ECE2150J Introduction to Circuits

Chapter 5. Operational Amplifiers

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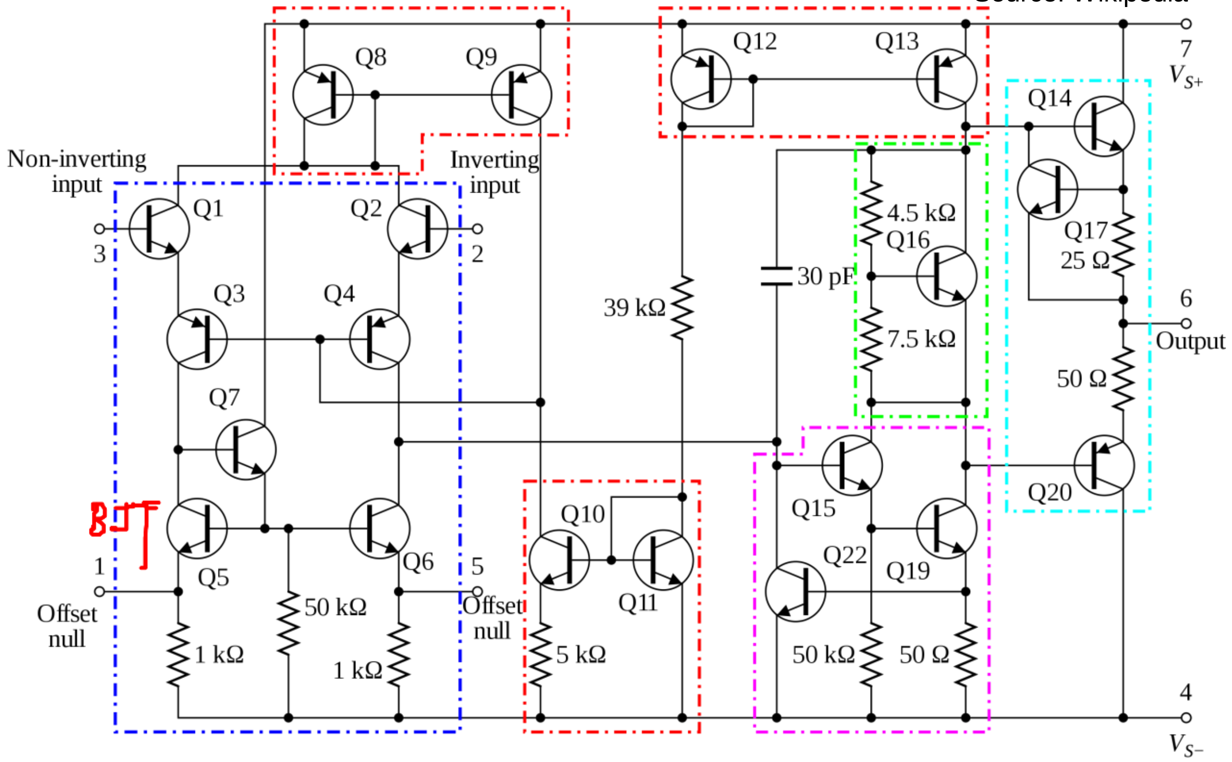
5.1 Introduction

The **operational amplifier**, or **op amp**, is an electronic device that behaves like a **voltage-controlled voltage source**. An op amp circuit can perform mathematical operations of *addition, subtraction, multiplication, division, differentiation, and integration*.

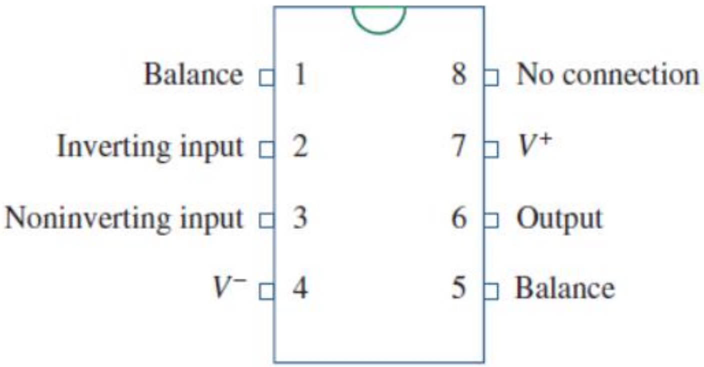
The op amp is an electronic device consisting of **a complex arrangement** of resistors, transistors, capacitors, and diodes

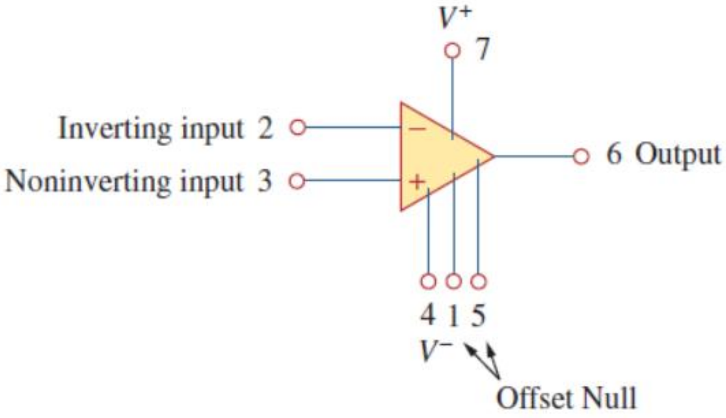
Source: Wikipedia



A component-level diagram of the common 741 op amp

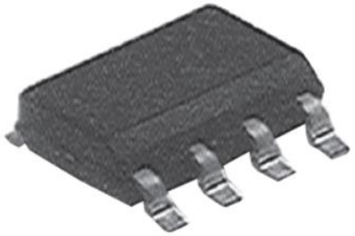
What we would see in this textbook is this.

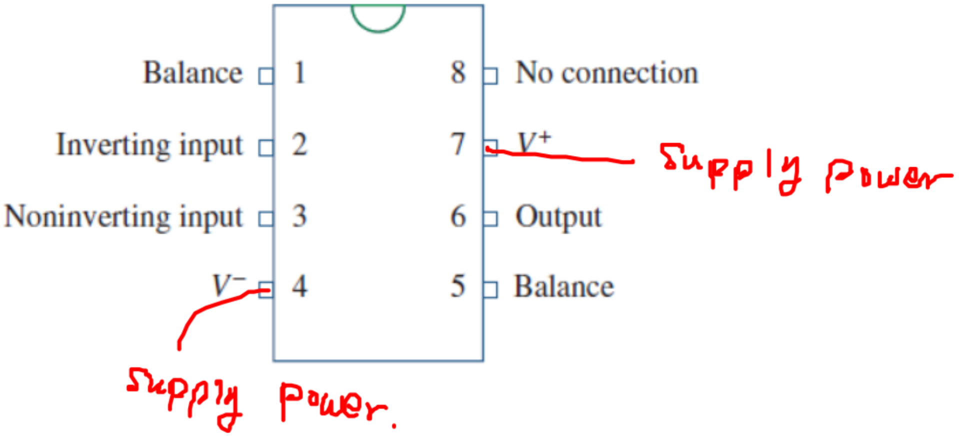




So don’t be afraid of op amp

5.2 Operational Amplifiers

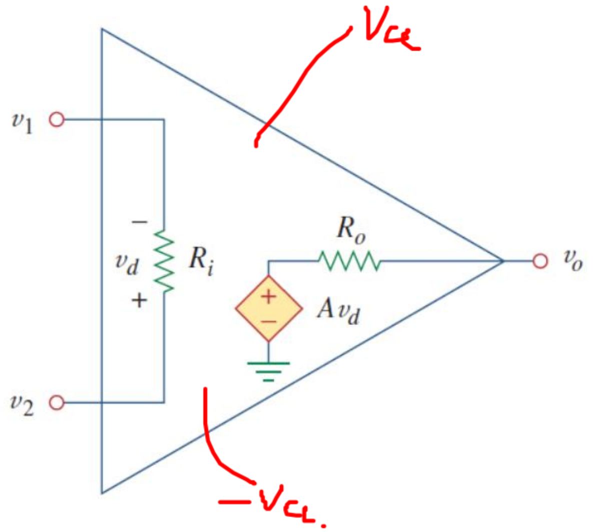




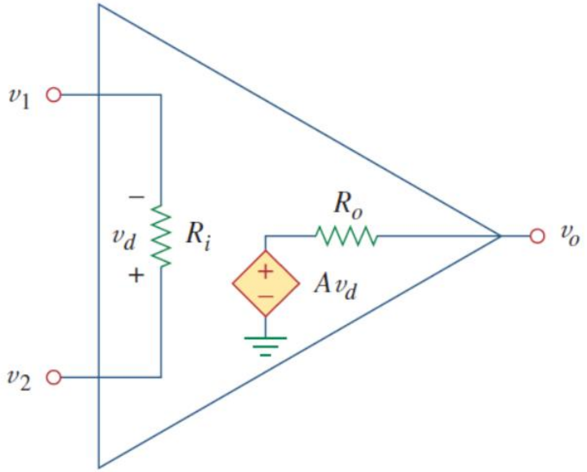
We treat the op amp as a circuit building block and **simply study what takes place at its terminals**. A full discussion of what is inside the op amp is in VE311.

What is Op Amp?

Op amp basically amplifies input voltage by the amount of a gain.



$$v_o = Av_d = A(v_2 - v_1)$$



$$v_o = Av_d = A(v_2 - v_1)$$

R_i is R_{Th} seen at the input.
R_o is R_{Th} seen at the output.

The output $v_o = Av_d = A(v_2 - v_1)$ where $\boldsymbol{v_d}$ is called the differential input voltage and ***A*** is called the open-loop voltage gain. It is the gain of the op amp without any external feedback from output to input.

TABLE 5.1 Typical ranges for op amp parameters

Parameter	*Typical range*	*Ideal values*
Open-loop gain, A	10^5 to 10^8	∞
Input resistance, R_i	10^5 to 10^{13} Ω	∞
Output resistance, R_o	10 to 100 Ω	0
Supply voltage, V_{CC}	5 to 24 V	NA

Practical Limitation of Op Amp

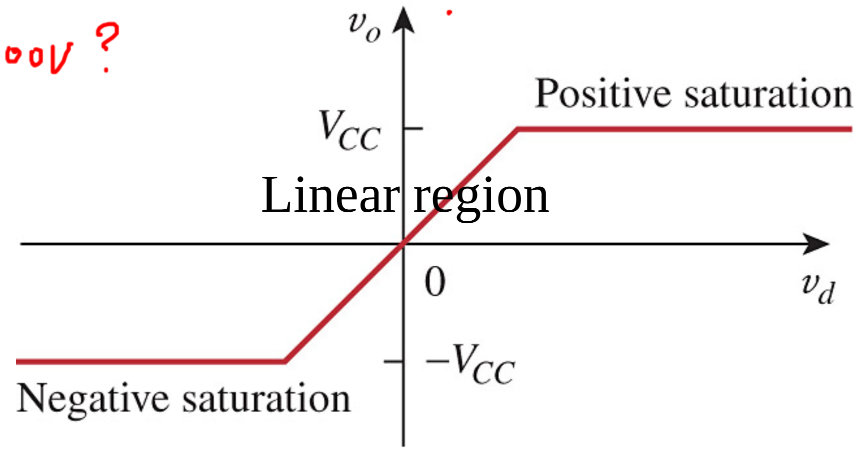
The magnitude of its output voltage cannot exceed $|V_{CC}|$. In other words, the output voltage is dependent on and is limited by the power supply voltage.

