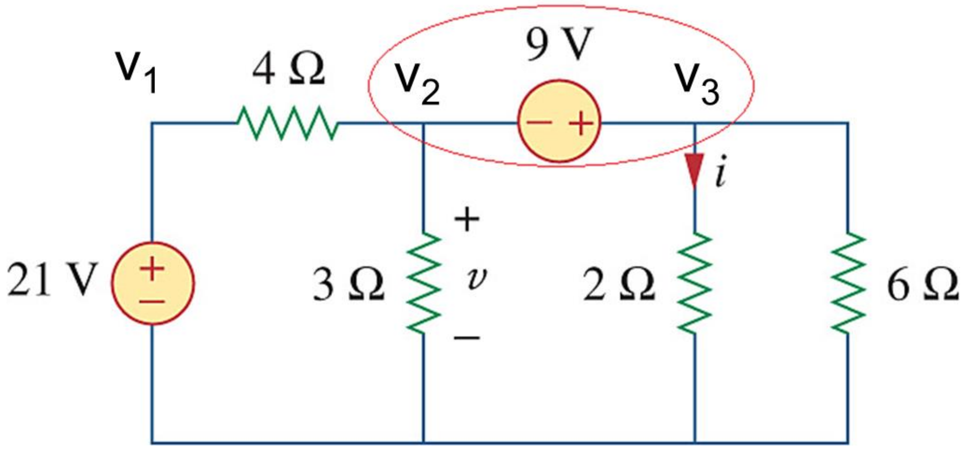
$v_2 = v_1+2$

$3v_1 = -22$

thus, $v_1 = -22/3 = -7.33$ V

$v_2 = -16/3 = -5.33$ V

Practice Problem 3.3 Find v and i in the circuit of Fig. 3.11.



Solution :

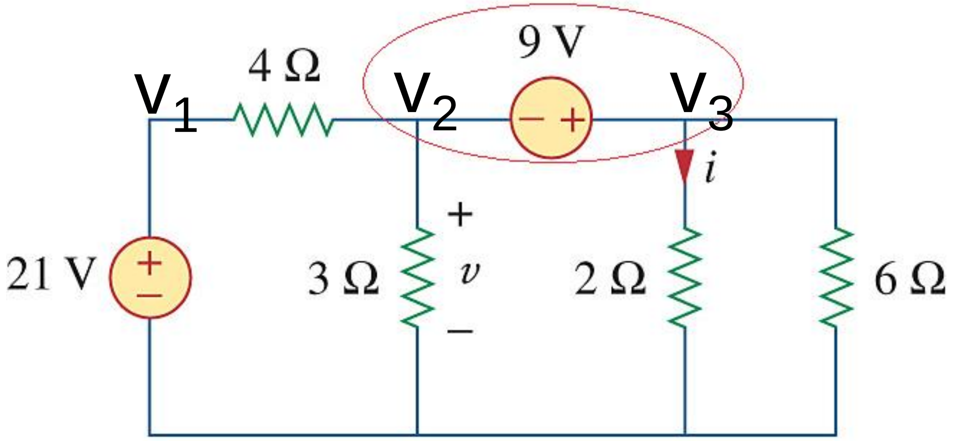


Figure 3.11

$$v_1 = 21$$

$$v_2 - v_3 = -9$$

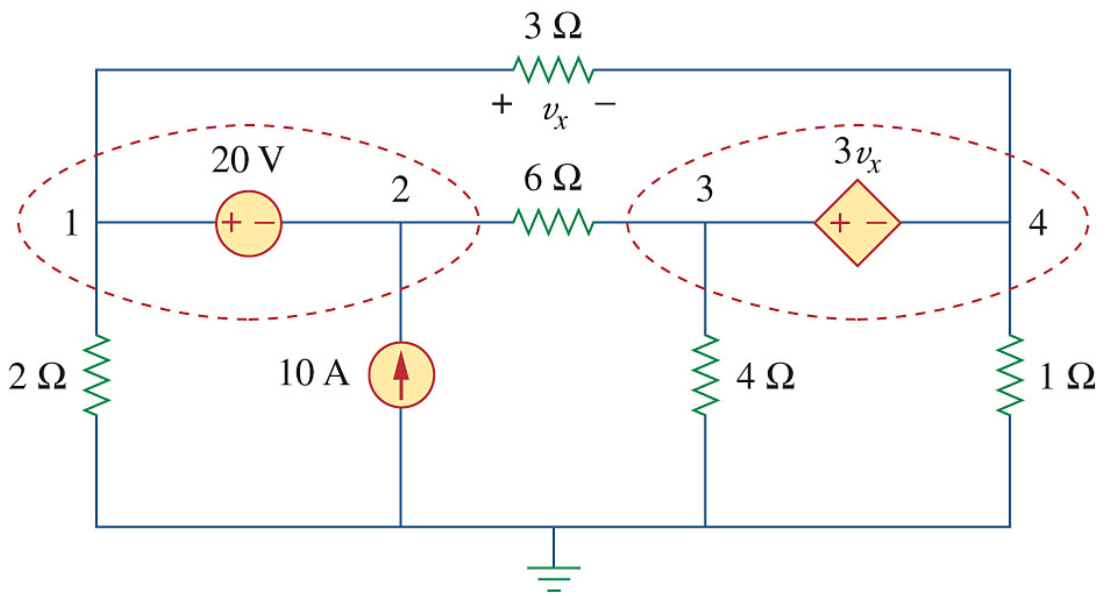
$$\frac{v_2 - v_1}{4} + \frac{v_2}{3} + \frac{v_3}{2} + \frac{v_3}{6} = 0$$

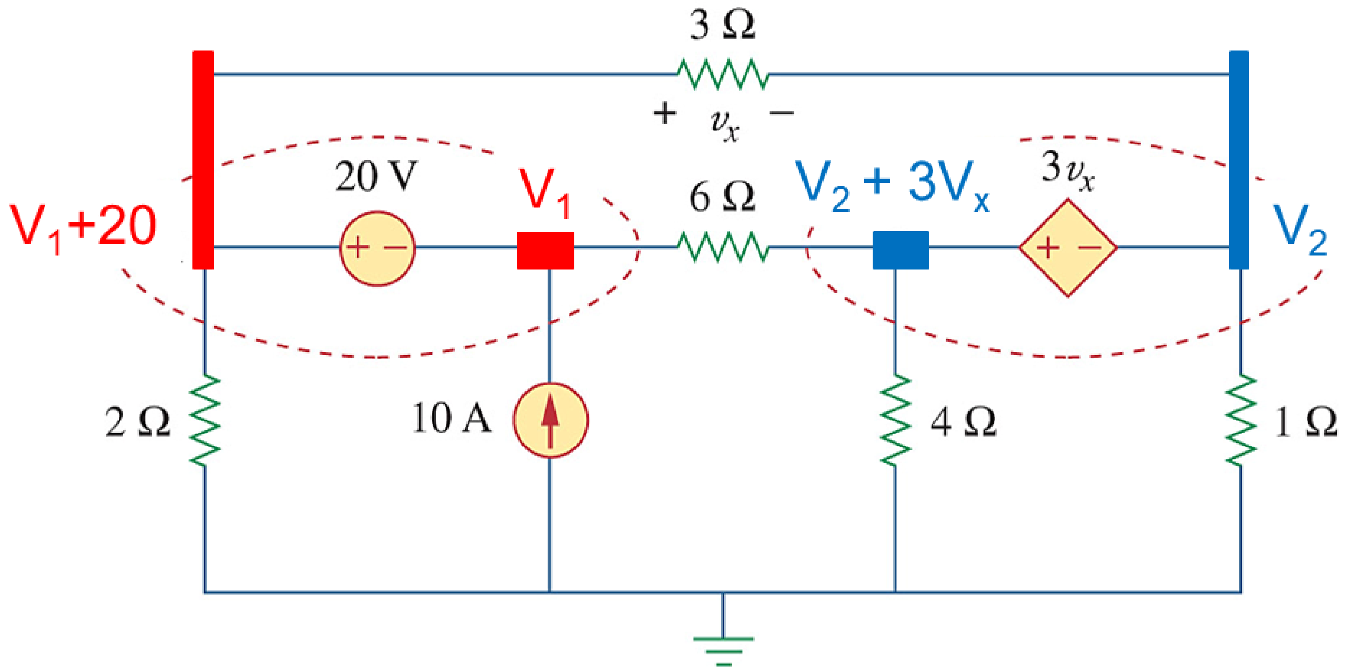
$$-3v_1 + 7v_2 + 8v_3 = 0$$

$$v_2 = -\frac{3}{5} = -0.6 \text{ (V)}, \; v_3 = \frac{42}{5} = 8.4 \text{ (V)}$$

$$v = v_2 = -0.6 \text{ V}, \; i = \frac{v_3}{2} = 4.2 \text{ (A)}$$

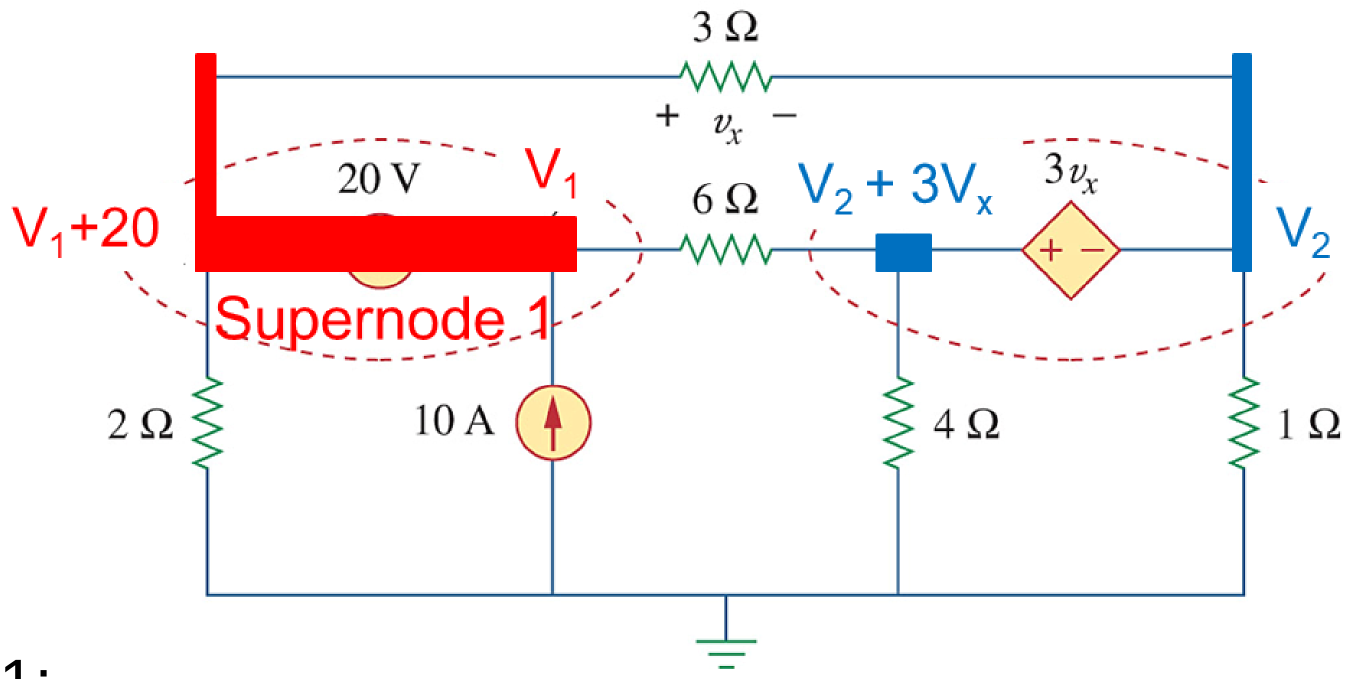
Example 3.4 Find the node voltages in the circuit of Fig. 3.12.





- We have three variables: V_1, V_2 and V_x

(1) $V_x = V_1 + 20 – V_2$
(2) By KCL at Supernode 1
(3) By KCL at Supernode 2



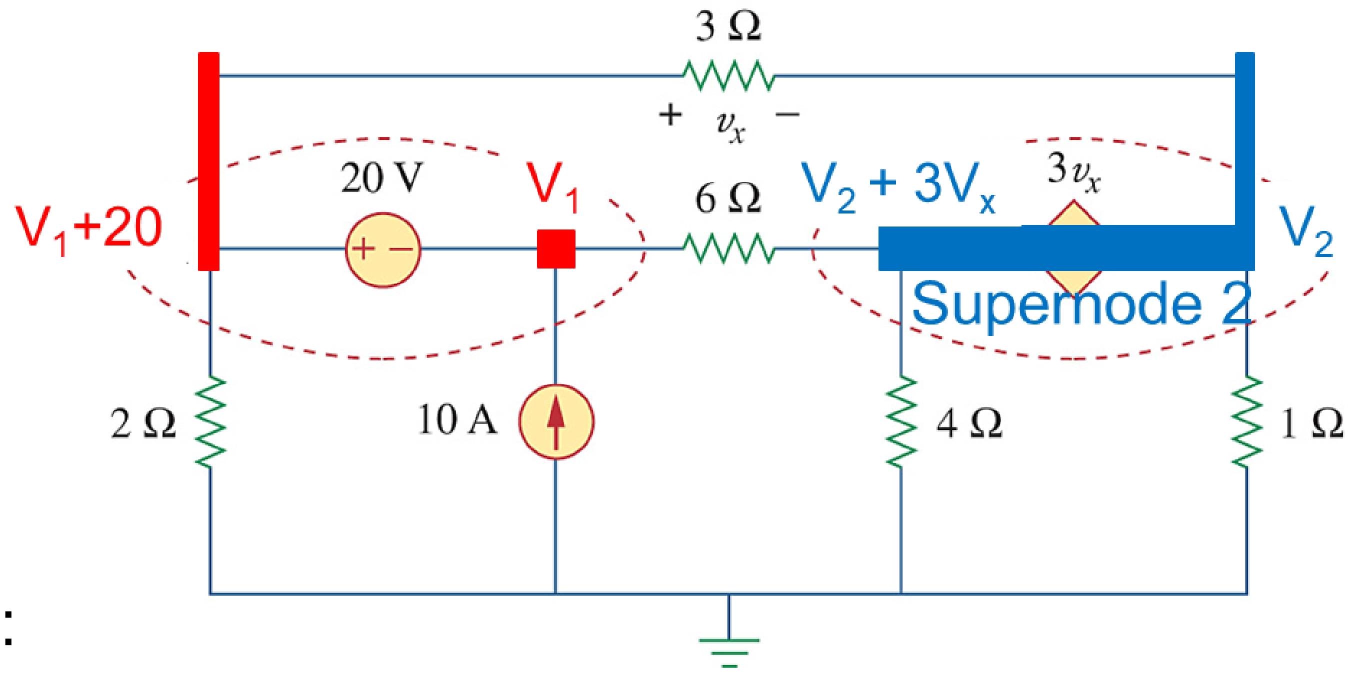
Supernode 1:

$$\frac{V_1 + 20}{2} + (-10) + \frac{V_1 - (V_2 + 3V_x)}{6} + \frac{V_1 + 20 - V_2}{3} = 0$$

Put $V_x = V_1 + 20 – V_2$ into the equation and solve

Node $V_1 = {}^{20}/_{3} \cong 6.67$ [V]

Node V_1+20 = 26.67 [V]

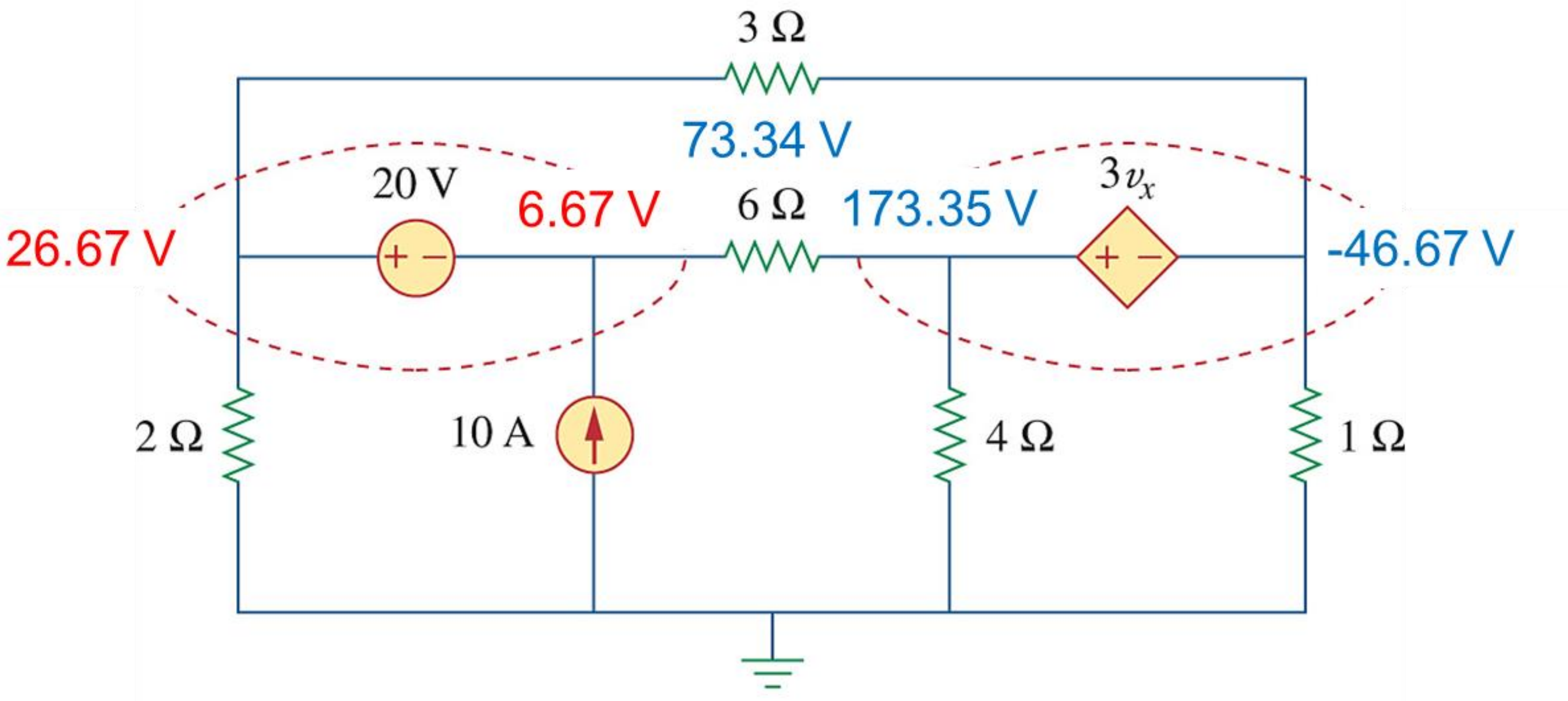


Supernode 2:

$$\frac{V_2 + 3V_x - V_1}{6} + \frac{V_2 + 3V_x}{4} + \frac{V_2}{1} + \frac{V_2 - (V_1 + 20)}{3} = 0$$

Put $V_x = V_1 + 20 - V_2$, $V_1 = 6.67$ into the equation and solve

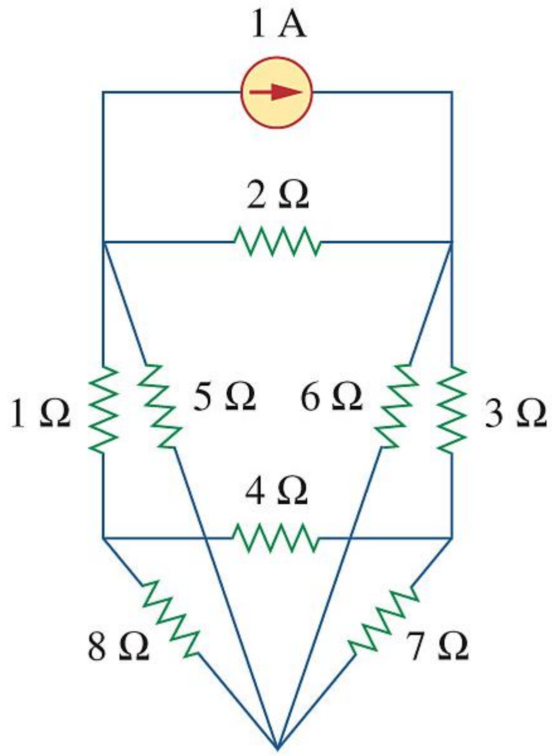
Node V_2 = -46.67 [V]
V_x = 73.34 [V]
Node $V_2 + 3V_x$ = 173.35 [V]


3 Ω
73.34 V
20 V
6.67 V
6 Ω
173.35 V
$3v_x$
26.67 V
-46.67 V
2 Ω
10 A
4 Ω
1 Ω

3.4 Mesh Analysis

- Mesh analysis: based on **KVL**
- Using **mesh currents** instead of element currents as circuit variables.
- Mesh analysis is only applicable to a circuit that is **planar**. A planar circuit is one that can be drawn in a plane with no branches crossing one another; otherwise it is **nonplanar**.

Planar Circuit Example



Planar Circuit Example

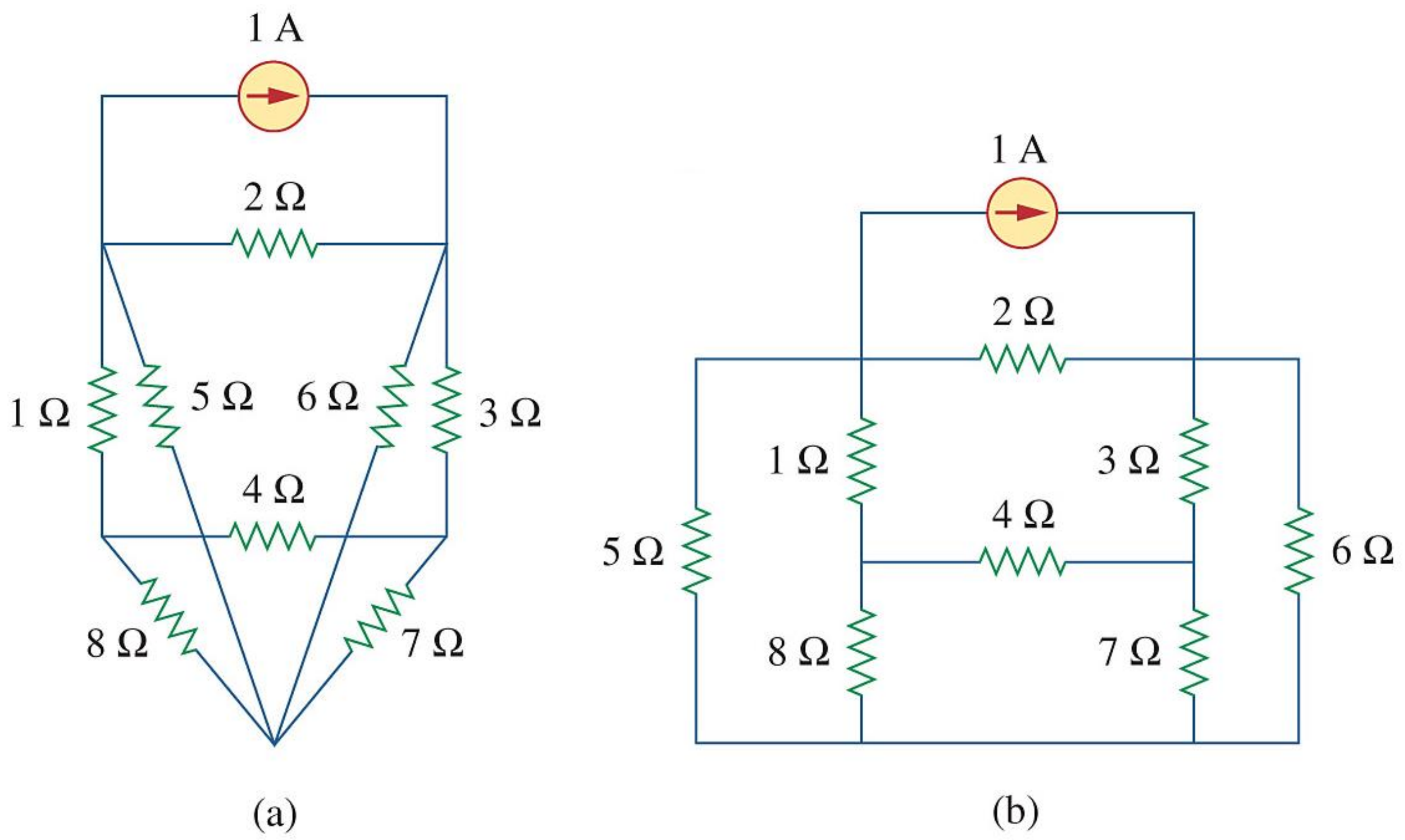


Figure 3.15 (a) A planar circuit with crossing branches, (b) the same circuit redrawn with no crossing branches.

Nonplanar Circuit Example

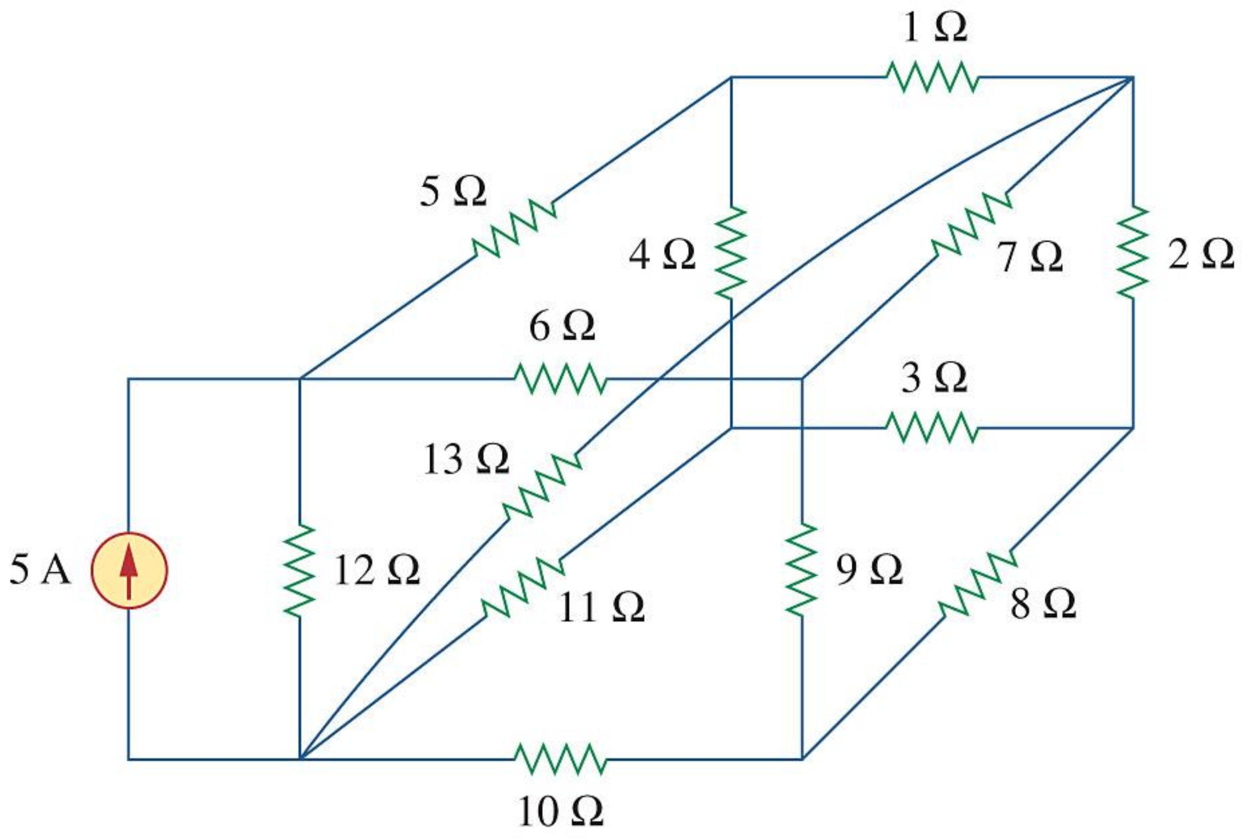


Figure 3.16 A nonplanar circuit.

