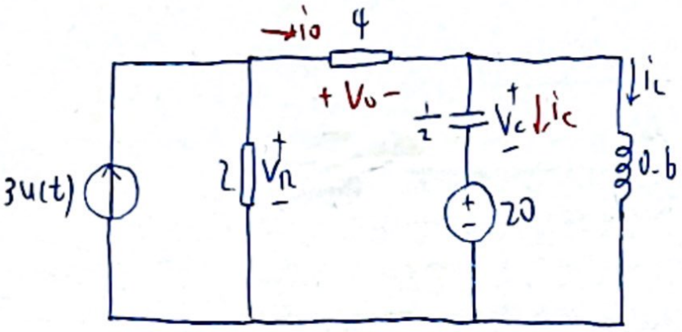
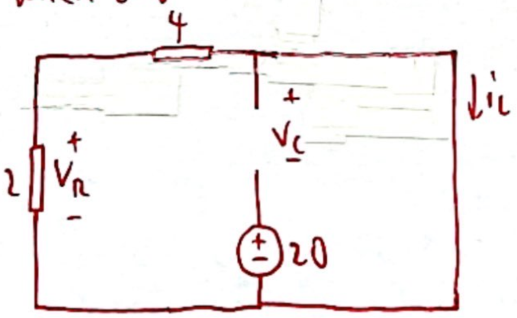
Example 8.2

Find $i_L(0), V_C(0), \frac{di_L}{dt}(0), \frac{dV_C}{dt}(0)$



when t<0



$i_L(0^-)=0$ $\quad V_C(0^-)=-20$

when t>0

$20+V_C(0^+)-V_L(0^+)=0$ (KVL)

$V_C(0^+)=-20$ (voltage across capacitor cannot change suddenly)

$V_L(0^+)=0$ $\quad \frac{di_L}{dt}(0^+)=\frac{V_L(0^+)}{L}=0$

$3-\frac{V_R(0^+)}{2}=\frac{V_o(0^+)}{4}$ (KCL)

$20+V_C(0^+)+V_o(0^+)-V_R(0^+)=0$ (KVL)

$V_o(0^+)=V_R(0^+)=4$

$3-\frac{V_R(0^+)}{2}=i_o(0^+)$ (KCL)

$i_o(0^+)=i_C(0^+)+i_L(0^+)$ (KCL)

$i_L(0^+)=0$ (current through inductor cannot change suddenly)

$i_C(0^+)=1$ $\quad \frac{dV_C}{dt}(0^+)=\frac{i_C(0^+)}{C}=2$

$i_L(0^+)=0$ $\quad V_C(0^+)=-20$ $\quad \frac{di_L}{dt}(0^+)=0$ $\quad \frac{dV_C}{dt}(0^+)=2$

Practice Problem 8.4

initial conditions

$i(0^+)=5 \qquad V(0^+)=0 \qquad \frac{di}{dt}(0^+)=-25$

$$\frac{d^2 i}{dt^2}+5\frac{di}{dt}+9i(t)=0$$

Method 1: Coefficient Comparison (待定系数)

eigen equation (特征方程) $\lambda^2+5\lambda+9=0$

eigen value (特征值) $\lambda=\frac{-5\pm j\sqrt{11}}{2}$

general solution to homogeneous equation (齐次方程的通解)

$$\bar{i}(t)=\tilde{C}_1 e^{\lambda_1 t}+\tilde{C}_2 e^{\lambda_2 t}$$
$$=\tilde{C}_1 e^{\frac{-5+j\sqrt{11}}{2}t}+\tilde{C}_2 e^{\frac{-5-j\sqrt{11}}{2}t}$$
$$=e^{-\frac{5}{2}t}\left(\tilde{C}_1\left(\cos\frac{\sqrt{11}}{2}t+j\sin\frac{\sqrt{11}}{2}t\right)+\tilde{C}_2\left(\cos\frac{\sqrt{11}}{2}t-j\sin\frac{\sqrt{11}}{2}t\right)\right)$$
$$=e^{-\frac{5}{2}t}\left(C_1\cos\frac{\sqrt{11}}{2}t+C_2\sin\frac{\sqrt{11}}{2}t\right)$$

particular solution (特解) $\tilde{i}(t)=0$

$$i(t)=e^{-\frac{5}{2}t}\left(C_1\cos\frac{\sqrt{11}}{2}t+C_2\sin\frac{\sqrt{11}}{2}t\right)$$

$$\frac{di}{dt}=-\frac{5}{2}e^{-\frac{5}{2}t}\left(C_1\cos\frac{\sqrt{11}}{2}t+C_2\sin\frac{\sqrt{11}}{2}t\right)+e^{-\frac{5}{2}t}\left(-\frac{\sqrt{11}}{2}C_1\sin\frac{\sqrt{11}}{2}t+\frac{\sqrt{11}}{2}C_2\cos\frac{\sqrt{11}}{2}t\right)$$