1V → 10V

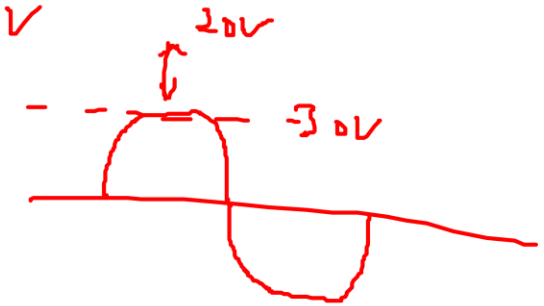
5V Sine x 10.

x10 100V ?

x100



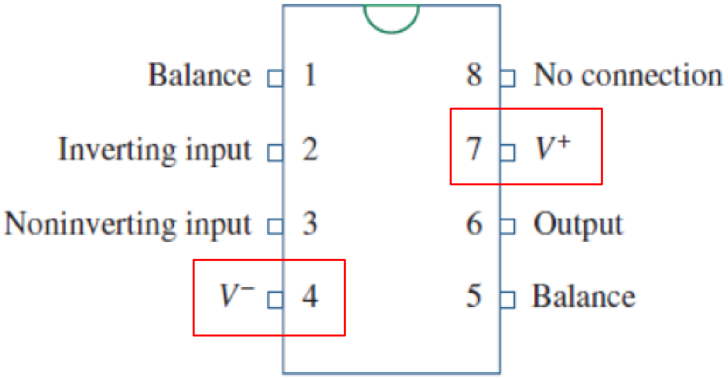
30V

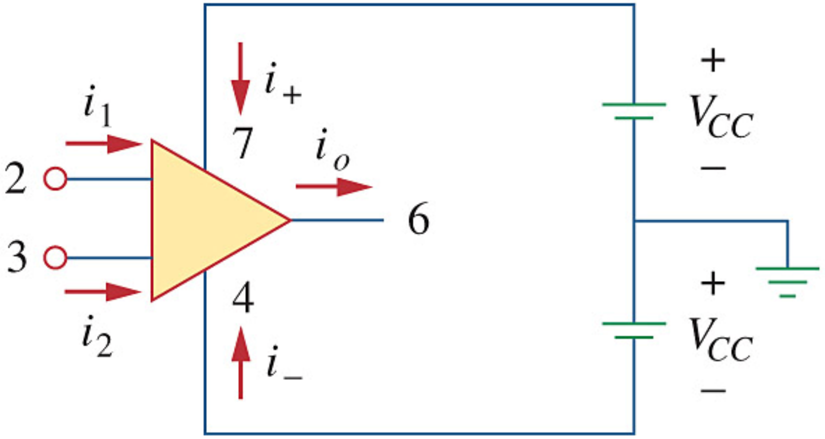


We will assume that our op amps operate in the linear mode

Powering Op Amp

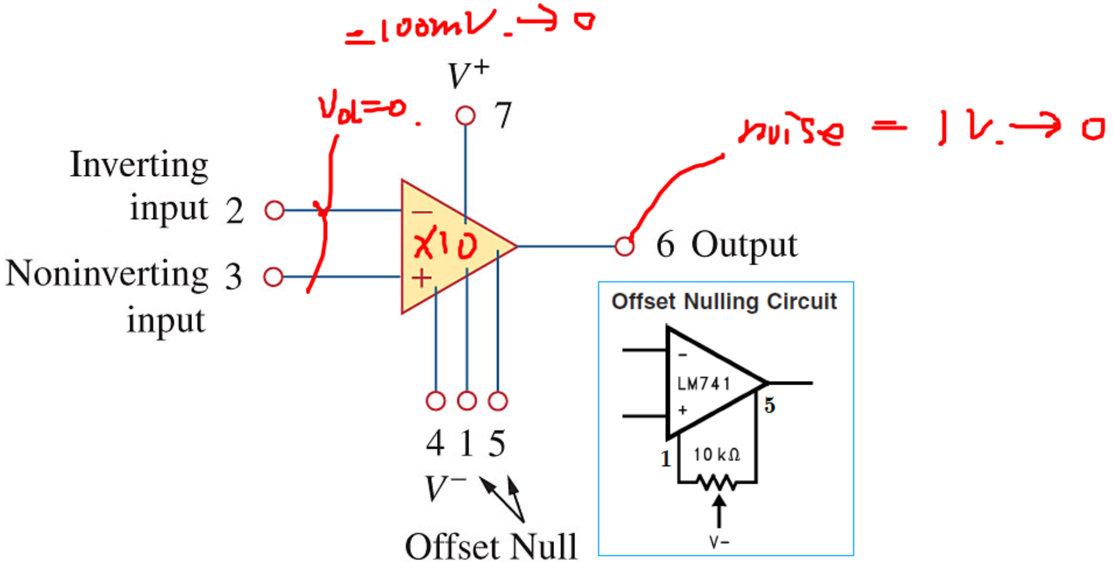
As an active element, the op amp must be powered by one or two voltage.





Although the power supplies are often ignored in op amp circuit diagrams for the sake of simplicity, the power supply currents must not be overlooked. By KCL, $i_o = i_1 + i_2 + i_+ + i_-$

Offset Null



The offset null pin is mainly used to remove the voltage difference between the inverting and non inverting pins.

If there exits some difference in the voltages between two inputs (ideally 0), it will be amplified by the op amp, which will distort output results.

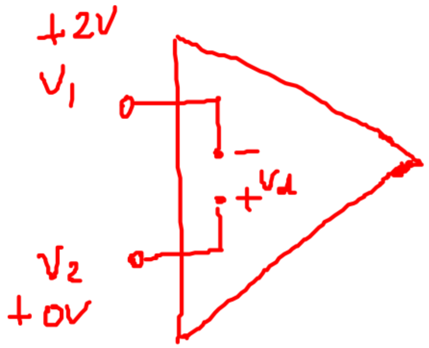
5.3 Ideal Op Amp

Op amp basically amplifies input voltage by the amount of a gain. From now on we will assume that Op amp is an ideal op amp.

Ideal op amp has the following characteristics:

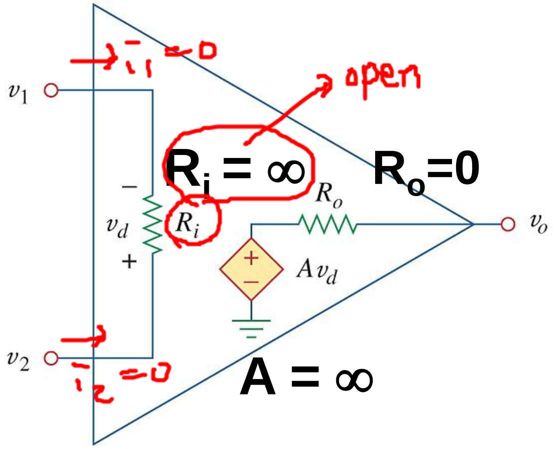
1. Infinite open-loop gain, $A = \infty$.
2. Infinite input resistance, $R_i = \infty$.
3. Zero output resistance, $R_o = 0$.

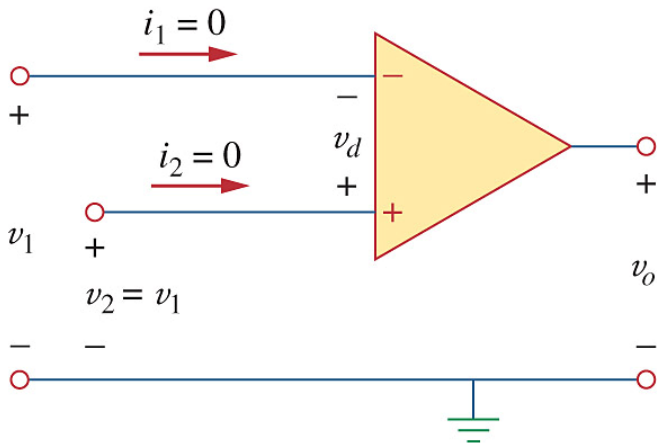
Important characteristics of ideal op amp



$-V_2 + V_d + V_1 = 0$

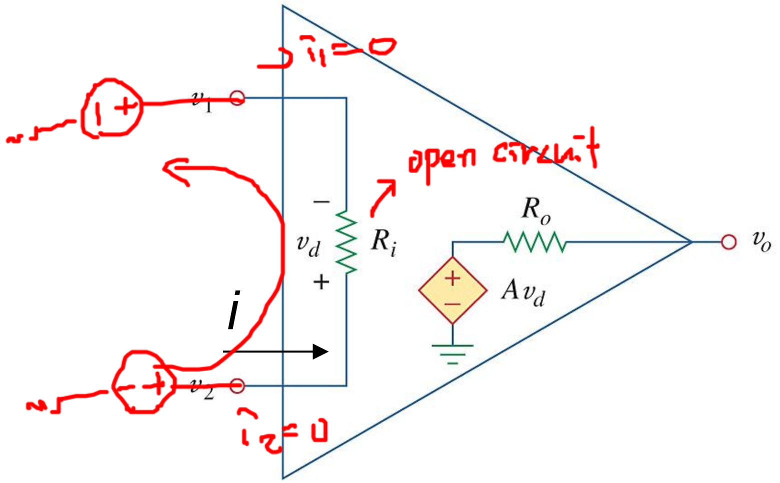
$V_d = 0.$





1. $i_1 = 0$ and $i_2 = 0$ due to infinite $R_i \rightarrow$ but the output current is not necessarily zero
2. $v_1 = v_2$ or $v_d = v_2 - v_1 = 0 \rightarrow$ **this does not mean $v_2 = 0$ and $v_1 = 0$**.

(i) $i_1 = i_2 = 0$

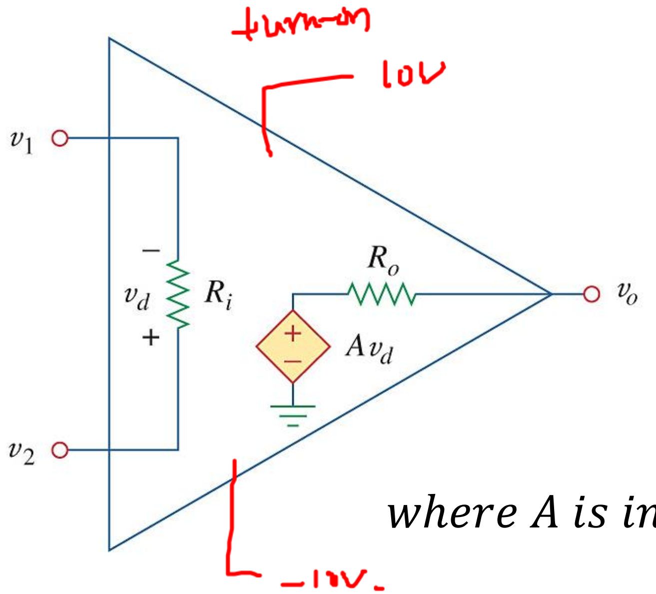


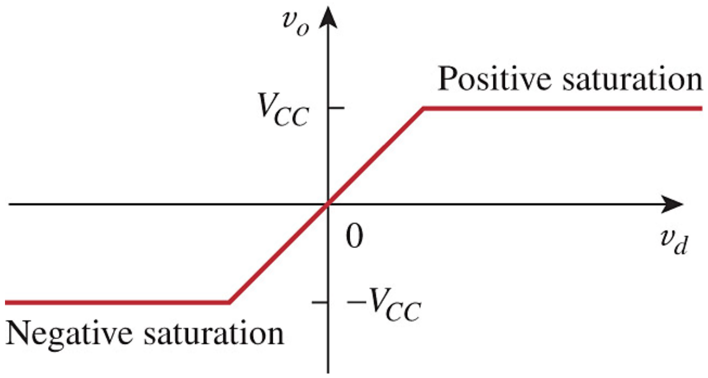
$= \infty$

- R_i is infinite → open circuit → no current flows

- By KVL: $-V_2 + \cancel{R_i i}^{\,\to 0} + V_1 = 0$ $\quad V_1 = V_2$, $V_d = 0$
  because R_i is infinite $i = 0$ → $i_1 = i_2 = 0$

(ii) $v_d = v_2 - v_1 = 0$



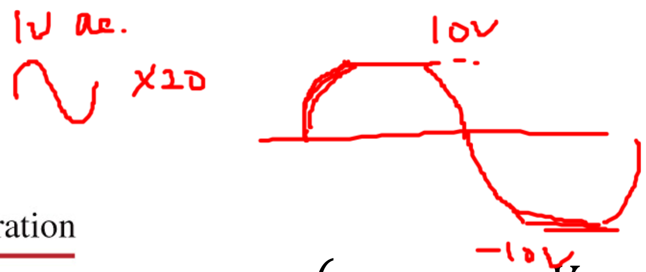


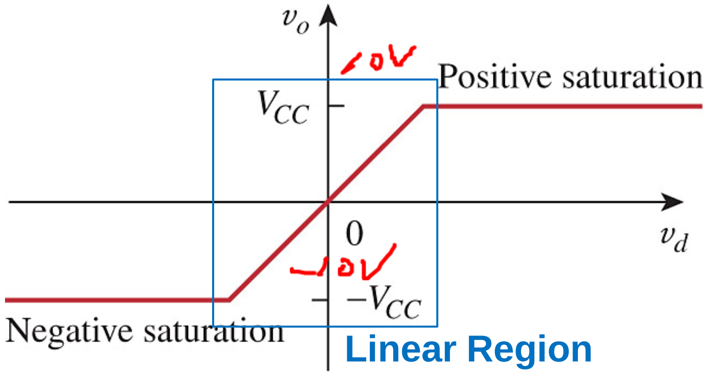
$$where\ A\ is\ infinite\ \rightarrow\ v_d = 0$$

∞.

$v_o = Av_d < \infty$ where A is infinite $\rightarrow$ $v_d = 0$

Op amp operates in the linear region and therefore v_o cannot exceed V_{CC}





$$v_o = \begin{cases} -V_{CC} & when\ A(v_2 - v_1) < -V_{CC} \\ \boldsymbol{A(v_2 - v_1)} & when - V_{CC} < A(v_2 - v_1) < +V_{CC} \\ +V_{CC} & when\ A(v_2 - v_1) > +V_{CC} \end{cases}$$

When the magnitude of the input voltage difference is small, the op amp behaves **as a linear device**. We use the op amp in the linear operating region, and thus **a constraint is imposed on the input voltages, v_1 and v_2**.

For example, V_{CC} is 20 V (generally limited to 20 V), and the gain A is 10,000.
In this case, the input voltage difference must be less than 2 mV (V_{CC}/A). Because node voltages in the circuits are much larger than 2 mV, a voltage difference of less than 2 mV means the two voltages are essentially equal, i.e. **$v_1 = v_2$**.

5.4 Inverting Amplifier (Inverter)

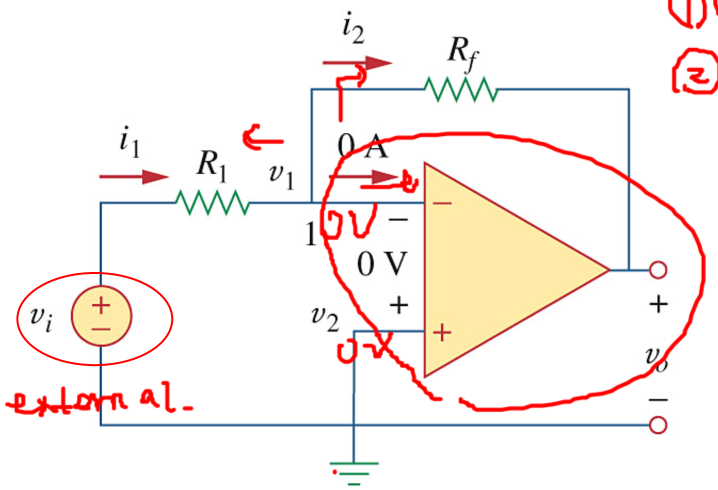
An inverting amplifier reverses the polarity of the input signal while amplifying it.

