ECE2150J Introduction to Circuits

Chapter 2 Basic Laws

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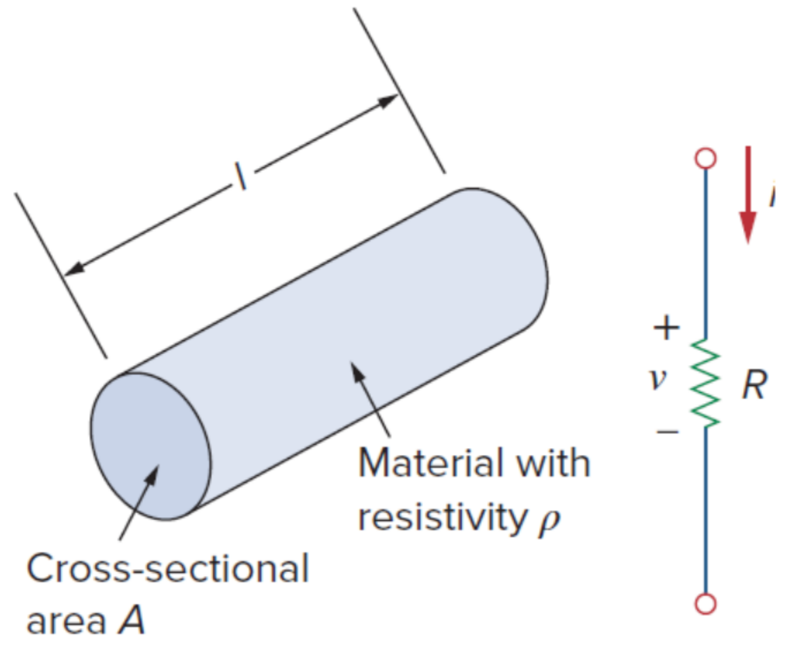
2.1 Introduction

- In this chapter, we study some fundamental laws that govern electric circuits, known as **Ohm's law** and **Kirchhoff's laws**, and discuss some techniques commonly applied in circuit analysis.

2.2 Ohm's Law

- Materials in general have a current-resisting behavior. This **physical property** is known as **resistance** and is represented by the symbol ***R***.
- The **element** used to model the current-resisting behavior of a material is the **resistor**.



Resistance $R = \rho \frac{l}{A}$

The resistance of any material depends on a uniform cross-sectional area A and its length l, where ρ is known as the *resistivity* of the material in ohm-meters.

Ohm’s law states that the **voltage *v* across a resistor is directly proportional to the current *i*** flowing through the resistor.

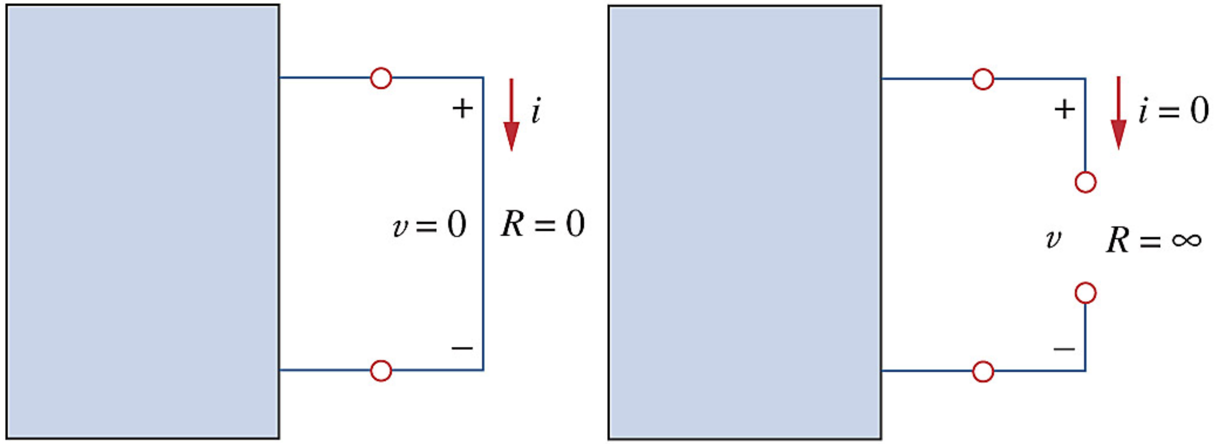
$$V = IR, or\ R = \frac{v}{i}, \text{thus } 1\ \Omega = 1\ \text{V/A}$$

The resistance R of an element denotes its ability to resist the flow of electric current; it is measured in **ohms (Ω)**.

Resistors with extreme values

(i) A short circuit: a circuit element with **resistance approaching zero**.

(ii) An open circuit: a circuit element with **resistance approaching infinity**.



$v = 0$
i can be any value

$i = 0$
v can be any value

A resistor is either fixed or variable.

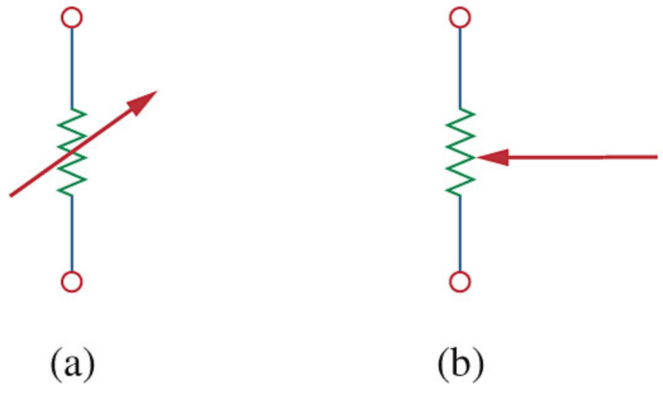
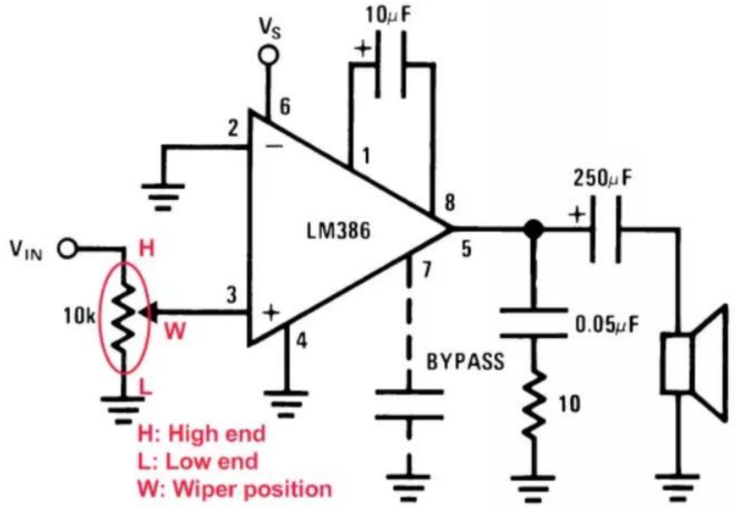


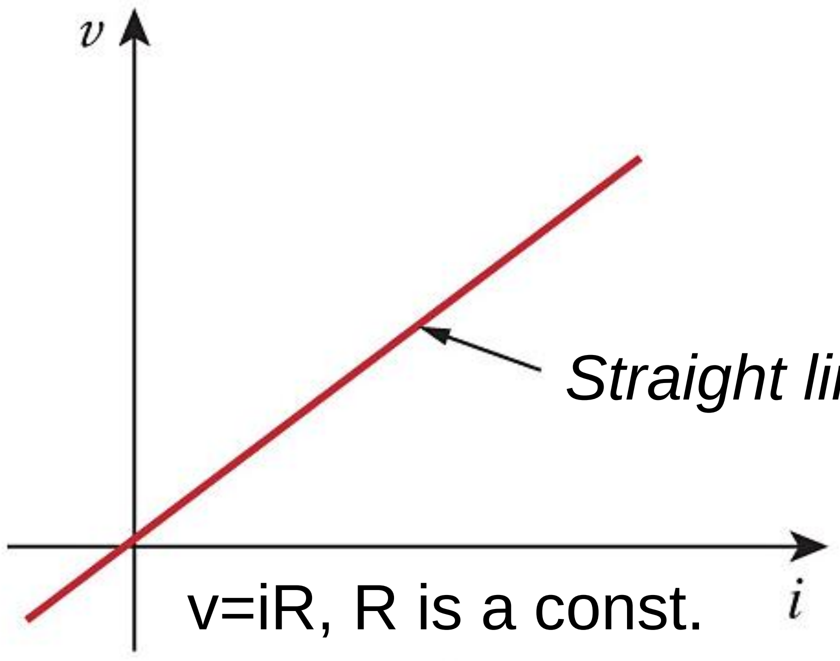
Figure 2.4 Circuit symbol for (a) a variable resistor in general, (b) a potentiometer.



e.g. the variable resistor (potentiometer) is used to control volumes in sound system.

- Surprisingly, **not all resistors obey Ohm’s law**.
- A resistor that obeys Ohm’s law is known as **a linear resistor**, i.e. a constant resistance.

iv curve of a linear resistor

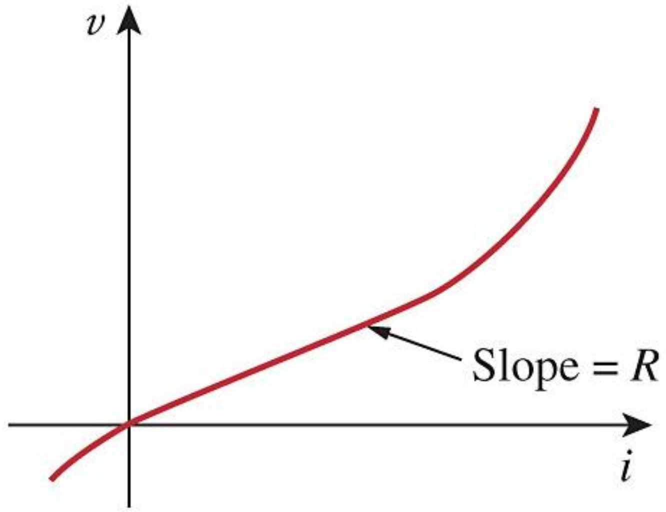


Straight line with slope R

v=iR, R is a const.

- **A nonlinear resistor** does not obey Ohm's law. Its resistance varies with current, e.g. a diode.

iv curve of a nonlinear resistor



v=i*R(i)

Conductance

Conductance (G): how well an element will conduct electric current. The unit of conductance is the *mho* (℧), or siemens (S). It is reciprocal of resistance *R.*

$$G = \frac{1}{R} = \frac{i}{v}, 1\,S = 1\,℧ = 1\,A/V$$

Ernst Werner Siemens, **von Siemens** was a German inventor and industrialist. Siemens' name has been adopted as the SI unit of electrical conductance, the siemens. He was also the founder of the electrical and telecommunications company Siemens.

The power dissipated by a resistor can be expressed in terms of *R* or *G*.

$$p = vi = i^2 R = \frac{v^2}{R}$$

$$p = vi = \frac{i^2}{G} = v^2 G$$

*Please note

1. The power dissipated in a resistor is a nonlinear function of either current or voltage.
2. *R* and *G > 0,* the power in a resistor is **always positive** → a resistor always absorbs power from the circuit (a passive element).

A resistor with high resistance → the more energy is wasted. Then, **high resistance = Bad**?

In an old-style light bulb, for example, electricity is made to flow through an extremely thin piece of wire called a filament. The wire is so thin that the electricity really has to fight to get through it. That makes the wire extremely hot, which, in fact, makes that it gives off light.

2.3 Nodes, Branches, and Loops

- A circuit is a *network, consisting of branches, nodes, and loops.*

(i) A *branch* represents a single element, i.e. any two-terminal element.

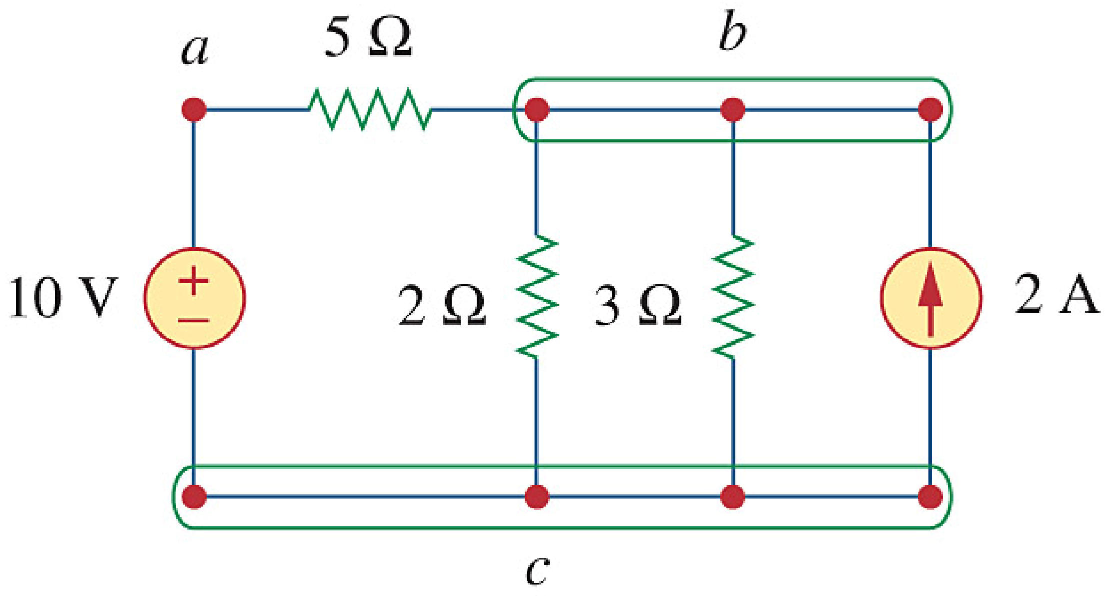


Figure 2.10 Nodes, branches, and loops.

(ii) A *node* is the point of connection between two or more branches.

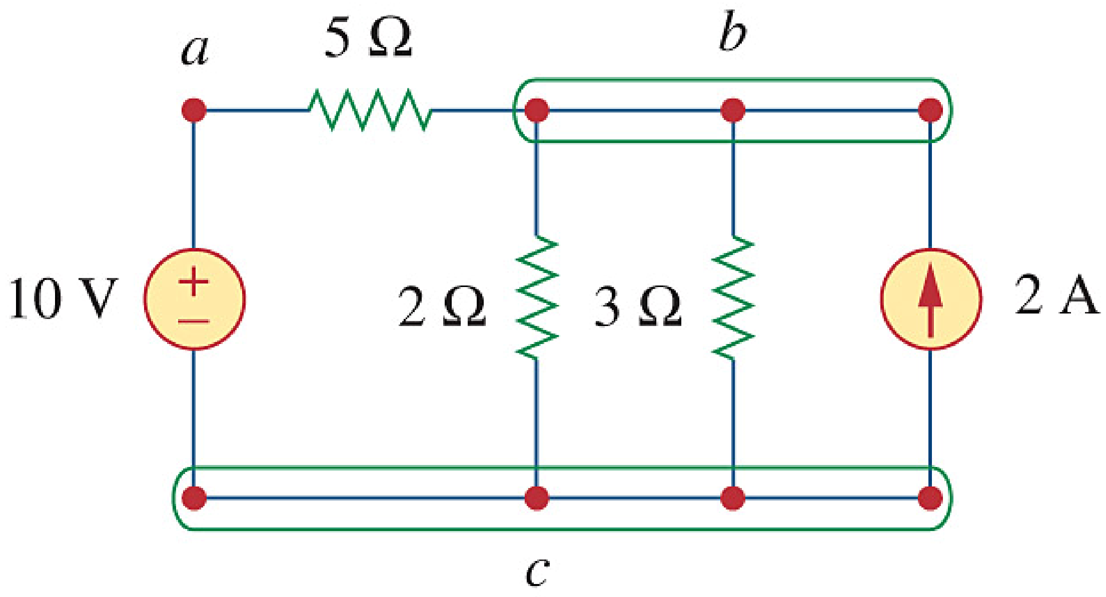


Figure 2.10 Nodes, branches, and loops.

*Node B is connected perfectly by conducting wires and therefore constitute **a single point**. The same for Node C.

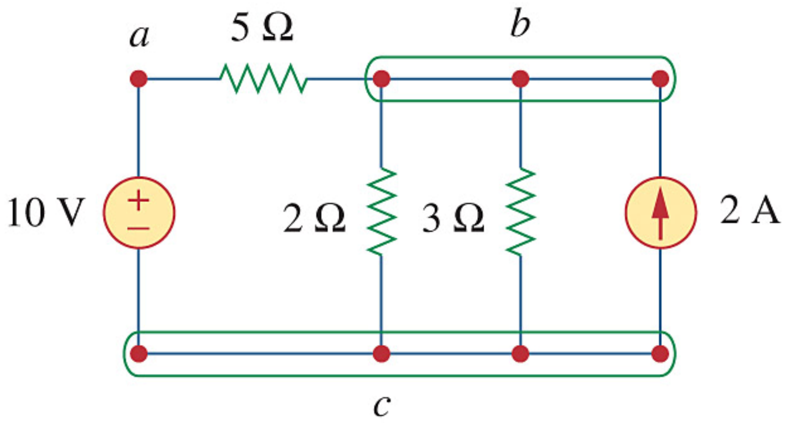


Figure 2.10 Nodes, branches, and loops.