$C_1=5 \qquad C_2=-\frac{25}{\sqrt{11}}$

$$i(t)=e^{-\frac{5}{2}t}\left(5\cos\frac{\sqrt{11}}{2}t-\frac{25}{\sqrt{11}}\sin\frac{\sqrt{11}}{2}t\right)$$

Method 2: Unilateral Laplace Transform (单边拉普拉斯变换) (optional)

$$s^2 I(s)-si(0)-i'(0)+5(sI(s)-i(0))+9I(s)=0$$
$$s^2 I(s)-5s+25+5(sI(s)-5)+9I(s)=0$$
$$(s^2+5s+9)I(s)=5s \qquad I(s)=\frac{5s}{s^2+5s+9}=\frac{5(s+\frac{5}{2})}{(s+\frac{5}{2})^2+\frac{11}{4}}-\frac{\frac{25}{\sqrt{11}}\times\frac{\sqrt{11}}{2}}{(s+\frac{5}{2})^2+\frac{11}{4}}$$

$$i(t)=e^{-\frac{5}{2}t}\left(5\cos\frac{\sqrt{11}}{2}t-\frac{25}{\sqrt{11}}\sin\frac{\sqrt{11}}{2}t\right)$$

Example 8.7.(a)

initial conditions

$i(0^+)=4 \quad v(0^+)=4 \quad \frac{dv}{dt}(0^+)=16$

$$\frac{d^2v}{dt^2}+5\frac{dv}{dt}+4v=96$$

Method 1: Coefficient Comparison (待定系数)

$\lambda^2+5\lambda+4=0 \quad (\lambda+1)(\lambda+4)=0 \quad \lambda_1=-1 \quad \lambda_2=-4$

general solution to homogeneous equation $\bar{v}(t)=C_1e^{-t}+C_2e^{-4t}$

particular solution $\tilde{v}(t)=k \quad 4k=96 \quad k=24 \quad \tilde{v}(t)=24$

$$v(t)=C_1e^{-t}+C_2e^{-4t}+24$$

$$\frac{dv}{dt}=-C_1e^{-t}-4C_2e^{-4t}$$

$$C_1=-\frac{64}{3} \quad C_2=\frac{4}{3}$$

$$v(t)=-\frac{64}{3}e^{-t}+\frac{4}{3}e^{-4t}+24$$

Method 2: Unilateral Laplace Transform (单边拉普拉斯变换) (optional)

$$s^2V(s)-sv(0)-v'(0)+5(sV(s)-v(0))+4V(s)=\frac{96}{s}$$

$$(s^2+5s+4)V(s)-4s-36=\frac{96}{s}$$

$$(s^2+5s+4)V(s)=\frac{4s^2+36s+96}{s}$$

$$V(s)=\frac{4s^2+36s+96}{s(s+1)(s+4)}=\frac{24}{s}-\frac{\frac{64}{3}}{s+1}+\frac{\frac{4}{3}}{s+4}$$

$$v(t)=24-\frac{64}{3}e^{-t}+\frac{4}{3}e^{-4t}$$

Example 8-7.(b)

initial conditions

$i(0^+)=4.8 \qquad v(0^+)=4.8 \qquad \frac{dv}{dt}(0^+)=19.2$

$$\frac{d^2v}{dt^2}+4\frac{dv}{dt}+4v=96$$

Method 1: Coefficient Comparison(待定系数)

$\lambda^2+4\lambda+4=0 \qquad (\lambda+2)^2=0 \qquad \lambda_1=\lambda_2=-2$

general solution to homogeneous equation $\bar{v}(t)=(C_1+C_2t)e^{-2t}$

particular solution $\tilde{v}(t)=24$

$v(t)=(C_1+C_2t)e^{-2t}+24$

$\frac{dv}{dt}=(-2C_1+C_2-2C_2t)e^{-2t}$

$C_1=C_2=-19.2$

$v(t)=(-19.2-19.2t)e^{-2t}+24$

Method 2: Unilateral Laplace Transform(单边拉普拉斯变换)(optional)

