Name of Candidate: .....

Student id: .....

Signature: .....

THE UNIVERSITY OF NEW SOUTH WALES

Term 2, 2020

COMP9417 Machine Learning and Data Mining – Sample Final Examination (SOLUTIONS)

1. I ACKNOWLEDGE THAT ALL OF THE WORK I SUBMIT FOR THIS EXAM WILL BE COMPLETED BY ME WITHOUT ASSISTANCE FROM ANYONE ELSE.
2. TIME ALLOWED — 24 hours
3. OPEN BOOK EXAM - LECTURE NOTES, TUTORIALS, AND ONLINE RESOURCES ARE PERMITTED. PLEASE USE REFERENCES WHERE NECESSARY.
4. SUBMISSION — YOU MUST SUBMIT A PDF FILE CONTAINING YOUR ANSWERS FOR EACH QUESTION ATTEMPTED. START EACH SUB-QUESTION ON A NEW PAGE. MARKS MAY BE DEDUCTED FOR UNCLEAR WORK. YOU MAY TYPE YOUR SOLUTIONS USING LATEX, OR TAKE CLEAR PHOTOS OF HANDWRITTEN WORK. FOR QUESTIONS THAT REQUIRE CODING, YOU MUST SUBMIT A .PY FILE (SEE TEMPLATE) CONTAINING YOUR CODE, THOUGH GENERATED PLOTS/TABLES MUST BE INCLUDED IN THE PDF.
5. DISCUSSION WITH OTHER STUDENTS IS STRICTLY PROHIBITED. CODE SUBMISSIONS WILL BE CHECKED FOR PLAGIARISM. CHEATING WILL RESULT IN A FAILING GRADE FOR THE COURSE AND POTENTIAL FURTHER DISCIPLINARY ACTION.
6. IF NEEDED, YOU ARE PERMITTED TO SEEK CLARIFICATION FROM COURSE STAFF ON THE WEBCMS FORUM. QUESTIONS SPECIFIC TO CONTENT WILL NOT BE ANSWERED.

Question 1 is on **Linear Regression** and requires you to refer to the following training data:

x	y
4	2
6	4
12	10
25	23
29	28
46	44
59	60

We wish to fit a linear regression model to this data, i.e. a model of the form:

$$\hat{y}_i = w_0 + w_1 x_i.$$

We consider the Least-Squares loss function:

$$L(w_0, w_1) = \sum_{i=1}^{n} (y_i - \hat{y}_i)^2,$$

where n is the total number of training points.

(a) Derive the least squares estiamtes of w_0 and w_1 (showing their working), and compute them for the provided data.

ANSWER

For (A), students are expected to derive the least squares estimates:

$$w_0 = \overline{y} - w_1 \overline{x}, \quad w_1 = \frac{\overline{xy} - \overline{x}\ \overline{y}}{\overline{x^2} - \overline{x}^2}$$

then, computing the specific different pieces, we get

$$w_0 = -2.4570, \quad w_1 = 1.0398.$$

(b) Based on your linear model, what is the prediction for a test point $x_\star = 50$?

ANSWER

For (B), they just need to compute

$$y_\star = \hat{w}_0 + \hat{w}_1 x_\star$$

using the estimates from their previous answer.

(c) Consider a new training point, $(x_{\text{new}}, y_{\text{new}}) = (45, 0)$. What are the new least squares parameters if we include all training data (including this new point)?

ANSWER

In (c), they need to recompute the estimates using the estimates in (A).

(d) The new training point in (c) can be considered to be what kind of point? What does such a point mean for your estimated parameters? How could you remedy the situation?

ANSWER

This is a clear outlier, and it has a large impact on our estimates under least squares error. This is not desirable as we would not want a single data point to have so much influence over our model. One way to remedy this is to screen for outliers, so this point would be removed from our analysis before computing the least squares estimates. Another approach would be to use a different loss function, namely, a robust loss such as the mean absolute error: $\sum_{i=1}^{n} |y_i - \hat{y}_i|$.

(e) Are you comfortable coding this question up in Numpy? What about using the scikit learn implementation of Linear Regression?

