VE215 RC1

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Overview

General Infomation

Basic Concepts

Basic Laws

Methods of Analysis

Course Structure

- Goal: analyze the circuits, from simple to complex.
- Structure:
    1. Chap. 1-8: DC circuits (the circuits driven by constant current/voltage sources)
       A variety of analysis tools
       $\rightarrow$ introducing some new circuits components
       $\rightarrow$ analyze circuits with those complex components added
    2. Chap. 9-14: AC circuits (the circuits driven by alternating current/voltage sources)

Overview

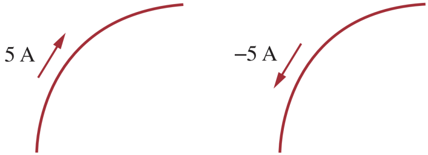
Current

$$i \triangleq \frac{\mathrm{d}q}{\mathrm{d}t}$$

$$Q \triangleq \int_{t_0}^{t} i \mathrm{d}t$$

Reference direction of current

In solving problems, it does not matter which direction we initially assume. If we obtain a result of negative current, it indicates that the actual direction is opposite to that we have initially assumed.



(a) (b)

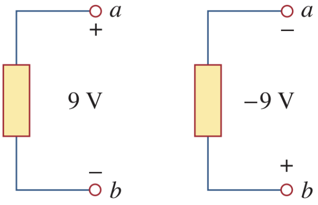
Voltage

$$v_{ab} = v_a - v_b$$

$$v_{ab} \triangleq \frac{\mathrm{d}w}{\mathrm{d}q}$$

Reference direction of voltage

In solving problems, it does not matter how we assign the "+/-" signs to two terminals of a circuit element. The two representations below are equivalent.



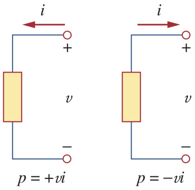
√

Power and Energy

$$p = \frac{\mathrm{d}w}{\mathrm{d}t} = vi \qquad w = \int_{t_0}^{t} vidt$$

Passive sign convention w.r.t. power:

- Currents enter through the positive terminal: $p = +vi$
- Currents enter through the negative terminal: $p = -vi$



Power and energy consumption:

- $p > 0$, element consumes energy.
- $p < 0$, element generates energy.

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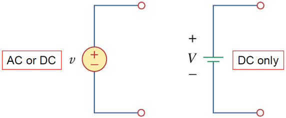
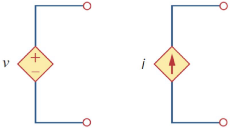
Circuit Elements

- **Active elements**: can generate energy
  e.g., generators, batteries, operational amplifiers
  - **independent source**: the source whose quantity is uninfluenced by its "surroundings".



  - **dependent source**: source quantity is controlled by another voltage or current in the circuit.



- **Passive elements**: cannot generate energy,
  e.g., resistors, capacitors, inductors

Overview

Nodes, Meshes and Loops

Branch: a single element, such as a voltage source or a resistor
Node: the point of connection between two or more branches
Loop: any closed path in a circuit

- **Mesh:** a loop that does not enclose any other loops, i.e., smallest loop
- **Independent loop:** a loop that contains at least one branch which is not a part of any other independent loop

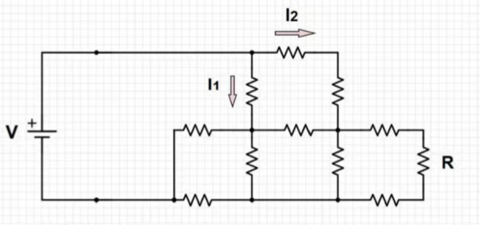
Fundamental theorem of network topology:

$$b \text{ (branches)} = l(\text{mesh}) + n \text{ (nodes)} - 1$$

Exercise 1.

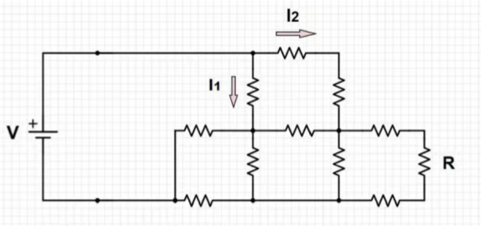
1. Suppose there are 3 meshes and 6 branches in one circuit. How many nodes in it?

2. Count the number of nodes, branches, meshes, loops in the following figure.

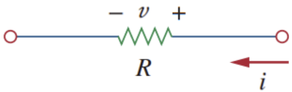


Exercise 1.