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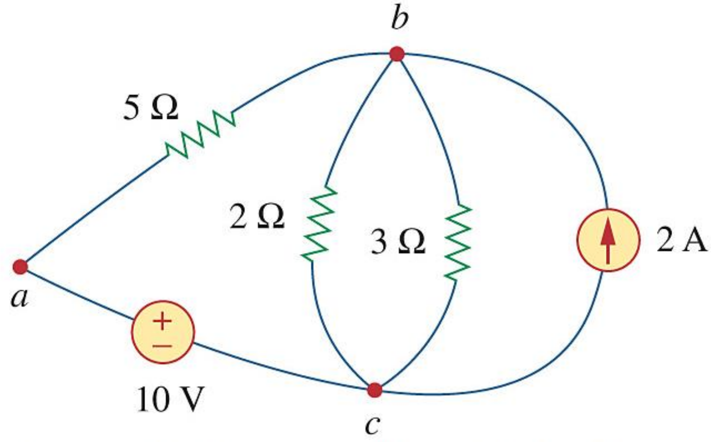


Figure 2.11 The circuit of Fig. 2.10 is redrawn.

(iii) A *loop* is any closed path in a circuit.

- *Independent loop:* the loop contains at least one branch which is not a part of any other independent loop.
- A *mesh* is a loop that does not enclose any other loops. (i.e., smallest loop)

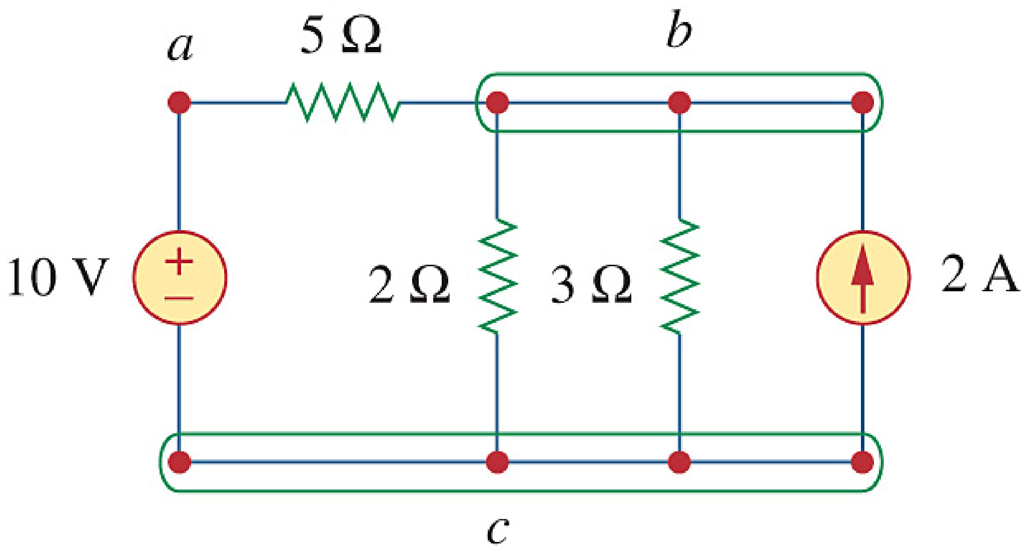


Figure 2.10 Nodes, branches, and loops.

- A network with b branches, n nodes, and l independent loops satisfies the fundamental theorem of network topology:

$$b \text{ (branches)} = l \text{ (independent loop)} + n \text{ (nodes)} - 1$$

Series and Parallel

- Two or more elements are in series if they exclusively share a single node and consequently carry the same current.
- Two or more elements are in parallel if they are connected to the same two nodes and consequently have the same voltage.

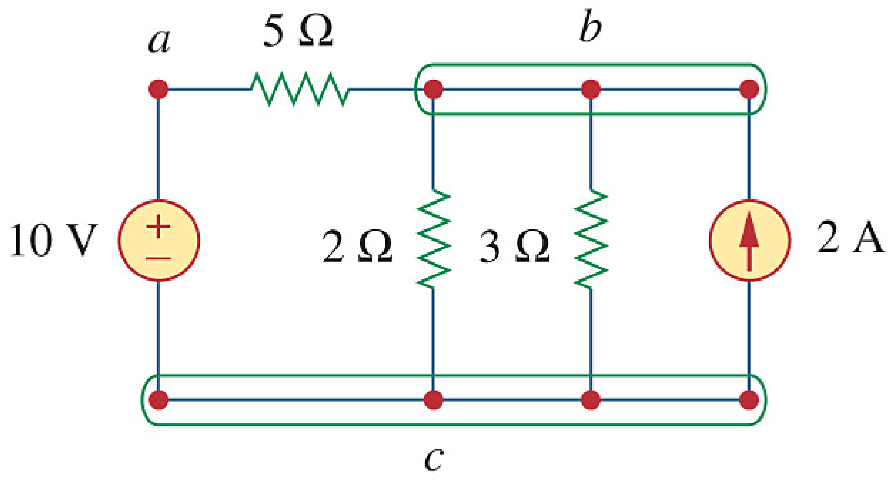


Figure 2.10 Nodes, branches, and loops.

Example 2.4 Determine the number of branches and nodes in the circuit shown in Fig. 2.12. Identity which elements are in series and which are in parallel.

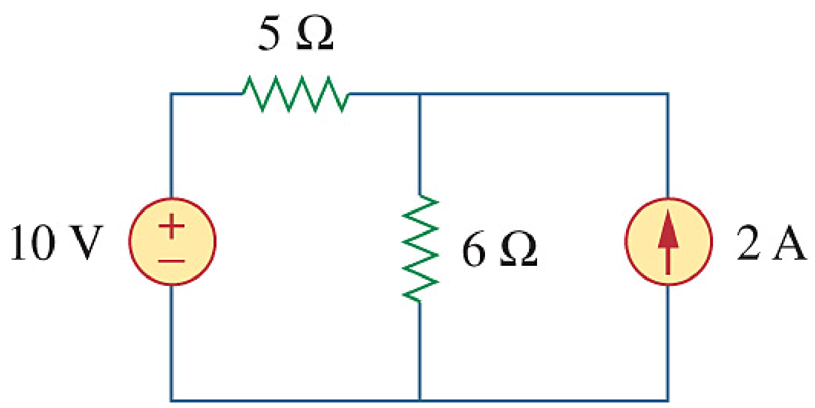


Figure 2.12

Solution :

Four branches and three nodes are identified

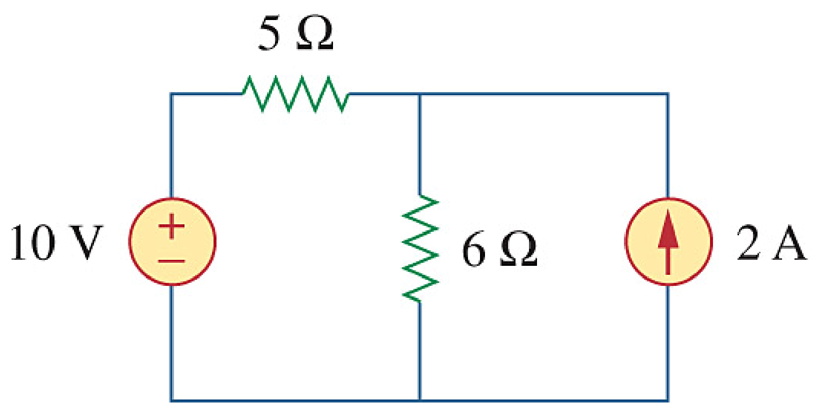


Figure 2.12

2.4 Kirchhoff’s Laws

- The Ohm’s law coupled with **Kirchhoff’s two laws** is a powerful set of tools for analyzing a large variety of electric circuits.

- **TWO Kirchhoff’s** laws

  (i) Kirchhoff’s current law, in short, KCL

  (ii) Kirchhoff’s voltage law, in short, KVL

Kirchhoff's current law (KCL)

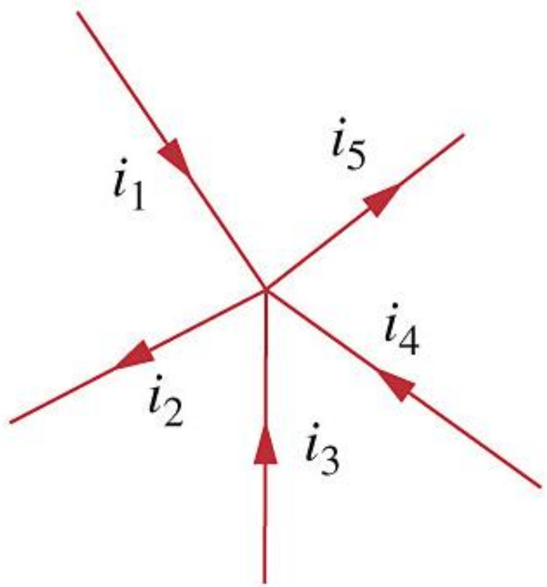
- Based on the law of conservation of charge.
- The algebraic sum of currents entering a node (or a closed boundary) is zero.
- The sum of the currents entering a node = the sum of the currents leaving the node

$$\sum_{n=1}^{N} i_n = 0$$

where N is the number of branches connected to the node and i_n is the nth current entering (or leaving) the node.

KCL at Node

(i) The algebraic sum of currents entering a node (or a closed boundary) is zero.
(ii) The sum of the currents entering a node = the sum of the currents leaving the node

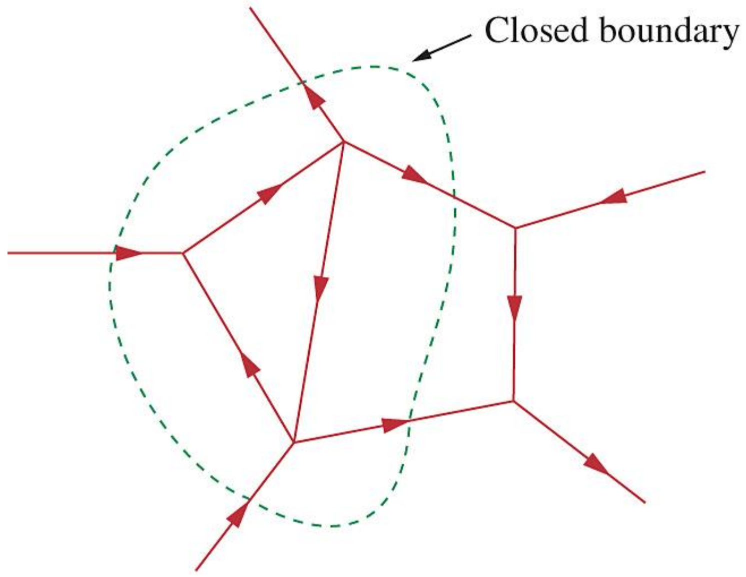


Statement (i): $i_1+(-i_2)+i_3+i_4+(-i_5) = 0$
Statement (ii): $i_1+i_3+i_4 = i_2+i_5$

Equations, upper and lower, are equal.

KCL in Closed Boundary

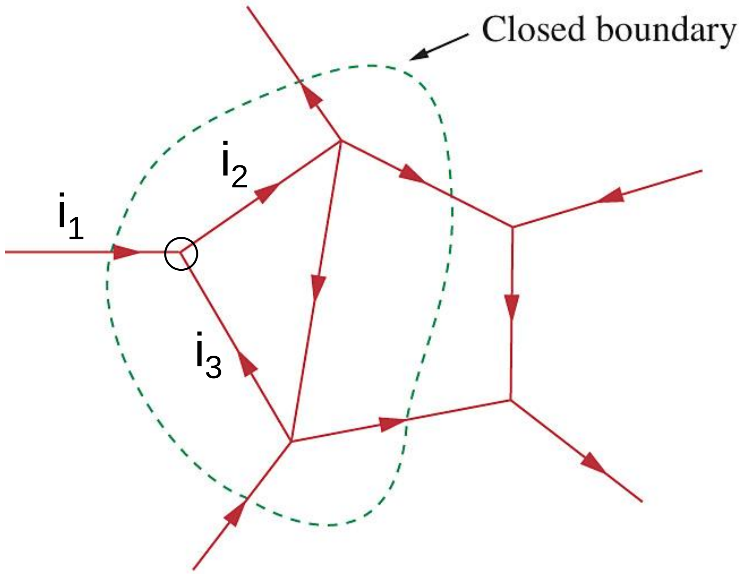
KCL also applies to a closed boundary.



The total current entering the closed surface is equal to the total current leaving the surface.

KCL in Closed Boundary

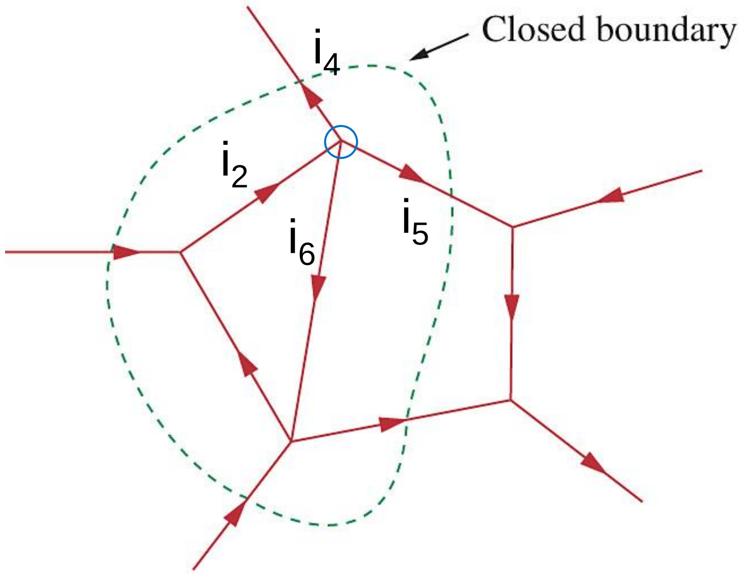
KCL also applies to a closed boundary.



Node 1:
$i_1 + i_3 = i_2$

KCL in Closed Boundary

KCL also applies to a closed boundary.

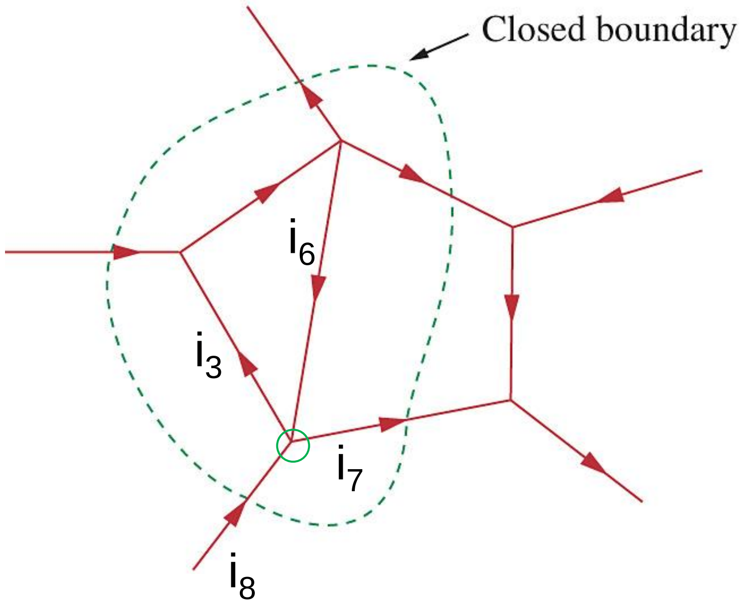


Node 2:

$i_2 = i_4 + i_5 + i_6$

KCL in Closed Boundary

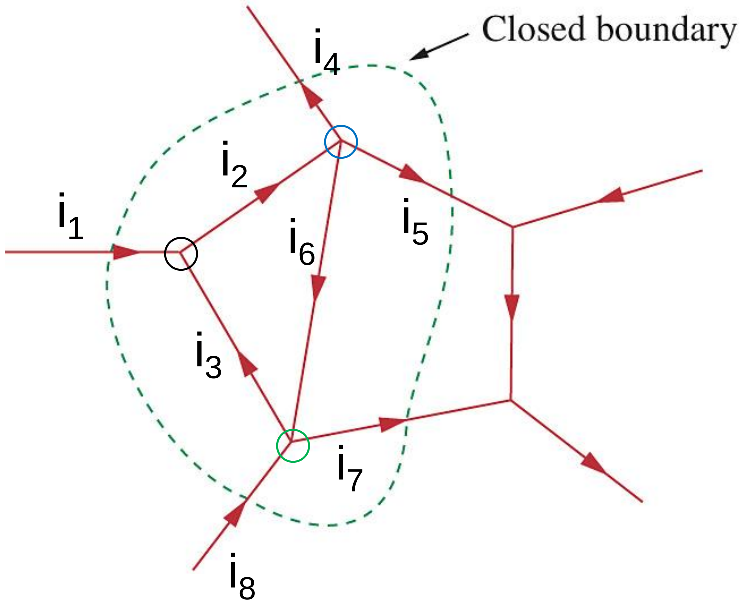
KCL also applies to a closed boundary.



Node 3:
$i_8 + i_6 = i_3 + i_7$

KCL in Closed Boundary

KCL also applies to a closed boundary.