(f) Compute the derivatives of the following functions (SHOW YOUR WORKING):

1. $f(x) = x^2 \ln(x)$
2. $g(x) = (1 + 2x^4)^3$
3. $h(x) = \frac{2\ln(x)+4x}{x^3}$

ANSWER

1. $f'(x) = 2x \ln(x) + x$
2. $g'(x) = 24x^3(1 + 2x^4)^2$
3. $h'(x) = \frac{\left(\frac{2}{x}+4\right)x^3 - (2\ln(x)+4x)3x^2}{x^6} = \frac{2-6\ln(x)-8x}{x^4}$.

Question 2 is on **Tree Learning** and requires you to refer to the following dataset containing a sample S of ten examples. Each example is described using two Boolean attributes A and B. Each is labelled (classified) by the target Boolean function.

A	B	Class
1	0	+
0	1	-
1	1	-
1	0	+
1	1	-
1	1	-
0	0	+
1	1	+
0	0	+
0	0	-

(a) What is the entropy of these examples with respect to the given classification?
(b) What is the Information gain of attribute A on sample S above?
(c) What is the information gain of attribute B on sample S above?
(d) What would be chosen as the 'best' attribute by a decision tree learner using the ifnromation gain splitting criterion? Why?
(e) What are ensembles? Discuss one example in which decision trees are used in an ensemble.

Questions 3 is on **Perceptron Training** and requires you to refer to the following training data:

x_1	x_2	y
-2	-1	-1
2	-1	1
1	1	1
-1	-1	-1
3	2	1

(a) Apply the Perceptron Learning Algorithm with starting values $w_0 = 5$, $w_1 = 1$ and $w_2 = 1$, and a learning rate $\eta = 0.4$. Be sure to cycle through the training data in the same order that they are presented in the table.

ANSWER

Running the Perceptron training algorithm results in the following iterations, where highlighted columns signify that the weight vector is changing on that particular iteration. The correct weights are therefore $w_0 = 3.8, w_1 = 2.6, w_2 = 2.2$. These weights first appear on iteration 9.

Iteration	$w^T x$	$yw^T x$	w
1	2.00	-2.00	$[4.6, 1.8, 1.4]^T$
2	6.80	6.80	$[4.6, 1.8, 1.4]^T$
3	7.80	7.80	$[4.6, 1.8, 1.4]^T$
4	1.40	-1.40	$[4.2, 2.2, 1.8]^T$
5	14.40	14.40	$[4.2, 2.2, 1.8]^T$
6	-2.00	2.00	$[4.2, 2.2, 1.8]^T$
7	6.80	6.80	$[4.2, 2.2, 1.8]^T$
8	8.20	8.20	$[4.2, 2.2, 1.8]^T$
9	0.20	-0.20	$[3.8, 2.6, 2.2]^T$
10	16.00	16.00	$[3.8, 2.6, 2.2]^T$
11	-3.60	3.60	$[3.8, 2.6, 2.2]^T$
12	6.80	6.80	$[3.8, 2.6, 2.2]^T$
13	8.60	8.60	$[3.8, 2.6, 2.2]^T$

(b) Consider a new point, $x_\star = (-5, 3)$. What is the predicted value and predicted class based on your learned perceptron for this point?

ANSWER

value: -2.6, class: $y_\star = -1$

(c) Consider adding a new point to the data set, $x_\star = (2, 2)$ and $y_\star = -1$. Will your perceptron converge on the new dataset? How might you remedy this?

(d) Consider the following three logical functions:

1. $A \wedge \neg B$
2. $\neg A \vee B$
3. $(A \vee B) \wedge (\neg A \vee \neg B)$

Which of these functions can a perceptron learn? Explain. What are two ways that you can extend a perceptron to learn all three functions ?

ANSWER

A Perceptron can only learn f_1 and f_2. we can extend via multilayer perceptrons, or by using a smart transformation (i.e. kernel perceptron).

Questions 4 covers **Unsupervised Learning** and require you to refer to the following information.