The closed-loop gain is $A_v = \dfrac{v_o}{v_i} = -\dfrac{R_f}{R_1}$

$$\frac{0 - v_i}{R_1} + \frac{0 - V_o}{R_f} + 0 = 0$$

① $V_d = 0$

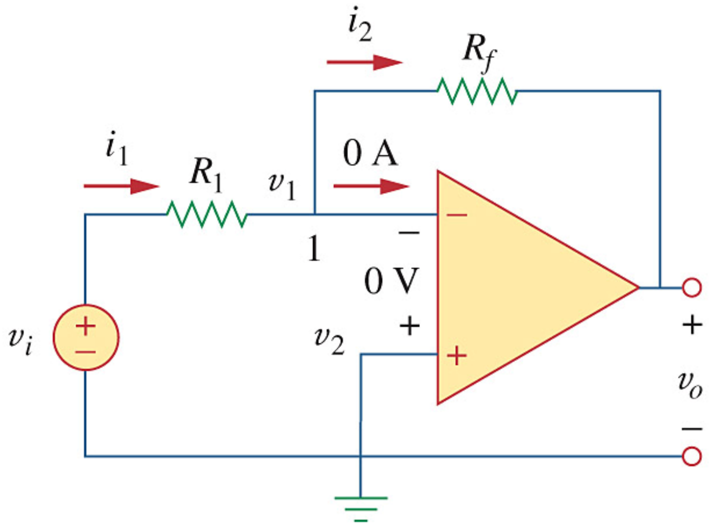
② $i_- = 0$



$$v_o = -\frac{R_f}{R_1} v_i$$

$\frac{R_f}{R_1}$ = gain

Input at the inverting terminal



Proof :

$$i_1 = i_2 \Rightarrow \frac{v_i - v_1}{R_1} = \frac{v_1 - v_o}{R_f}$$

$$v_1 = v_2 = 0$$

$$\frac{v_i}{R_1} = -\frac{v_o}{R_f} \Rightarrow v_o = -\frac{R_f}{R_1} v_i \Rightarrow A_v = \frac{v_o}{v_i} = -\frac{R_f}{R_1}$$

The gain is the feedback resistance divided by the input resistance $\rightarrow$ the gain depends only on the external elements connected to the op amp.

Inverting amplifier equivalent circuit

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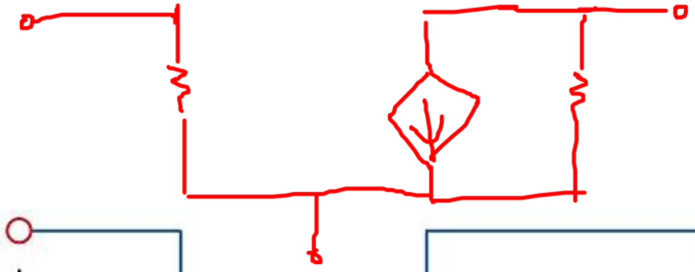
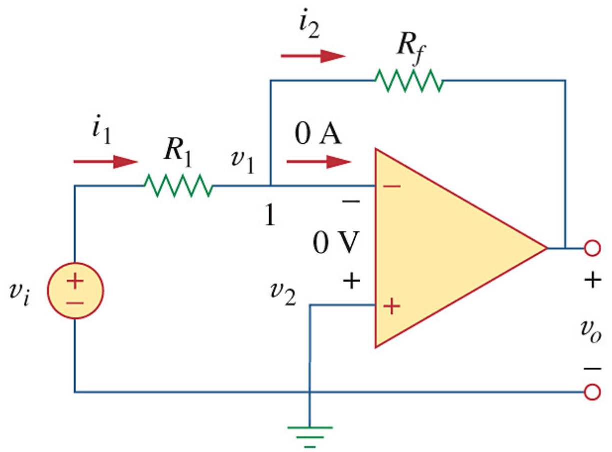
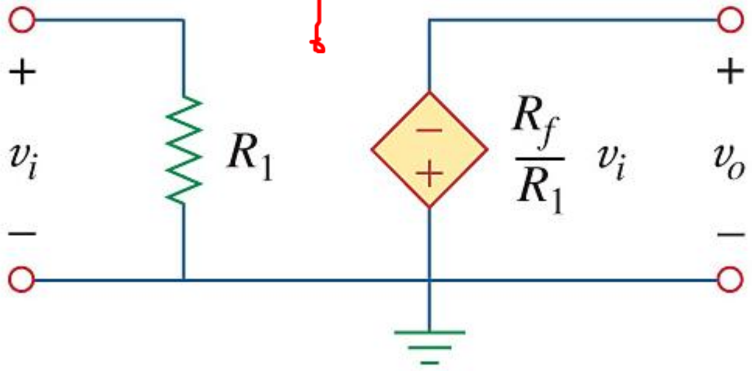






$$v_o = -\frac{R_f}{R_1} v_i$$

$$v_o = -\frac{R_f}{R_1} v_i$$

Two circuits are equivalent

Example 5.3 Refer to the circuit in Fig. 5.12. If $v_i = 0.5$ V, calculate (a) the output voltage v_o, and (b) the current in the 10-kΩ resistor.

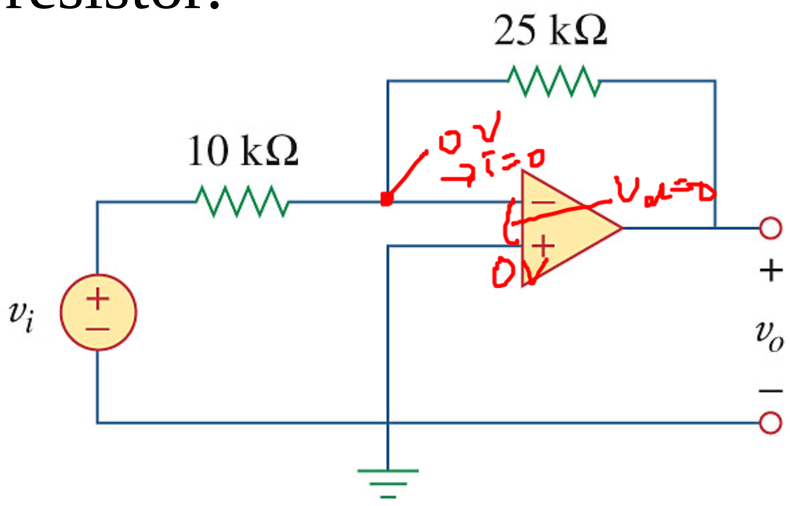


Figure 5.12

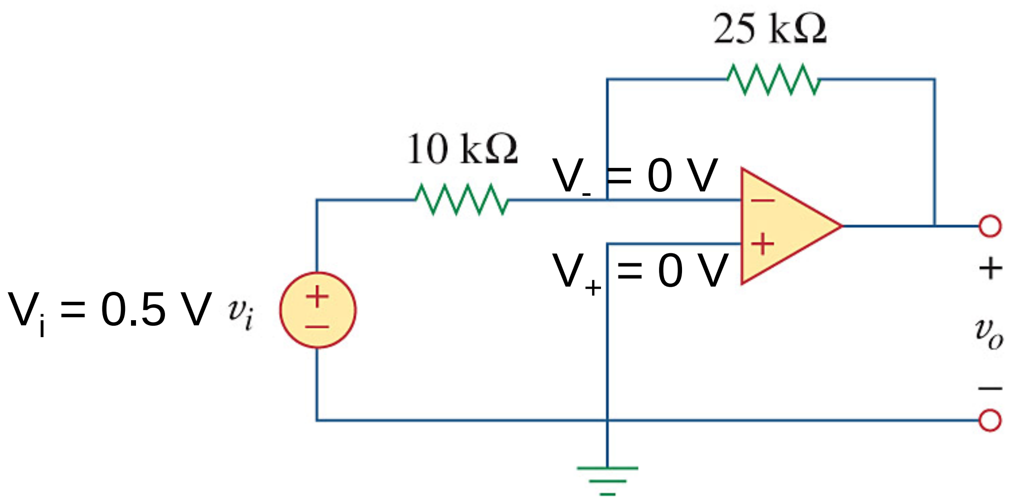
$$\frac{0-V_i}{10k} + \frac{0-V_o}{25k} = 0$$

Using the equation

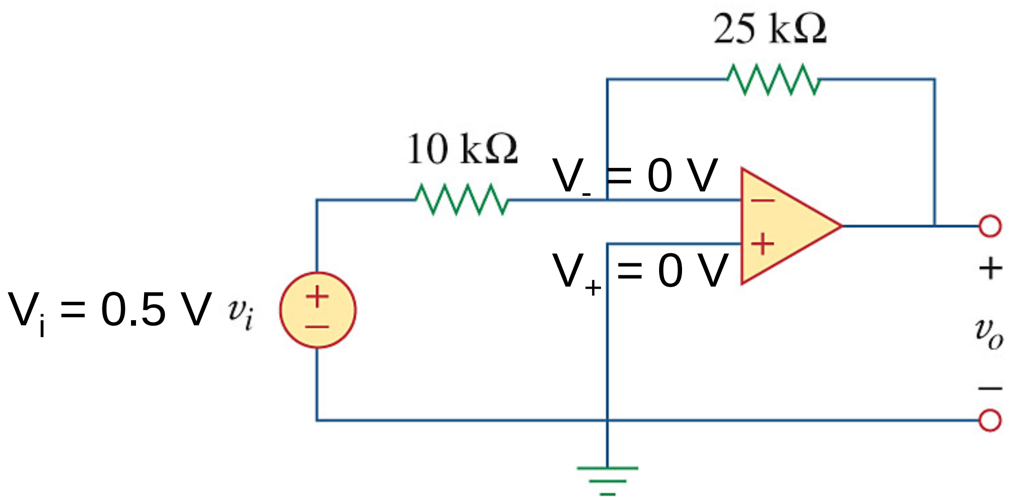
$$v_o = -\frac{R_f}{R_1} v_i = -\frac{25}{10} \times 0.5 = -1.25 \text{ (V)}$$

$$i = \frac{v_i}{R_1} = \frac{0.5}{10 \times 10^3} = 5 \times 10^{-5} \text{ (A)} = 50\ \mu\text{A}$$

What if you do not remember the equation?



What if you do not remember the equation?



$$\frac{0-0.5}{10k} + i_{-} + \frac{0-v_0}{25k} = 0 \;\rightarrow -4v_0 = 5$$

Therefore v_o = -1.25 [V]
Current in the 10kΩ = 0.5/10k = 0.5 μA

Note: Do not apply KCL at output node (output current is unknown)

Practical Problem 5.4

Two kinds of current-to-voltage converters (also known as transresistance amplifier) are shown in Fig. 5.15.

(a) Show that for the converter in Fig. 5.15(a), $v_o / i_s = -R$

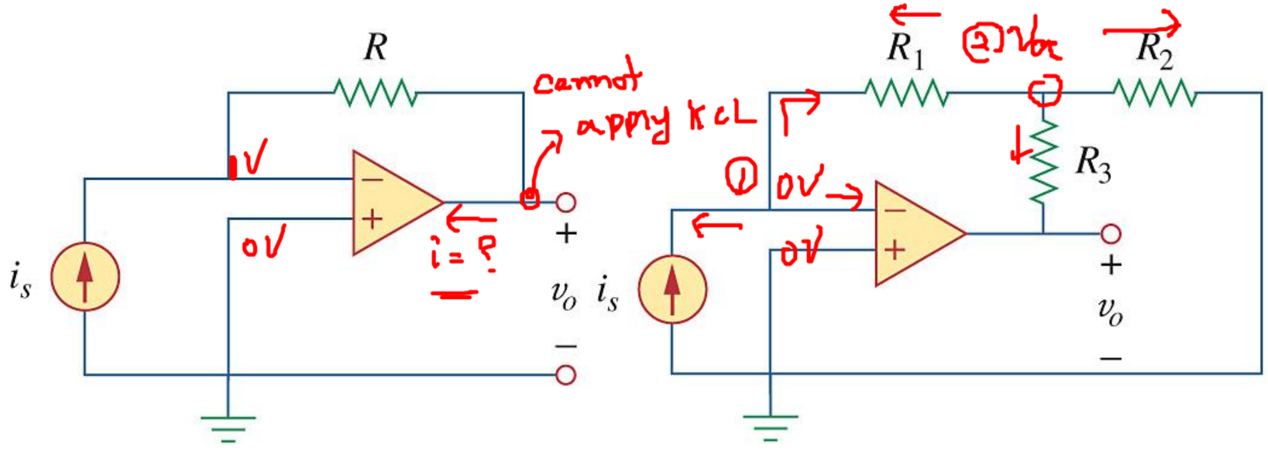
(b) Show that for the converter in Fig. 5.15(b), $v_o / i_s = -R_1(1 + R_3 / R_2 + R_3 / R_1)$