Node 1:
$i_1 + i_3 = i_2$

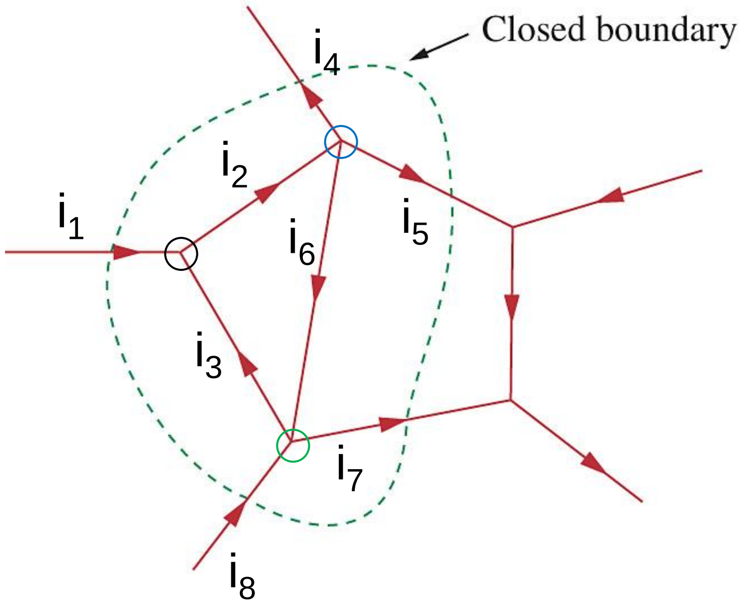
Node 2:
$i_2 = i_4 + i_5 + i_6$

Node 3:
$i_8 + i_6 = i_3 + i_7$

Node 1 + 2 + 3
$i_1 + i_2 + i_3 + i_8 + i_6 = i_2 + i_4 + i_5 + i_6 + i_3 + i_7$

KCL in Closed Boundary

KCL also applies to a closed boundary.



Node 1:
$i_1 + i_3 = i_2$

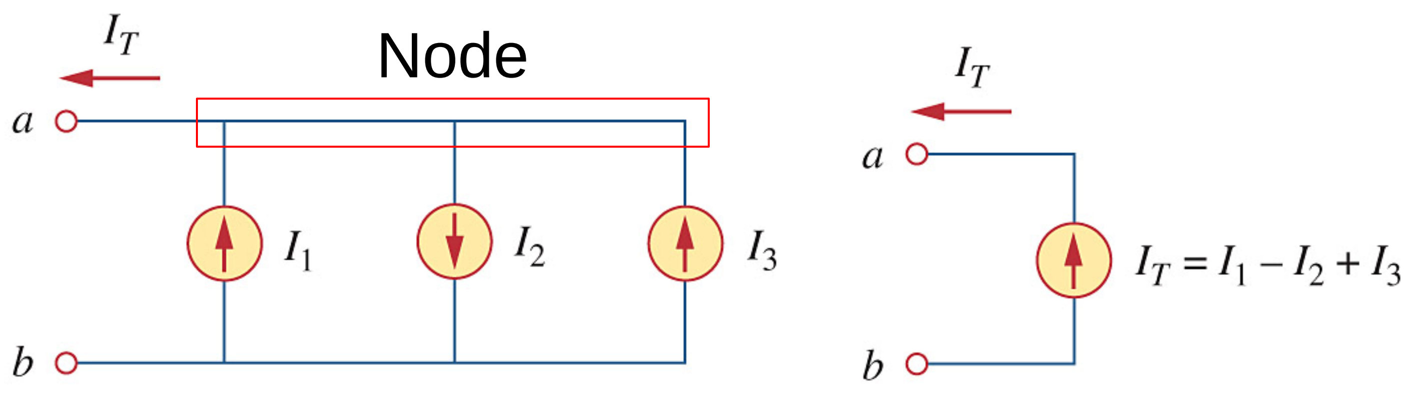
Node 2:
$i_2 = i_4 + i_5 + i_6$

Node 3:
$i_8 + i_6 = i_3 + i_7$

Node 1 + 2 + 3
$i_1 + i_2 + i_3 + i_8 + i_6 = i_2 + i_4 + i_5 + i_6 + i_3 + i_7$
$\rightarrow$ **$i_1 + i_8 = i_4 + i_5 + i_7$**
Current entering the closed boundary = current leaving the boundary

KCL Applications

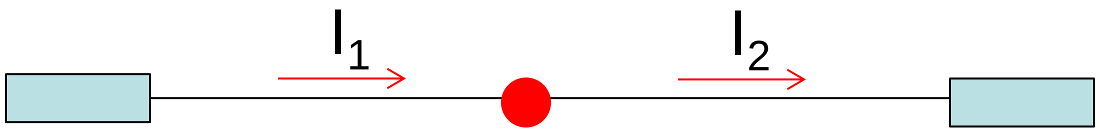
Combining current sources in parallel. The combined current is the algebraic sum of the current supplied by the individual sources.



The algebraic sum of currents entering a node is zero.

$\rightarrow$ $-I_T+I_1+(-I_2)+I_3 = 0$, and thus, $\mathbf{I_T = I_1+(-I_2)+I_3}$

- A circuit cannot contain two different currents, $\mathbf{I_1}$ **and** $\mathbf{I_2}$, in a series circuit, i.e. $I_1 = I_2$; otherwise, KCL will be violated.



MUST be $\mathbf{I_1 = I_2}$

Kirchhoff’s voltage law (KVL)

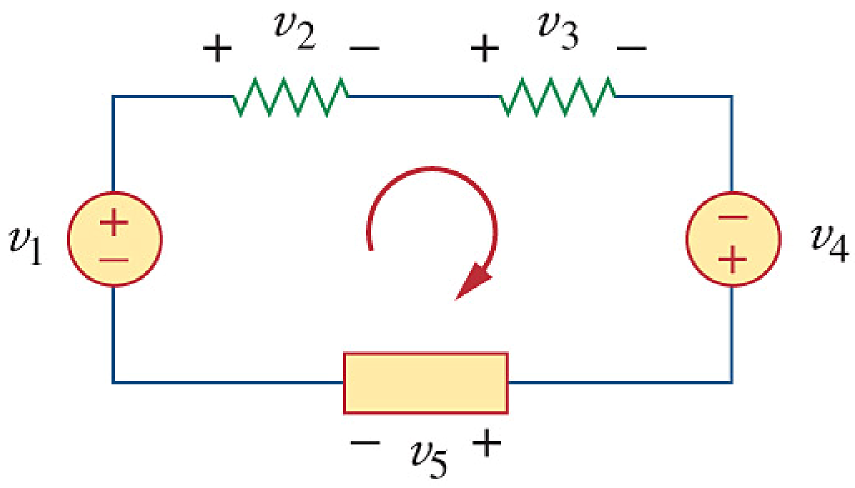
- Based on the principle of conservation of energy
- The algebraic sum of all voltages around a closed path (or loop) is zero.
- The sum of voltage drops = the sum of voltage rises.

$$\sum_{m=1}^{M} v_m = 0$$

where *M* is the number of voltages in the loop (or the number of branches in the loop) and is the *m*th voltage.

KVL Example

We can start with any branch and go around the loop either clockwise or counterclockwise

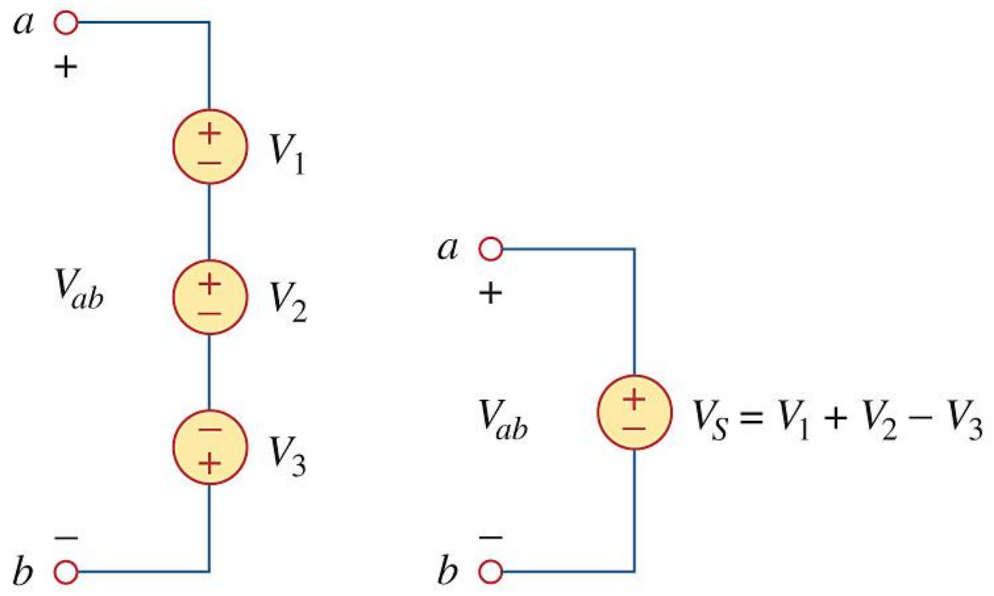


The algebraic sum of all voltages around a closed path (or loop) is zero.

$\rightarrow -v_1+v_2+v_3-v_4+v_5 = 0$

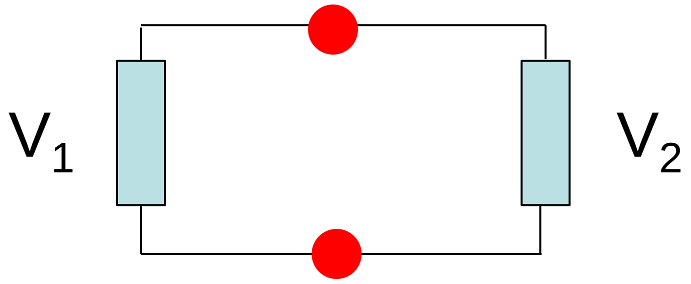
KCL Applications

Combining voltage sources in series. The combined voltage is the algebraic sum of the voltages supplied by the individual sources.



The algebraic sum of all voltages: $-V_{ab}+V_1+V_2+(-V_3) = 0$, and therefore, $V_{ab} = V_1+V_2+(-V_3)$

- A circuit cannot contain two different voltages, ***V_1* and *V_2***, in a parallel circuit, i.e. $V_1 = V_2$; otherwise, KVL will be violated.



MUST be **$V_1 = V_2$**

Practice Problem 2.5 Find v_1 and v_2 in the circuit of Fig. 2.22.

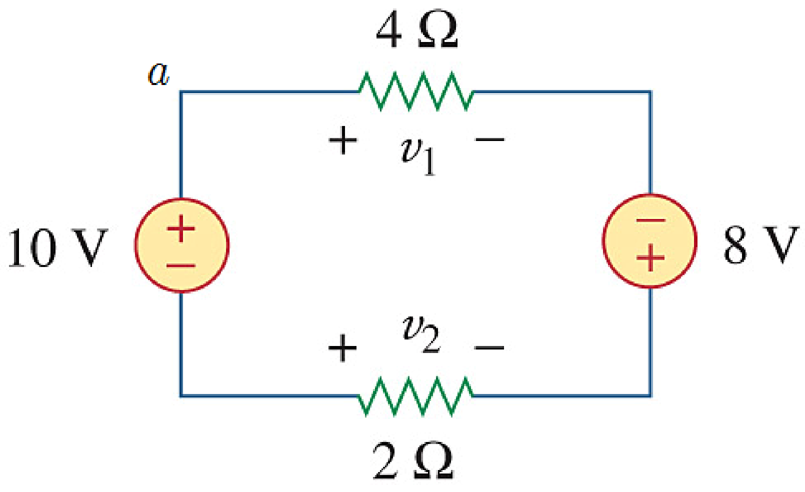


Figure 2.22

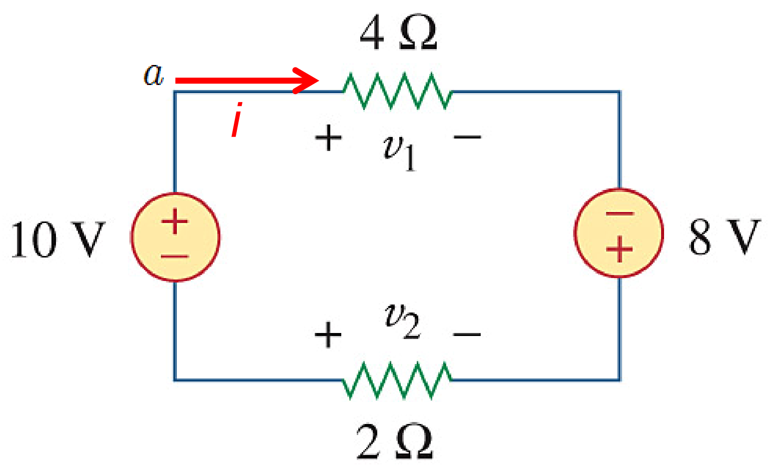


Figure 2.22

We set a current i flowing in the circuit, then, **by KVL**
-10 + 4i -8 +2i = 0, which gives the value i = 3 A

By ohm's law,
V_1 = 4i and V_2 = -2i

Thus, V_1 = 12 V
V_2 = -6 V

Example 2.6 Determine v_o and i in the circuit shown in Fig. 2.23(a).

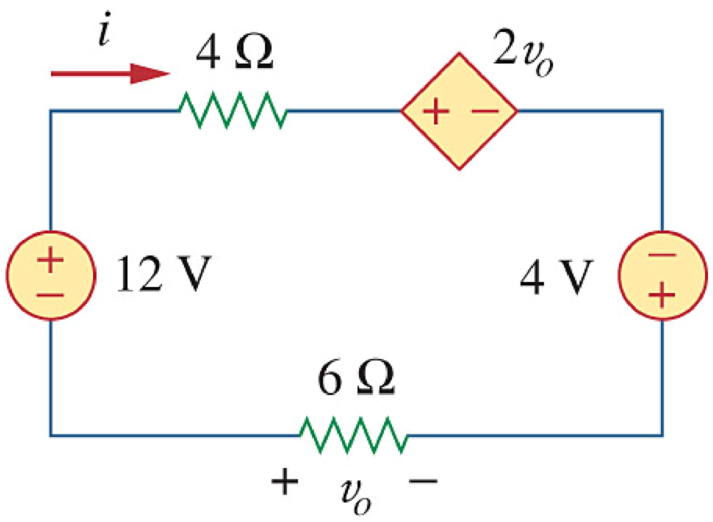


Figure 2.23 (a)

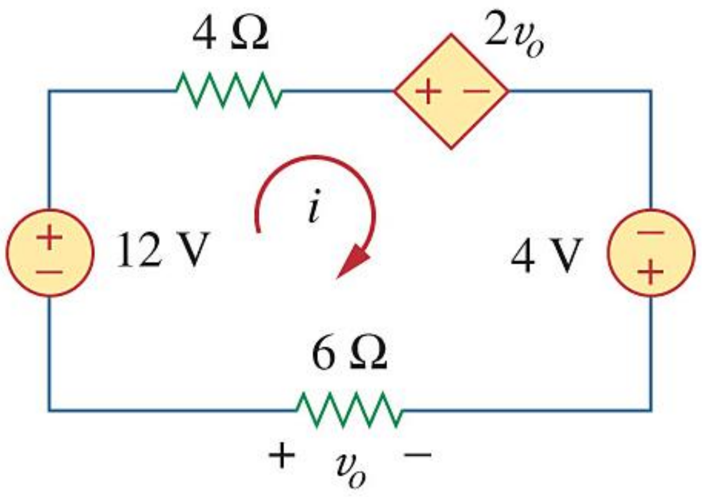


Figure 2.23 (b)

By KVL:

(1) $-12+4i+2v_o-4+6i=0 \rightarrow$ two variables

By ohm's law:

(2) $v_o=6(-i)$

$-12+4i-12i-4+6i=0$

$-2i=16$

$i=-8A$

$v_o=48V$

Practice Problem 2.7 Find v_o and i_o in the circuit of Fig. 2.26.

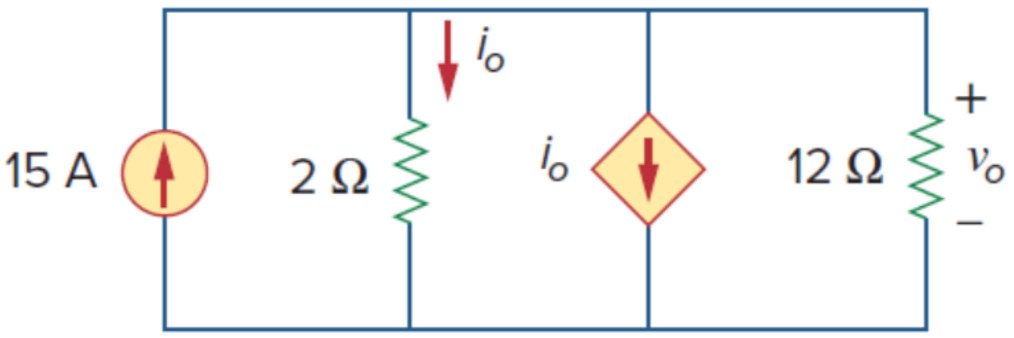


Figure 2.26

Answer: $V_o = \frac{180}{13}, i_o = \frac{90}{13}$

Example 2.8 Find currents and voltages in the circuit shown in Fig. 2.27(a).

