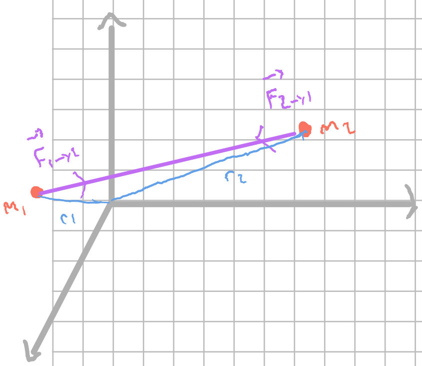
T5 - DINÁMICA DE SISTEMA DE PARTÍCULAS



- 3ª NEWT $\left[\vec{F}_{1\to2} = -\vec{F}_{2\to1}\right]$
- 2ª NEWT: $\vec{F}_{1\to2} = m_1 \dfrac{d\vec{v}_1}{dt} \longrightarrow \dfrac{\vec{F}_{12}}{m_1} = \dfrac{d\vec{v}_1}{dt}$

  $\vec{F}_{2\to1} = m_2 \dfrac{d\vec{v}_2}{dt} \longrightarrow \dfrac{\vec{F}_{21}}{m_2} = \dfrac{d\vec{v}_2}{dt}$

  $\rightarrow \left[\vec{F}_{12}\left(\dfrac{1}{m_1}+\dfrac{1}{m_2}\right) = \dfrac{d(\vec{v}_1-\vec{v}_2)}{dt} = \dfrac{d\vec{v}_{12}}{dt}\right]$

- $\left[r_{12} = r_1 - r_2\right]$ $\left[F_{12} = \dfrac{m_1+m_2}{m_1 m_2}\cdot\dfrac{d\vec{v}_{12}}{dt}\right]$

→ CENTRO DE MASAS

$$\left[r_{CM} = \frac{\Sigma m_i \vec{r}_i}{M_{TOTAL}}\right] \longrightarrow \left[M\cdot r_{CM} = \Sigma m_i \vec{r}_i\right]$$

masa · $\vec{r}$ de cada partícula

→ SIST. NO AISLADO: acel. del centro de masas $= 0$

→ CONS. MOMENTO LINEAL (SIEMPRE)

$$\left[\vec{v}_{CM} = \frac{\Sigma m_i \vec{v}_i}{M_T}\right] = \frac{\vec{P}}{M_T} \qquad \left[\frac{d\vec{P}}{dt} = F\right]$$

MOMENTO LINEAL

vel. sistema

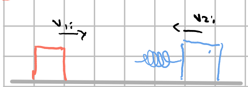
[COLISIÓN] {
- Elástica → se conserva Ec
- Inelástica → no se conserva Ec
  - ↳ Perfectamente inelástica → cuerpos se juntan

$$\left[m_1\vec{v}_{1i} + m_2\vec{v}_{2i} = m_1\vec{v}_{1F} + m_2\vec{v}_{2F}\right]$$

$$\left[\underbrace{\tfrac{1}{2}m_1 v_{1i}^2 + \tfrac{1}{2}m_2 v_{2i}^2}_{\text{ANTES DE COLISIÓN}} = \underbrace{\tfrac{1}{2}m_1 v_{1F}^2 + \tfrac{1}{2}m_2 v_{2F}^2 + Q}_{\text{DESPUÉS DE COLISIÓN}}\right]$$

sólo si

EJ



$m_1 = 1'5$ kg, $m_2 = 2'1$ kg

$v_1 = 4'6$ m/s, $v_2 = 2'5$ m/s

$(v_1' = v_{1F})$

a) $v_1' = 3$ m/s ¿v_2'?

$m_1v_1 - m_2v_2 = m_1v_1' + m_2v_2' \Rightarrow \boxed{v_2' = -1'74 \text{ m/s}}$

b) $\tfrac{1}{2}m_1v_1^2 + \tfrac{1}{2}m_2v_2^2 = \tfrac{1}{2}m_1v_1'^2 + \tfrac{1}{2}m_2v_2'^2 + Q$

en este caso, Q es la Ep elástica (la E que queda), es decir → $\tfrac{1}{2}kx^2$

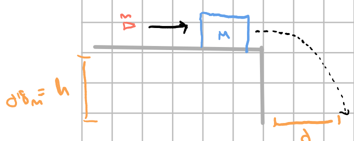
$\Rightarrow \boxed{x = 0'173 \text{ m}}$

c) ¿v_1'? $v_2' = 0$

$m_1v_1 - m_2v_2 = m_1v_1' + m_2\cdot 0 \Rightarrow \boxed{v_1' = 0'719 \text{ m/s}}$

$\tfrac{1}{2}m_1v_1^2 + \tfrac{1}{2}m_2v_2^2 = \tfrac{1}{2}m_1v_1'^2 + \tfrac{1}{2}m_2v_2'^2 + \tfrac{1}{2}kx^2 \Rightarrow \boxed{x = \quad \text{m}}$

Ej. 1



a.) ¿d si se incrusta? (perf. inelást.)
b.) ¿d si rebota? (elástico)