RC, RL → 1st ODE

RLC → 2nd ODE

① $-i_s + \frac{0 - v_a}{R_1} = 0$ $v_a = -R_1 i_s$

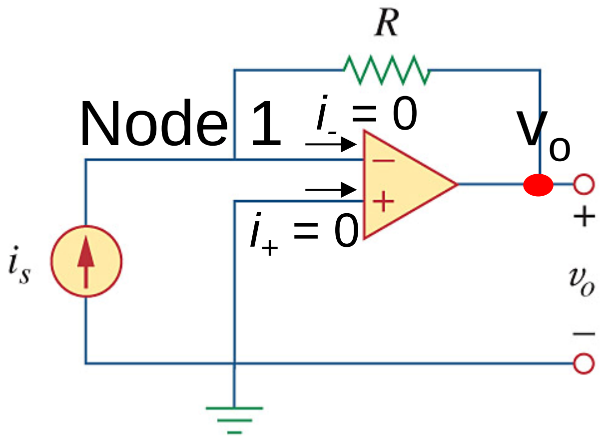
② $\frac{v_a - 0}{R_1} + \frac{v_a - v_o}{R_3} + \frac{v_a - 0}{R_2} = 0$



(a) (b)

Figure 5.15

(a)



(i) $V_+ = 0$ because it is grounded

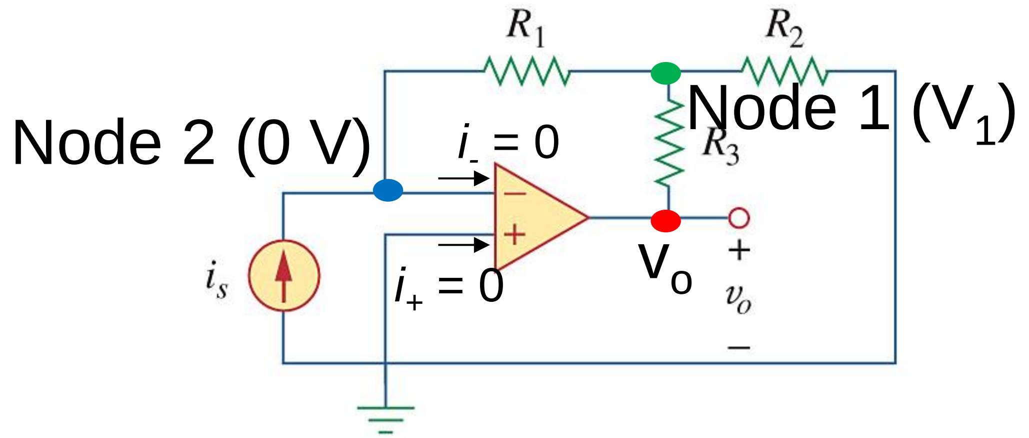
(ii) Therefore, $V_- = 0$

Nodal analysis from the Node 1 (V = 0)

$$-i_s + i_- + \frac{0 - v_o}{R} = 0 \; where \; i_- = 0$$

$$i_s = \frac{-v_o}{R} \rightarrow \boldsymbol{\frac{v_o}{i_s} = -R}$$

(b)



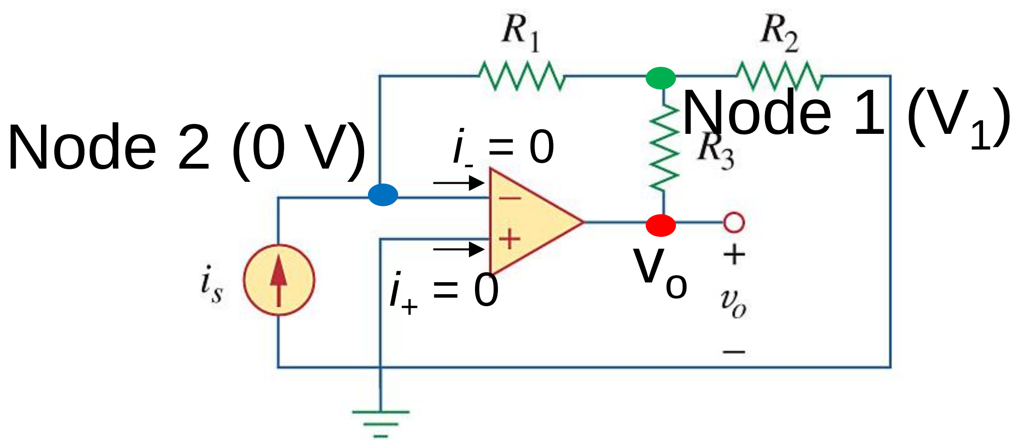
(i) $V_+ = 0$ because it is grounded
(ii) Therefore, $V_- = 0$

Nodal analysis from the Node 1 (V_1) and Node 2 (0 V)

$$\frac{V_1 - 0}{R_1} + \frac{V_1 - v_o}{R_3} + \frac{V_1 - 0}{R_2} = 0$$

$$-i_s + i_- + \frac{0 - V_1}{R_1} = 0 \rightarrow \ -i_s R_1 = V_1 \ (put\ into\ the\ first\ eq.)$$

(b)



$$\frac{-i_s R_1 - 0}{R_1} + \frac{-i_s R_1 - v_o}{R_3} + \frac{-i_s R_1 - 0}{R_2} = 0$$

$$-i_s(R_1 R_2 + R_1 R_3 + R_2 R_3) = R_2 v_o$$

$$\frac{v_o}{i_s} = -\frac{R_1 R_2 + R_1 R_3 + R_2 R_3}{R_2}$$

$$= -R_1\left(1 + \frac{R_3}{R_2} + \frac{R_3}{R_1}\right)$$

5.5 Noninverting Amplifier

Noninverting amplifier circuit: the input v_i is applied directly at the noninverting input terminal. This provides **positive voltage gain**.

$$\frac{v_i - 0}{R_1} + \frac{v_i - v_o}{R_f} + \cancel{i_-}^{\,0} = 0$$

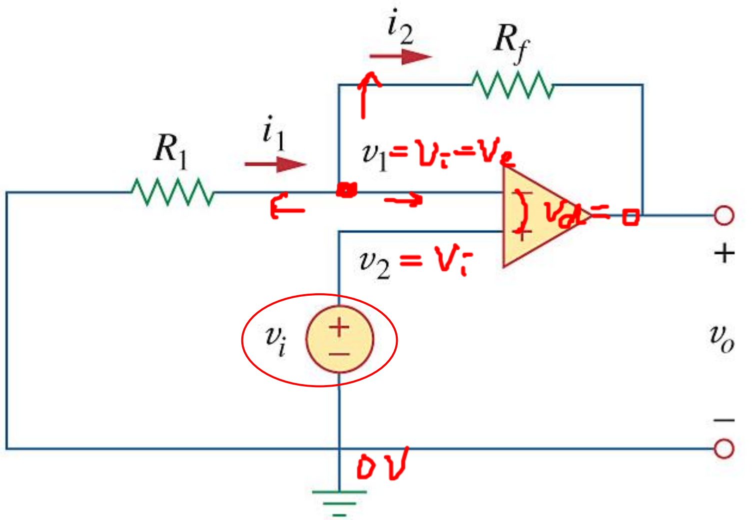
① $i_- = 0$

② $v_1 = v_i$

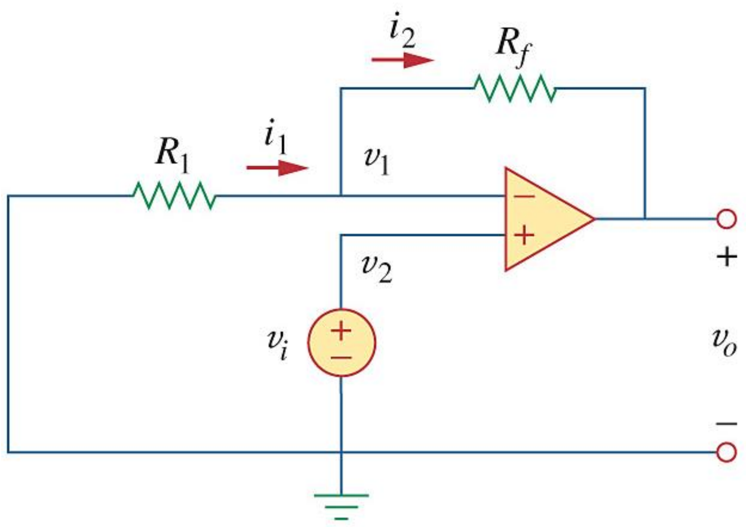
The closed-loop gain is

$$A_v = \frac{v_o}{v_i} = 1 + \frac{R_f}{R_1}$$

$$v_o = \left(1 + \frac{R_f}{R_1}\right) v_i$$



Input at the noninverting terminal



Proof :

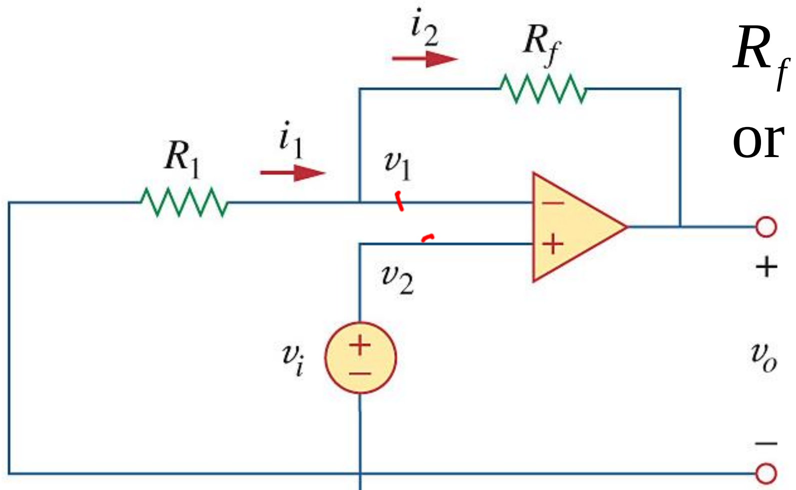
$$i_1 = i_2 \Rightarrow \frac{0 - v_1}{R_1} = \frac{v_1 - v_o}{R_f} \Rightarrow \frac{v_o}{v_1} = 1 + \frac{R_f}{R_1}$$

$$v_1 = v_2 = v_i$$

$$A_v = \frac{v_o}{v_i} = 1 + \frac{R_f}{R_1}$$

Again, the gain depends only on the external resistors

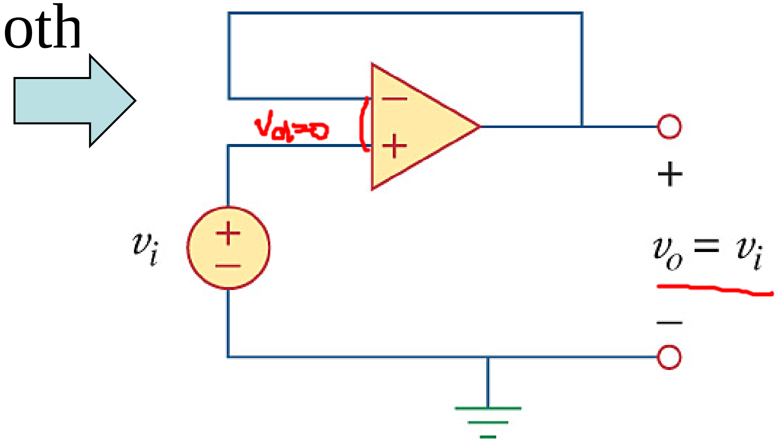
Voltage follower circuit



$$v_o = \left(1 + \frac{R_f}{R_1}\right) v_i$$

$R_f = 0$ or $R_1 = \infty$

or both

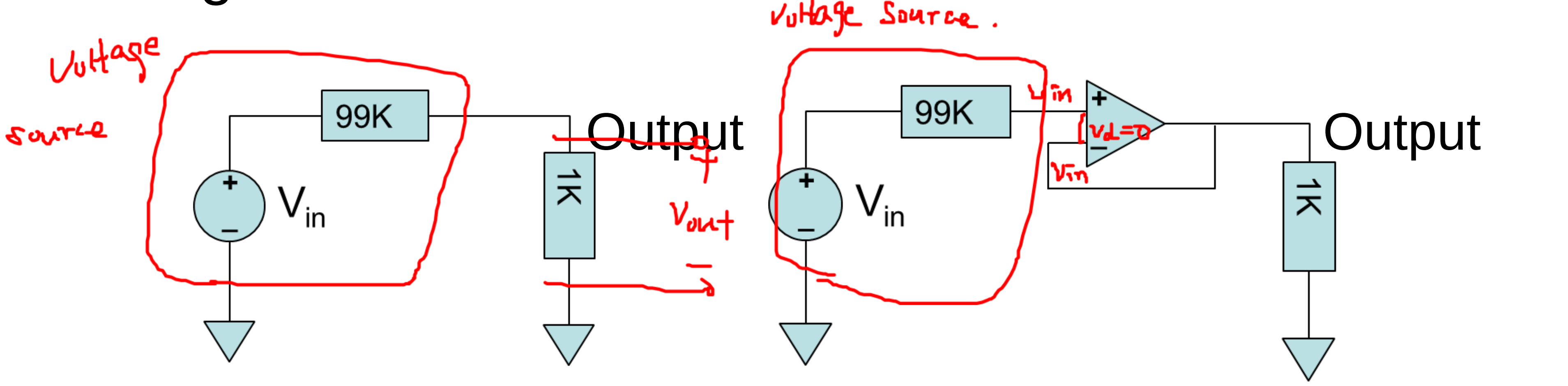


$$v_o = v_i$$

When R_f = 0 or R_1 = ∞ or both, then the gain is 1, which means **$v_i = v_o$** → **voltage follower**

Voltage follower circuit

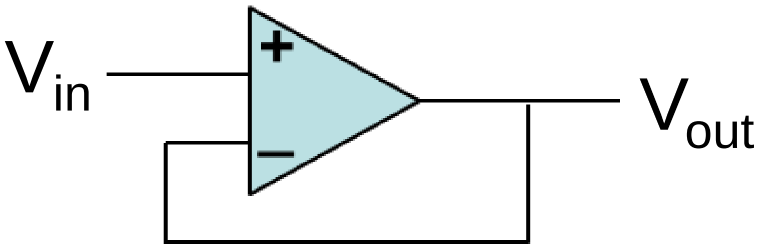
Advantage: It can supply a large current at the output while drawing almost no current from the input when the source has a high resistance.