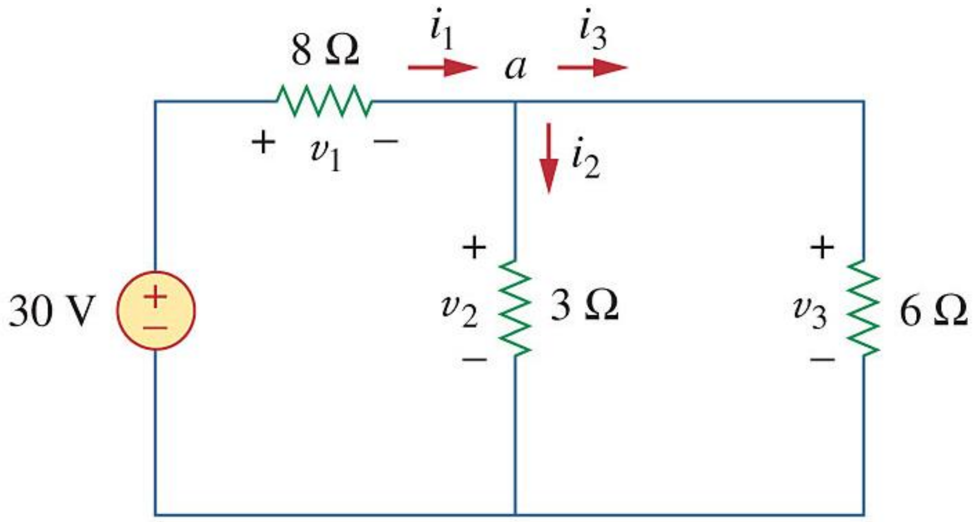


Figure 2.27(a)

(1) KVL
(2) Ohm's Law
(3) KCL

Answer : $i_1 = 3$ A, $i_2 = 2$ A, $i_3 = 1$ A,
$v_1 = 24$ V, $v_2 = v_3 = 6$ V.

Example 2.8 (continue)

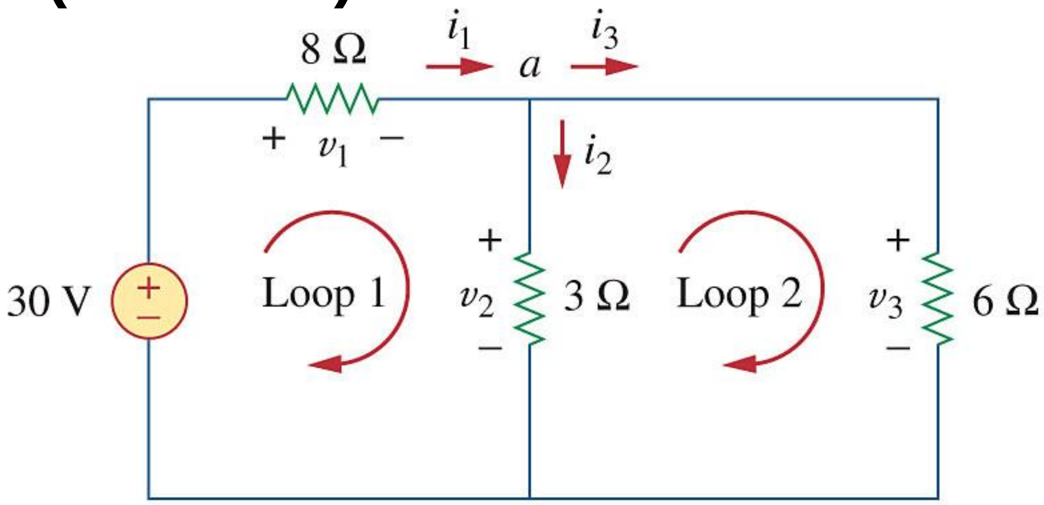


Figure 2.27 (b)

(1) KVL:

- Loop 1: $-30+v_1+v_2=0$
- Loop 2: $-v_2+v_3=0$

(2) Ohm's law:

- $v_1 = 8i_1$; $v_2 = 3i_2$; $v_3 = 6i_3$

(3) KCL:

- $i_1=i_2+i_3$

Example 2.8 (continue)

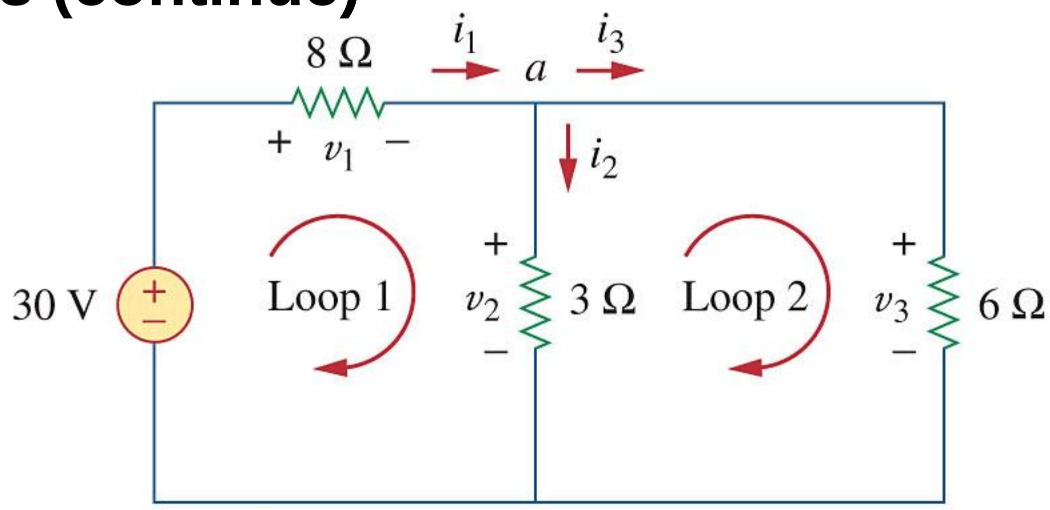


Figure 2.27 (b)

(1) KVL:

- Loop 1: $-30+v_1+v_2=0$ → $\mathbf{8i_1 + 3i_2 = 30}$
- Loop 2: $-v_2+v_3=0$ → $\mathbf{-3i_2 + 6i_3 = 0}$

(2) Ohm's law:

- $v_1 = 8i_1$; $v_2 = 3i_2$; $v_3 = 6i_3$

(3) KCL:

- $i_1=i_2+i_3$ → $\mathbf{-i_1 + i_2 + i_3 = 0}$

Three variables and three equations

Example 2.8 (continue)

(1) KVL:

- Loop 1: $-30+v_1+v_2=0$ $\rightarrow$ $\mathbf{8i_1 + 3i_2 = 30}$
- Loop 2: $-v_2+v_3=0$ $\rightarrow$ $\mathbf{-3i_2 + 6i_3 = 0}$

(2) Ohm's law:

- $v_1 = 8i_1$; $v_2 = 3i_2$; $v_3 = 6i_3$

(3) KCL:

- $i_1=i_2+i_3$ $\rightarrow$ $\mathbf{-i_1 + i_2 + i_3 = 0}$

Alternatively, these three equations can be written as..

$$\begin{bmatrix} 8 & 3 & 0 \\ 0 & -3 & 6 \\ 1 & -1 & -1 \end{bmatrix} \begin{bmatrix} i_1 \\ i_2 \\ i_3 \end{bmatrix} = \begin{bmatrix} 30 \\ 0 \\ 0 \end{bmatrix}$$

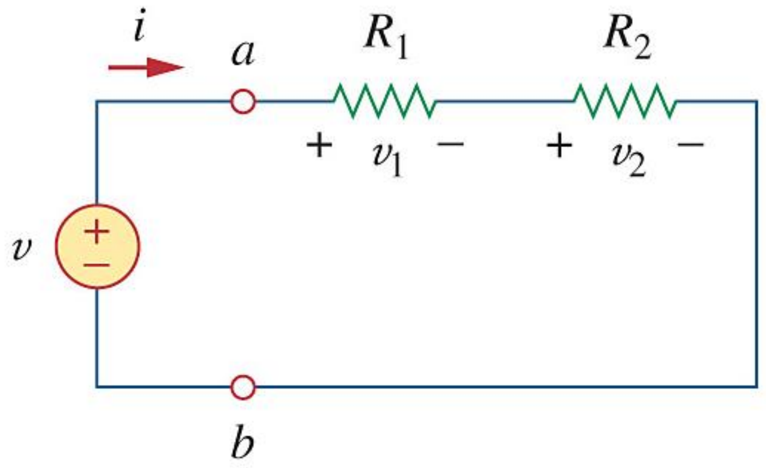
By using Cramer's rule, we can get the answers.
We will learn this method in Chapter 3.

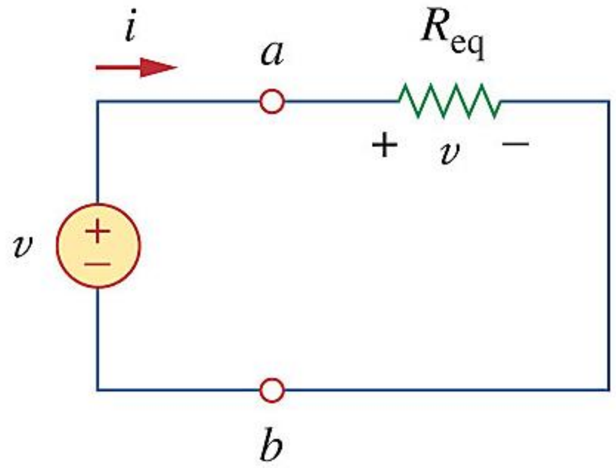
2.5 Series Resistors and Voltage Division

- The equivalent resistance of *N* resistors connected in series is the sum of their individual resistances.

$$R_{eq} = R_1 + R_2 + \cdots + R_N = \sum_{n=1}^{N} R_n$$

$$R_{eq} = R_1 + R_2 + \cdots + R_N = \sum_{n=1}^{N} R_n$$





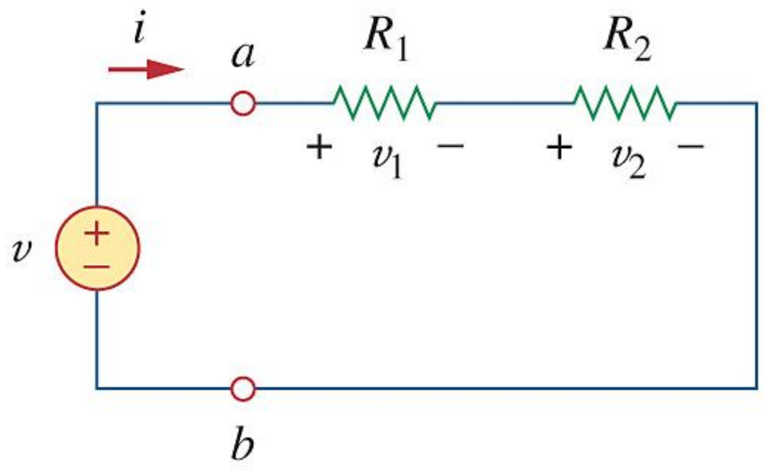
By KVL: $v = v_1 + v_2$

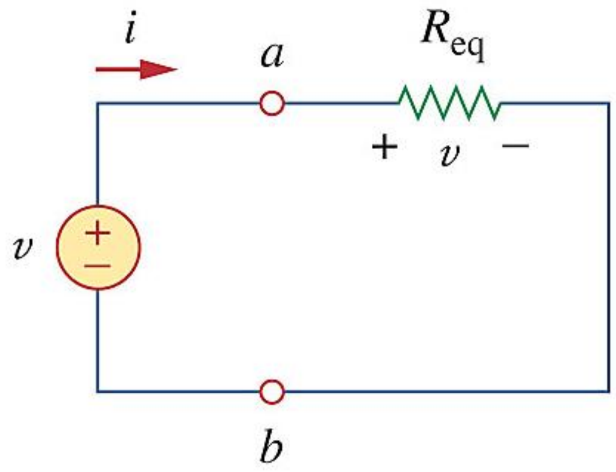
By Ohm's Law: $v_1 = iR_1$; $v_2 = iR_2$

Thus, $v = iR_1 + iR_2 = i \times (\mathbf{R_1+R_2}) = i \times \mathbf{R_{eq}}$

$\mathbf{R_{eq} = R_1+R_2}$

Voltage Division





From equation v = i(R_1+R_2), we get $i = \frac{v}{R_1+R_2}$

Put i into v_1 = iR_1 and v_2 = iR_2, then we get

$$v_1 = \frac{R_1}{R_1 + R_2} v \; and \; v_2 = \frac{R_2}{R_1 + R_2} v$$

The source voltage v is divided among the resistors in direct proportion to their resistances.

In general, the voltage across each resistor is

$$v_n = \frac{R_n}{\sum_{n=1}^{N} R_n} v, \quad n = 1, 2, \ldots, N$$

In series, a current i is the same $\rightarrow i = \frac{v}{\sum R_n}$

Voltage at resistance n: $v_n = iR_n$

Therefore, $v_n = \frac{R_n}{\sum R_n} v$

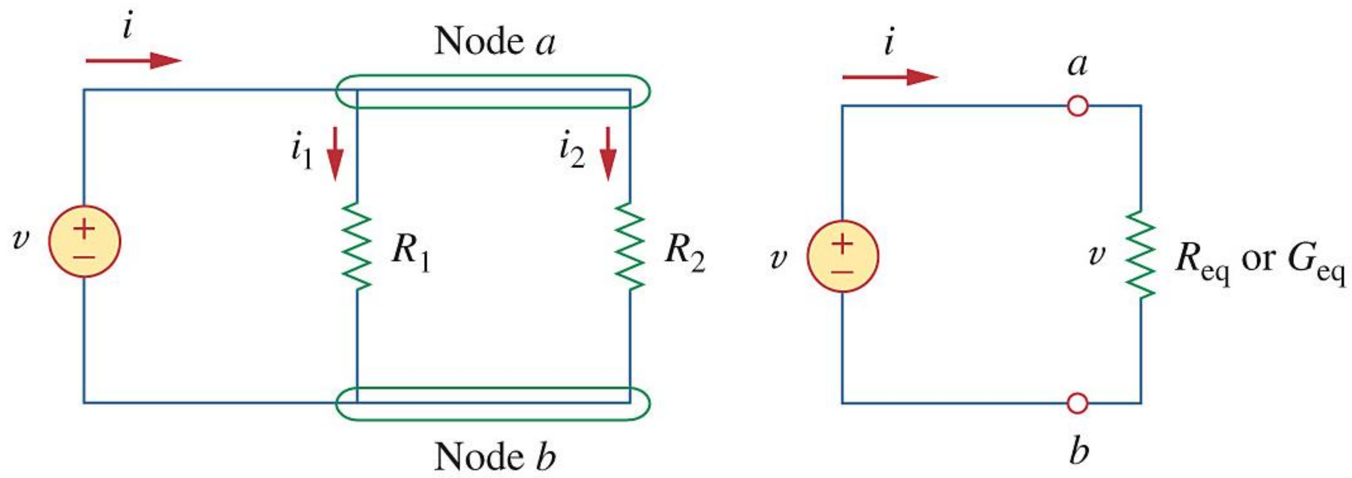
2.6 Parallel Resistors and Current Division

- The equivalent conductance of N resistors connected in parallel is the sum of their individual conductance.

$$\frac{1}{R_{eq}} = \frac{1}{R_1} + \frac{1}{R_2} + \cdots + \frac{1}{R_N} = \sum_{n=1}^{N} \frac{1}{R_n}$$

As 1/R = G, $$G_{eq} = G_1 + G_2 + \cdots + G_N = \sum_{n=1}^{N} G_n$$

$$\frac{1}{R_{eq}} = \frac{1}{R_1} + \frac{1}{R_2} + \cdots + \frac{1}{R_N} = \sum_{n=1}^{N} \frac{1}{R_n}$$

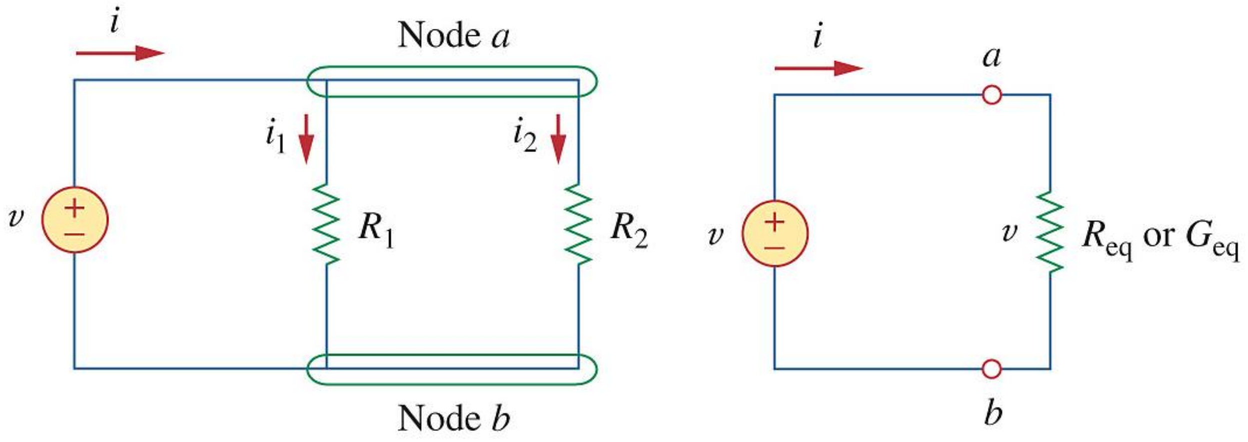


By KCL: $i = i_1 + i_2$
By Ohm's Law: $i_1 = v/R_1$; $i_2 = v/R_2$

Thus, $i = v(1/R_1 + 1/R_2) = v \times (1/R_{eq})$

$1/R_{eq} = 1/R_1 + 1/R_2$

Current Division (Resistance)