$m = 0'3$ kg $M = 4$ kg

$v_1 = 400$ m/s

$h = 0'8$ m

a.) $m\vec{v}_b + \cancel{M\vec{v}_B}^{\,0} = m\vec{v}' + M\vec{v}' = \vec{v}'(m+M)$

v_f es la misma, pq. se incrusta

$\vec{v}' = \frac{m \cdot v_b}{m+M}\vec{i} = V_{ox}\vec{i}$

$\boxed{y:}$ $y' = y_0 + \cancel{v_{oy}t}^{\,0} - \frac{1}{2}gt^2$

$y' - y_0 = 0'8 = -\frac{1}{2}gt^2 \Rightarrow \boxed{t = \sqrt{\frac{2h}{g}}}$

$\boxed{x:}$ $x = \cancel{x_0}^{\,0} + V_{ox}t$

$x - x_0 = d \Rightarrow \boxed{d = 11'27\text{ m}}$

b.) $mv_b + \cancel{Mv_B}^{\,0} = mv_b' + Mv_B'$

$[\, m(v_b - v_b') = Mv_B' \,]$

elást.: $\rightarrow \cancel{\tfrac{1}{2}}mv_b^2 = \cancel{\tfrac{1}{2}}mv_b'^2 + \cancel{\tfrac{1}{2}}Mv_B'^2$

$m(v_b^2 - v_b'^2) = Mv_B'^2$

$\frac{(v_b + v_b')\cancel{(v_b - v_b')}}{\cancel{(v_b - v_b')}} = \frac{v_B'^2}{\cancel{v_B'}} \Rightarrow \boxed{v_b = v_B + v_b'}$

$\rightarrow m(v_b - v_b') = M(v_b + v_b')$

$v_b' = \frac{m-M}{m+M}v_b$

$v_B' = v_b + \frac{m-M}{m+M}v_b = \frac{2m}{m+M}v_b$

~

$y' - y_0 = h = 0'8 = -\frac{1}{2}gt^2$

$y' = y_0 + \cancel{v_{oy}t} - \frac{1}{2}gt^2$ $(y' = 0)$

$\boxed{x:}$ $d = v_B't \longrightarrow = \boxed{22'5\text{ m}}$

→ Mov. curvilíneo

ANALOGÍA CON MOV. LINEAL
- Fuerza ↔ Torque
- Momento lineal ↔ Momento angular
- Masa ↔ Momento de inercia

→ Torque:

Lo que da a lugar a la rotación en un sist. También se llama momento de la fuerza. $[\,\vec{\tau} = \vec{r} \times \vec{F}\,]$

$\rightarrow [\, |\vec{\tau}| = r \cdot F \cdot \operatorname{sen}\theta \,]$

$\rightarrow [\, \vec{\tau}_{TOT} = \sum \vec{\tau}_i \,]$

$$\left[\, \vec{\tau}_{ext} = \frac{d\vec{L}}{dt} \,\right]$$

(N·m)

(en un sist. aislado, L = cte, τ=0)

$\rightarrow [\, \vec{\tau} = \cancel{(\vec{r}_1 - \vec{r}_2) \times \vec{F}_{12}}^{\,0} + \underbrace{\vec{r}_1 \times \vec{F}_1^{ext} + \vec{r}_2 \times \vec{F}_2^{ext}}_{\text{sólo influyen Fext.}} \,]$

(regla mano dcha.)

→ Momento angular:

$$[\, \vec{L} = \vec{r} \times \vec{p} = m\vec{r} \times \vec{v} \,]$$

(kg·m²/s)

$[\, |\vec{L}| = rp\operatorname{sen}\theta \,]$

$[\, \vec{L}_{TOT} = \sum \vec{L}_i \,]$

$(p = mv)$

$[\, L = I\omega \,]$ $[\, I = \sum m_i r_i^2 \,]$

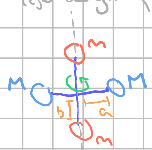
→ Momento de inercia:

$[\, I = \sum m_i r_i^2 \,]$ ← dist. al eje de giro

EJ

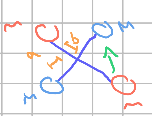
(eje de giro)

a.)



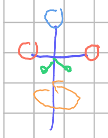
$I = Ma^2 + Ma^2 + \cancel{m \cdot 0} + \cancel{m \cdot 0} = \boxed{2Ma^2}$

b.)



$I = Ma^2 + Ma^2 + mb^2 + mb^2 = \boxed{2Ma^2 + 2mb^2}$

(7)



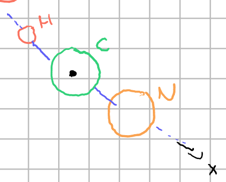
$w_1 = 2\ r.p.s. = 2(2\pi)\ rad/s$

$I_1 = 5\ kg m^2$

$I_2 = 2\ kg m^2$

$I_1 w_1 = I_2 w_2 \rightarrow w_2$

(10)