Output = $0.01V_{in}$ Output = V_{in}

Although the voltage gain is only 1, the power gain is much larger.

Negative Feedback

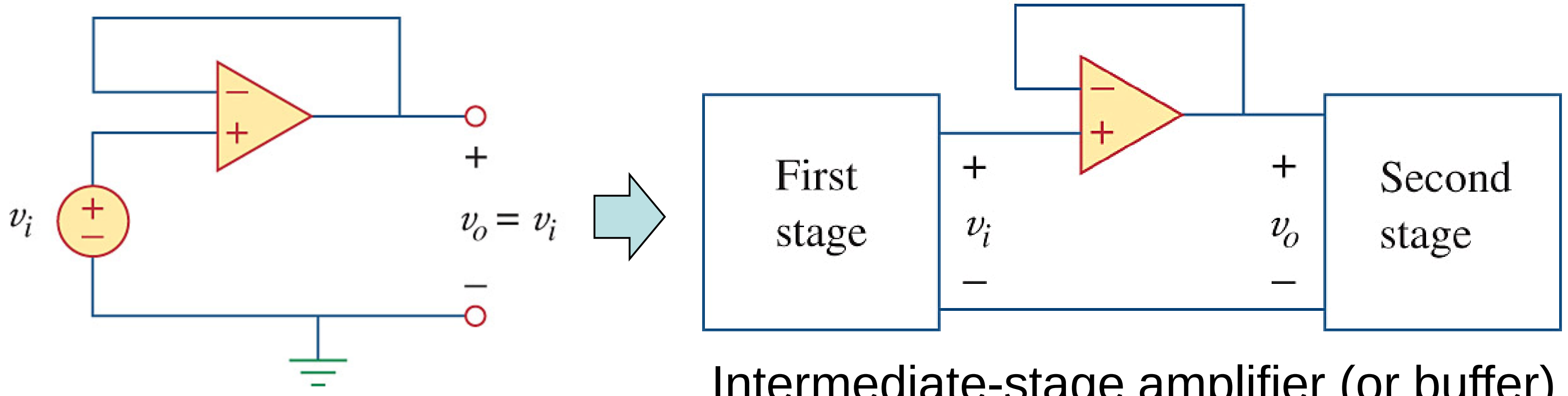


Negative feedback is when the occurrence of an event causes something to happen that counteracts the original event.
If op amp output V_{out} falls then V_- will fall by the same amount, and thus, $(V_+ - V_-)$ will increase. Negative feedback adjusts the output to make $V_+ \cong V_-$

$$V_{out}\downarrow = AV_d = A(V_+ - V_-\downarrow) = A(\overbrace{V_{in} - V_{out}\downarrow}^{V_d\uparrow}) \rightarrow V_{out}\uparrow$$

$$V_{out}(1 + A) = AV_{in} \rightarrow V_{out} = \frac{1}{1 + {}^{1}/_{A}} V_{in} \rightarrow V_{in}\ for\ large\ A$$

Voltage follower circuit as a buffer



Intermediate-stage amplifier (or buffer)

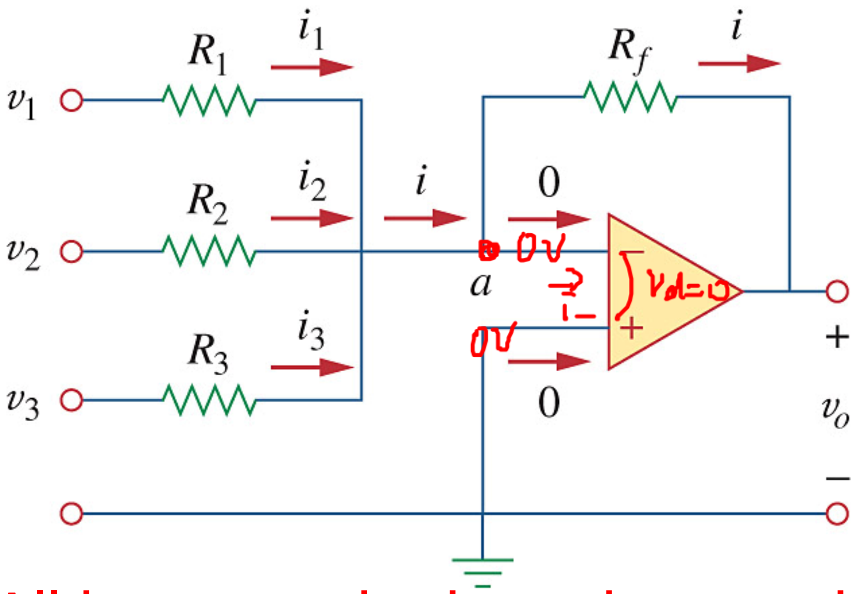
Due to **very high input impedance,** the voltage follower circuit is useful as an intermediate-stage (or **buffer**) amplifier to isolate one circuit from another.
→ minimizes interaction between the two stages and eliminates interstage loading.

5.6 Summing Amplifier (Adder)

The op amp also can perform addition and subtraction.

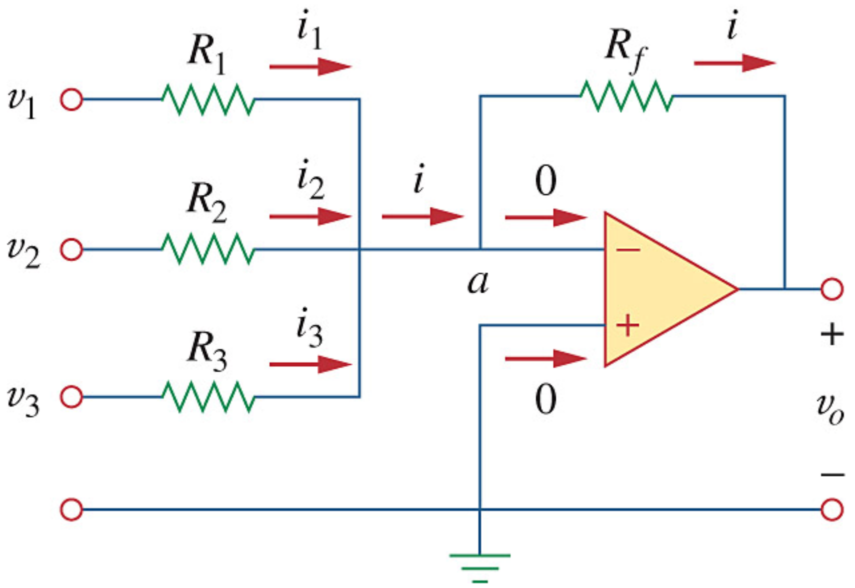
A summing amplifier is an op amp circuit that combines several inputs and produces **an output that is the weighted sum of the inputs**.

$\frac{0-v_1}{R_1}+\frac{0-v_2}{R_2}+\frac{0-v_3}{R_3}+\frac{0-v_o}{R_f}+i_- = 0$



$$v_o = -\left(\frac{R_f}{R_1}v_1 + \frac{R_f}{R_2}v_2 + \frac{R_f}{R_3}v_3\right)$$

All inputs at the inverting terminal

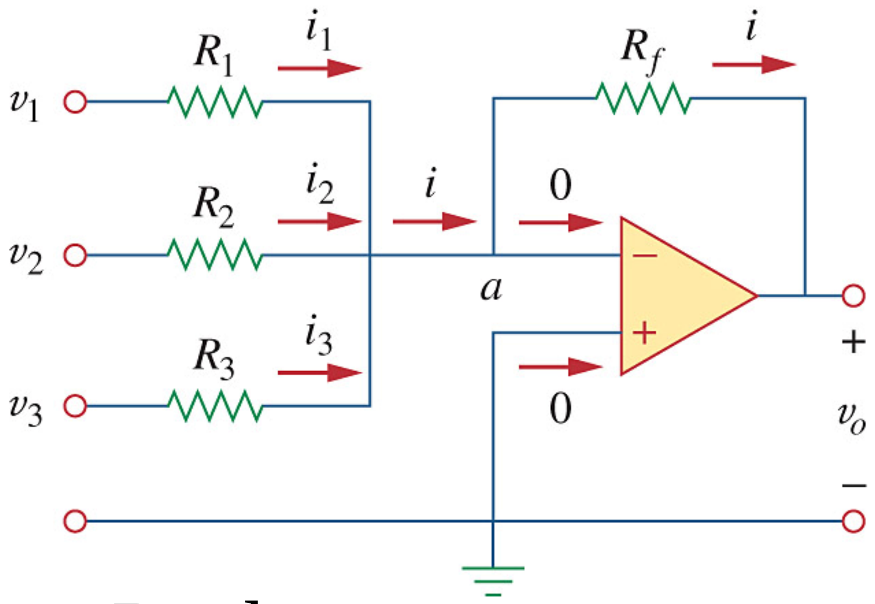


Proof :

$$i = i_1 + i_2 + i_3 \Rightarrow$$

$$\frac{0 - v_o}{R_f} = \frac{v_1 - 0}{R_1} + \frac{v_2 - 0}{R_2} + \frac{v_3 - 0}{R_3}$$

$$v_o = -\left(\frac{R_f}{R_1} v_1 + \frac{R_f}{R_2} v_2 + \frac{R_f}{R_3} v_3 \right)$$



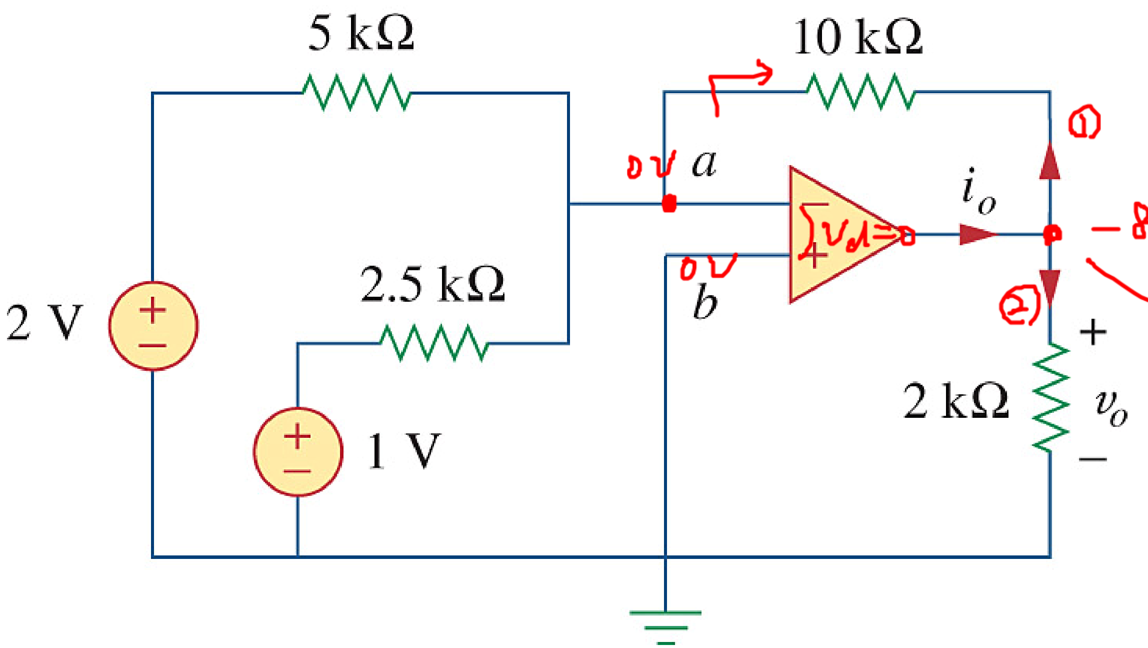
If $R_3 = R_2 = R_1$, then

$$v_o = -\frac{R_f}{R_1}\left(v_1 + v_2 + v_3\right)$$

If $R_3 = R_2 = R_1 = R_f$, then

$$v_o = -\left(v_1 + v_2 + v_3\right)$$

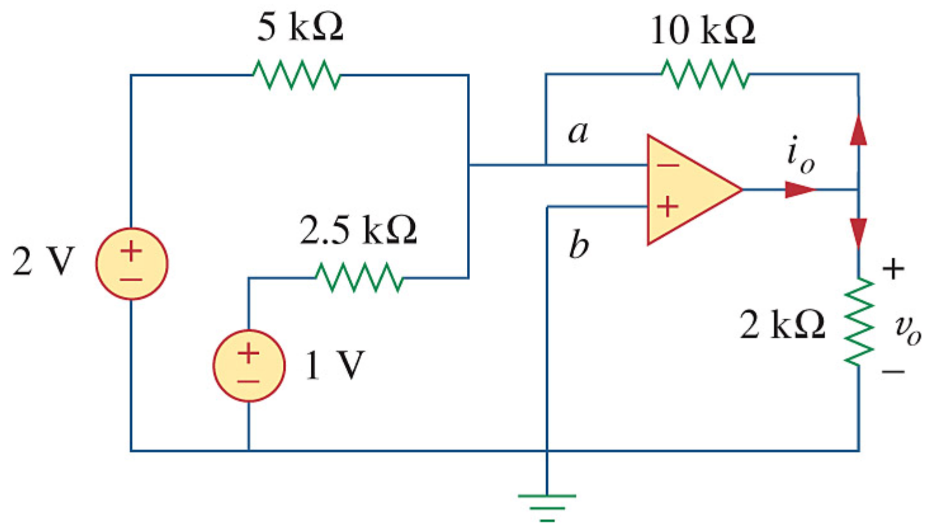
Example 5.6 Calculate v_o and i_o in the op amp circuit in Fig. 5.22.