Definitions on Independent Loop and Mesh

- **Independent loop:** A loop is said to be independent if it contains at least one branch which is **not a part of any other independent loop**.

- **Mesh:** A mesh is a loop that does not contain any other loop within it. (Smallest loops)

Max. # of independent loops

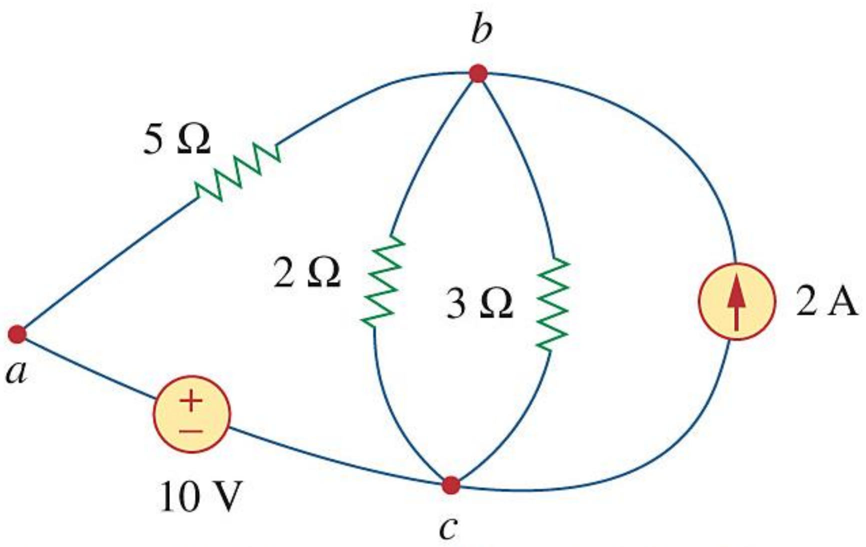


Figure 2.11 The circuit of Fig. 2.10 is redrawn.

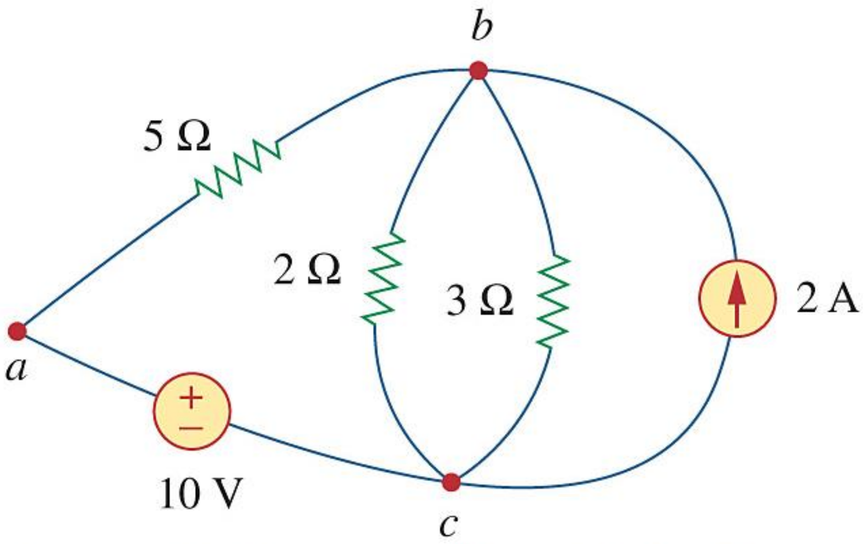


Figure 2.11 The circuit of Fig. 2.10 is redrawn.

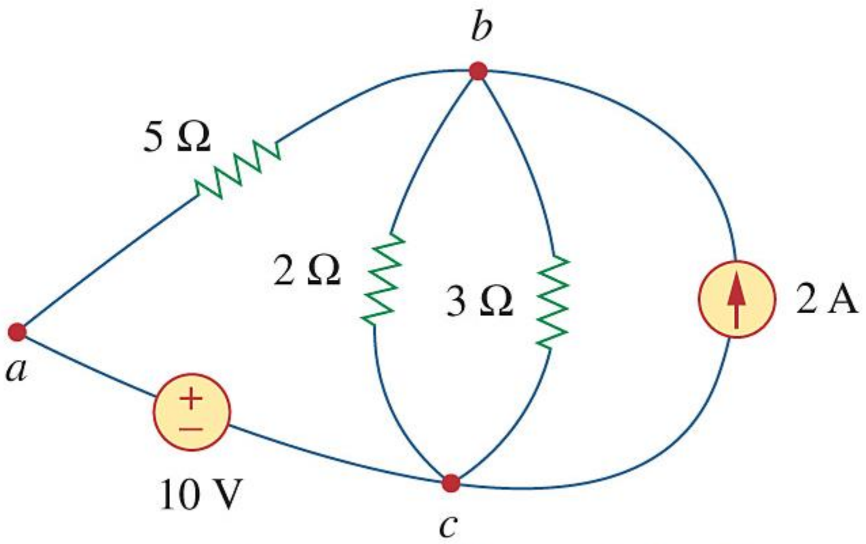


Figure 2.11 The circuit of Fig. 2.10 is redrawn.

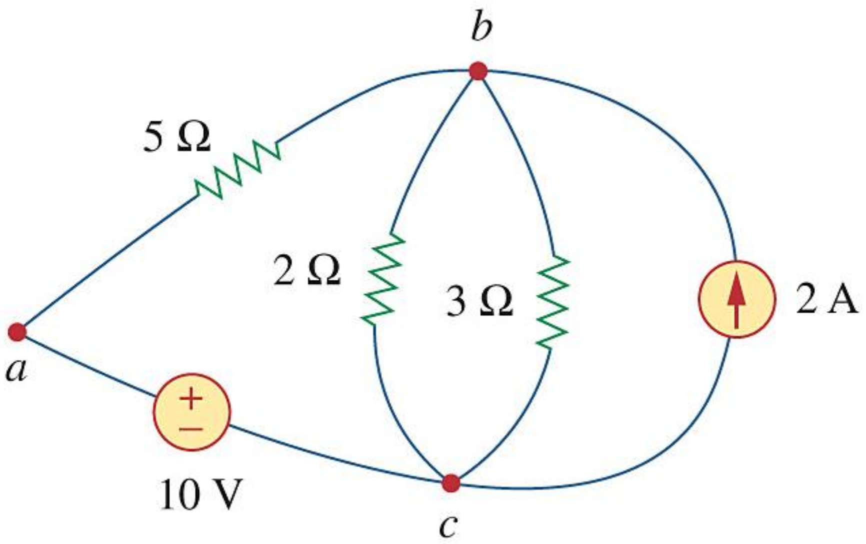


Figure 2.11 The circuit of Fig. 2.10 is redrawn.

Max. # of independent loops

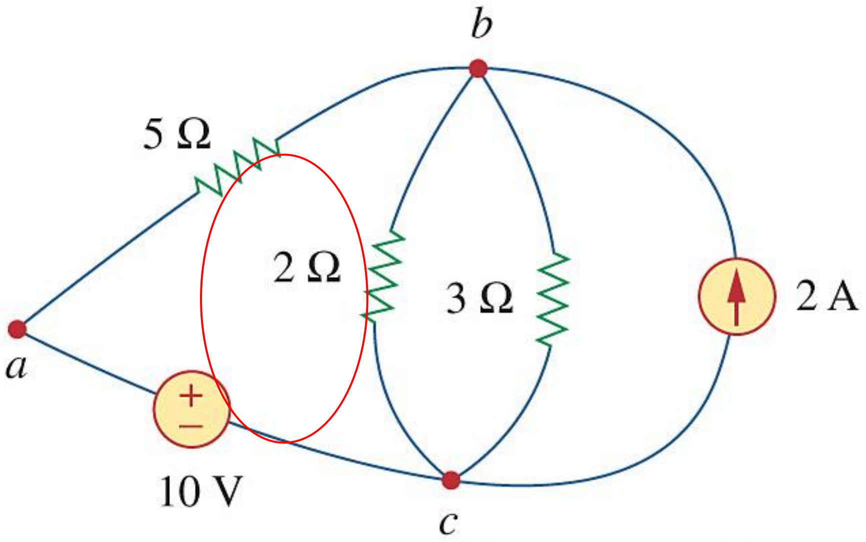


Figure 2.11 The circuit of Fig. 2.10 is redrawn.

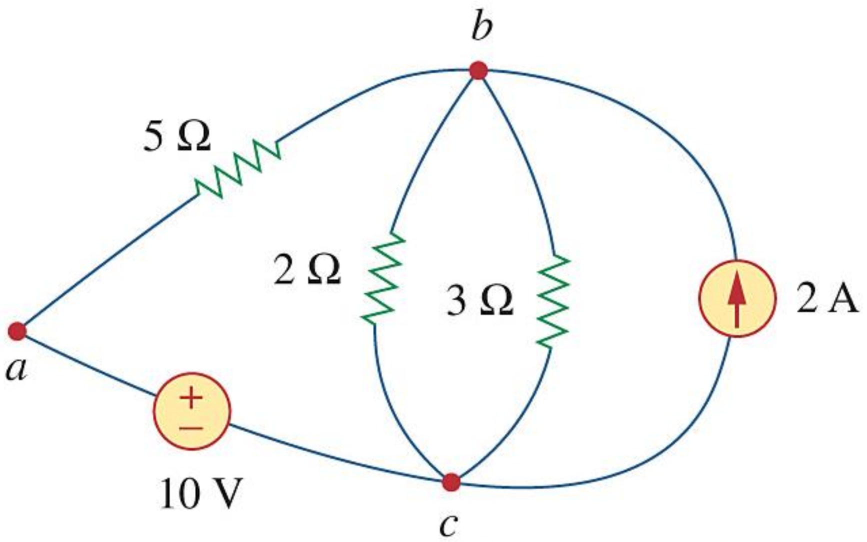


Figure 2.11 The circuit of Fig. 2.10 is redrawn.

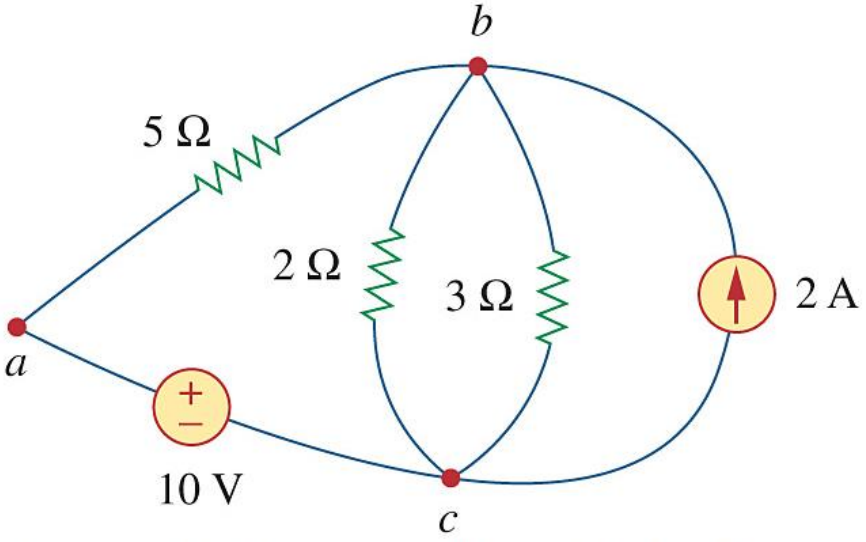


Figure 2.11 The circuit of Fig. 2.10 is redrawn.

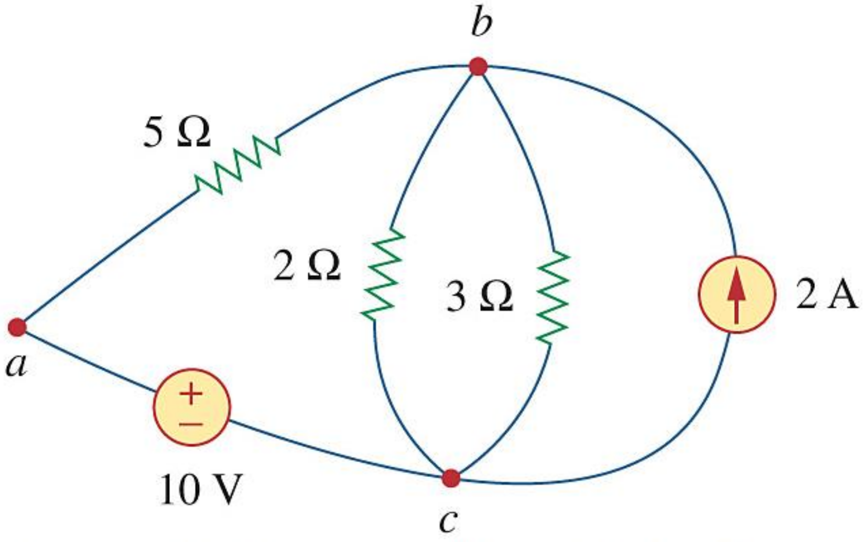


Figure 2.11 The circuit of Fig. 2.10 is redrawn.

Max. # of independent loops

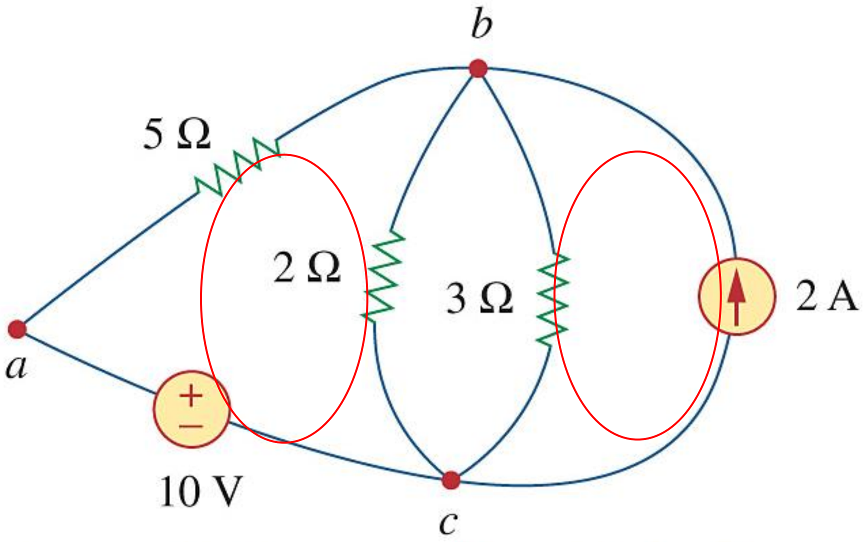


Figure 2.11 The circuit of Fig. 2.10 is redrawn.

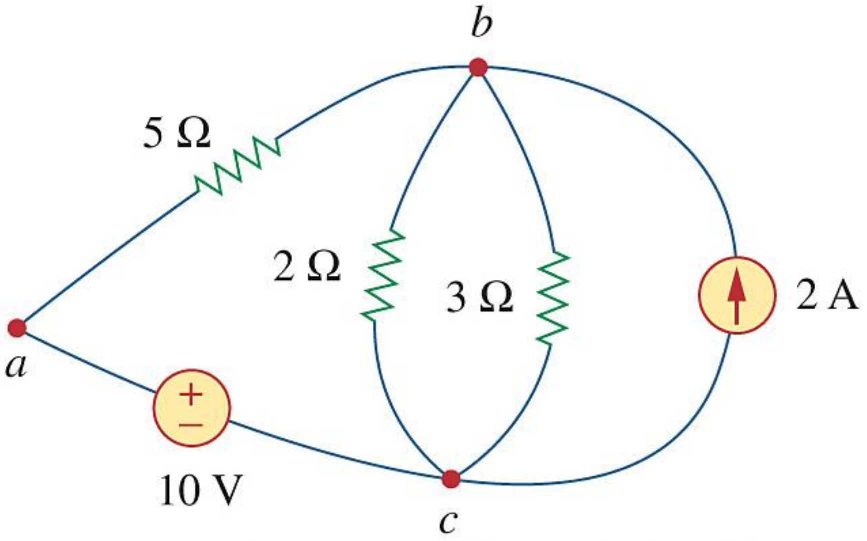


Figure 2.11 The circuit of Fig. 2.10 is redrawn.

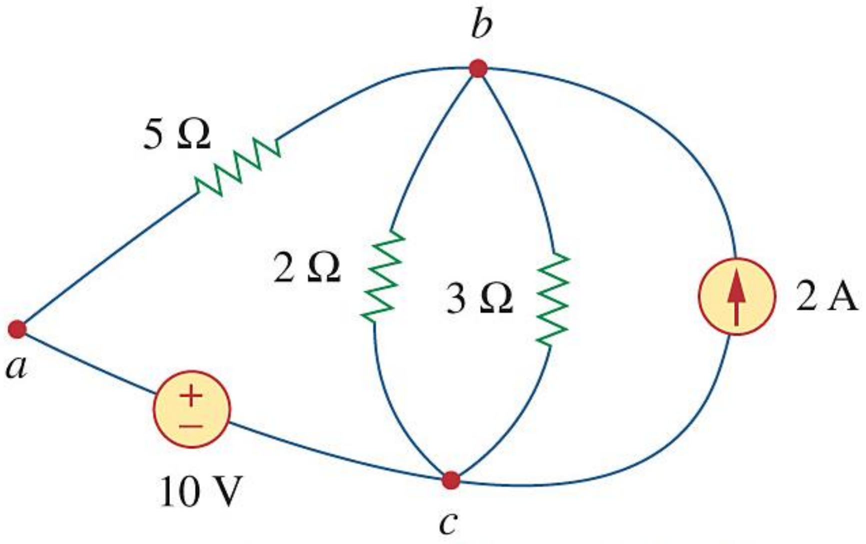


Figure 2.11 The circuit of Fig. 2.10 is redrawn.

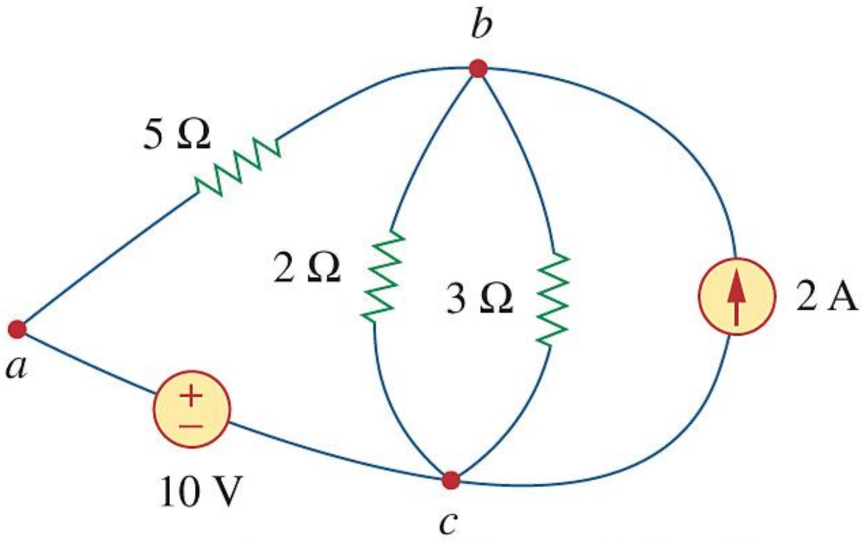


Figure 2.11 The circuit of Fig. 2.10 is redrawn.

Max. # of independent loops

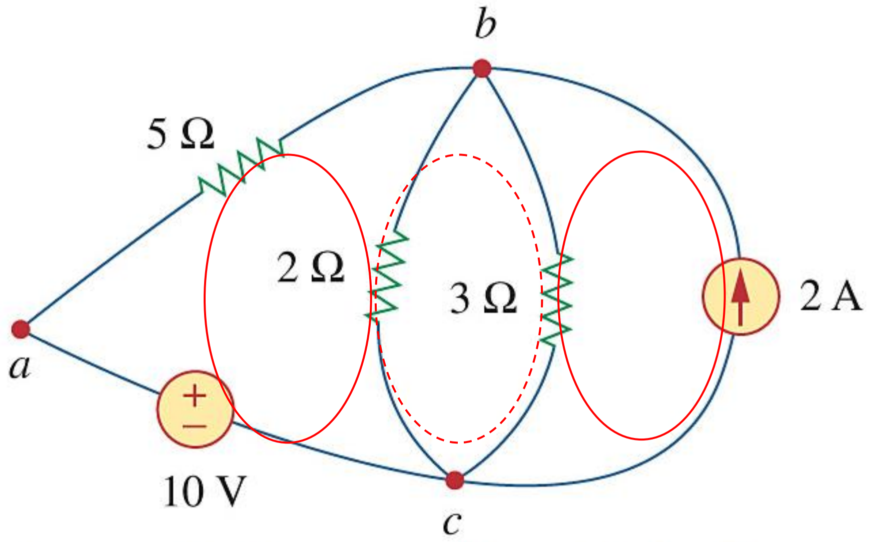


Figure 2.11 The circuit of Fig. 2.10 is redrawn.

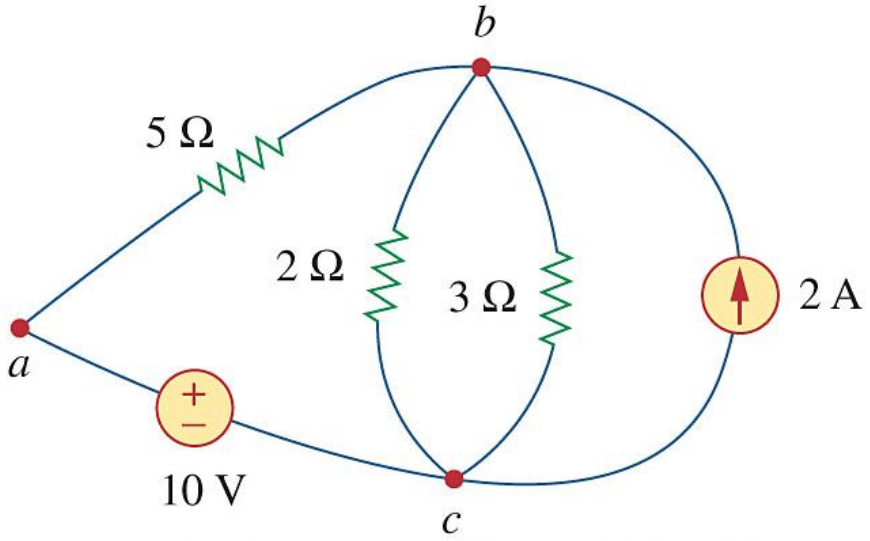


Figure 2.11 The circuit of Fig. 2.10 is redrawn.

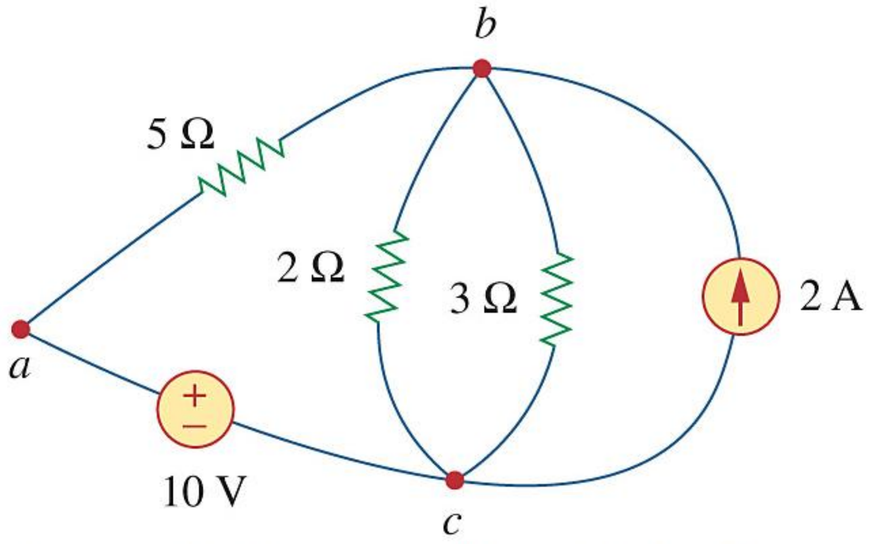


Figure 2.11 The circuit of Fig. 2.10 is redrawn.