$s^2V(s)-sv(0)-v'(0)+4(sV(s)-v(0))+4V(s)=\frac{96}{s}$

$(s^2+4s+4)V(s)-\frac{24}{5}s-\frac{192}{5}=\frac{96}{s}$

$(s^2+4s+4)V(s)=\dfrac{\frac{24}{5}s^2+\frac{192}{5}s+96}{s}$

$V(s)=\dfrac{\frac{24}{5}s^2+\frac{192}{5}s+96}{s(s+2)^2}=\dfrac{24}{s}-\dfrac{\frac{96}{5}}{s+2}-\dfrac{\frac{96}{5}}{(s+2)^2}$

$v(t)=24-\frac{96}{5}e^{-2t}-\frac{96}{5}te^{-2t}$

Example 8.7.(c)

initial conditions

$i(0^+)=12 \quad v(0^+)=12 \quad \frac{dv}{dt}(0^+)=48$

$$\frac{d^2v}{dt^2}+\frac{dv}{dt}+4v=96$$

Method 1: Coefficient Comparison(待定系数)

$\lambda^2+\lambda+4=0 \quad \lambda=\frac{-1\pm j\sqrt{15}}{2}$

general solution to homogeneous equation $\tilde{v}(t)=\tilde{C}_1e^{\frac{-1+j\sqrt{15}}{2}t}+\tilde{C}_2e^{\frac{-1-j\sqrt{15}}{2}t}$

$$=e^{-\frac{1}{2}t}\left(C_1\cos\frac{\sqrt{15}}{2}t+C_2\sin\frac{\sqrt{15}}{2}t\right)$$

particular solution $\tilde{v}(t)=24$

$$v(t)=e^{-\frac{1}{2}t}\left(C_1\cos\frac{\sqrt{15}}{2}t+C_2\sin\frac{\sqrt{15}}{2}t\right)+24$$

$$\frac{dv}{dt}=-\frac{1}{2}e^{-\frac{1}{2}t}\left(C_1\cos\frac{\sqrt{15}}{2}t+C_2\sin\frac{\sqrt{15}}{2}t\right)+e^{-\frac{1}{2}t}\left(-\frac{\sqrt{15}}{2}C_1\sin\frac{\sqrt{15}}{2}t+\frac{\sqrt{15}}{2}C_2\cos\frac{\sqrt{15}}{2}t\right)$$

$C_1=-12 \quad C_2=\frac{84}{\sqrt{15}}$

$$v(t)=e^{-\frac{1}{2}t}\left(-12\cos\frac{\sqrt{15}}{2}t+\frac{84}{\sqrt{15}}\sin\frac{\sqrt{15}}{2}t\right)+24$$

Method 2: Unilateral Laplace Transform(单边拉普拉斯变换)(optional)

$$s^2V(s)-sv(0)-v'(0)+sV(s)-v(0)+4V(s)=\frac{96}{s}$$

$$(s^2+s+4)V(s)-12s-60=\frac{96}{s}$$

$$(s^2+s+4)V(s)=\frac{12s^2+60s+96}{s}$$

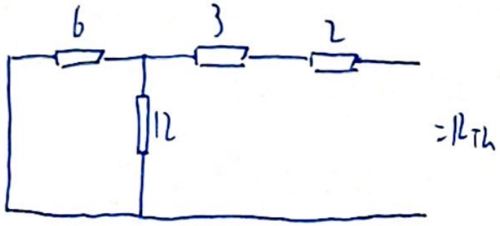
$$V(s)=\frac{12s^2+60s+96}{s(s^2+s+4)}=\frac{24}{s}+\frac{-12s+36}{(s+\frac{1}{2})^2+\frac{15}{4}}=\frac{24}{s}+\frac{-12(s+\frac{1}{2})}{(s+\frac{1}{2})^2+\frac{15}{4}}+\frac{\frac{84}{\sqrt{15}}\times\frac{\sqrt{15}}{2}}{(s+\frac{1}{2})^2+\frac{15}{4}}$$

$$v(t)=24+e^{-\frac{1}{2}t}\left(-12\cos\frac{\sqrt{15}}{2}t+\frac{84}{\sqrt{15}}\sin\frac{\sqrt{15}}{2}t\right)$$

Exercise (a)

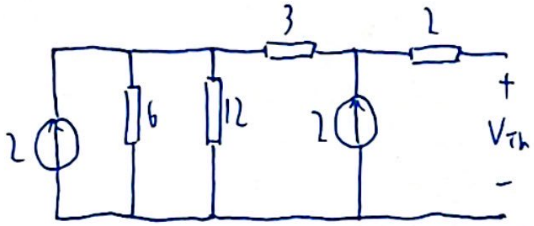
We try to find a Thevenin equivalent circuit.

