COMP9414 Tutorial

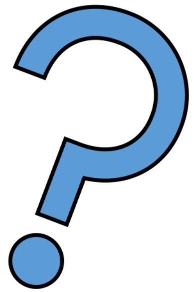
Week 5

News

- Assignment 1 is due this week (Friday, 3rd July)
    - Wayne will do one final consultation session on it
    - Make sure you’re printing to standard output so that piping will work (program > output1.txt)

- Assignment 2 will be released next week
    - Will only know how to complete it after the week 7 lecture
    - Not expected to start it

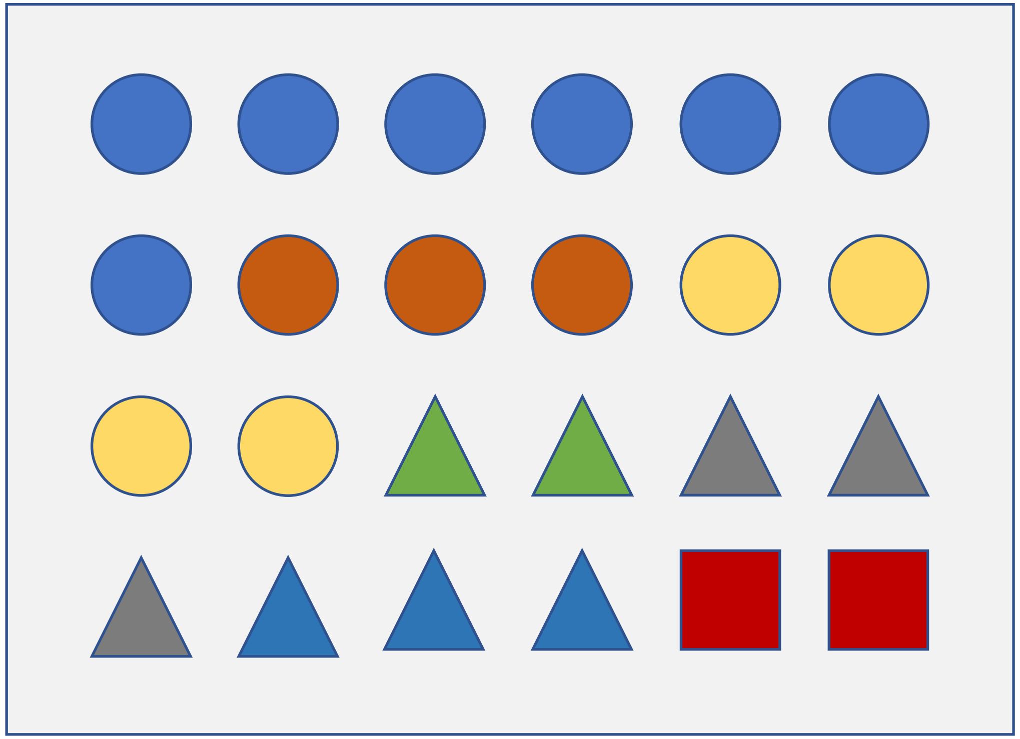
- Week 6 will be a week break for us
    - Consultations only



Background – Base Example

- 24 Shapes in total
    - 14 Circles
    - 8 Triangles
    - 2 Squares

- P(Circle) = $^{14}/_{24}$ = 0.58 = 58%
- P(Triangle) = $^{8}/_{24}$ = 0.33 = 33%
- P(Square) = $^{2}/_{24}$ = 0.08 = 8%

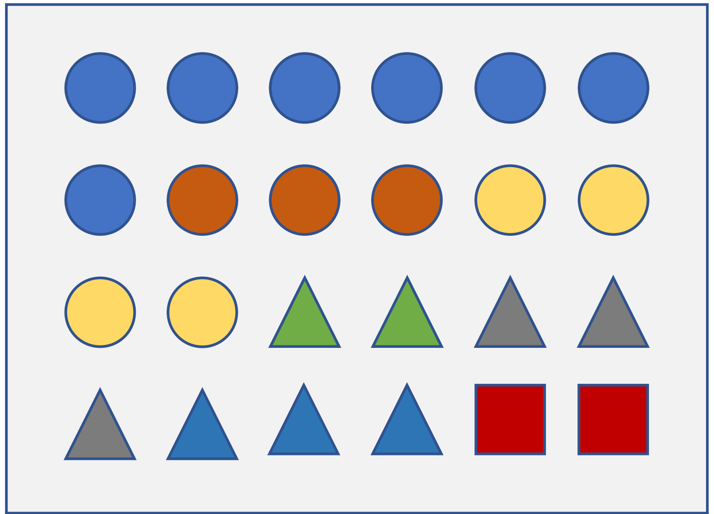


Background – Conditional Probability

$P(A \mid B)$
$= P(A \wedge B) / P(B)$

Given a circle, what is the probability that it is blue:
P(Blue | Circle)

- P(Blue) $= {}^{10}/_{24} = 0.42$
- P(Circle) $= {}^{14}/_{24} = 0.58$
- P(Blue ∧ Circle) $= {}^{7}/_{24} = 0.29$
- P(Blue | Circle) $= {}^{P(Blue \wedge Circle)}/_{P(Circle)}$
  $= 0.29 / 0.58 = 0.5$