In these two questions you will apply the k-MEANS algorithm. You will use a univariate (one-variable) dataset containing the following 12 instances:

$$\text{Dataset} = \{\ 2.01,\ 3.49,\ 4.58,\ 4.91,\ 4.99,\ 5.01,\ 5.32,\ 5.78,\ 5.99,\ 6.21,\ 7.26,\ 8.00\ \}$$

Use the *Manhattan* or *city-block* distance, i.e., the distance between two instances x_i and x_j is the absolute value of the difference $x_i - x_j$. For example, if $x_i = 2$ and $x_j = 3$ then the distance between x_i and x_j is $|2 - 3| = 1$. Use the arithmetic mean to compute the centroids.

Apply the k-MEANS algorithm to the above dataset of examples. Let $k = 2$. Let the two centroids (means) be initialised to $\{3.33, 6.67\}$. On each iteration of the algorithm record the centroids.

After two iterations of the algorithm you should have recorded two sets of two centroids.

Centroids	**After 1 iteration**	**After 2 iterations**
Centroids 1	{ 2.75, 5.81 }	{ 3.24, 6.01 }
Centroids 2	{ 4.00, 6.22 }	{ 4.17, 6.43 }
Centroids 3	{ 4.51, 6.87 }	{ 4.33, 6.65 }
Centroids 4	{ 4.67, 7.16 }	{ 4.51, 6.87 }
Centroids 5	{ 4.83, 7.03 }	{ 4.28, 6.79 }

(a) After applying your algorithm to the dataset for two iterations, which of the sets of centroids in the table above has been learned ?
(select the row of the table with values closest to your centroids)

(a) Centroids 1
(b) Centroids 2
(c) Centroids 3
(d) Centroids 4
(e) Centroids 5

ANSWER

(b)

Now apply the algorithm for one more iteration. Record the new centroids after iteration 3 and answer the following question.

(b) After 3 iterations it is clear that:
(a) due to randomness in the data, the centroids could change on further iterations
(b) due to randomness in the algorithm, the centroids could change on further iterations
(c) k-MEANS converges in probability to the true centroids
(d) the algorithm has converged and the clustering will not change on further iterations
(e) the algorithm has not converged and the clustering will change on further iterations

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ANSWER

(d)

Question 5 is on **Learning Theory** and requires you to apply a mistake-bounded learner to the following dataset.

This dataset has 6 binary features, $x_1, x_2, \ldots x_6$. The class variable y can be either 1, denoting a positive example of the concept to be learned, or 0, denoting a negative example.

Example	$\mathbf{x_1}$	$\mathbf{x_2}$	$\mathbf{x_3}$	$\mathbf{x_4}$	$\mathbf{x_5}$	$\mathbf{x_6}$	**Class**
1)	0	0	0	0	1	1	1
2)	1	0	1	1	0	1	1
3)	0	1	0	1	0	1	0
4)	0	1	1	0	0	1	0
5)	1	1	0	0	0	0	1

Apply the WINNOW2 algorithm to the above dataset of examples **in the order in which they appear**. Use the following values for the WINNOW2 parameters: threshold $t = 2$, $\alpha = 2$. Initialise all weights to have the value 1.

Learned Weights	$\mathbf{w_1}$	$\mathbf{w_2}$	$\mathbf{w_3}$	$\mathbf{w_4}$	$\mathbf{w_5}$	$\mathbf{w_6}$
Weight vector 1	2.000	1.000	1.000	0.000	2.000	1.000
Weight vector 2	3.000	0.000	1.000	1.000	2.000	1.000
Weight vector 3	2.000	2.000	2.000	2.000	2.000	2.000
Weight vector 4	2.000	0.500	0.500	0.500	2.000	0.500
Weight vector 5	2.000	0.250	0.500	0.500	4.000	0.125

(a) After one epoch, i.e., one pass through the dataset, which of the above weight configurations has been learned ?

(a) Weight vector 1
(b) Weight vector 2
(c) Weight vector 3
(d) Weight vector 4
(e) Weight vector 5

ANSWER

(d)

(b) On which of the examples did the algorithm ***not*** make a mistake ?