$$\frac{0-2}{5K} + \frac{0-1}{2.5K} + \frac{0-v_0}{10K} + \cancel{i_-}^{0} = 0$$

$$v_0 = -8V.$$

by KCL : $i_0 = ① + ②$



$$v_o = -\left(\frac{R_f}{R_1}v_1 + \frac{R_f}{R_2}v_2\right) \quad = -\frac{10k}{5k}2 - \frac{10k}{2.5k}1 = -8\,V$$

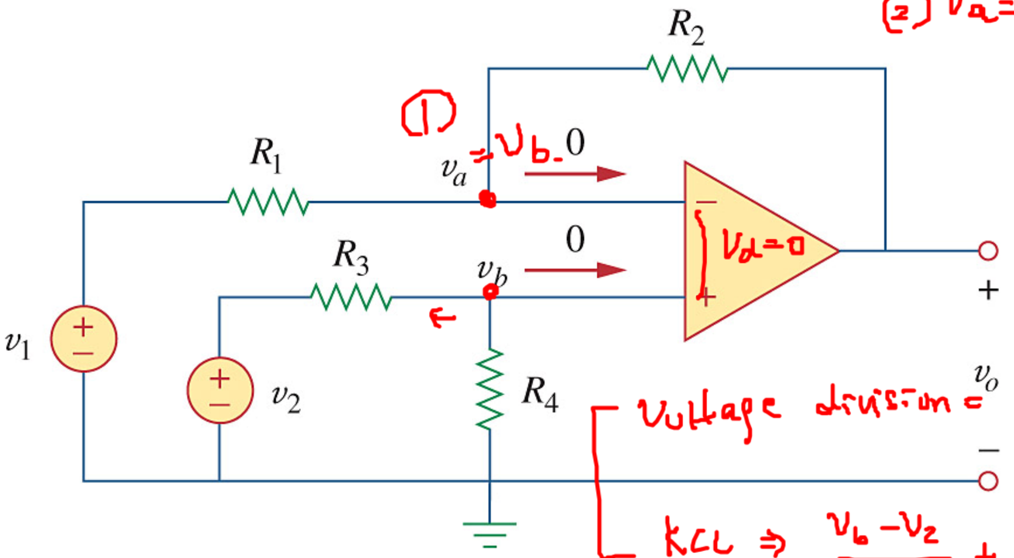
$$i_o = \frac{v_o}{2k} + \frac{v_o - 0}{10k} = \frac{-8}{2k} + \frac{-8}{10k} = -0.0048\,A = -4.8\,mA$$

5.7 Difference Amplifier (Subtractor)

Difference (or differential) amplifiers are used in various applications where there is a need to **amplify the difference between two input signals**.

① KCL: $\frac{v_a - v_1}{R_1} + \frac{v_a - v_o}{R_2} + \cancel{i_-}^{\,0} = 0$

② $v_a = v_b$



$$v_o = \left(\frac{R_2}{R_1} + 1\right)\frac{R_4}{R_3 + R_4}v_2 - \frac{R_2}{R_1}v_1$$

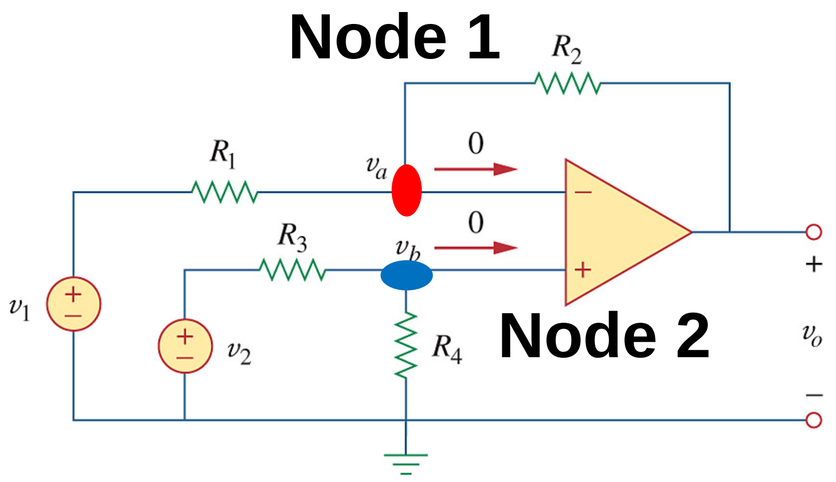
$$v_o = \frac{R_2(1 + R_1/R_2)}{R_1(1 + R_3/R_4)}v_2 - \frac{R_2}{R_1}v_1$$

Voltage division: $\frac{R_4}{R_3+R_4} v_2 = v_b$.

KCL ⇒ $\frac{v_b - v_2}{R_3} + \frac{v_b}{R_4} + \cancel{i_+}^{\,0} = 0$

Inputs at both inverting and noninverting terminal

Proof



Node 1:

$$\frac{v_a - v_1}{R_1} + \frac{v_a - v_o}{R_2} = 0 \rightarrow R_2\left(\frac{1}{R_1} + \frac{1}{R_2}\right)v_a - \frac{R_2}{R_1}v_1 = v_o$$

Node 2:

$$\frac{v_b - v_2}{R_3} + \frac{v_b - 0}{R_4} = 0, because\ v_a = v_b$$

$$v_a = v_b = \frac{R_4}{R_3 + R_4}v_2$$

Put into the equation

$$R_2\left(\frac{1}{R_1}+\frac{1}{R_2}\right)v_a-\frac{R_2}{R_1}v_1=v_o$$

$$v_a=v_b=\frac{R_4}{R_3+R_4}v_2$$

$$\rightarrow \left(\mathbf{1}+\frac{\boldsymbol{R_2}}{\boldsymbol{R_1}}\right)\left(\frac{\boldsymbol{R_4}}{\boldsymbol{R_3}+\boldsymbol{R_4}}\right)\boldsymbol{v_2}-\frac{\boldsymbol{R_2}}{\boldsymbol{R_1}}\boldsymbol{v_1}=\boldsymbol{v_o}$$

$$v_o=\frac{R_2(1+R_1/R_2)}{R_1(1+R_3/R_4)}v_2-\frac{R_2}{R_1}v_1$$

Because a difference amplifier must reject a signal common to the two inputs, the amplifier must have the property that $v_o = 0$ when $v_1 = v_2$.

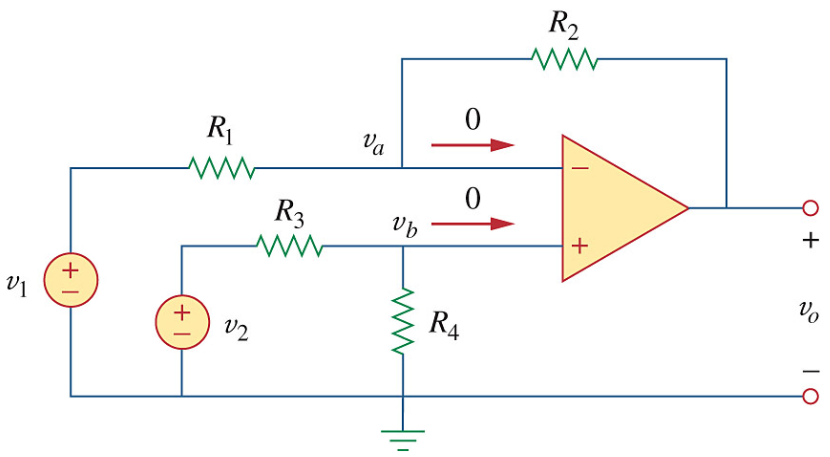
$$v_o = \frac{R_2(1 + R_1/R_2)}{R_1(1 + R_3/R_4)} v_2 - \frac{R_2}{R_1} v_1$$

If $R_4 / R_3 = R_2 / R_1$, then

$$v_o = \frac{R_2}{R_1}(v_2 - v_1)$$

If $R_2 = R_1$ and $R_3 = R_4$ then

$$v_o = v_2 - v_1 = 0$$



5.8 Cascaded Op Amp Circuits

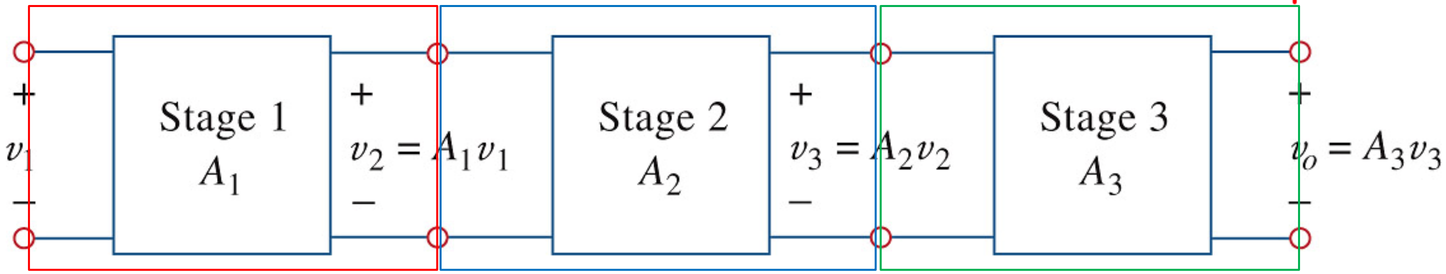
A cascade connection is a head-to-tail arrangement of several op amp circuits such that **the output of one is the input of the next.**

$v_2 = A_1 v_1$ $v_3 = A_2 v_2$ $v_o = A_3 v_3$

$= A_3 A_2 v_2$

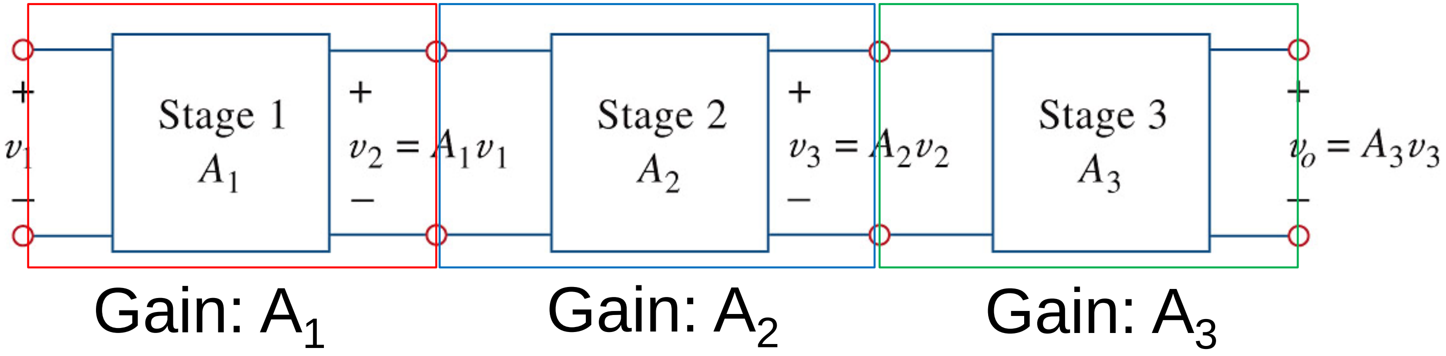
$= A_3 A_2 A_1 v_1$

$v_o / v_1 = A_3 A_2 A_1$



Each circuit is called Stage

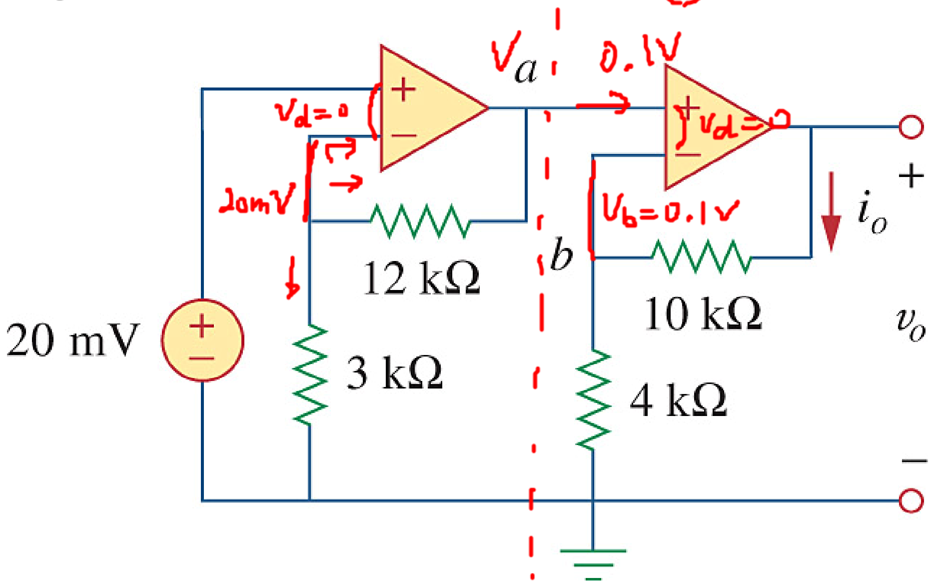
Gain of Cascaded Op Amp



Original input signal is increased by the **gain of the individual stage**, and the final gain is **the product of all gains at each stage**.

$$A = \frac{v_o}{v_1} = \frac{v_2}{v_1} \cdot \frac{v_3}{v_2} \cdot \frac{v_o}{v_3} = A_1 A_2 A_3$$

Example 5.9 Find v_o and i_o in the circuit in Fig. 5.29.