P(A | B) $= {}^{P(A \wedge B)}/_{P(B)}$

Background – Product Rule

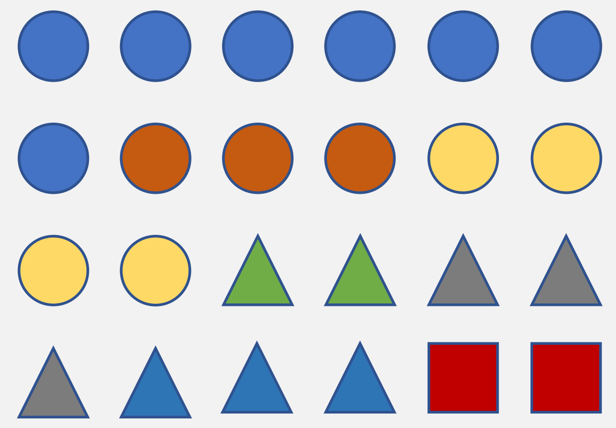
$$P(A \wedge B) = P(A \mid B) \times P(B)$$
$$P(B \wedge A) = P(B \mid A) \times P(A)$$

What is the probability of a shape being blue and a circle:
P(Blue ∧ Circle)

- P(Blue | Circle) $= {}^{P(Blue \wedge Circle)}/_{P(Circle)}$
  = 0.29 / 0.58 = 0.5

- P(Circle) = $^{14}/_{24}$ = 0.58

- P(Blue ∧ Circle) = 0.5 * 0.58 = 0.29



P(A ∧ B) = $P(A \mid B) \times P(B)$
P(B ∧ A) = $P(B \mid A) \times P(A)$

Bayes Theorem

$$P(H \mid E) = \frac{P(E \mid H) \times P(H)}{P(E)}$$

Probability of a given event occurring based on our prior knowledge or evidence

$$P(Hypothesis \mid Evidence) = \frac{P(Evidence \mid Hypothesis) \times P(Hypothesis)}{P(Evidence)}$$

Bayes Theorem - Walkthrough

Evaluate a probability based on evidence that may increase or decrease it whilst being known.

- Low chance of a general fire
  - P(Fire) = 0.01 = 1%
- Will this change given that we see smoke however?

Fire

Bayes Theorem - Walkthrough

Smoke is relatively common

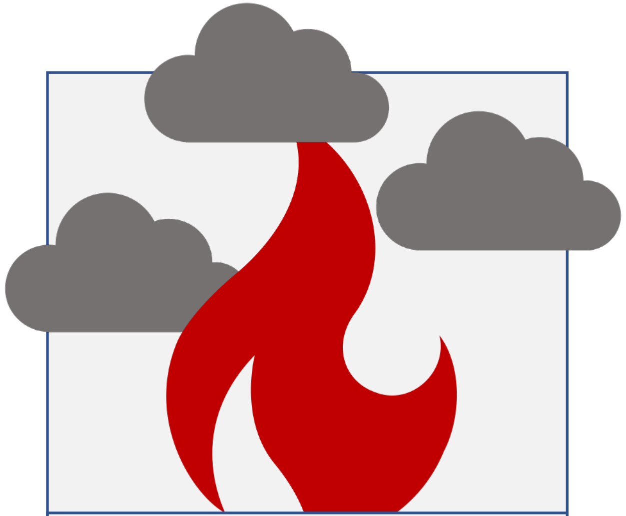
- P(Smoke) = 0.1 = 10%

High chance of there being smoke if we see a fire

- P(Smoke | Fire) = 0.9 = 90%

$$P(Fire \mid Smoke) = \frac{P(Smoke \mid Fire) \times P(Fire)}{P(Smoke)} = \frac{0.9 \times 0.01}{0.1}$$

$$= \frac{0.009}{0.1} = 0.09 = 9\%$$



Fire + Smoke

Question 1 - Proof

$$P(A \wedge B) = P(A \mid B) \times P(B)$$
$$P(B \wedge A) = P(B \mid A) \times P(A)$$

Show how to derive Bayes' Rule from the definition: $P(A \wedge B) = P(A \mid B) \times P(B)$

$$P(A \mid B) = \frac{P(B \mid A) \times P(A)}{P(B)}$$

Question 1 - Proof

$$P(A \wedge B) = P(A \mid B) \times P(B)$$
$$P(B \wedge A) = P(B \mid A) \times P(A)$$

Show how to derive Bayes' Rule from the definition: $P(A \wedge B) = P(A \mid B) \times P(B)$

$$P(A \wedge B) = P(A|B) \times P(B)$$
$$P(B \wedge A) = P(B|A) \times P(A)$$

$$P(A \wedge B) = P(B \wedge A)$$
$$P(A|B) \times P(B) = P(B|A) \times P(A)$$

$$P(A|B) = \frac{P(B \mid A) \times P(A)}{P(B)}$$

$$P(A \mid B) = \frac{P(B \mid A) \times P(A)}{P(B)}$$

Question 2 – Conditional Probability

Determine the conditional probability of a patient suffering from mumps given that they have don’t have a fever:

$$P(Mumps \mid \neg Fever)$$

- Mumps causes fever 75% of the time (0.75)
- The chance of a patient having mumps is ${}^{1}/_{15000}$
- The chance of a patient having fever is ${}^{1}/_{1000}$

Question 2 – Conditional Probability

$P(Mumps \mid \neg Fever)$

$P(Mumps) = 0.000066$	$P(Fever) = 0.001$	$P(Fever \mid Mumps) = 0.75$

Question 2 – Conditional Probability

$P(Mumps \mid \neg Fever)$

$P(Mumps) = 0.000066$	$P(Fever) = 0.001$	$P(Fever \mid Mumps) = 0.75$

$P(Mumps \mid \neg Fever)$	$\frac{P(\neg Fever \mid Mumps) \times P(Mumps)}{P(\neg Fever)}$	
	$\frac{(1 - P(Fever \mid Mumps)) \times 0.000066}{1 - P(Fever)}$	
	$\frac{0.25 \times 0.000066}{0.999}$	0.0000165