1. Suppose there are 3 meshes and 6 branches in one circuit. How many nodes in it?
   Answer: 4
2. Count the number of nodes, branches, meshes in the following figure.
   Answer: 8,12,5



Ohm's Law



Ohm's law:

$$V = IR \qquad R = \frac{v}{i}$$

Passive sign convention for Ohms's law:

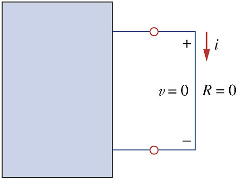
- i enters through the positive terminal: $v = iR$
- i enters through the negative terminal: $v = -iR$

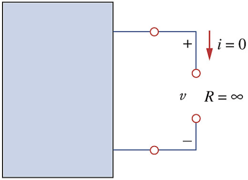
Not all resistors obey Ohm's law!

A resistor that obeys Ohm's law is known as **a linear resistor**, i.e., a constant resistance.

Resistance with extreme values

1. Short circuit: a circuit element with resistance approaching zero.
2. Open circuit: a circuit element with resistance approaching infinity.





$v = 0$
i can be any value

$i = 0$
v can be any value

Conductance

$$G = \frac{1}{R} = \frac{i}{v},\ 1S = 1\mho = 1A/V$$

where **G** is the conductance, **S** (siemens) is the SI unit of conductance and $\mho$ is the reciprocal ohm.
some useful formula:

$$i = Gv, p = vi = i^2 R = \frac{v^2}{R} = v^2 G = \frac{i^2}{G}$$

Kirchhoff's Law

Kirchhoff's Law	Expression	Based on
KCL	$\sum i_k = 0$ for a node	Conservation of charge
KVL	$\sum v_k = 0$ for a mesh	Conservation of energy

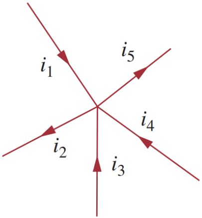


Figure: KCL

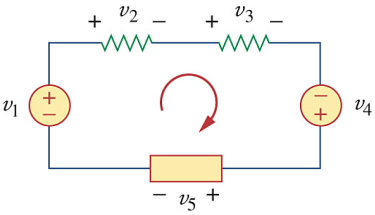


Figure: KVL

KCL

KCL: the algebraic sum of currents entering **a node (or a closed boundary)** is zero.
Steps of applying KCL:

1. Find out all branches connected to the node of interest.
2. Specify **reference** direction for current on each branch.
3. Find all $i_k (k = 1, 2, \cdots, n)$ (Ohm's law $i = \frac{v_a - v_b}{R}$ for linear resistors).
4. List the KCL equation $\sum_k i_k = 0$.

KVL

KVL: the algebraic sum of all voltages around **a closed path (or loop)** is zero.
Steps of applying KVL:

1. Select reference KVL direction (clockwise by convention).
2. Confirm/specify the +/- terminal of each branch.
3. Find $v_k (k = 1, 2, \cdots, n)$ for each branch.
4. List the KVL equation $\sum_k v_k = 0$. Mind that by passive sign convention, the sign in front of a certain term v_k is
   - "+" if the reference KVL direction enters through the positive terminal of the branch.
   - "-" if the reference KVL direction enters through the negative terminal of the branch.

Series connection and Parallel connection

R_{eq}: the **equivalent resistance**

1. **Series** connection:

$$R_{eq} = R_1 + R_2 + \ldots + R_N = \sum_{n=1}^{N} R_n$$

   Principle of voltage division: $v_n = \frac{R_n}{\sum_{n=1}^{N} R_n} v$

2. **Parallel** connection:

$$G_{eq} = \frac{1}{R_{eq}} = G_1 + G_2 + \ldots + G_N = \sum_{n=1}^{N} G_n$$

   Principle of voltage division: $i_n = \frac{G_n}{\sum_{n=1}^{N} G_n} i$

Wye-Delta Transformation

- Motivation: simplify the circuits for easier calculation.
- Two forms of special circuit connections:

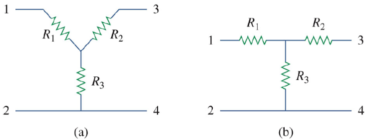


Figure: Wye (Y or T)

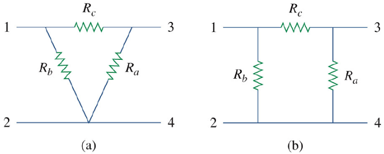


Figure: Delta (Δ or π)

- Goal: transform one type of connection into another.

