AI Tutorial

Week 4

Background - Definitions

Conjunctions

Predicates on the left and right must both be true for the clause to be true

$A \wedge B$ = True if $A = B$ = True, else False

A	B	A ∧ B
T	T	T
T	F	F
F	T	F
F	F	F

Background - Definitions

Disjunctions

A predicate on the left or right must be true for the clause to be true

$A \vee B$ = True if A = True or B = True, else False if A = B = False

A	B	$A \vee B$
T	T	T
T	F	T
F	T	T
F	F	F

Background - Definitions

Negation

The value of a predicate is basically flipped

$\neg A$ = True if A = False else $\neg A$ = False if A = True

A	¬A
T	F
F	T

Background - Definitions

Entail

Predicates on the right are True when all predicates on the left are True

$\forall P$, P |= Q if when P = True, Q = True

P	Q	P |= Q
T	Must be True	
F	Can be True or False	

Background - Definitions

→ is equivalent to ⇒

Implication

First predicate implies the second

A → B is False if A = True and B = False, otherwise True

A	B	A → B
T	T	T
T	F	F
F	T	T
F	F	T

Background - Definitions

$\leftrightarrow$ is equivalent to $\Leftrightarrow$

Bi-implication

True if both side are the same

$A \leftrightarrow B$ is True if $A = B$

A	B	$A \leftrightarrow B$
T	T	T
T	F	F
F	T	F
F	F	T

Question 1i

If Jane and John are not in town we will play tennis

Question 1i

If Jane and John are not in town we will play tennis

$(\neg Jane \land \neg John) \rightarrow Tennis$

Jane	Jane is in town
John	John is in town
Tennis	We will play tennis

Question 1iii

You will not pass this course unless you study

Question 1iii

You will not pass this course unless you study

$\neg S \rightarrow \neg P$
$\neg(P \wedge \neg S)$

S	You study
P	You pass

Background – Conjunctive Normal Form (CNF)

Conjunction of disjunctive literals

$(A \vee B) \wedge (C \vee D) \wedge (E \vee F \vee G) \wedge H$

Grammar = {literals, conjunctions, disjunctions, internal negations}

Example	Conjunctive Normal Form
$(A \vee B) \wedge (C \vee D)$	Yes
$(A \vee \neg B) \wedge C \wedge (\neg D \vee E)$	Yes
$(A \vee \neg B) \wedge C \vee (\neg D \vee E)$	No
$(A \rightarrow \neg B) \wedge \neg(D \vee E)$	No

Background – Disjunctive Normal Form (DNF)

Disjunction of conjunctive literals

$(A \wedge B) \vee (C \wedge D) \vee (E \wedge F \wedge G) \vee H$

Grammar = {literals, conjunctions, disjunctions, internal negations}

Example	Conjunctive Normal Form
$(A \wedge B) \vee (C \wedge D)$	Yes
$(A \wedge \neg B) \vee C \vee (\neg D \wedge E)$	Yes
$(A \wedge \neg B) \vee C \wedge (\neg D \wedge E)$	No
$(A \rightarrow \neg B) \vee \neg(D \wedge E)$	No

Background - Equivalencies

Distribution Changes

Change a conjunction into a disjunction and vice-versa

Flip centre connective and distribute left predicate literals with right predicate

Example	Distribution Change
$(A \land B) \lor C$	$(A \lor C) \land (B \lor C)$
$(A \lor B) \land C$	$(A \land C) \lor (B \land C)$

Background - Equivalencies

Double Negative

Negative values cancel each other out, $\neg\neg A = A$

A	¬A	¬¬A
T	F	T
F	T	F

Background - Equivalencies

Implication

A → B is equivalent to ¬A V B

A	B	A → B	¬A V B
T	T	T	T
T	F	F	F
F	T	T	T
F	F	T	T

Background - Equivalencies

De Morgan

¬(A ∧ B) is equivalent to ¬A ∨ ¬B

¬(A ∨ B) is equivalent to ¬A ∧ ¬B

A	B	¬(A ∧ B)	¬A ∨ ¬B
T	T	F	F
T	F	T	T
F	T	T	T
F	F	T	T

Question 2i

$P \rightarrow Q$

Operation	Clause

Question 2i

$P \rightarrow Q$

Operation	Clause
Implication	$\neg P \land Q$

Question 2iii

$\neg(P \wedge \neg Q) \rightarrow (\neg R \vee \neg Q)$

Operation	Clause

Question 2iii

$\neg(P \land \neg Q) \rightarrow (\neg R \lor \neg Q)$

Operation	Clause
Implication	$\neg\neg(P \land \neg Q) \lor (\neg R \lor \neg Q)$
Double Negation	$(P \land \neg Q) \lor (\neg R \lor \neg Q)$
Distribution	$(P \lor \neg R \lor \neg Q) \land (\neg Q \lor \neg R \lor \neg Q)$
Simplification	$(P \lor \neg R \lor \neg Q) \land (\neg Q \lor \neg R)$
	$P \lor \neg Q \lor \neg R$