Question 3i

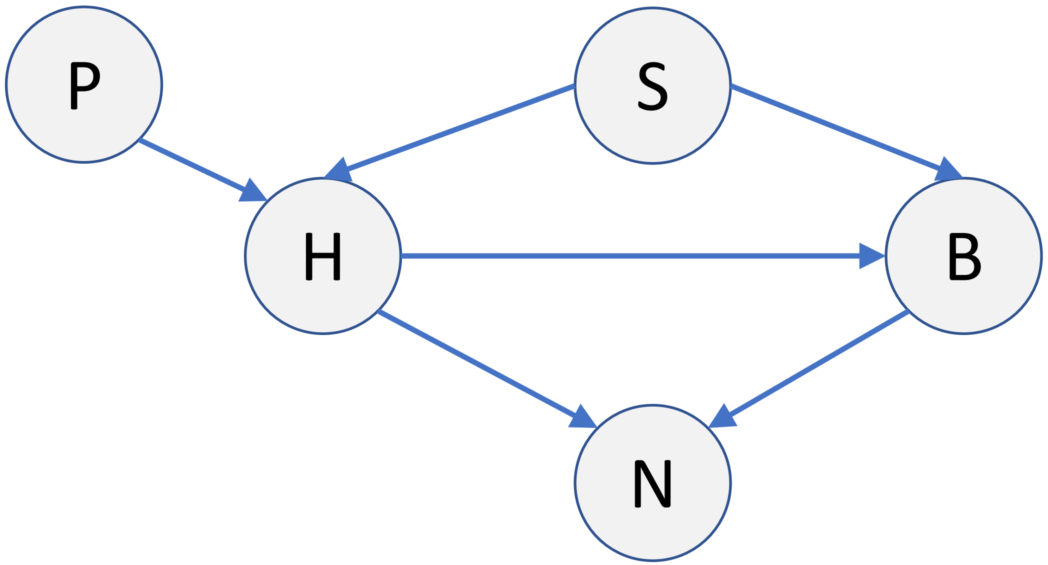
Headaches and blurred vision may be the result of sitting too close to a monitor. Headaches may also be caused by bad posture. Headaches and blurred vision may cause nausea. Headaches may also lead to blurred vision.

H	Headache
B	Blurred Vision
S	Sitting to close to a monitor
P	Bad posture
N	Nausea

Question 3i

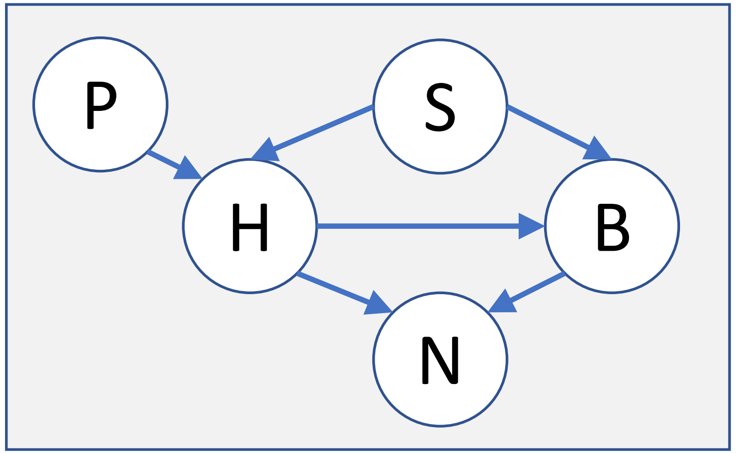
Headaches and blurred vision may be the result of sitting too close to a monitor. Headaches may also be caused by bad posture. Headaches and blurred vision may cause nausea. Headaches may also lead to blurred vision.

H	Headache
B	Blurred Vision
S	Sitting to close to a monitor
P	Bad posture
N	Nausea



Question 3i

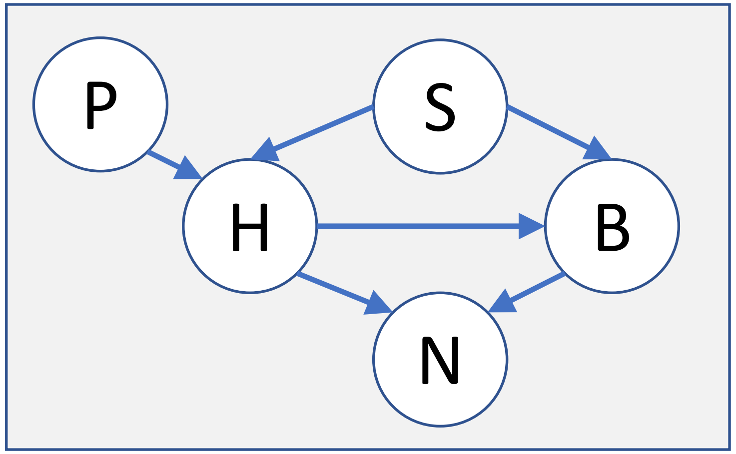
$$P(H \wedge B \wedge S \wedge P \wedge N) =$$



Question 3i

$P(H \wedge B \wedge S \wedge P \wedge N) =$

$P(H \vert P \wedge S)$	$P(B \vert S \wedge H)$	$P(S)$
$P(P)$	$P(N \vert H \wedge B)$	



$$P(H \wedge B \wedge S \wedge P \wedge N) = P(H|P \wedge S) \times P(B|S \wedge H) \times P(S) \times P(P) \times P(N|H \wedge B)$$

Background – Joint Probabilities

Enumerate through all possible probabilities based on the value distribution of the variables involved.