(a) Examples 1), 2) and 5)
(b) Example 5)
(c) Example 4)
(d) Examples 4) and 5)
(e) None of the above

ANSWER

(e)

(c) The algorithm has learned a consistent concept on the training data:

(a) True
(b) False
(c) It is not possible to determine this

ANSWER

(a)

(d) Assume the target concept from which this dataset was generated is defined by exactly two features. The worst-case mistake bound for the algorithm on this dataset is approximately:

(a) 1.79
(b) 2.58
(c) 3.58
(d) 4.67
(e) 10.75

ANSWER

(c)

Question 6 is on Boosting Theory and requires you to implement the Adaptive Boosting Algorithm from lectures. Use the following code to generate a toy binary classification dataset:

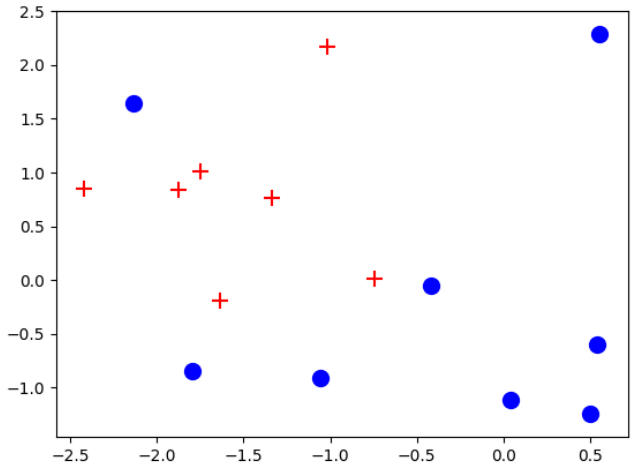
```
import numpy as np
import matplotlib.pyplot as plt
from matplotlib.colors import ListedColormap
import warnings
warnings.simplefilter(action='ignore', category=FutureWarning)

from sklearn.tree import DecisionTreeClassifier
from sklearn.datasets import make_blobs

np.random.seed(2)
n_points = 15
X, y = make_blobs(n_points, 2, centers=[(0,0), (-1,1)])
y[y==0] = -1        # use -1 for negative class instead of 0

plt.scatter(*X[y==1].T, marker="+", s=100, color="red")
plt.scatter(*X[y==-1].T, marker="o", s=100, color="blue")
plt.show()
```

Your data should look like:

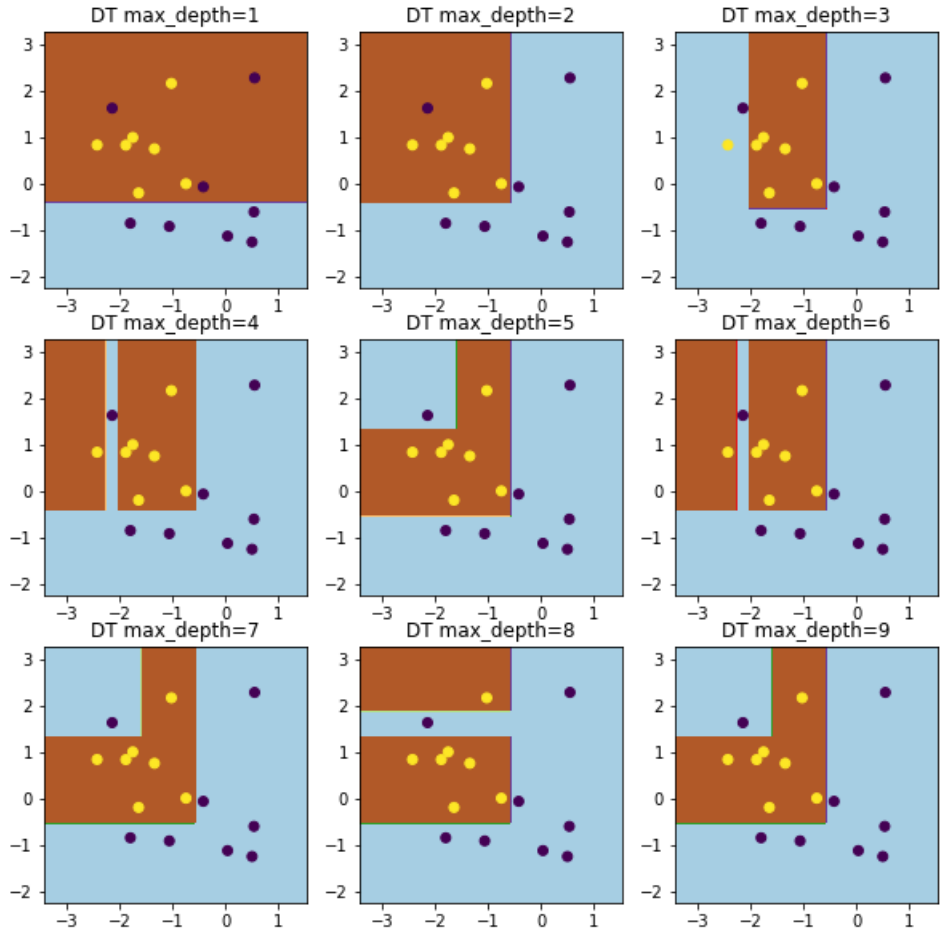


(a) By now, you should be familiar with the scikitlearn DecisionTreeClassifier class. Fit Decision trees of increasing maximum depth for depths ranging from 1 to 9. Plot the decision boundaries of each of your models in a 3×3 grid. You may find the following helper function useful:

```
def plotter(classifier, X, y, title, ax=None):
```

```
# plot decision boundary for given classifier
plot_step = 0.02
x_min, x_max = X[:, 0].min() - 1, X[:,0].max() + 1
y_min, y_max = X[:, 1].min() - 1, X[:,1].max() + 1
xx, yy = np.meshgrid(np.arange(x_min, x_max, plot_step),
            np.arange(y_min, y_max, plot_step))
Z = classifier.predict(np.c_[xx.ravel(),yy.ravel()])
Z = Z.reshape(xx.shape)
if ax:
  ax.contourf(xx, yy, Z, cmap = plt.cm.Paired)
  ax.scatter(X[:, 0], X[:, 1], c = y)
  ax.set_title(title)
else:
  plt.contourf(xx, yy, Z, cmap = plt.cm.Paired)
  plt.scatter(X[:, 0], X[:, 1], c = y)
  plt.title(title)
```

ANSWER



The code used is:

```
fig, ax = plt.subplots(3,3, figsize=(10,10))
titles = [f"DT max_depth={i}" for i in range(1,10)]
for i, ax in enumerate(ax.flat):
    dt = DecisionTreeClassifier(max_depth=i+1).fit(X, y)
    plotter(dt, X, y, titles[i], ax)
plt.savefig("boosting_a.png")
plt.tight_layout()
```

```
plt.show()
```

(b) Comment on your results in (a). What do you notice as you increase the depth of the trees? What do we mean when we say that trees have low bias and high variance?

ANSWER
something along the lines of: low bias since the shape of the learned function is highly flexible and can adapt to the shape of the data. DTs can fit any noise in the data so are high variance. Some discussion of the bias variance decomposition would also be nice.

(c) We now restrict attention to trees of depth 1. These are the most basic decision trees and are commonly referred to as decision stumps. Consider the adaptive boosting algorithm presented in the ensemble methods lecture notes on slide 50/70. In adaptive boosting, we build a model composed of T weak learners from a set of weak learners. At step t, we pick a model from the set of weak learners that minimises weighted error:

$$\epsilon_t = \sum_{i=1}^{n} w_{t-1,i} \mathbb{I}\{y_i \neq \hat{y}_i\}$$

where $w_{t-1,i}$ is the weight at the previous step for observation i, and $\mathbb{I}\{y_i \neq \hat{y}_i\}$ is equal to 1 if $y_i \neq \hat{y}_i$ and zero otherwise. We do this for a total of T steps, which gives us a boosted model composed of T base classifiers:

$$M(x) = \sum_{t=1}^{T} \alpha_t M_t(x)$$

where α_t is the weight assigned to the t-th model. Classification is then carried out by assigning a point to the positive class if $M(x) > 0$ or to the negative class if $M(x) < 1$. Here we will take the class of weak learners to be the class of Decision stumps. You may make use of the 'sample_weight' argument in the 'fit()' method to assign weights to the individual data points. Write code to build a boosted classifier for $T = 15$. Demonstrate the performance of your model on the generated dataset by printing out a list of your predictions versus the true class labels. (**note:** you may be concerned that the decision tree implementation in scikit learn does not actually minimise ϵ_t even when weights are assigned, but we will ignore this detail for the current question).