From the equation i = v($1/R_1+1/R_2$) $\rightarrow v = \frac{R_1R_2}{R_1+R_2} i$

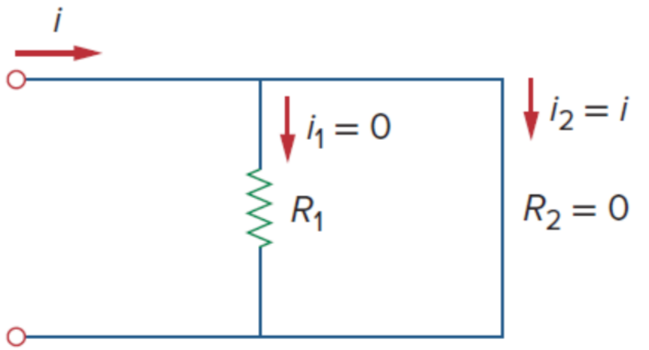
Put v into the equations $i_1 = v/R_1$; $i_2 = v/R_2$, then we get

$$i_1 = \frac{R_1R_2}{R_1+R_2}\frac{1}{R_1}i = \frac{R_2}{R_1+R_2}i$$
$$i_2 = \frac{R_1R_2}{R_1+R_2}\frac{1}{R_2}i = \frac{R_1}{R_1+R_2}i$$

The total current i is shared by the resistors in inverse proportion to their resistances

Examples in extreme cases

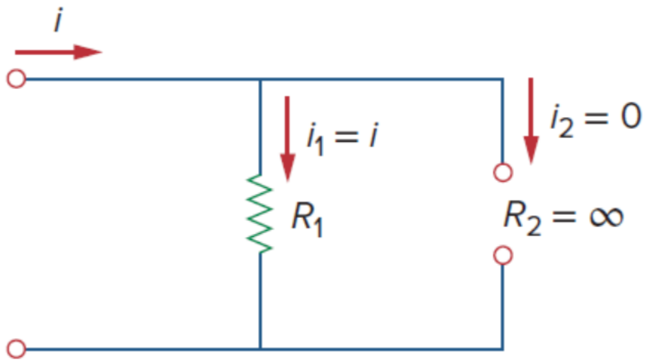
(i) A short circuit



$$i_1 = \frac{R_2\, i}{R_1 + R_2}, \qquad i_2 = \frac{R_1\, i}{R_1 + R_2}$$

R_2 = 0 and thus i_1 = 0, i_2 = i

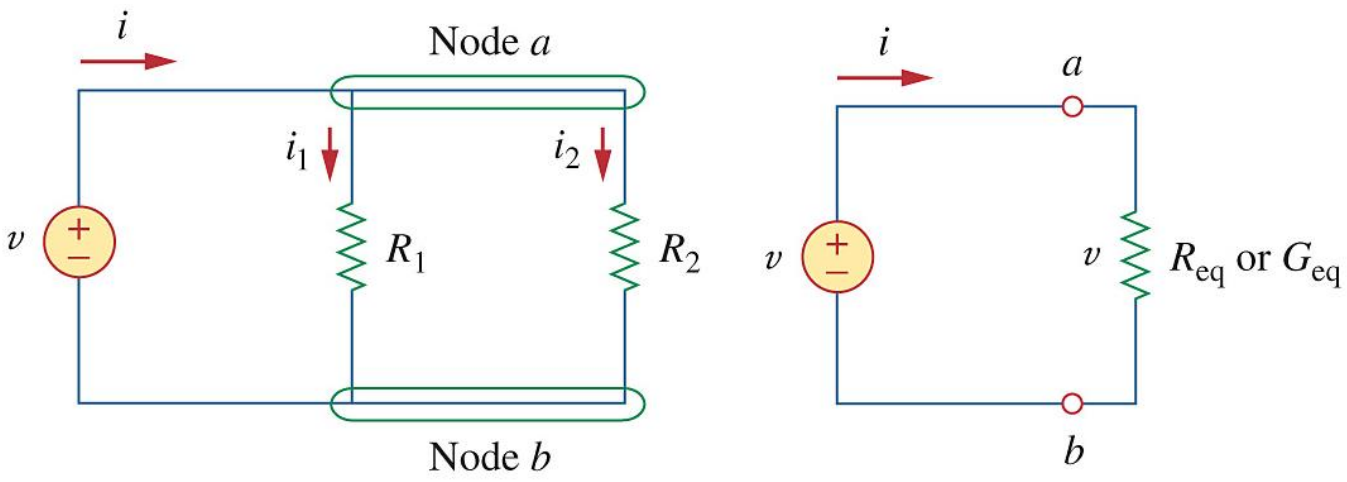
(ii) An open circuit



$$i_1 = \frac{R_2\, i}{R_1 + R_2}, \qquad i_2 = \frac{R_1\, i}{R_1 + R_2}$$

$R_2 = \infty$ and thus i_1 = i, i_2 = 0

Current Division (Conductance)



Using conductance G, from the equation $i = v(1/R_1+1/R_2)$ $= v(G_1 + G_2)$

Put v into the equations $i_1 = vG_1$; $i_2 = vG_2$, then we get

$$i_1 = \frac{G_1}{G_1 + G_2} i, and\ i_2 = \frac{G_2}{G_1 + G_2} i$$

In general, if a current divider has N conductors in parallel with the source current i, the nth conductor (G_n) will have current

$$i_n = \frac{G_n}{\sum_{n=1}^{N} G_n} i, \quad n = 1, 2, \ldots, N$$

In parallel, a voltage *v* is the same $\rightarrow \mathrm{v} = \frac{i}{\sum G_n}$

Current at the nth conductor: $i_n = vG_n$

Therefore, $i_n = \frac{G_n}{\sum G_n} i$

Example 2.9 Find R_{eq} for the circuit shown in Fig. 2.34.

Answer : 14.4 Ω.

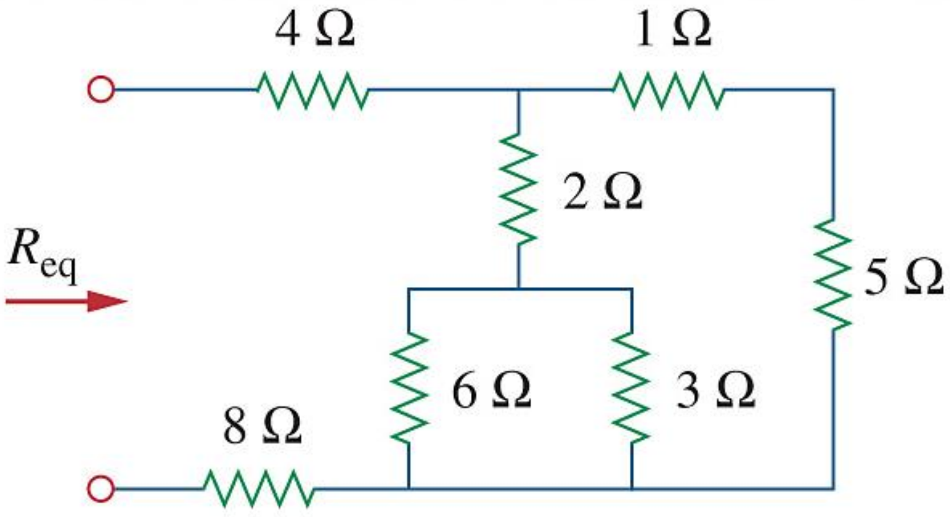


Figure 2.34

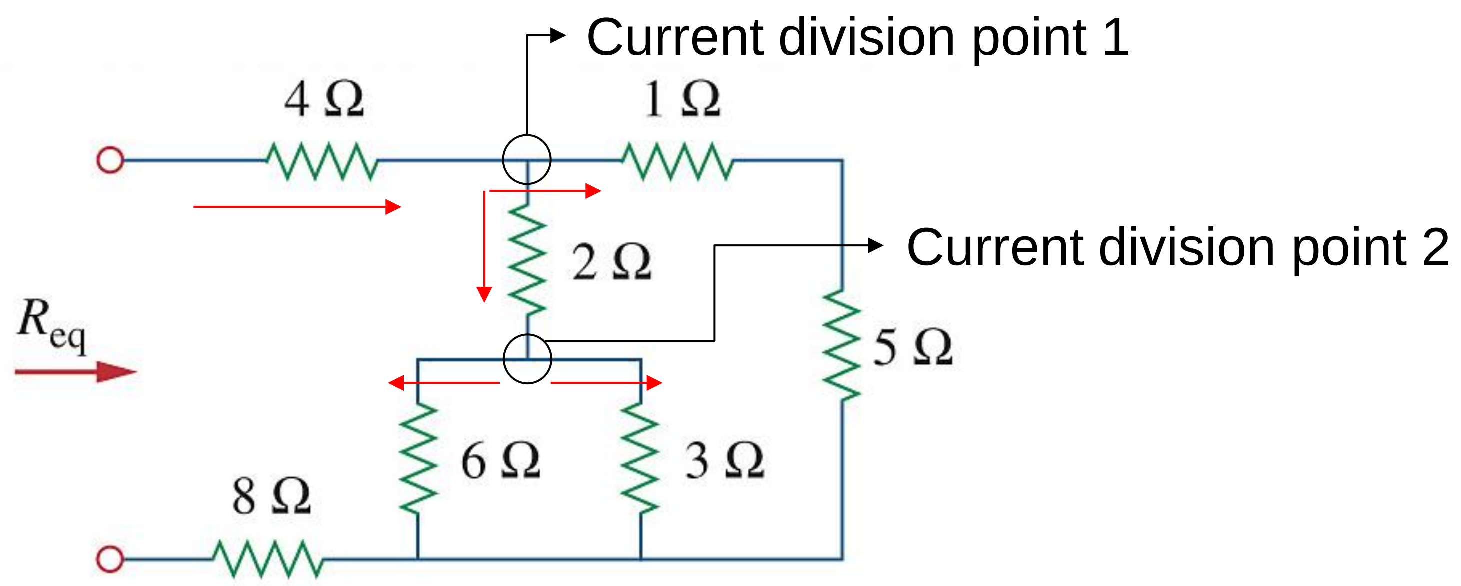
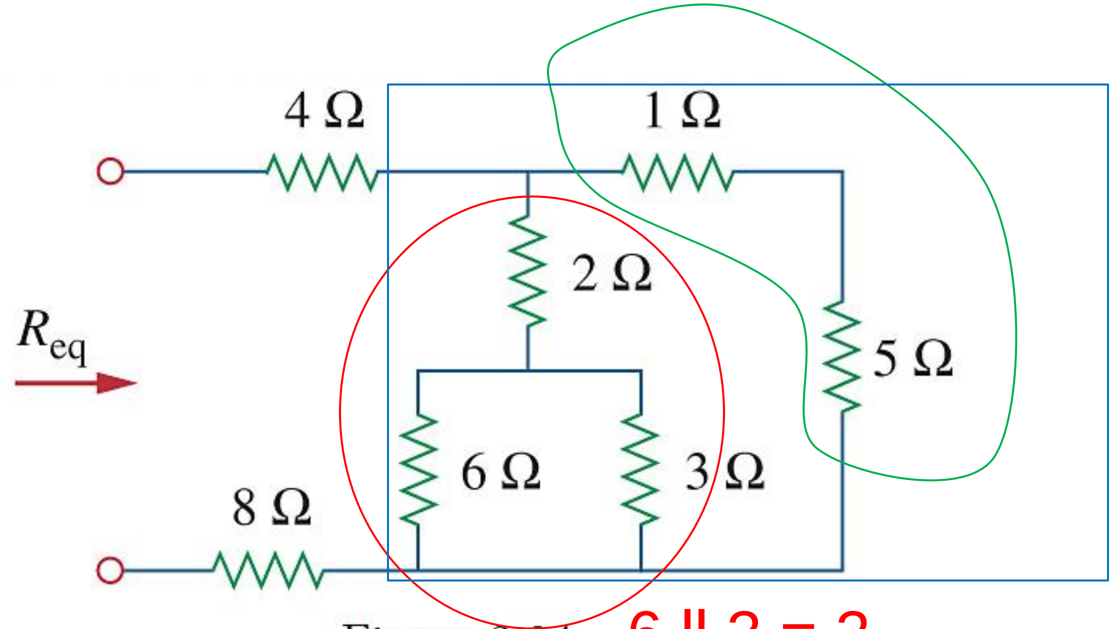


Figure 2.34

Let's assume that we flow a current to this circuit.
→ There are two points where the current is divided (parallel).



4 ll 6 = 2.4 Ω

Figure 2.34

6 ll 3 = 2

First, need to solve the parallel resistors 6 and 3 Ω because red and green are parallel.

Then, blue part (red and green parallel) → 4 ll 6 = 2.4 Ω
Finally, three series resistance → 4 + 2.4 + 8 = **14.4 Ω**

Example 2.10 Calculate the equivalent resistance R_{ab} in the circuit in Fig. 2.37.

Answer : 11.2 Ω.