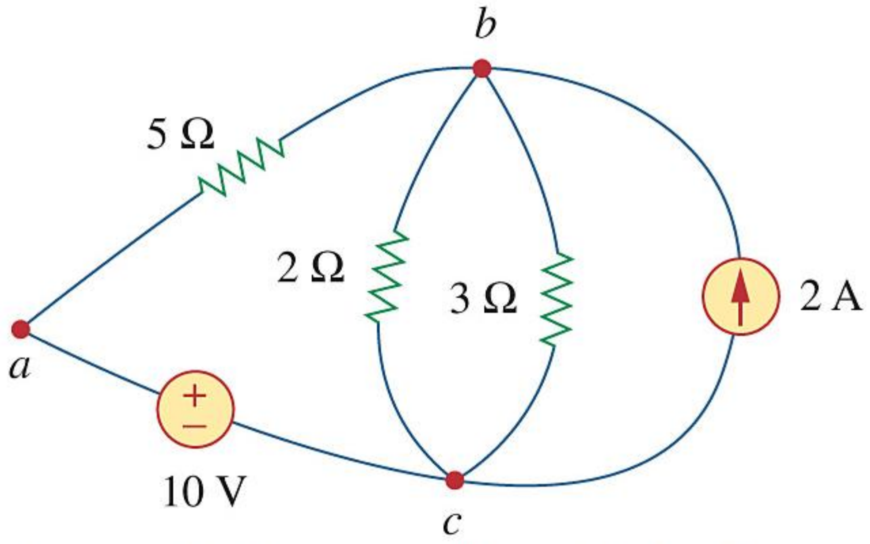
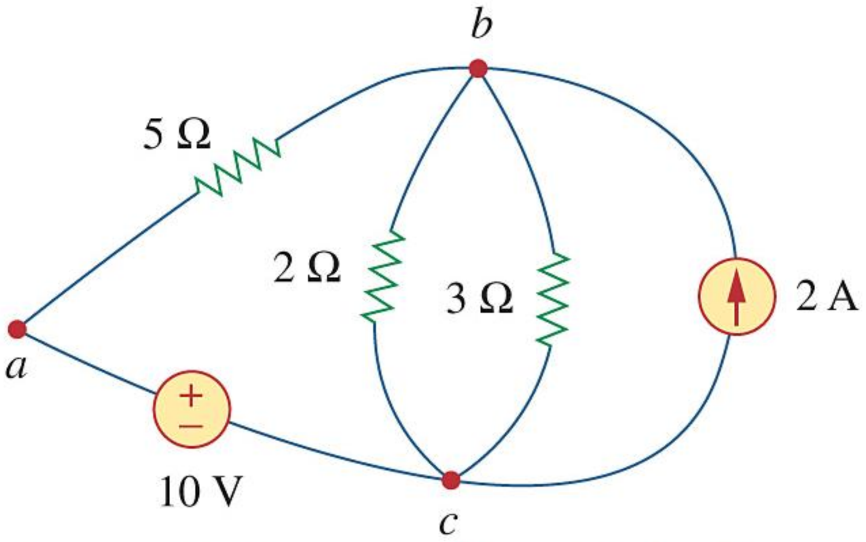


Figure 2.11 The circuit of Fig. 2.10 is redrawn.

Max. # of independent loops



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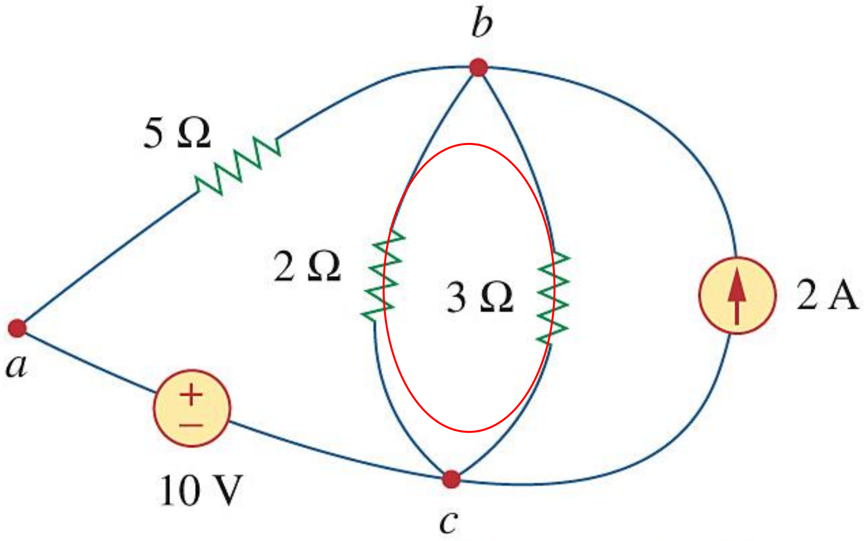

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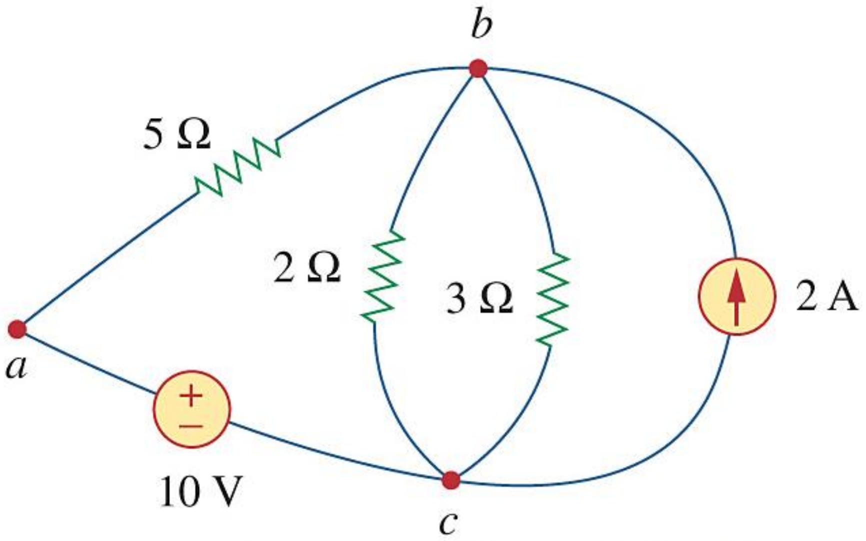

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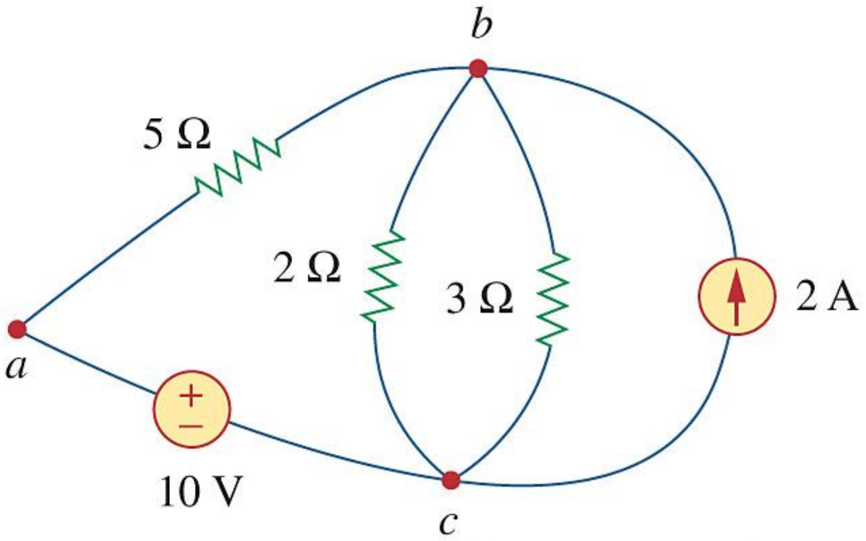

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Max. # of independent loops

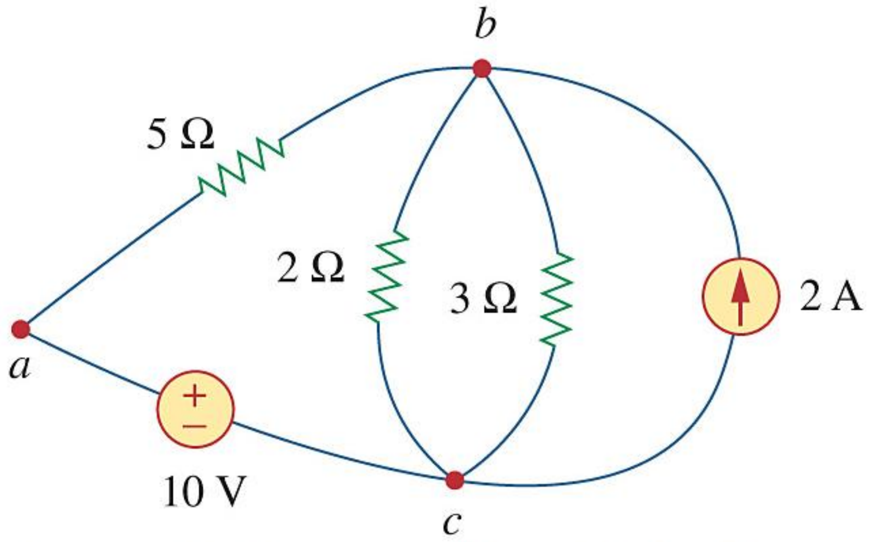


Figure 2.11 The circuit of Fig. 2.10 is redrawn.

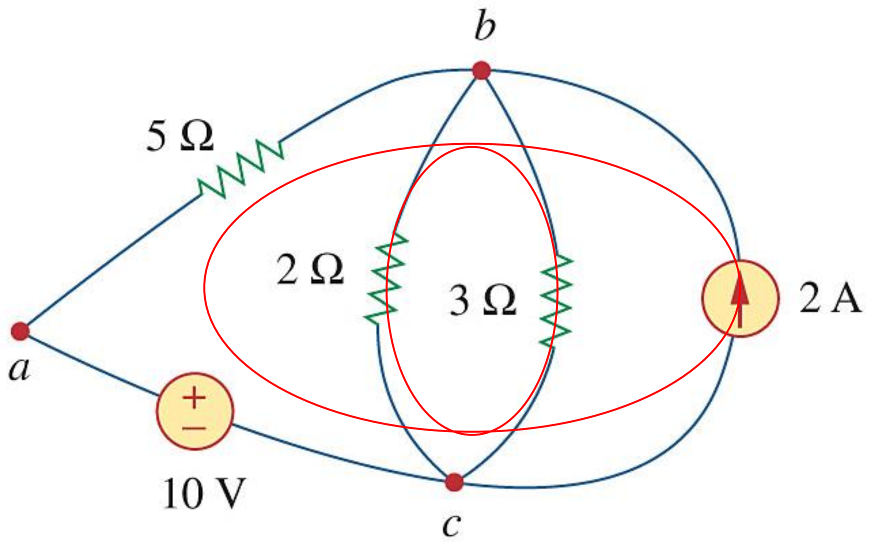


Figure 2.11 The circuit of Fig. 2.10 is redrawn.

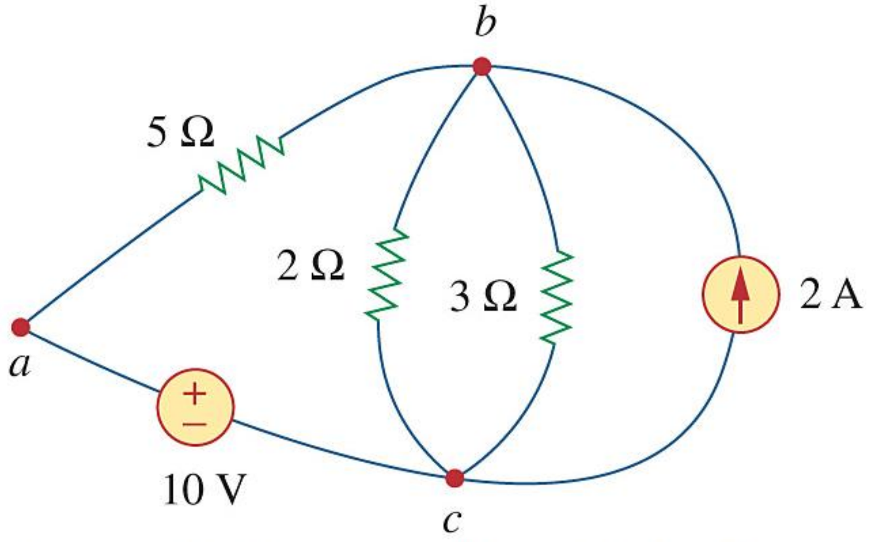


Figure 2.11 The circuit of Fig. 2.10 is redrawn.

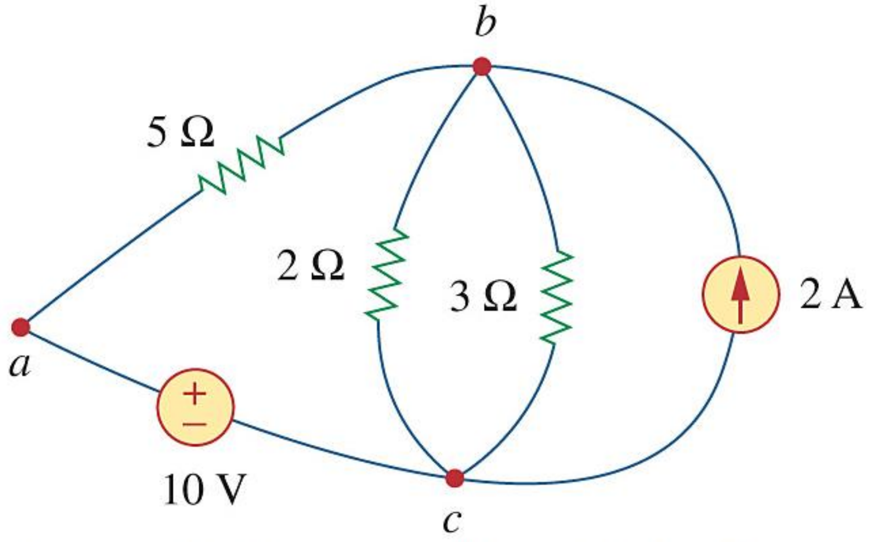
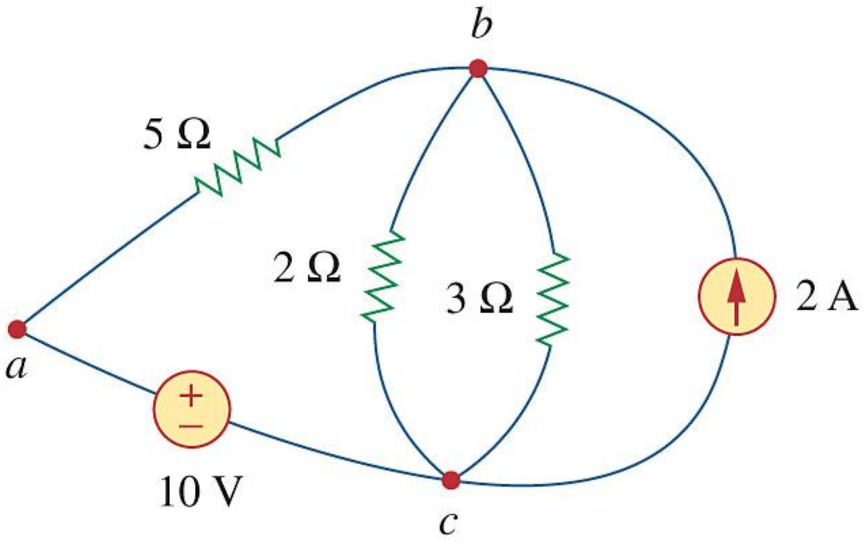


Figure 2.11 The circuit of Fig. 2.10 is redrawn.

Max. # of independent loops



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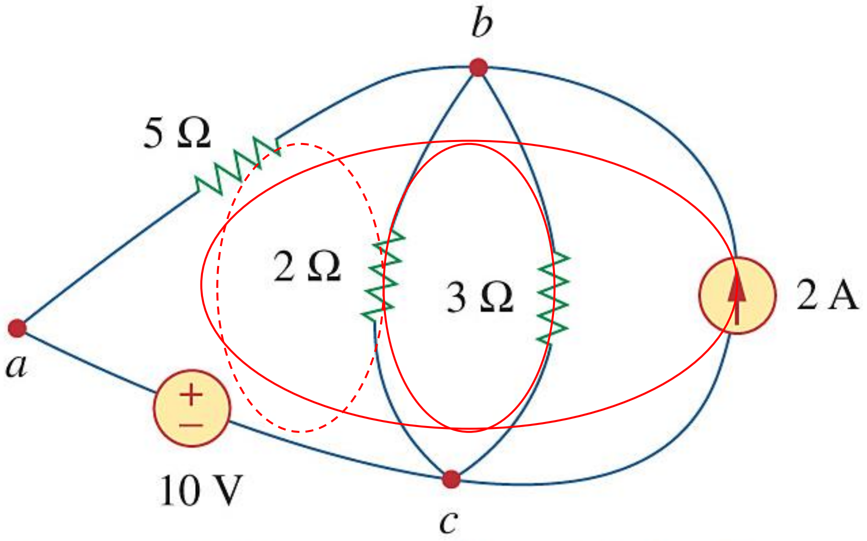


Figure 2.11 The circuit of Fig. 2.10 is redrawn.

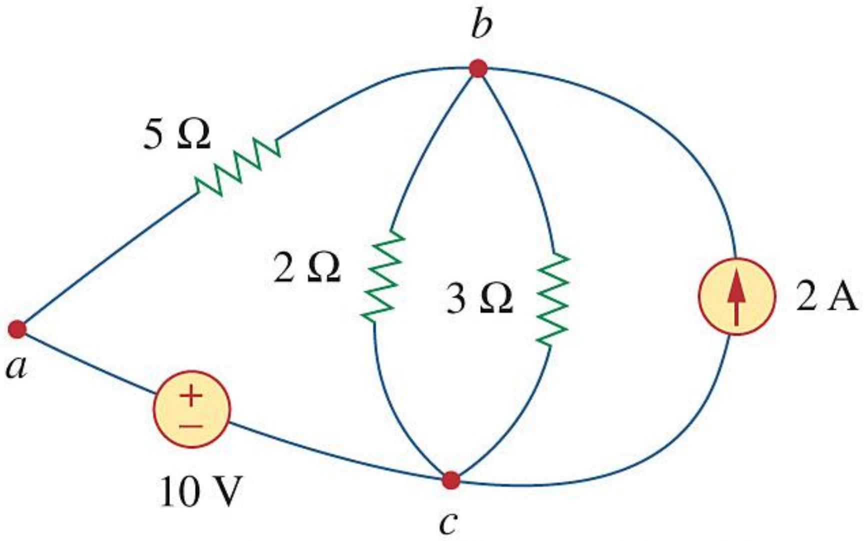
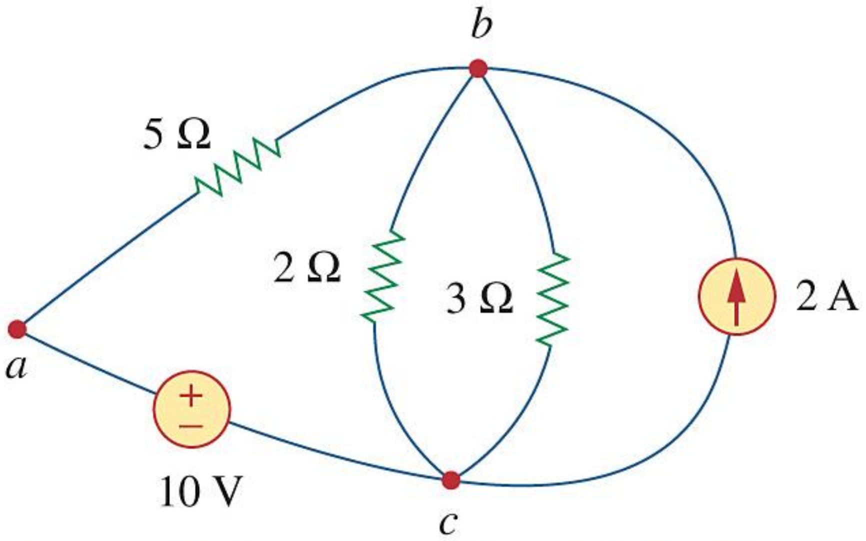


Figure 2.11 The circuit of Fig. 2.10 is redrawn.



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Max. # of independent loops

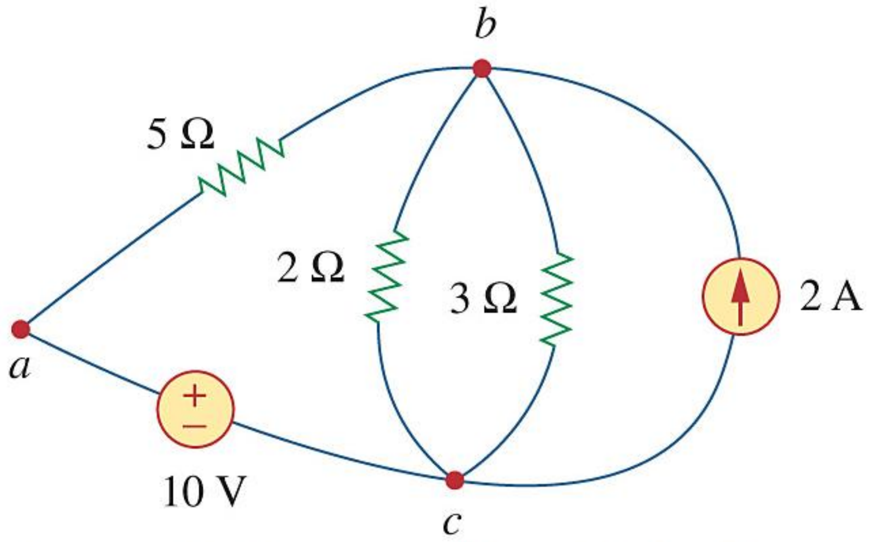


Figure 2.11 The circuit of Fig. 2.10 is redrawn.

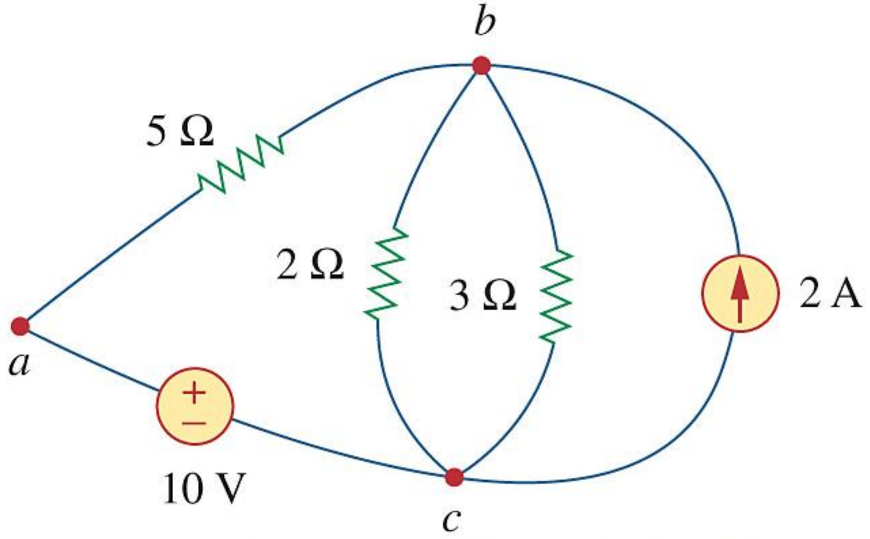


Figure 2.11 The circuit of Fig. 2.10 is redrawn.

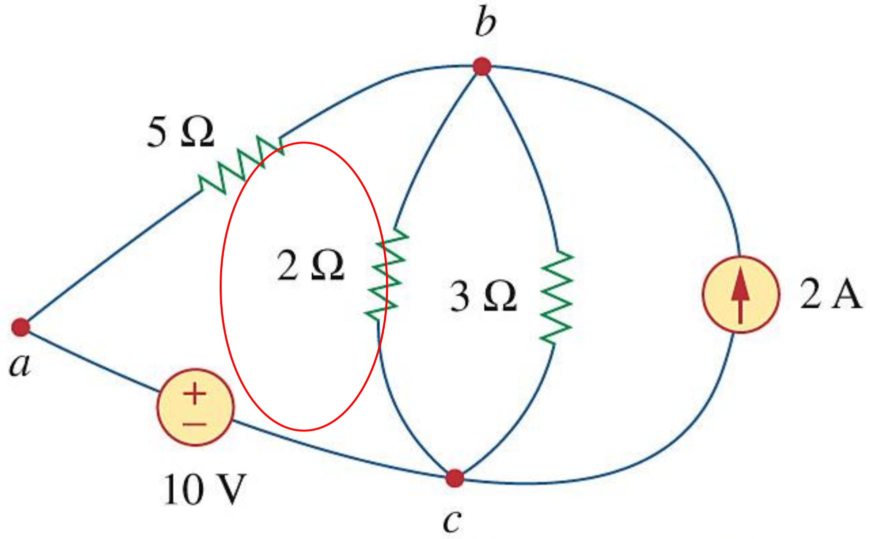


Figure 2.11 The circuit of Fig. 2.10 is redrawn.

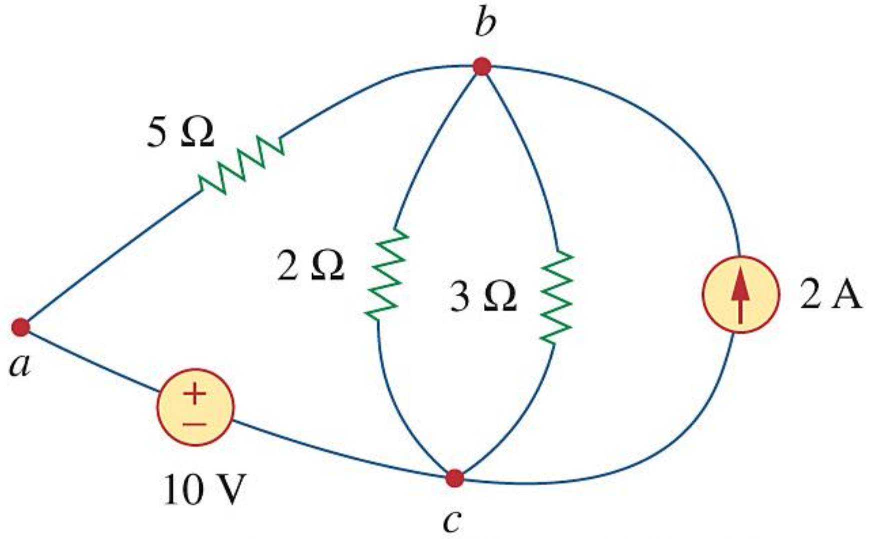


Figure 2.11 The circuit of Fig. 2.10 is redrawn.