Background - Definitions

Valid (Tautology)

Predicate results in True regardless of the value of its variables

A	¬A	A ∨ ¬A
T	F	T
F	T	T

Question 3i

$P \rightarrow Q, \neg Q \models \neg P$

P	Q	$\neg P$	$P \rightarrow Q$	$\neg Q$	$\models \neg P$

Question 3i

$P \rightarrow Q, \neg Q \models \neg P$

P	Q	$\neg P$	$P \rightarrow Q$	$\neg Q$	$\models \neg P$
T	T	F	T	F	F
T	F	F	F	T	T
F	T	T	T	F	F
F	F	T	T	T	T

Question 3iii

$P \to Q, Q \to R \models P \to R$

P	Q	R	$P \to Q$	$Q \to R$	$\models P \to R$

Question 3iii

$P \to Q, Q \to R \models P \to R$

P	Q	R	$P \to Q$	$Q \to R$	$\models P \to R$
T	T	T	T	T	T
T	T	F	T	F	F
T	F	T	F	T	T
F	T	T	T	T	T
T	F	F	F	T	F
F	T	F	T	F	T
F	F	T	T	T	T
F	F	F	T	T	T

Background - Definitions

Complimentary literals

Two literals are complimentary if one is a negation of the other

A	¬A

Background - Definitions

Resolution rule

A new clause is created from two other clauses containing complementary literals

Clause 1	Clause 2	New Clause
¬Q V P	Q V P	P
¬Q V R	¬R	¬Q

Background - Definitions

Resolution rule

A new clause is created from two other clauses containing complementary literals

Clause 1	Clause 2	New Clause
{¬Q, P}	{Q, P}	{P}
{Q, R}	{¬R}	{¬Q}

Background - Definitions

Resolution Refutation

Determine what predicates in the clauses refute the proof

1. Negate the derivation

2. Convert all clauses to conjunctive normal form (CNF)
   - Including the derivation

3. Continually apply the resolution rule
   - Basically just combining clauses until all predicates are cancelled

Resolution Rule Example

$(P \vee Q), P \rightarrow R, Q \rightarrow R \vdash R$

#	Original	Operation	Result
1	R	Conclusion Negation	$\neg R$
2	$(P \vee Q)$	CNF Conversion	$P \vee Q$
3	$P \rightarrow R$	CNF Conversion	$\neg P \vee R$
4	$Q \rightarrow R$	CNF Conversion	$\neg Q \vee R$