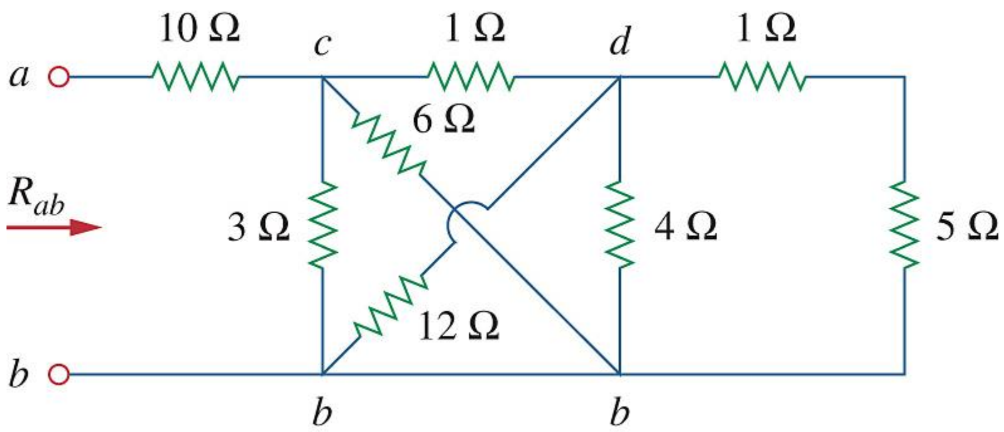


Figure 2.37

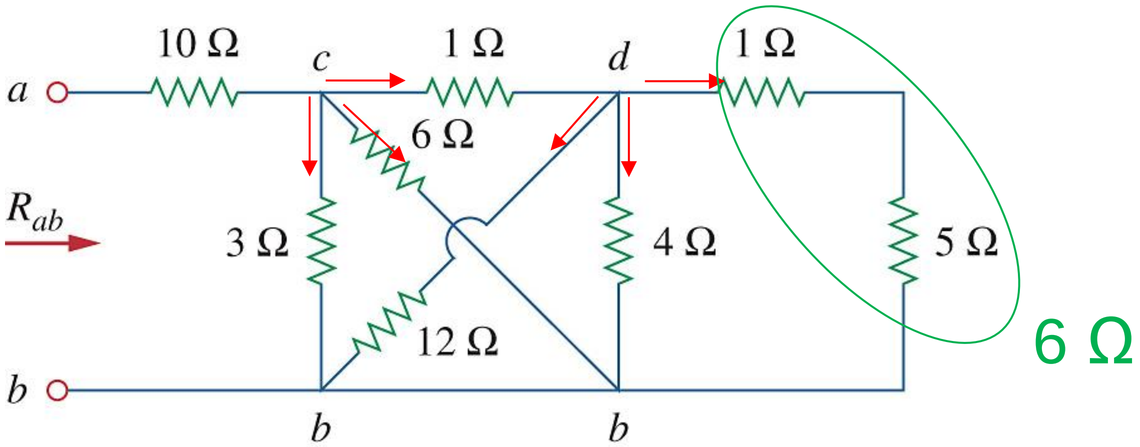


Figure 2.37

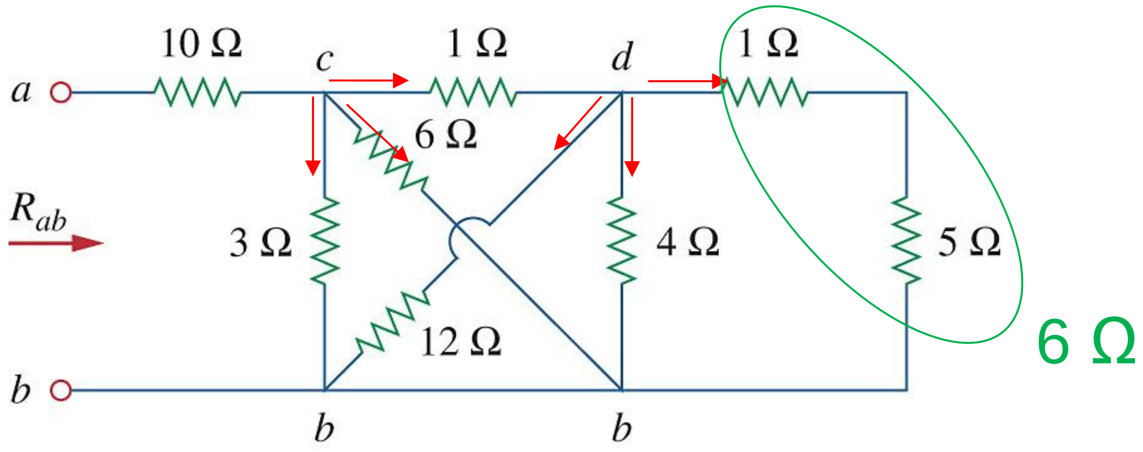
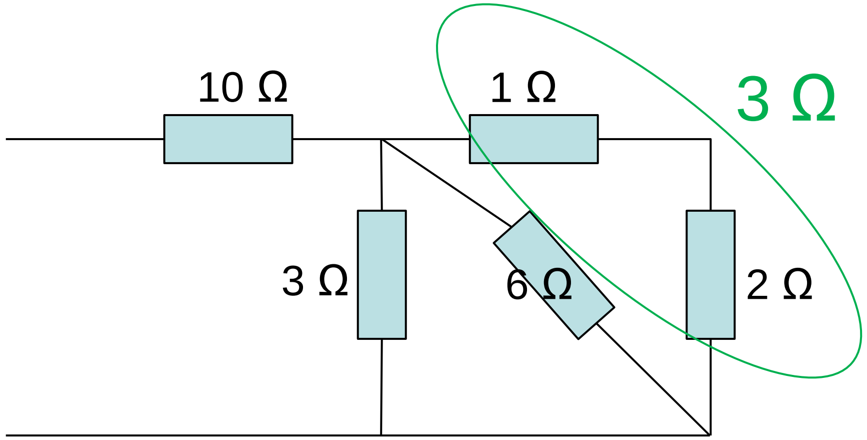
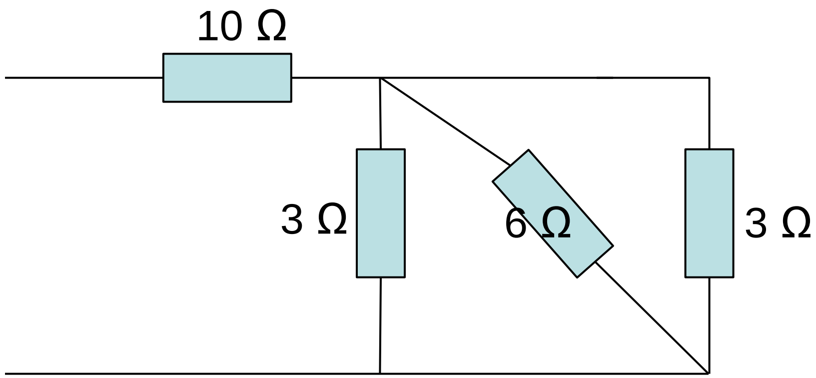


Figure 2.37

First, 12 ll 4 ll 6 parallel → 2 Ω
Then, the circuit becomes





Second, 3 ll 6 ll 3 parallel → 1.2 Ω

Finally, 10 + 1.2 in series → 11.2 Ω

2.7 Wye-Delta Transformations

- Situations often arise in circuit analysis when resistors are neither in parallel nor in series: How do we combine resistors like these.

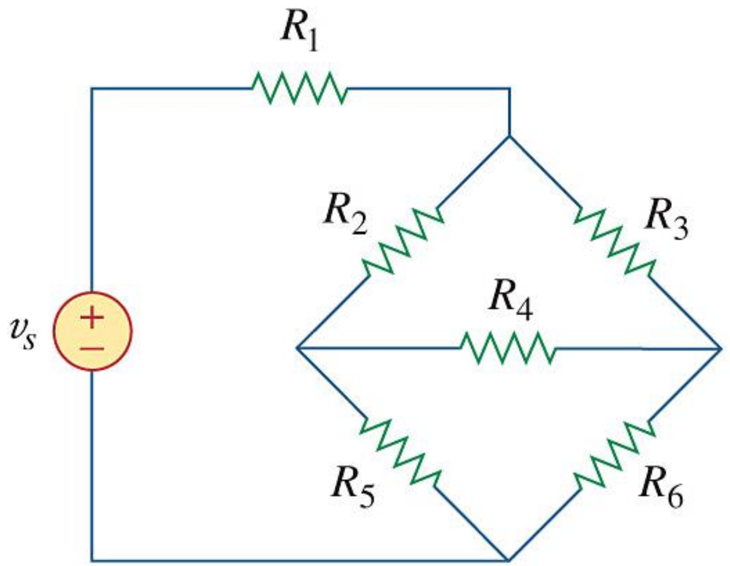


Figure 2.46 The bridge network.

- These are the wye or tee network and the delta or pi network.

Y = T

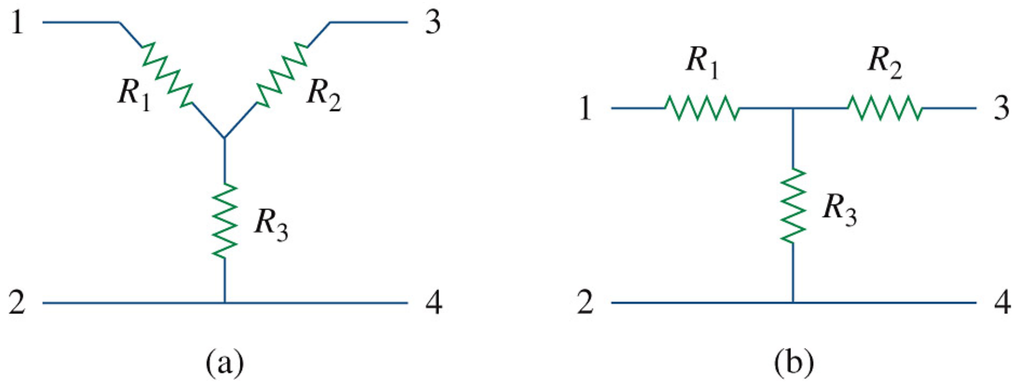


Figure 2.47 Two forms of the same network: (a) Y, (b) T.

Δ = π

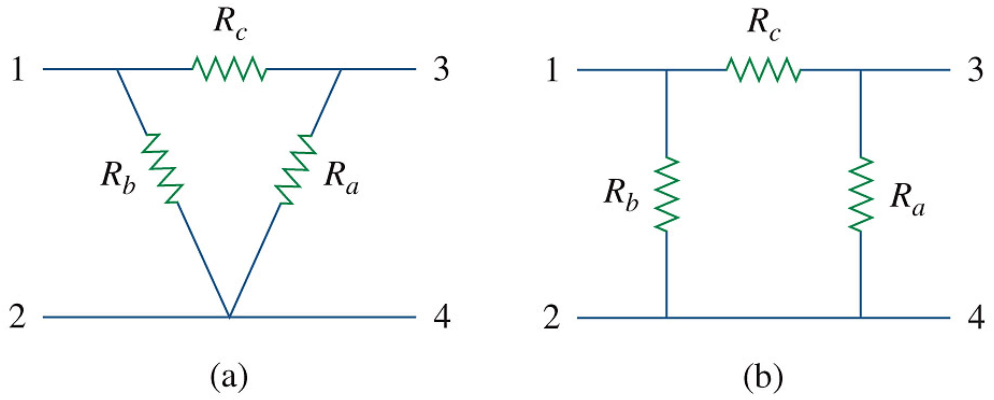


Figure 2.48 Two forms of the same network: (a) $\mathbf{\Delta}$, (b) $\mathbf{\Pi}$.

- How to identify them when they occur as part of a network and how to apply wye-delta transformation in the analysis of that network. There are two types of transformation:
  (i) Delta to wye conversion
  (ii) Wye to delta conversion

(i) Delta to wye conversion

Each resistor in the Y network is the product of the resistors in the two adjacent Δ branches, divided by the sum of the three Δ resistors.

$$\begin{cases} R_1 = \dfrac{R_b R_c}{R_a + R_b + R_c} \\[2ex] R_2 = \dfrac{R_c R_a}{R_a + R_b + R_c} \\[2ex] R_3 = \dfrac{R_a R_b}{R_a + R_b + R_c} \end{cases}$$

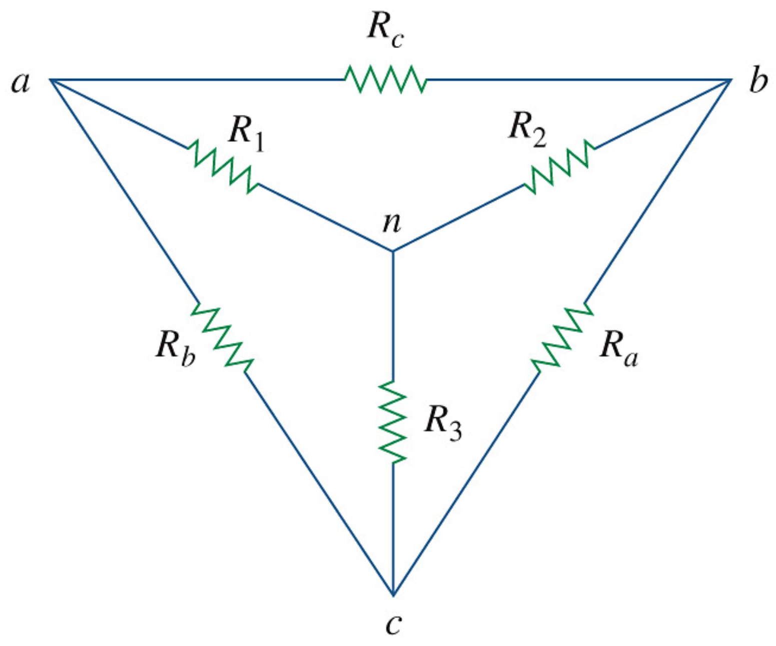


Figure 2.49 Superposition of wye and delta networks as an aid in transforming one to the other.

(ii) Wye to delta conversion

Each resistor in the network is the sum of all possible products of Y resistors taken two at a time, divided by the opposite Y resistor.

$$R_a = \frac{R_1R_2 + R_2R_3 + R_3R_1}{R_1}$$

$$R_b = \frac{R_1R_2 + R_2R_3 + R_3R_1}{R_2}$$

$$R_c = \frac{R_1R_2 + R_2R_3 + R_3R_1}{R_3}$$

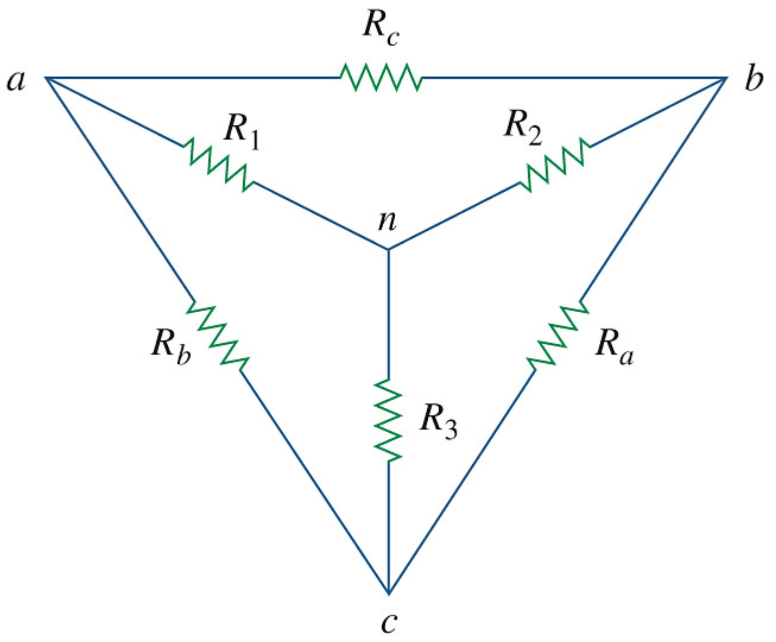
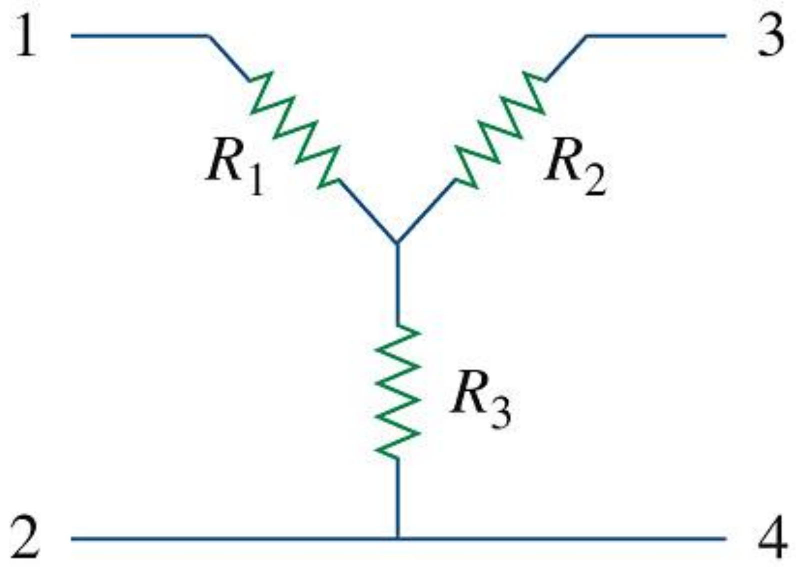
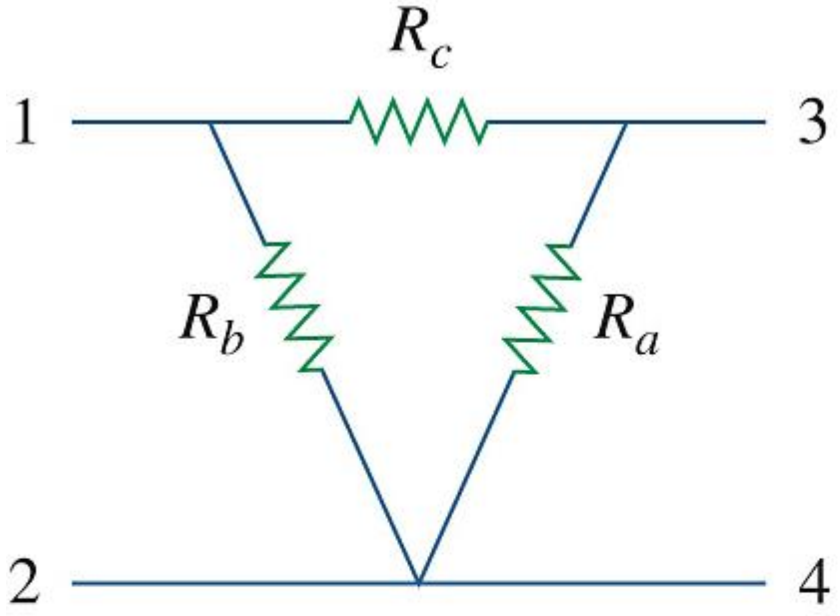


Figure 2.49 Superposition of wye and delta networks as an aid in transforming one to the other.

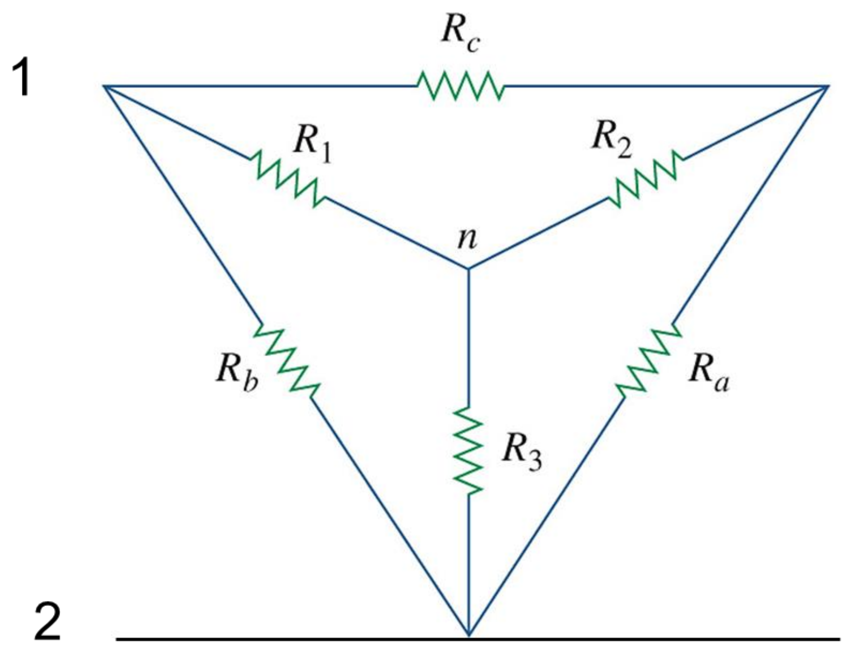
Proof: Delta to Y

To obtain the equivalent resistances, the resistance between each pair of nodes in the Δ (or π) network is the same the resistance between the same pair of the nodes in the Y (or T) network.