Max. # of independent loops

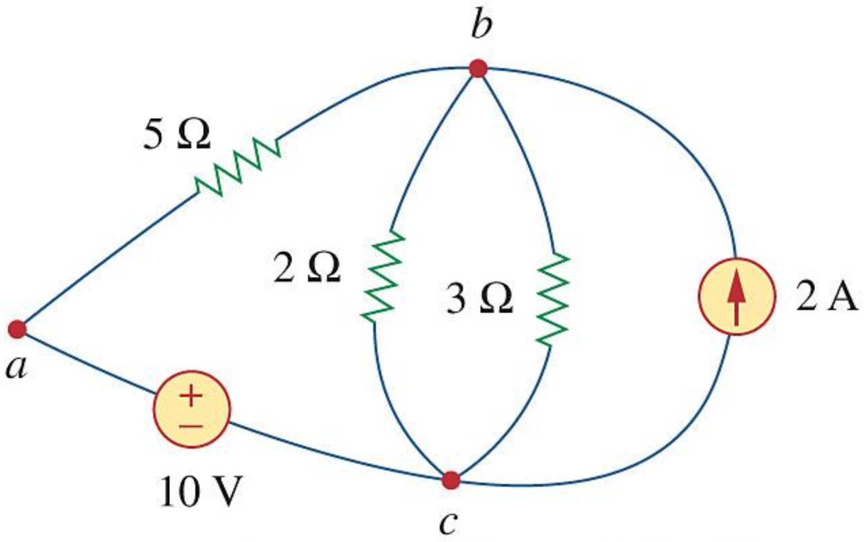
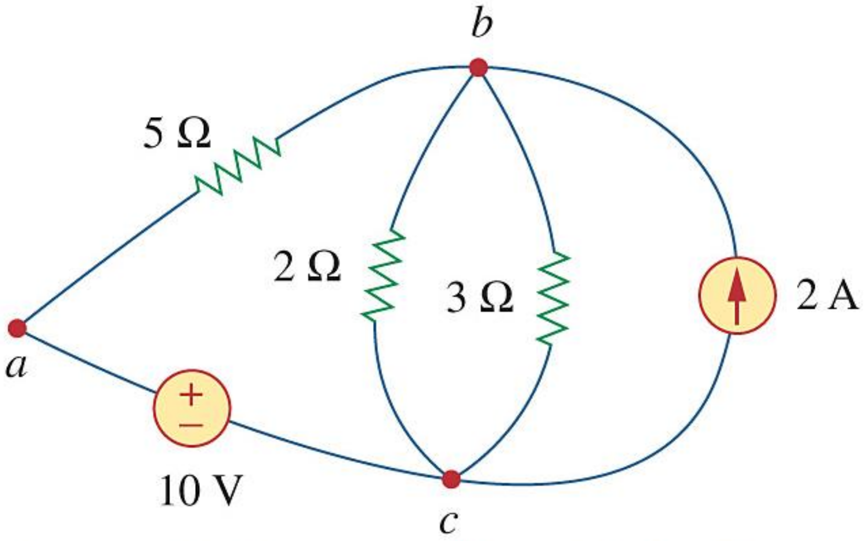


Figure 2.11 The circuit of Fig. 2.10 is redrawn.



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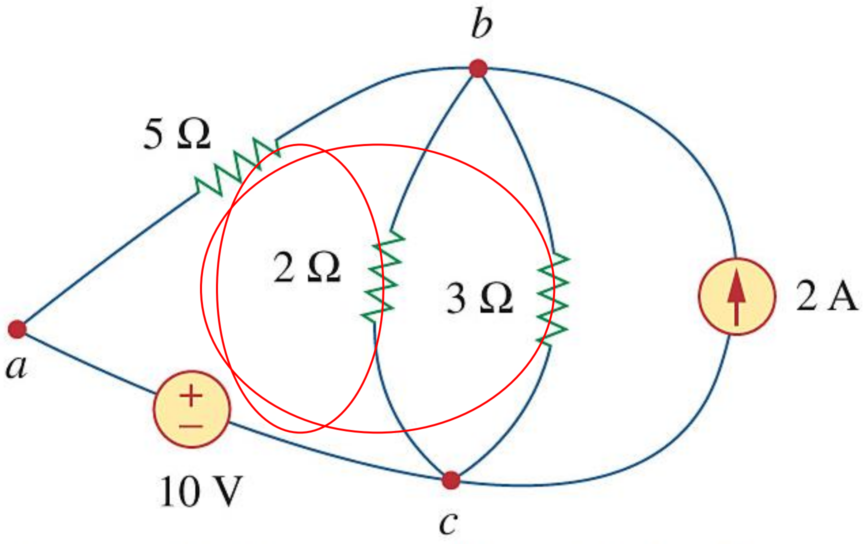


Figure 2.11 The circuit of Fig. 2.10 is redrawn.

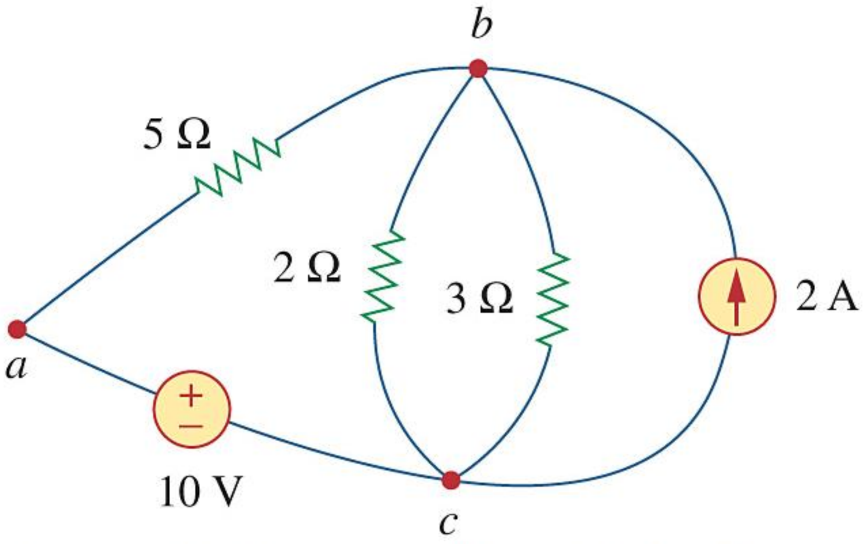
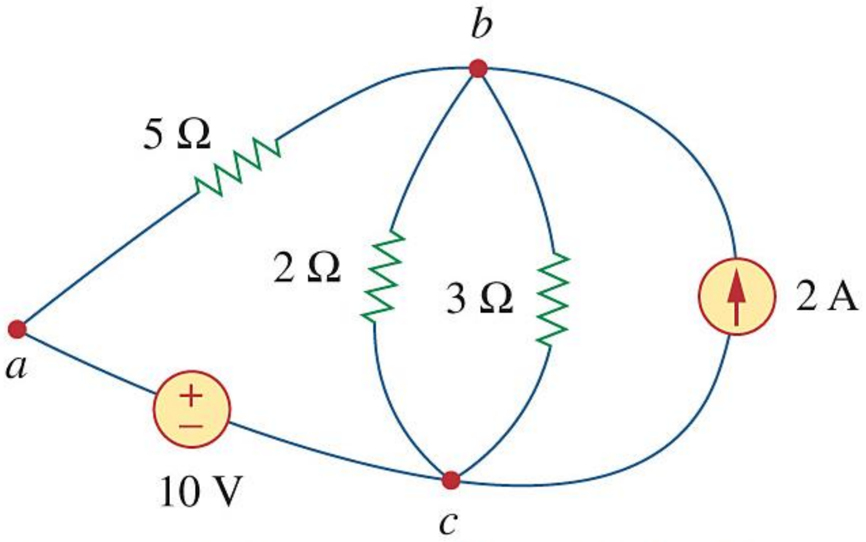


Figure 2.11 The circuit of Fig. 2.10 is redrawn.

Max. # of independent loops



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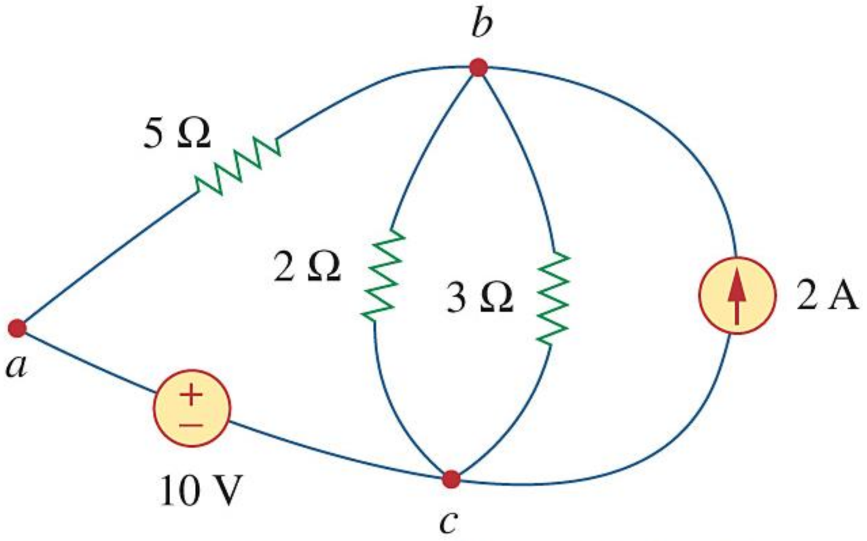

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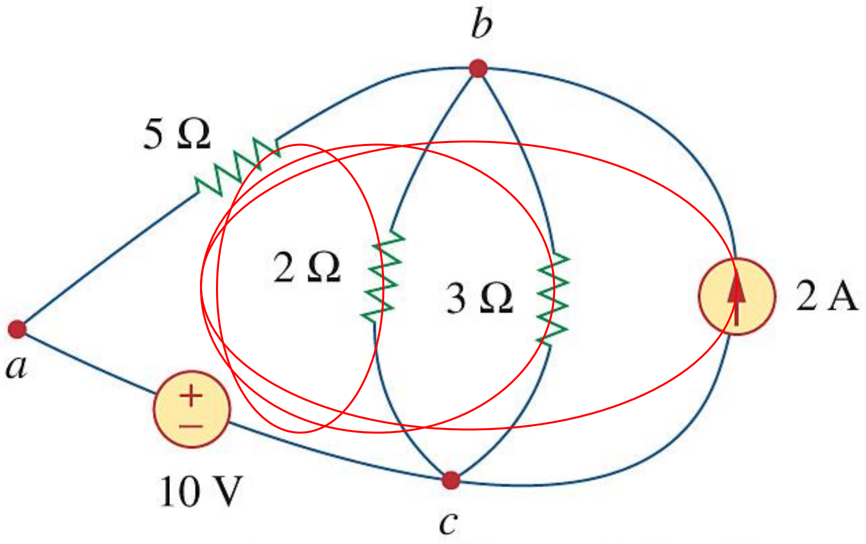

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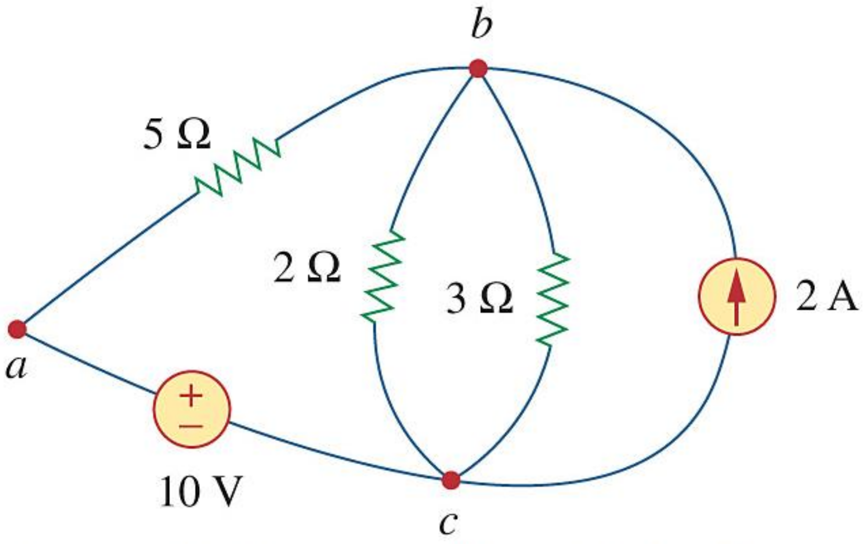

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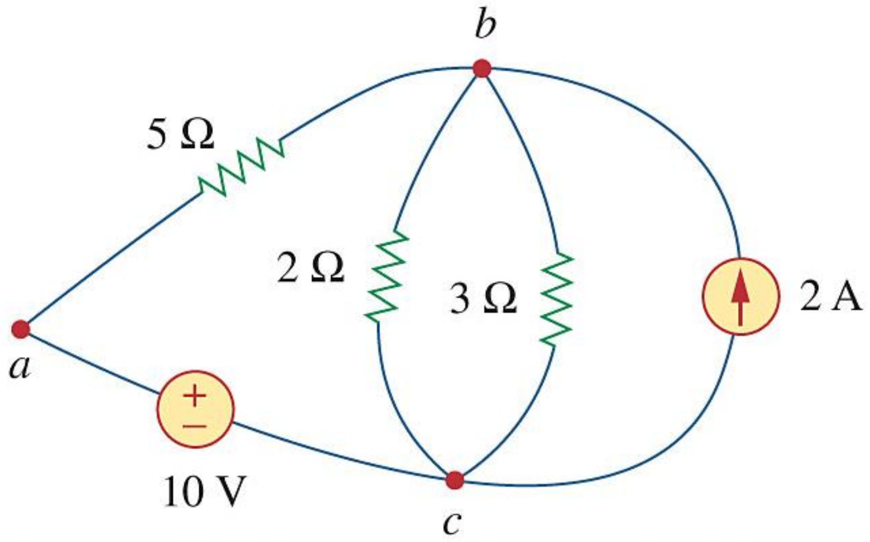
Max. # of independent loops



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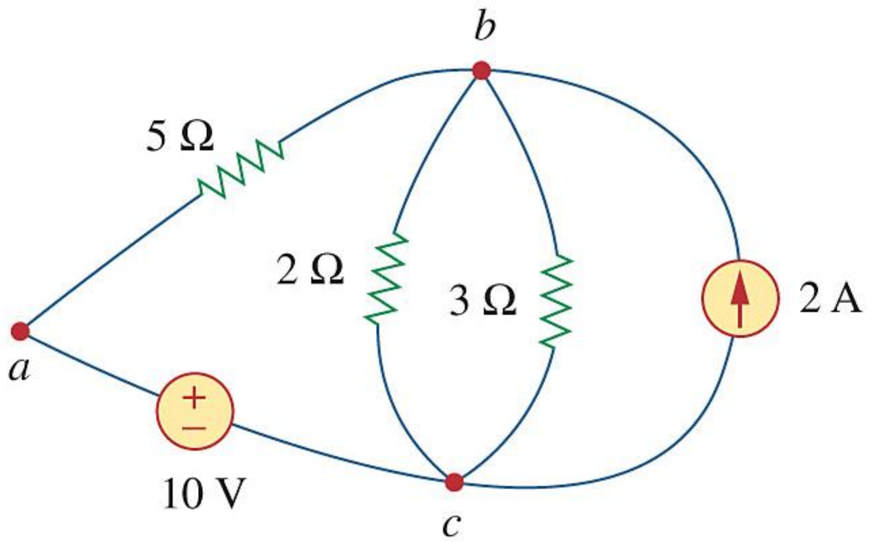

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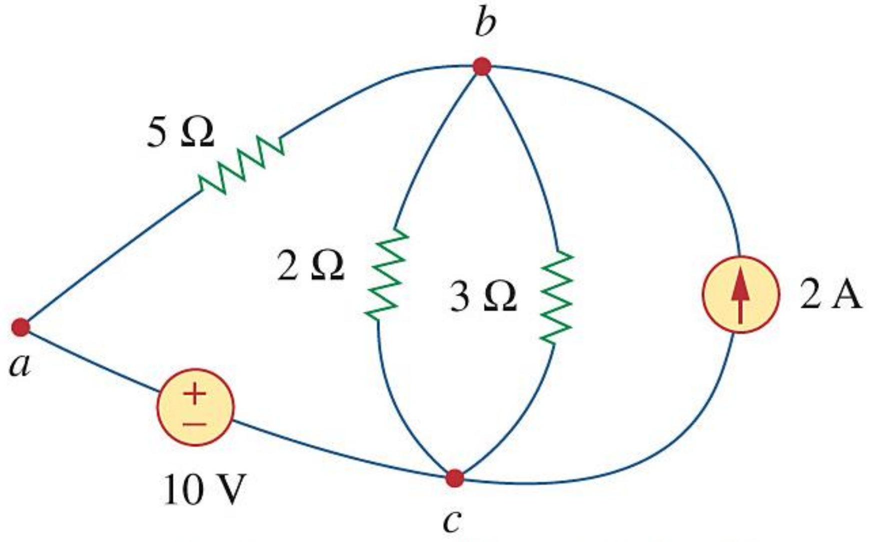


Figure 2.11 The circuit of Fig. 2.10 is redrawn.

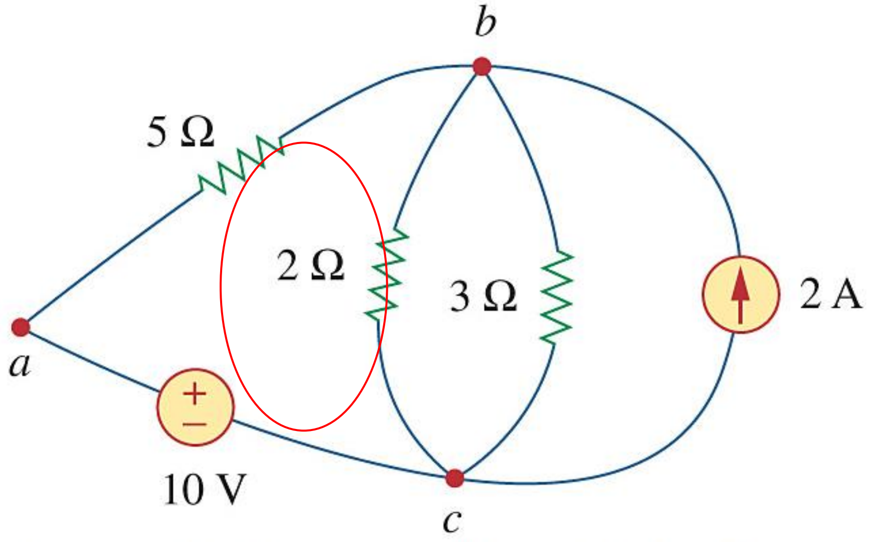


Figure 2.11 The circuit of Fig. 2.10 is redrawn.

Max. # of independent loops

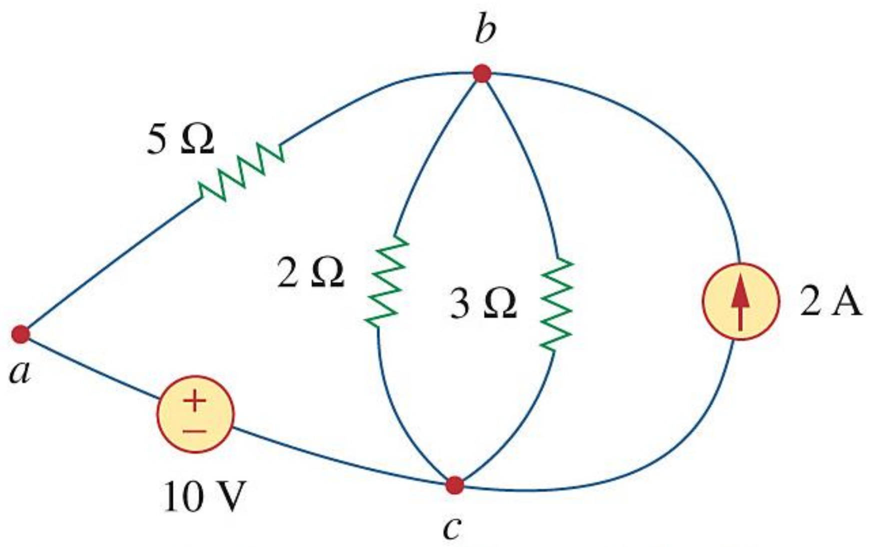


Figure 2.11 The circuit of Fig. 2.10 is redrawn.

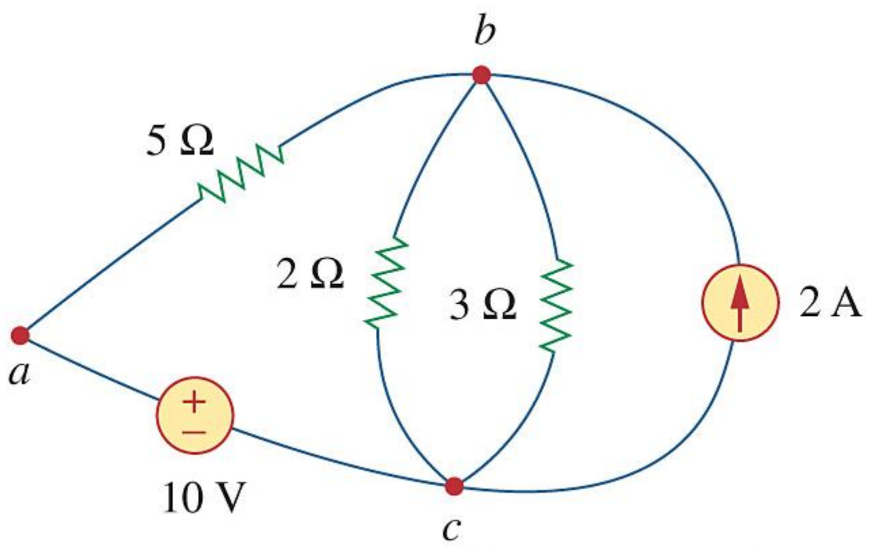


Figure 2.11 The circuit of Fig. 2.10 is redrawn.

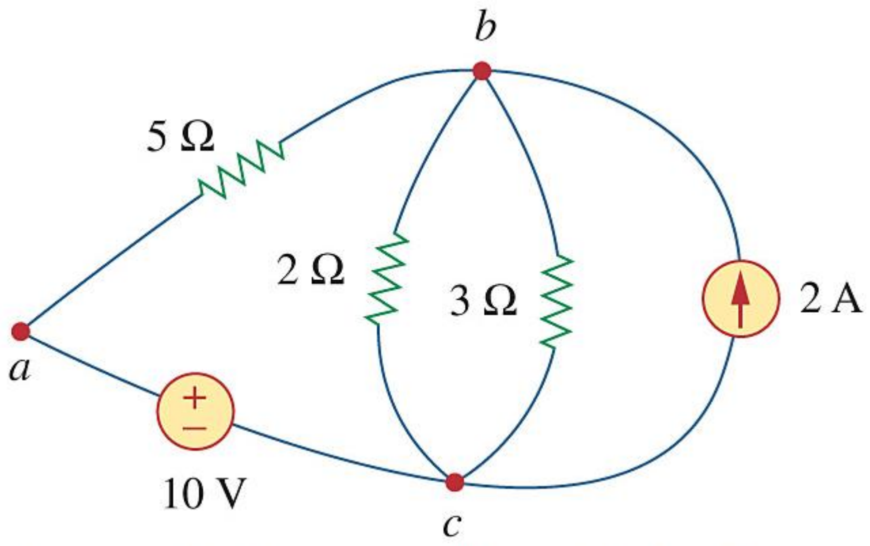


Figure 2.11 The circuit of Fig. 2.10 is redrawn.

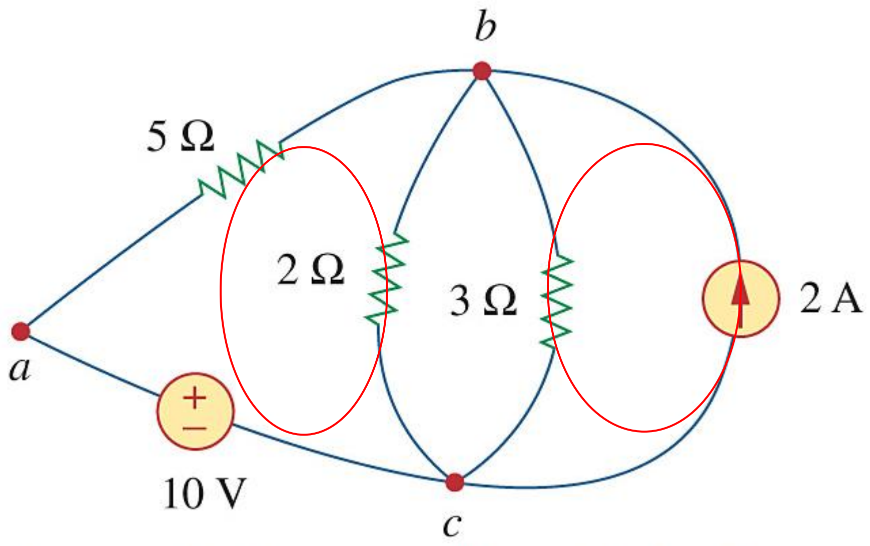


Figure 2.11 The circuit of Fig. 2.10 is redrawn.

Max. # of independent loops

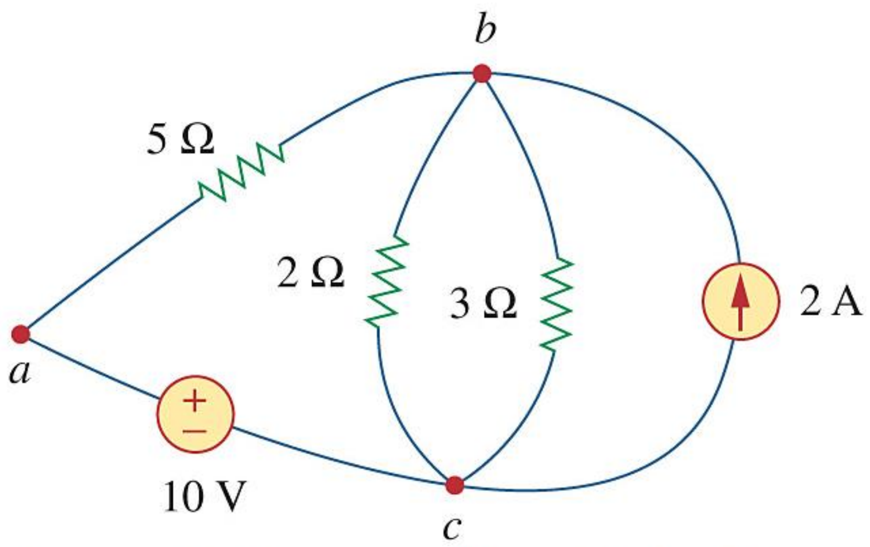


Figure 2.11 The circuit of Fig. 2.10 is redrawn.