$\{\neg R, P \vee Q, \neg P \vee R, \neg Q \vee R\}$

Resolution Rule Example

(P ∨ Q), P → R, Q → R ⊢ R

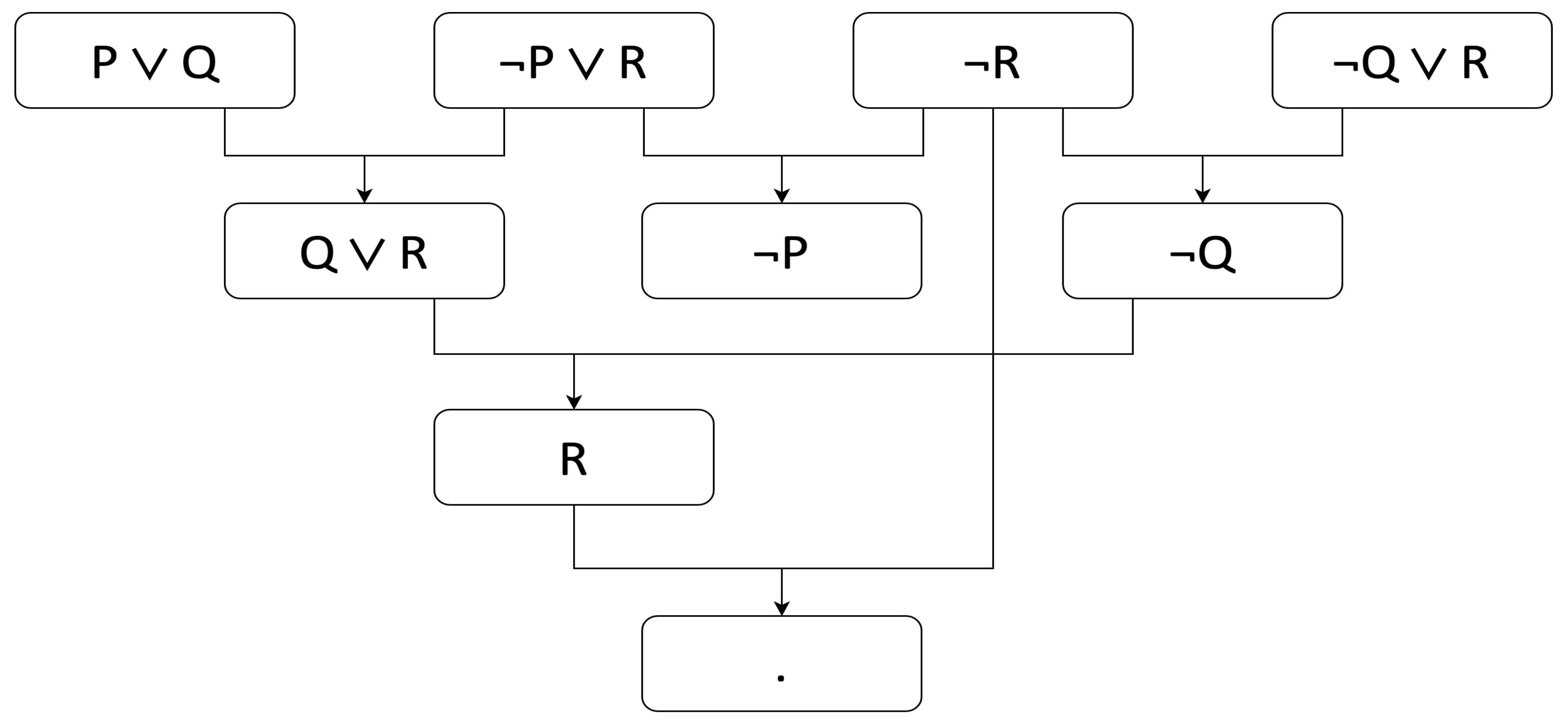
#	Original	Operation	Result
5	P ∨ Q, ¬P ∨ R	Resolution rule 2, 3	Q ∨ R
6	¬R, ¬P ∨ R	Resolution rule 1, 3	¬P
7	¬R, ¬Q ∨ R	Resolution rule 1, 4	¬Q
8	Q ∨ R, ¬Q	Resolution rule, 5, 7	R
9	¬R, R	Resolution rule, 1, 8	.

{¬R, P ∨ Q, ¬P ∨ R, ¬Q ∨ R}

Resolution Rule Example

(P ∨ Q), P → R, Q → R ⊢ R

{P ∨ Q, ¬P ∨ R, ¬Q ∨ R, ¬R}



Question 4i

$P \rightarrow Q, \neg Q \vdash \neg P$

#	Original	Operation	Result
1			
2			
3			
4			
5			
6			

Question 4i

$P \to Q, \neg Q \vdash \neg P$

#	Original	Operation	Result
1	$\neg P$	Conclusion Negation	$\neg\neg P$
	$\neg\neg P$	CNF Conversion	P
2	$P \to Q$	CNF Conversion	$\neg P \lor Q$
3	$\neg Q$	CNF Conversion	$\neg Q$
4	$P, \neg P \lor Q$	Resolution rule 1, 2	Q
5	$\neg Q, Q$	Resolution rule 3, 4	.

Question 4iii

$P \to Q, Q \to R \vdash P \to R$

#	Original	Operation	Result
1			
2			
3			
4			

Question 4iii

$P \rightarrow Q, Q \rightarrow R \vdash P \rightarrow R$

#	Original	Operation	Result
5			
6			
7			
8			

Question 4iii

$P \to Q, Q \to R \vdash P \to R$

#	Original	Operation	Result
	$P \to R$	Conclusion Negation	$\neg(P \to R)$
	$\neg(P \to R)$	CNF Conversion	$\neg(\neg P \lor R)$
			$P \land \neg R$
			$P, \neg R$
1			P
2			$\neg R$
3	$P \to Q$	CNF Conversion	$\neg P \lor Q$
4	$Q \to R$	CNF Conversion	$\neg Q \lor R$

Question 4iii

$P \rightarrow Q, Q \rightarrow R \vdash P \rightarrow R$

#	Original	Operation	Result
5	P, ¬P V Q	Resolution rule 1, 3	Q
6	¬Q V R, Q	Resolution rule 4, 5	R
7	¬R, R	Resolution rule 2, 6	.