For example, terminal 1 and 2



R_1 and R_a
R_2 and R_b
R_3 and R_c
Face each other

From 1 to reach 2
Y is only possible through the series resistance $R_1 + R_3$
whereas for Δ, R_b is in parallel with $(R_c + R_a)$

$$R_{12}(Y) = R_1 + R_3$$

$$R_{12}(\Delta) = R_b \parallel (R_a + R_c)$$

Because we want those two resistances to be equal, we set R_{12} (Y) = R_{12} (Δ)

$$R_{12}(Y) = R_1 + R_3 \quad = \quad R_{12}(\Delta) = R_b \parallel (R_a + R_c)$$

$$R_b \| (R_a + R_c) = \frac{1}{\frac{1}{R_b} + \frac{1}{R_a + R_c}} = \frac{1}{\frac{R_a + R_b + R_c}{R_b(R_a + R_c)}} = \frac{R_b(R_a + R_c)}{R_a + R_b + R_c}$$

$$\rightarrow \quad R_{12} = R_1 + R_3 = \frac{R_b(R_a + R_c)}{R_a + R_b + R_c}$$

Do the same procedures for R_{13} and R_{34}, then we get

$$R_{13} = R_1 + R_2 = \frac{R_c(R_a + R_b)}{R_a + R_b + R_c}$$

$$R_{34} = R_2 + R_3 = \frac{R_a(R_b + R_c)}{R_a + R_b + R_c}$$

To get R_1,calculate $R_{12} – R_{34} + R_{13}$

$$R_{12} = R_1 + R_3 = \frac{R_b(R_a + R_c)}{R_a + R_b + R_c}$$

$$- \quad R_{34} = R_2 + R_3 = \frac{R_a(R_b + R_c)}{R_a + R_b + R_c}$$

gives $$R_1 - R_2 = \frac{R_c(R_b - R_a)}{R_a + R_b + R_c}$$

And then, add R_{13}

$$R_1 - R_2 = \frac{R_c(R_b - R_a)}{R_a + R_b + R_c}$$

$$+ \quad R_{13} = R_1 + R_2 = \frac{R_c(R_a + R_b)}{R_a + R_b + R_c}$$

$$\boxed{R_1 = \frac{R_b R_c}{R_a + R_b + R_c}}$$

To get R_2,

$$R_1 - R_2 = \frac{R_c(R_b - R_a)}{R_a + R_b + R_c}$$

$$- \quad R_1 + R_2 = \frac{R_c(R_a + R_b)}{R_a + R_b + R_c}$$

$$\boxed{R_2 = \frac{R_c R_a}{R_a + R_b + R_c}}$$

To get R_3,

$$R_1 + R_3 = \frac{R_b(R_a + R_c)}{R_a + R_b + R_c}$$

$$- \quad R_1 = \frac{R_b R_c}{R_a + R_b + R_c}$$

$$\boxed{R_3 = \frac{R_a R_b}{R_a + R_b + R_c}}$$

Proof: Y to Delta

From the equations that we attained

$$R_1 = \frac{R_b R_c}{R_a + R_b + R_c} \quad R_2 = \frac{R_c R_a}{R_a + R_b + R_c} \quad R_3 = \frac{R_a R_b}{R_a + R_b + R_c}$$

We get the equation

$$R_1 R_2 + R_2 R_3 + R_3 R_1 = \frac{R_a R_b R_c (R_a + R_b + R_c)}{(R_a + R_b + R_c)^2} = \frac{R_a R_b R_c}{R_a + R_b + R_c}$$

and then, get R_a

$$R_a = \frac{\dfrac{R_a R_b R_c}{R_a + R_b + R_c}}{\dfrac{R_b R_c}{R_a + R_b + R_c}} = \frac{R_1 R_2 + R_2 R_3 + R_3 R_1}{R_1}$$

Similarly, from two equations below

$$R_2 = \frac{R_c R_a}{R_a + R_b + R_c} \qquad R_1R_2 + R_2R_3 + R_3R_1 = \frac{R_aR_bR_c}{R_a + R_b + R_c}$$

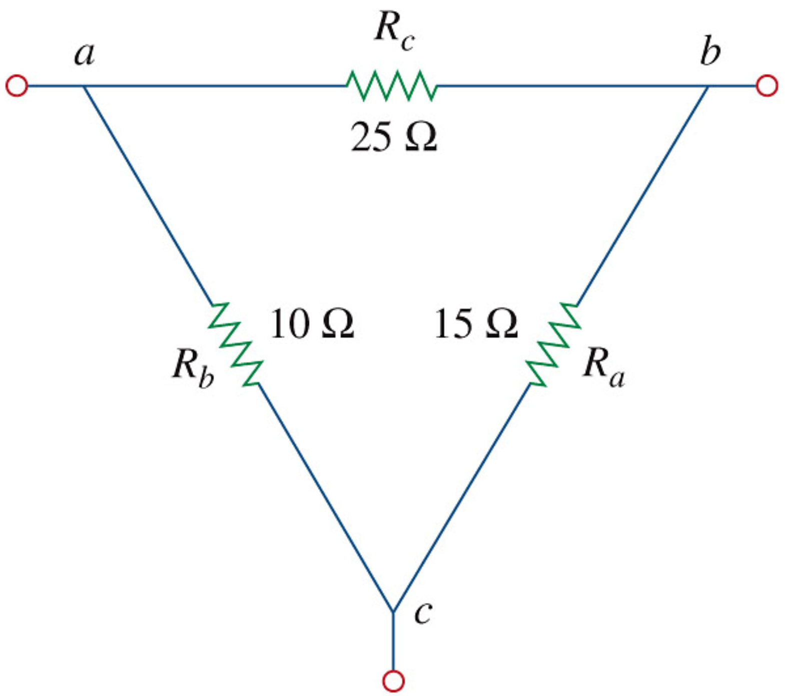
$$R_b = \frac{\dfrac{R_aR_bR_c}{R_a + R_b + R_c}}{\dfrac{R_cR_a}{R_a + R_b + R_c}} = \frac{R_1R_2 + R_2R_3 + R_3R_1}{R_2}$$

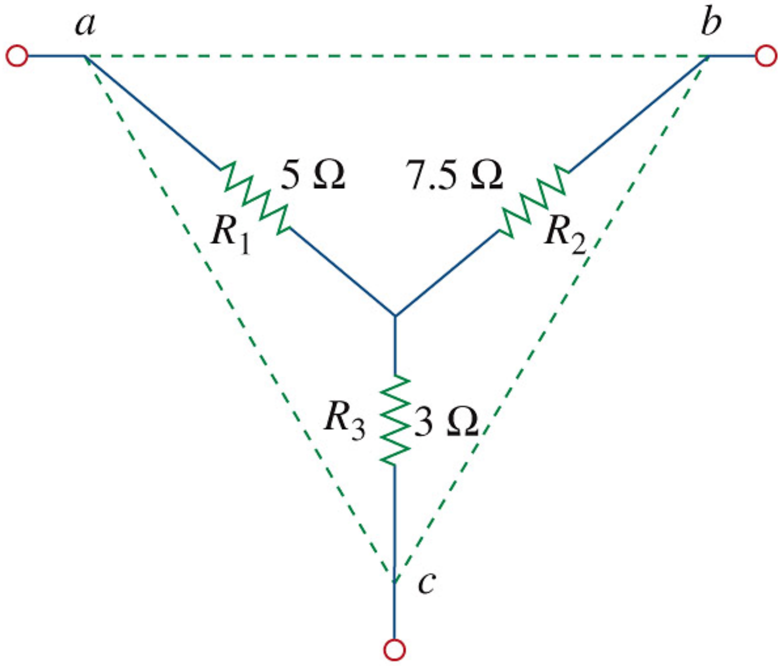
Similarly, from two equations below

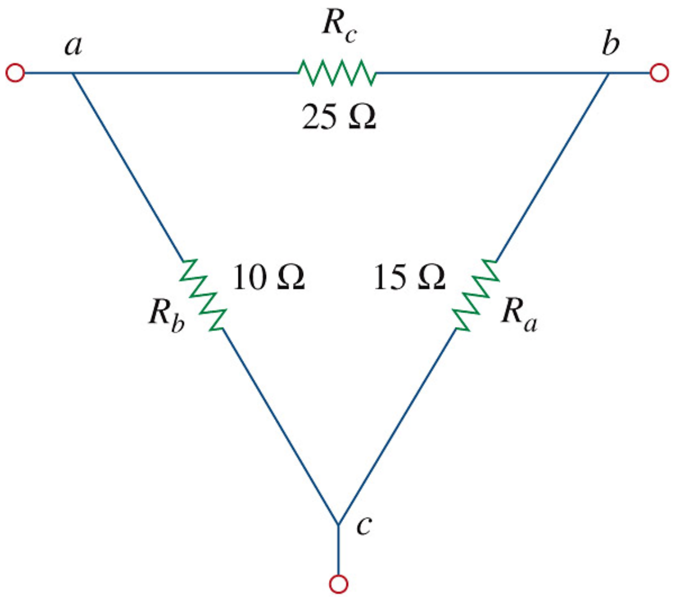
$$R_3 = \frac{R_a R_b}{R_a + R_b + R_c} \qquad R_1R_2 + R_2R_3 + R_3R_1 = \frac{R_a R_b R_c}{R_a + R_b + R_c}$$

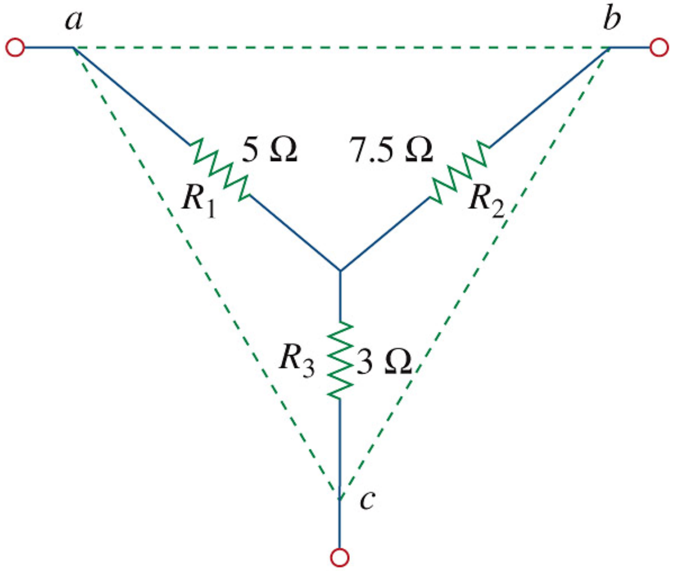
$$R_c = \frac{\dfrac{R_a R_b R_c}{R_a + R_b + R_c}}{\dfrac{R_a R_b}{R_a + R_b + R_c}} = \frac{R_1R_2 + R_2R_3 + R_3R_1}{R_3}$$

Example 2.14 Convert the Δ network in Fig. 2.50(a) to an equivalent Y network.

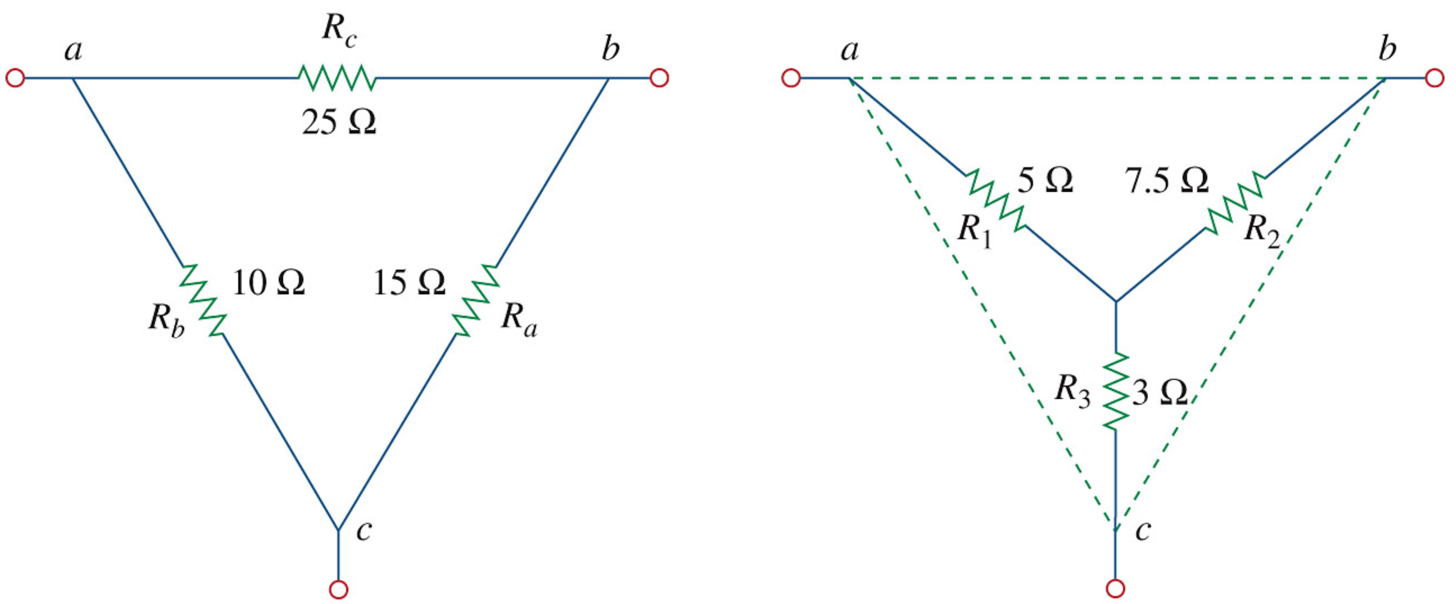






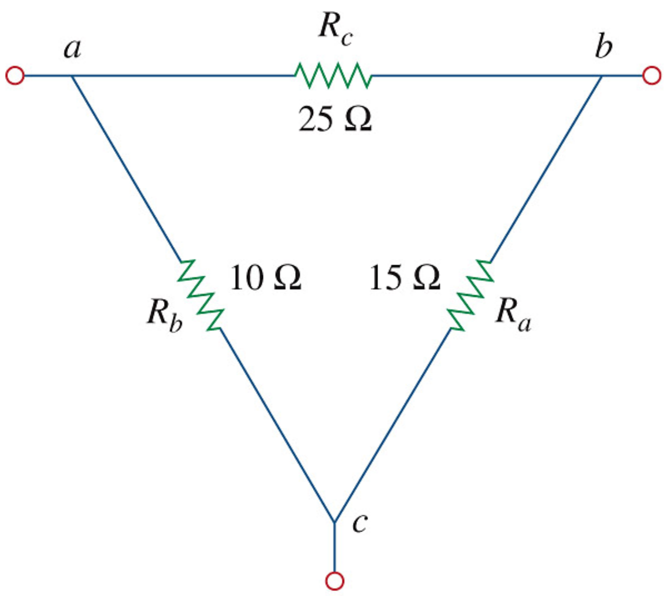


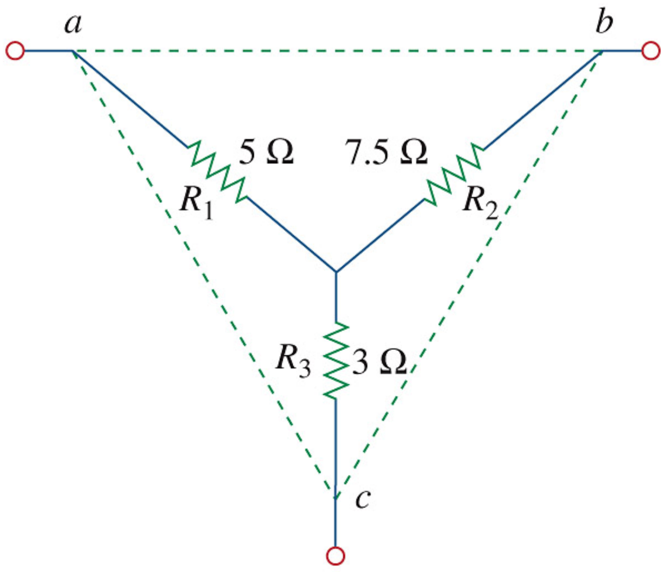
$$R_1 = \frac{R_b R_c}{R_a + R_b + R_c} \quad R_2 = \frac{R_c R_a}{R_a + R_b + R_c} \quad R_3 = \frac{R_a R_b}{R_a + R_b + R_c}$$



$$R_1 = \frac{R_b R_c}{R_a + R_b + R_c} \quad R_2 = \frac{R_c R_a}{R_a + R_b + R_c} \quad R_3 = \frac{R_a R_b}{R_a + R_b + R_c}$$