Wye-Delta Transformation

$$\begin{cases} R_1 = \dfrac{R_b R_c}{R_a + R_b + R_c} \\ R_2 = \dfrac{R_c R_a}{R_a + R_b + R_c} \\ R_3 = \dfrac{R_a R_b}{R_a + R_b + R_c} \end{cases}$$

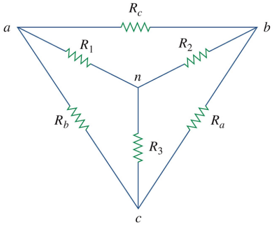


Figure 2.49 Superposition of wye and delta networks as an aid in transforming one to the other.

Figure: $\Delta - Y$

Intuition: parallel $\rightarrow$ series, resistance for each element decreases.

Wye-Delta Transformation

$$R_a = \frac{R_1R_2 + R_2R_3 + R_3R_1}{R_1}$$

$$R_b = \frac{R_1R_2 + R_2R_3 + R_3R_1}{R_2}$$

$$R_c = \frac{R_1R_2 + R_2R_3 + R_3R_1}{R_3}$$

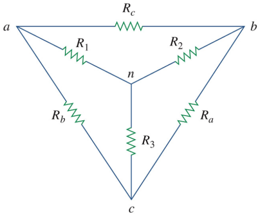


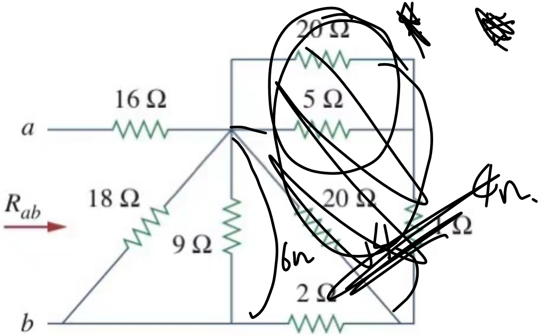
Figure 2.49 Superposition of wye and delta networks as an aid in transforming one to the other.

Figure: Y-Δ

Intuition: series → parallel, resistance for each element increases.

Exercise 2.

Calculate the equivalent resistance R_{ab} in the circuit

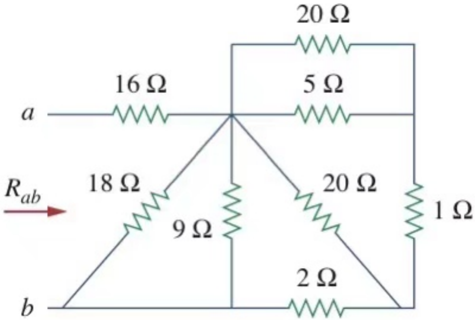


8Ω

$$\frac{1}{R} = \frac{1}{18} + \frac{1}{9} + \frac{1}{6} = \frac{1+2+3}{18} = \frac{\cancel{6}\,1}{\cancel{18}\,3}$$

Exercise 2.

Calculate the equivalent resistance R_{ab} in the circuit

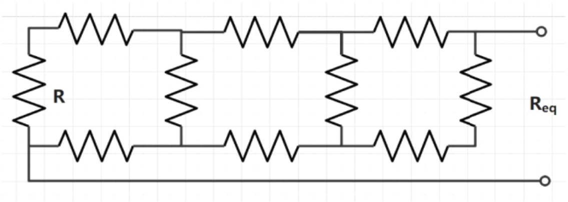


Answer: 19Ω

Exercise 3.

6. Suppose the resistance of all the resistors is **R**, what's the equivalent resistance $\mathbf{R_{eq}}$?

__________________________________ (4 marks)



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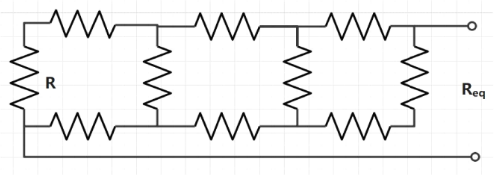


Answer: $0.5 + \dfrac{0.5 + 1 + 0.25}{2} + 0.5 = \dfrac{15}{8} R$

Overview

Methods of Analysis

Nodal Analysis:

1. Select a reference node (ground)
2. Apply KCL
3. Solve the equations

Mesh Analysis:

1. Mark the current of all the meshes
2. Apply KVL
3. Solve the equations

Analysis by Inspection

$$\begin{bmatrix} G_{11} & G_{12} & \dots & G_{1N} \\ G_{21} & G_{22} & \dots & G_{2N} \\ \vdots & \vdots & \vdots & \vdots \\ G_{N1} & G_{N2} & \dots & G_{NN} \end{bmatrix} \begin{bmatrix} v_1 \\ v_2 \\ \vdots \\ v_N \end{bmatrix} = \begin{bmatrix} i_1 \\ i_2 \\ \vdots \\ i_N \end{bmatrix}$$

G_{kk} = Sum of the conductances connected to node k

$G_{kj} = G_{jk}$ = Negative of the sum of the conductances directly connecting nodes k and j, $k \neq j$

v_k = Unknown voltage at node k

i_k = Sum of all ~~independent~~ current sources directly connected to node k, with currents entering the node treated as positive

For Nodal Analysis

(only current source in circuit)

$$\begin{bmatrix} R_{11} & R_{12} & \dots & R_{1N} \\ R_{21} & R_{22} & \dots & R_{2N} \\ \vdots & \vdots & \vdots & \vdots \\ R_{N1} & R_{N2} & \dots & R_{NN} \end{bmatrix} \begin{bmatrix} i_1 \\ i_2 \\ \vdots \\ i_N \end{bmatrix} = \begin{bmatrix} v_1 \\ v_2 \\ \vdots \\ v_N \end{bmatrix}$$

R_{kk} = Sum of the resistances in mesh k

$R_{kj} = R_{jk}$ = Negative of the sum of the resistances in common with meshes k and j, $k \neq j$

i_k = Unknown mesh current for mesh k in the clockwise direction

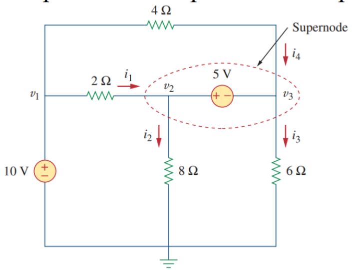
v_k = Sum taken clockwise of all ~~independent~~ voltage sources in mesh k, with voltage rise treated as positive

For Mesh Analysis

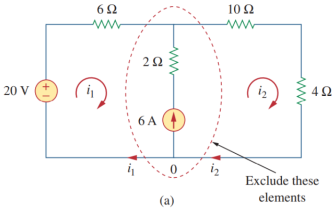
(only voltage source in circuit)

Supernode & Supermesh

- Supernode & Supermesh – simplify the equation

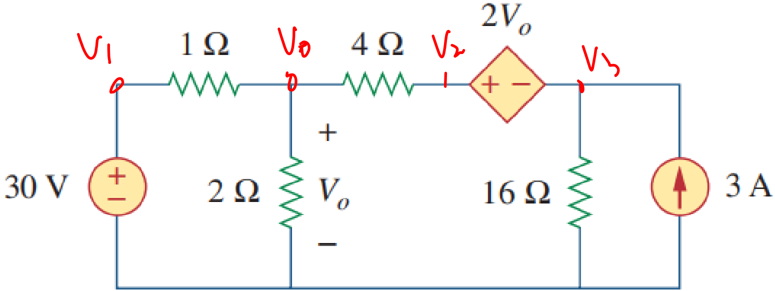


$$i_1 - i_2 - i_3 + i_4 = 0$$

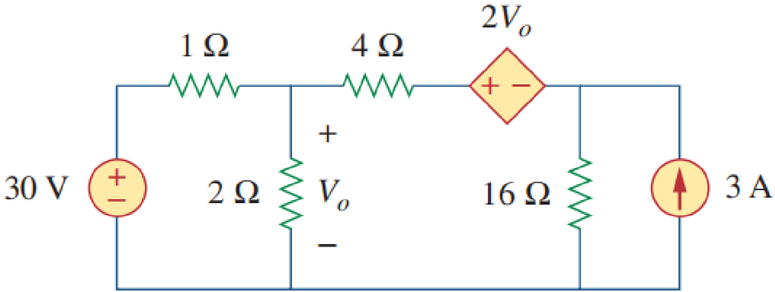


$$20 - 6i_1 - 14i_2 = 0$$

Exercise 4.



Exercise 4.



Answer: $\frac{648}{29}\,V \approx 22.34\,V$

References

1. 2024Fall VE215 slides
2. 2023Fall RC1,RC2, Chongye Yang

Thank you!