• $g_1(x) = \dfrac{xe^x}{e^x + e^{-x}}$

$\rightarrow$ AV: $\boxed{\nexists}$ (nunca se anula denominador)

$\rightarrow$ A.H.: $\lim\limits_{x\to\infty} g_1(x) = \dfrac{\infty \cdot e^\infty}{e^\infty + 0} = \dfrac{\infty \cdot \cancel{e^\infty}}{\cancel{e^\infty}} = \infty$ $\boxed{\nexists}$

$\rightarrow$ A.O.: $\lim\limits_{x\to\infty} \left(\dfrac{g_1(x)}{x}\right) = \lim\limits_{x\to\infty} \dfrac{xe^x}{xe^x + 0} = 1 = b$

$\Rightarrow \lim\limits_{x\to} (g(x) - 1x) = \lim\limits_{x\to\infty} \left(\dfrac{xe^x}{e^x + e^{-x}} - 1x\right) = \lim\limits_{x\to\infty} \dfrac{xe^x - x(e^x + e^{-x})}{e^x + e^{-x}} = \lim \dfrac{-xe^{-x}}{e^x + e^{-x}} = 0 = a$

$y = a + bx \Rightarrow \boxed{y = x}$ (es una A.O. por la dcha)

exp. va más rápido que $x$, y $e^{\infty} = 0$

$\rightarrow \lim\limits_{x\to\infty} \left(\dfrac{g(x)}{x}\right) = \lim\limits_{x\to-\infty} x \cdot e^{2x} = \boxed{0}$ ← asíntota horizontal

• $g_2(x) = \dfrac{x^2 - 1}{(x^2+1)^{1/2}}$

$\rightarrow$ A.V.: $\nexists$

$\rightarrow$ A.H.: $\lim\limits_{x\to\infty} g_2(x) = \lim\limits_{x\to\infty} \dfrac{x^2-1}{(x^2+1)^{1/2}} = \lim\limits_{x\to\infty} \dfrac{x^2}{x} = \infty$ $\boxed{\nexists}$

$\lim\limits_{x\to-\infty} g_2(x) = \infty$ $\boxed{\nexists}$

$\rightarrow$ A.O.: $\lim\limits_{x\to\infty} \left(\dfrac{g_2(x)}{x}\right) = \lim\limits_{x\to\infty} \dfrac{x^2}{x^2} = 1 = b$

$\lim\limits_{x\to\infty} \left(\dfrac{x^2-1}{(x^2+1)^{1/2}} - 1x\right) = \dfrac{(x^2-1) - x(x^2+1)^{1/2}}{(x^2+1)^{1/2}} \overset{\text{CONJUGADO}}{=} \lim\limits_{x\to\infty} \dfrac{(x^2-1)^2 - \left(x(x^2+1)^{1/2}\right)^2}{(x^2+1)^{1/2}\left((x^2-1) + x(x^2+1)^{1/2}\right)} =$

$= \lim\limits_{x\to\infty} \dfrac{\cancel{x^4} - 2x^2 + 1 - x^2(\cancel{x^2}+1)}{x^3} = 0 = a \longrightarrow \boxed{y = x}$ (dcha.)

$\lim\limits_{x\to-\infty} \dfrac{g_2(x)}{x} = \lim \dfrac{x^2-1}{\underbrace{x(x^2+1)^{1/2}}_{\text{tiene que ser positivo } (\nexists \sqrt{-x})}} = \lim\limits_{x\to-\infty} \dfrac{x^2}{x|x|} = -1 = b$

$\ominus \cdot \ominus$

$\lim\limits_{x\to-\infty} (g_2(x) + x) = 0 \longrightarrow \boxed{y = -x}$ (izq.)

MISMO LIM QUE ANTES PERO DISTINTO SIGNO → DA LO MISMO

- $\lim_{x\to 0} \frac{\sqrt{x+1}-1}{x} = \frac{0}{0} \to \lim_{x\to 0} \frac{(\sqrt{x+1}-1)(\sqrt{x+1}+1)}{x(\sqrt{x+1}+1)} = \lim_{x\to 0} \frac{\cancel{x+1-1}}{x(\sqrt{x+1}+1)} = \lim_{x\to 0} \frac{1}{\sqrt{x+1}+1} = \boxed{\frac{1}{2}}$

- $\lim_{x\to -1} \left(\frac{3}{x+1} - \frac{12}{x^2+6x-5}\right) \to \lim_{x\to -1} \left(\frac{3}{x+1} - \frac{12}{(x+1)(x+5)}\right) = \lim_{x\to -1} \left(\frac{3(x+5)-12}{(x+1)(x+5)}\right) = \lim_{x\to -1} \frac{3(x+1)}{(x+1)(x+5)} = \lim_{x\to -1} \frac{3}{x+5} = \boxed{\frac{3}{4}}$

- $\lim_{x\to +\infty} \frac{4x^3+8x-1}{7x^3+6x} = \boxed{\frac{4}{7}}$   - $\lim_{x\to +\infty} \frac{x^2+1}{2x^3+3x+1} = \boxed{0}$

- $\lim_{x\to 0} \frac{\operatorname{sen} x}{x} = 1$   - $\lim_{x\to 0} \frac{\operatorname{sen}(x/2)}{x} \xrightarrow{[t=x/2]} \lim_{x\to 0} \frac{\operatorname{sen} t}{2t} = \lim_{x\to 0} \frac{1}{2} \cdot \frac{\operatorname{sen} t}{t} = \boxed{\frac{1}{2}}$