$I = \sum m_i r_i^2$

$r_{CM} = \frac{\sum m_i r_i}{\sum m_i}$

a.) ¿centro masa? b.) ¿I? c.) ¿Ec? $L=$

a.) $r_{CM} = \frac{m_H(-1 Å) + m_C(0) + m_N(1 Å)}{m_H + m_C + m_N}$ ← 27 uma

$x_{CM} = 0'48\ Å = \boxed{0'48 \cdot 10^{-10}\ m}$

b.) $I = m_H(1'48\ Å)^2 + m_C(0'48\ Å)^2 + m_N(1 - 0'48)^2 =$

c.) $E_C = \frac{1}{2} I w^2 = \frac{1}{2} I \left(\frac{L}{I}\right)^2$

(8)

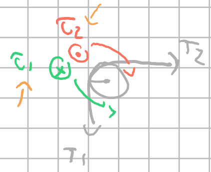
$\sum \vec{F} = m\vec{a}$

$\boxed{m_1}$: $m_1 g - T_1 = m_1 a_1$

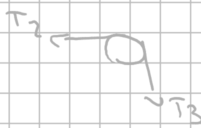
$\boxed{m_2}$: $-m_2 g + T_3 = m_2 a_2$

$(a_1 = a_2 = a)$

POLEAS: $\sum \tau = I\alpha$ ↻ ⊖ , ↺ ⊕

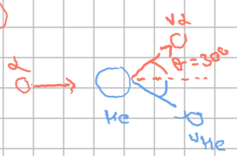


$\left[\oplus R T_1 \ominus R T_2 = I \frac{a}{R}\right]$



$\left[R T_2 - R T_3 = I \frac{a}{R}\right]$

(5)



$m_{He} = m_\alpha = m$

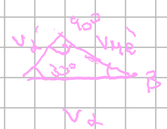
$v_\alpha = 300\ m/s$

a.) $\frac{1}{2} m v_\alpha^2 + 0 = \frac{1}{2} m v_\alpha'^2 + \frac{1}{2} m v_{He}'^2$

$v_\alpha^2 = v_\alpha'^2 + v_{He}'^2$

b.) $m\vec{v}_\alpha = m\vec{v}_\alpha' + m\vec{v}_{He}'$

→ PITÁGORAS:



→ $\beta = 60°$

$\boxed{x}$: $v_\alpha = v_\alpha' \cos\theta + v_{He}' \cos\theta$

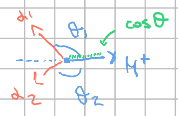
$\boxed{y}$: $0 = v_\alpha' \sin\theta - v_{He}' \sin\theta$

$v_\alpha' =$

$v_{He}' =$

(no se mueve en "y" inicialmente)

⑥



$\vec{p} = cte$ (aislado)

a.) $Ec_T = Ec_{\alpha_1}' + Ec_{\alpha_2}' + Ec_p' =$

$= 2Ec_{\alpha}' + K_p' =$ ($Ec_{\alpha_1} = Ec_{\alpha_2}$)

$= 2\left(\frac{1}{2} m_\alpha' v_\alpha'^2\right) + \frac{1}{2} m_p' v_p'^2 \Rightarrow \boxed{v_\alpha' = 1'46 \cdot 10^6 \ m/s}$

b.) ($\vec{p} = cte$) $0 = m_p \vec{v}_p + m_\alpha \vec{v}_{\alpha 1} + m_\alpha \vec{v}_{\alpha 2}$

$0 = \vec{v}_p + 4\vec{v}_{\alpha 1} + 4\vec{v}_{\alpha 2}$

$$\begin{cases} \boxed{x:} \ 0 = v_p + 4v_{\alpha 1}\cos\theta_1 + 4v_{\alpha 2}\cos\theta_2 \\ \boxed{y:} \ 0 = 0 + 4v_{\alpha 1}\,\text{sen}\,\theta_1 + 4v_{\alpha 2}\,\text{sen}\,\theta_2 \end{cases}$$

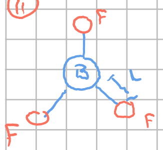
$\downarrow v_{\alpha 1} = v_{\alpha 2}$

$$\begin{cases} 0 = v_p + 4v_\alpha \cos\theta_1 + 4v_\alpha \cos\theta_2 \\ 0 = 4v_{\alpha 1}\,\text{sen}\,\theta_1 + 4v_\alpha\,\text{sen}\,\theta_2 \end{cases} \quad \begin{cases} -4v_{\alpha 1}\,\text{sen}\,\theta_1 = 4v_{\alpha 1}\,\text{sen}\,\theta_2 \\ \longrightarrow \theta_1 = -\theta_2 \end{cases}$$

$\downarrow$

$0 = v_p + 8v_\alpha \cos\theta_1 \Rightarrow$

⑪



$L = 1$ Å

a.) $\vec{r}_{CM} = \frac{\sum m_i \vec{r}_i}{\sum m_i}$ ⟶ y no hace falta, por geom: $\boxed{\text{Boro es CM}}$

$I_{CM} = \sum m_i r_i^2 = \cancel{m_B r_B^2}^{\,0} + 3(m_F r_F^2)$

b.) (L_{cm} dato) $L = I\omega \Rightarrow \omega = \frac{I_{cm}}{L_{cm}} =$

$Ec = \frac{1}{2} I \omega^2$