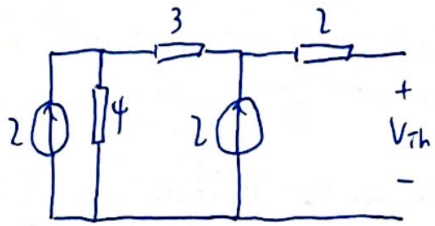
For R_{Th}

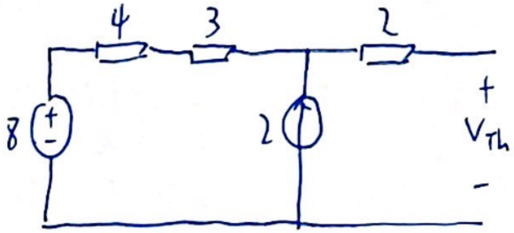


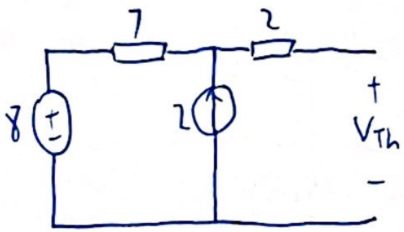
$R_{Th}=2+3+6//12=9$

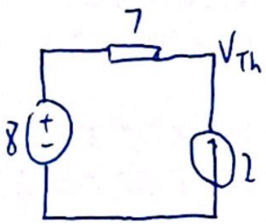
For V_{Th}











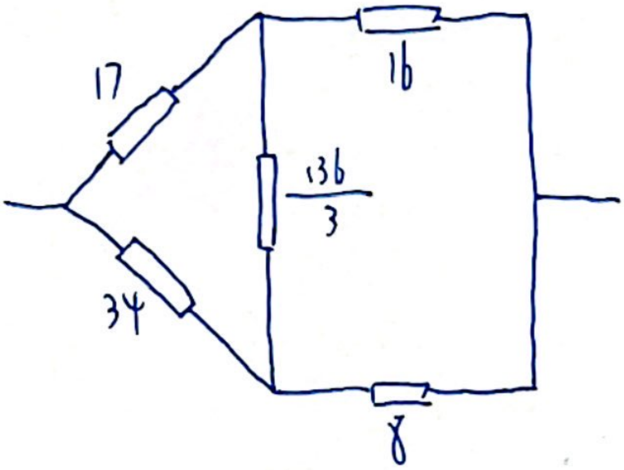
$\frac{V_{Th}-8}{7}=2 \Rightarrow V_{Th}=22$

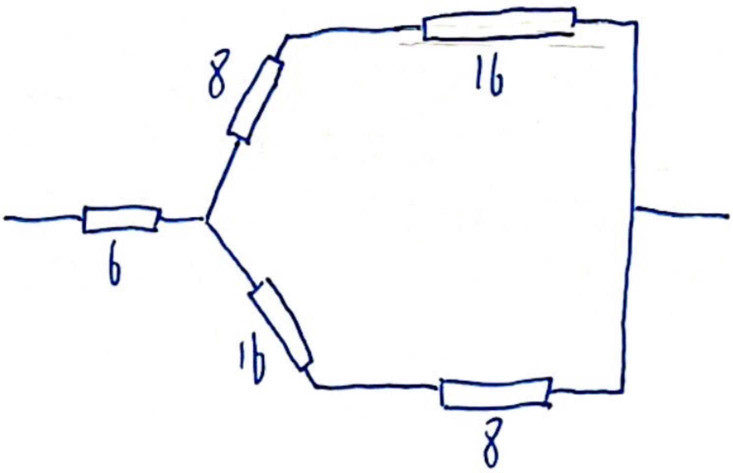
$P_{max}=\frac{V_{Th}^2}{4R_{Th}}=\frac{22^2}{4\times9}=\frac{121}{9}$