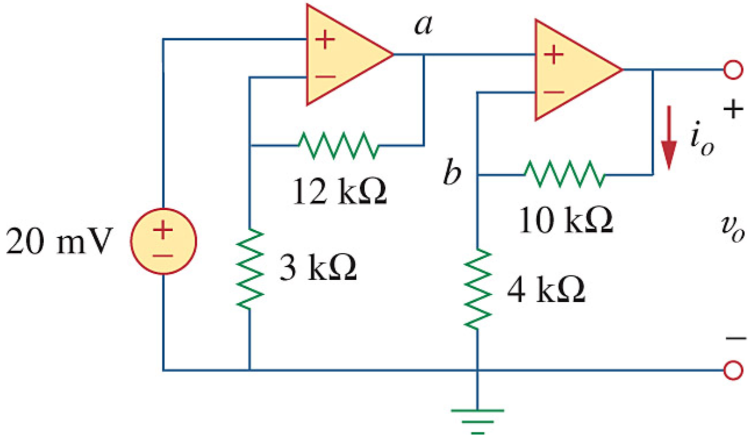
① $\frac{20mV-0}{3K} + \frac{20mV-V_a}{12K} + \cancel{i}^{\,0} = 0$

$V_a = 0.1V.$

② $\frac{0.1-0}{4k} + \frac{0.1-V_o}{10K} + \cancel{i}^{\,0} = 0$

$V_o = 0.35V$

$i_o = \frac{0.35-0.1}{10K.}$

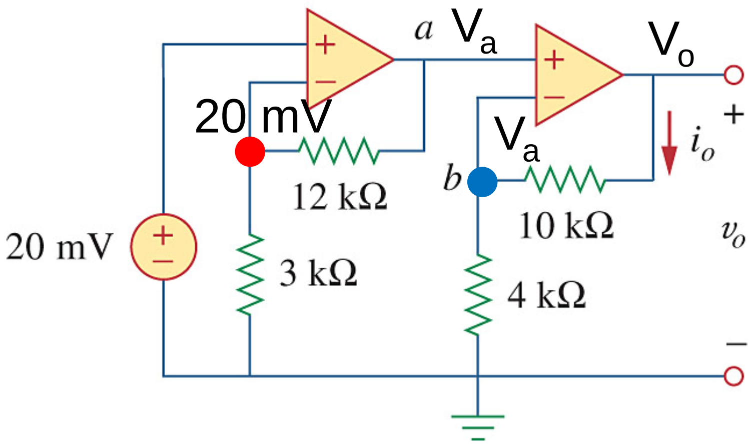


Input is to $V_+ \rightarrow$ noninverting amplifier $A_v = \dfrac{v_o}{v_i} = 1 + \dfrac{R_f}{R_1}$

$$v_a = \left(1 + \frac{12}{3}\right) \times 20 \times 10^{-3} = 0.1\ \text{(V)}$$

$$v_o = \left(1 + \frac{10}{4}\right) v_a = \left(1 + \frac{10}{4}\right) \times 0.1 = 0.35\ \text{(V)}$$

$$i_o = \frac{v_o}{10 + 4} = \frac{0.35}{14} = 0.025\ \text{(mA)} = 25\ \mu\text{A}$$



I don't remember any equation.. then,

(1) First op amp

By KCL: $\frac{20mV - V_a}{12k} + \frac{20mV}{3k} = 0 \rightarrow V_a = 0.1\,V$

(2) Second op amp

By KCL: $\frac{0.1 - V_o}{10k} + \frac{0.1}{4k} = 0 \rightarrow V_o = 0.35\,V$

$i_o = V_o/14k = 0.025$ mA = 25 μA

Example 5.10 If $v_1 = 1$ V and $v_2 = 2$ V, find v_o in the op amp circuit of Fig. 5.31.

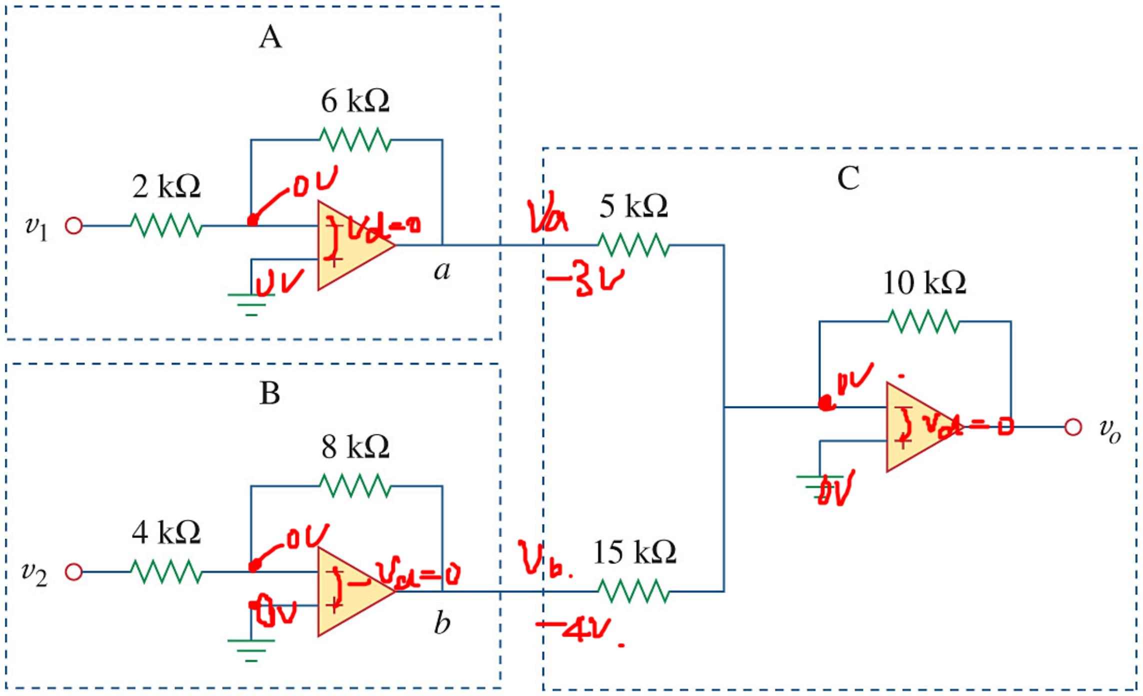


Figure 5.31

A: $\frac{0-v_1}{2k} + \frac{0-V_a}{6k} + \cancel{i}^{0} = 0$

$v_1 = 1V.$

$V_a = -3V$

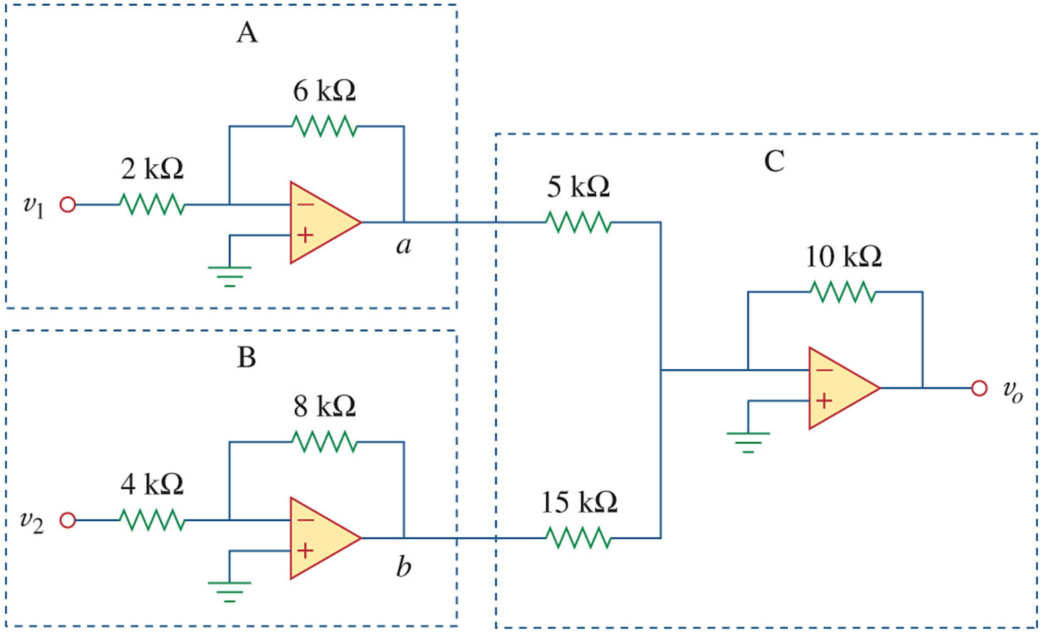
B: $\frac{0-V_2}{4k} + \frac{0-V_b}{8k} + \cancel{i}^{0} = 0$

$V_2 = 2V$

$V_b = -4V$

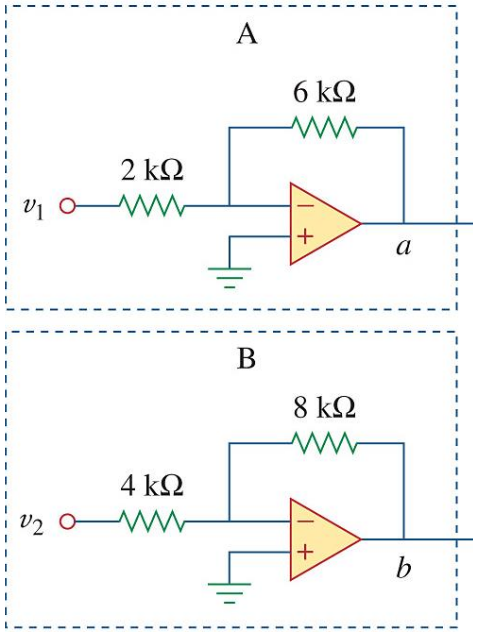
C: $\frac{0-(-3)}{5k} + \frac{0-(-4)}{15k} + \frac{0-V_o}{10k} + \cancel{i}^{0} = 0$

$V_o = 8.67$ V



We have this scary circuit.. Take a look at **each stage**

- A and B are noninverting amplifiers.
- C is a summing amplifier.
- Solve it stage by stage.

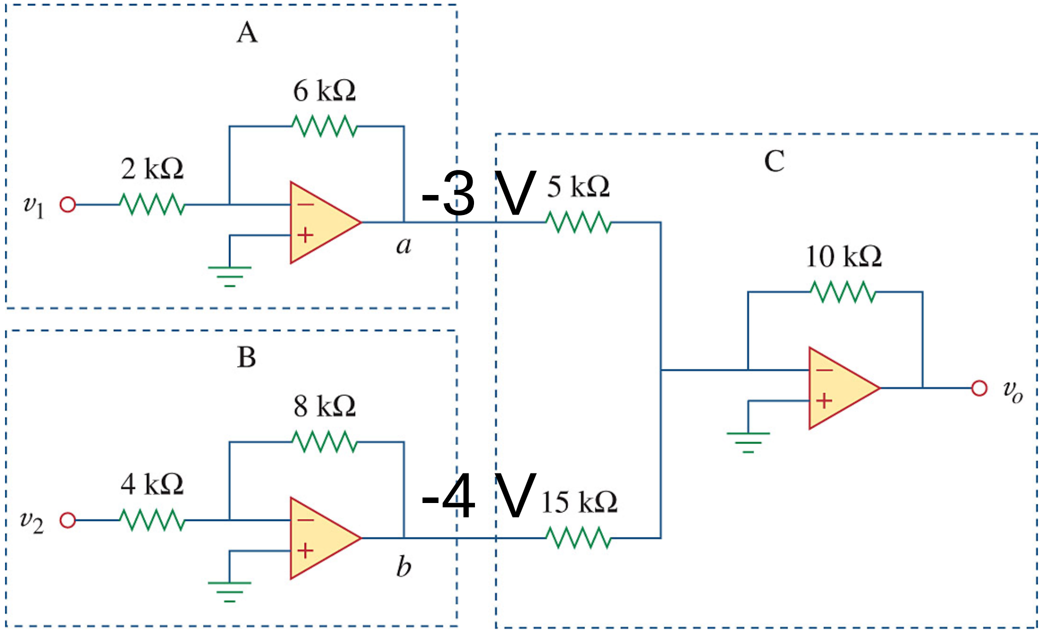


$$v_o = -\frac{R_f}{R_1} v_i$$

A and B are inverting amplifiers

$$v_a = -\frac{6}{2} v_1 = -\frac{6}{2} \times 1 = -3 \text{ (V)}$$

$$v_b = -\frac{8}{4} v_2 = -\frac{8}{4} \times 2 = -4 \text{ (V)}$$



C is a summing amplifier

$$v_o = -\left(\frac{R_f}{R_1} v_1 + \frac{R_f}{R_2} v_2 + \frac{R_f}{R_3} v_3 \right)$$

$$v_o = -\left(\frac{10}{5} v_a + \frac{10}{15} v_b \right)$$

$$= -\left(\frac{10}{5} \times (-3) + \frac{10}{15} \times (-4) \right) = \frac{26}{3} \approx 8.67 \text{ (V)}$$

Again, you can solve it by using KCL without using equations

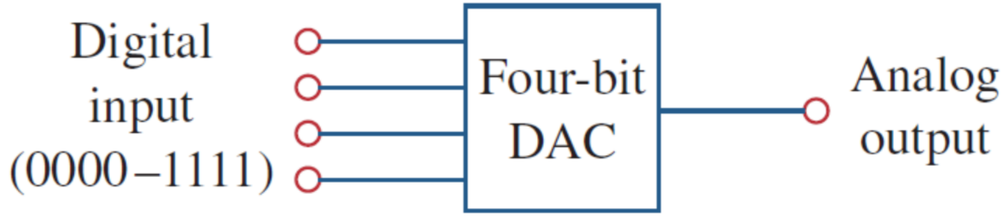
5.10. Applications

- The op amp is a fundamental building block in modern electronic instrumentation.
- e.g. digital-to-analog converters, analog computers, etc.