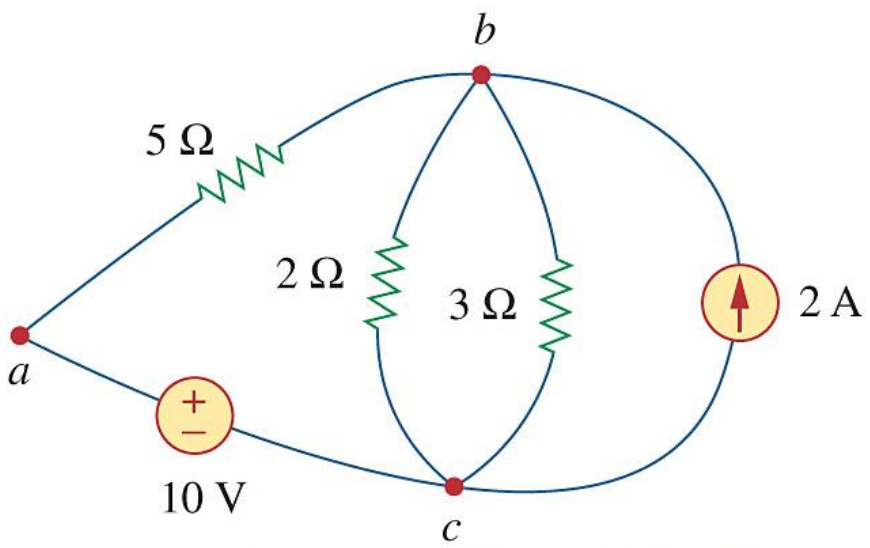


Figure 2.11 The circuit of Fig. 2.10 is redrawn.

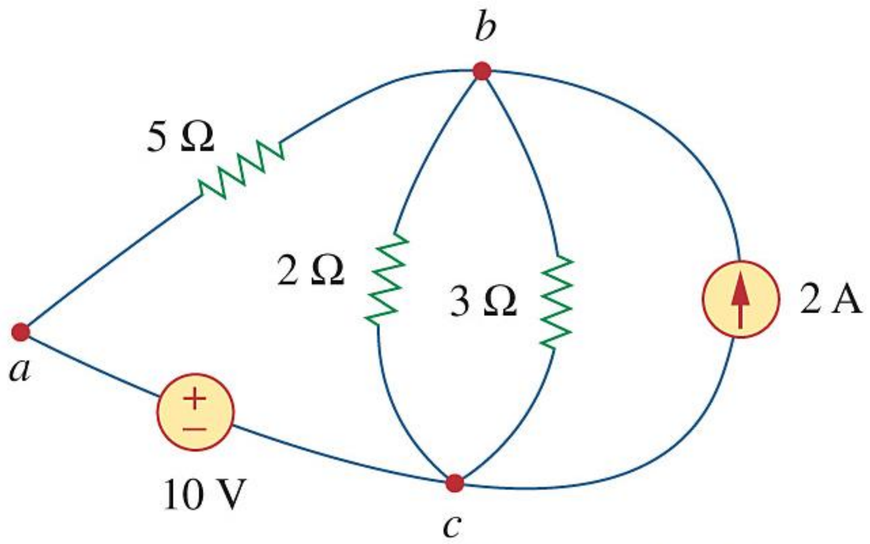


Figure 2.11 The circuit of Fig. 2.10 is redrawn.

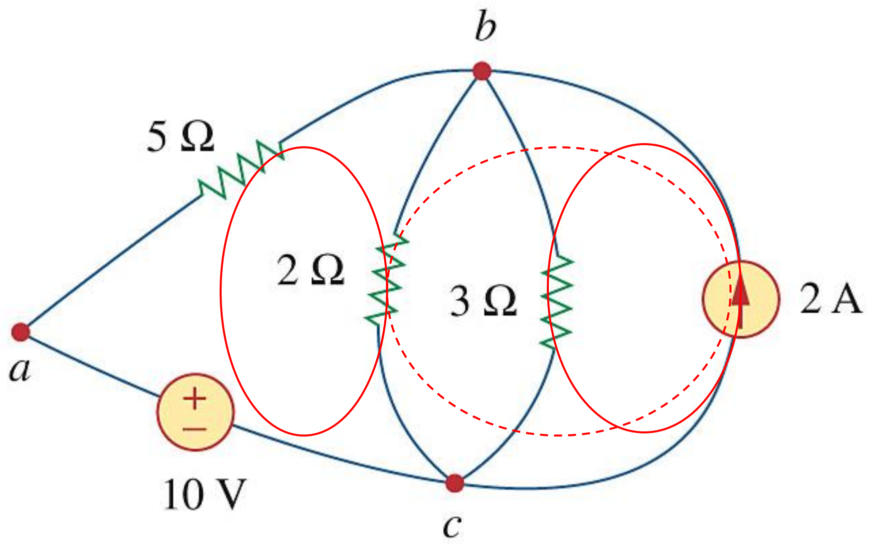


Figure 2.11 The circuit of Fig. 2.10 is redrawn.

Max. # of independent loops

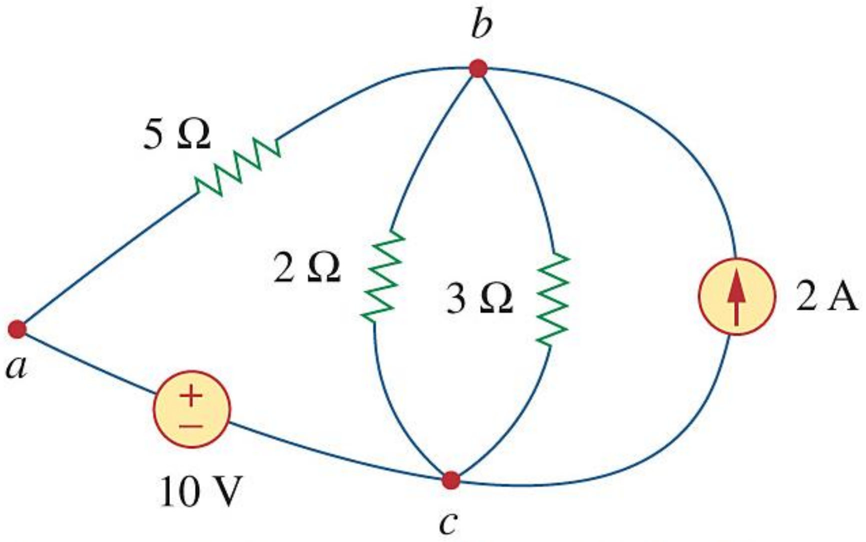


Figure 2.11 The circuit of Fig. 2.10 is redrawn.

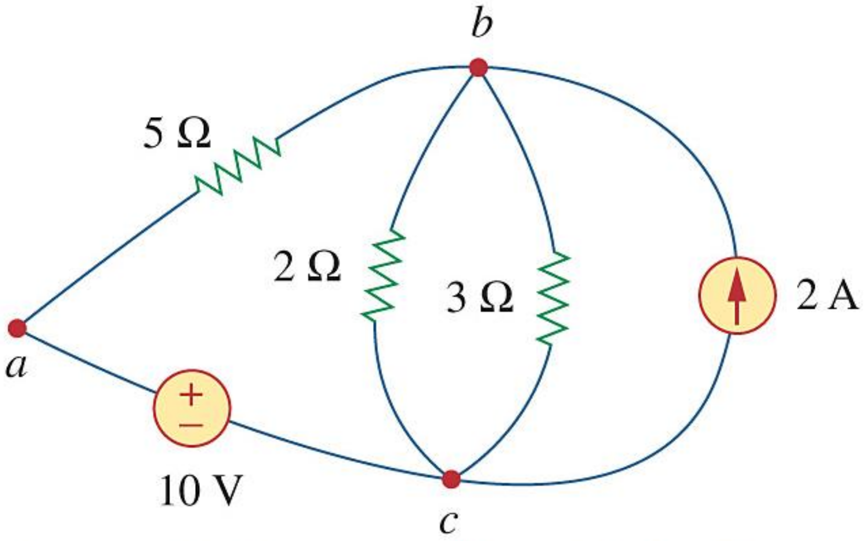


Figure 2.11 The circuit of Fig. 2.10 is redrawn.

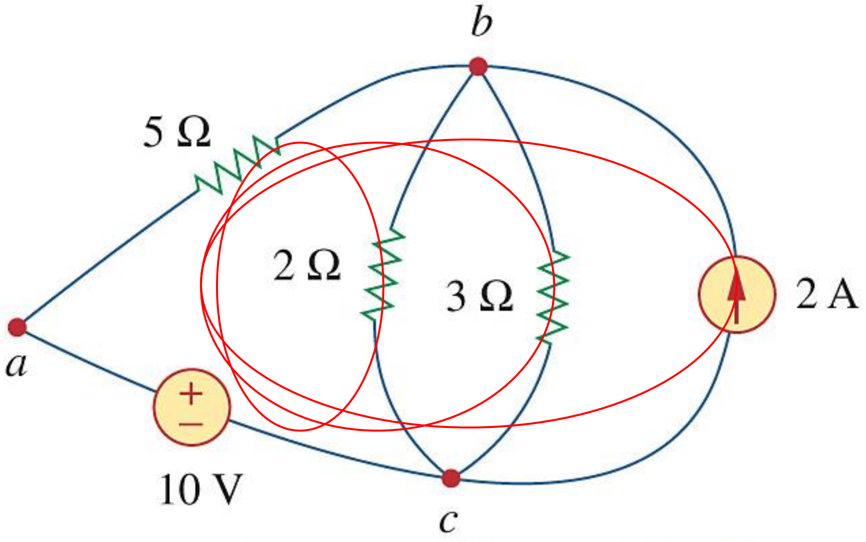


Figure 2.11 The circuit of Fig. 2.10 is redrawn.

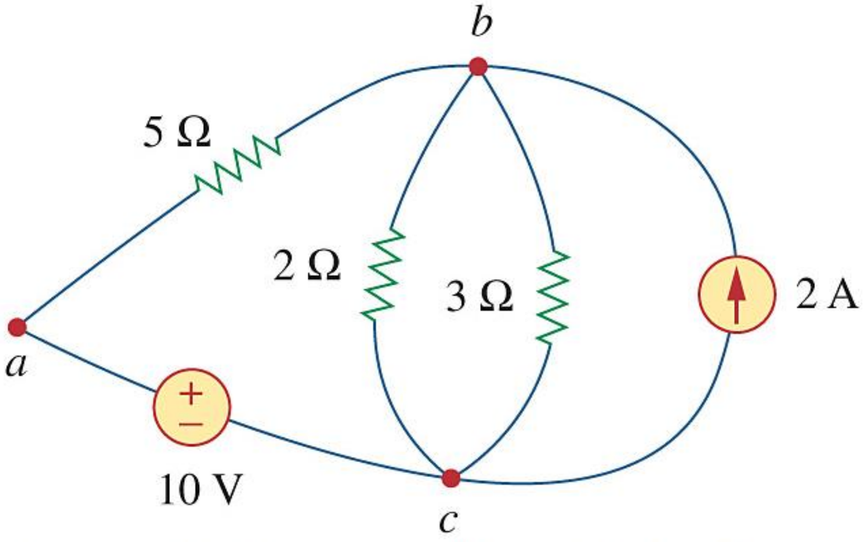


Figure 2.11 The circuit of Fig. 2.10 is redrawn.

(Max.) # of meshes

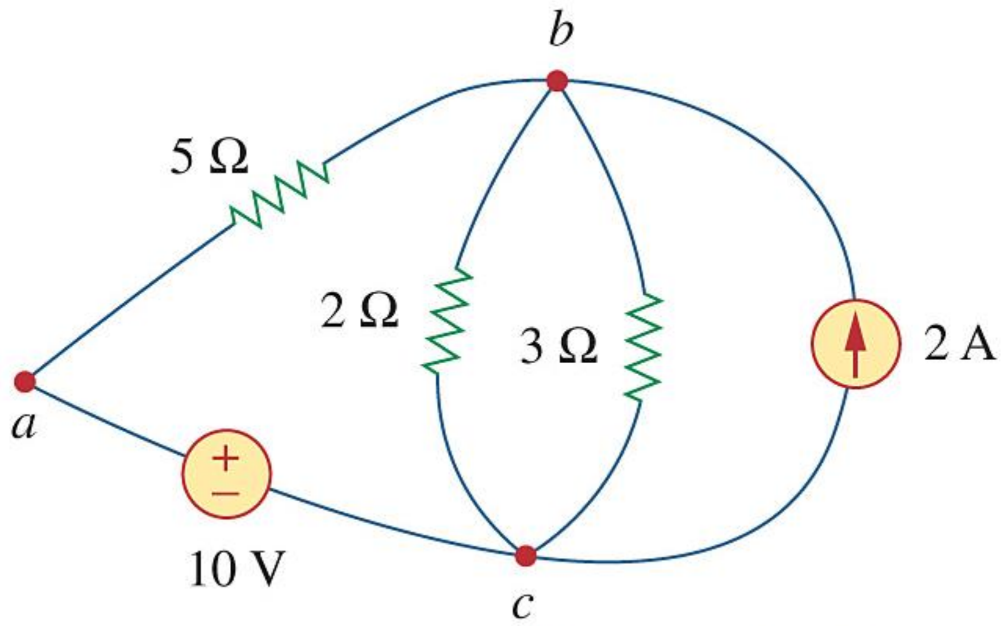


Figure 2.11 The circuit of Fig. 2.10 is redrawn.

Mesh: A mesh is a loop that does not contain any other loop within it. (Smallest loops)

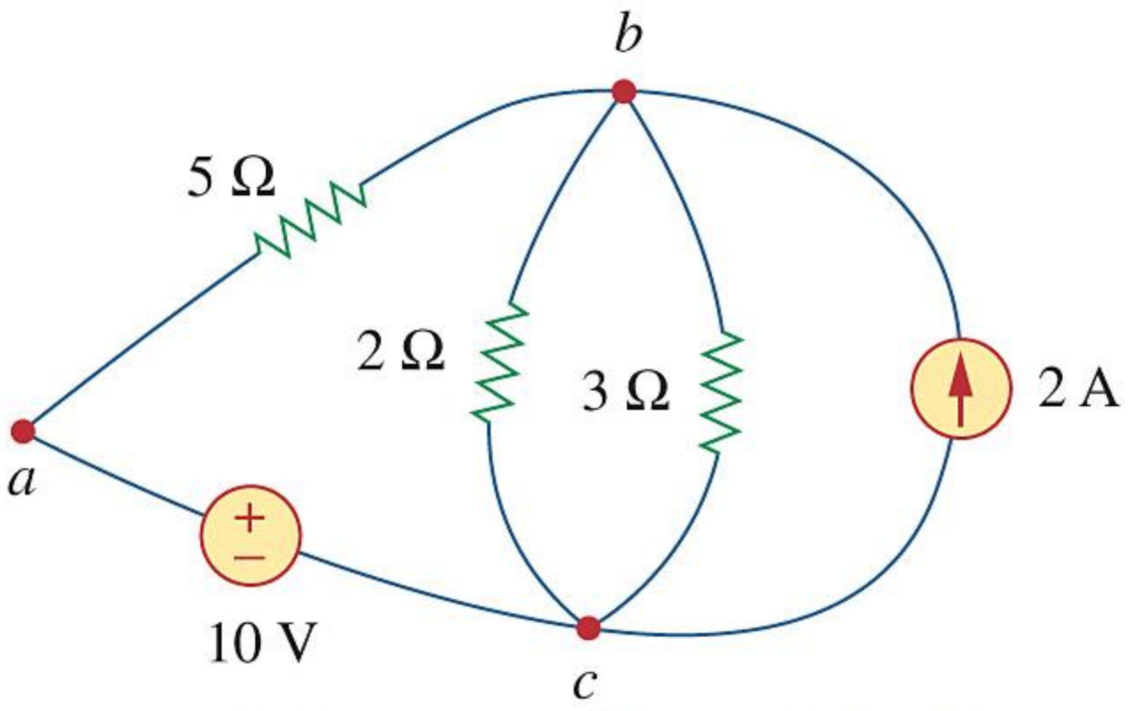


Figure 2.11 The circuit of Fig. 2.10 is redrawn.

# of meshes = max. # of independent loops

Mesh $\neq$ Independent Loop

- If a circuit has n nodes, b branches, and l independent loops, then $b = l + n - 1$.
- This applied to meshes, too: $b = m + n - 1$ where m is the number of meshes.

Independent Loop and Meshes: Definitions are different.
The numbers of them are the same.

b = l (or m) + n – 1

n nodes, b branches, l independent loops, and m meshes:
b = l (m) + n - 1

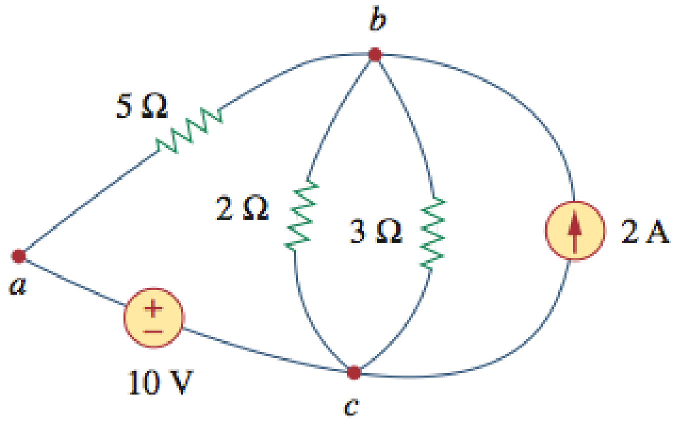


Figure 2.11
The three-node circuit of Fig. 2.10 is redrawn.

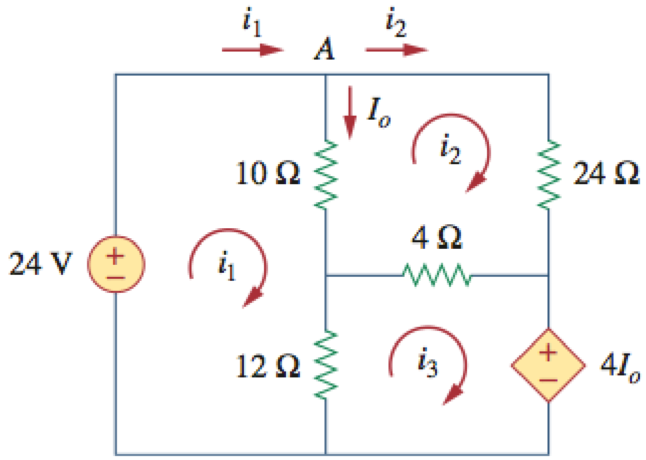


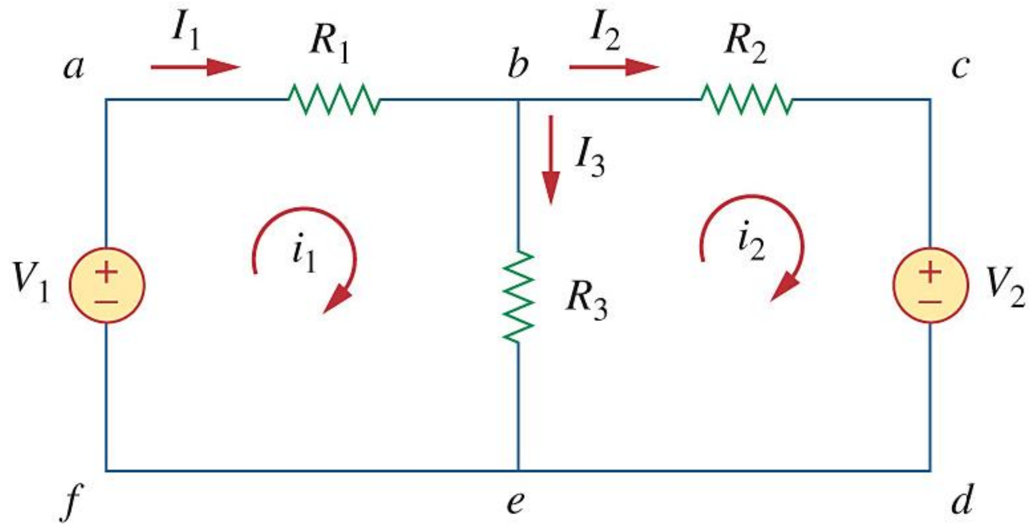
Figure 3.20
For Example 3.6.

5 = l +3 – 1
l = 3

6 = l +4 – 1
l = 3

Mesh Analysis

The current through a mesh is known as mesh current. In figure below, for example, i_1 and i_2 are mesh currents whereas I_1, I_2, and I_3 are branch currents.

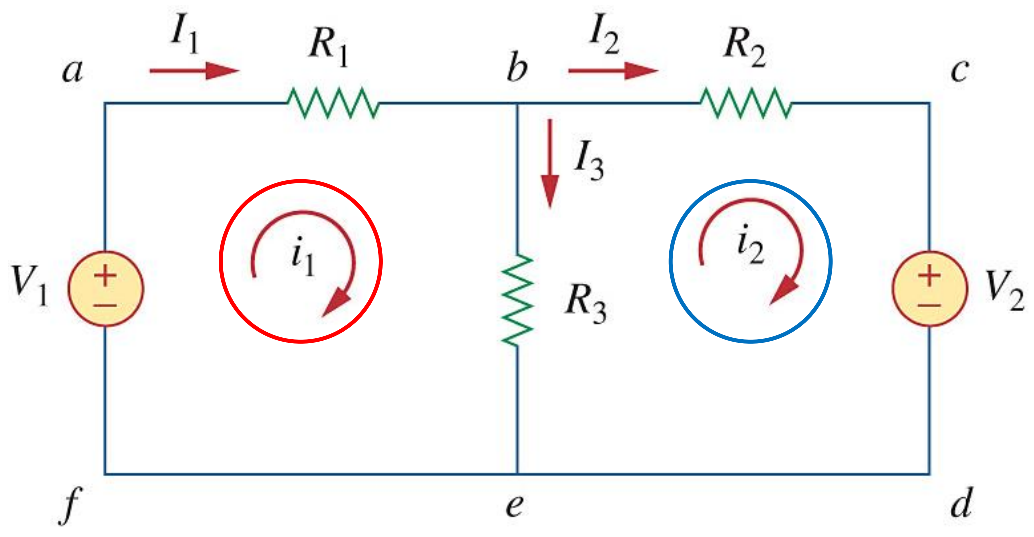


Mesh analysis: find i_1 and i_2 using KVL

Steps to Determine Mesh Currents

- A planar circuits with n meshes, and there are no current sources

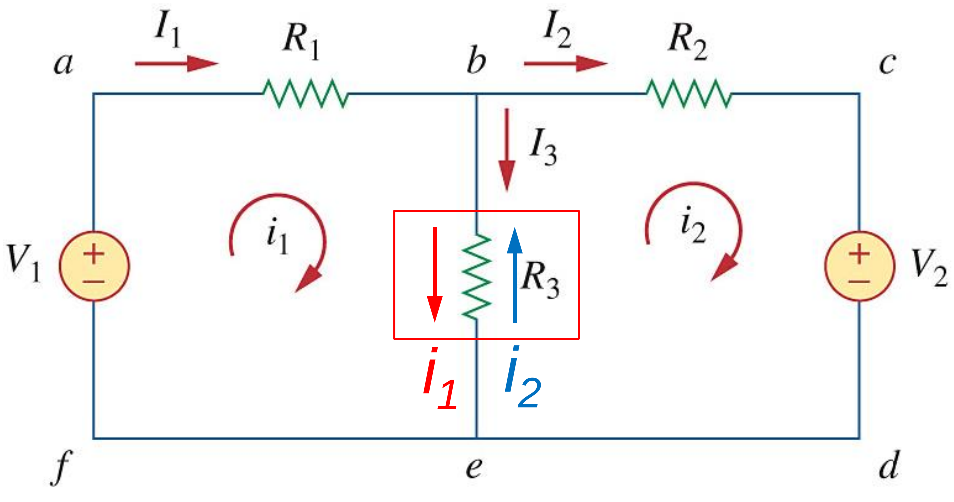
(1) Assign mesh currents i_1, i_2, …, i_n to the n meshes.



Although a mesh current may be assigned in an arbitrary direction, it is conventional to assume that **each mesh current flows clockwise**.

Steps to Determine Mesh Currents

(2) Apply KVL to each of the n meshes. Use Ohm's law to express the voltages in terms of the mesh currents



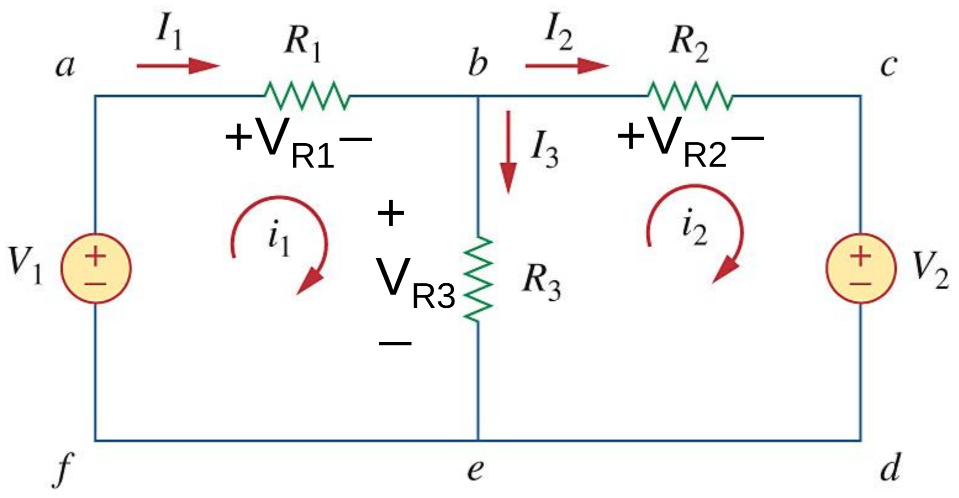
Mesh 1: $-V_1 + R_1 i_1 + \boxed{R_3(i_1 - i_2)} = 0 \longrightarrow (R_1 + R_3)i_1 - R_3 i_2 = V_1$

Mesh 2: $R_2 i_2 + V_2 + \boxed{R_3(i_2 - i_1)} = 0 \longrightarrow -R_3 i_1 + (R_2 + R_3)i_2 = -V_2$

From the previous slide

Mesh 1: $-V_1 + R_1 i_1 + \boxed{R_3(i_1 - i_2)} = 0$

Mesh 2: $R_2 i_2 + V_2 + \boxed{R_3(i_2 - i_1)} = 0$



By KVL

Mesh 1: $-V_1 + V_{R1} + V_{R3} = 0$

Mesh 2: $-V_{R3} + V_{R2} + V_2 = 0$

$V_{R3} = R_3(i_1\text{-}i_2)$, therefore, $-V_{R3} = R_3(i_2\text{-}i_1)$

Steps to Determine Mesh Currents

(3) Solve the resulting n simultaneous equations to get the mesh currents.