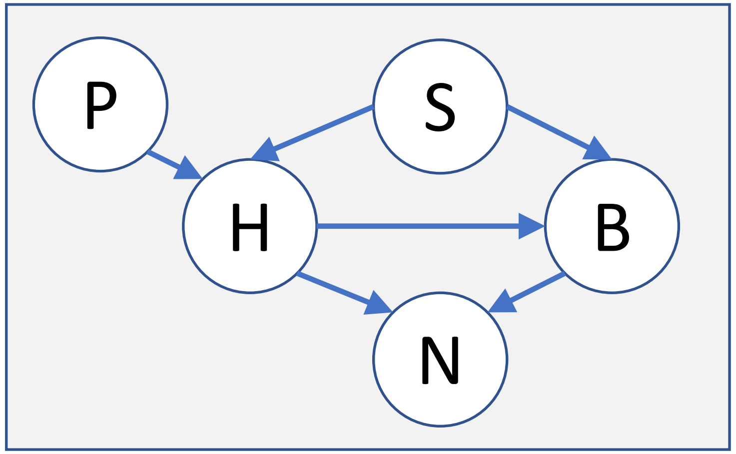
	Smoke	¬Smoke	Total
Fire	0.009	0.001	0.01
¬Fire	0.01	0.899	0.99
Total	0.1	0.9	1.0

$$P(Fire \mid Smoke) = {}^{P(Fire \land Smoke)}/_{P(Smoke)} = {}^{0.009}/_{0.1} = 0.09$$

Question 3ii

$P(H \mid P \wedge S) = 0.8$	$P(H \mid P \wedge \neg S) = 0.4$
$P(H \mid \neg P \wedge S) = 0.6$	$P(H \mid \neg P \wedge \neg S) = 0.02$

$P(B \mid S \wedge H) = 0.4$	$P(B \mid S \wedge \neg H) = 0.2$
$P(B \mid \neg S \wedge H) = 0.3$	$P(B \mid \neg S \wedge \neg H) = 0.01$



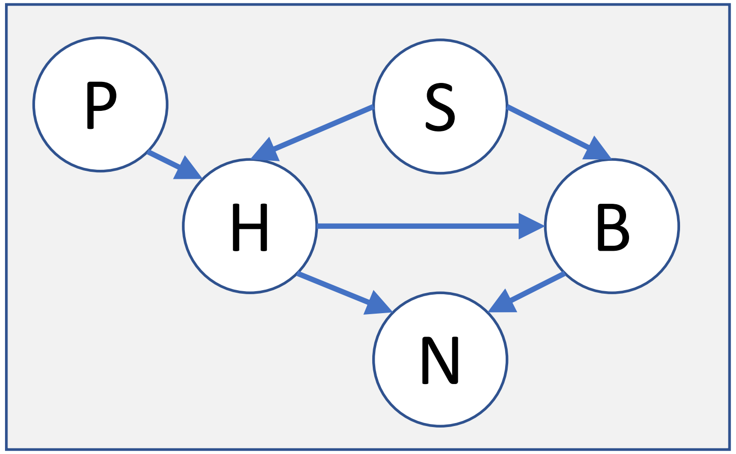
$P(N \mid H \wedge B) = 0.9$	$P(N \mid H \wedge \neg B) = 0.5$
$P(N \mid \neg H \wedge B) = 0.3$	$P(N \mid \neg H \wedge \neg B) = 0.7$

$P(S) = 0.1$
$P(P) = 0.2$

Question 3ii

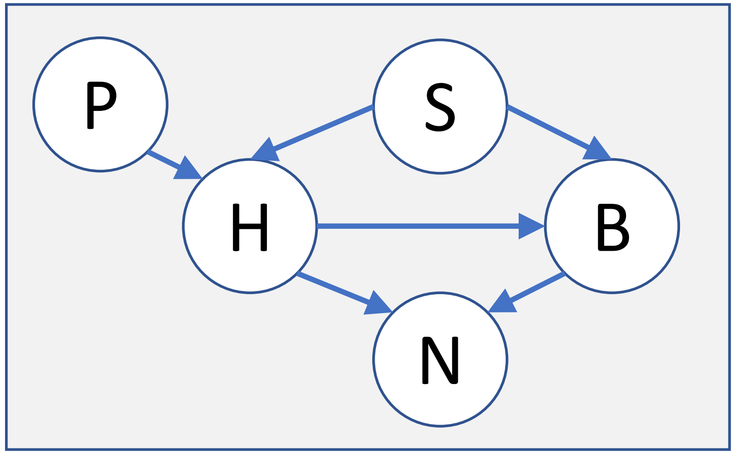
$P(H) = True$	$P(N) = False$

$P(H \land \neg N \land B \land S \land P)$	$P(H \land \neg N \land \neg B \land \neg S \land P)$
$P(H \land \neg N \land \neg B \land S \land P)$	$P(H \land \neg N \land \neg B \land S \land \neg P)$
$P(H \land \neg N \land B \land \neg S \land P)$	$P(H \land \neg N \land B \land \neg S \land \neg P)$
$P(H \land \neg N \land B \land S \land \neg P)$	$P(H \land \neg N \land \neg B \land \neg S \land \neg P)$