ANSWER

One approach is as follows:

```
def weighted_error(w, y, yhat):
    return np.sum(w*(y!=yhat))

T = 15
w = np.zeros((T, n_points))
w[0] = np.ones(n_points)/n_points

# storage for boosted model
alphas = np.zeros(T-1); component_models = []

def boosted_model(x, alphas, component_models):
    individual_preds = np.array([m.predict(x) for m in component_models])
    alphaM = alphas * individual_preds.T
    return np.sign(alphaM.sum(axis=1))

for t in range(1, T):

    # select weak classifier that minimises weighted error
    dt = DecisionTreeClassifier(max_depth=1).fit(X, y, sample_weight=w[t-1])

    # compute predictions from current model
    yhat = dt.predict(X)

    # comptue weighted error epsilon_t
    eps_t = weighted_error(w[t-1], y, yhat)

    # compute alpha_t (model weight)
    alpha_t = 0.5 * np.log((1-eps_t)/eps_t)

    # update weights for next round
    for i in range(n_points):
        if y[i] == yhat[i]:
            # correctly classified instances
            w[t, i] = w[t-1, i]/(2 * (1-eps_t))
        else:
            # misclassified instances
            w[t, i] = w[t-1, i]/(2 * eps_t)

    # save model
    alphas[t-1] = alpha_t
  component_models.append(dt)

  boosted_preds = boosted_model(X, alphas, component_models)
  np.all((boosted_preds * y) == 1)     # check all classified correctly
```

(d) In this question, we will extend our implementation in (c) to be able to use the plotter

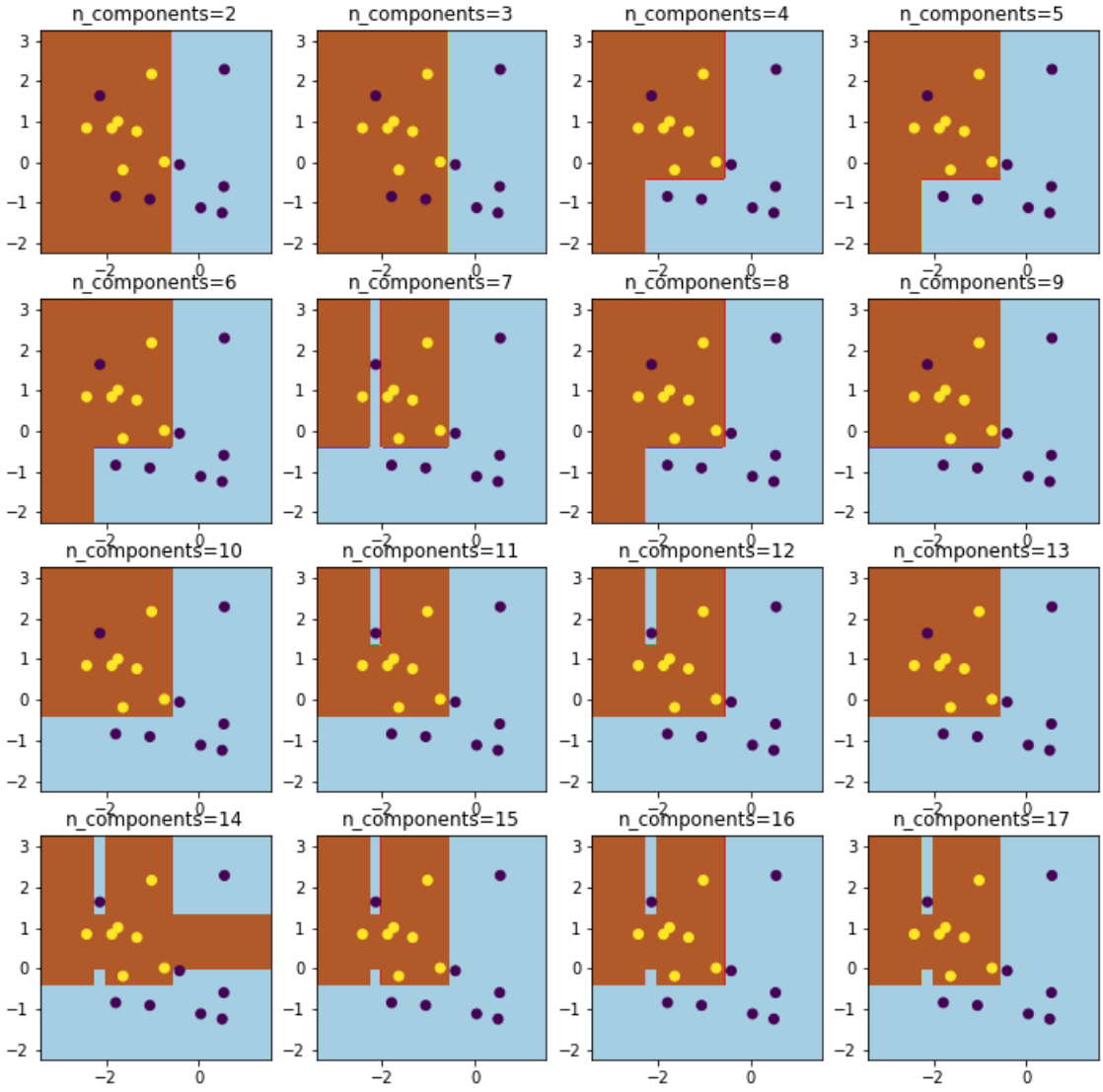
function in (b). To do this, we need to implement a boosting model class that has a 'predict' method. Once you do this, repeat (c) for $T = [2, \ldots, 17]$. Plot the decision boundary of your 16 models in a 4×4 grid. The following template may be useful:

```
class boosted_model:
    def __init__(self, T):
      self.alphas = # YOUR CODE HERE
      # YOUR CODE HERE

    def predict(self, x):
        # YOUR CODE HERE
```