To solve this, we use either

(i) the elimination method

Mesh 1: $(R_1 + R_3)i_1 - R_3 i_2 = V_1$

Mesh 2: $-R_3 i_1 + (R_2 + R_3)i_2 = -V_2$

(ii) Cramer's rule

$$\begin{bmatrix} R_1 + R_3 & -R_3 \\ -R_3 & R_2 + R_3 \end{bmatrix} \begin{bmatrix} i_1 \\ i_2 \end{bmatrix} = \begin{bmatrix} V_1 \\ -V_2 \end{bmatrix}$$

Example 3.5 For the circuit in Fig. 3.18, find the branch currents I_1, I_2, and I_3 using mesh analysis.

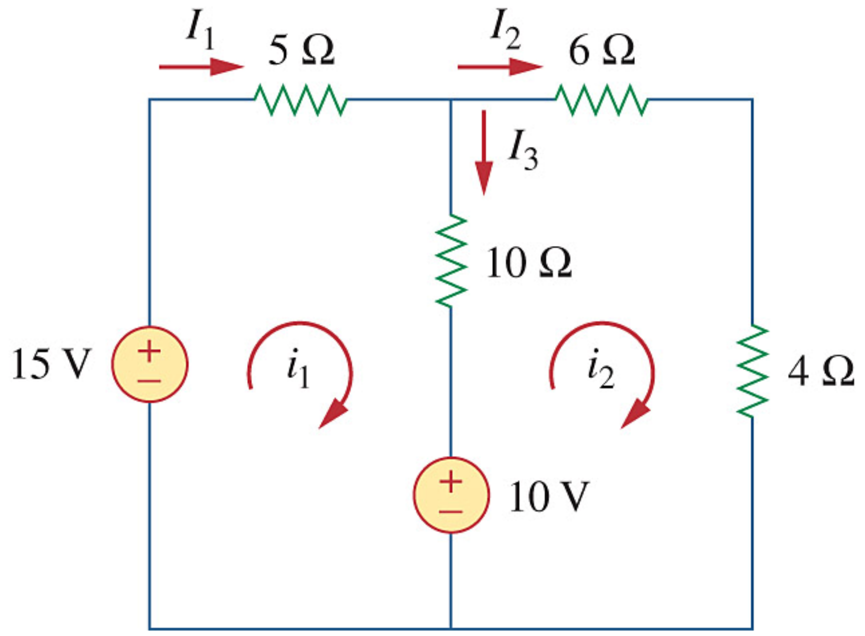


Figure 3.18

By KVL,

Mesh 1: $-15 + 5i_1 + 10(i_1 - i_2) + 10 = 0$

$$\rightarrow 15i_1 - 10i_2 = 5$$
$$\rightarrow 3i_1 - 2i_2 = 1$$

Mesh 2: $-10 + 10(i_2 - i_1) + 6i_2 + 4i_2 = 0$

$$\rightarrow 10i_1 - 20i_2 = -10$$
$$\rightarrow i_1 - 2i_2 = -1$$

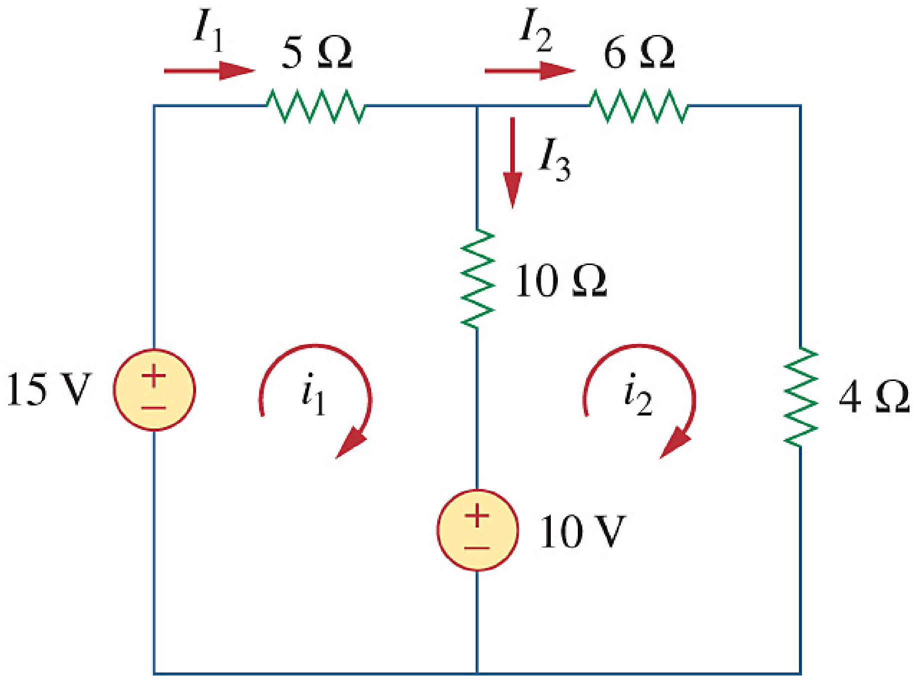


Figure 3.18

Write two equations into the matrix form

$$\begin{bmatrix} 3 & -2 \\ 1 & -2 \end{bmatrix} \begin{bmatrix} i_1 \\ i_2 \end{bmatrix} = \begin{bmatrix} 1 \\ -1 \end{bmatrix}$$

i_1 = 1 [A]
i_2 = 1 [A]

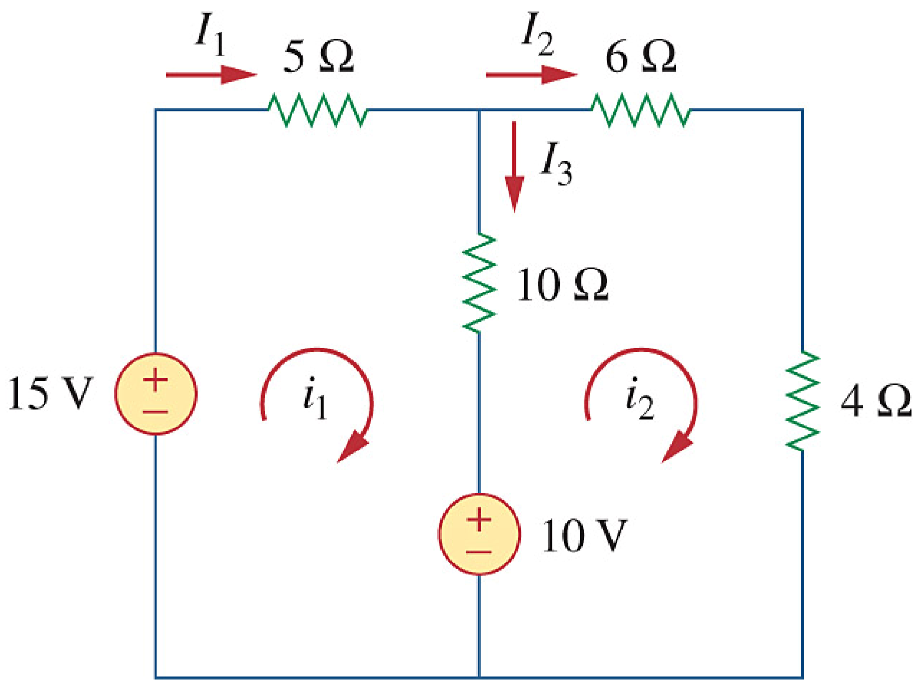


Figure 3.18

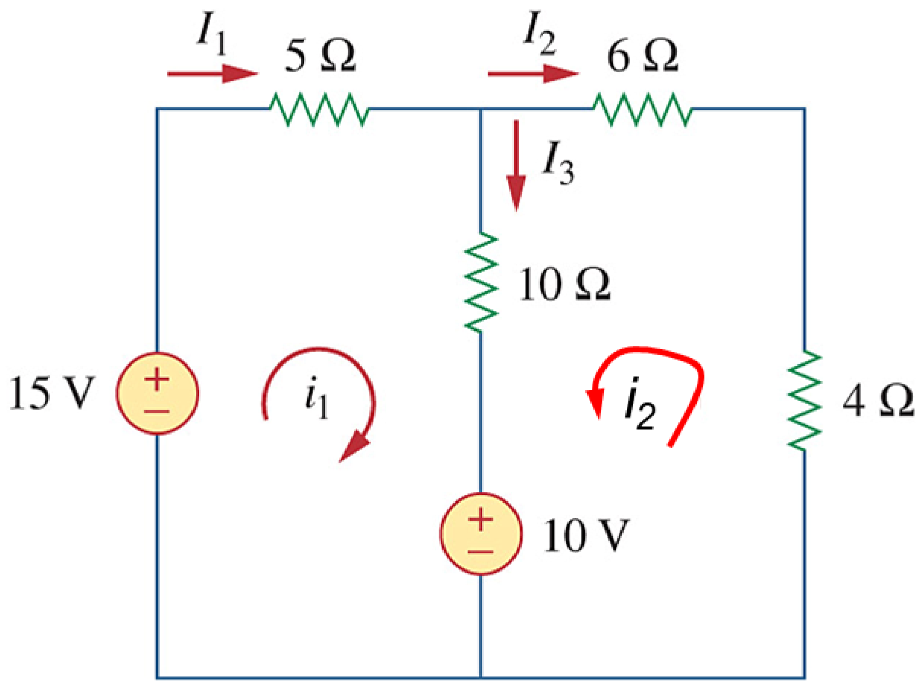


Figure 3.18

Write two equations into the matrix form

$$\begin{bmatrix} 3 & -2 \\ 1 & -2 \end{bmatrix} \begin{bmatrix} i_1 \\ i_2 \end{bmatrix} = \begin{bmatrix} 1 \\ -1 \end{bmatrix}$$

i_1 = 1 [A]
i_2 = 1 [A]

What happens if we choose one of the mesh currents opposite way?

By KVL,

Mesh 1: $-15 + 5i_1 + 10(i_1 + i_2) + 10 = 0$

$\rightarrow 15i_1 + 10i_2 = 5$

$\rightarrow 3i_1 + 2i_2 = 1$

Mesh 2: $4i_2 + 6i_2 + 10(i_1 + i_2) + 10 = 0$

$\rightarrow 10i_1 + 20i_2 = -10$

$\rightarrow i_1 + 2i_2 = -1$

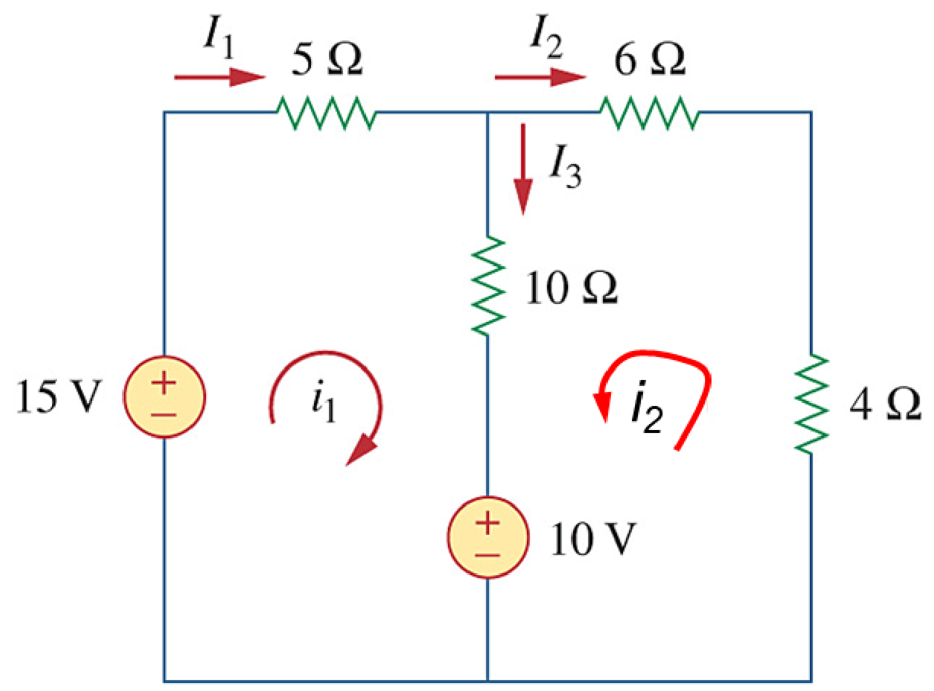


Figure 3.18

Write two equations into the matrix form

$$\begin{bmatrix} 3 & 2 \\ 1 & 2 \end{bmatrix} \begin{bmatrix} i_1 \\ i_2 \end{bmatrix} = \begin{bmatrix} 1 \\ -1 \end{bmatrix}$$

i_1 = 1 [A]

i_2 = -1 [A]

Mesh Analysis by Inspection (Section 3.6)

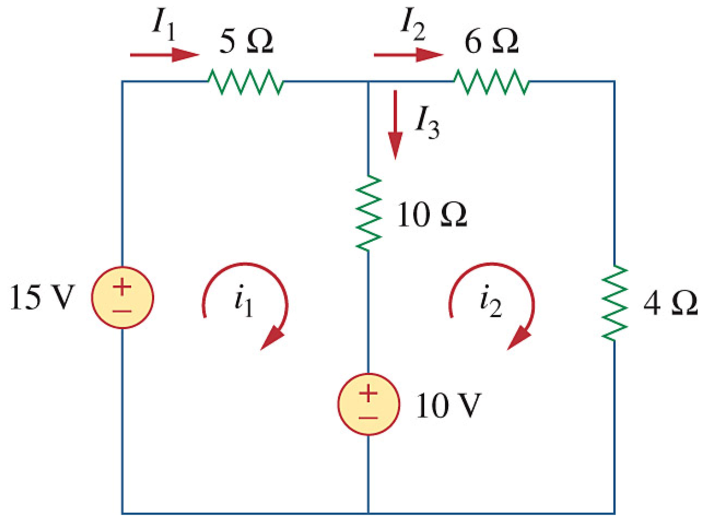


Figure 3.18

$$\begin{bmatrix}15 & -10\\ -10 & +20\end{bmatrix}\begin{bmatrix}i_1\\ i_2\end{bmatrix} = \begin{bmatrix}15-10\\ +10\end{bmatrix}$$

Diagonal: the sum of the resistances in the related mesh.

Off-diagonal: the negative of the resistance common to meshes.

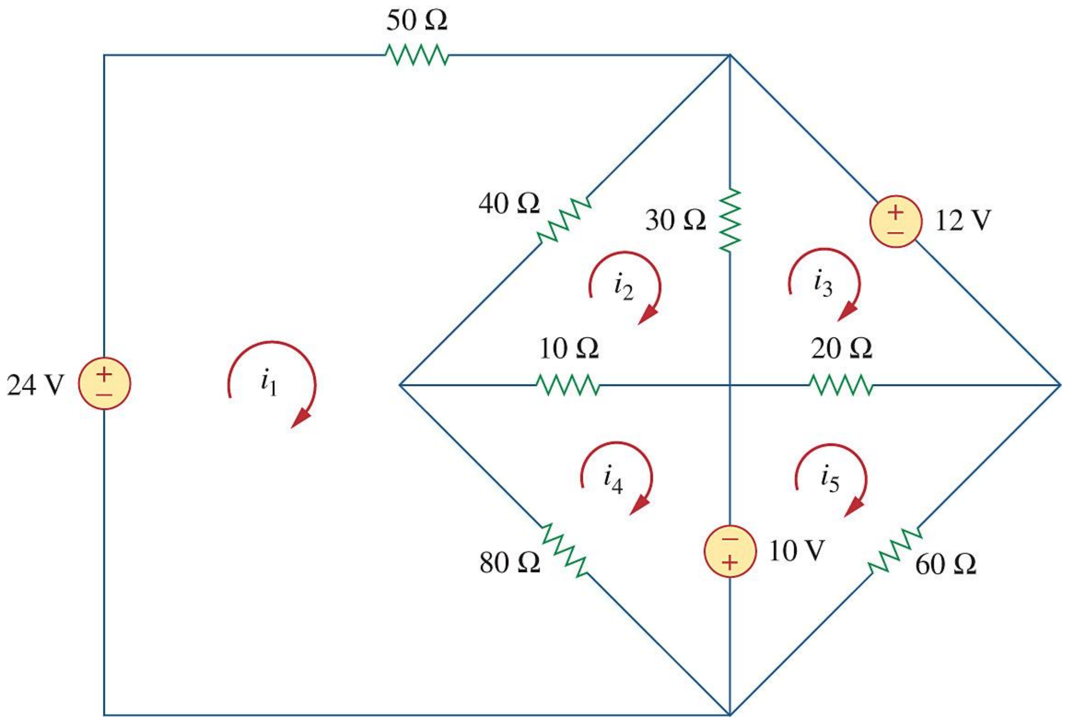
Voltage sources: the algebraic sum taken clockwise of all independent voltage sources in the related mesh, with voltage rise treated as positive.

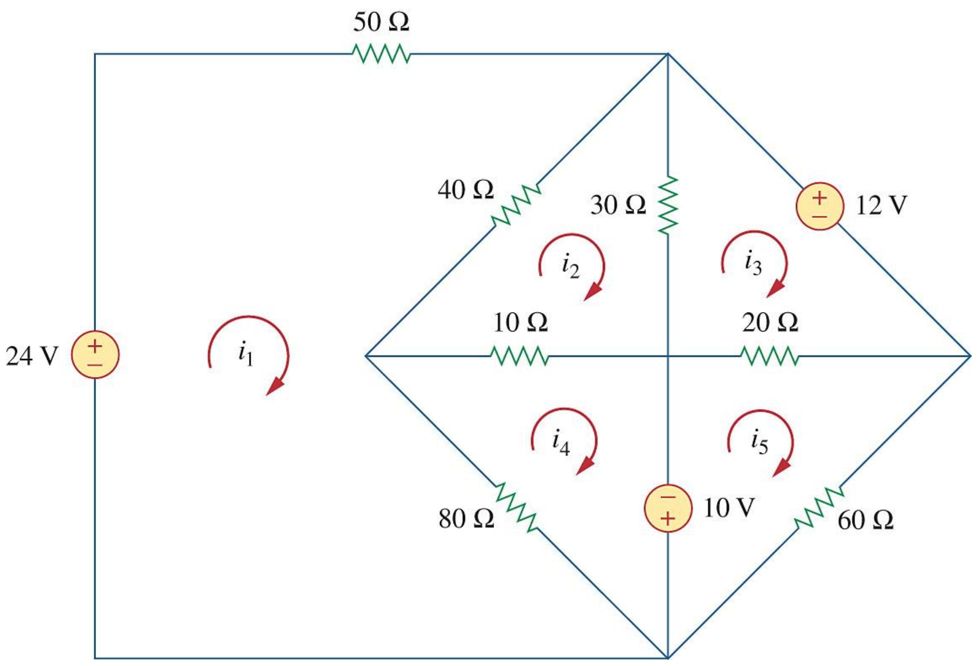
In general, if a circuit with only independent voltage sources has N meshes, the node-current equations can be written as

$$\begin{bmatrix} R_{11} & R_{12} & \dots & R_{1N} \\ R_{21} & R_{22} & \dots & R_{2N} \\ \vdots & \vdots & \vdots & \vdots \\ R_{N1} & R_{N2} & \dots & R_{NN} \end{bmatrix} \begin{bmatrix} i_1 \\ i_2 \\ \vdots \\ i_N \end{bmatrix} = \begin{bmatrix} v_1 \\ v_2 \\ \vdots \\ v_N \end{bmatrix}$$

$\mathbf{R_{kk}}$ = Sum of the resistances in mesh k
$\mathbf{R_{kj}} = \mathbf{R_{jk}}$ = Negative of the sum of the resistances in common with meshes k and j, $k \neq j$
$\mathbf{i_k}$ = Unknown current for mesh k in the clockwise direction
$\mathbf{v_k}$ = Sum taken clockwise of all independent voltage sources in mesh k, with voltage rise treated as positive

Practice Problem 3.9 By inspection, obtain the mesh-current equations for the circuit in Fig. 3.30.





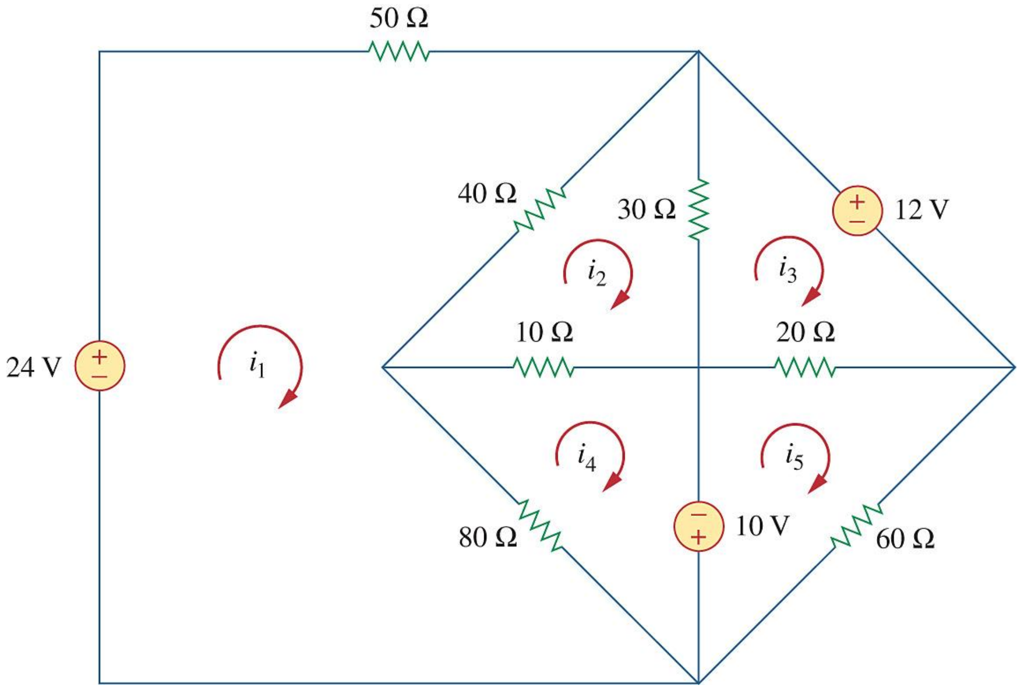
R_{kk} = Sum of the resistances in mesh k

R_{kj} = R_{jk} = Negative of the sum of the resistances in common with meshes k and j, k≠j

i_k = Unknown current for mesh k in the clockwise direction

v_k = Sum taken clockwise of all independent voltage sources in mesh k, with voltage rise treated as positive

$$\begin{bmatrix} \quad & \quad & \quad & \quad & \quad \\ & & & & \\ & & & & \\ & & & & \\ & & & & \end{bmatrix} \begin{bmatrix} i_1 \\ i_2 \\ i_3 \\ i_4 \\ i_5 \end{bmatrix} = \begin{bmatrix} \quad \\ \\ \\ \\ \\ \end{bmatrix}$$



$$\begin{bmatrix} 50+40+80 & -40 & 0 & -80 & 0 \\ -40 & 40+30+10 & -30 & -10 & 0 \\ 0 & -30 & 30+20 & 0 & -20 \\ -80 & -10 & 0 & 10+80 & 0 \\ 0 & 0 & -20 & 0 & 20+60 \end{bmatrix} \begin{bmatrix} i_1 \\ i_2 \\ i_3 \\ i_4 \\ i_5 \end{bmatrix} = \begin{bmatrix} 24 \\ 0 \\ -12 \\ 10 \\ -10 \end{bmatrix}$$

Example 3.6 Use mesh analysis to find the current I_o in the circuit of Fig. 3.20.