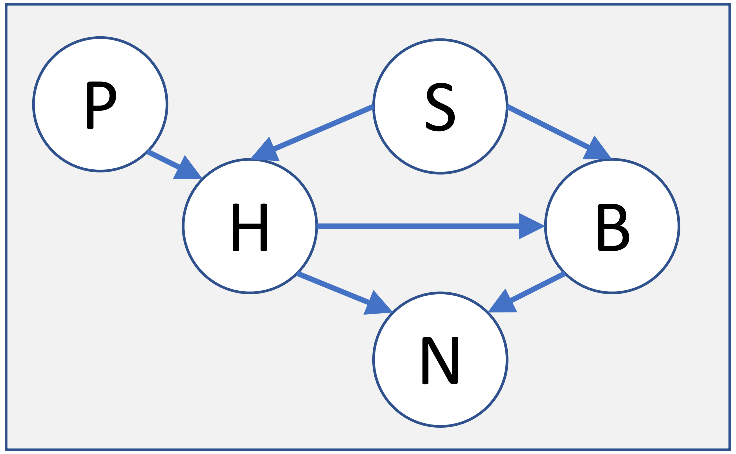
Question 3ii

$P(H) = True$	$P(N) = False$
$P(H \wedge \neg N \wedge B \wedge S \wedge P)$	
$P(H\|P,S) = 0.8$	$P(S) = 0.1$
$P(\neg N\|H,B) = 0.1$	$P(P) = 0.2$
$P(B\|S,H) = 0.4$	



Question 3ii

$P(H) = True$	$P(N) = False$
$P(H \wedge \neg N \wedge B \wedge S \wedge P)$	
$P(H\|P,S) = 0.8$	$P(S) = 0.1$
$P(\neg N\|H,B) = 0.1$	$P(P) = 0.2$
$P(B\|S,H) = 0.4$	

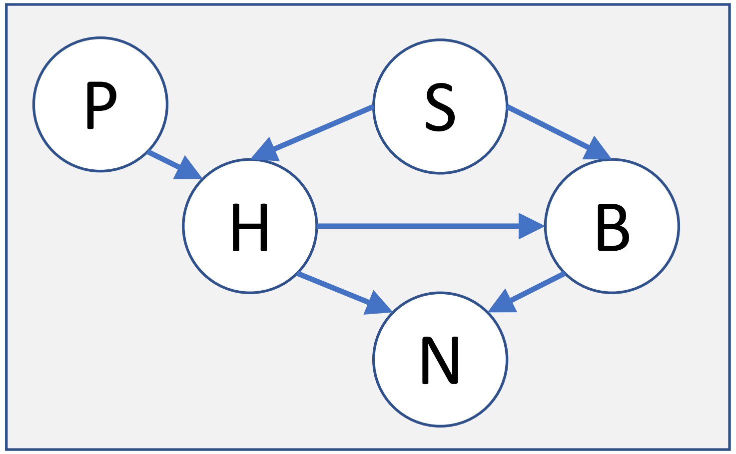


$$P(H \wedge \neg N \wedge B \wedge S \wedge P) = P(H|P,S) \times P(\neg N|H,B) \times P(B|S,H) \times P(S) \times P(P)$$

$P(H \wedge \neg N \wedge B \wedge S \wedge P) = 0.8 \times 0.1 \times 0.4 \times 0.1 \times 0.2$	0.00064

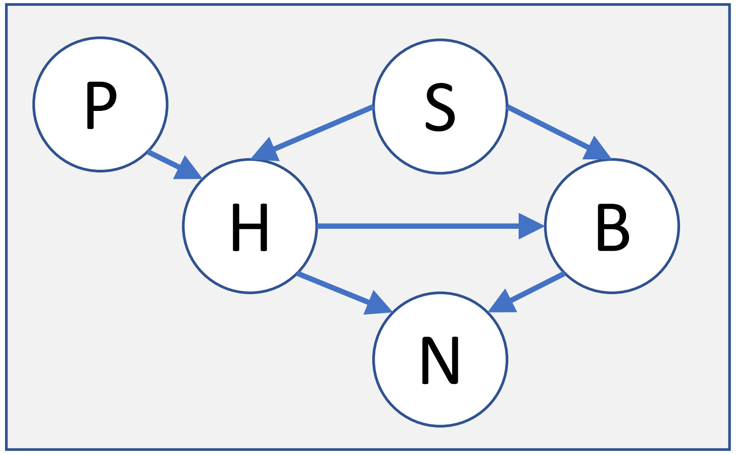
Question 3ii

$P(H) = True$	$P(N) = False$
$P(H \wedge \neg N \wedge \neg B \wedge \neg S \wedge \neg P)$	
$P(H \mid \neg P, \neg S) = 0.02$	$P(\neg S) = 0.9$
$P(\neg N \mid H, \neg B) = 0.5$	$P(\neg P) = 0.8$
$P(\neg B \mid \neg S, H) = 0.7$	



Question 3ii

$P(H) = True$	$P(N) = False$
$P(H \wedge \neg N \wedge \neg B \wedge \neg S \wedge \neg P)$	
$P(H \mid \neg P, \neg S) = 0.02$	$P(\neg S) = 0.9$
$P(\neg N \mid H, \neg B) = 0.5$	$P(\neg P) = 0.8$
$P(\neg B \mid \neg S, H) = 0.7$	



$P(H \wedge \neg N \wedge \neg B \wedge \neg S \wedge \neg P) = P(H \mid \neg P, \neg S) \times P(\neg N \mid H, \neg B) \times P(\neg B \mid \neg S, H) \times P(\neg S) \times P(\neg P)$	
$P(H \wedge \neg N \wedge B \wedge S \wedge P) = 0.02 \times 0.5 \times 0.7 \times 0.9 \times 0.8$	0.00504

Question 3iii

What is the probability that the patient suffers from bad posture given that they are suffering from headaches but not from nausea?