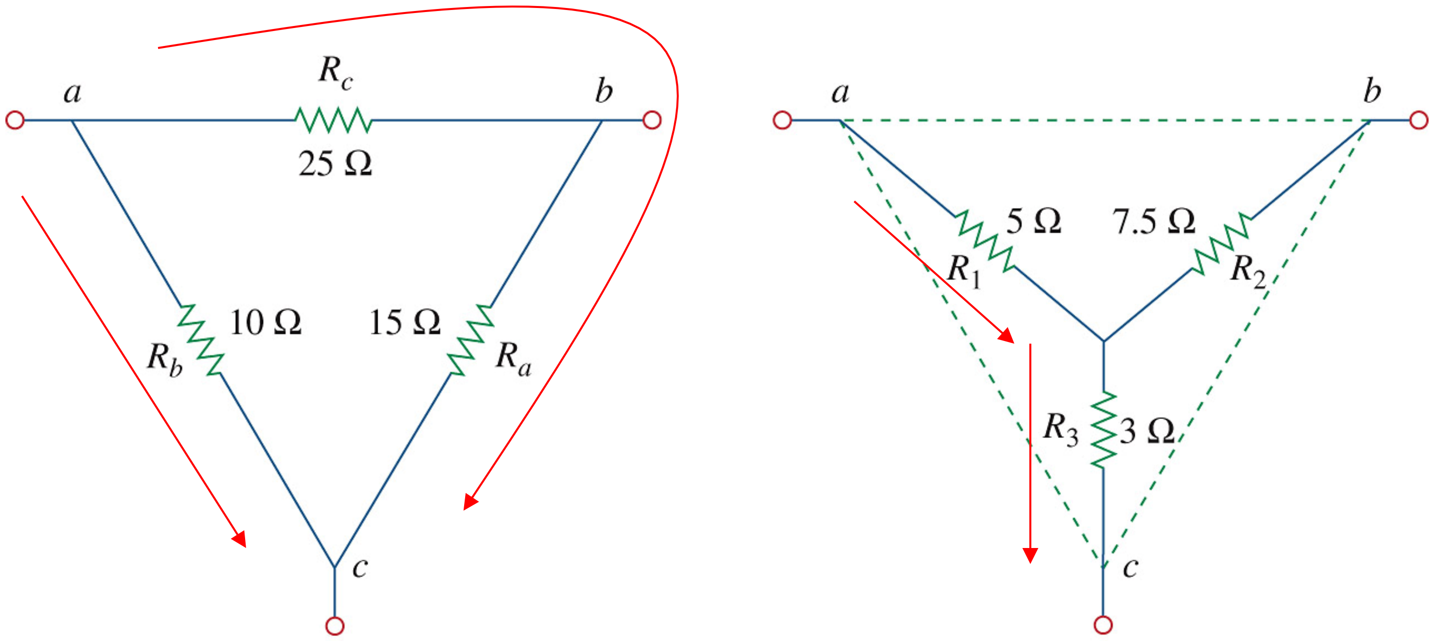
$$R_1 = \frac{10 * 25}{15 + 10 + 25} = 5 \quad R_2 = \frac{25 * 15}{15 + 10 + 25} = 7.5 \quad R_3 = \frac{15 * 10}{15 + 10 + 25} = 3$$





R_Y is less than $R_\Delta \rightarrow$ Are they really equivalent..?

Y-connection is like a series whereas the Δ-connection is like a parallel connection.



- a-to-c in Δ *vs* a-to-c in Y: R_b ll R_c+R_a vs R_1+R_3
  Comparing R_b ll R_c+R_a with R_1+R_3, both are smaller than R_b

- Same for a-to-b: R_c ll R_b+R_a *vs* R_1+R_2 smaller than R_c
- Same for b-to-c: R_a ll R_c+R_b *vs* R_2+R_3 smaller than R_a

Values look larger in Δ, but they are actually equivalent.

Practice Problem 2.14 Transform the wye network in Fig. 2.51 to a delta network.

Answer : $R_a = 140\ \Omega,\ R_b = 70\ \Omega,\ R_c = 35\ \Omega.$

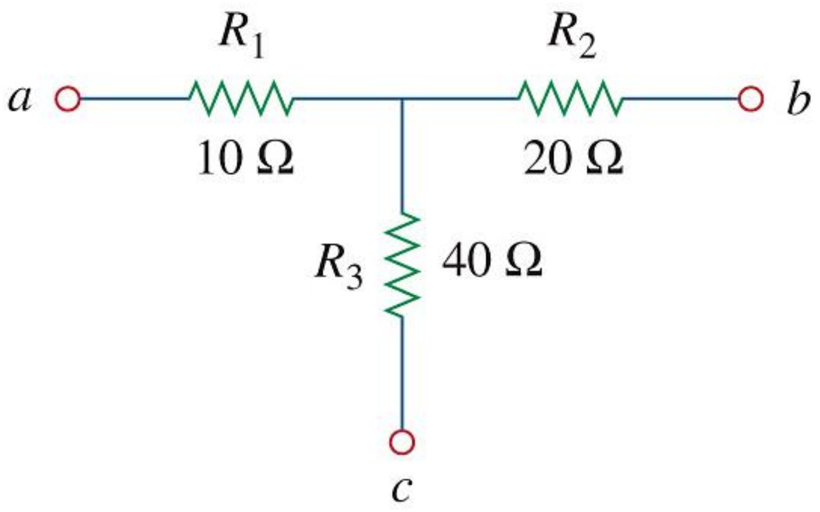
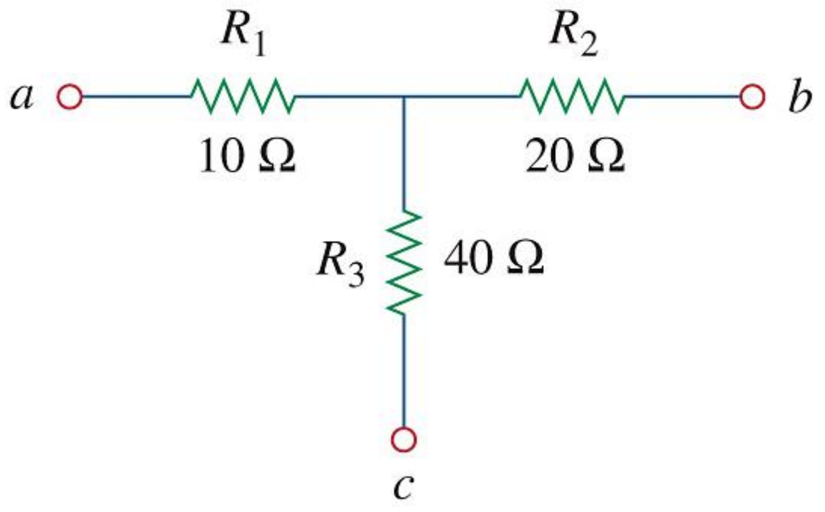


Figure 2.51



= 1400

$$R_a = \frac{R_1R_2 + R_2R_3 + R_3R_1}{R_1}$$

= 140

$$R_b = \frac{R_1R_2 + R_2R_3 + R_3R_1}{R_2}$$

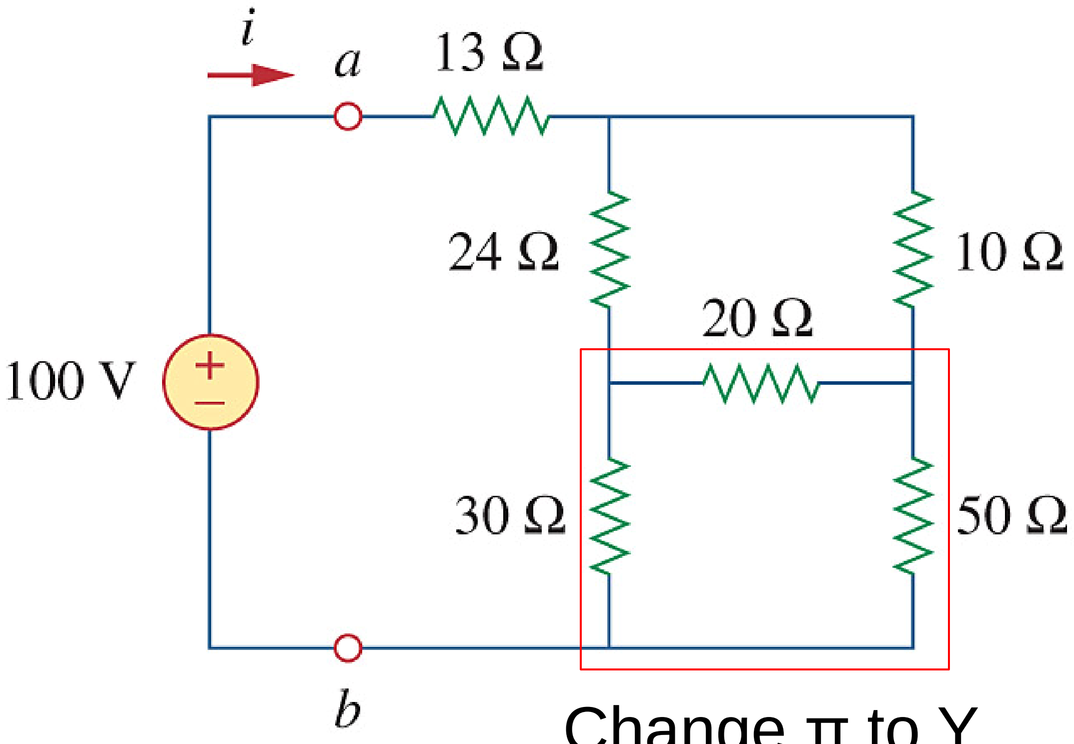
= 70

$$R_c = \frac{R_1R_2 + R_2R_3 + R_3R_1}{R_3}$$

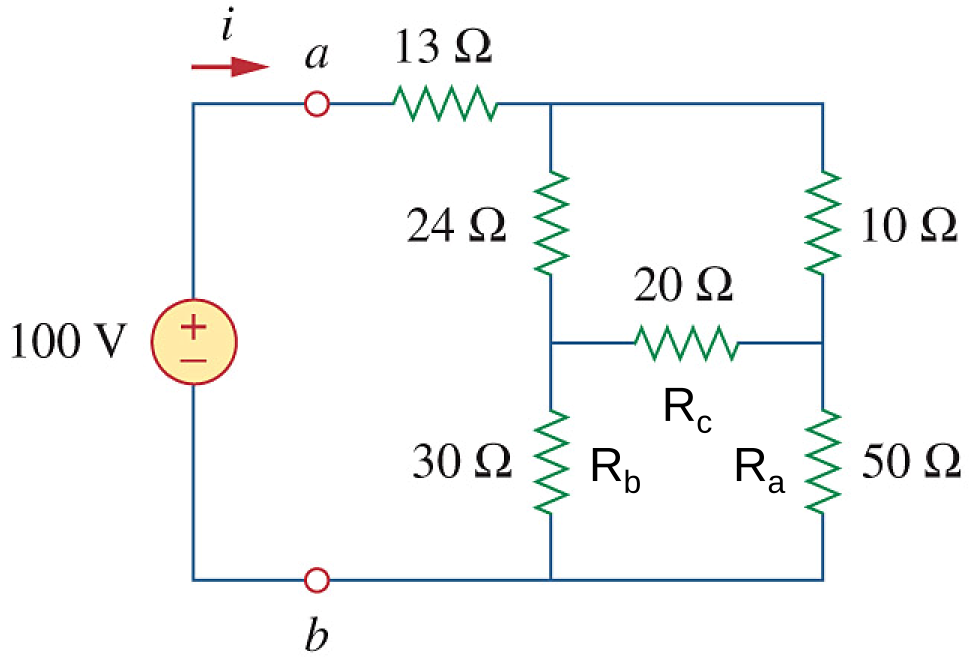
= 35

Practice Problem 2.15 For the bridge network in Fig. 2.54, find R_{ab} and i.

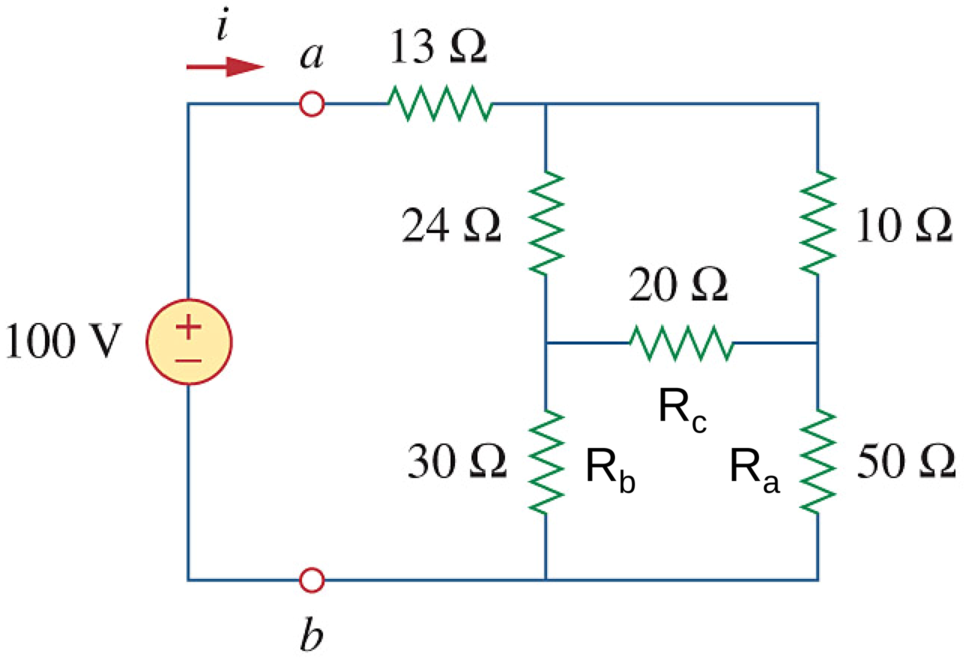
Answer : $R_{ab} = 40\ \Omega,\ i = 2.5$ A.



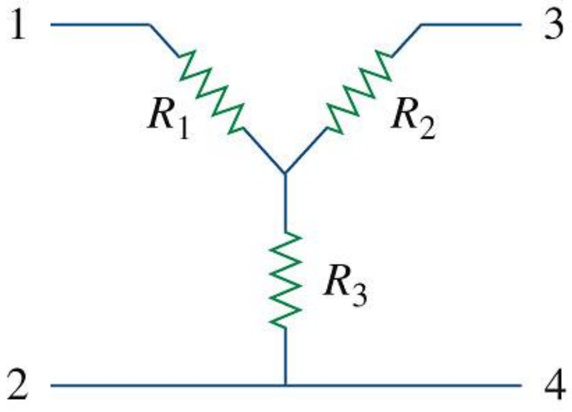
Change π to Y



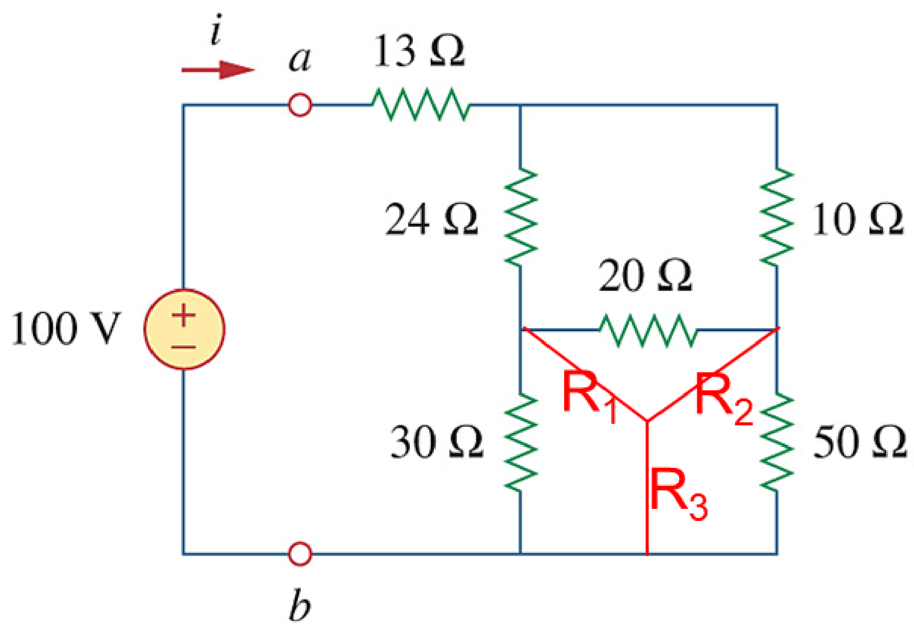
$$R_1 = \frac{R_b R_c}{R_a + R_b + R_c} \quad R_2 = \frac{R_c R_a}{R_a + R_b + R_c} \quad R_3 = \frac{R_a R_b}{R_a + R_b + R_c}$$



$$R_1 = \frac{R_b R_c}{R_a + R_b + R_c} \quad R_2 = \frac{R_c R_a}{R_a + R_b + R_c} \quad R_3 = \frac{R_a R_b}{R_a + R_b + R_c}$$



R_1 = 30*20 / (30 + 20 + 50) = 6
R_2 = 20*50 / (30 + 20 + 50) = 10
R_3 = 50*30 / (30 + 20 + 50) = 15



R_{eq} = 13 + (30 ll 20) + 15 = 40 Ω
$i = V/R_{eq} = 100/40 = 2.5A$