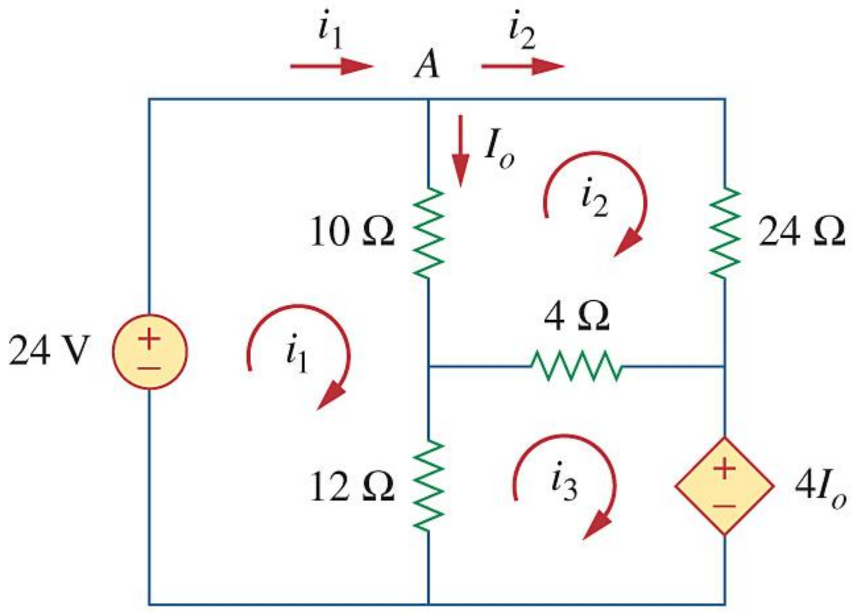


Figure 3.20

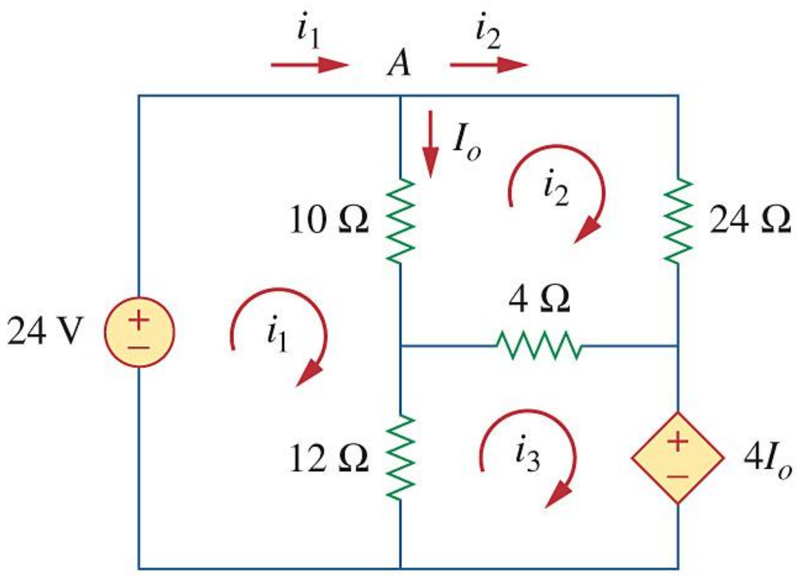


Figure 3.20

By KVL,

Mesh 1: $-24 + 10(i_1 - i_2) + 12(i_1 - i_3) = 0$

Mesh 2: $10(i_2 - i_1) + 24i_2 + 4(i_2 - i_3) = 0$

Mesh 3: $12(i_3 - i_1) + 4(i_3 - i_2) + 4I_o = 0$

Mesh 1: $22i_1 - 10i_2 - 12i_3 = 24$

Mesh 2: $-10\mathrm{i}_1 + 38\mathrm{i}_2 - 4\mathrm{i}_3 = 0$

Mesh 3: $-8\mathrm{i}_1 - 8\mathrm{i}_2 + 16\mathrm{i}_3 = 0$

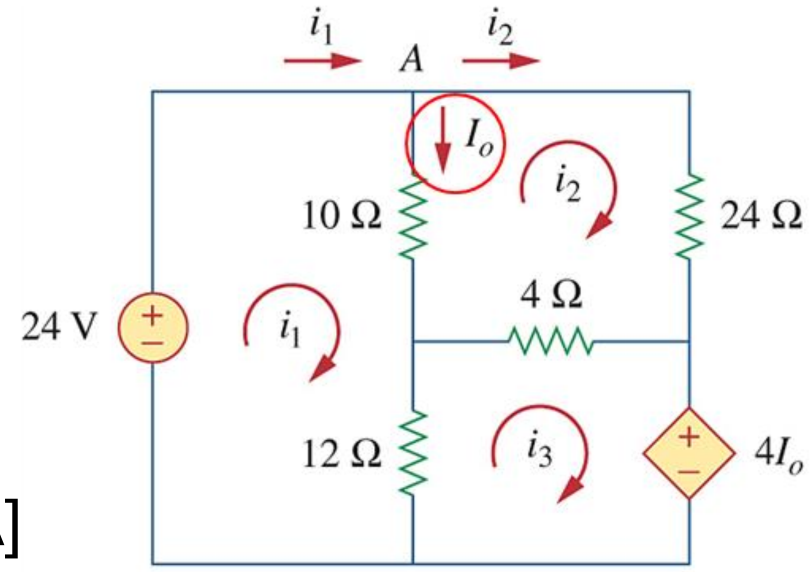
Mesh 1: $22i_1 - 10i_2 - 12i_3 = 24$ / 2

Mesh 2: $-10\mathrm{i}_1 + 38\mathrm{i}_2 - 4\mathrm{i}_3 = 0$ / 2

Mesh 3: $-8\mathrm{i}_1 - 8\mathrm{i}_2 + 16\mathrm{i}_3 = 0$ / 8

Put the equations into the matrix form and use Cramer's rule to solve

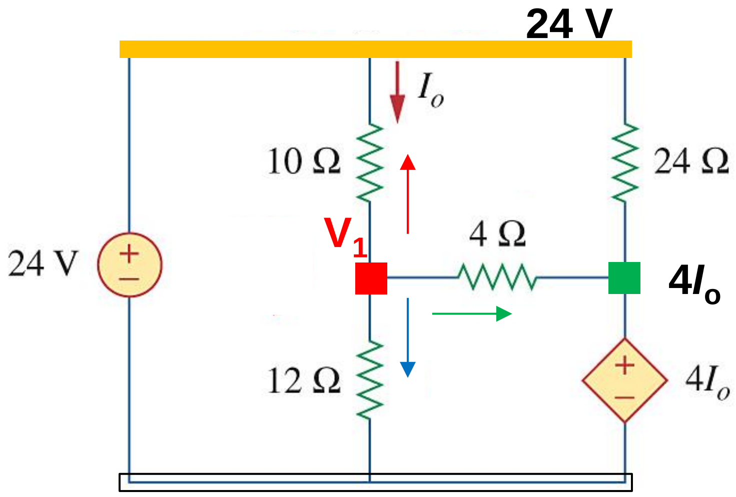
$$\begin{bmatrix} 11 & -5 & -6 \\ -5 & 19 & -2 \\ -1 & -1 & 2 \end{bmatrix} \begin{bmatrix} i_1 \\ i_2 \\ i_3 \end{bmatrix} = \begin{bmatrix} 12 \\ 0 \\ 0 \end{bmatrix}$$



$i_1 = 2.25$ [A]; $i_2 = 0.75$ [A]; $i_3 = 1.5$ [A]

Finally, $\mathbf{I_o = i_1 - i_2} = 2.25 - 0.75 = 1.5$ [A]

Actually, nodal analysis for this circuit is easier.



By KCL at node V_1

$$\frac{V_1 - 24}{10} + \frac{V_1}{12} + \frac{V_1 - 4I_o}{4} = 0$$

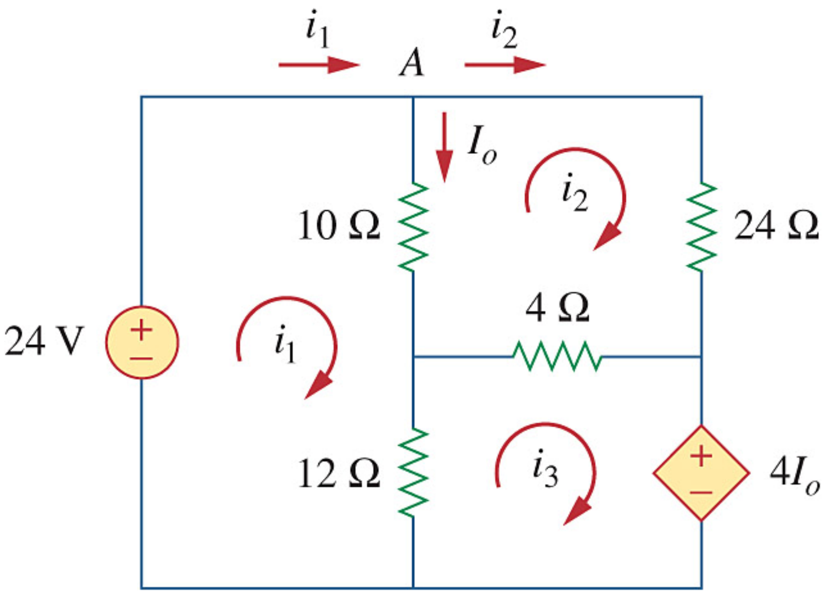
Also, by Ohm's law

$$I_o = \frac{24 - V_1}{10}$$

V_1 = 9 [V]

I_o = (24-9)/10 = 1.5 [A]

Alternatively, by the inspection method

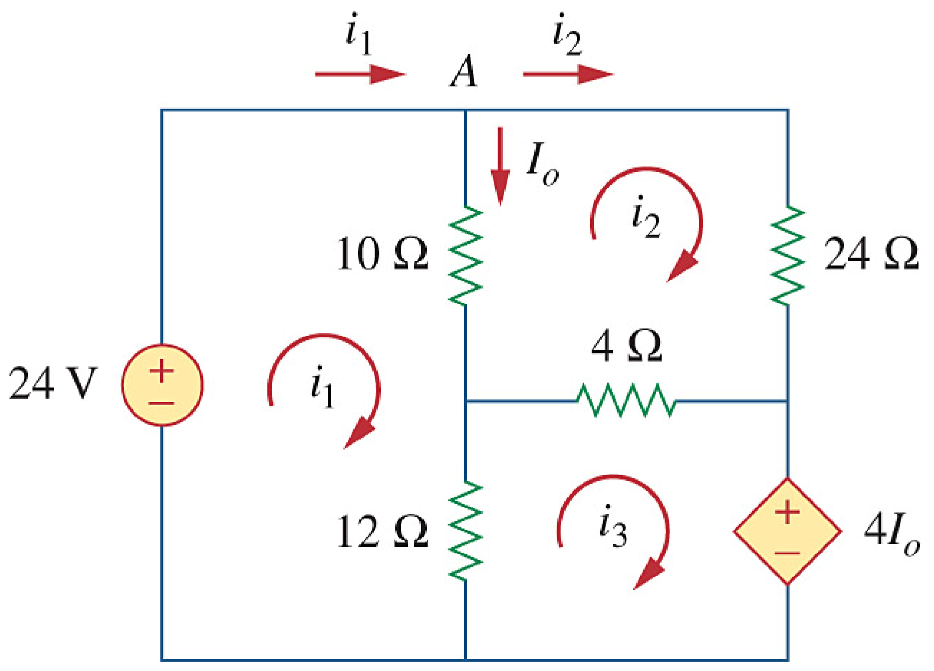


R_{kk} = Sum of the resistances in mesh k
R_{kj} = R_{jk} = Negative of the sum of the resistances in common with meshes k and j, k≠j
i_k = Unknown current for mesh k in the clockwise direction
v_k = Sum taken clockwise of all independent voltage sources in mesh k, with voltage rise treated as positive

$$\begin{bmatrix} & & \\ & & \\ & & \end{bmatrix}\begin{bmatrix} i_1 \\ i_2 \\ i_3 \end{bmatrix} = \begin{bmatrix} \\ \\ \\ \end{bmatrix}$$

$$I_o = i_1 - i_2$$

Alternatively, by the inspection method



$$\begin{bmatrix} 10+12 & -10 & -12 \\ -10 & 10+24+4 & -4 \\ -12 & -4 & 12+4 \end{bmatrix}\begin{bmatrix} i_1 \\ i_2 \\ i_3 \end{bmatrix} = \begin{bmatrix} 24 \\ 0 \\ -4I_o \end{bmatrix}$$

$$I_o = i_1 - i_2$$

$$\begin{bmatrix} 10+12 & -10 & -12 \\ -10 & 10+24+4 & -4 \\ -12 & -4 & 12+4 \end{bmatrix}\begin{bmatrix} i_1 \\ i_2 \\ i_3 \end{bmatrix} = \begin{bmatrix} 24 \\ 0 \\ -4(i_1 - i_2) \end{bmatrix}$$

$$\begin{bmatrix} 10+12 & -10 & -12 \\ -10 & 10+24+4 & -4 \\ -12+4 & -4-4 & 12+4 \end{bmatrix} \begin{bmatrix} i_1 \\ i_2 \\ i_3 \end{bmatrix} = \begin{bmatrix} 24 \\ 0 \\ 0 \end{bmatrix}$$

$$\begin{bmatrix} 22 & -10 & -12 \\ -10 & 38 & -4 \\ -8 & -8 & 16 \end{bmatrix} \begin{bmatrix} i_1 \\ i_2 \\ i_3 \end{bmatrix} = \begin{bmatrix} 24 \\ 0 \\ 0 \end{bmatrix}$$

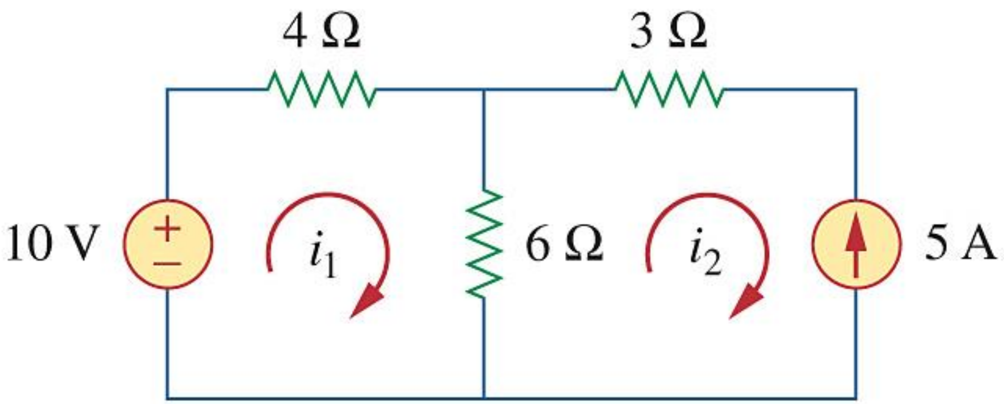
$$\boxed{\begin{bmatrix} 11 & -5 & -6 \\ -5 & 19 & -2 \\ -1 & -1 & 2 \end{bmatrix} \begin{bmatrix} i_1 \\ i_2 \\ i_3 \end{bmatrix} = \begin{bmatrix} 12 \\ 0 \\ 0 \end{bmatrix}}$$

We get the same result

3.5 Mesh Analysis with Current Source

Now, let's apply mesh analysis to circuits containing current sources (dependent or independent).

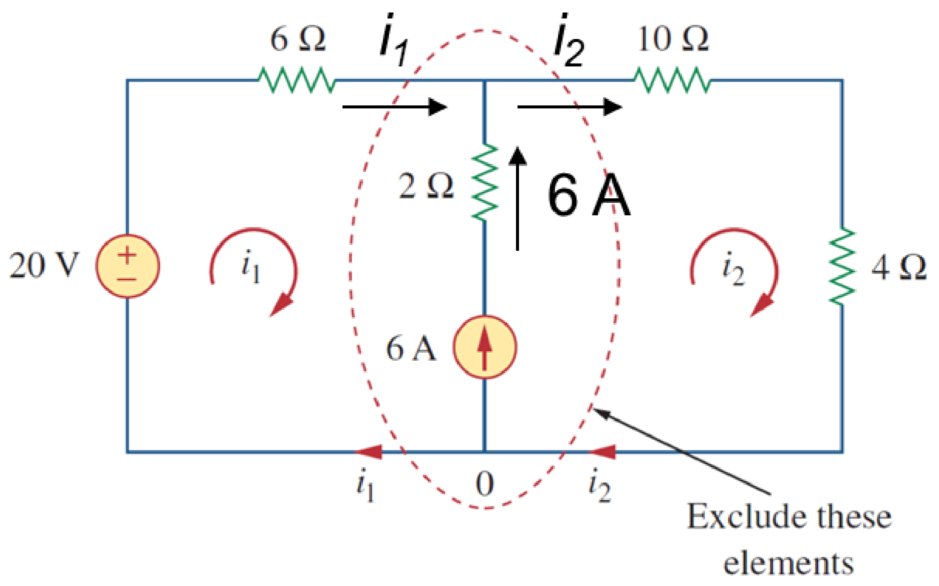
Case 1: A current source exists only in one mesh



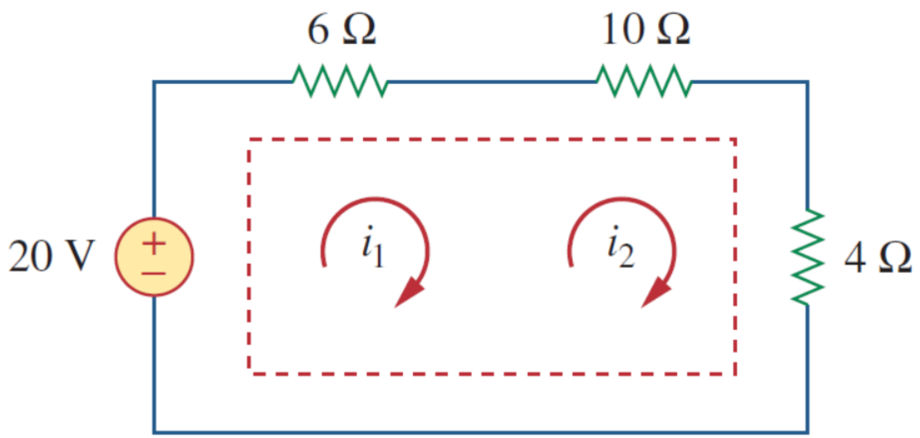
i_2 is simply -5 A, and thus we can reduce one equation

Case 2: A current source exists between two meshes. We create a supermesh by excluding the current source and **any elements connected in series with it**.

supermesh $i_2 = i_1 + 6$



We treat the supermesh differently, because mesh analysis applied KVL, and we do not know the voltage across a current source in advance.

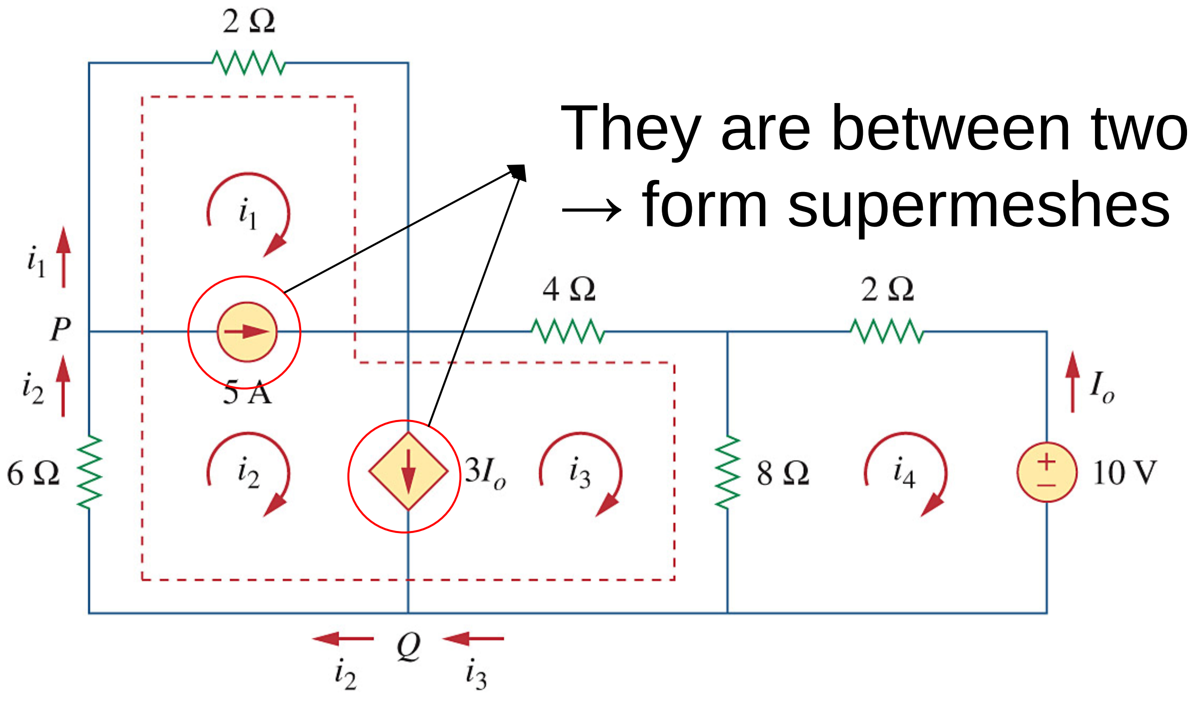


We consider it as one mesh

$-20 + 6i_1 + 10(i_1 + 6) + 4(i_1 + 6) = 0$

Or, $-20 + 6i_1 + 10i_2 + 4i_2 = 0, and\ i_2 = i_1 + 6$

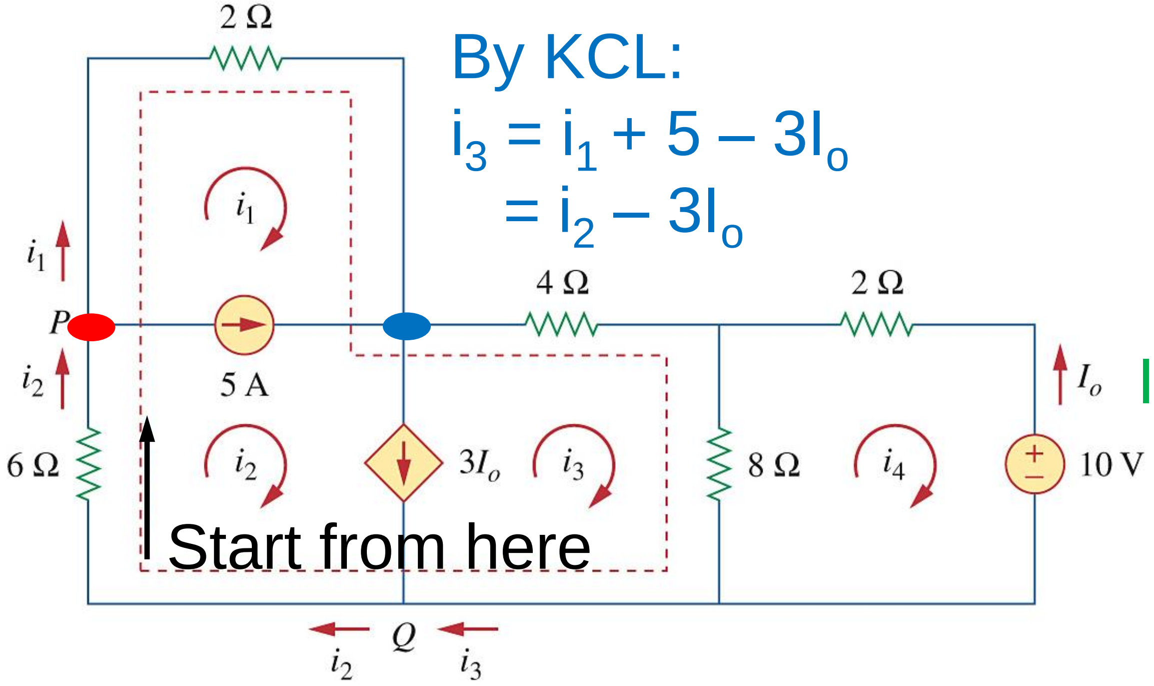
Example 3.7 For the circuit in Fig. 3.24, find i_1 to i_4 using mesh analysis.



They are between two meshes → form supermeshes

Figure 3.24

By KCL:
$i_1 = i_2 - 5$



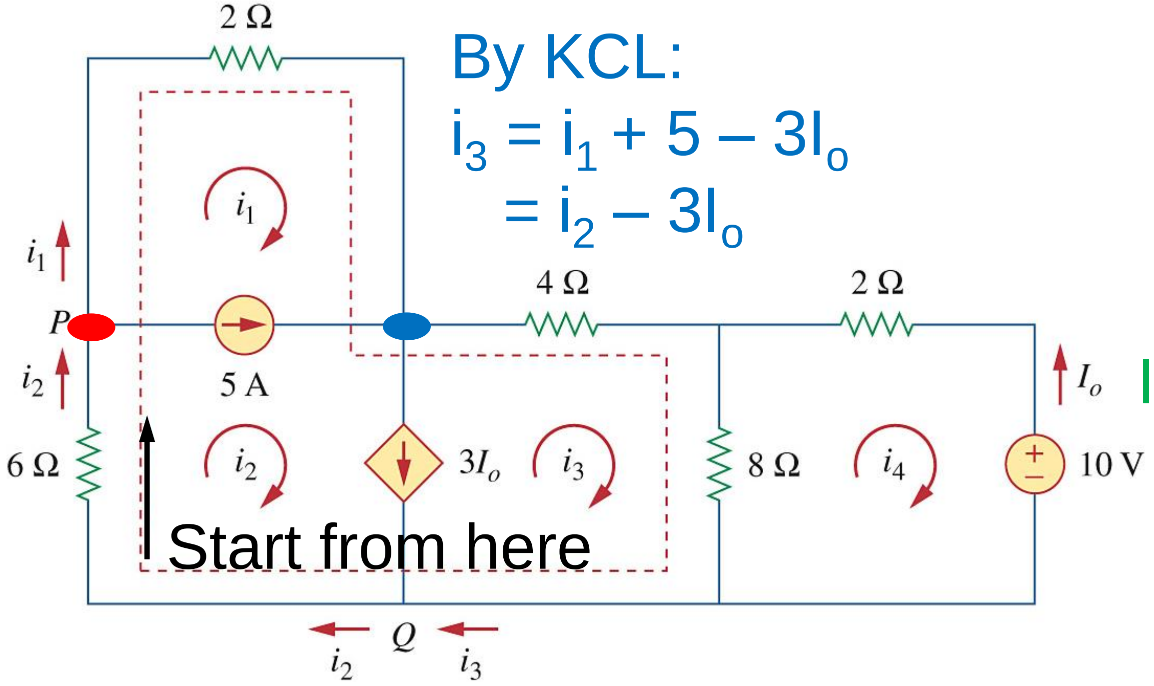
By KCL:
$i_3 = i_1 + 5 - 3I_o$
$= i_2 - 3I_o$

$I_o = -\, i_4$

(1) Supermesh:

$$6i_2 + 2(i_2 - 5) + 4(i_2 - 3I_o) + 8(i_2 - 3I_o - i_4) = 0$$
$$\rightarrow 20i_2 - 12I_o - 24I_o - 8i_4 = 10 \; where \; I_o = -i_4$$
$$\rightarrow 20i_2 - 36(-i_4) - 8i_4 = 10$$
$$\rightarrow 20i_2 + 28i_4 = 10 \rightarrow \mathbf{10}\boldsymbol{i_2} + \mathbf{14}\boldsymbol{i_4} = \mathbf{5}$$

By KCL:
$i_1 = i_2 - 5$



By KCL:
$i_3 = i_1 + 5 - 3I_o$
$= i_2 - 3I_o$

$I_o = - i_4$

(2) Mesh 4:

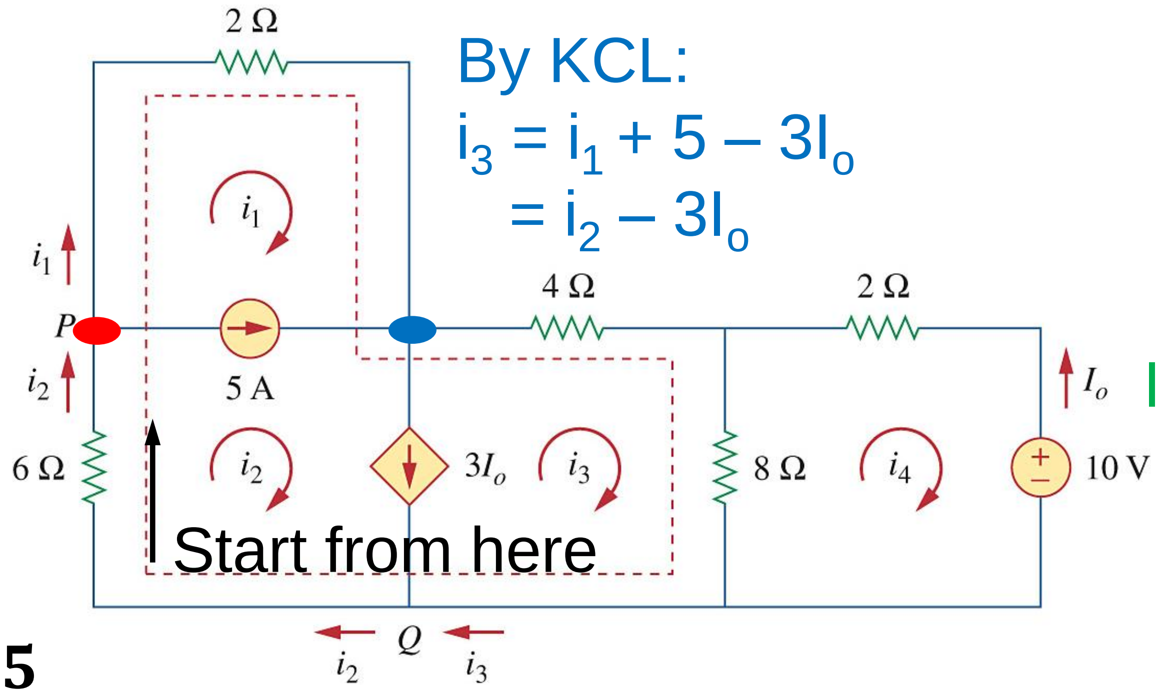
$8\left(i_4 - (i_2 - 3I_o)\right) + 2i_4 + 10 = 0$