	Product Rule
$P(P \mid H \wedge \neg N)$	$\dfrac{P(P \wedge H \wedge \neg N)}{P(H \wedge \neg N)}$

Question 3iii

$$\frac{P(P \wedge H \wedge \neg N)}{P(H \wedge \neg N)}$$

What is the probability that the patient suffers from bad posture given that they are suffering from headaches but not from nausea?

$$P(P \mid H \wedge \neg N)$$

$P(H \wedge \neg N \wedge B \wedge S \wedge P) = 0.00064$	$P(H \wedge \neg N \wedge \neg B \wedge \neg S \wedge P) = 0.02520$
$P(H \wedge \neg N \wedge \neg B \wedge S \wedge P) = 0.00480$	$P(H \wedge \neg N \wedge \neg B \wedge S \wedge \neg P) = 0.0144$
$P(H \wedge \neg N \wedge B \wedge \neg S \wedge P) = 0.00216$	$P(H \wedge \neg N \wedge B \wedge \neg S \wedge \neg P) = 0.000432$
$P(H \wedge \neg N \wedge B \wedge S \wedge \neg P) = 0.00192$	$P(H \wedge \neg N \wedge \neg B \wedge \neg S \wedge \neg P) = 0.00504$

Question 3iii

$P(P \wedge H \wedge \neg N)$

$$\sum_{b,s} P(H \wedge \neg N \wedge b \wedge s \wedge P)$$

$$\left.\begin{array}{l} P(H \wedge \neg N \wedge B \wedge S \wedge P) = 0.00064 \\ P(H \wedge \neg N \wedge \neg B \wedge \neg S \wedge P) = 0.02520 \\ P(H \wedge \neg N \wedge \neg B \wedge S \wedge P) = 0.00480 \\ P(H \wedge \neg N \wedge B \wedge \neg S \wedge P) = 0.00216 \end{array}\right\}$$

$$0.00064 + 0.02520 + 0.00480 + 0.00216$$

$$0.0328$$

Question 3iii

$P(H \wedge \neg N)$

$$\sum_{b,s,p} P(H \wedge \neg N \wedge b \wedge s \wedge p)$$

$P(H \wedge \neg N \wedge B \wedge S \wedge P) = 0.00064$	$P(H \wedge \neg N \wedge \neg B \wedge S \wedge P) = 0.00480$
$P(H \wedge \neg N \wedge B \wedge \neg S \wedge P) = 0.00216$	$P(H \wedge \neg N \wedge B \wedge S \wedge \neg P) = 0.00192$
$P(H \wedge \neg N \wedge \neg B \wedge \neg S \wedge P) = 0.02520$	$P(H \wedge \neg N \wedge \neg B \wedge S \wedge \neg P) = 0.0144$
$P(H \wedge \neg N \wedge B \wedge \neg S \wedge \neg P) = 0.000432$	$P(H \wedge \neg N \wedge \neg B \wedge \neg S \wedge \neg P) = 0.00504$

$$\left.\begin{array}{l} 0.00064 + 0.00480 + 0.00216 + 0.00192 \\ + 0.02520 + 0.0144 + 0.000432 + 0.00504 \end{array}\right\} \boxed{0.054592}$$

Question 3iii

$$\frac{P(P \wedge H \wedge \neg N)}{P(H \wedge \neg N)}$$

$P(P \mid H \wedge \neg N)$ =	$\frac{P(P \wedge H \wedge \neg N)}{P(H \wedge \neg N)}$	$\frac{0.0328}{0.054592} = 0.6008$

Background – Chain Rule

$$P(A \wedge B) = P(B \mid A) \times P(A)$$

The probability of observing two events A and B is equivalent to the probability of observing A multiplied by the probability of observing B given A