$R_L=R_{Th}=9$

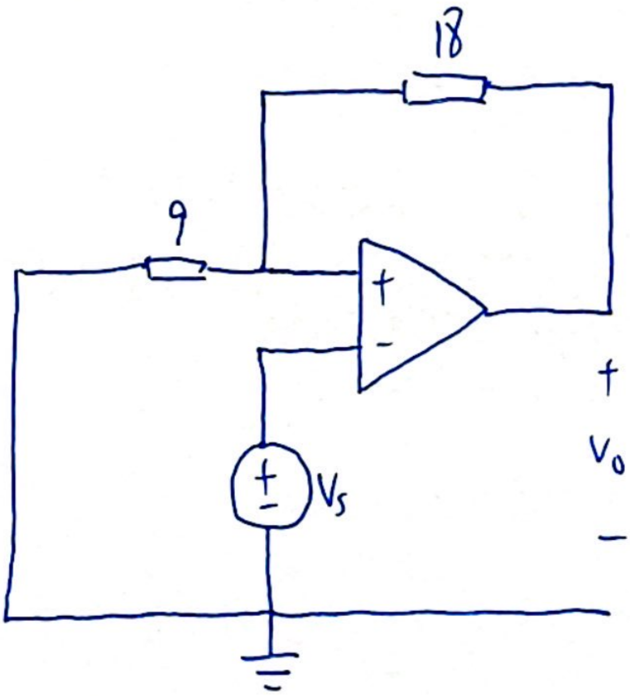
Exercise (b)

We apply Y-Δ transformation to simplify the "bridge".





$R_{bridge} = 6 + (8+16) // (16+8) = 18$



$\frac{V_s}{9} + \frac{V_s - V_o}{18} = 0$ (KCL)

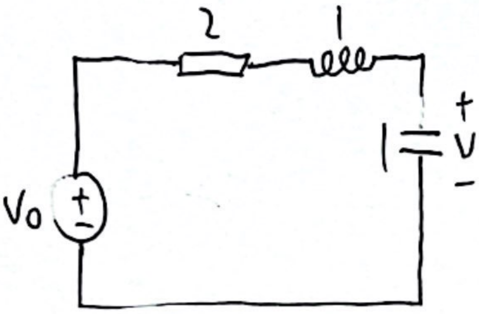
$V_o = 3V_s$

$A_v = 3$

Exercise (c)

$V_s = \frac{1}{3}(e^{-t}+e^{-2t})u(t) \Rightarrow V_0 = (e^{-t}+e^{-2t})u(t)$

We have a series RLC circuit at initial rest.



$V(0^+)=0 \qquad \frac{dv}{dt}(0^+)=0$

$\frac{d^2v}{dt^2}+2\frac{dv}{dt}+v=e^{-t}+e^{-2t}$

$\lambda^2+2\lambda+1=0 \qquad (\lambda+1)^2=0 \qquad \lambda_1=\lambda_2=-1$

general solution to the homogeneous equation $\bar{v}(t)=(c_1+c_2t)e^{-t}$

particular solution $\tilde{v}(t)=d_1t^2e^{-t}+d_2e^{-2t}$

$\frac{d\tilde{v}}{dt}=d_1(2t-t^2)e^{-t}-2d_2e^{-2t}$

$\frac{d^2\tilde{v}}{dt^2}=d_1(t^2-4t+2)e^{-t}+4d_2e^{-2t}$

$d_1(t^2-4t+2)e^{-t}+4d_2e^{-2t}+d_1(4t-2t^2)e^{-t}-4d_2e^{-2t}+d_1t^2e^{-t}+d_2e^{-2t}=e^{-t}+e^{-2t}$

$2d_1e^{-t}+d_2e^{-2t}=e^{-t}+e^{-2t} \qquad d_1=\frac{1}{2} \qquad d_2=1 \qquad \tilde{v}(t)=\frac{1}{2}t^2e^{-t}+e^{-2t}$

$v(t)=(c_1+c_2t)e^{-t}+\frac{1}{2}t^2e^{-t}+e^{-2t}$

$0=c_1+1 \qquad c_1=-1 \qquad v(t)=(-1+c_2t)e^{-t}+\frac{1}{2}t^2e^{-t}+e^{-2t}$

$\frac{dv}{dt}=(-\frac{1}{2}t^2+(1-c_2)t+c_2+1)e^{-t}-2e^{-2t}$

$0=c_2+1-2 \qquad c_2=1 \qquad v(t)=(-1+t)e^{-t}+\frac{1}{2}t^2e^{-t}+e^{-2t}$

$v(t)=(\frac{1}{2}t^2+t-1)e^{-t}+e^{-2t}$