ANSWER

The plot should look like:



```
class boosted_model:
    def __init__(self, T):
        self.alphas = np.zeros(T-1)
        self.component_models = []

    def predict(self, x):
        individual_preds = np.array([m.predict(x) for m in self.
    component_models])
        alphaM = self.alphas * individual_preds.T
        return np.sign(alphaM.sum(axis=1))

bms = []

for T in range(2, 18):
    w = np.zeros((T, n_points))
    w[0] = np.ones(n_points)/n_points

    bm = boosted_model(T)
    for t in range(1, T):

        # select weak classifier that minimises weighted error
        dt = DecisionTreeClassifier(max_depth=1).fit(X, y, sample_weight=w[t
    -1])

        # compute predictions from current model
        yhat = dt.predict(X)

        # comptue weighted error epsilon_t
        eps_t = weighted_error(w[t-1], y, yhat)

        # compute alpha_t (model weight)
        alpha_t = 0.5 * np.log((1-eps_t)/eps_t)

        # update weights for next round
        for i in range(n_points):
            if y[i] == yhat[i]:
                # correctly classified instances
                w[t, i] = w[t-1, i]/(2 * (1-eps_t))
            else:
                # misclassified instances
                w[t, i] = w[t-1, i]/(2 * eps_t)

        # save model
        bm.alphas[t-1] = alpha_t
        bm.component_models.append(dt)

```

```
    # store current boosted model
    bms.append(bm)




fig, ax = plt.subplots(4,4, figsize=(12,12))
titles = [f"n_components={i}" for i in range(2,18)]
for i, ax in enumerate(ax.flat):
    plotter(bms[i], X, y, titles[i], ax)
#plt.savefig("myplot.png")
plt.tight_layout()
plt.show()
```

(e) Discuss the differences between bagging and boosting.

Question 7 Some more suggestions - Make sure you are comfortable with the following

1. Naive Bayes Classification example from lectures/tutorials
2. Decision trees
3. Understanding the Bias Variance trade off
4. SVM calculation
5. K Means clustering
6. VC dimension
7. have worked through the labs and are comfortable with scikitlearn/numpy.

END OF PAPER