{P, ¬R, ¬P V Q, ¬Q V R}

Question 5i

$((P \vee Q) \wedge \neg P) \rightarrow Q$

P	Q	$(P \vee Q)$	$\neg P$	$(P \vee Q) \wedge \neg P$	$\rightarrow Q$

Question 5i

((P V Q) Λ ¬P) → Q

P	Q	(P V Q)	¬P	(P V Q) Λ ¬P	→ Q
T	T	T	F	F	T
T	F	T	F	F	T
F	T	T	T	T	T
F	F	F	T	F	T

Question 5iv

$(P \vee Q) \rightarrow \neg(\neg P \wedge \neg Q)$

P	Q	$(P \vee Q)$	$\neg P$	$\neg Q$	$(\neg P \wedge \neg Q)$

Question 5iv

$(P \vee Q) \rightarrow \neg(\neg P \wedge \neg Q)$

$(P \vee Q)$	$(\neg P \wedge \neg Q)$	$\neg(\neg P \wedge \neg Q)$	$\rightarrow$

Question 5iv

$(P \vee Q) \rightarrow \neg(\neg P \wedge \neg Q)$

P	Q	$(P \vee Q)$	$\neg P$	$\neg Q$	$(\neg P \wedge \neg Q)$
T	T	T	F	F	F
T	F	T	F	T	F
F	T	T	T	F	F
F	F	F	T	T	T

Question 5iv

$(P \vee Q) \rightarrow \neg(\neg P \wedge \neg Q)$

$(P \vee Q)$	$(\neg P \wedge \neg Q)$	$\neg(\neg P \wedge \neg Q)$	$\rightarrow$
T	F	T	T
T	F	T	T
T	F	T	T
F	T	F	T

Question 6i

$\vdash ((P \vee Q) \wedge \neg P) \rightarrow Q$

Operation	Clause

Question 6i

$\vdash ((P \lor Q) \land \neg P) \to Q$

#	Original	Operation	Result
1		CNF Conversion	
2		CNF Conversion	
3		CNF Conversion	
4			
5			

Question 6i

$\vdash ((P \lor Q) \land \neg P) \rightarrow Q$

Operation	Clause
Implication	$\neg(\neg((P \lor Q) \land \neg P) \lor Q)$
De Morgan	$\neg\neg((P \lor Q) \land \neg P) \land \neg Q$
Double Negation	$(P \lor Q) \land \neg P \land \neg Q$

Question 6i

$\vdash ((P \lor Q) \land \neg P) \rightarrow Q$

#	Original	Operation	Result
1	$((P \lor Q) \land \neg P) \rightarrow Q$	CNF Conversion	$(P \lor Q)$
2	$((P \lor Q) \land \neg P) \rightarrow Q$	CNF Conversion	$\neg P$
3	$((P \lor Q) \land \neg P) \rightarrow Q$	CNF Conversion	$\neg Q$
4	$(P \lor Q), \neg Q$	Resolution rule 1, 3	P
5	$P, \neg P$	Resolution rule 2, 4	.

Question 6iv

$\vdash (P \vee Q) \rightarrow \neg(\neg P \wedge \neg Q)$

Operation	Clause

Question 6iv

$\vdash (P \lor Q) \rightarrow \neg(\neg P \land \neg Q)$

#	Original	Operation	Result
1		CNF Conversion	
2		CNF Conversion	
3		CNF Conversion	
4			
5			

Question 6iv

$\vdash (P \lor Q) \rightarrow \neg(\neg P \land \neg Q)$

Operation	Clause
Implication	$\neg(\neg(P \lor Q) \lor \neg(\neg P \land \neg Q))$
De Morgan	$\neg\neg(P \lor Q) \land \neg\neg(\neg P \land \neg Q)$
Double Negation	$(P \lor Q) \land \neg P \land \neg Q$

Question 6iv

$\vdash (P \vee Q) \rightarrow \neg(\neg P \wedge \neg Q)$

#	Original	Operation	Result
1	$(P \vee Q) \rightarrow \neg(\neg P \wedge \neg Q)$	CNF Conversion	$(P \vee Q)$
2	$(P \vee Q) \rightarrow \neg(\neg P \wedge \neg Q)$	CNF Conversion	$\neg P$
3	$(P \vee Q) \rightarrow \neg(\neg P \wedge \neg Q)$	CNF Conversion	$\neg Q$
4	$(P \vee Q), \neg P$	Resolution rule 1, 2	Q
5	$Q, \neg Q$	Resolution rule 3, 4	.

Notes

- All python files for question 3 can be found at:
  - https://artint.info/AIPython/
- Small running example on the resolution rule:
  - https://ocw.mit.edu/courses/electrical-engineering-and-computer-science/6-825-techniques-in-artificial-intelligence-sma-5504-fall-2002/lecture-notes/Lecture7FinalPart1.pdf