$\rightarrow 8i_4 - 8i_2 + 24I_o + 2i_4 + 10 = 0$

$\rightarrow 8i_4 - 8i_2 - 24i_4 + 2i_4 = -10$

$\rightarrow \boldsymbol{-8i_2 - 14i_4 = -10}$

By KCL:
$i_1 = i_2 - 5$



By KCL:
$i_3 = i_1 + 5 - 3I_o$
$= i_2 - 3I_o$

$I_o = - i_4$

$$\begin{array}{rl} & \mathbf{10}\boldsymbol{i_2} + \mathbf{14}\boldsymbol{i_4} = \mathbf{5} \\ + & -\mathbf{8}\boldsymbol{i_2} - \mathbf{14}\boldsymbol{i_4} = -\mathbf{10} \end{array}$$

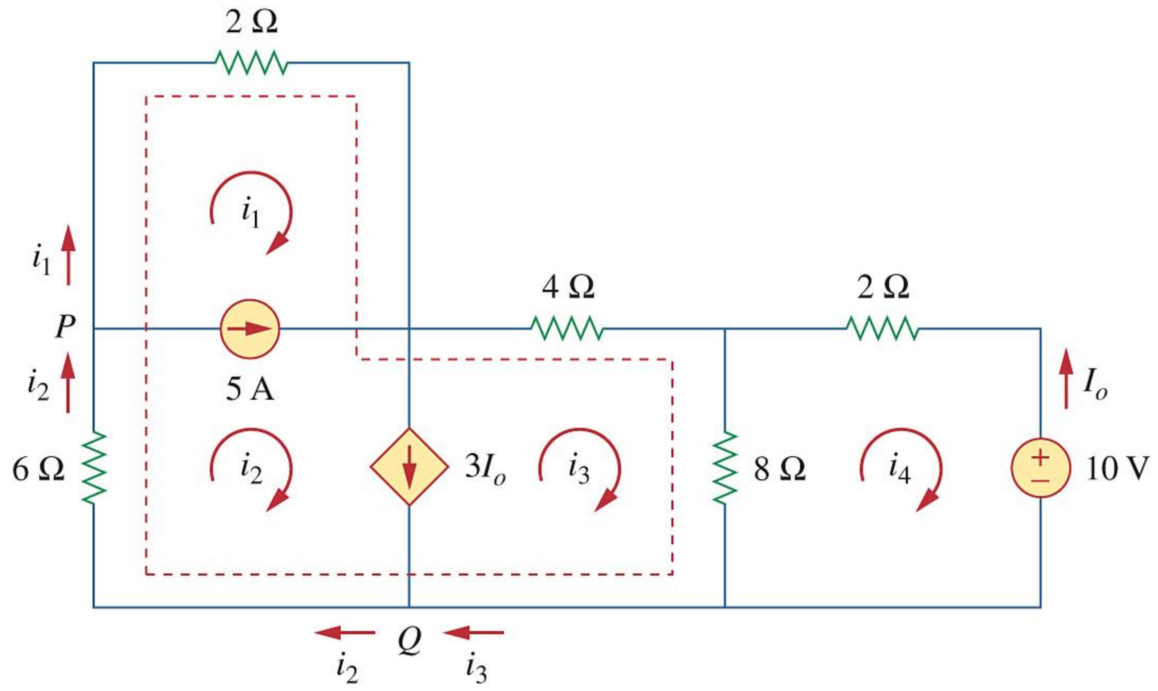
$2i_2 = -5$, thus, $i_2 = -2.5$ [A]

$i_1 = i_2 - 5 = -7.5$ [A]

$i_4 = (5 - 10(-2.5))/14 = 2.14$ [A]

$i_3 = -2.5 - 3(-2.14) = 3.92$ [A]

Alternative way



(1) Supermesh:

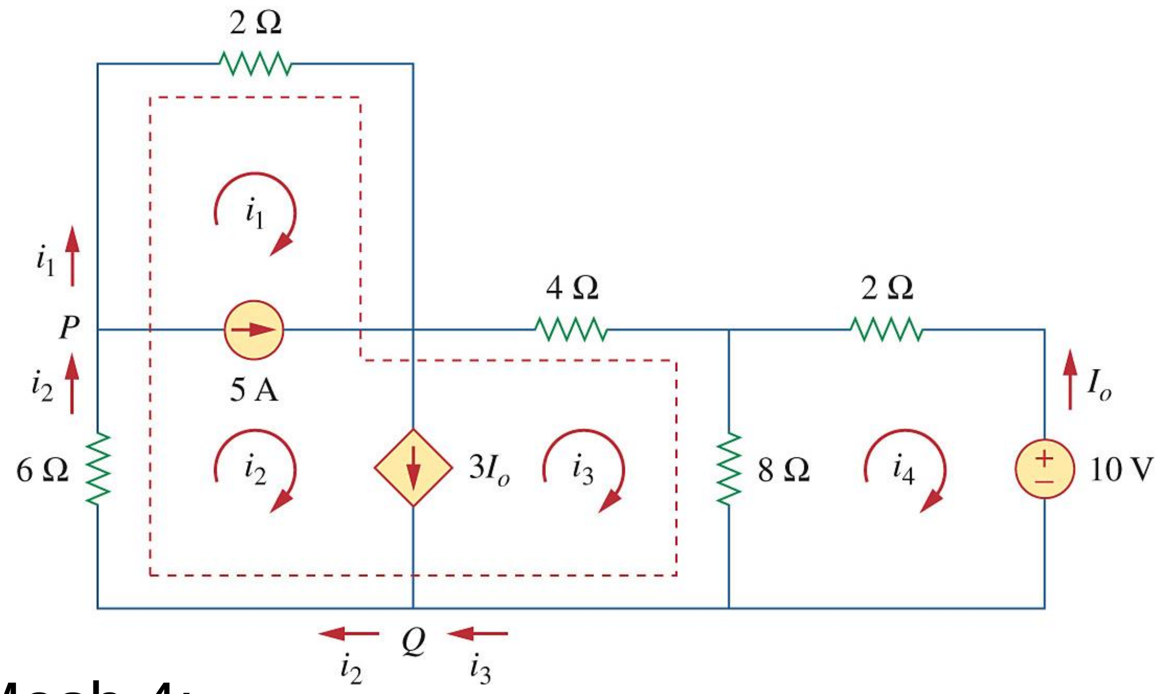
$$6i_2 + 2i_1 + 4i_3 + 8(i_3 - i_4) = 0$$

We have two conditions from the supermesh.

(i) $i_2 - i_1 = 5$

(ii) $i_2 - i_3 = 3I_o$

Alternative way (continue)



(2) Mesh 4:

$$8(i_4 - i_3) + 2i_4 + 10 = 0, where\ I_o = -i_4$$

By reducing parameters, i_1 and i_2, we will get two equations composed of only i_3 and i_4

Alternative way to calculate

We have five conditions:

(i) $6i_2 + 2i_1 + 4i_3 + 8(i_3 - i_4) = 0$

(ii) $i_2 - i_1 = 5$

(iii) $i_2 - i_3 = 3I_o$

(iv) $8(i_4 - i_3) + 2i_4 + 10 = 0$

(v) $I_o = -i_4$

Little bit confusing which parameter to reduce.. then,

(i) $2i_1 + 6i_2 + 12i_3 - 8i_4 = 0$

(ii) $-i_1 + i_2 = 5$

(iii) $i_2 - i_3 + 3i_4 = 0$

(iv) $-8i_3 + 10i_4 = -10$

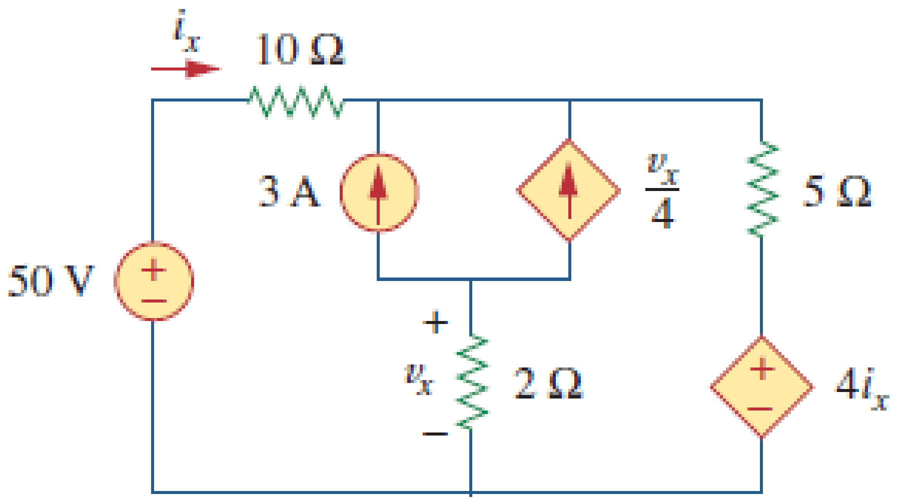
$$\begin{bmatrix} 2 & 6 & 12 & -8 \\ -1 & 1 & 0 & 0 \\ 0 & 1 & -1 & 3 \\ 0 & 0 & -8 & 10 \end{bmatrix} \begin{bmatrix} i_1 \\ i_2 \\ i_3 \\ i_4 \end{bmatrix} = \begin{bmatrix} 0 \\ 5 \\ 0 \\ -10 \end{bmatrix}$$

i_1 = = – 7.5 [A]; i_2 = – 2.5 [A]; i_3 = 3.93 [A]; i_4 = 2.14 [A]

3.7 Nodal *vs* Mesh Analysis

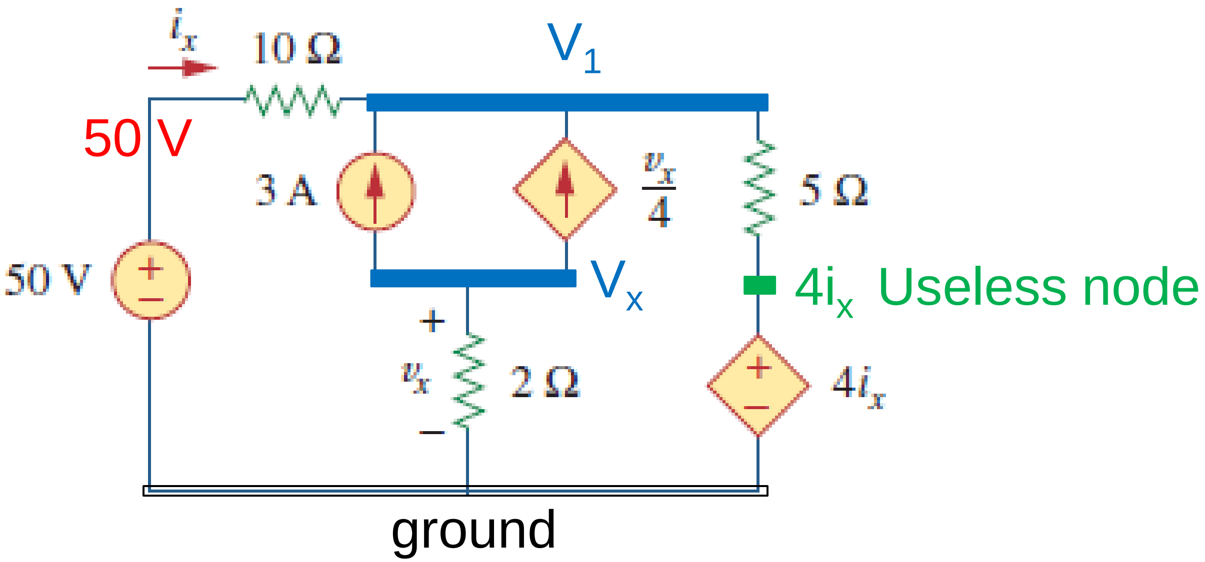
Nodal Analysis	Mesh Analysis
• Contain many parallel-connected elements • Current sources, or supernodes	• Contain many series-connected elements • voltage sources, or super- meshes
• Fewer nodes than meshes	• Fewer meshes than nodes
• If node voltages are required	• If branch or mesh currents are required

Practice Problem



3 meshes vs 5 nodes (4 nodes variables)

Nodal Analysis



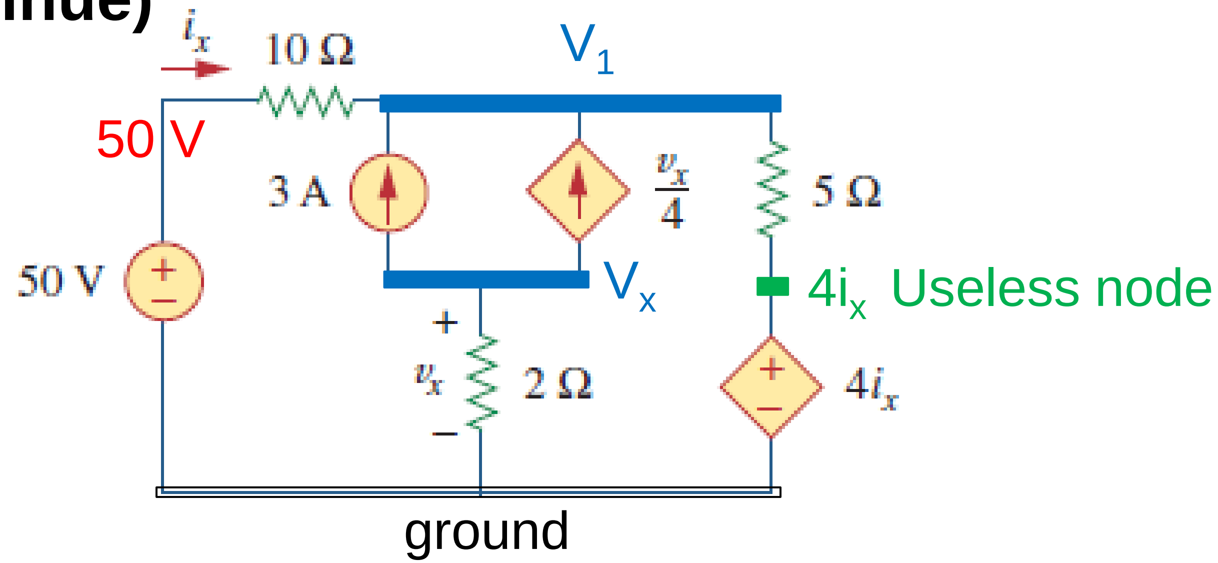
Variables to find themselves are nodal voltages

V_1 node: $\frac{V_1-50}{10} - 3 - \frac{V_x}{4} + \frac{V_1-4i_x}{5} = 0$

V_x node: $3 + \frac{V_x}{4} + \frac{V_x}{2} = 0 \rightarrow V_x = -4\ [V]$

$$i_x = \frac{50 - V_1}{10}$$

Nodal Analysis (Continue)



Variables to find themselves are nodal voltages

$V_x = -4$ V

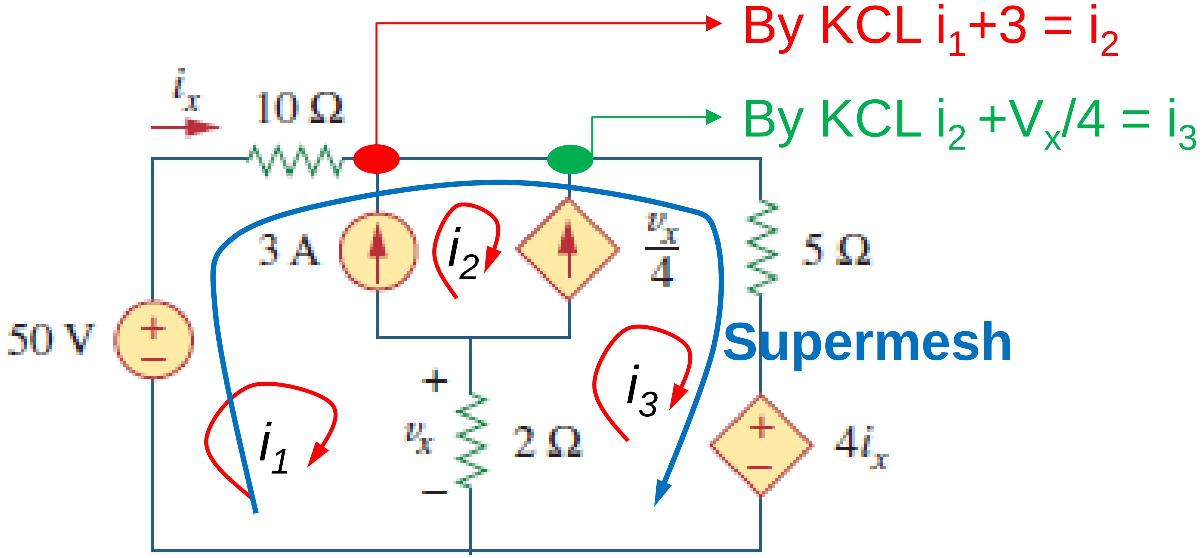
$$\frac{V_1 - 50}{10} - 3 - \frac{V_x}{4} + \frac{V_1 - 4i_x}{5} = 0$$

$$i_x = \frac{50 - V_1}{10}$$

These two equations give V_1 = 28.95 [V]

Then, i_x = 2.105 [A]

Mesh Analysis



Supermesh: $-50 + 10i_1 + 5i_3 + 4i_x = 0$

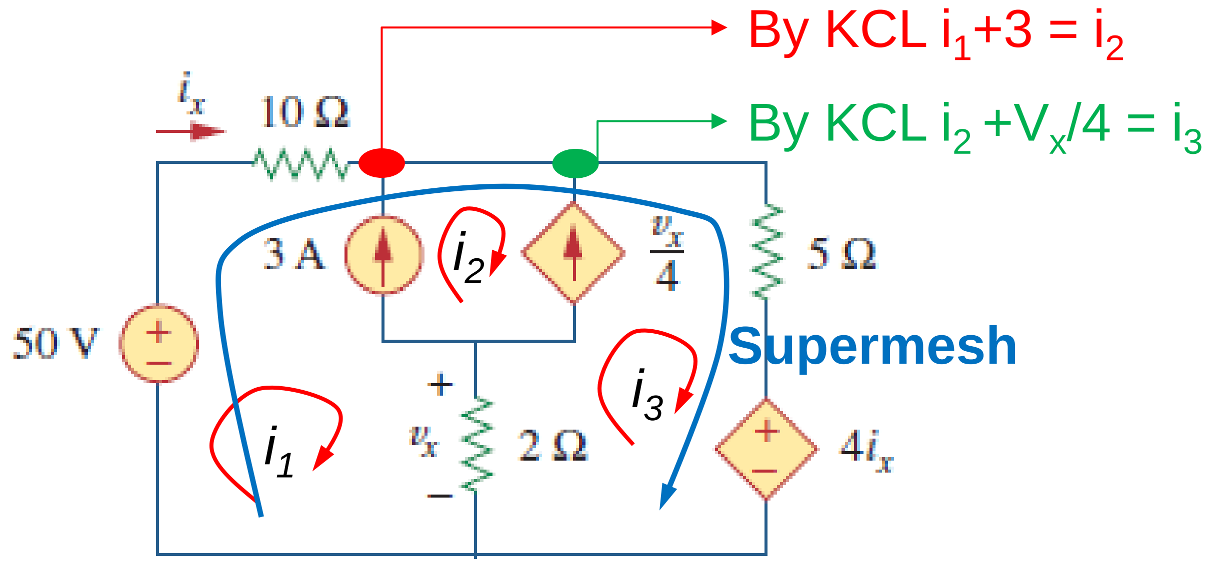
Because i_x = i_1, and $i_3 = i_2 + {}^{V_x}/_4 = i_1 + 3 + {}^{V_x}/_4$

The equation above becomes

$$-50 + 10i_1 + 5\left(i_1 + 3 + {}^{V_x}/_4\right) + 4i_1 = 0$$

$V_x = 2(i_1 - i_3) = 2(i_1 - (i_1 + 3 + {}^{V_x}/_4)) \rightarrow V_x = -4\text{ V}$

Mesh Analysis (Continue)



$$-50 + 10i_1 + 5\left(i_1 + 3 + {}^{V_x}/_{4}\right) + 4i_1 = 0$$

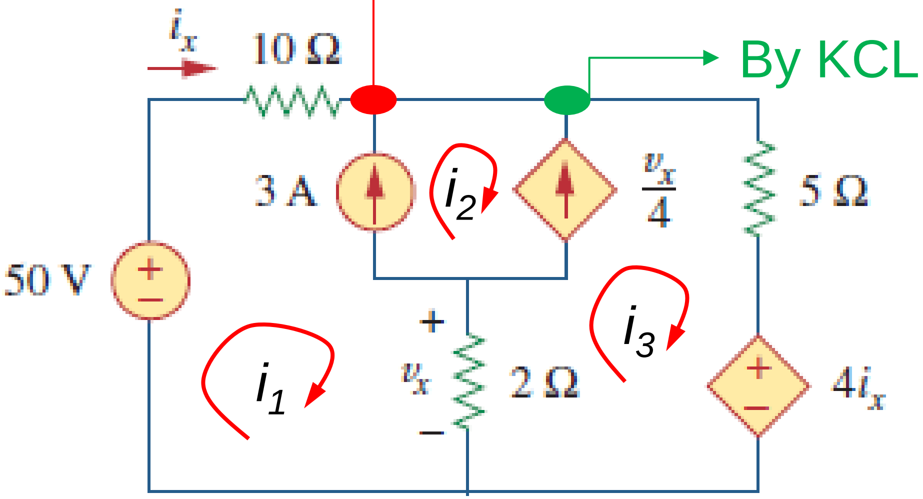
and $V_x = -4$ V

Therefore, $i_1 = i_x = 2.105$ [A]

Mesh Analysis (alternative)

By KCL $i_1 + 3 = i_2$

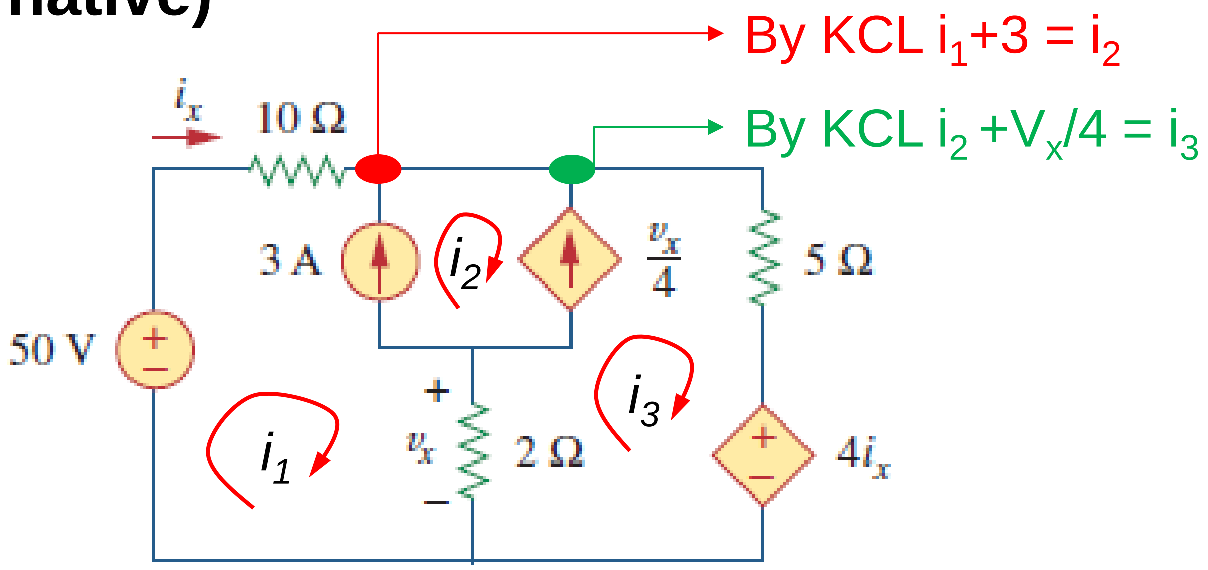
By KCL $i_2 + V_x/4 = i_3$



Supermesh: $-50 + 10i_1 + 5i_3 + 4ix = 0$
Because $i_X = i_1$,
Above equation becomes $14i_1 + 5i_3 = 50$

Thus, we have three conditions
(1) $14i_1 + 0i_2 + 5i_3 = 50$
(2) $i_1 - i_2 = -3$
(3) $i_2 - i_3 = -V_x/4$, and $V_x = 2(i_1 - i_3) \rightarrow 2i_1 + 4i_2 - 6i_3 = 0$

Mesh Analysis (alternative)



(1) $14i_1 + 0i_2 + 5i_3 = 50$

(2) $i_1 - i_2 = -3$

(3) $i_1 + 2i_2 - 3i_3 = 0$

$$\begin{bmatrix} 14 & 0 & 5 \\ 1 & -1 & 0 \\ 1 & 2 & -3 \end{bmatrix} \begin{bmatrix} i_1 \\ i_2 \\ i_3 \end{bmatrix} = \begin{bmatrix} 50 \\ -3 \\ 0 \end{bmatrix}$$

Solving the matrix, then we will get the same result.

Notes

- Nodal analysis by Inspection can only be applied to the case **without voltage sources** (i.e., without supernodes)

- Mesh analysis by Inspection can only be applied to the case **without current sources** (i.e., without supermeshes)