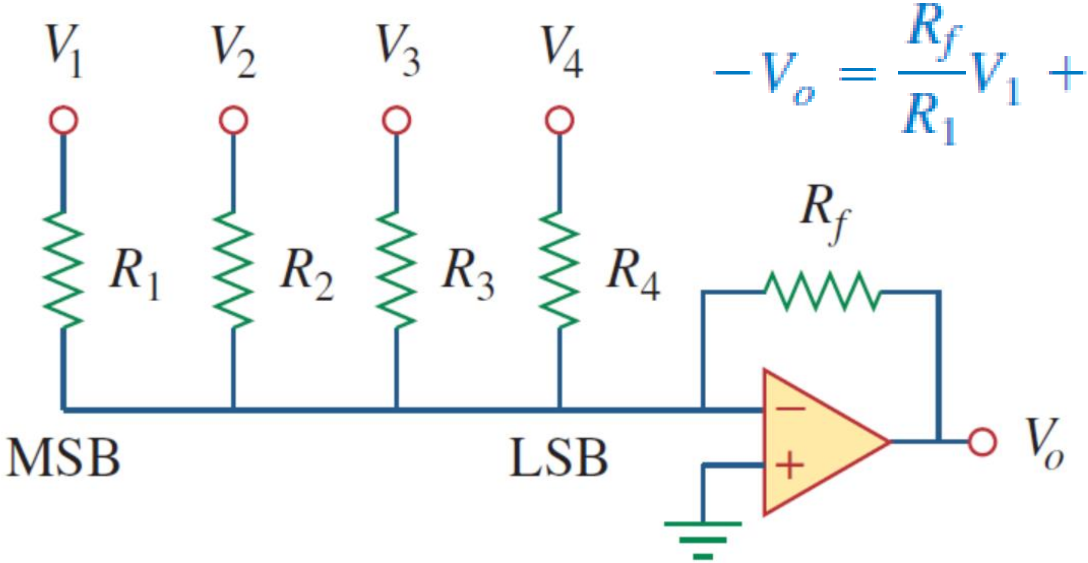
DAC.

Digital-to-Analog Converter

- The digital-to-analog converter (DAC) transforms digital signals into analog form.
- Example: a four-bit DAC



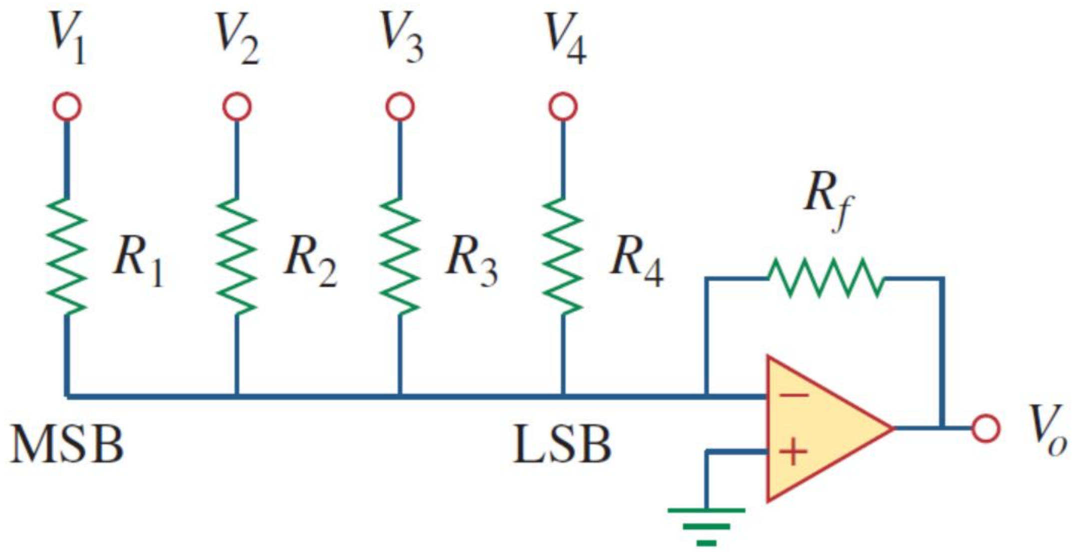
DAC: binary weighted ladder



$$-V_o = \frac{R_f}{R_1}V_1 + \frac{R_f}{R_2}V_2 + \frac{R_f}{R_3}V_3 + \frac{R_f}{R_4}V_4$$

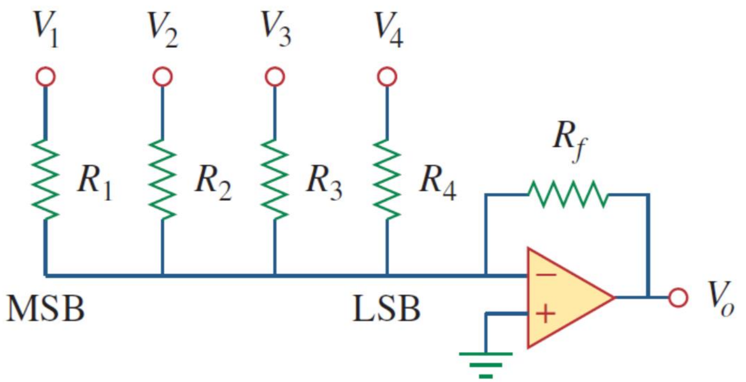
- An inverting summing amplifier
- Bits are weighted by descending value of R_f/R_n to produce 2 times difference for adjacent bits

DAC: binary weighted ladder



- Digital inputs: V_1 – V_4 is 0 or 1V
- Input = $[V_1V_2V_3V_4]$
- V_1: most significant bit (MSB)
- V_4: least significant bit (LSB)

DAC: binary weighted ladder



×2

$$-V_o = \frac{R_f}{R_1}V_1 + \frac{R_f}{R_2}V_2 + \frac{R_f}{R_3}V_3 + \frac{R_f}{R_4}V_4$$

1k 1k. 2k.

- **Bits are weighted** by descending value of R_f/R_n to produce 2 times difference for adjacent bits

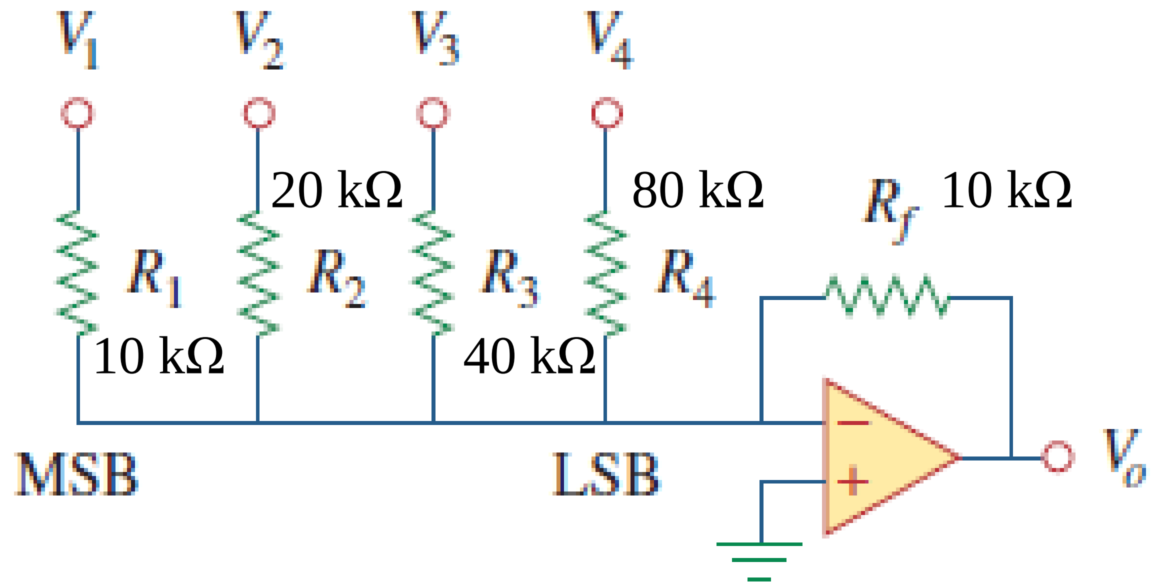
1

$$\boxed{-V_o=k(2^3V_1+2^2V_2+2^1V_3+2^0V_4)}$$

KV4

Example 5.12

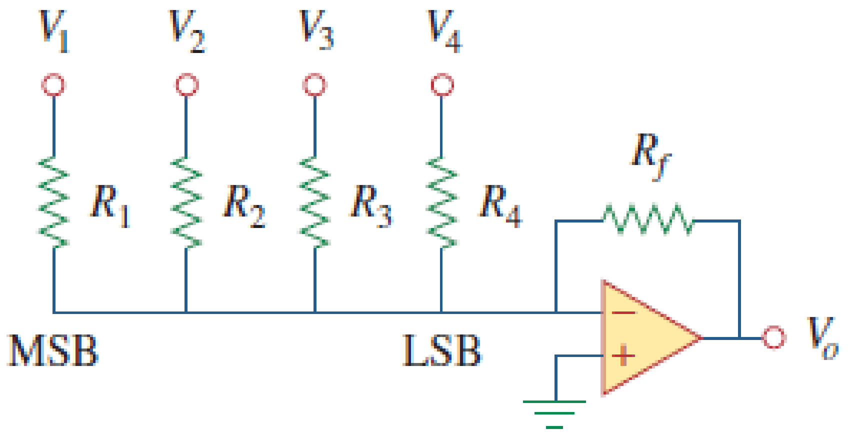
In the op amp circuit of Fig. 5.36(b), let $R_f = 10\text{ k}\Omega$, $R_1 = 10\text{ k}\Omega$, $R_2 = 20\text{ k}\Omega$, $R_3 = 40\text{ k}\Omega$, and $R_4 = 80\text{ k}\Omega$. Obtain the analog output for binary inputs [0000], [0001], [0010], . . . , [1111].



1. Output voltage

$$-V_o = \frac{\overset{10K}{R_f}}{R_1}\underset{10K}{}V_1 + \frac{R_f}{R_2}\underset{20K}{}V_2 + \frac{R_f}{R_3}\underset{40K}{}V_3 + \frac{R_f}{R_4}\underset{80K}{}V_4$$

$$= V_1 + 0.5V_2 + 0.25V_3 + 0.125V_4$$



1. Output voltage
$$-V_o = \frac{R_f}{R_1}V_1 + \frac{R_f}{R_2}V_2 + \frac{R_f}{R_3}V_3 + \frac{R_f}{R_4}V_4$$
$$= V_1 + 0.5V_2 + 0.25V_3 + 0.125V_4$$

2. Digital input [$V_1V_2V_3V_4$] = [0000] produces analog output $-V_o$ = 0 V

   [0001] → $-V_o$ = 0.125 V   increase 0.125 = LSB = resolution

   [0010] → $-V_o$ = 0.250 V

   [0011] → $-V_o$ = 0.375 V

TABLE 5.2

Input and output values of the four-bit DAC.

Binary input $[V_1V_2V_3V_4]$	Decimal value	Output $-V_o$
0000	0	0
0001	1	0.125
0010	2	0.25
0011	3	0.375
0100	4	0.5
0101	5	0.625
0110	6	0.75
0111	7	0.875
1000	8	1.0
1001	9	1.125
1010	10	1.25
1011	11	1.375
1100	12	1.5
1101	13	1.625
1110	14	1.75
1111	15	1.875 ≈ 2V

÷ 0.125

× 0.125

Output 2V

resolution 0.125 ↑ ×16. $= 2^4$

1.1V

1.05V

- Resolution (the smallest resolvable analog output) = 0.125 V = V_{LSB}

Question: In practice, for 1 V range, if you want to produce a resolution of 1mV, roughly how many bits do you need?

output = 1000mV

resolution = 1mV

x1000 = 2^{10}

Question: In practice, for 1 V range, if you want to produce a resolution of 1mV, roughly how many bits do you need?

For an n-bit DAC it can be given as

Resolution = $\frac{V_{oFS}}{2^n - 1}$ where V_{oFS} = full scale output voltage.

$$0.001 = \frac{1}{2^n - 1}$$