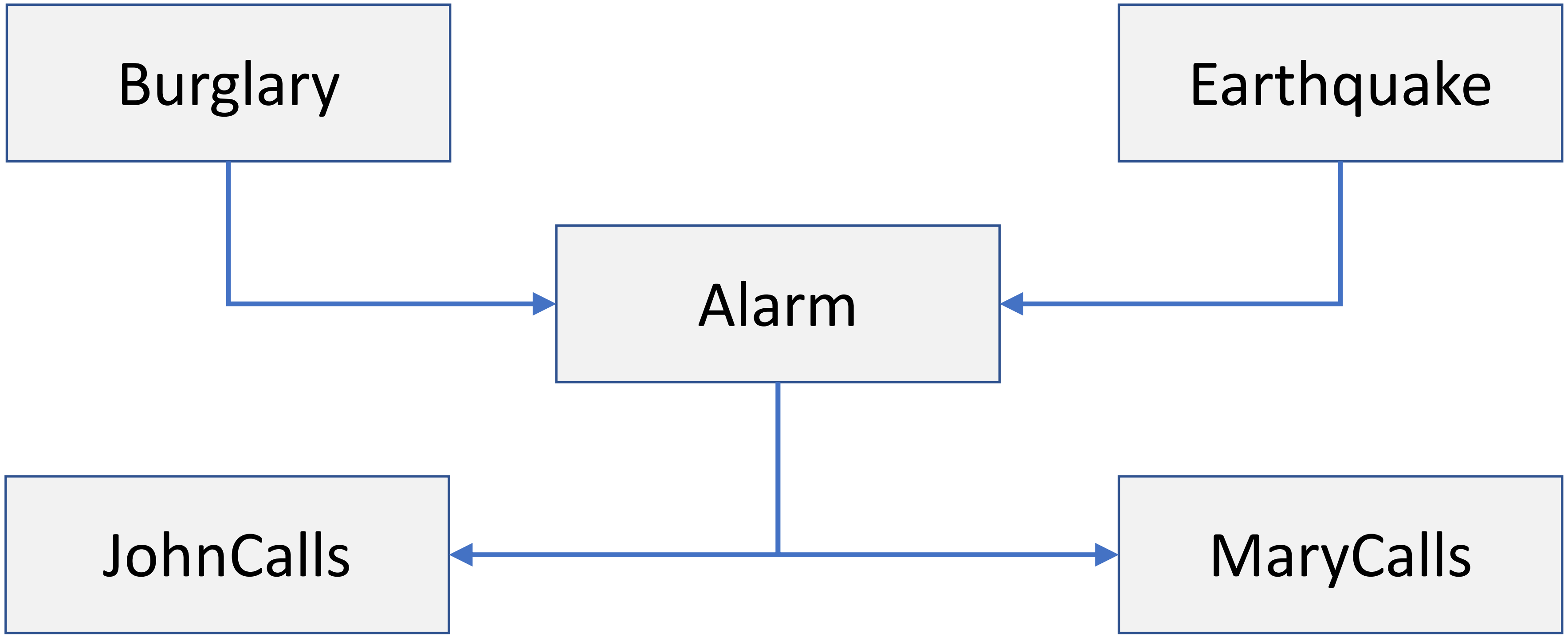
$$P(A \wedge B \wedge C \wedge D) = P(A \mid B, C, D)$$

$$P(A \wedge B \wedge C \wedge D) = P(A \mid B, C, D) \times P(B \mid C, D)$$

$$P(A \wedge B \wedge C \wedge D) = P(A \mid B, C, D) \times P(B \mid C, D) \times P(C \mid D)$$

$$P(A \wedge B \wedge C \wedge D) = P(A \mid B, C, D) \times P(B \mid C, D) \times P(C \mid D) \times P(D)$$

Question 4

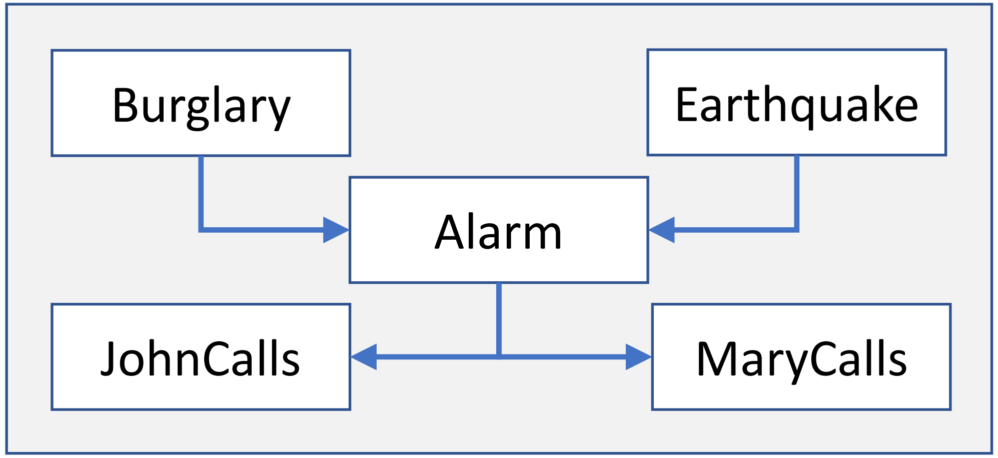


Question 4

$P(Burglary) = 0.001$
$P(Earthquake) = 0.002$

$P(Alarm \mid Burglary \wedge Earthquake) = 0.95$
$P(Alarm \mid Burglary \wedge \neg Earthquake) = 0.94$
$P(Alarm \mid \neg Burglary \wedge Earthquake) = 0.29$
$P(Alarm \mid \neg Burglary \wedge \neg Earthquake) = 0.001$

$P(JohnCalls \mid Alarm) = 0.90$
$P(JohnCalls \mid \neg Alarm) = 0.05$

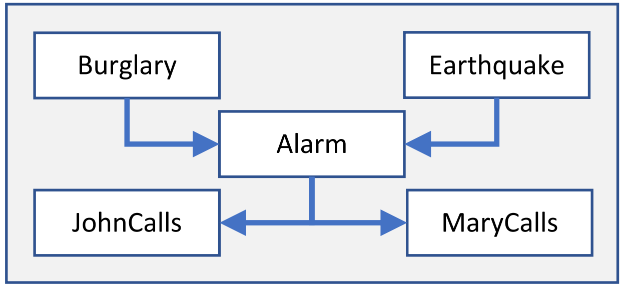


$P(MaryCalls \mid Alarm) = 0.70$
$P(MaryCalls \mid \neg Alarm) = 0.01$

Question 4

$$\frac{P(Alarm \mid Burglary) \times P(Burglary)}{P(Alarm)}$$

$$P(Burglary \mid Alarm) = \frac{P(Alarm \mid Burglary) \times P(Burglary)}{P(Alarm)}$$