- $\lim_{x\to 0} \frac{\operatorname{sen}(3x)}{\operatorname{sen}(7x)} = \lim_{x\to 0} \underbrace{\frac{\operatorname{sen}(3x)}{3x}}_{\to 1} \cdot \underbrace{\frac{7x}{\operatorname{sen}(7x)}}_{\to 1} \cdot \underbrace{\frac{3x}{7x}}_{\to 3/7} = \boxed{\frac{3}{7}}$

- $\lim_{x\to 0} \frac{1-\cos x}{x} = \lim_{x\to 0} \frac{(1-\cos x)(1+\cos x)}{x(1+\cos x)} = \lim_{x\to 0} \frac{1-\cos^2 x}{x(1+\cos x)} = \frac{\operatorname{sen}^2 x}{x(1+\cos x)} =$
$= \lim_{x\to 0} \underbrace{\frac{\operatorname{sen} x}{x}}_{\to 1} \cdot \underbrace{\frac{\operatorname{sen} x}{x}}_{\to 1} \cdot \underbrace{\frac{x}{(1+\cos x)}}_{\to \frac{0}{2}=0} = \boxed{0}$

es un nº neg., añado ⊖ al lado de raíz

- $\lim_{x\to 0^+} x\sqrt{1+\frac{1}{x^2}} = \lim_{x\to 0^+} \sqrt{x^2+1} = \lim_{x\to 0^+} \sqrt{1} = \boxed{1}$   $\lim_{x\to 0^-} x\sqrt{1+\frac{1}{x^2}} = -\sqrt{x^2}\sqrt{1+\frac{1}{x^2}} = \boxed{-1}$

- $\lim_{x\to 0} x \operatorname{sen}\left(\frac{1}{x}\right)$   [Para $x>0$] $\lim_{x\to 0^+} x\operatorname{sen}\left(\frac{1}{x}\right) = \boxed{0}$   [Para $x<0$] $\lim_{x\to 0^-} x\operatorname{sen}\left(\frac{1}{x}\right) = \boxed{0}$

$-x \le x\operatorname{sen}\left(\frac{1}{x}\right) \le x$

## Hoja 3

$\frac{e^t-1}{t^2} = \frac{1}{t} + \frac{1}{2!} + \frac{t}{3!} \dots$

(6) $\lim_{x\to 0^+} \frac{\sqrt{x}}{\ln(x+1)} \to \lim_{t\to 0^+} \frac{\sqrt{x}}{t} = \lim_{t\to 0^+} \frac{(e^t-1)^{1/2}}{t} = \lim_{t\to 0^+} \left(\frac{e^t-1}{t^2}\right)^{1/2} = \boxed{\infty}$

$\ln(x+1)=t \Rightarrow x+1=e^t$
$\Rightarrow x = e^t-1$

$\lim_{x\to +\infty} \frac{\ln x}{\sqrt{x}} \xrightarrow{\ln x = t} \lim_{t\to\infty} \frac{t}{e^{t/2}} = \boxed{0}$

exp

$\lim_{x\to\infty} \frac{\ln x}{x} \xrightarrow{t=\ln x} \lim_{t\to\infty} \frac{t}{e^t} = 0$

$\lim_{x\to\infty} x^{1/x} = \lim_{x\to\infty} e^{\frac{1}{x}\ln x} = \boxed{0}$

(7) Valores de $a$ para que sea continua

$$f(x) = \begin{cases} x^3 & \text{si } x \le 2 \\ ax^2 & \text{si } x > 2 \end{cases}$$

← Ya es continua

El único punto donde no podría ser continua es 2. Hago lím

$\lim_{x\to 2^+} ax^2 = 4a$   $\lim_{x\to 2^-} x^3 = 8$

Para que sea continua tienen que ser iguales

$4a = 8$

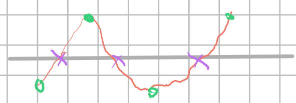
$\boxed{a=2}$

$$g(x)=\begin{cases} \frac{4\,\text{sen}\,x}{x} & \text{si } x<0 \\ b-7x & \text{si } x\geq 0 \end{cases}$$

$$\lim_{x\to 0^+}(b-2x)=b \qquad \lim_{x\to 0^-}\frac{4\,\text{sen}\,x}{x}=4\lim_{x\to 0^-}\frac{\text{sen}\,x}{x}=1\cdot 4=4$$

$$\rightarrow \boxed{b=4}$$

(8) T. VALORES INTERM.

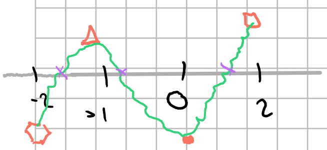


probar para buscar valor posit. y otro neg.

$P(x)=x^3-3x-1$

$p(0)=-1<0$

$p(2)=1>0$



$\hookrightarrow$ p. contin. en $\underline{[0,2]}$ ∃ HAY UNA RAÍZ

$p(-1)=1>0$

$\hookrightarrow$ otra raíz entre $\underline{[-1,0]}$

$p(-2)=-3<0$

$\hookrightarrow$ otra entre $\underline{[-2,-1]}$

CADA CRUCE ES 1 RAÍZ. ES DE GRADO 3 → $\underline{3 \text{ RAÍCES}}$

$\bullet\ f(x)=\frac{|x-1|}{x-1}$ si $x\neq 1$

$f(1)=0$

$$\rightarrow f(x)=\begin{cases} \frac{+x-1}{x-1}=1 & \text{si } x>1 \\ \frac{-x+1}{x-1}=-1 & \text{si } x<1 \\ 0 & \text{si } x=1 \end{cases}$$