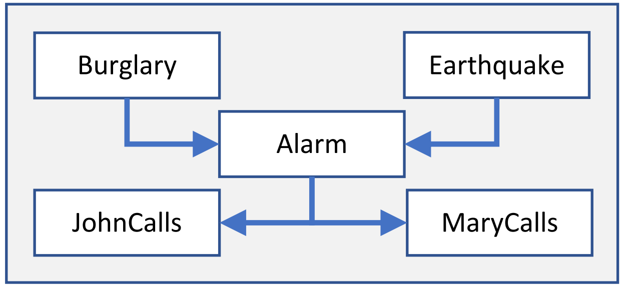
$P(Alarm \mid Burglary)$	$P(Burglary)$	
$P(Alarm \mid Burglary \land Earthquake) \times P(Earthquake)$	$0.95 \times 0.002 \times 0.001$	0.0000019
$P(Alarm \mid Burglary \land \neg Earthquake) \times P(\neg Earthquake)$	$0.94 \times 0.998 \times 0.001$	0.0009381

$0.0000019 + 0.0009381 = 0.00094$

Question 4

$$\frac{P(Alarm\,|Burglary) \times P(Burglary)}{P(Alarm)}$$

$$P(Burglary\,|\,Alarm) = \frac{P(Alarm\,|Burglary) \times P(Burglary)}{P(Alarm)}$$



$$P(Alarm)$$

$$P(Alarm\,|\,Burglary \wedge Earthquake) \times P(Burglary) \times P(Earthquake)$$

$$P(Alarm\,|\,Burglary \wedge \neg Earthquake) \times P(Burglary) \times P(\neg Earthquake)$$

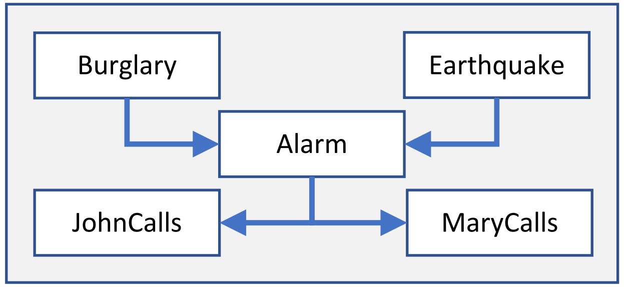
$$P(Alarm\,|\neg Burglary \wedge Earthquake) \times P(\neg Burglary) \times P(Earthquake)$$

$$P(Alarm\,|\neg Burglary \wedge \neg Earthquake) \times P(\neg Burglary) \times P(\neg Earthquake)$$

Question 4

$$\frac{P(Alarm \mid Burglary) \times P(Burglary)}{P(Alarm)}$$

$$P(Burglary \mid Alarm) = \frac{P(Alarm \mid Burglary) \times P(Burglary)}{P(Alarm)}$$



$P(Alarm)$	
$0.95 \times 0.02 \times 0.001$	0.000019
$0.94 \times 0.998 \times 0.001$	0.000938
$0.29 \times 0.999 \times 0.002$	0.000579
$0.001 \times 0.998 \times 0.999$	0.000997

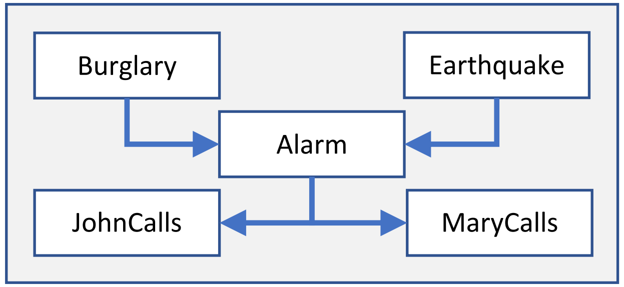
$$0.000019 + 0.000938 + 0.000579 + 0.000997 = 0.002533$$

Question 4

$$\frac{P(Alarm\,|Burglary) \times P(Burglary)}{P(Alarm)}$$

$$P(Burglary\,|\,Alarm) = \frac{P(Alarm\,|Burglary) \times P(Burglary)}{P(Alarm)}$$

$$\frac{0.00094}{0.002533} = 0.3711$$



Question 5 - Proof

$$P(B \mid A, C) = \frac{P(A \mid B, C) \times P(B \mid C)}{P(A \mid C)}$$

Chain Rule

$$P(A \land B \land C) = P(B \mid A, C) \times P(A \mid C) \times P(C)$$

$$= P(A \mid B, C) \times P(B \mid C) \times P(C)$$

Both are equivalent

Question 5 - Proof

$$P(B \mid A, C) = \frac{P(A \mid B, C) \times P(B \mid C)}{P(A \mid C)}$$

$$P(B \mid A, C) \times P(A \mid C) \times P(C) = P(A \mid B, C) \times P(B \mid C) \times P(C)$$

$$P(B \mid A, C) \times P(A \mid C) = P(A \mid B, C) \times P(B \mid C)$$

$$P(B \mid A, C) = \frac{P(A \mid B, C) \times P(B \mid C)}{P(A \mid C)}$$

Notes

- All python files for the assignment can be found at:
  - https://artint.info/AIPython/
- Some nice videos on Bayes Theorem:
  - https://www.youtube.com/watch?v=OqmJhPQYRc8
  - https://www.youtube.com/watch?v=HZGCoVF3YvM
  - https://www.youtube.com/watch?v=U_85TaXbeIo